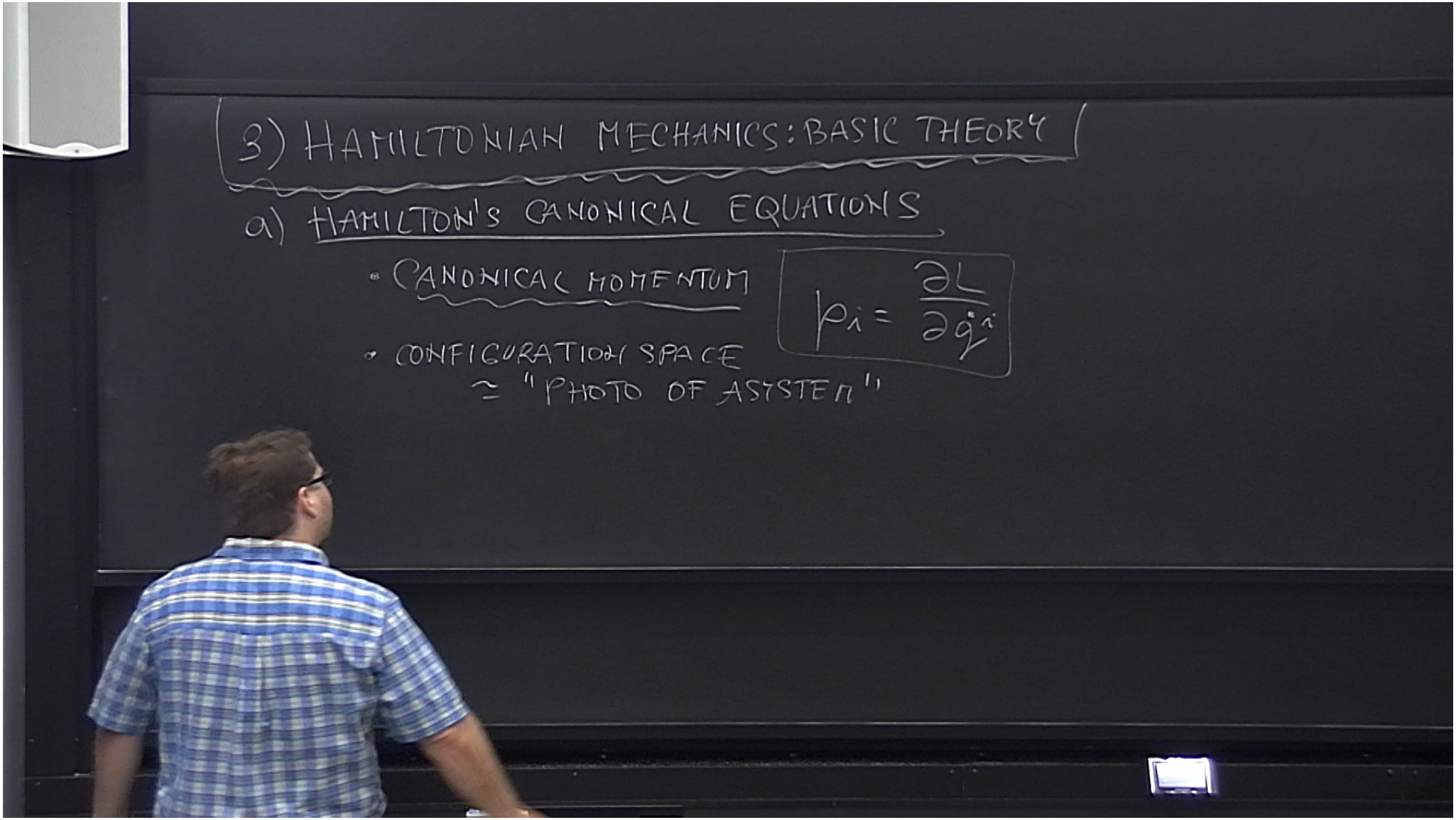
Title: PSI 17/18 Front End - Theoretical Mechanics (Kubiznak)

Date: Aug 15, 2017 09:00 AM

URL: http://pirsa.org/17080046

Abstract:




3) HAMILTONIAN MECHANICS: BASIC THEORY
a) HAMILTON'S CANONICAL EQUATIONS
• CANONICAL MOMENTUM
$p_i = \frac{\partial L}{\partial \dot{q}^i}$
• CONFIGURATION SPACE
≃ "PHOTO OF SYSTEM"

3) HAMILTONIAN MECHANICS: BASIC THEORY

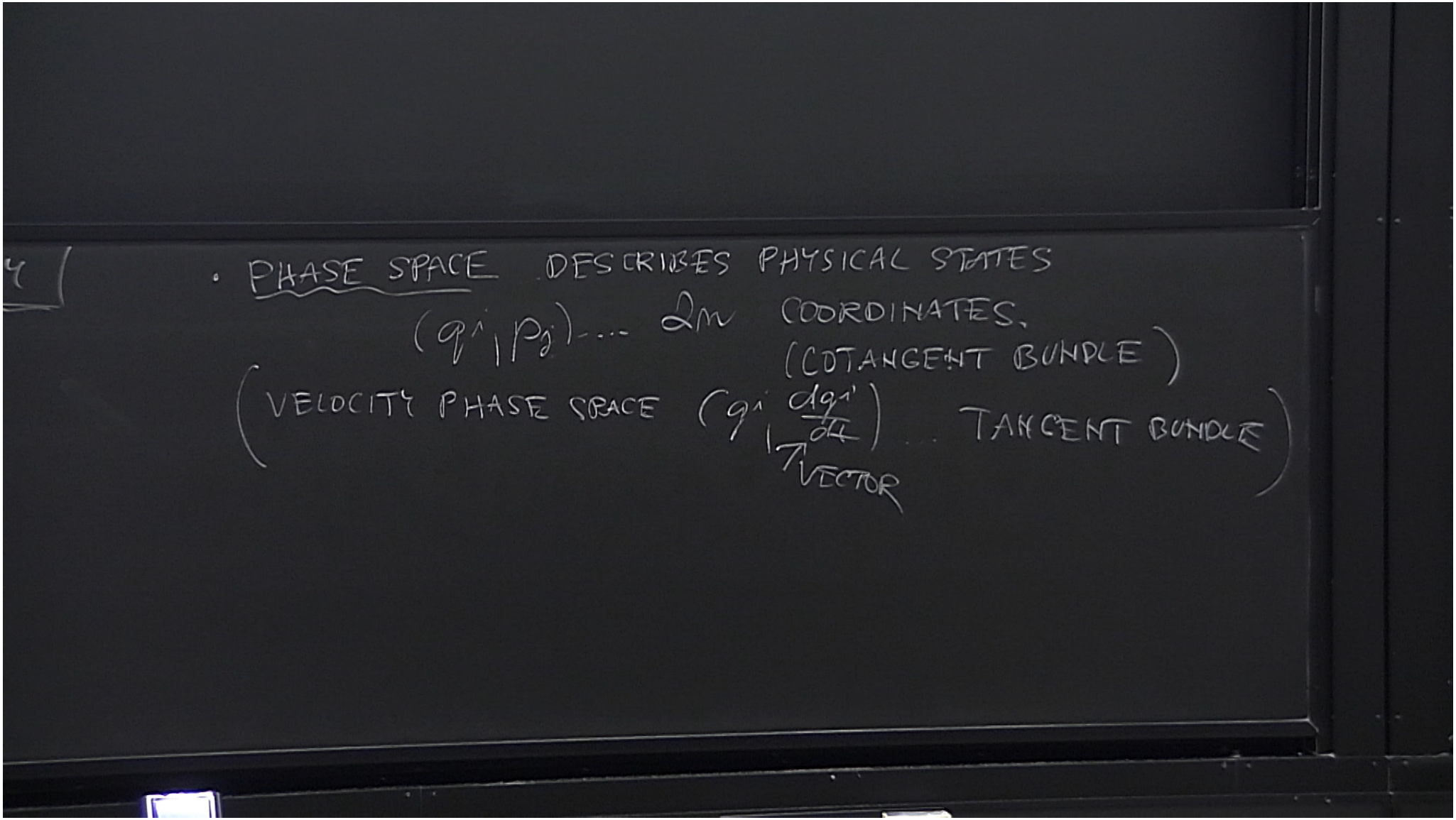
a) HAMILTON'S CANONICAL EQUATIONS

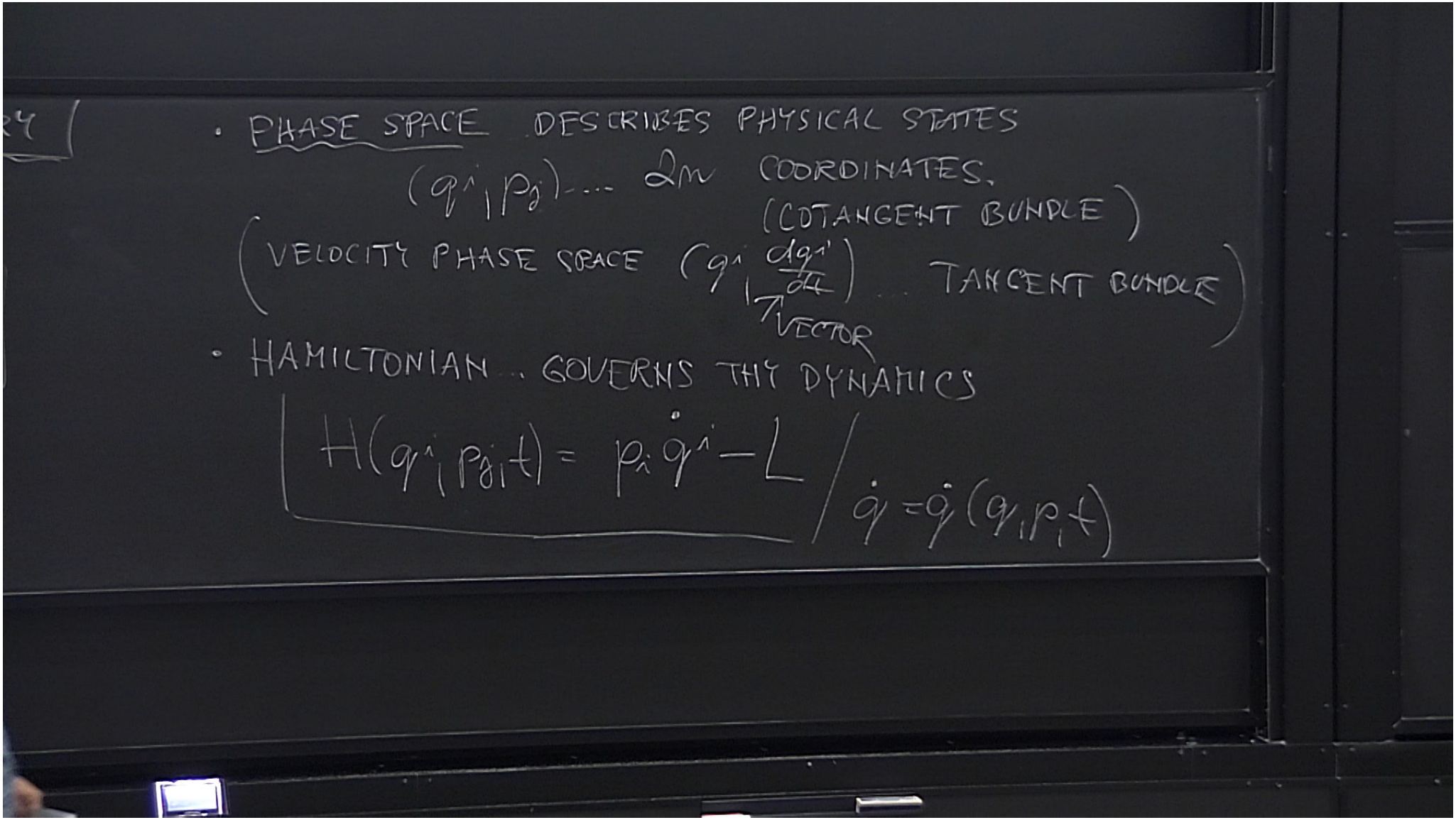
- CANONICAL MOMENTUM

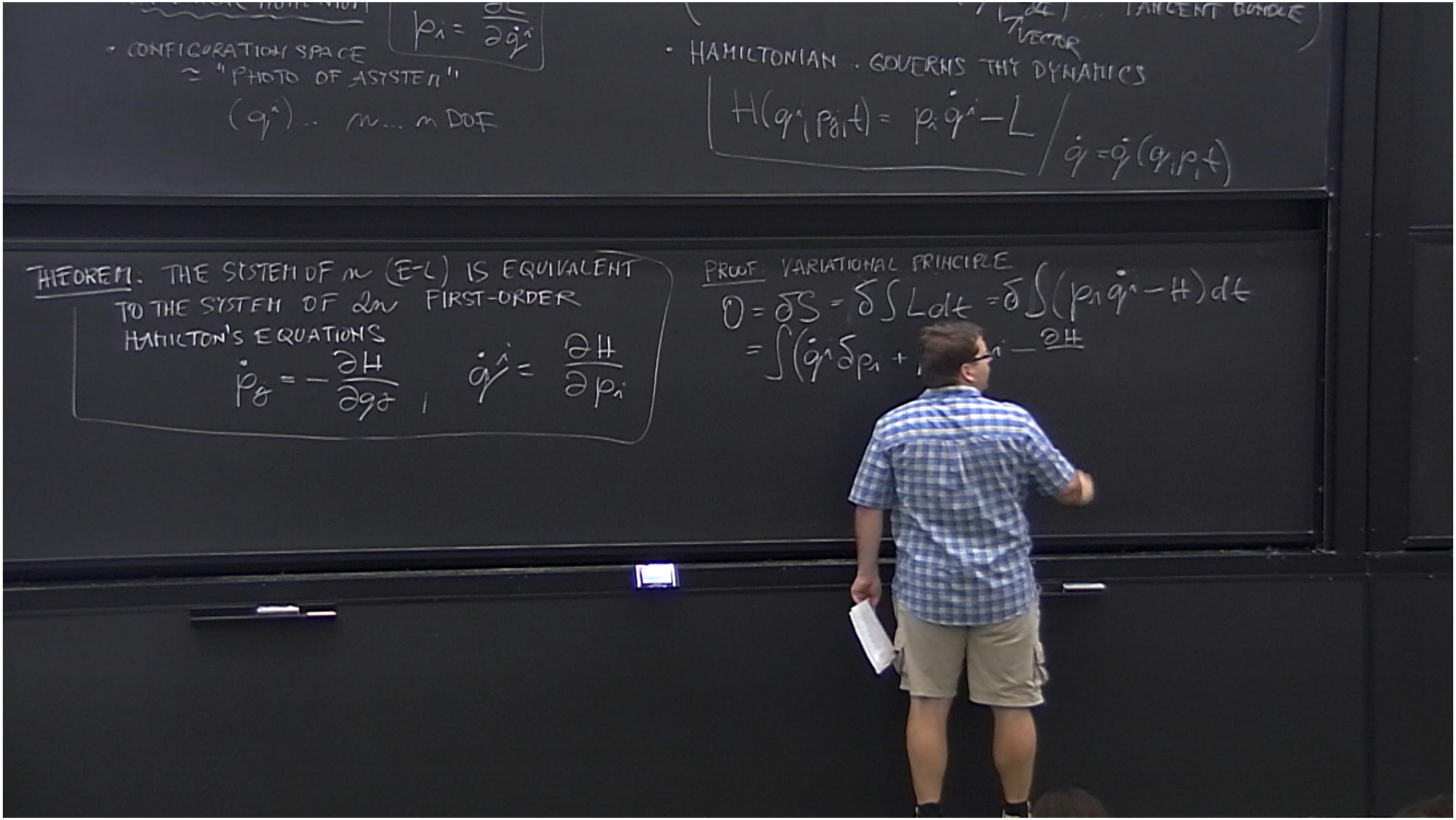
$$p_i = \frac{\partial L}{\partial \dot{q}^i}$$

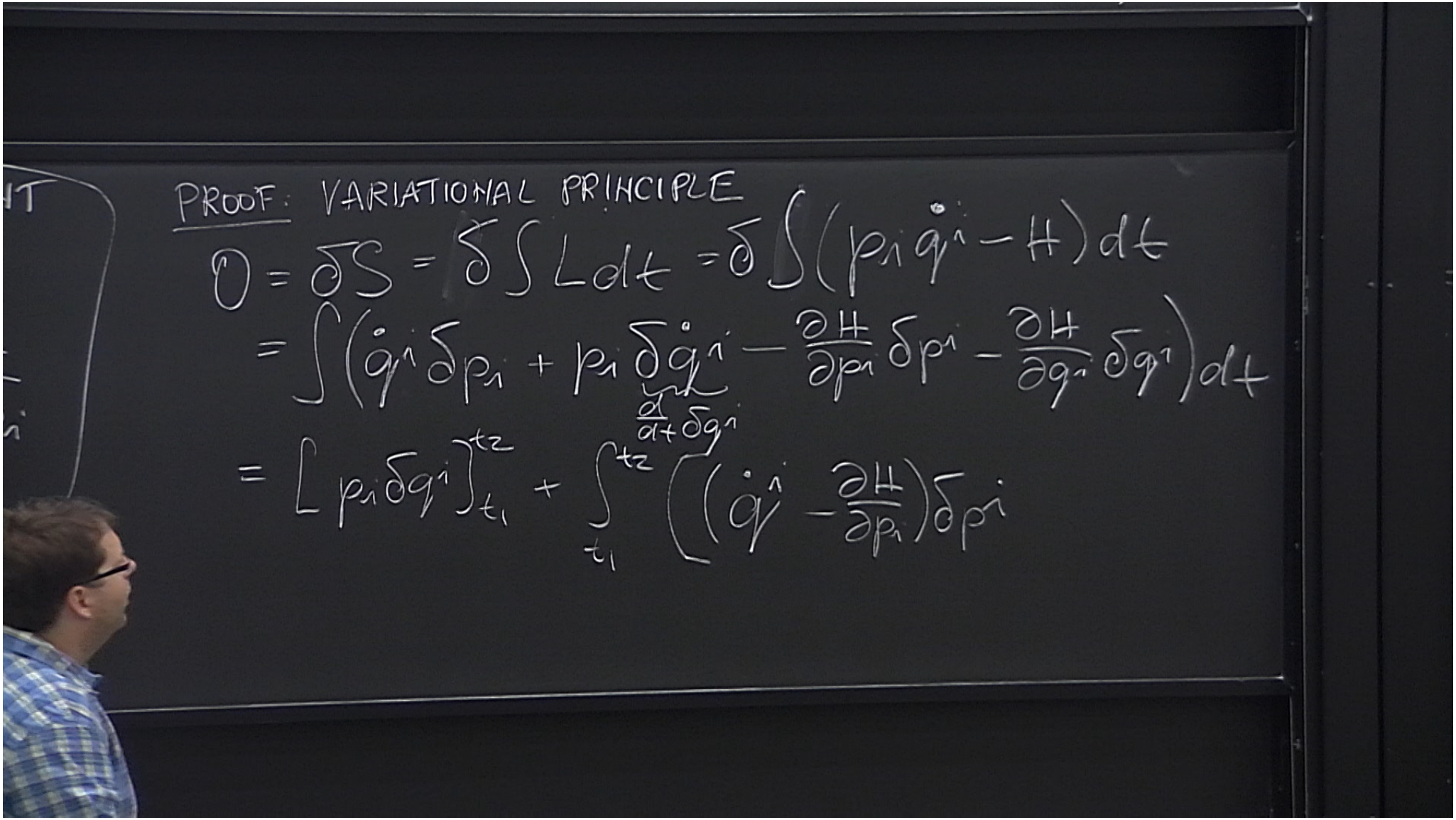
- CONFIGURATION SPACE
  ≃ "PHOTO OF SYSTEM"

(q^i) ... n ... n DOF


· PHASE SPACE DESCRIBES PHYSICAL STATES
(q^i, p_j) ... $2n$ COORDINATES.
(COTANGENT BUNDLE)
(VELOCITY PHASE SPACE $(q^i, \frac{dq^i}{dt})$ ... TANGENT BUNDLE)
↑ VECTOR


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↑VECTOR
• HAMILTONIAN . GOVERNS THE DYNAMICS
$H(q^i, p_i, t) = p_i \dot{q}^i - L$ / $\dot{q} = \dot{q}(q, p, t)$


• CONFIGURATION SPACE
≃ "PHOTO OF A SYSTEM"
(q^i) ... n ... n DOF
$p_i = \frac{\partial L}{\partial \dot{q}^i}$
TANGENT BUNDLE
VECTOR
• HAMILTONIAN . GOVERNS THE DYNAMICS
$H(q^i, p_i, t) = p_i \dot{q}^i - L$
$\dot{q} = \dot{q}(q, p, t)$
THEOREM. THE SYSTEM OF n (E-L) IS EQUIVALENT TO THE SYSTEM OF $2n$ FIRST-ORDER HAMILTON'S EQUATIONS
$\dot{p}_i = -\frac{\partial H}{\partial q^i}$, $\dot{q}^i = \frac{\partial H}{\partial p_i}$
PROOF VARIATIONAL PRINCIPLE
$0 = \delta S = \delta \int L dt = \delta \int (p_i \dot{q}^i - H) dt$
$= \int (\dot{q}^i \delta p_i +$, $\dot{} - \frac{\partial H}{}$


PROOF: VARIATIONAL PRINCIPLE
$0 = \delta S = \delta \int L\,dt = \delta \int (p_i \dot{q}^i - H)\,dt$
$= \int \left(\dot{q}^i \delta p_i + p_i \delta \dot{q}^i - \frac{\partial H}{\partial p_i} \delta p^i - \frac{\partial H}{\partial q^i} \delta q^i \right) dt$
$\frac{d}{dt} \delta q^i$
$= \left[p_i \delta q^i \right]_{t_1}^{t_2} + \int_{t_1}^{t_2} \left(\left(\dot{q}^i - \frac{\partial H}{\partial p_i} \right) \delta p^i \right.$

PROOF: VARIATIONAL PRINCIPLE

$$0 = \delta S = \delta \int L\,dt = \delta \int (p_i \dot{q}^i - H)\,dt$$

$$= \int \left(\dot{q}^i \delta p_i + p_i \delta \dot{q}^i - \frac{\partial H}{\partial p_i} \delta p^i - \frac{\partial H}{\partial q^i} \delta q^i \right) dt$$

$$\delta \dot{q}^i = \frac{d}{dt} \delta q^i$$

$$= \left[p_i \delta q^i \right]_{t_1}^{t_2} + \int_{t_1}^{t_2} \left[\left(\dot{q}^i - \frac{\partial H}{\partial p_i} \right) \delta p^i - \left(p_i + \frac{\partial H}{\partial q^i} \right) \delta q^i \right] dt$$

PROOF: VARIATIONAL PRINCIPLE

$$0 = \delta S = \delta \int L\,dt = \delta \int (p_i \dot{q}^i - H)\,dt$$

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PROOF: VARIATIONAL PRINCIPLE

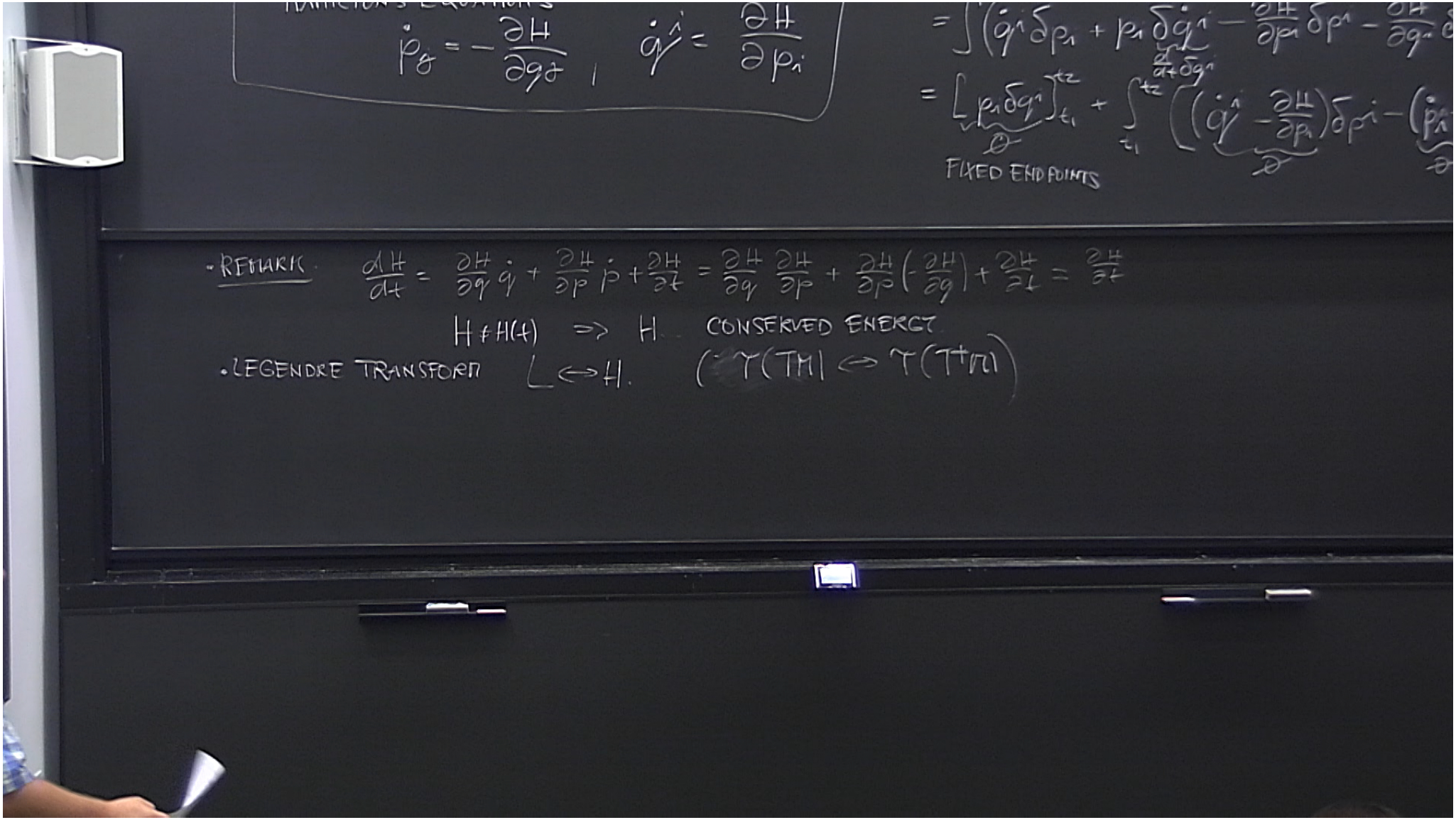
$$0 = \delta S = \delta \int L\,dt = \delta \int (p_i \dot{q}^i - H)\,dt$$

$$= \int \left(\dot{q}^i \delta p_i + p_i \delta \dot{q}^i - \frac{\partial H}{\partial p_i}\delta p^i - \frac{\partial H}{\partial q^i}\delta q^i \right) dt$$

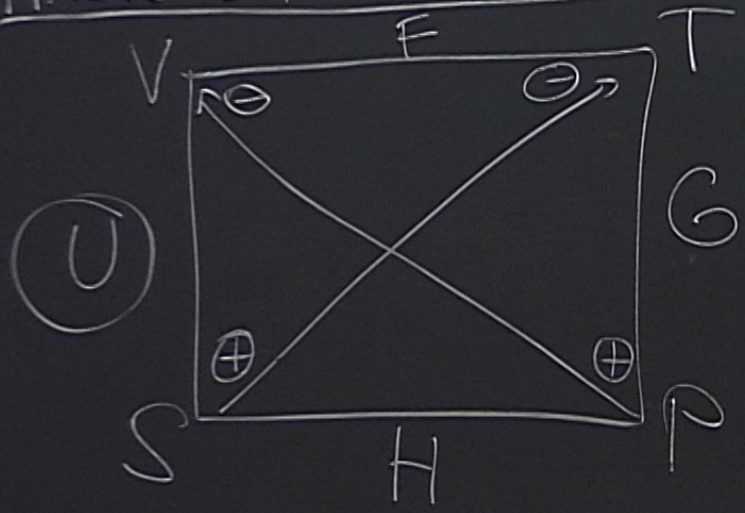
$$\delta \dot{q}^i = \frac{d}{dt}\delta q^i$$

$$= \underbrace{\left[p_i \delta q^i \right]_{t_1}^{t_2}}_{0} + \int_{t_1}^{t_2} \left[\underbrace{\left(\dot{q}^i - \frac{\partial H}{\partial p_i} \right)}_{0} \delta p^i - \underbrace{\left(\dot{p}_i + \frac{\partial H}{\partial q^i} \right)}_{0} \delta q^i \right] dt$$

FIXED END POINTS


$\dot{p}_\sigma = -\frac{\partial H}{\partial q_\sigma}$, $\dot{q}^i = \frac{\partial H}{\partial p_i}$
$= \int (\dot{q}^i \delta p_i + p_i \delta \dot{q}^i - \frac{\partial H}{\partial p^i}\delta p^i - \frac{\partial H}{\partial q^i}$
$= [p_i \delta q^i]_{t_1}^{t_2} + \int_{t_1}^{t_2} \left((\dot{q}^i - \frac{\partial H}{\partial p_i}) \delta p^i - (\dot{p}_i$
$\frac{d}{dt}\delta q^i$
0
FIXED END POINTS
• REMARK $\frac{dH}{dt} = \frac{\partial H}{\partial q}\dot{q} + \frac{\partial H}{\partial p}\dot{p} + \frac{\partial H}{\partial t} = \frac{\partial H}{\partial q}\frac{\partial H}{\partial p} + \frac{\partial H}{\partial p}\left(-\frac{\partial H}{\partial q}\right) + \frac{\partial H}{\partial t} = \frac{\partial H}{\partial t}$
$H \neq H(t) \Rightarrow H$ CONSERVED ENERGY
• LEGENDRE TRANSFORM $L \leftrightarrow H$. $(T(TM) \leftrightarrow T(T^*M))$

- MAGIC SQUARE IN TDS



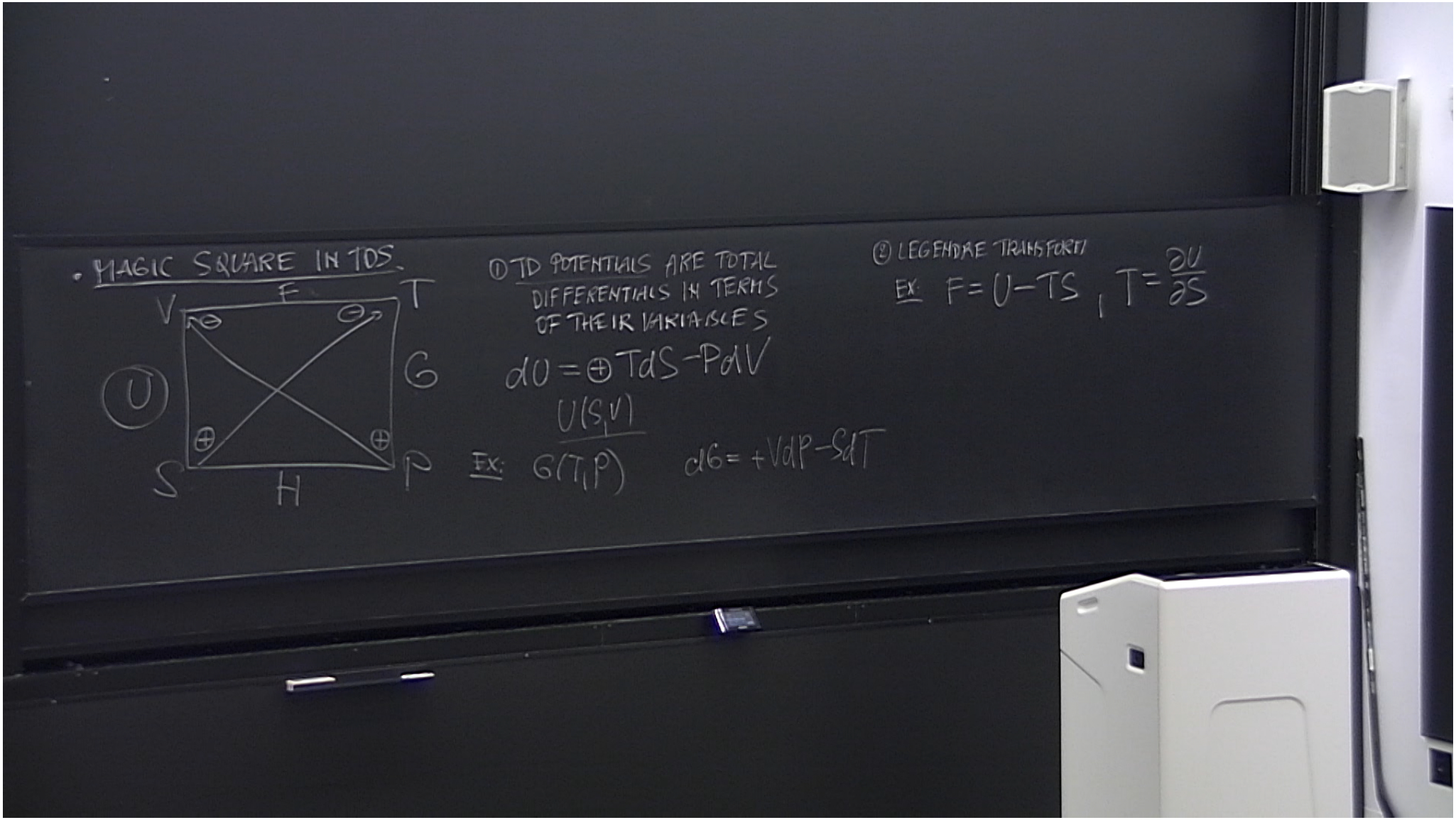
① TD POTENTIALS ARE TOTAL DIFFERENTIALS IN TERMS OF THEIR VARIABLES

$$dU = \oplus TdS - PdV$$

$U(S,V)$

Ex: $G(T,P)$ $\quad dG = +VdP - SdT$

② LEGENDR


• MAGIC SQUARE IN TDS
V
F
T
U
G
S
H
P
① TD POTENTIALS ARE TOTAL DIFFERENTIALS IN TERMS OF THEIR VARIABLES
dU = ⊕TdS − PdV
U(S,V)
Ex: G(T,P) dG = +VdP − SdT
② LEGENDRE TRANSFORM
Ex: F = U − TS , T = ∂U/∂S

① TD POTENTIALS ARE TOTAL DIFFERENTIALS IN TERMS OF THEIR VARIABLES

$$dU = \oplus TdS - PdV$$

$$U(S,V)$$

Ex: $G(T,P)$ $\quad dG = +VdP - SdT$

② LEGENDRE TRANSFORM

Ex: $F = U - TS$, $T = \frac{\partial U}{\partial S}$

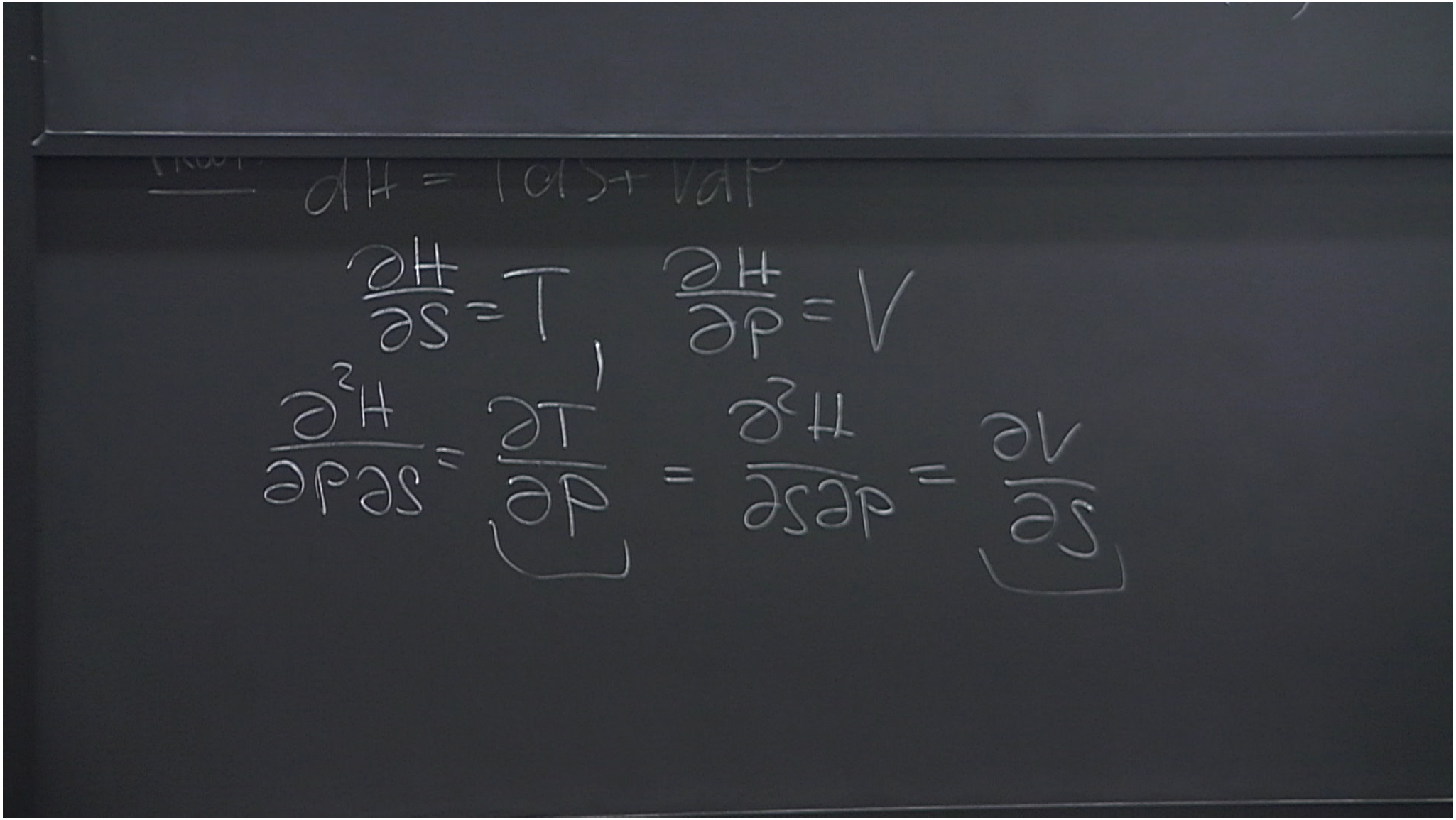
③ MAXWELL'S RELATIONS

$$\frac{\partial^2}{\partial \cdot \partial \cdot} = \frac{\partial^2}{\partial \cdot \partial \cdot}$$

SIGN CONST SIGN CONST

$$\left(\frac{\partial V}{\partial S}\right)_P = \left(\frac{\partial T}{\partial P}\right)_S$$

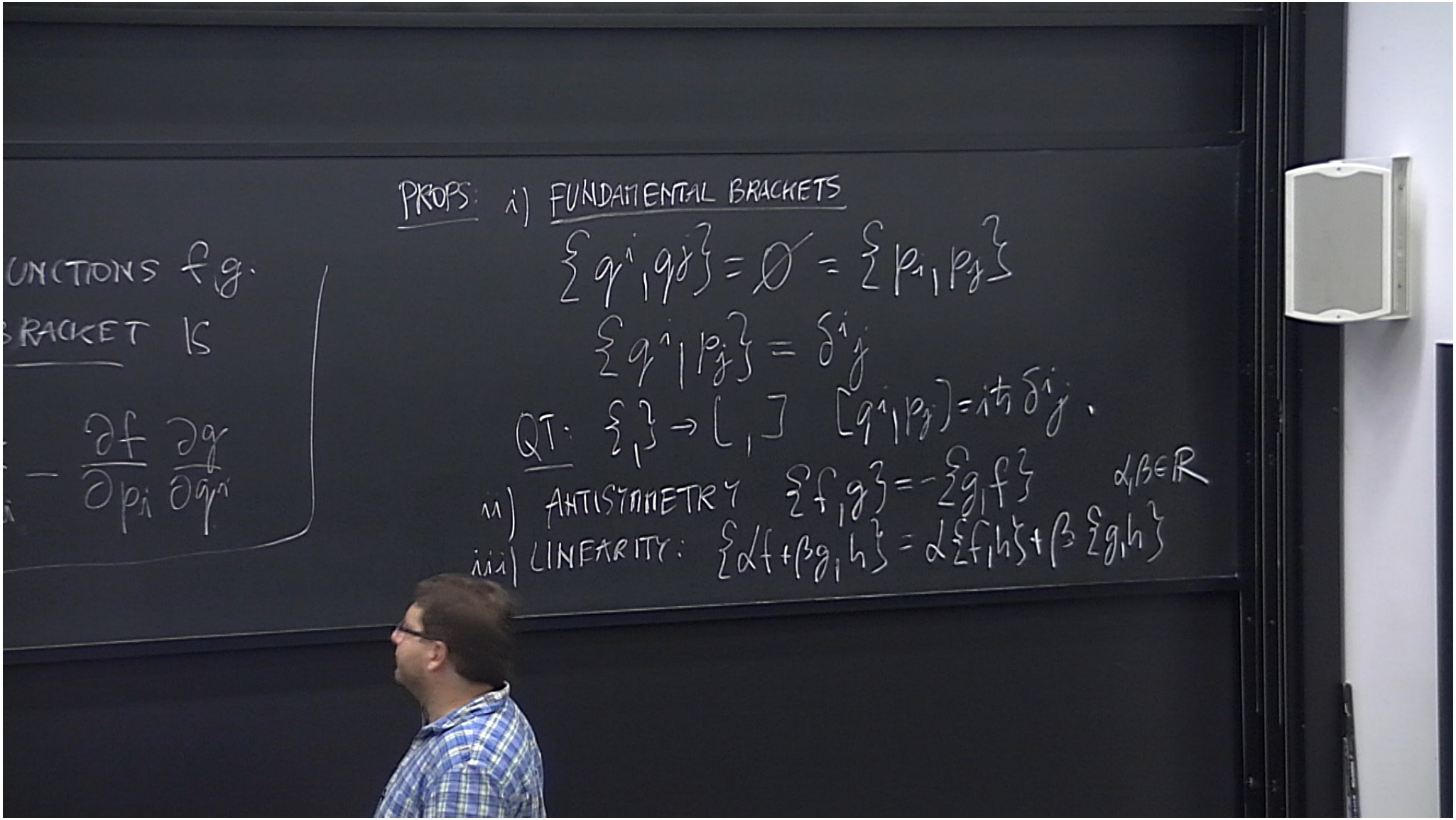
$$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$$

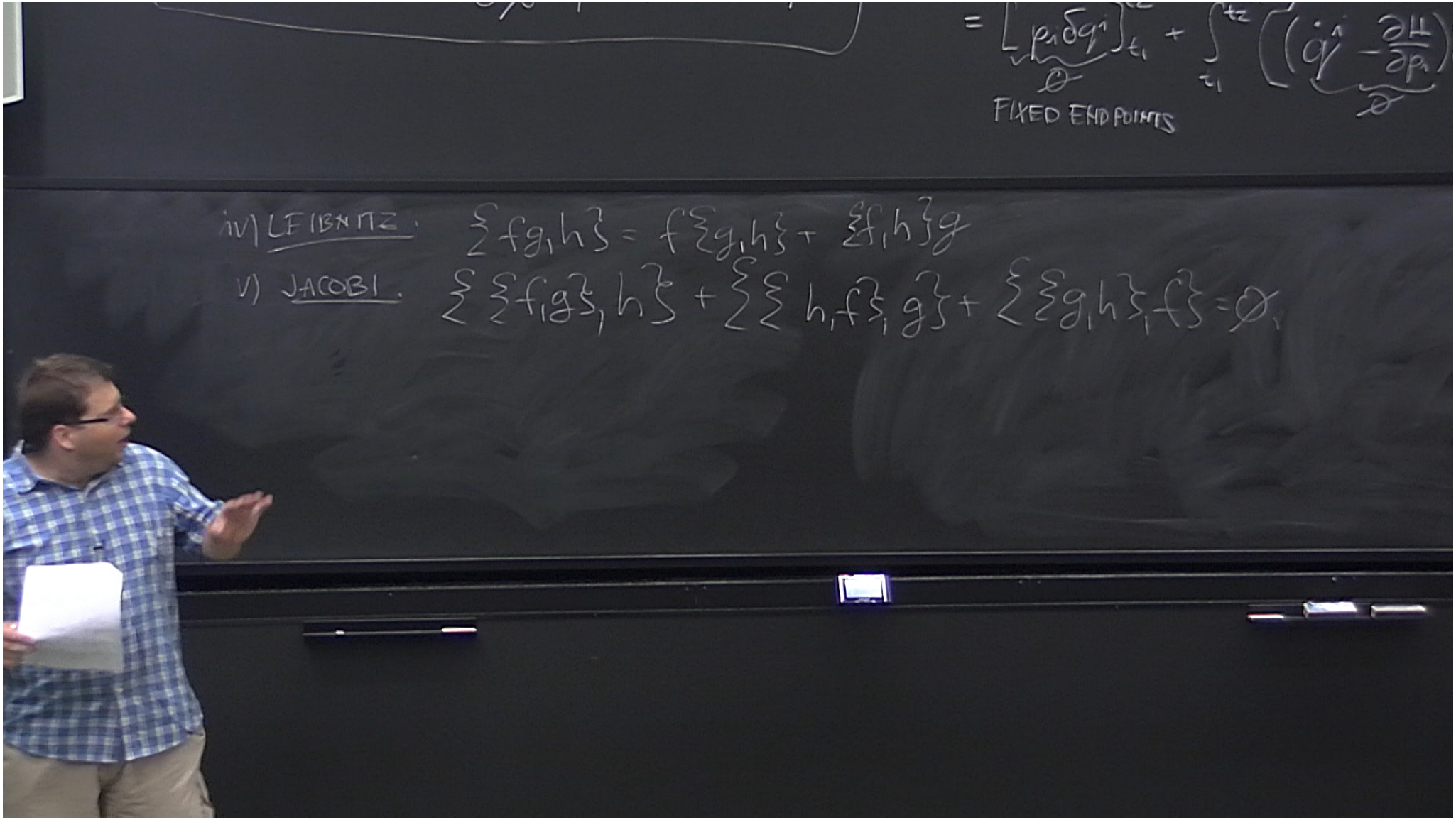

dH = TdS + VdP
∂H/∂S = T
∂H/∂P = V
∂²H/∂P∂S = ∂T/∂P = ∂²H/∂S∂P = ∂V/∂S

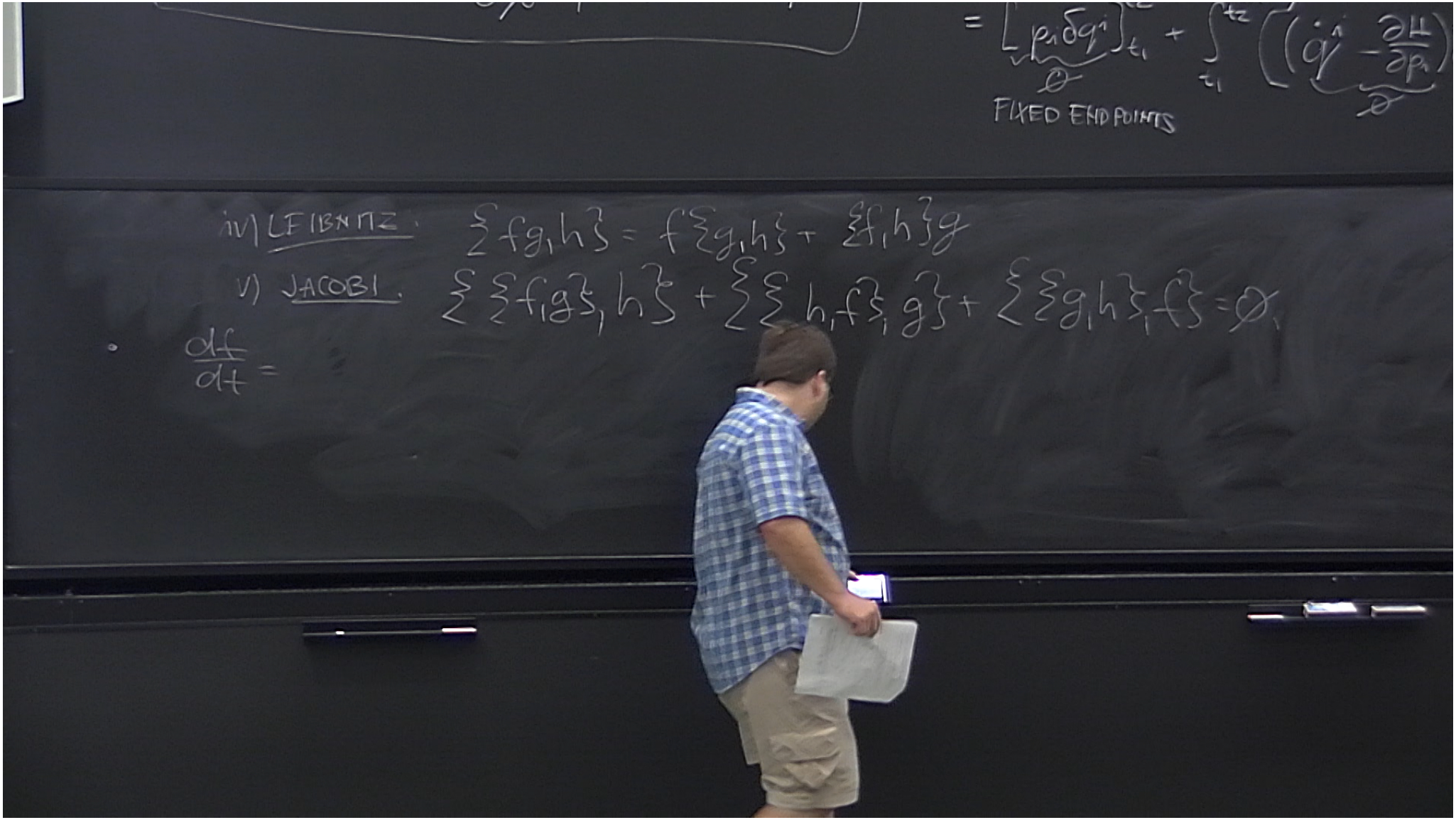
b) POISSON BRACKETS

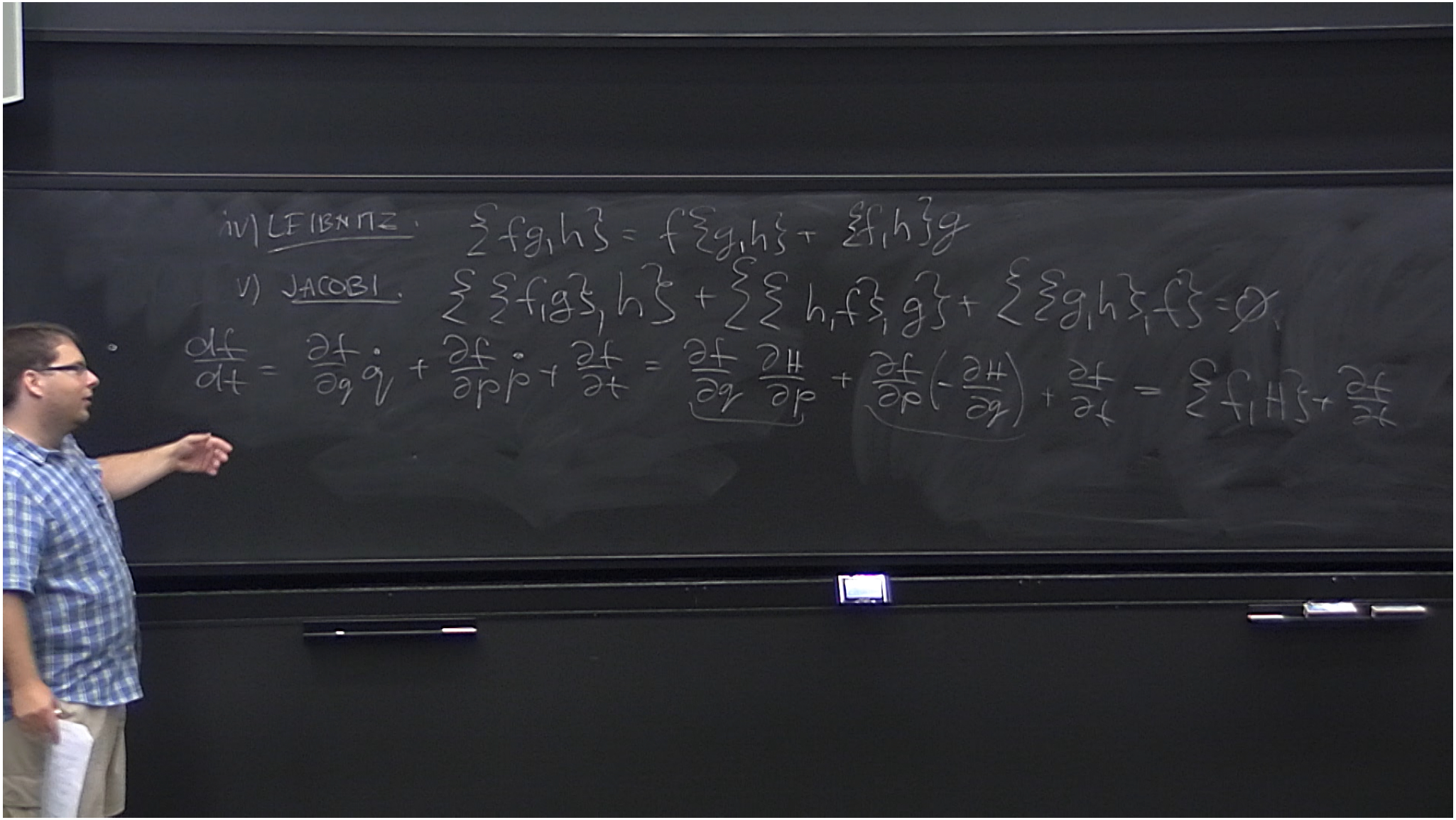
DEF: FOR TWO PHASE SPACE FUNCTIONS f, g: THEIR CANONICAL POISSON BRACKET IS A NEW P-S FUNCTION,

$$\{f, g\} = \frac{\partial f}{\partial q^i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q^i}$$


UNCTIONS f,g.
BRACKET IS
$-\frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q^i}$
PROPS: i) FUNDAMENTAL BRACKETS
$\{q^i, q^j\} = 0 = \{p_i, p_j\}$
$\{q^i, p_j\} = \delta^i_j$
QT: $\{,\} \to [,]$ $[q^i, p_j] = i\hbar\,\delta^i_j$.
ii) ANTISYMMETRY $\{f,g\} = -\{g,f\}$ $\alpha,\beta \in \mathbb{R}$
iii) LINEARITY: $\{\alpha f + \beta g, h\} = \alpha\{f,h\} + \beta\{g,h\}$


FIXED END POINTS
iv) LEIBNITZ. $\{fg,h\} = f\{g,h\} + \{f,h\}g$
v) JACOBI. $\{\{f,g\},h\} + \{\{h,f\},g\} + \{\{g,h\},f\} = 0$


FIXED END POINTS
iv) LEIBNITZ
v) JACOBI


iv) LEIBNITZ. $\{fg,h\} = f\{g,h\} + \{f,h\}g$
v) JACOBI. $\{\{f,g\},h\} + \{\{h,f\},g\} + \{\{g,h\},f\} = 0$
$\frac{df}{dt} = \frac{\partial f}{\partial q}\dot{q} + \frac{\partial f}{\partial p}\dot{p} + \frac{\partial f}{\partial t} = \frac{\partial f}{\partial q}\frac{\partial H}{\partial p} + \frac{\partial f}{\partial p}\left(-\frac{\partial H}{\partial q}\right) + \frac{\partial f}{\partial t} = \{f,H\} + \frac{\partial f}{\partial t}$

FIXED END POINTS

iv) LEIBNITZ: $\{fg,h\} = f\{g,h\} + \{f,h\}g$

v) JACOBI: $\{\{f,g\},h\} + \{\{h,f\},g\} + \{\{g,h\},f\} = 0$

- $\frac{df}{dt} = \frac{\partial f}{\partial q}\dot{q} + \frac{\partial f}{\partial p}\dot{p} + \frac{\partial f}{\partial t} = \frac{\partial f}{\partial q}\frac{\partial H}{\partial p} + \frac{\partial f}{\partial p}\left(-\frac{\partial H}{\partial q}\right) + \frac{\partial f}{\partial t} = \boxed{\{f,H\} + \frac{\partial f}{\partial t} = \frac{df}{dt}}$
- INTEGRAL OF MOTION $\boxed{\frac{dI}{dt} = 0}$

FIXED END POINTS

iv) LEIBNITZ: $\{fg,h\} = f\{g,h\} + \{f,h\}g$

v) JACOBI: $\{\{f,g\},h\} + \{\{h,f\},g\} + \{\{g,h\},f\} = 0$

- $\frac{df}{dt} = \frac{\partial f}{\partial q}\dot{q} + \frac{\partial f}{\partial p}\dot{p} + \frac{\partial f}{\partial t} = \frac{\partial f}{\partial q}\frac{\partial H}{\partial p} + \frac{\partial f}{\partial p}\left(-\frac{\partial H}{\partial q}\right) + \frac{\partial f}{\partial t} = \boxed{\{f,H\} + \frac{\partial f}{\partial t} = \frac{df}{dt}}$
- INTEGRAL OF MOTION $\boxed{\frac{dI}{dt} = 0} \iff \boxed{\{I,H\} = 0}$

$p_\dot{q} = -\frac{\partial}{\partial \dot{q}^i}$, $\dot{q}^i = \frac{\partial}{\partial p_i}$

$= \underbrace{[p_i \delta q^i]_{t_1}^{t_2}}_{0} + \int_{t_1}^{t_2} \left(\left(\dot{q}^i - \frac{\partial H}{\partial p_i} \right) \delta p_i - \left(\dot{p}_i + \frac{\partial H}{\partial q^i} \right) \delta q^i \right) dt$

FIXED END POINTS

iv) LEIBNITZ: $\{fg, h\} = f\{g, h\} + \{f, h\}g$

v) JACOBI: $\{\{f, g\}, h\} + \{\{h, f\}, g\} + \{\{g, h\}, f\} = 0$

$\frac{df}{dt} = \frac{\partial f}{\partial q}\dot{q} + \frac{\partial f}{\partial p}\dot{p} + \frac{\partial f}{\partial t} = \frac{\partial f}{\partial q}\frac{\partial H}{\partial p} + \frac{\partial f}{\partial p}\left(-\frac{\partial H}{\partial q}\right) + \frac{\partial f}{\partial t} = \boxed{\{f, H\} + \frac{\partial f}{\partial t} = \frac{df}{dt}}$

- INTEGRAL OF MOTION $\boxed{\frac{dI}{dt} = 0} \iff \boxed{\{I, H\} = 0}$

PROP: $\{I_1, I_2\} = I_3$ "NEW" INTEGRAL OF MOTION

iv) LEIBNIZ: $\{fg,h\} = f\{g,h\} + \{f,h\}g$

v) JACOBI: $\{\{f,g\},h\} + \{\{h,f\},g\} + \{\{g,h\},f\} = 0$

$$\frac{df}{dt} = \frac{\partial f}{\partial q}\dot{q} + \frac{\partial f}{\partial p}\dot{p} + \frac{\partial f}{\partial t} = \frac{\partial f}{\partial q}\frac{\partial H}{\partial p} + \frac{\partial f}{\partial p}\left(-\frac{\partial H}{\partial q}\right) + \frac{\partial f}{\partial t} = \boxed{\{f,H\} + \frac{\partial f}{\partial t} = \frac{df}{dt}}$$

- INTEGRAL OF MOTION $\boxed{\frac{dI}{dt} = 0} \iff \boxed{\{I,H\} = 0}$

PROP: $\{I_1, I_2\} = I_3$ "NEW" INTEGRAL OF MOTION

C) CANONICAL TRANSFORMATIONS

DEF: A CANONICAL TRANSFORMATION

$$Q^\alpha = Q^\alpha(q^\beta, p_\beta, t), \quad P_\alpha = P_\alpha(q^\beta, p_\beta, t)$$

TRANSF. SO THAT IT PRESERVES THE FORM OF HAMILTON'S EQUATIONS"

THAT, WE CAN FIND NEW HAMILTONIAN $H' = H'(Q^\alpha, P_\alpha, t)$ SUCH THAT

$$\frac{\partial H'}{\partial P_\alpha} = \dot{Q}^\alpha, \quad \frac{\partial H'}{\partial Q_\alpha} = -\dot{P}_\alpha$$

- INTEGRAL OF MOTION $\boxed{\frac{dI}{dt}=0}$ $\Leftrightarrow$ $\boxed{\{I, H\}=0}$

C) CANONICAL TRANSFORMATIONS

DEF: A CANONICAL TRANSFORMATION

$$Q^j = Q^j(q^i, p_i, t), \quad P_j = P_j(q^i, p_i, t)$$

IS SUCH TRANSF. SO THAT IT PRESERVES "THE FORM OF HAMILTON'S EQUATIONS"



$\frac{\partial f}{\partial q}\frac{\partial p}{\partial t} + \frac{\partial f}{\partial p}\left(-\frac{\partial H}{\partial q}\right) + \frac{\partial f}{\partial t} = \{f, H\} + \frac{\partial f}{\partial t} = \frac{df}{dt}$

$\Leftrightarrow \boxed{\{I, H\} = 0}$

PROP: $\{I_1, I_2\} = I_3$ "NEW" INTEGRAL OF MOTION

THAT, WE CAN FIND NEW HAMILTONIAN $H' = H'(Q_\beta, P_\beta, t)$

SUCH THAT

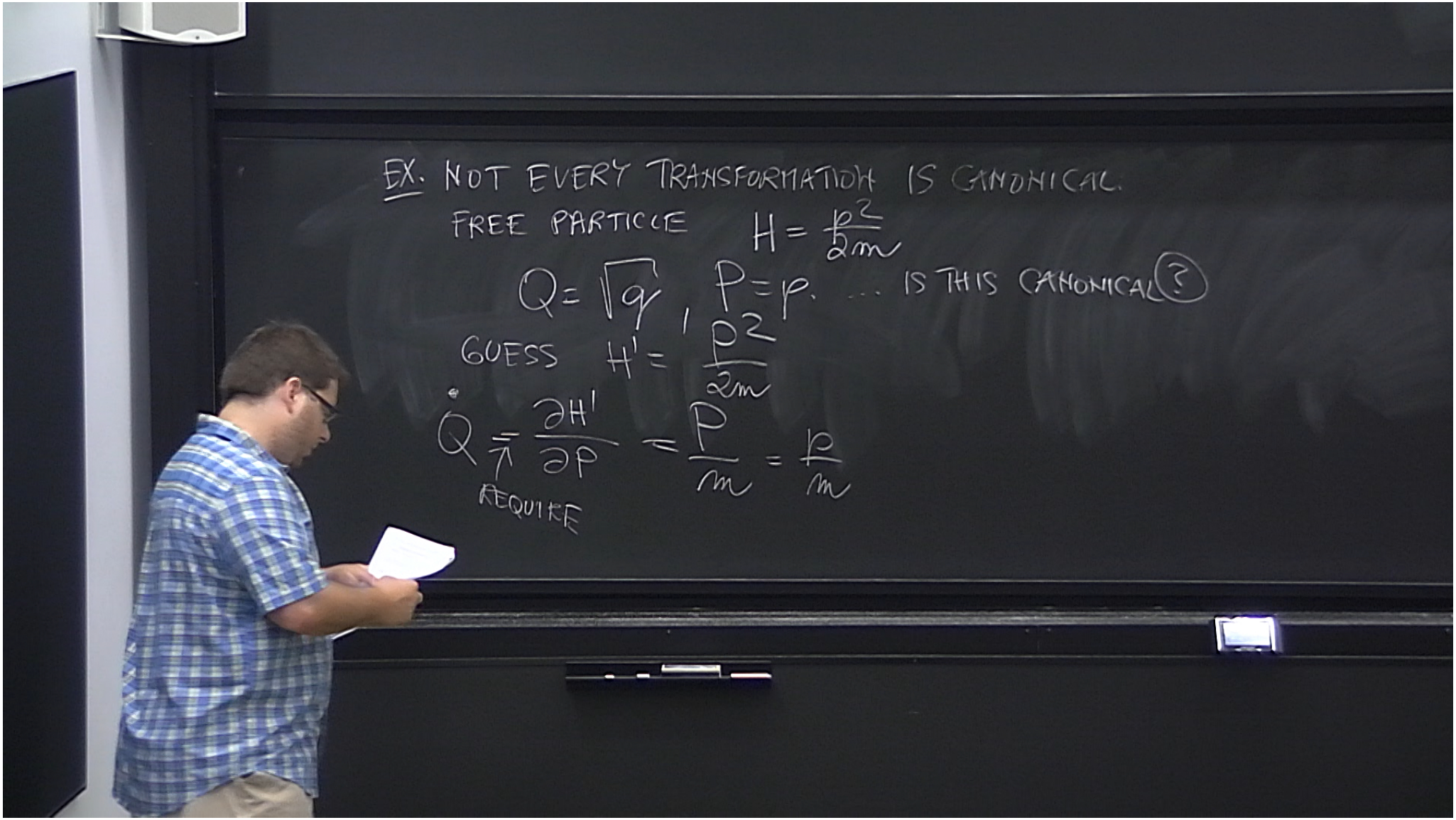
$$\frac{\partial H'}{\partial P_\beta} = \dot{Q}_\beta, \quad \frac{\partial H'}{\partial Q_\beta} = -\dot{P}_\beta$$

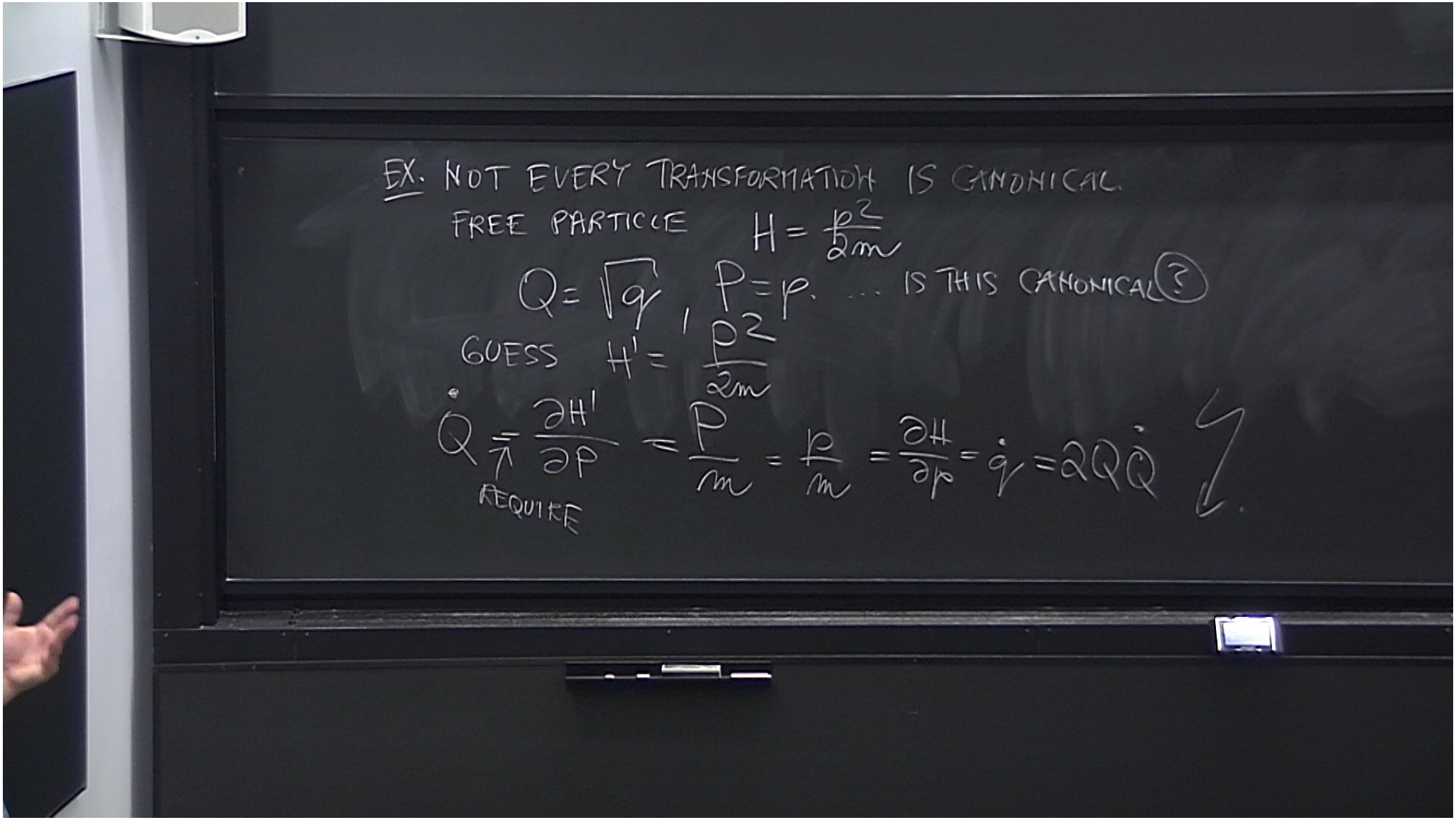
DESCRIBES DYNAMICS.

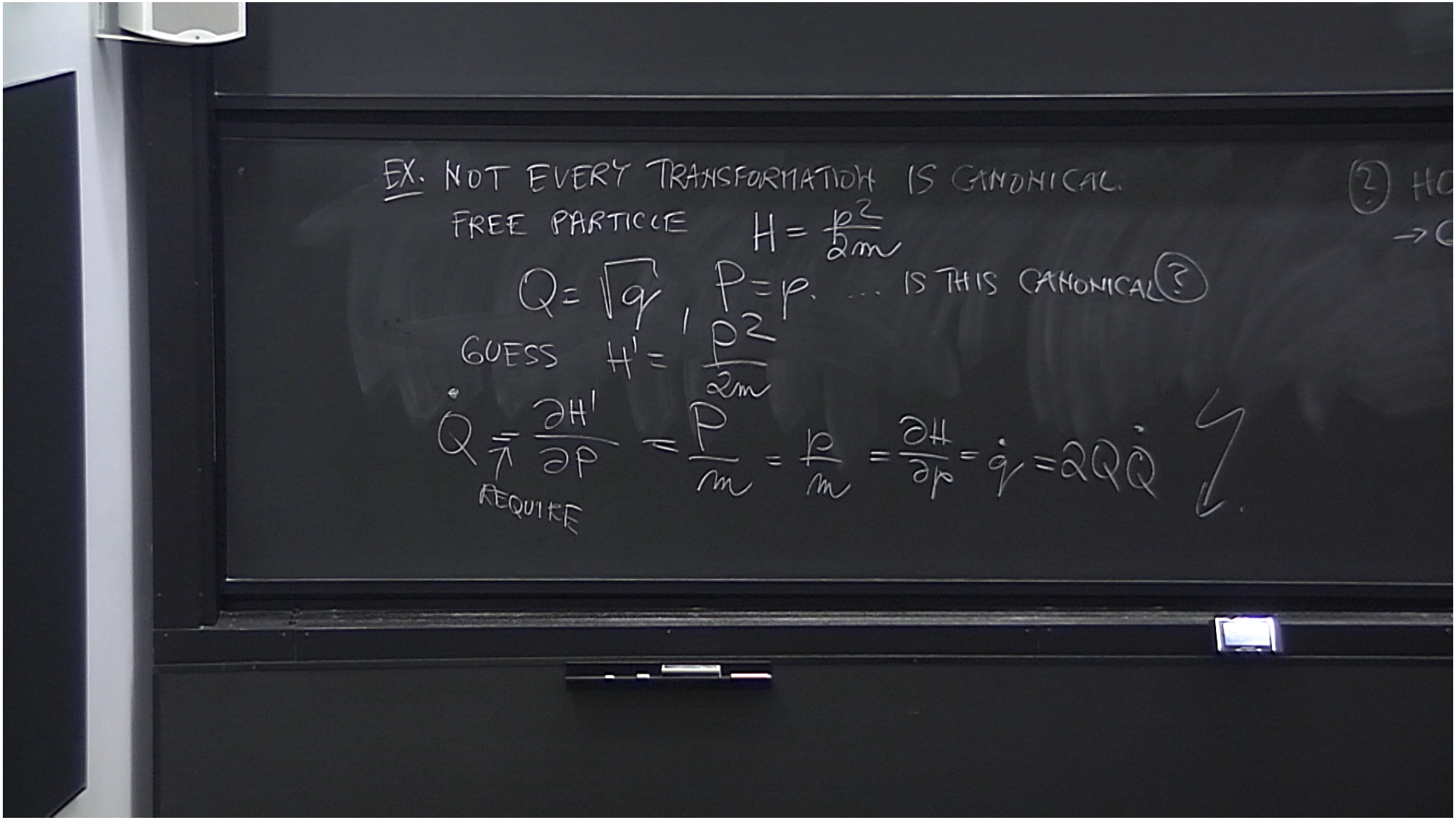
$= P_\beta(q, p, t)$

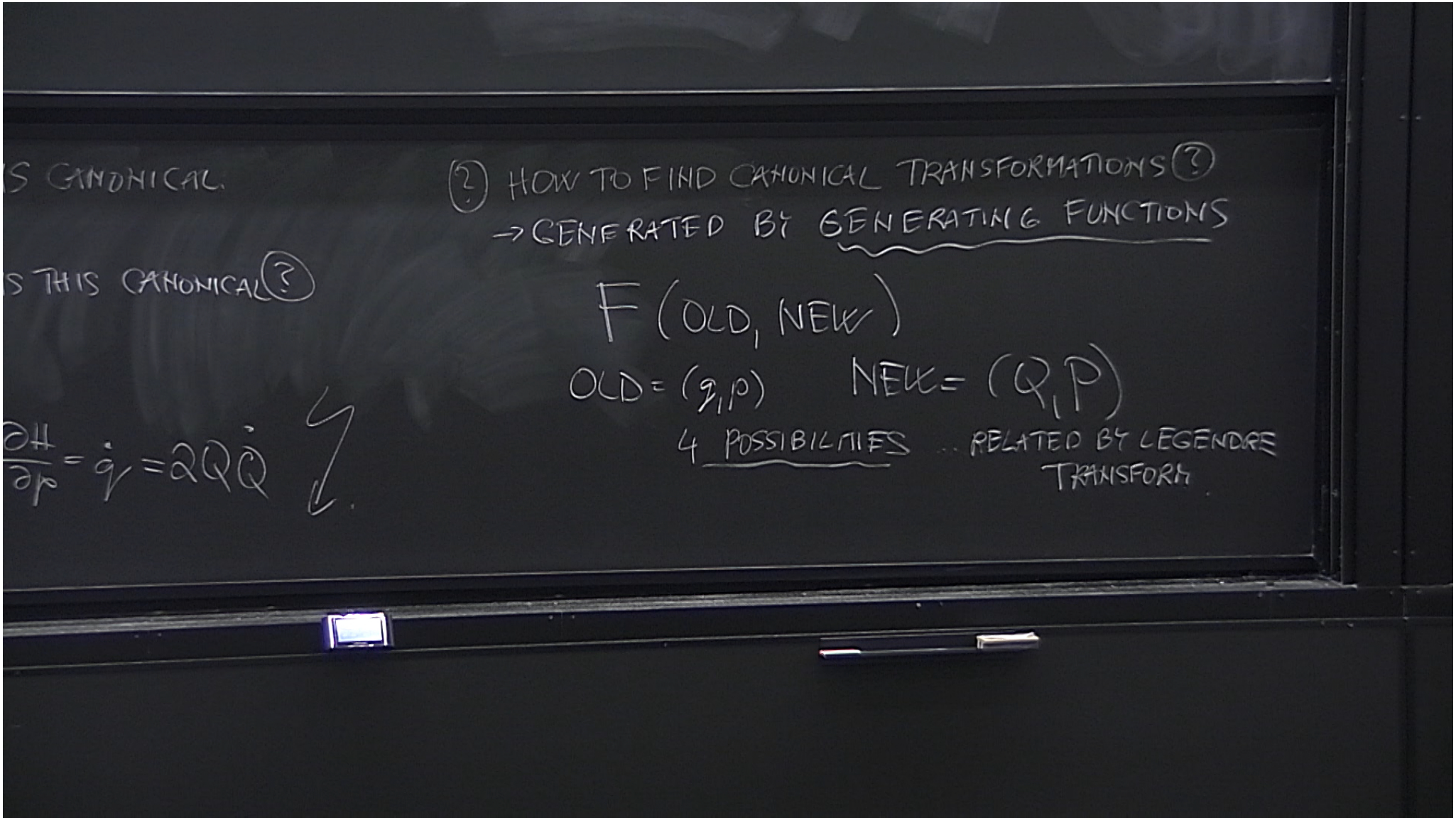
SERVES

EQUATIONS"


EX. NOT EVERY TRANSFORMATION IS CANONICAL.
FREE PARTICLE $H = \frac{p^2}{2m}$
$Q = \sqrt{q}$, $P = p$. ... IS THIS CANONICAL?
GUESS $H' = \frac{P^2}{2m}$
$\dot{Q} = \frac{\partial H'}{\partial P} = \frac{P}{m} = \frac{p}{m}$
REQUIRE


EX. NOT EVERY TRANSFORMATION IS CANONICAL.
FREE PARTICLE $H = \frac{p^2}{2m}$
$Q = \sqrt{q}$, $P = p$. ... IS THIS CANONICAL?
GUESS $H' = \frac{P^2}{2m}$
$\dot{Q} = \frac{\partial H'}{\partial P} = \frac{P}{m} = \frac{p}{m} = \frac{\partial H}{\partial p} = \dot{q} = 2Q\dot{Q}$
REQUIRE


EX. NOT EVERY TRANSFORMATION IS CANONICAL.
FREE PARTICLE $H = \frac{p^2}{2m}$
$Q = \sqrt{q}$, $P = p$. ... IS THIS CANONICAL?
GUESS $H' = \frac{P^2}{2m}$
$\dot{Q} = \frac{\partial H'}{\partial P} = \frac{P}{m} = \frac{p}{m} = \frac{\partial H}{\partial p} = \dot{q} = 2Q\dot{Q}$
REQUIRE


S CANONICAL.
S THIS CANONICAL?
$\frac{\partial H}{\partial p} = \dot{q} = 2Q\dot{Q}$
? HOW TO FIND CANONICAL TRANSFORMATIONS ?
→ GENERATED BY GENERATING FUNCTIONS
F (OLD, NEW)
OLD = (q, p) NEW = (Q, P)
4 POSSIBILITIES .. RELATED BY LEGENDRE TRANSFORM

b) POISSON BRACKETS

DEF: FOR TWO PHASE SPACE FUNCTIONS f, g. THEIR CANONICAL POISSON BRACKET IS ANEW PHASE FUNCTION.

PROPS: i) FUNDAMENTAL BRACKETS

$$\{q^i, q^j\} = 0 = \{p_i, p_j\}$$

$$\{q^i, p_j\} = \delta^i_j$$

EX: CONSIDER $F(q, Q, t)$ ← FREEDOM.

$$S = \int \left(p\dot{q} - H - \frac{dF}{dt} \right) dt$$

$$= \int \left(p\dot{q} - H - \frac{\partial F}{\partial q}\dot{q} - \frac{\partial F}{\partial Q}\dot{Q} - \frac{\partial F}{\partial t} \right) dt = \int \left(\dot{Q}\left(-\frac{\partial F}{\partial Q}\right) + \dot{q}\left(p - \frac{\partial F}{\partial q}\right) - \left(H + \frac{\partial F}{\partial t}\right) \right) dt$$

WANT $= \int \left(P\dot{Q} - H' \right) dt$

b) POISSON BRACKETS

DEF: FOR TWO PHASE SPACE FUNCTIONS f, g. THEIR CANONICAL POISSON BRACKET IS ANALYTIC PHASE FUNCTION

PROPS: i) FUNDAMENTAL BRACKETS

$$\{q^i, q^j\} = 0 = \{p_i, p_j\}$$

$$\{q^i, p_j\} = \delta^i_j$$

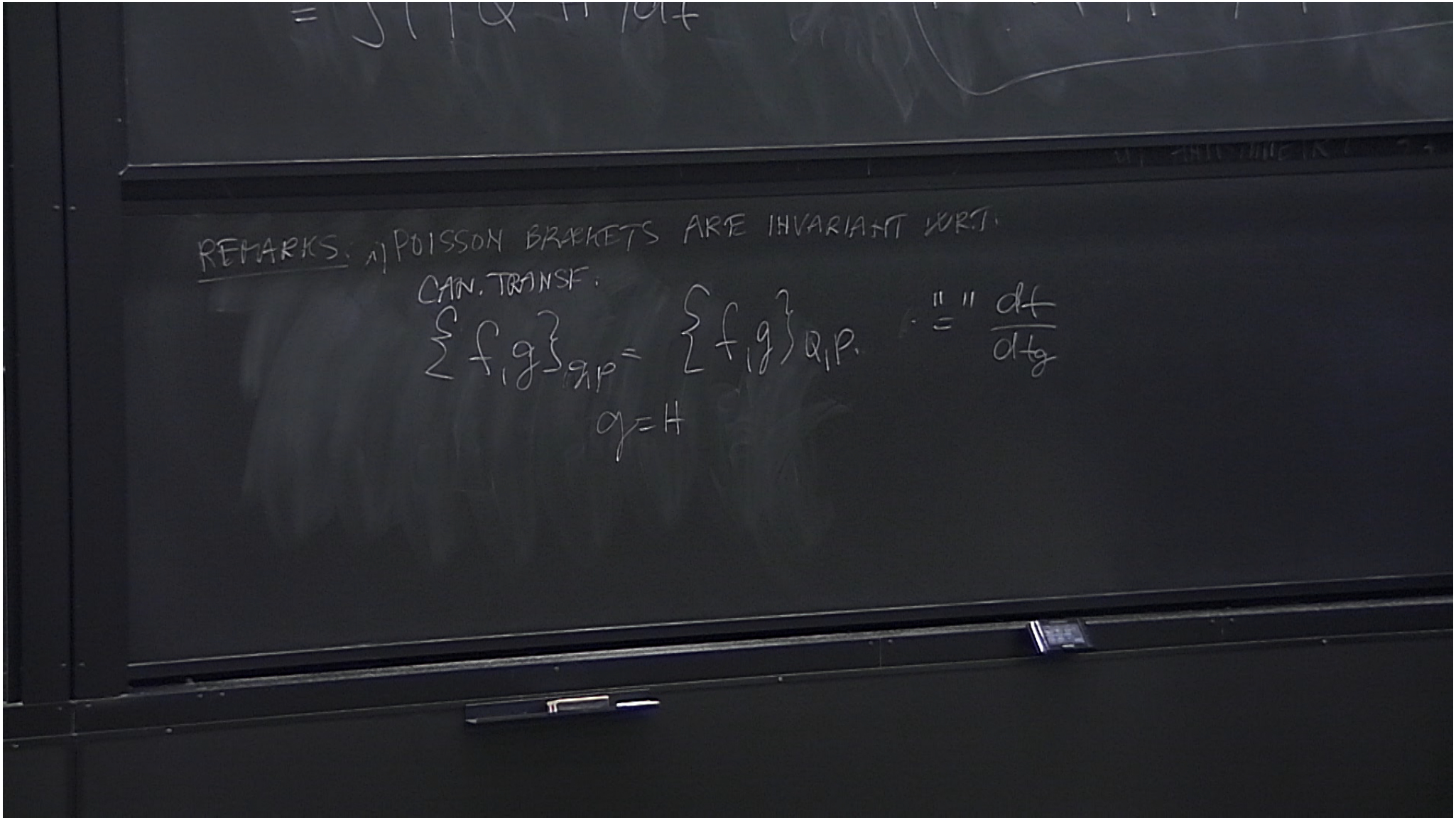
EX: CONSIDER $F(q, Q, t)$ ← FREEDOM.

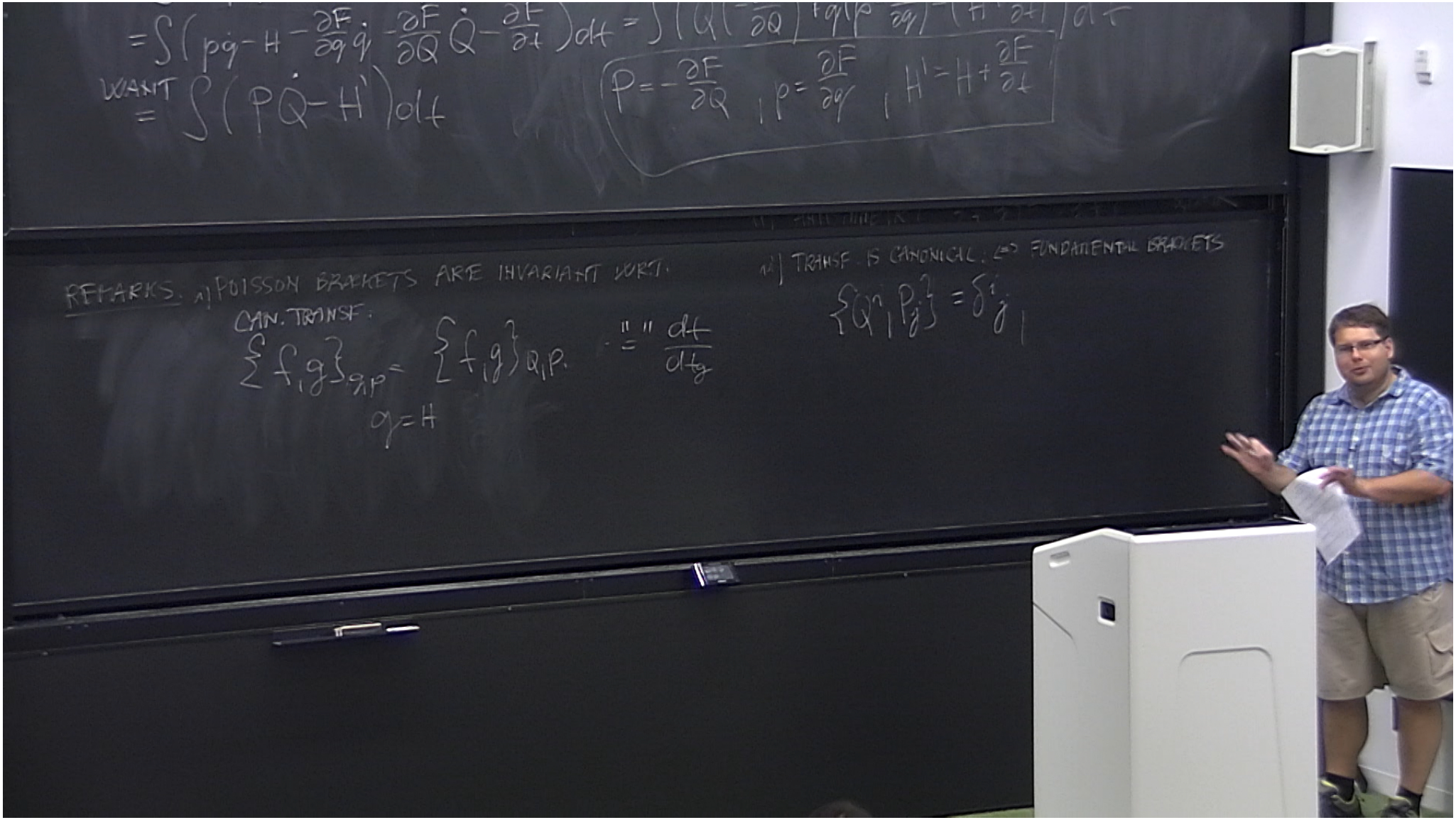
$$S = \int \left(p\dot{q} - H - \frac{dF}{dt} \right) dt$$

$$= \int \left(p\dot{q} - H - \frac{\partial F}{\partial q}\dot{q} - \frac{\partial F}{\partial Q}\dot{Q} - \frac{\partial F}{\partial t} \right) dt = \int \left(\dot{Q}\underbrace{\left(-\frac{\partial F}{\partial Q}\right)}_{P} + \dot{q}\underbrace{\left(p - \frac{\partial F}{\partial q}\right)}_{\text{0}} - \left(H + \frac{\partial F}{\partial t}\right) \right) dt$$

WANT $= \int \left(P\dot{Q} - H' \right) dt$

$$P = -\frac{\partial F}{\partial Q}, \quad p = \frac{\partial F}{\partial q}, \quad H' = H + \frac{\partial F}{\partial t}$$


REMARKS. 1) POISSON BRACKETS ARE INVARIANT W.R.T.
CAN. TRANSF.
$\{f,g\}_{q,p} = \{f,g\}_{Q,P}$ "=" $\frac{dt}{dt_g}$
$q_f = H$


$= \int \left(pq - H - \frac{\partial F}{\partial q}\dot{q} - \frac{\partial F}{\partial Q}\dot{Q} - \frac{\partial F}{\partial t} \right) dt$
WANT $= \int (P\dot{Q} - H')\, dt$
$P = -\frac{\partial F}{\partial Q}$, $p = \frac{\partial F}{\partial q}$, $H' = H + \frac{\partial F}{\partial t}$
REMARKS. a) POISSON BRACKETS ARE INVARIANT UNDER CAN. TRANSF:
$\{f,g\}_{q,p} = \{f,g\}_{Q,P}$
$q = H$
$= " \frac{df}{dt_g}$
b) TRANSF. IS CANONICAL $\Leftrightarrow$ FUNDAMENTAL BRACKETS
$\{Q^i, P_j\} = \delta^i_j$

THEIR CANONICAL POISSON BRACKET K

REMARKS: i) POISSON BRACKETS ARE INVARIANT WRT. CAN. TRANSF:

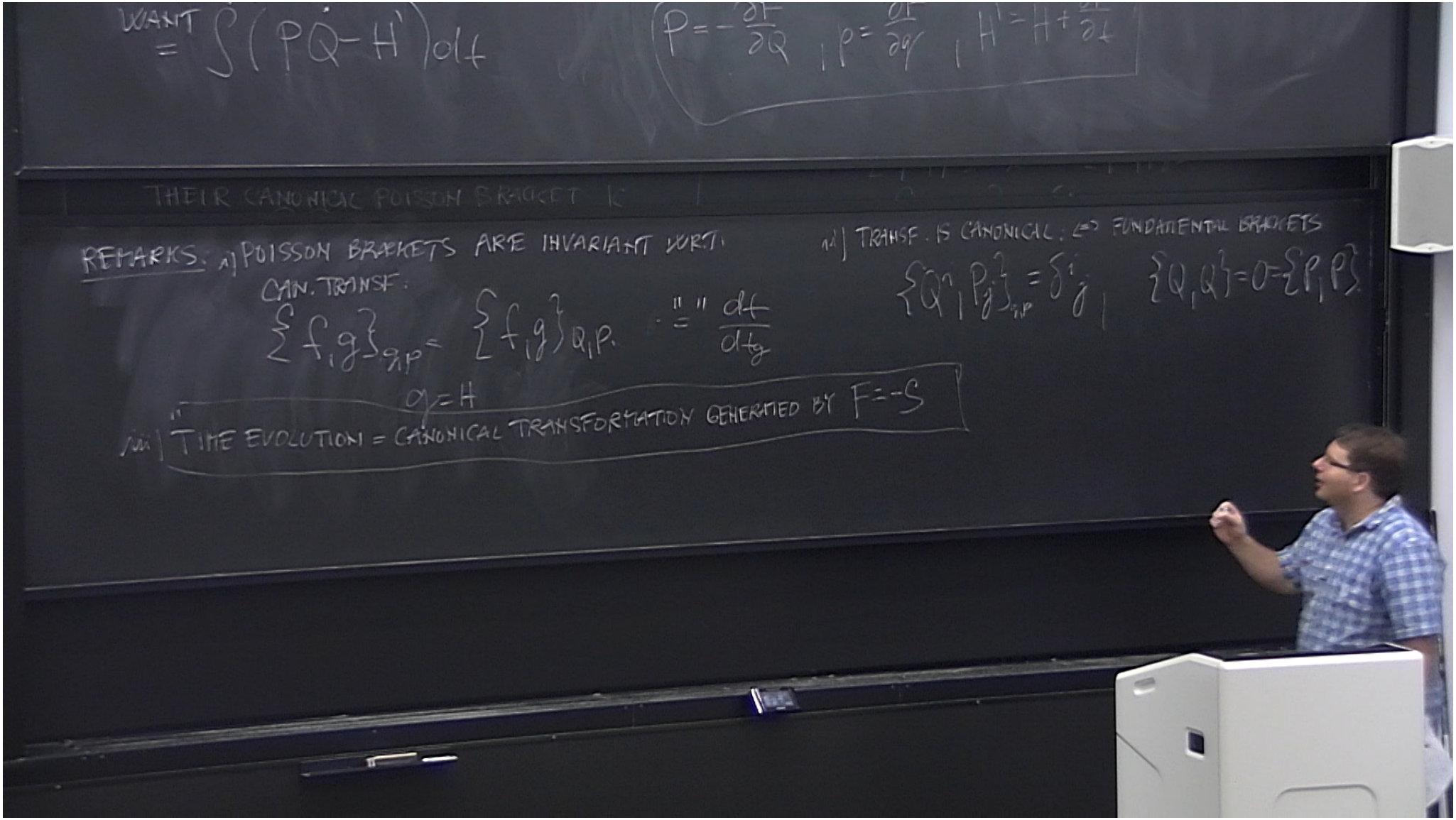
$$\{f,g\}_{q,p} = \{f,g\}_{Q,P}$$

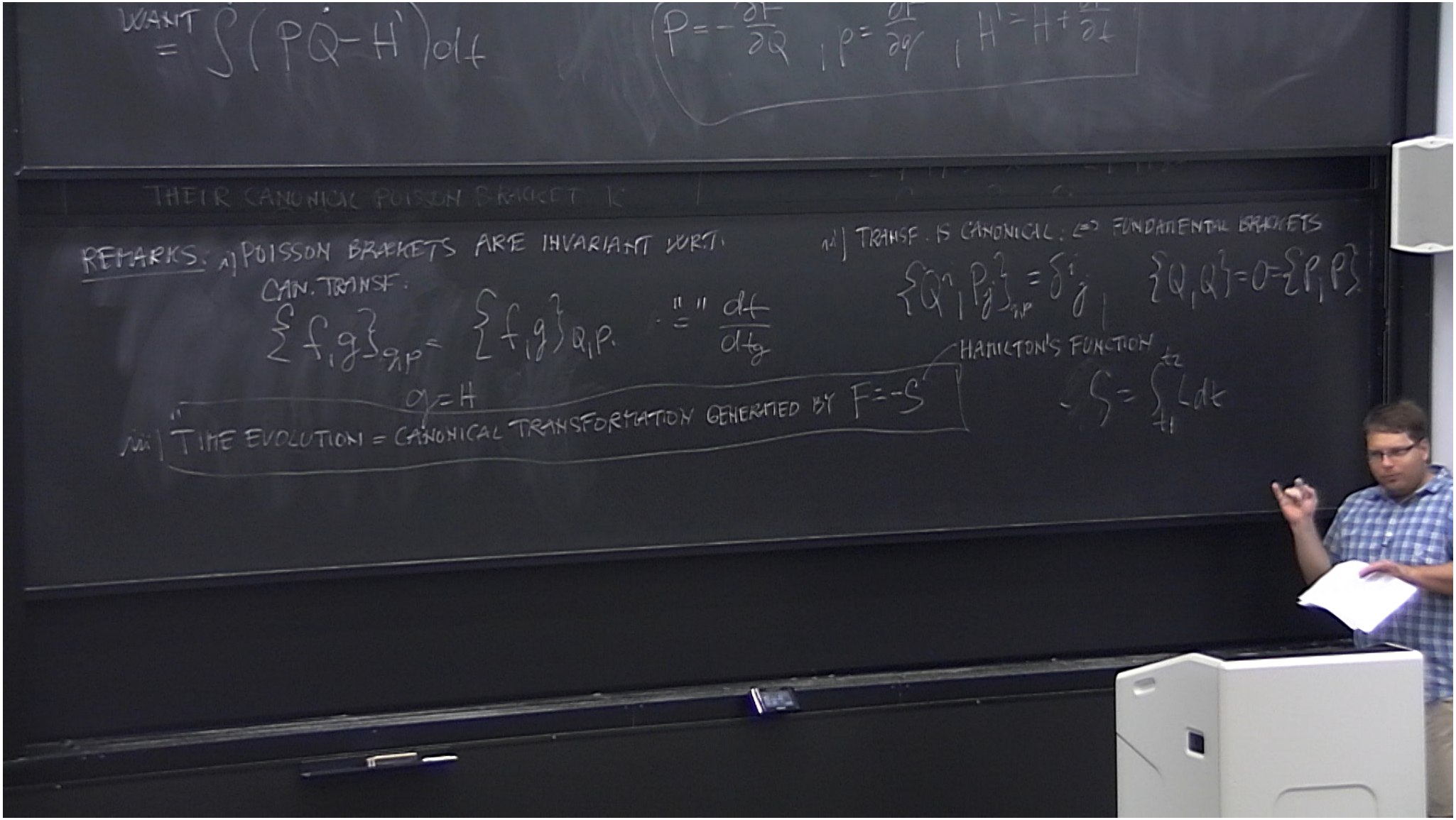
ii) "TIME EVOLUTION = CANONICAL TRANSFORMATION

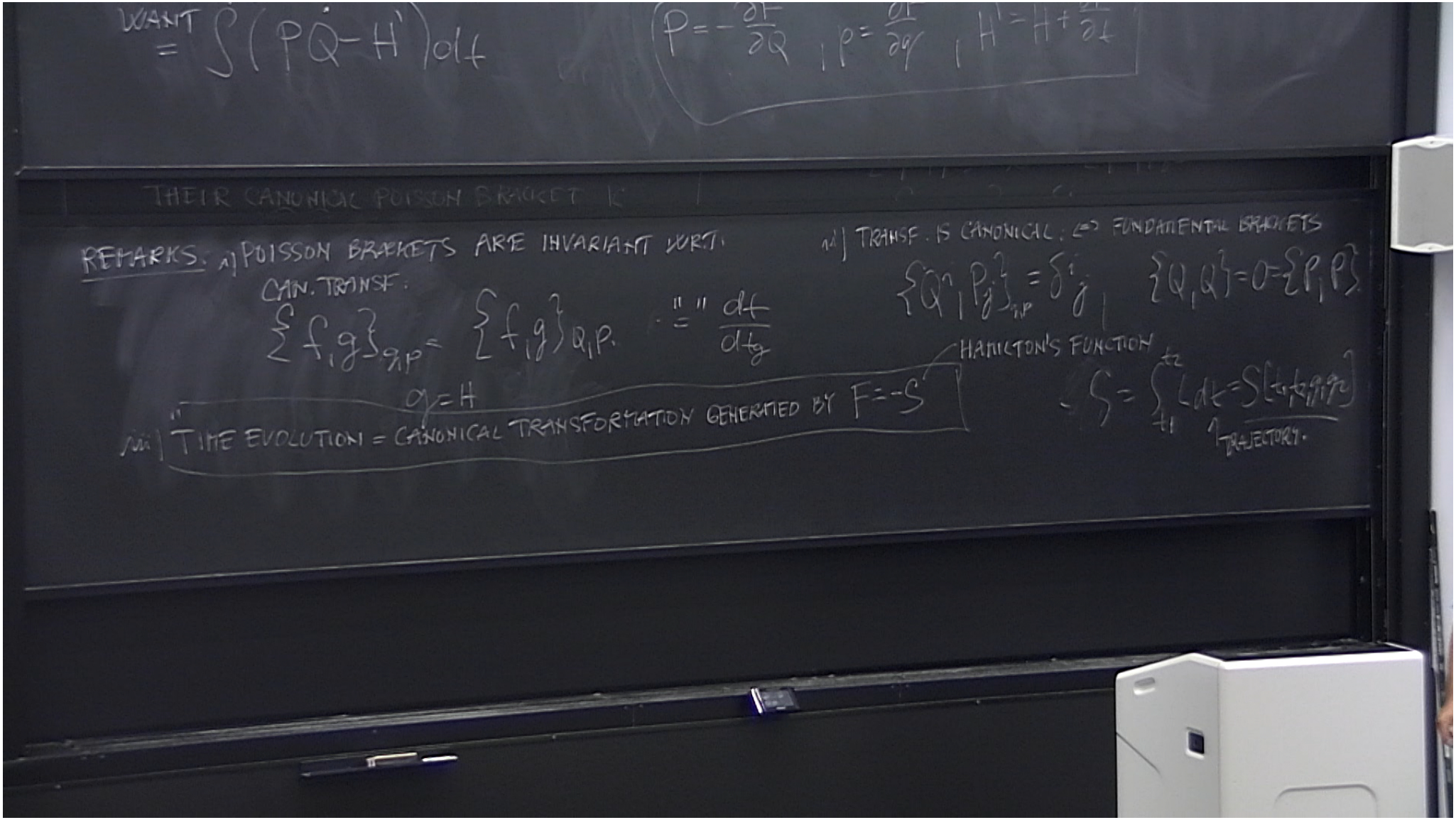
$$Q = H$$

$$\dot{f} = \{f,H\}_{q,p} = \{f,H\}_{Q,P} = \frac{df}{dt_Q}$$

iii) TRANSF.


WANT = $\int (PQ - H')\,dt$
$P = -\frac{\partial F}{\partial Q}$, $p = \frac{\partial F}{\partial q}$, $H' = H + \frac{\partial F}{\partial t}$
THEIR CANONICAL POISSON BRACKET IS
REMARKS: i) POISSON BRACKETS ARE INVARIANT WRT. CAN. TRANSF:
$\{f,g\}_{q,p} = \{f,g\}_{Q,P}$ "$= \frac{df}{dt_g}$"
$g = H$
ii) TRANSF. IS CANONICAL $\Leftrightarrow$ FUNDAMENTAL BRACKETS
$\{Q^i, P_j\}_{q,p} = \delta^i_j$, $\{Q,Q\} = 0 = \{P,P\}$
iii) "TIME EVOLUTION = CANONICAL TRANSFORMATION GENERATED BY $F = -S$"


WANT
$= \int (P\dot{Q} - H')\,dt$
$P = -\frac{\partial F}{\partial Q}$, $p = \frac{\partial F}{\partial q}$, $H' = H + \frac{\partial F}{\partial t}$
THEIR CANONICAL POISSON BRACKET IS
REMARKS: i) POISSON BRACKETS ARE INVARIANT UNDER CAN. TRANSF:
$\{f,g\}_{q,p} = \{f,g\}_{Q,P}$ "$=$" $\frac{df}{dt_g}$
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iii) "TIME EVOLUTION = CANONICAL TRANSFORMATION GENERATED BY $F = -S$"
HAMILTON'S FUNCTION
$S = \int_{t_1}^{t_2} L\,dt$


WANT = $\int (P\dot{Q} - H')\,dt$
$P = -\frac{\partial F}{\partial Q}$, $p = \frac{\partial F}{\partial q}$, $H' = H + \frac{\partial F}{\partial t}$
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REMARKS: i) POISSON BRACKETS ARE INVARIANT WRT. CAN. TRANSF.:
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iii) "TIME EVOLUTION = CANONICAL TRANSFORMATION GENERATED BY $F = -S$"
HAMILTON'S FUNCTION
$S = \int_{t_1}^{t_2} L\,dt = S[t_2, t_1, q_2, q_1]$
TRAJECTORY.