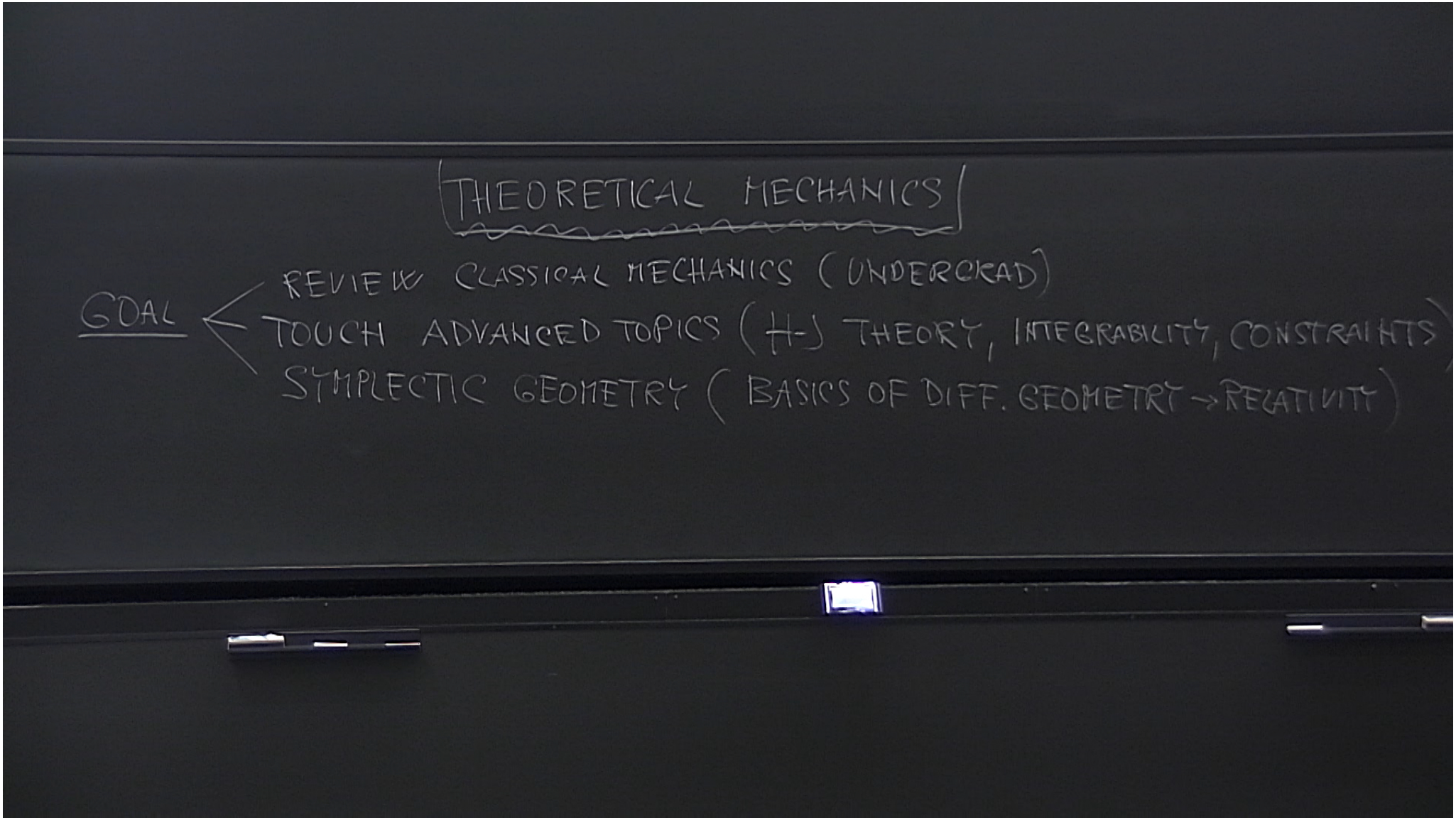
Title: PSI 17/18 Front End - Theoretical Mechanics (Kubiznak)

Date: Aug 09, 2017 09:00 AM

URL: http://pirsa.org/17080042

Abstract:

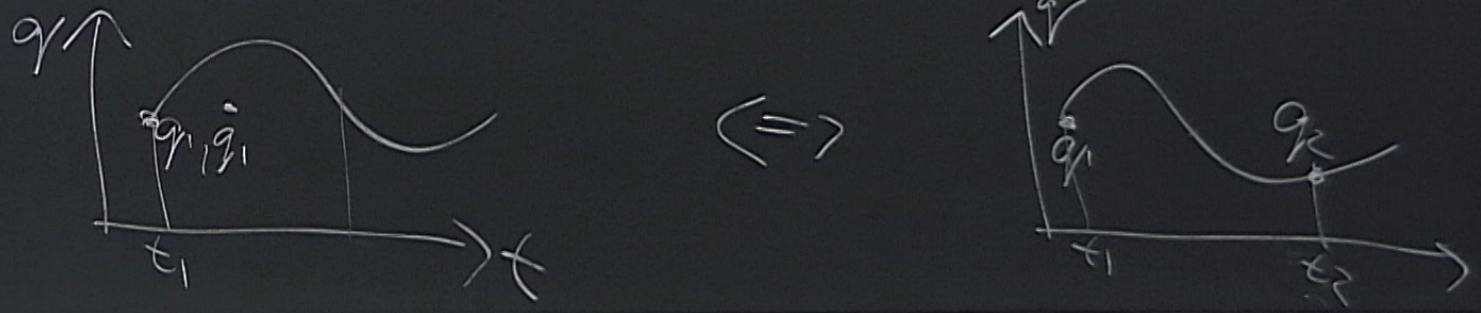

THEORETICAL MECHANICS
GOAL
REVIEW CLASSICAL MECHANICS (UNDERGRAD)
TOUCH ADVANCED TOPICS (H-J THEORY, INTEGRABILITY, CONSTRAINTS)
SYMPLECTIC GEOMETRY (BASICS OF DIFF. GEOMETRY → RELATIVITY)

1) LAGRANGIAN MECHANICS

a) HAMILTON'S PRINCIPLE OF LEAST ACTION

MOTION: GENERALIZED COORDINATE $q(t)$

"INITIAL VALUE PROBLEM" $\Leftrightarrow$ "BOUNDARY VALUE PROBLEM"



PRINCIPLE OF [illegible] (HAMILTON)

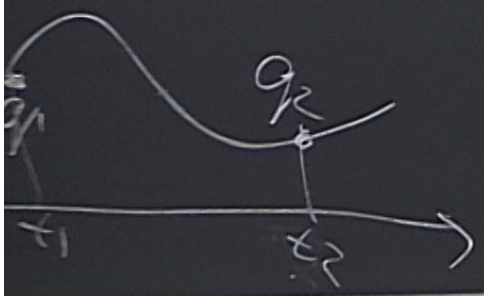
MOTIONS OF MECHANICAL SYSTEM IN $t \in (t_1, t_2)$ ARE GIVEN BY EXTREMALS OF THE ACTION FUNCTIONAL

$$S = S[q(t)] = \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt$$

L .. LAGRANGIAN .. DETERMINES DYNAMICAL SYSTEM

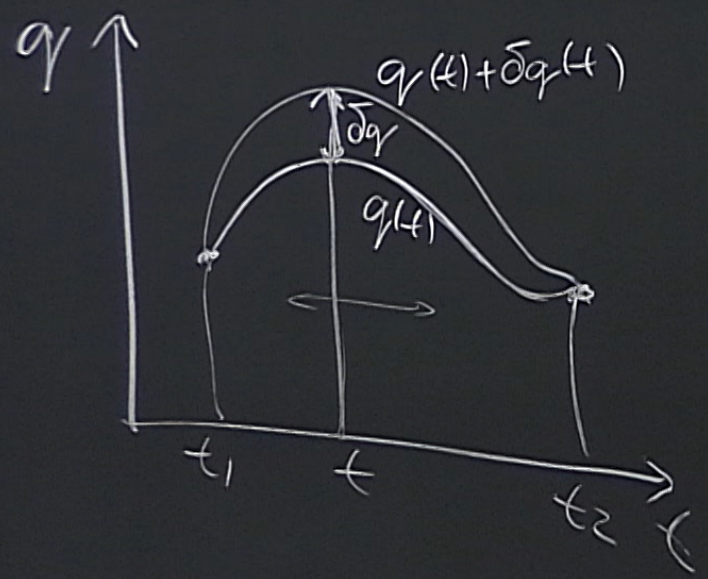
$q(t)$

DARY VALUE PROBLEM"



- TO DERIVE EOM : CONSIDER "FIXED POINTS"

$$\delta q(t_1) = 0 = \delta q(t_2)$$

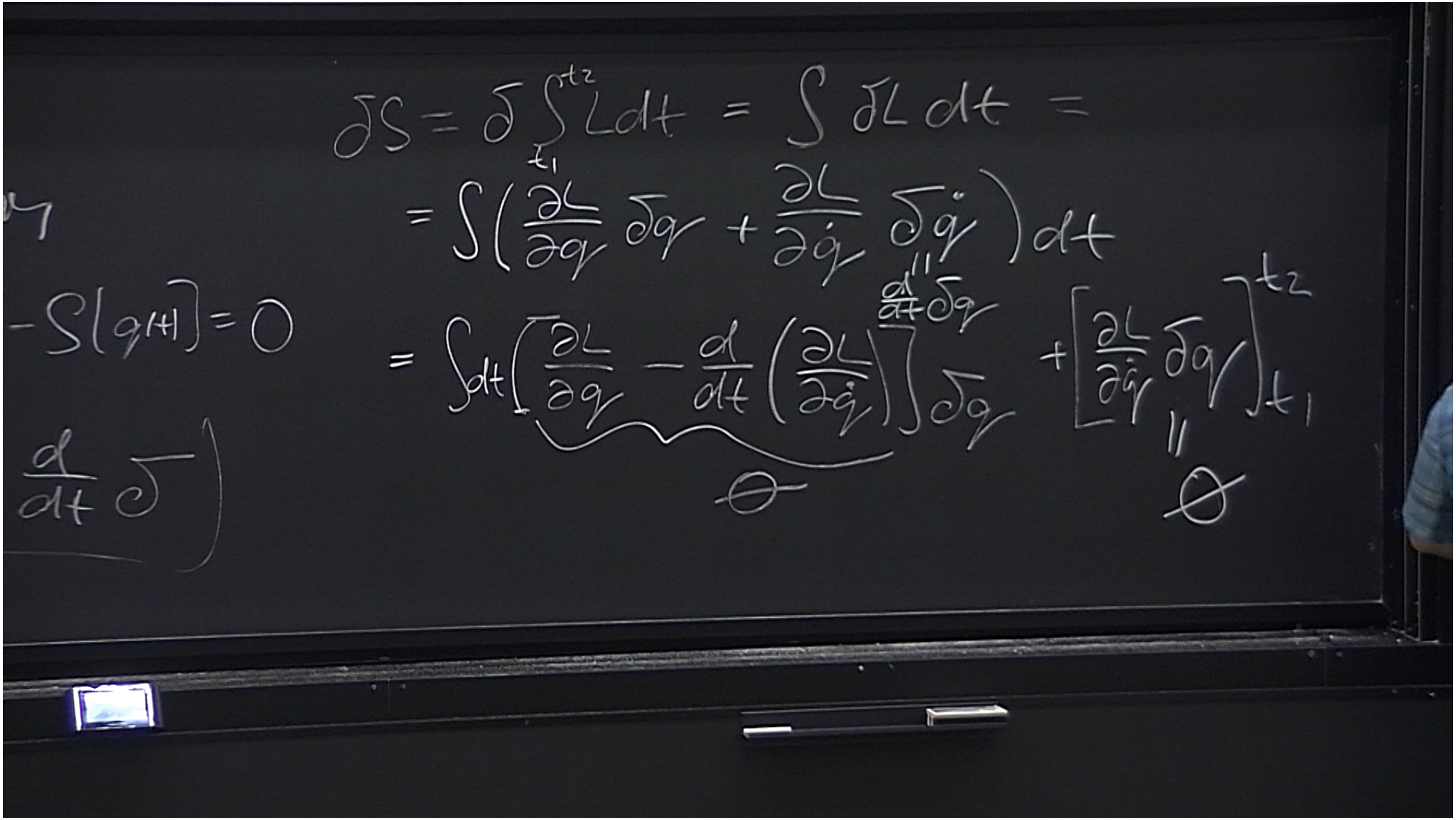


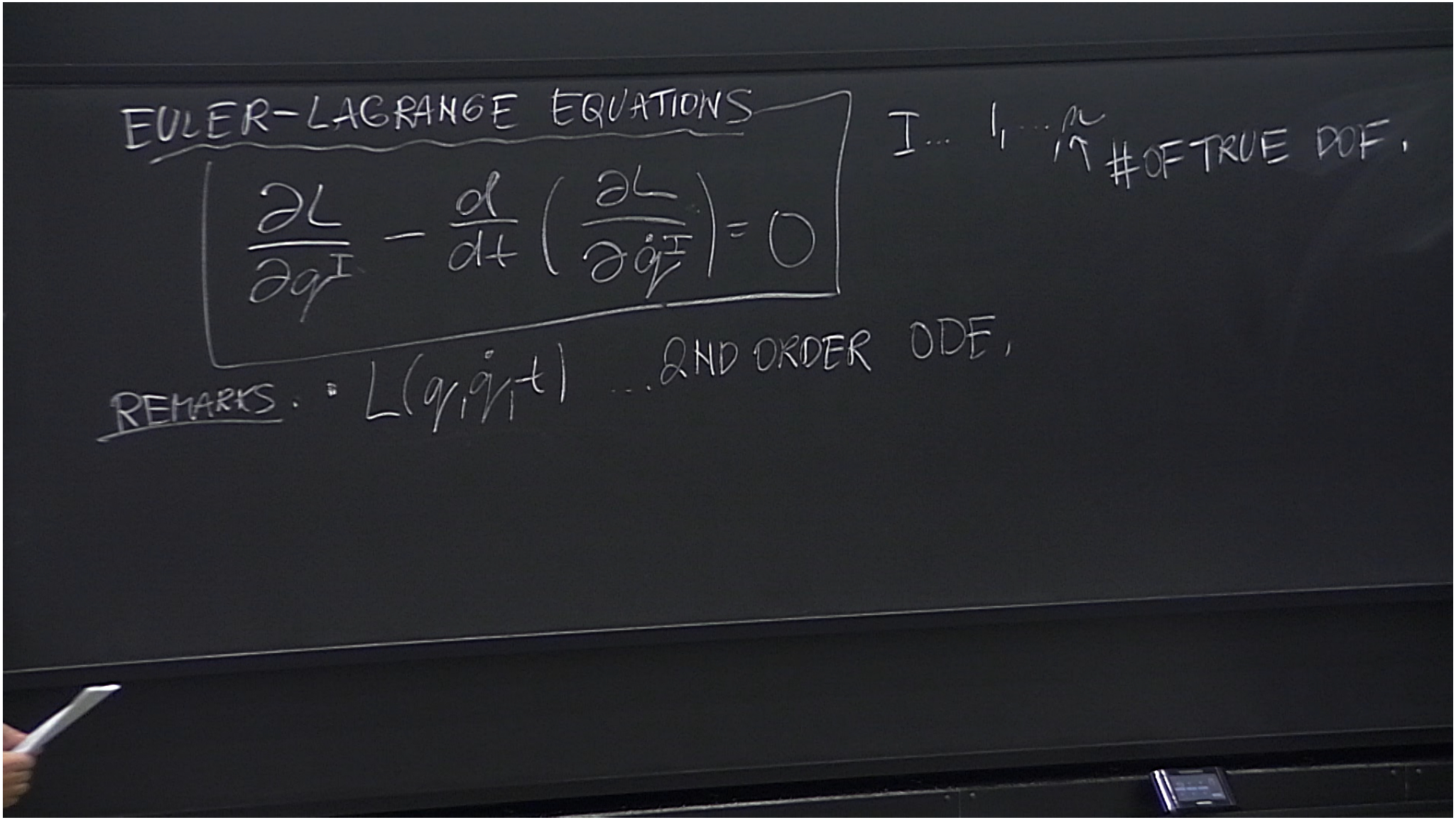
WE SEEK EXTREMUM

$$\delta S = S[q(t)+\delta q(t)] - S[q(t)] = 0$$

WE HAVE IDENTITY

$$\boxed{\delta \frac{d}{dt} = \frac{d}{dt}\delta}$$


$\delta S = \delta \int_{t_1}^{t_2} L\,dt = \int \delta L\,dt =$
$= \int \left(\frac{\partial L}{\partial q} \delta q + \frac{\partial L}{\partial \dot{q}} \delta \dot{q} \right) dt$
$= \int dt \left[\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}} \right) \right] \delta q + \left[\frac{\partial L}{\partial \dot{q}} \delta q \right]_{t_1}^{t_2}$
$\frac{d}{dt}\delta q$
δq
$= 0$
$= 0$
$-S[q+\epsilon] = 0$
$\frac{d}{dt}\delta$


EULER-LAGRANGE EQUATIONS
$\frac{\partial L}{\partial q^I} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^I}\right) = 0$
I ... 1, ... n
↑ # OF TRUE DOF.
REMARKS. • $L(q, \dot{q}, t)$ ... 2ND ORDER ODE,

I... $l_1 \dots \tilde{l}$ # OF TRUE DOF.

• L IS NOT UNIQUE: $L'(q,\dot{q},t) = L(q,\dot{q},t) + \frac{d\Lambda(q,t)}{dt}$

GIVES SAME EOM.

R ODE, PROOF. $S' = \int L' dt = \int L dt + \int \frac{d\Lambda}{dt} dt = S + [\Lambda]_{t_1}^{t_2}$

$\delta S' = \delta S + \left[\frac{\partial \Lambda}{\partial q} \delta q\right]_{t_1}^{t_2}$

0.. FIXED ENDPOINTS

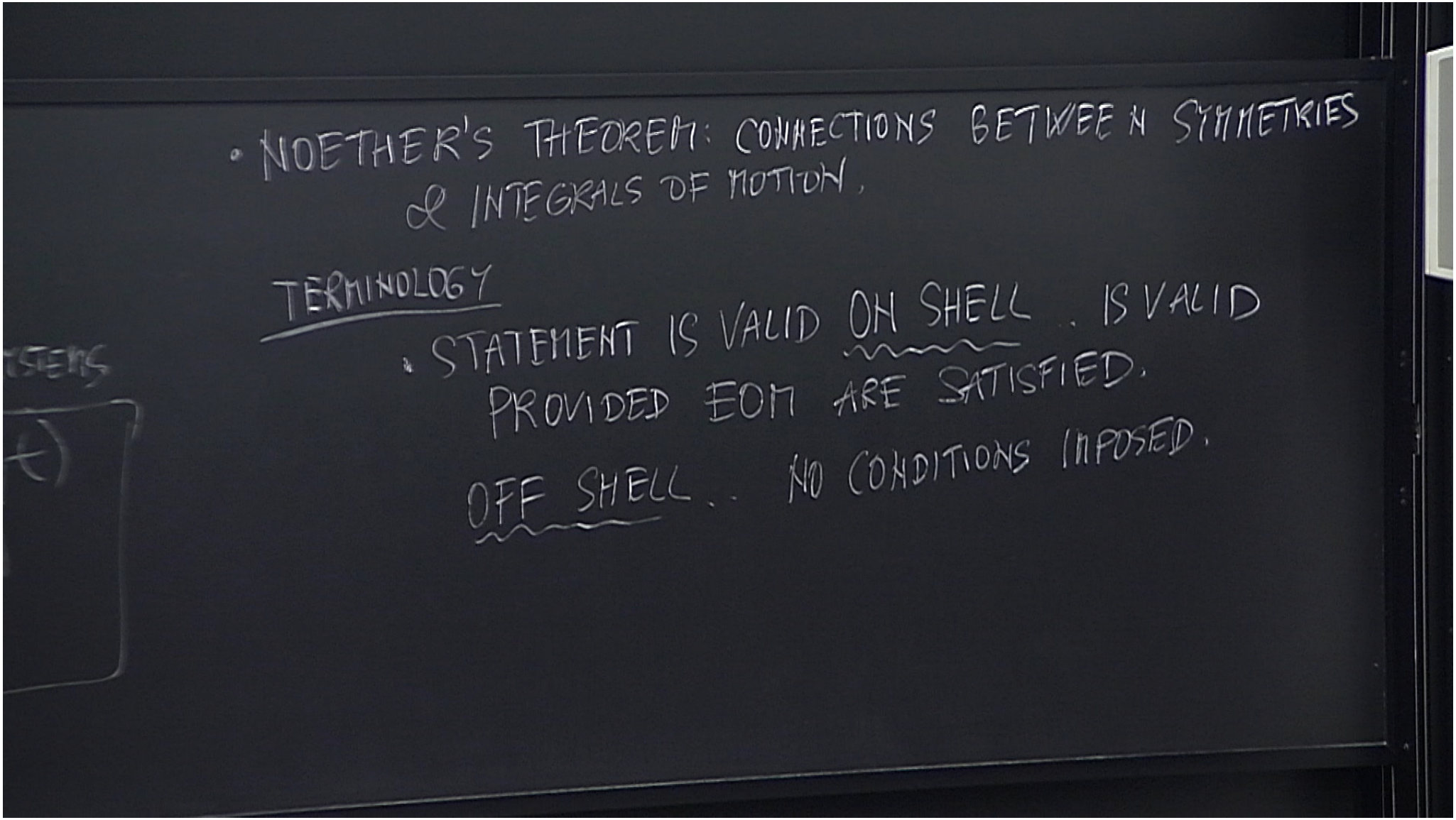
TIAL

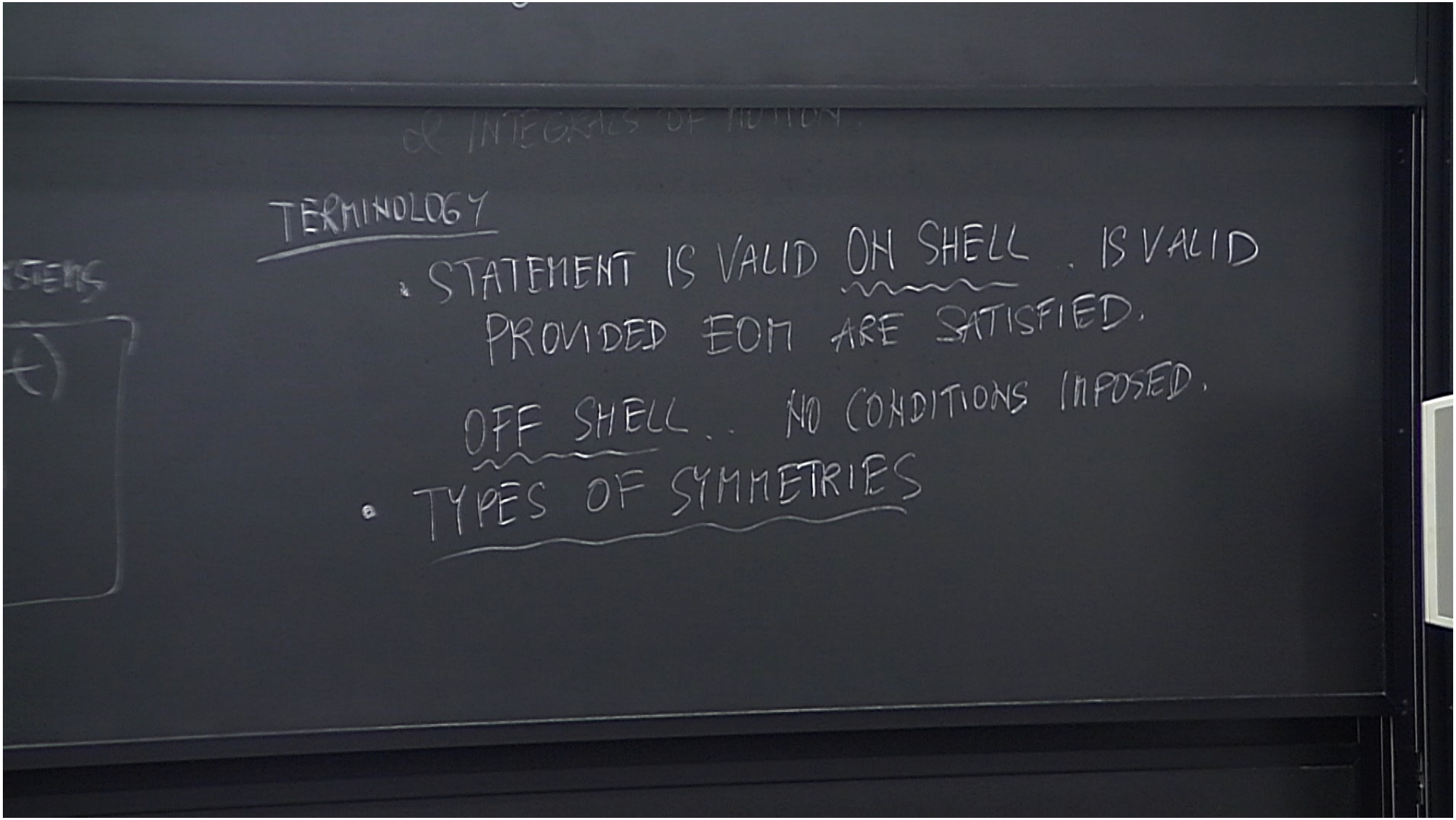
b) INTEGRALS OF MOTION

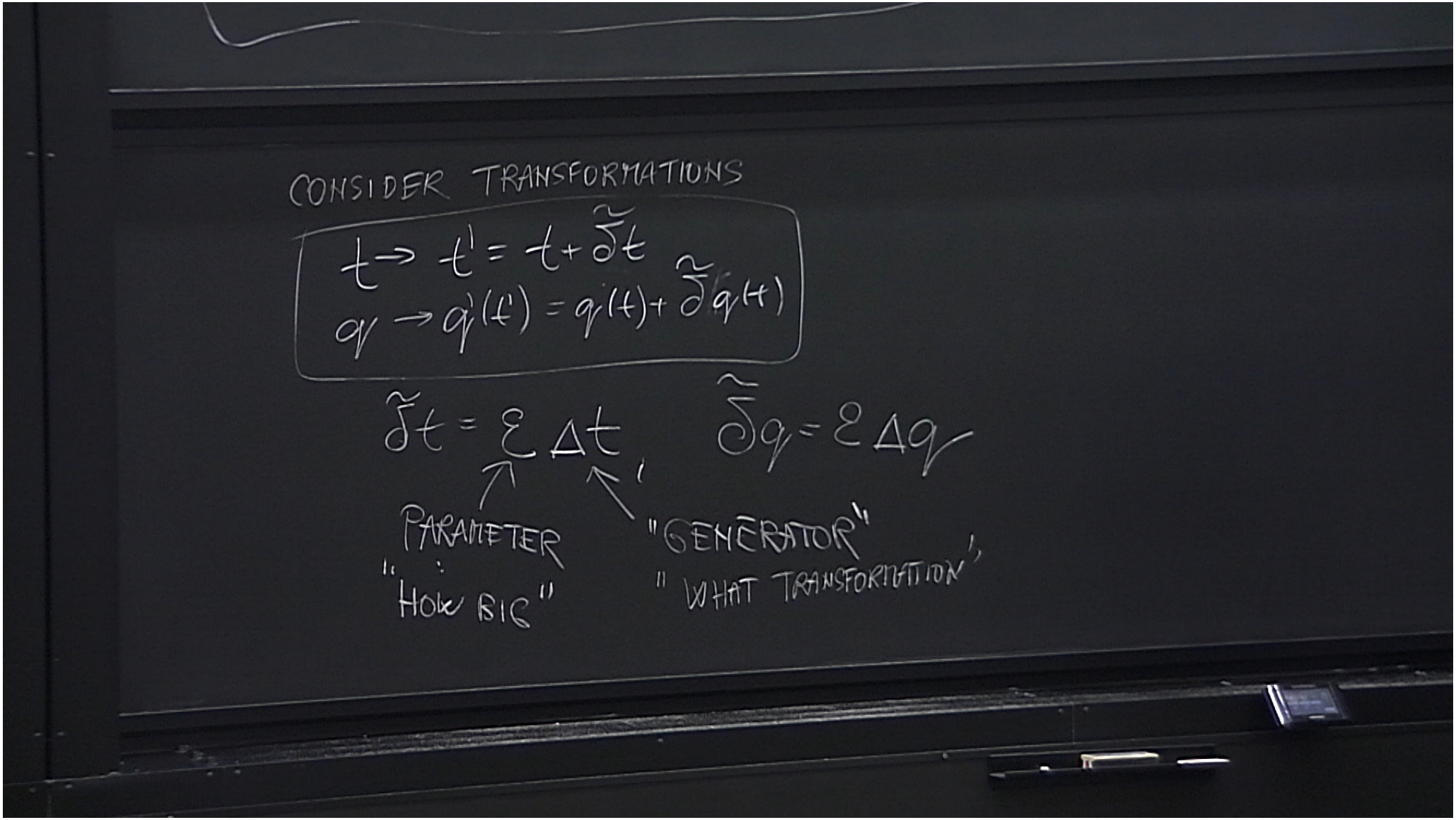
SOLVING (E-L) <
- NUMERICALLY
- PERTURBATIVELY
- ANALYTICALLY – INTEGRABLE SYSTEMS

DEF: INTEGRAL (CONSTANT) OF MOTION IS $I = I(q, \dot{q}, t)$ SUCH THAT $\frac{dI}{dt} = 0$ FOR ANY $q(t)$

SOLVING EOM.

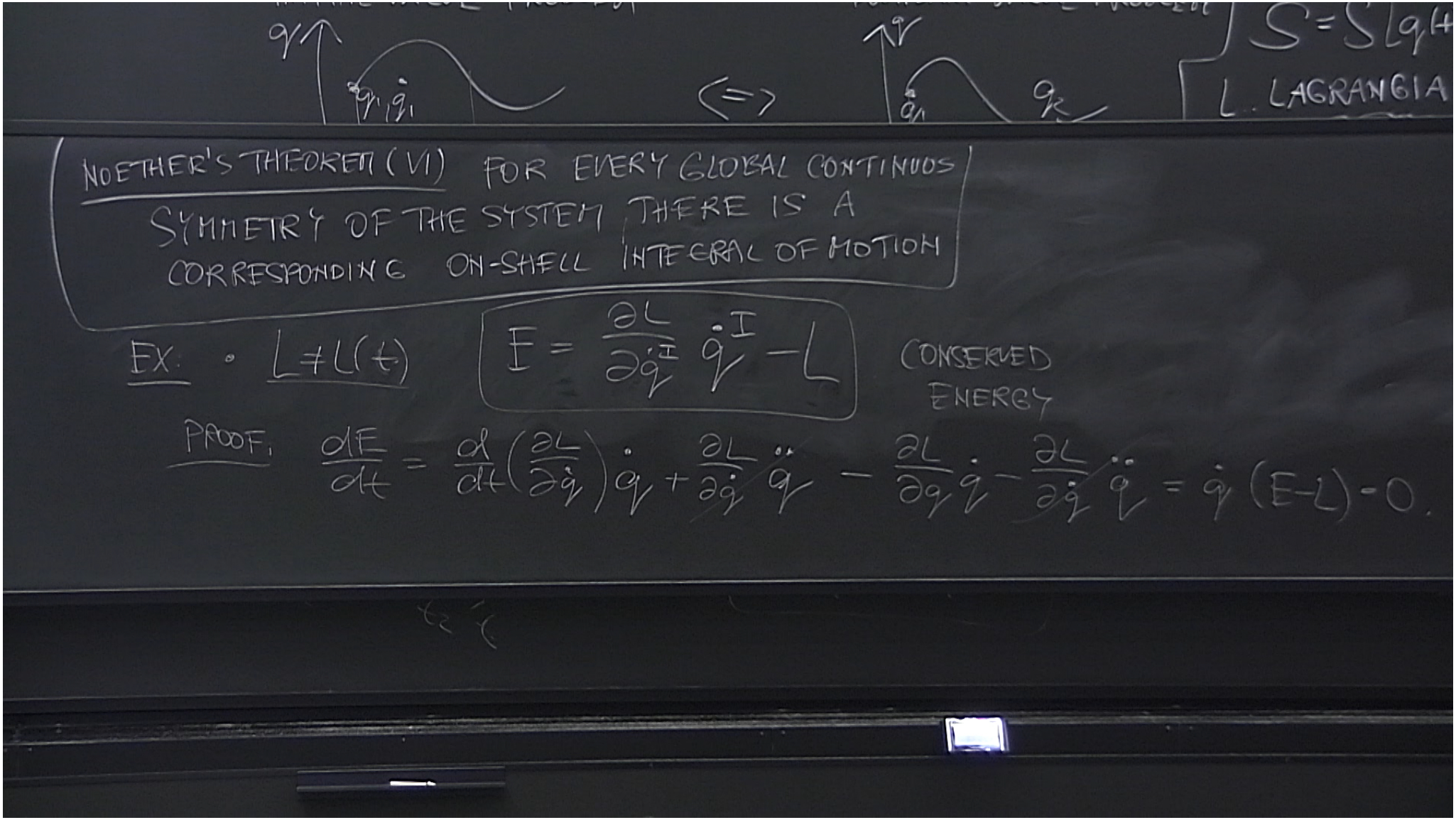

• NOETHER'S THEOREM: CONNECTIONS BETWEEN SYMMETRIES & INTEGRALS OF MOTION.
TERMINOLOGY
• STATEMENT IS VALID ON SHELL. IS VALID PROVIDED EOM ARE SATISFIED.
OFF SHELL.. NO CONDITIONS IMPOSED.


TERMINOLOGY
• STATEMENT IS VALID ON SHELL . IS VALID
PROVIDED EOM ARE SATISFIED.
OFF SHELL .. NO CONDITIONS IMPOSED.
• TYPES OF SYMMETRIES


CONSIDER TRANSFORMATIONS
$t \to t' = t + \tilde{\delta} t$
$q_r \to q_r'(t') = q_r(t) + \tilde{\delta} q_r(t)$
$\tilde{\delta} t = \varepsilon \Delta t$, $\tilde{\delta} q = \varepsilon \Delta q_r$
PARAMETER
"HOW BIG"
"GENERATOR"
"WHAT TRANSFORMATION"

SYMMETRIES
- DISCRETE — 90° … ε DISCRETE … a (NO CONSERVED QUANTITIES)
- CONTINUOUS … ε ARBITRARILY SMALL

SYMMETRIES
- GLOBAL … ε = CONST … 1ST NOETHER'S THEOREM … CONSERVED QUANTITIES
- LOCAL (GAUGE) … ε = ε(t) … 2ND — " — … BIANCHI IDENTITIES

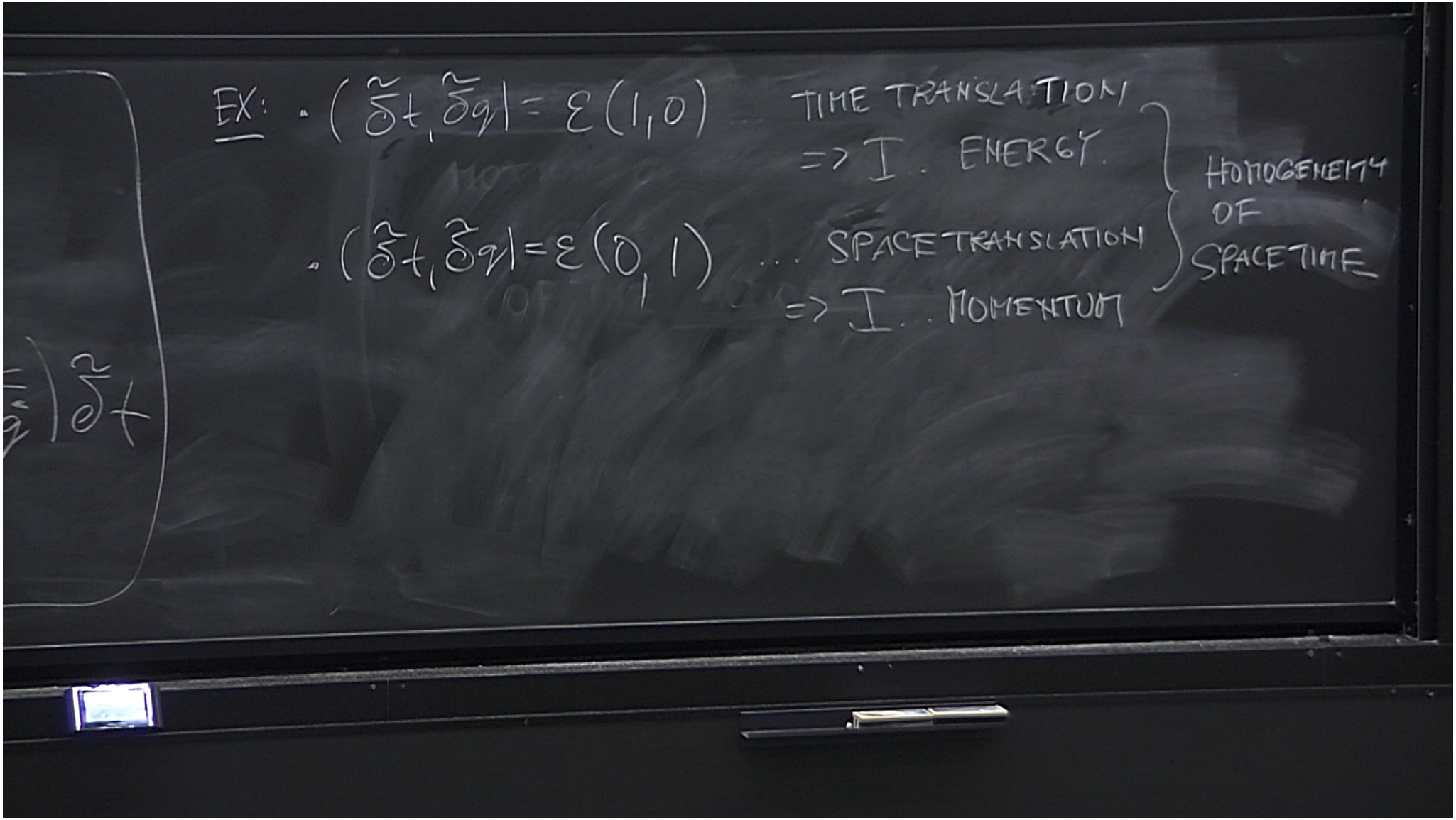

q_1
$q_1, \dot{q}_1$
$\Leftrightarrow$
$\dot{q}_2$
q_2
$S = S[q_i]$
L. LAGRANGIA
NOETHER'S THEOREM (VI) FOR EVERY GLOBAL CONTINUOS SYMMETRY OF THE SYSTEM THERE IS A CORRESPONDING ON-SHELL INTEGRAL OF MOTION
EX. • $L \neq L(t)$
$E = \frac{\partial L}{\partial \dot{q}^I} \dot{q}^I - L$
CONSERVED ENERGY
PROOF. $\frac{dE}{dt} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\dot{q} + \frac{\partial L}{\partial \dot{q}}\ddot{q} - \frac{\partial L}{\partial q}\dot{q} - \frac{\partial L}{\partial \dot{q}}\ddot{q} = \dot{q}(E-L) - 0.$

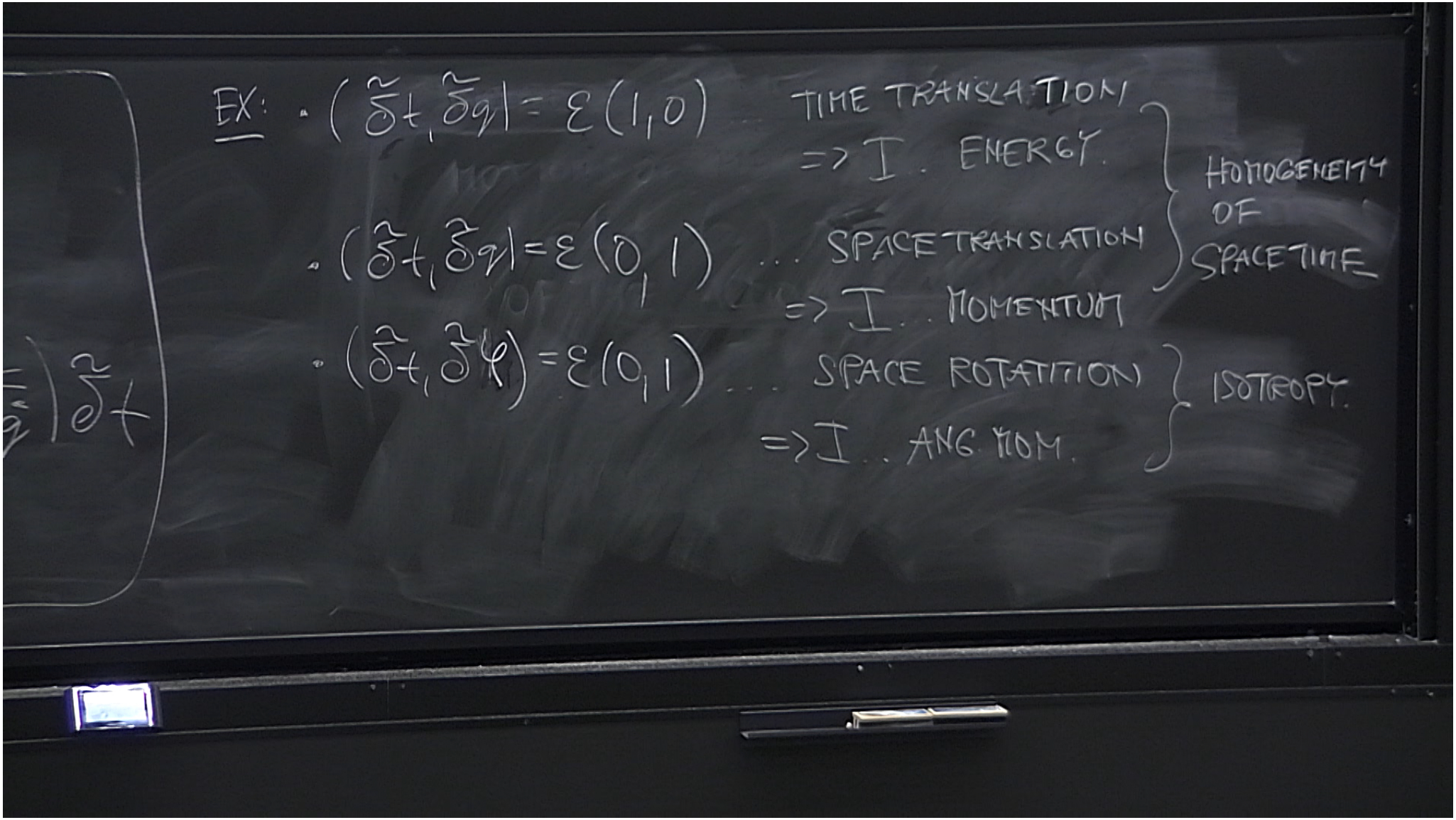
NOETHER'S THEOREM (V2 - EXPLICIT FORMULA)

LET $\tilde{\delta}$ BE A GLOBAL CONT. SYMMETRY, THAT IS OFF-SHELL WE FIND $\tilde{\delta}t, \tilde{\delta}q$ SUCH THAT

$$\tilde{\delta}S = 0 \Rightarrow I = \frac{\partial L}{\partial \dot{q}}\tilde{\delta}q + \left(L - \dot{q}\frac{\partial L}{\partial \dot{q}}\right)\tilde{\delta}t$$

IS AN ON-SHELL INTEGRAL OF MOTION


EX:
TIME TRANSLATION
=> I. ENERGY
SPACE TRANSLATION
=> I. MOMENTUM
HOMOGENEITY OF SPACETIME


EX:
TIME TRANSLATION
=> I. ENERGY
SPACE TRANSLATION
=> I. MOMENTUM
SPACE ROTATION
=> I. ANG MOM.
HOMOGENEITY OF SPACETIME
ISOTROPY

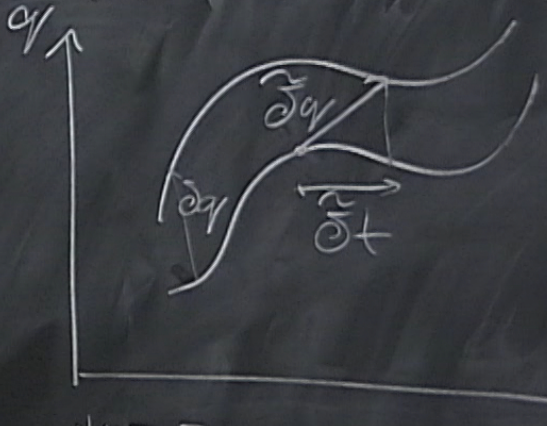
PROOF: - DISTINGUISH δ & $\bar{\delta}$

EOM

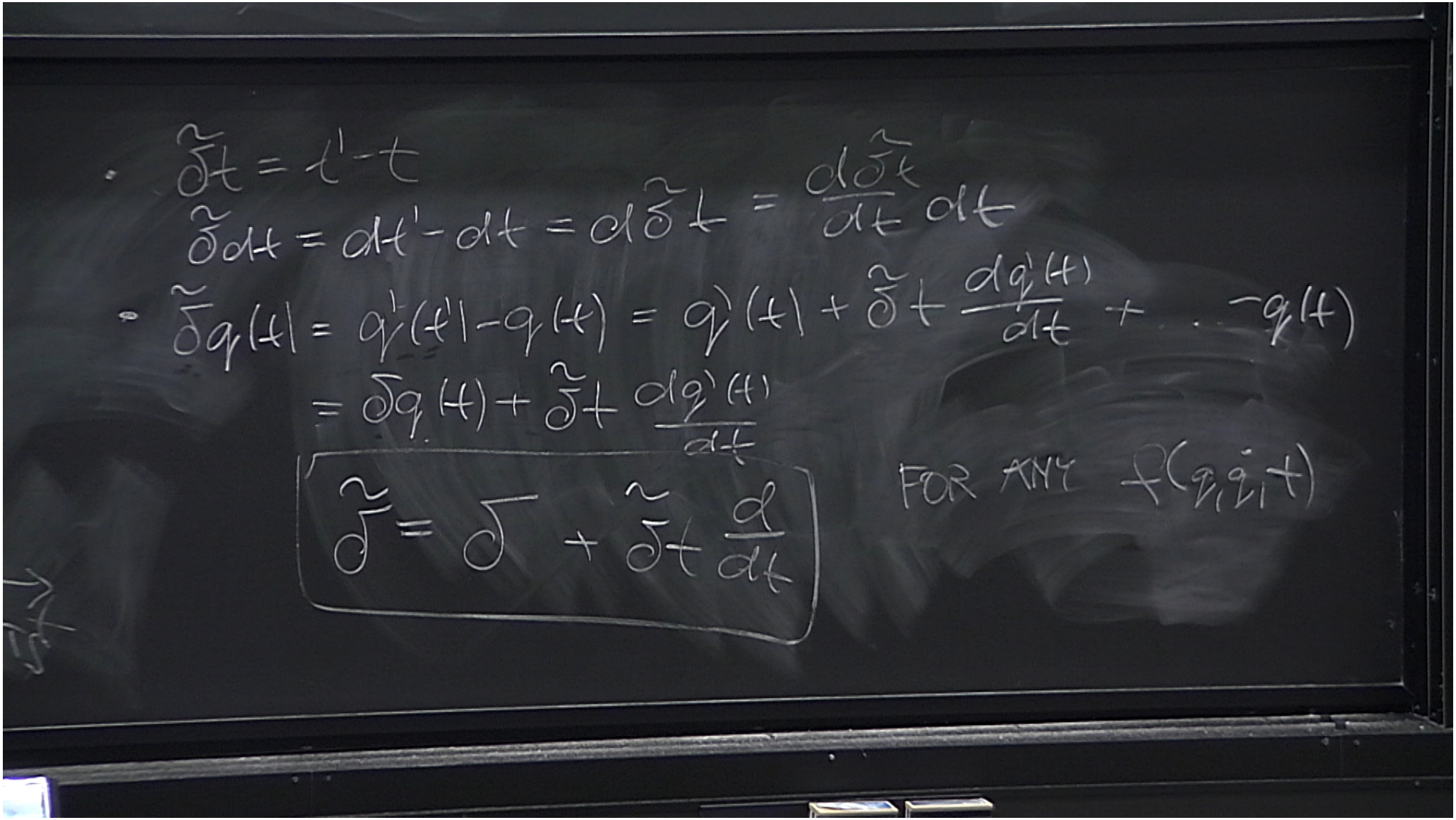


FIXED END POINTS

NOETHER



NOT FIXED END POINTS


$\tilde{\delta}t = t' - t$
$\tilde{\delta}dt = dt' - dt = d\tilde{\delta}t = \frac{d\tilde{\delta}t}{dt} dt$
$\tilde{\delta}q(t) = q'(t') - q(t) = q'(t) + \tilde{\delta}t \frac{dq'(t)}{dt} + \ldots - q(t)$
$= \delta q(t) + \tilde{\delta}t \frac{dq'(t)}{dt}$
$\tilde{\delta} = \delta + \tilde{\delta}t \frac{d}{dt}$
FOR ANY $f(q, \dot{q}, t)$