Title: Bounding and building fault-tolerant quantum circuits on stabilizer codes

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Abstract: <p>Transversality is one of the most desirable features of fault-tolerant circuits because it automatically limits the propagation of errors. However, it was shown by Eastin & Knill that no universal set of quantum gates on any quantum code is transversal. In this talk, we strengthen this result for stabilizer codes to say that transversal gates must in fact be contained in the Clifford hierarchy. Moreover, we present new circuits on Bacon-Shor codes that saturate our bounds. In particular, we show how a k-qubit controlled-Z gate can be implemented by a transversal circuit on m-by-m^k Bacon-Shor codes and provide some estimates of its performance in terms of pseudo-thresholds and resource overhead.</p>

Building and bounding fault-tolerant quantum circuits on stabilizer codes

Ted Yoder

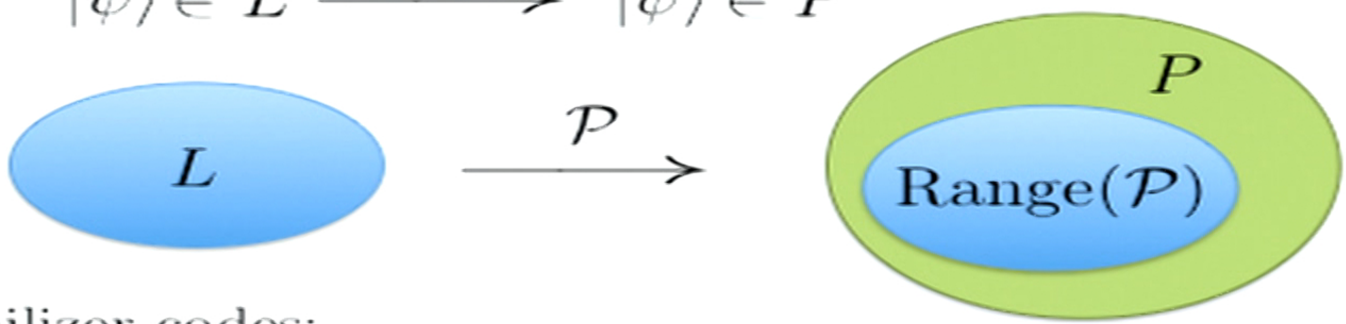
arXiv:1705.01686

+ Jochym-O'Connor, Kubica, TJY, in prep.

Protecting quantum states

Encoding states

$$|\psi\rangle \in L \xrightarrow{\mathcal{P}} |\bar{\psi}\rangle \in P$$



Stabilizer codes:

$$W|\bar{0}\rangle = |\bar{0}\rangle, \forall W \in \mathcal{S}, W = \bar{Z}$$

$$W|\bar{1}\rangle = |\bar{1}\rangle, \forall W \in \mathcal{S}, W = -\bar{Z}$$

$$\bar{X}|\bar{0}\rangle = |\bar{1}\rangle$$

e.g. $$\mathcal{S} = \left\langle \begin{matrix} ZZXIX \\ XZZXI \\ IXZZX \\ XIXZZ \end{matrix} \right\rangle$$

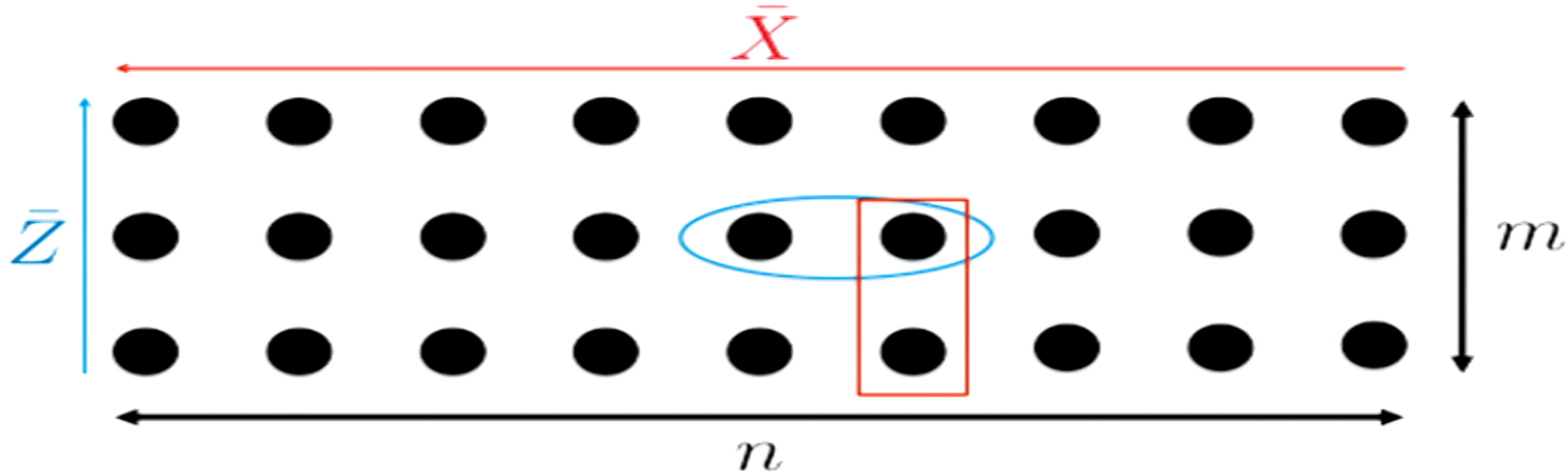
$$\bar{Z} = ZZZZZ$$

$$\bar{X} = XXXXX$$

Bacon-Shor codes:

Subsystem codes on qubits arranged in an $m \times n$ lattice in 2D

$\mathcal{G} = \langle Z_{i,j} Z_{i,j+1}, X_{i,j} X_{i+1,j} \rangle$ $\qquad d = \min(m, n)$

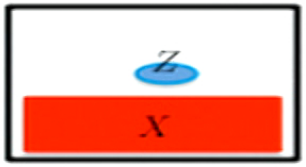


- Important gauges:

  X-gauge: take X-type gauge operators as stabilizers

  ⇒ Z-gauge: take Z-type gauge operators as stabilizers ⇐

  Surface code: take a checkerboard of X-gauge & Z-gauge pairs as stabilizers



- Aliferis switch: go from surface to Bacon-Shor by measuring every other row

Protecting quantum states

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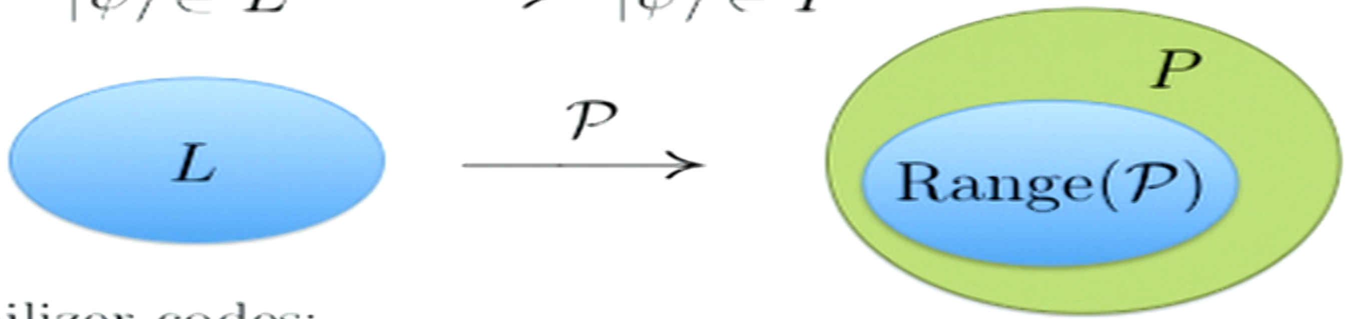
$$|\psi\rangle \in L \xrightarrow{\mathcal{P}} |\bar{\psi}\rangle \in P$$



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$$W|\bar{1}\rangle = |\bar{1}\rangle, \forall W \in \mathcal{S}, W = -\bar{Z}$$

Encoding operators

$$U \xrightarrow{\mathcal{P}} \bar{U}$$

Universal gate sets: $\{H, T, \mathrm{CX}\}$

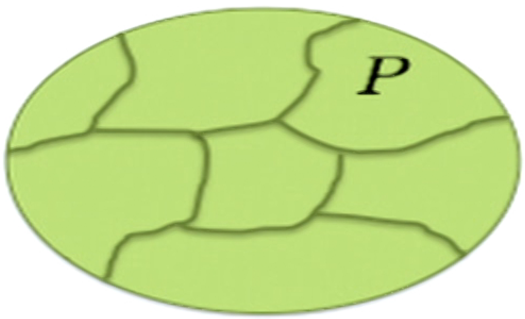
$\{H, \mathrm{CCZ}\}$

Q: what encoded operators are fault-tolerant?

What is fault-tolerance?

Sufficient criteria: "transversality" of $\bar{U}$

1. A decomposition of P into correctable pieces



$$P = P_1 \otimes P_2 \otimes \cdots \otimes P_n$$

distance d

=

"can recover from $d-1$ missing pieces"

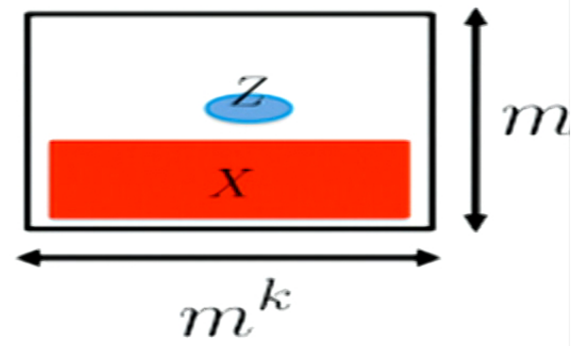
2. A circuit for $\bar{U}$ has the same decomposition

$$\bar{U} = \bar{U}_1 \otimes \bar{U}_2 \otimes \cdots \otimes \bar{U}_n$$

(Most common decomposition: qubits)

Main result: A depth-1 (i.e. transversal) circuit for C^kZ on $m \times m^k$ Z-gauge Bacon-Shor codes

Lemma 1: Round-robin logical gates
arXiv:1603.03948



Corollary 1: A very efficient scheme for fault-tolerant computing at low distance, esp. $d = 3$

Lemma 2: Non-transversal fault-tolerance
Lemma 3: Error-correction with clifford recovery

} $3 \times 9 \rightarrow 3 \times 3$ for CCZ

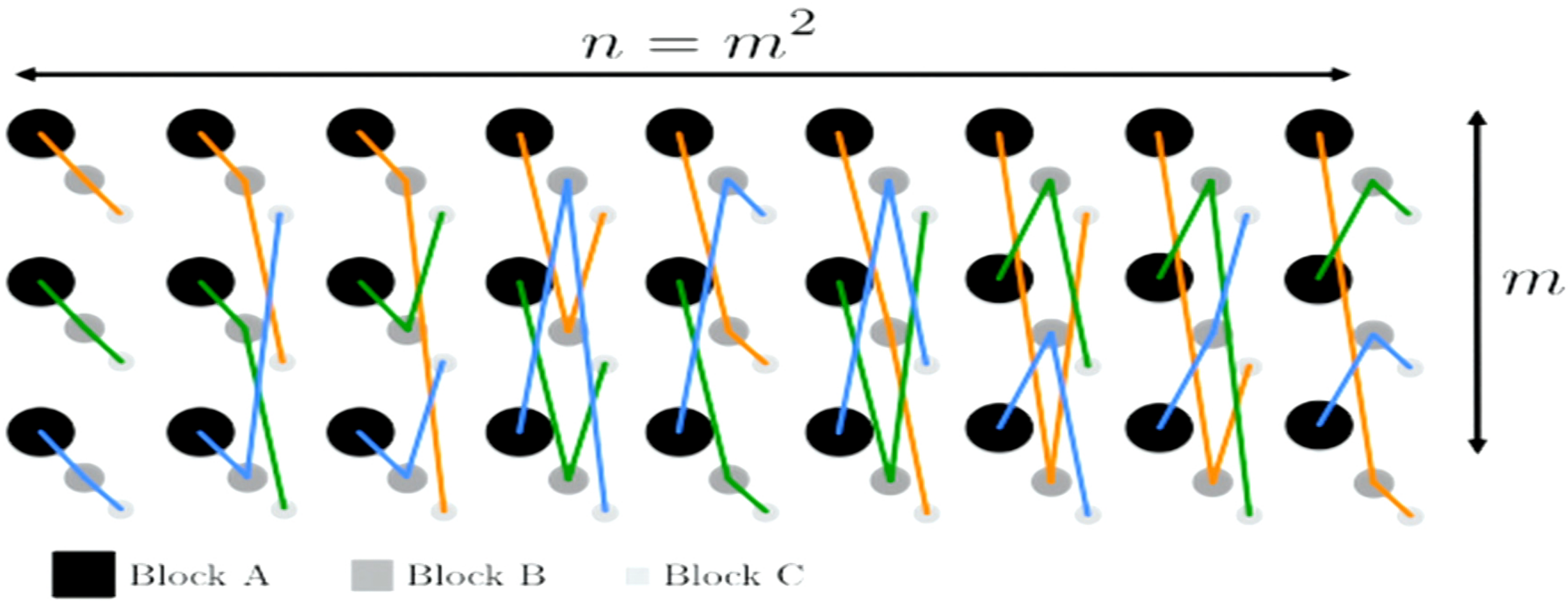
2nd Main result: transversal gates on any stabilizer code are limited to the Clifford hierarchy.

Corollary 2: asymmetry in logical operators is necessary to have transversal gates high in the hierarchy.

Contrast with local circuits in Bravyi-König:
D-dimensions $\Rightarrow$ D-level

Main Result:

transversal $\overline{\text{CCZ}}$



Likewise, a logical C^kZ gate is depth-1 (i.e. transversal) on $m \times m^k$ codes

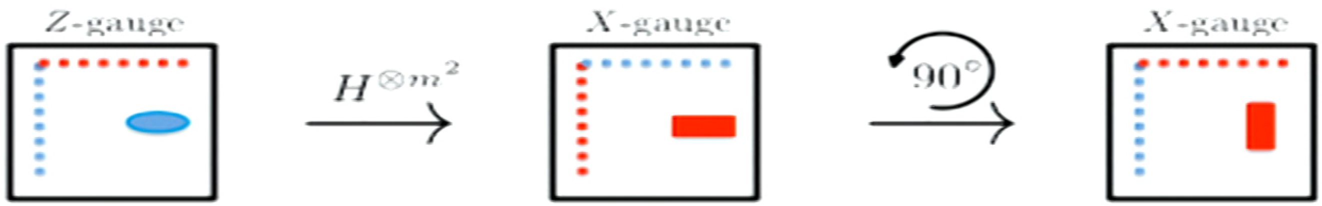
Remark: gate range required is $O(m)$

No-go theorems

Eastin-Knill: for any transversal partitioning of a quantum code, the set of unitary logical gates respecting that partitioning is finite

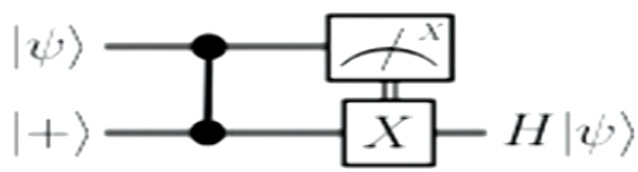
We change codes!

- H is transversal only by changing gauge and on symmetric $m \times m$ codes



- CCZ is transversal only on $m \times m^2$ codes (can extend by gauge-fixing)

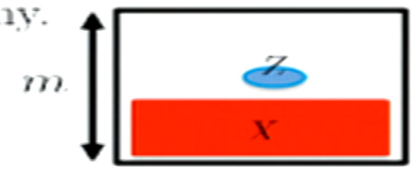
- Teleporting H into an asymmetric code is not unitary



(Note Z-error correction is "free")

No-go theorems

→ Bravyi-König: a local stabilizer code in D-dimensions only has depth-$O(1)$ local circuits for logical operators in level D of the Clifford hierarchy. (e.g. $D=1$: Paulis, $D=2$: Cliffords, $D=3$: universal)



- Our circuits use gates with range $O(m) = O(d)$
- Technically, we have a local subsystem code, or a non-local stabilizer code

→ Pastawski-Yoshida: Bravyi-König for local subsystem codes with a threshold

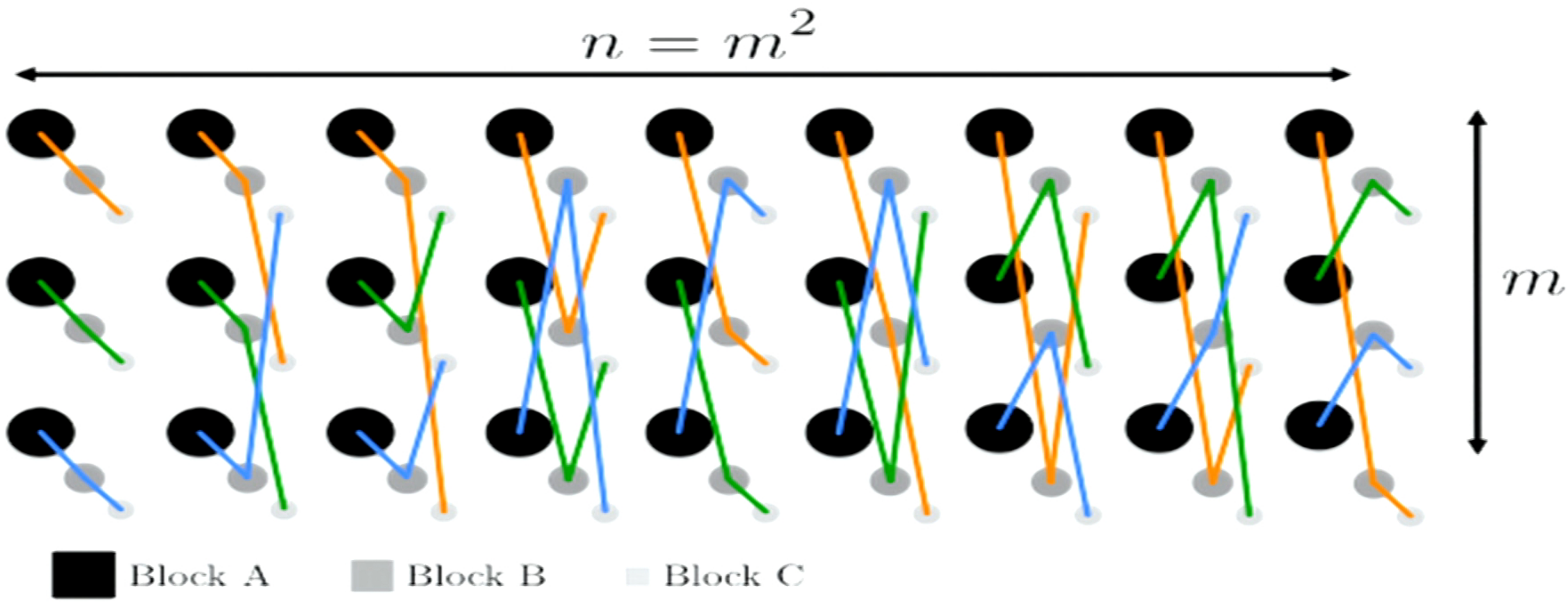
- But Bacon-Shor has no threshold

→ Prove our own bound on Bacon-Shor codes: 2D Bacon-Shor codes cannot have depth-$O(1)$ circuits for non-Clifford gates unless they contain gates with ranges R_x, R_y in the $x-$ and $y-$dimensions and $(R_x+1)(R_y+1) = O(d)$.

- Ranged gates are a necessary part of our circuits
- Circuits with $R_y = d-1$ and $R_x = 0$ are optimal

Main Result:

transversal $\overline{\text{CCZ}}$



Likewise, a logical C^kZ gate is depth-1 (i.e. transversal) on $m \times m^k$ codes

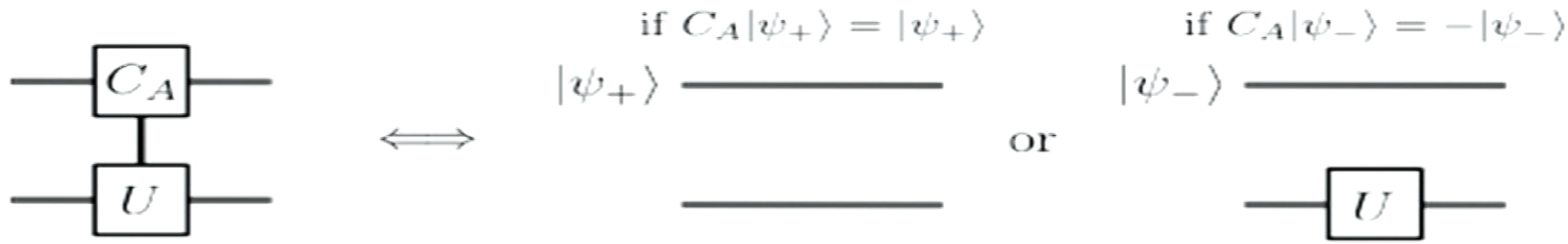
Remark: gate range required is $O(m)$

A circuit identity

Consider Hermitian, unitary operators C_A, C_B (e.g. Paulis)

$\Rightarrow$ eigvals(C_A), eigvals$(C_B) \subseteq \{-1, +1\}$

We can "control" an operation U on C_A.



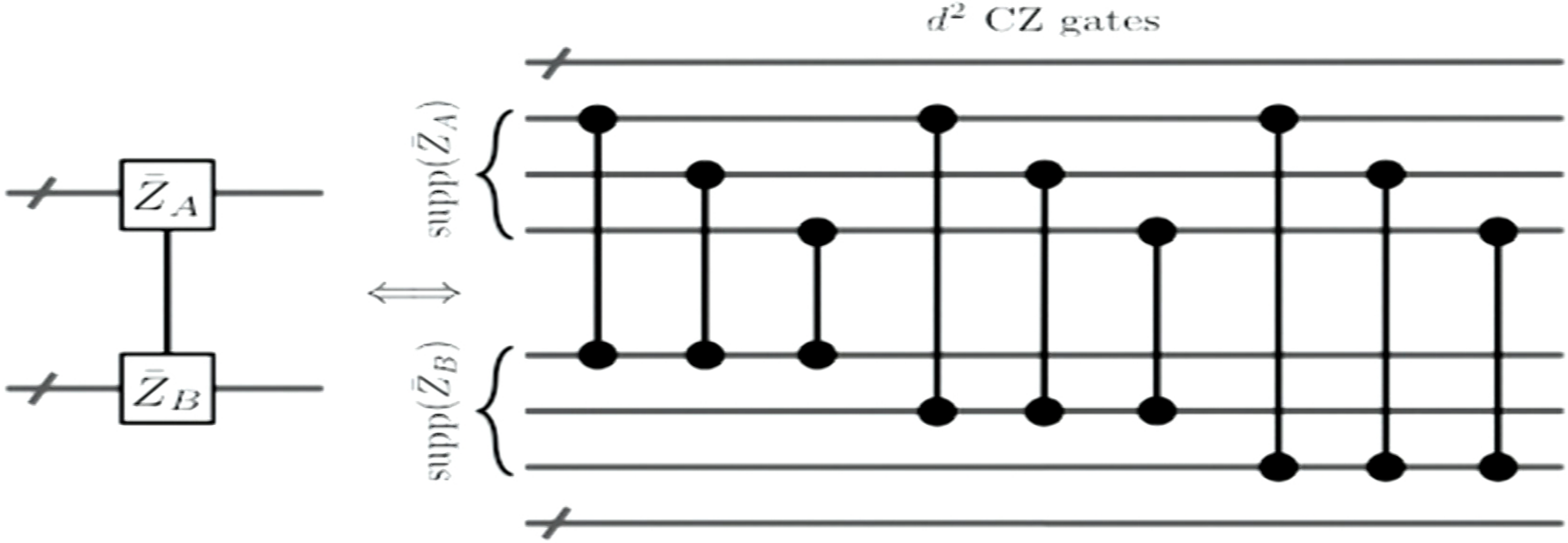
Then,

Use more ancillas and induction to extend to multi-controlled operations.

The round-robin trick for logical gates

Let's make $\overline{\mathrm{CZ}}_{AB} = \Gamma(\bar{Z}_A, \bar{Z}_B)$

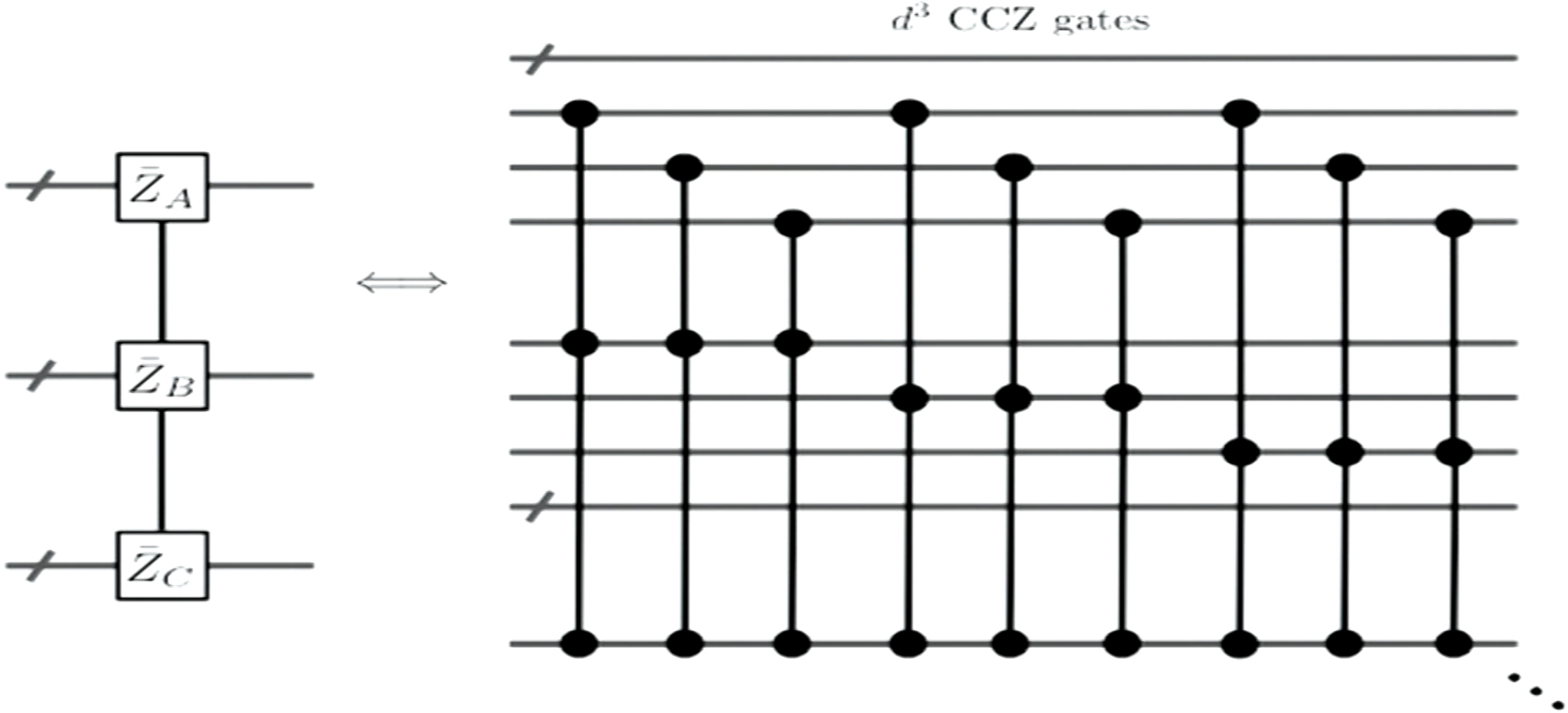
Take $C_A = \bar{Z}_A$ and $C_B = \bar{Z}_B$



The round-robin trick for logical gates

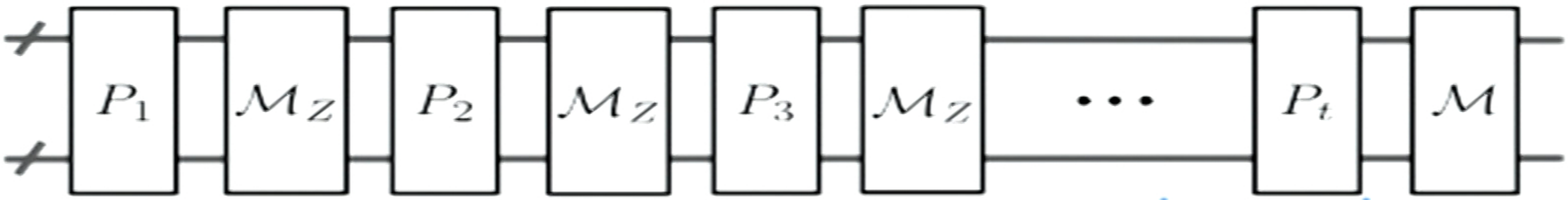
Let's make $\overline{\mathrm{CCZ}}_{ABC}$ (computationally universal paired with $\overline{H}$)

Take $C_A = \bar{Z}_A$, $C_B = \bar{Z}_B$, $C_C = \bar{Z}_C$

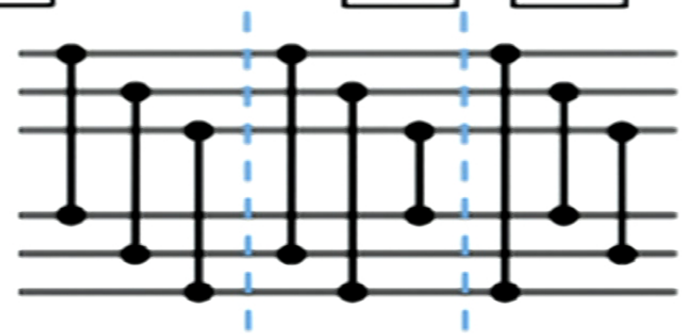


Make round-robin gates fault-tolerant

$\overline{\mathrm{CZ}} = P_t P_{t-1} \dots P_2 P_1, \quad t \geq d/2$ Correct X-errors intermediate in the circuit



$\overline{\mathrm{CCZ}} = P_t P_{t-1} \dots P_2 P_1, \quad t \geq d^2/4$



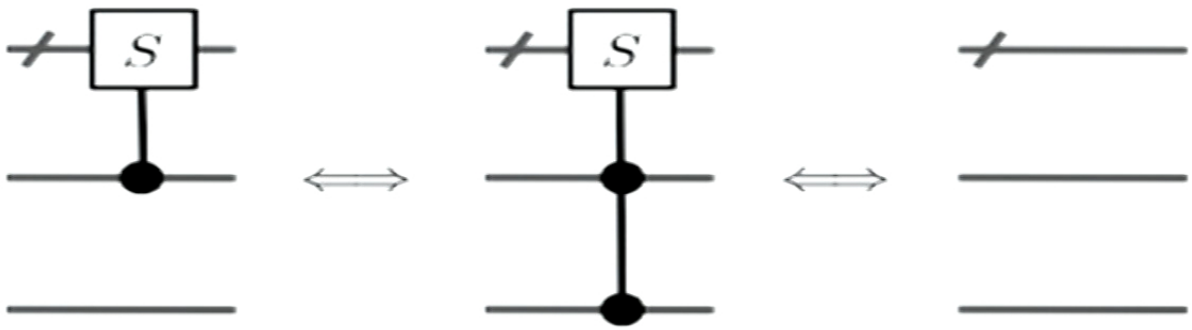
But
(1) this takes many rounds of intermediate correction
(2) no (asymptotic) threshold without concatenation

Can we simplify round-robin first, then make it fault-tolerant?

Compiling in codespace

Stabilizers implement logical identity $S|\bar{\psi}\rangle = |\bar{\psi}\rangle$

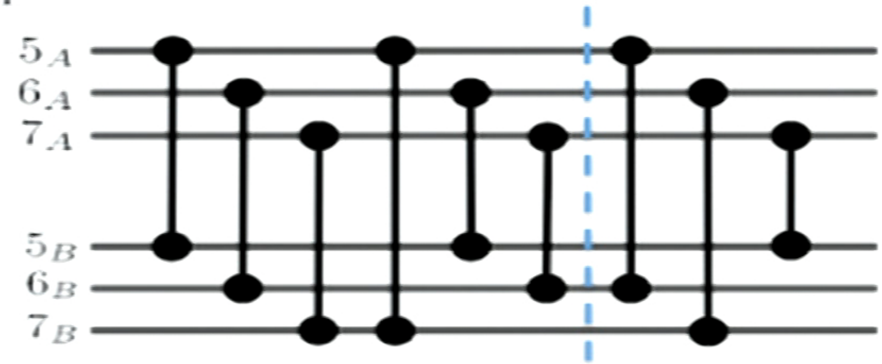
But so do (multi-)controlled stabilizers



- When S is Z-type these are circuits of CZs and CCZs.
- Multiply round-robin circuits by identities to cancel/move CZ and CCZ gates.
- A linear algebraic condition efficiently decides whether two circuits are related by these identities.

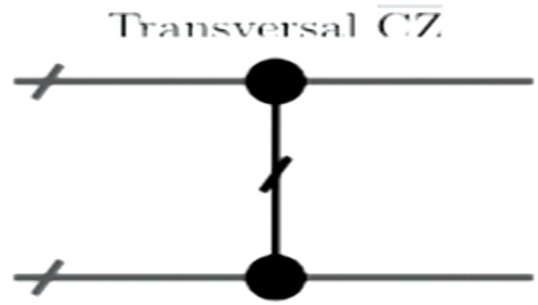
Examples of codespace compiling

7-qubit code:

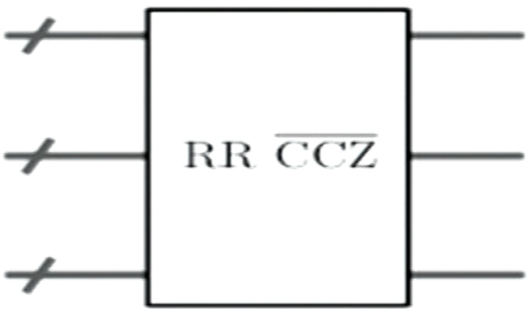


9 CZs, 2 pieces

$\Longleftrightarrow$

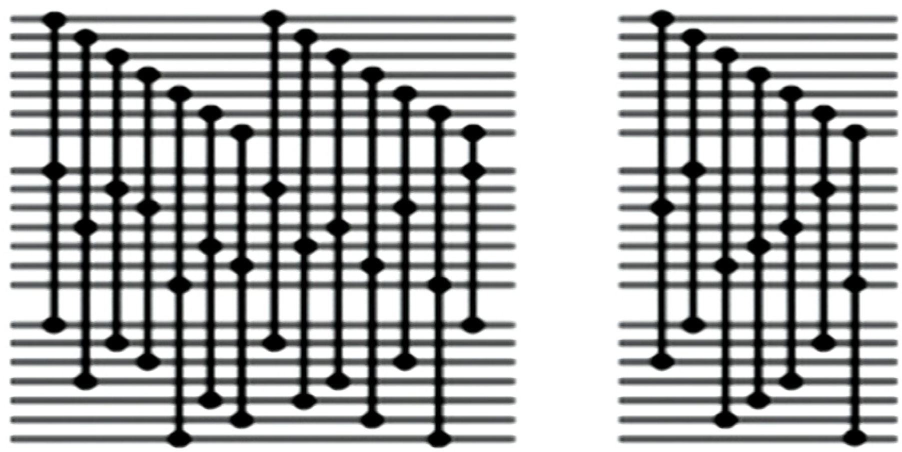


7 CZs, 1 piece



27 CCZs, 4 pieces

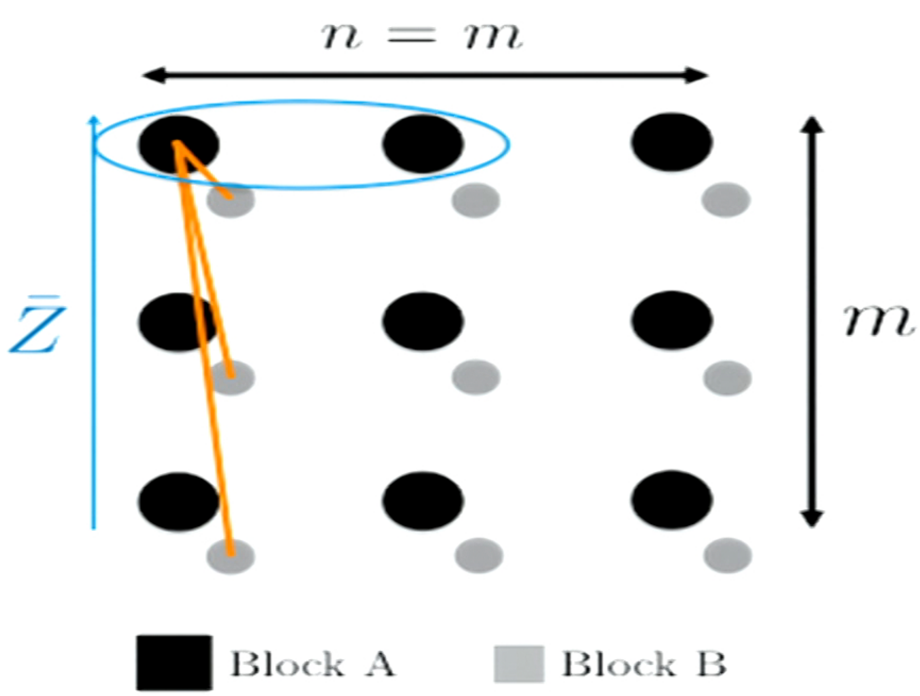
$\Longleftrightarrow$

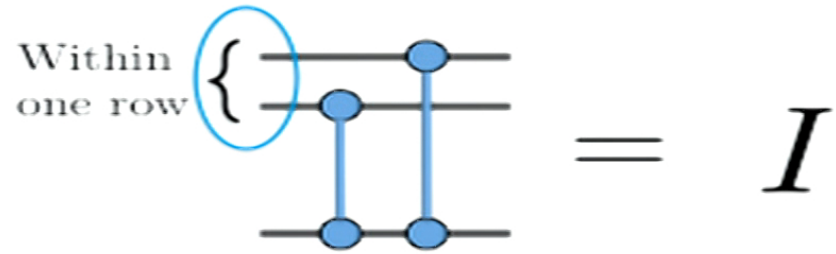


21 CCZs, 2 pieces

Examples of codespace compiling

Bacon-Shor codes: $\overline{\mathrm{CZ}}$

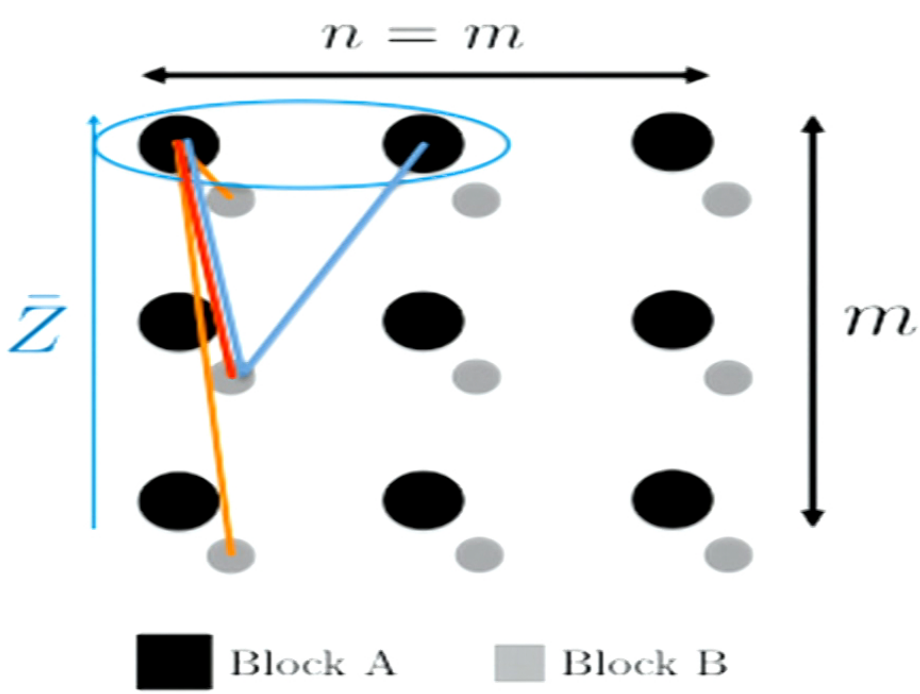


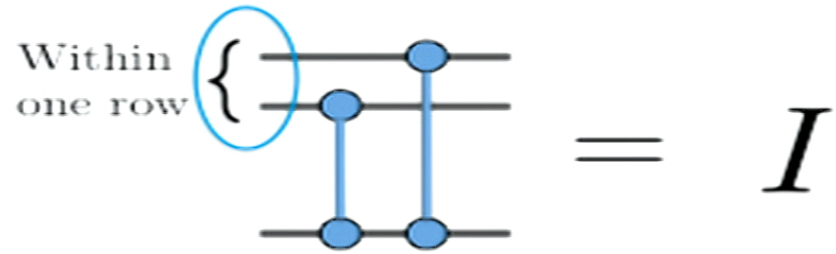


- Use stabilizers to slide CZs along rows
- Depth-1 (transversal) CZ

Examples of codespace compiling

Bacon-Shor codes: $\overline{\mathrm{CZ}}$





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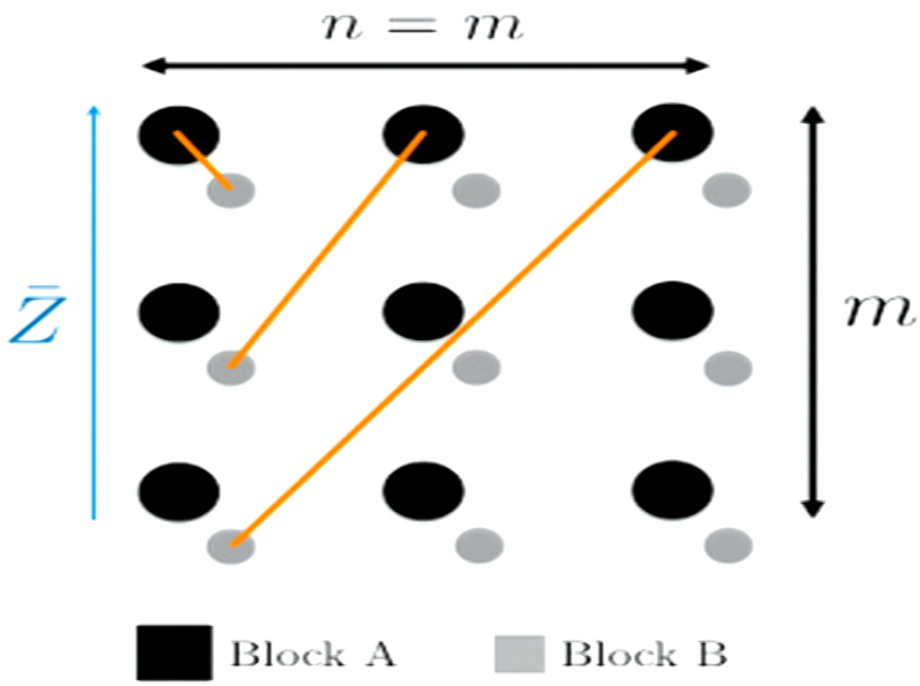
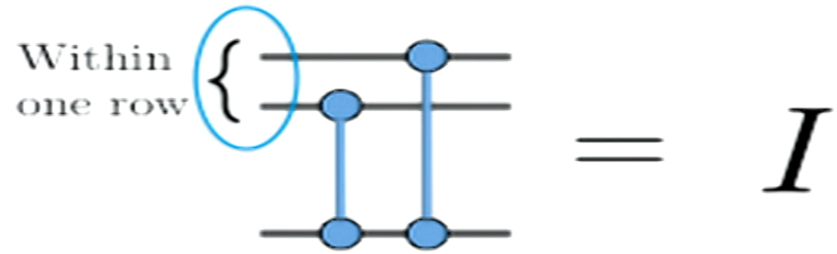
Examples of codespace compiling

Bacon-Shor codes: $\overline{\mathrm{CZ}}$





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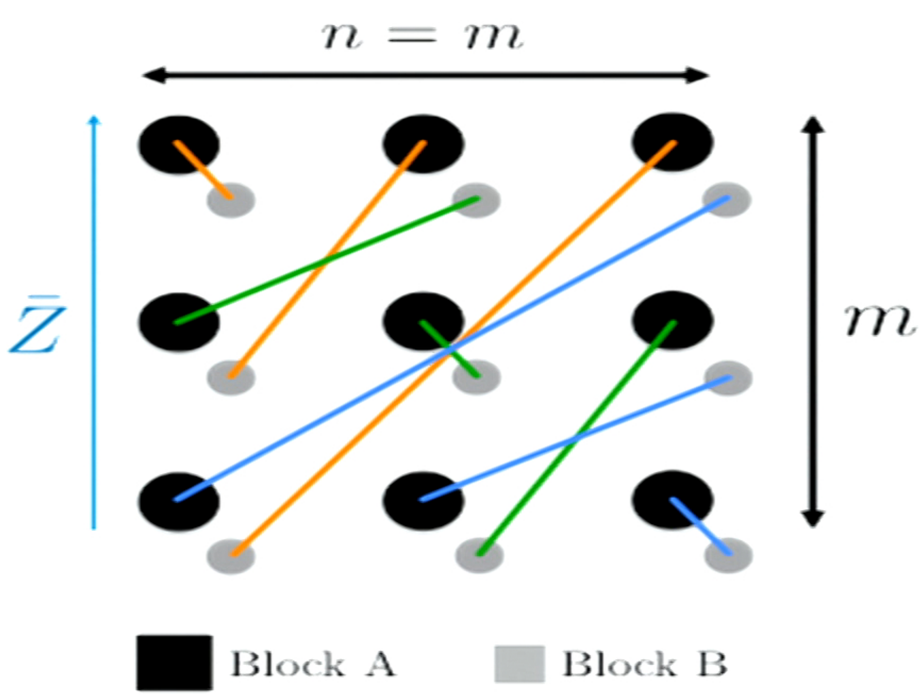
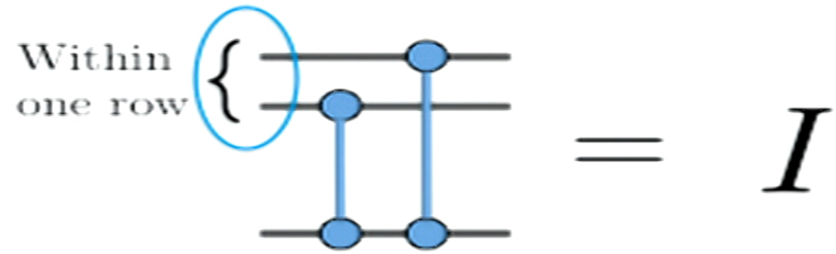
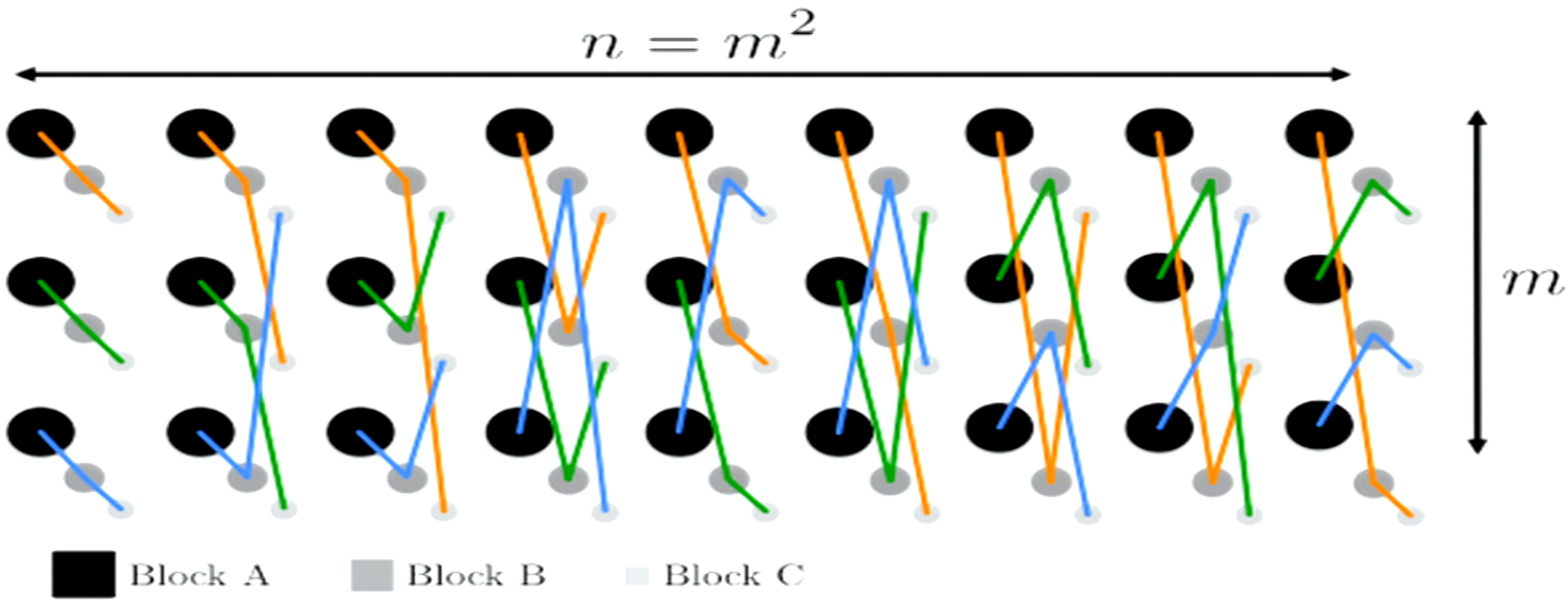




- Use stabilizers to slide CZs along rows
- Depth-1 (transversal) CZ

Main Result:

transversal $\overline{\mathrm{CCZ}}$



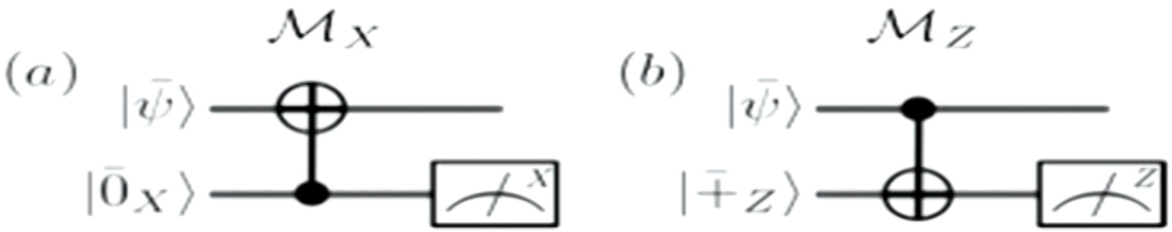
Likewise, a logical $C^k Z$ gate is depth-1 (i.e. transversal) on $m \times m^k$ codes

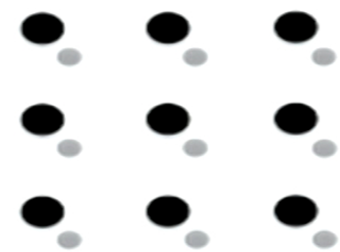
Remark: gate range required is $O(m)$

Very efficient encoding and decoding

$$|\bar{+}_Z\rangle = (|0\rangle^{\otimes n} + |1\rangle^{\otimes n})^{\otimes m}$$
$$|\bar{0}_X\rangle = (|+\rangle^{\otimes m} + |-\rangle^{\otimes m})^{\otimes n}$$

- Steane EC just requires CAT states (and no repeats)



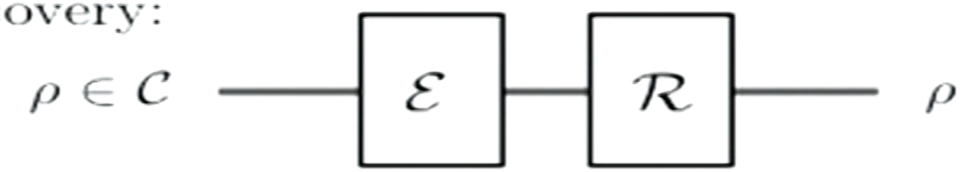


$m = n = 3$ case: 18 physical qubits for a logical qubit, one-shot EC
No postselection for 3-qubit CATs

- Measuring logical operators of a CSS code is transversal
- Does $\overline{\text{CCZ}}$ really take 3×9 qubits?

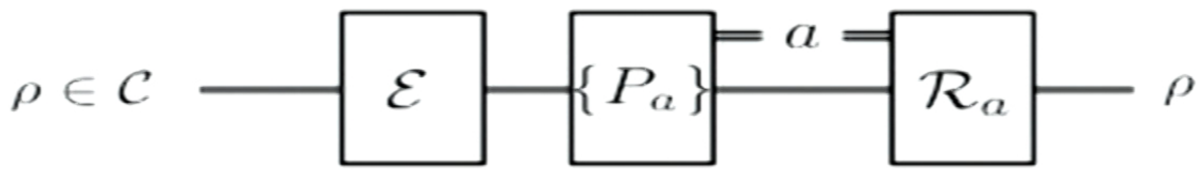
The error-correcting conditions (revisited)

- Generic recovery:



  Knill-Laflamme: For code with projector P, there is a recovery operation $\mathcal{R}$ for error channel $\mathcal{E}$ iff $PE_i^\dagger E_j P = \alpha_{ij} P, \forall i, j$.

- Stabilizer code "standard":

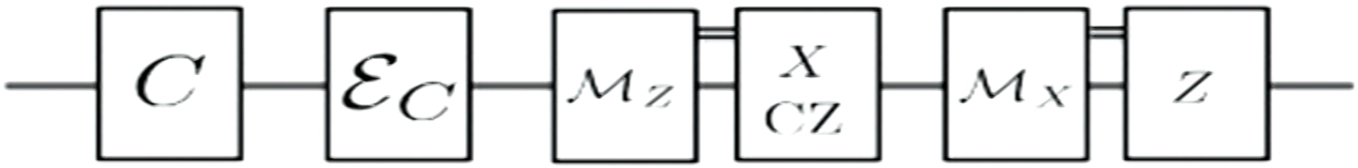


  Stabilizer projective: For stabilizer code with projectors onto syndrome cosets P_a ($P = P_0$), there are recovery operations $\mathcal{R}_a$ for error channel $\mathcal{E}$ iff $PE_i^\dagger P_a E_j P = \alpha_{ij}^{(a)} P$.

- Stabilizer projective recovery collapses all errors to Paulis, perhaps rendering them uncorrectable

Non-Pauli error-correction

- A circuit C has structure $\Longrightarrow$ error channel $\mathcal{E}_C$ has structure

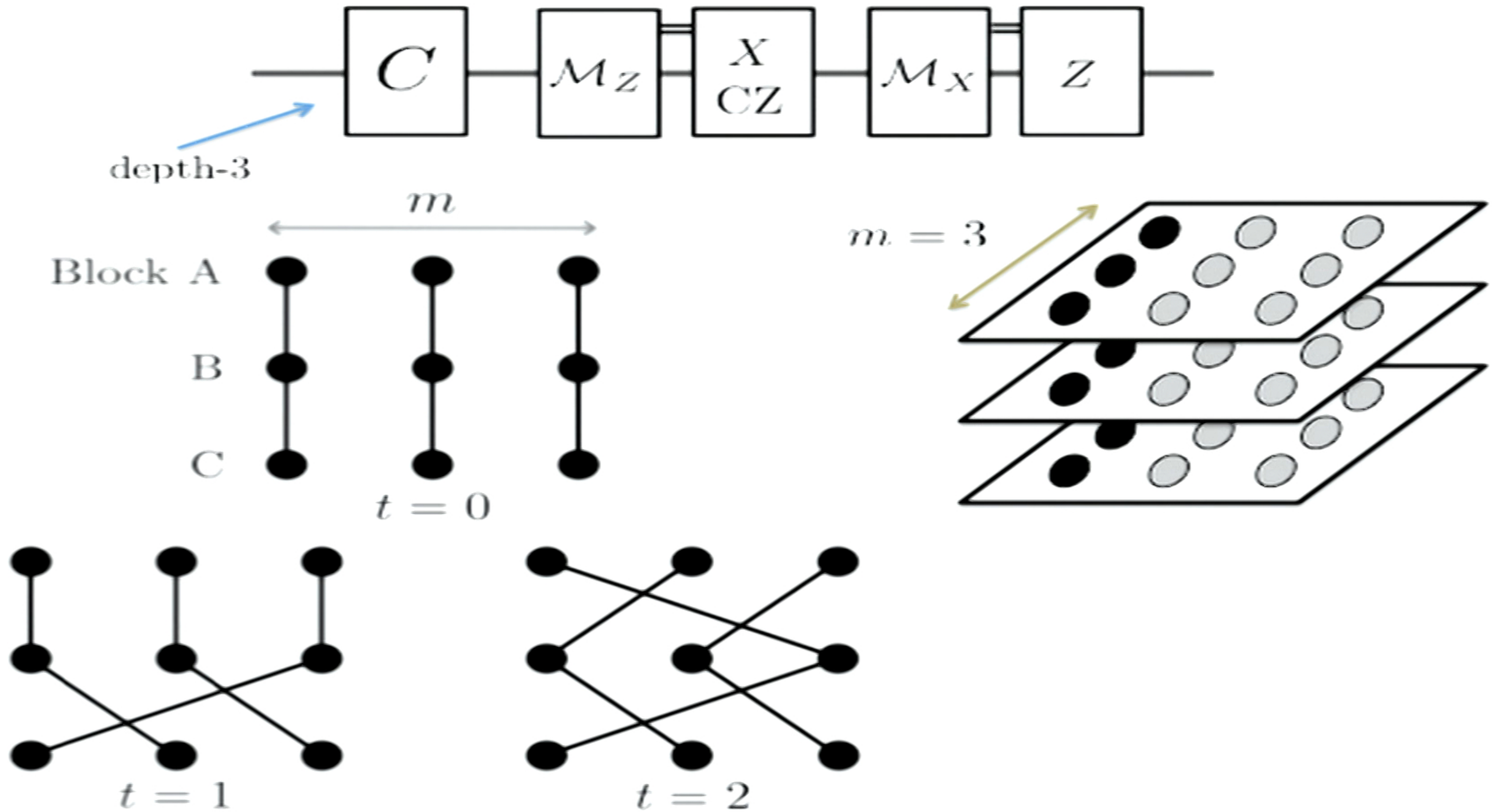


- Worst-case circuit noise

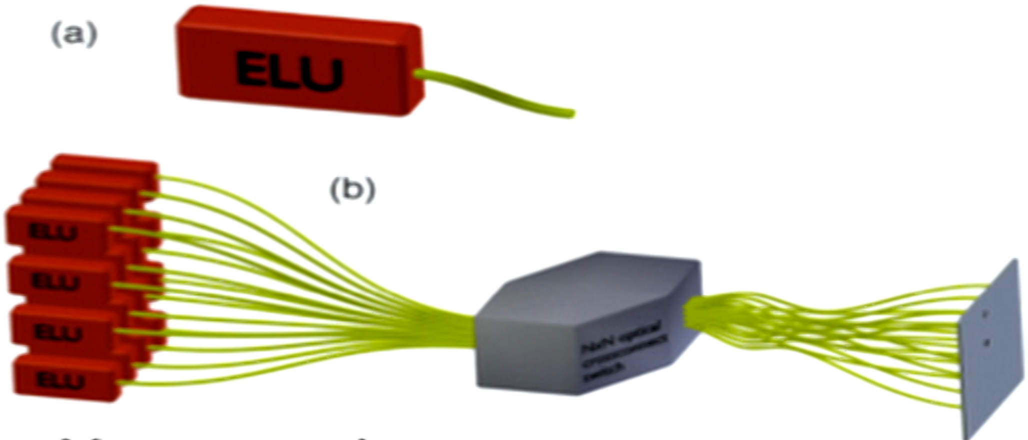
- Detecting multiple X errors $\Rightarrow$ a CCZ failed $\Rightarrow$ possible CZ errors

$$\mathrm{CZ} = \tfrac{1}{2}(II + IZ + ZI - ZZ)$$

CCZ on 3 × 3 Bacon-Shor



MUSIQC architecture



Monroe, et. al.

Operation	Time
1-qubit	$1\mu s$
CX	$10\mu s$
CCZ	$10\mu s$
init.	$1\mu s$
meas.	$30\mu s$

- $N_q \sim 100$ qubits per elementary logical unit (ELU)
- $C_q \sim N_q/2$ are communication qubits, entangled with other ELUs
- Gates may be non-local!

  Up to 12 parallel operations per timestep

MUSIQC architecture (2014) vs. Innsbruck (2017)

Operation	Time
1-qubit	$1\mu s$
CX	$10\mu s$
CCZ	$10\mu s$
init.	$1\mu s$
meas.	$30\mu s$

Operation	Current duration	Current infidelity	Anticipated duration	Anticipated Infidelity
Single-qubit gates	$5\mu s$	$5 \cdot 10^{-5}$	$1\mu s$	$1 \cdot 10^{-5}$
Entangling (2 qubits)	$40\mu s$	$1 \cdot 10^{-2}$	$15\mu s$	$2 \cdot 10^{-4}$
Entangling (5 qubits)	$60\mu s$	$5 \cdot 10^{-2}$	$15\mu s$	$1 \cdot 10^{-3}$
Dual species entangling (2 qubits)	$60\ \mu s$	$3 \cdot 10^{-2}$	$15\ \mu s$	$4 \cdot 10^{-4}$
Dual species entangling (3 qubits)	$80\ \mu s$	$5 \cdot 10^{-2}$	$15\ \mu s$	$6 \cdot 10^{-4}$
Dual species entangling (5 qubits)	-	-	$15\ \mu s$	$2 \cdot 10^{-3}$
Measurement	$400\mu s$	$1 \cdot 10^{-3}$	$30\mu s$	$1 \cdot 10^{-4}$
Re-cooling	$400\mu s$	$\bar{n} < 0.1$	$100\mu s$	$\bar{n} < 0.1$
Qubit reset	$50\mu s$	$5 \cdot 10^{-3}$	$10\mu s$	$5 \cdot 10^{-3}$ *

Blatt et. al.

Overhead comparison

	Circ. Vol.	Spacetime	Time	Qubits
Magic 7	1,400	19,900 $\mu s \times$ qub.	940μs	66
Magic 9	1,100	15,800 $\mu s \times$ qub.	910μs	81
BS 3×3	440	5,540 $\mu s \times$ qub.	190μs	54

Circuit volume:

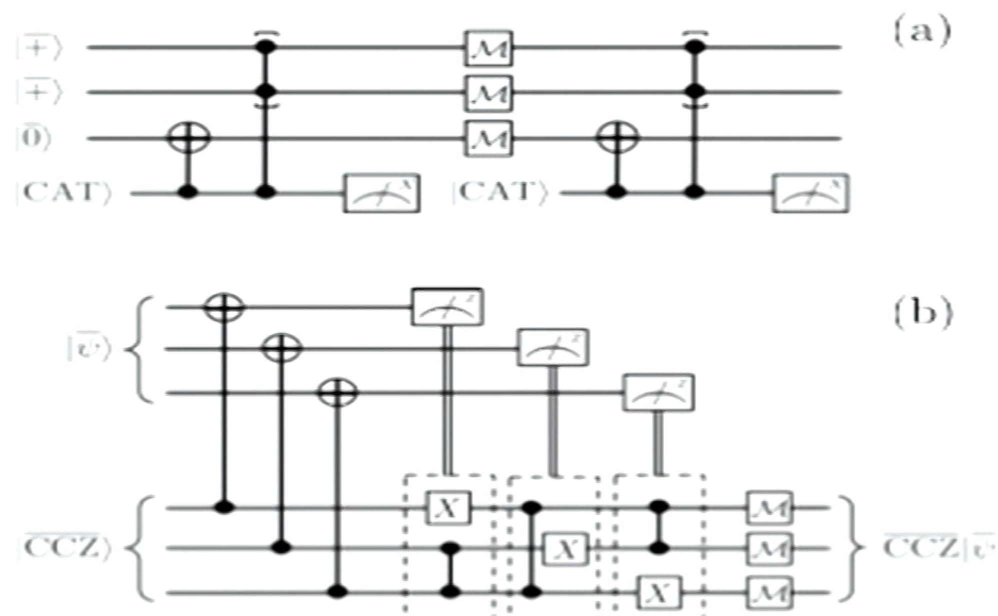
- Count gates weighted by qubits

Spacetime volume:

- Weight by qubits and time

Magic states at $d = 3$

- Create $|\overline{\mathrm{CCZ}}\rangle$ (a)
- Inject $\overline{\mathrm{CCZ}}$ (b)

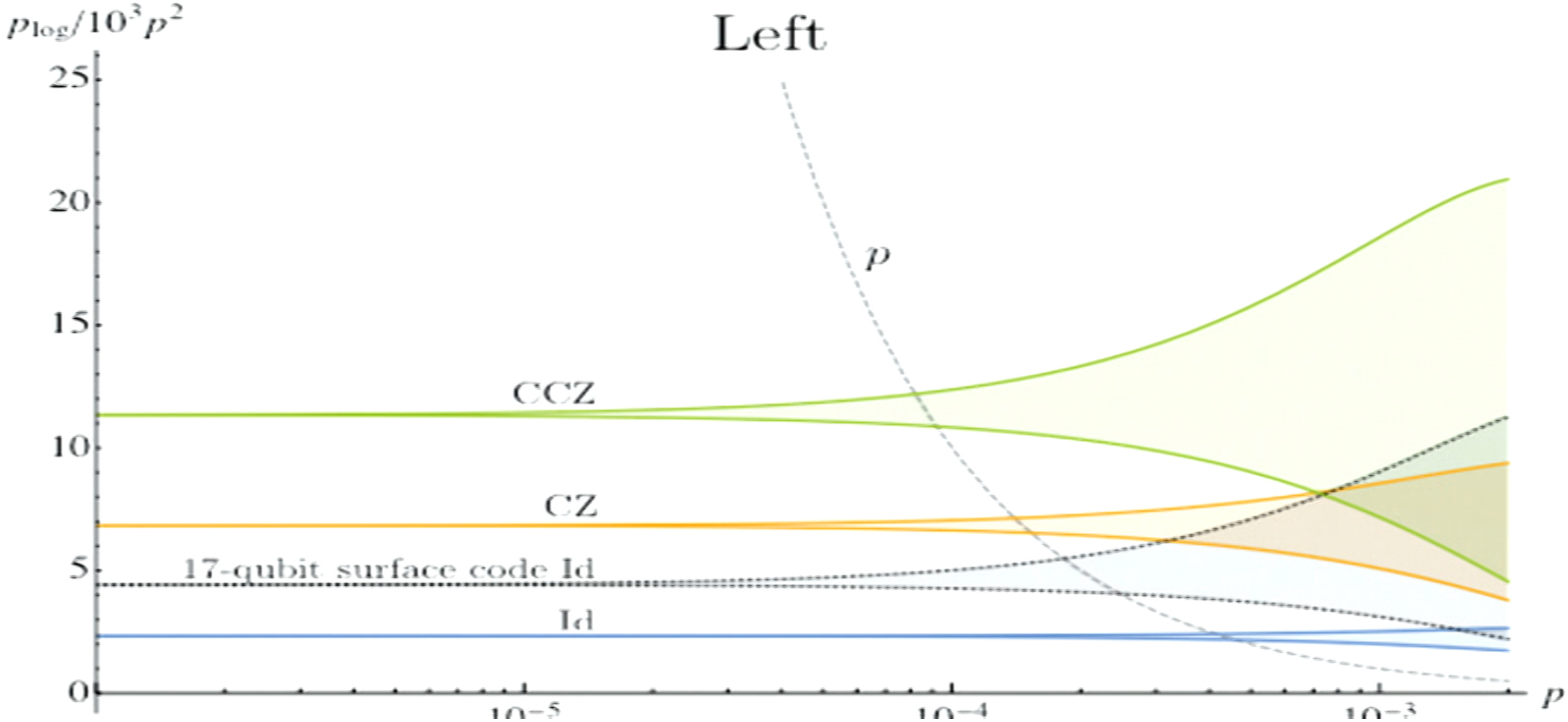


exREC pseudothresholds

Left: $p_{CCZ} = p_{CX} = p_1 = p$

Right: $p_{CCZ}/10 = p_{CX} = 10p_1 = p$

Gate	p	$p/10^{2-q}$
I & H	4.1×10^{-4}	1.9×10^{-4}
CNOT	1.4×10^{-4}	5.3×10^{-4}
CCZ	8.2×10^{-5}	6.1×10^{-4}



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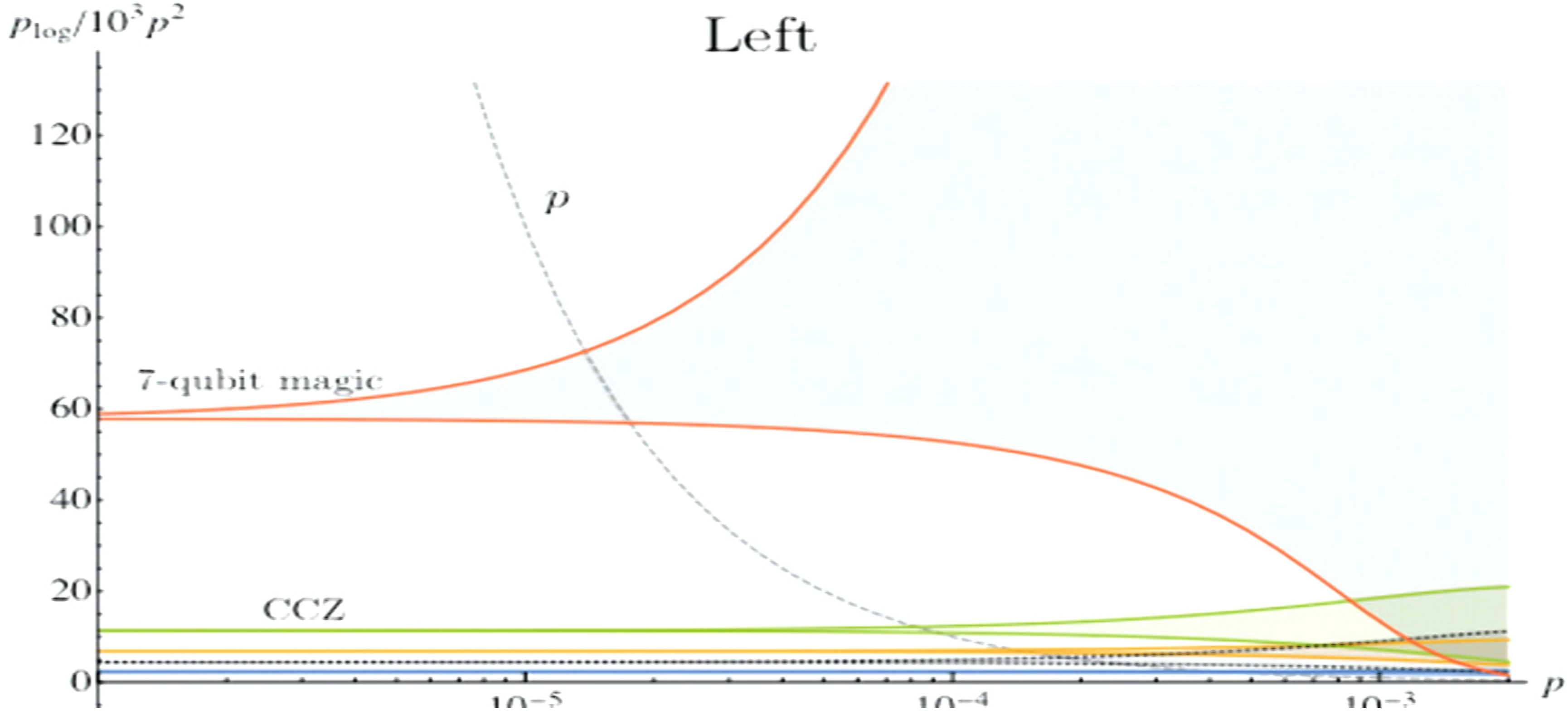
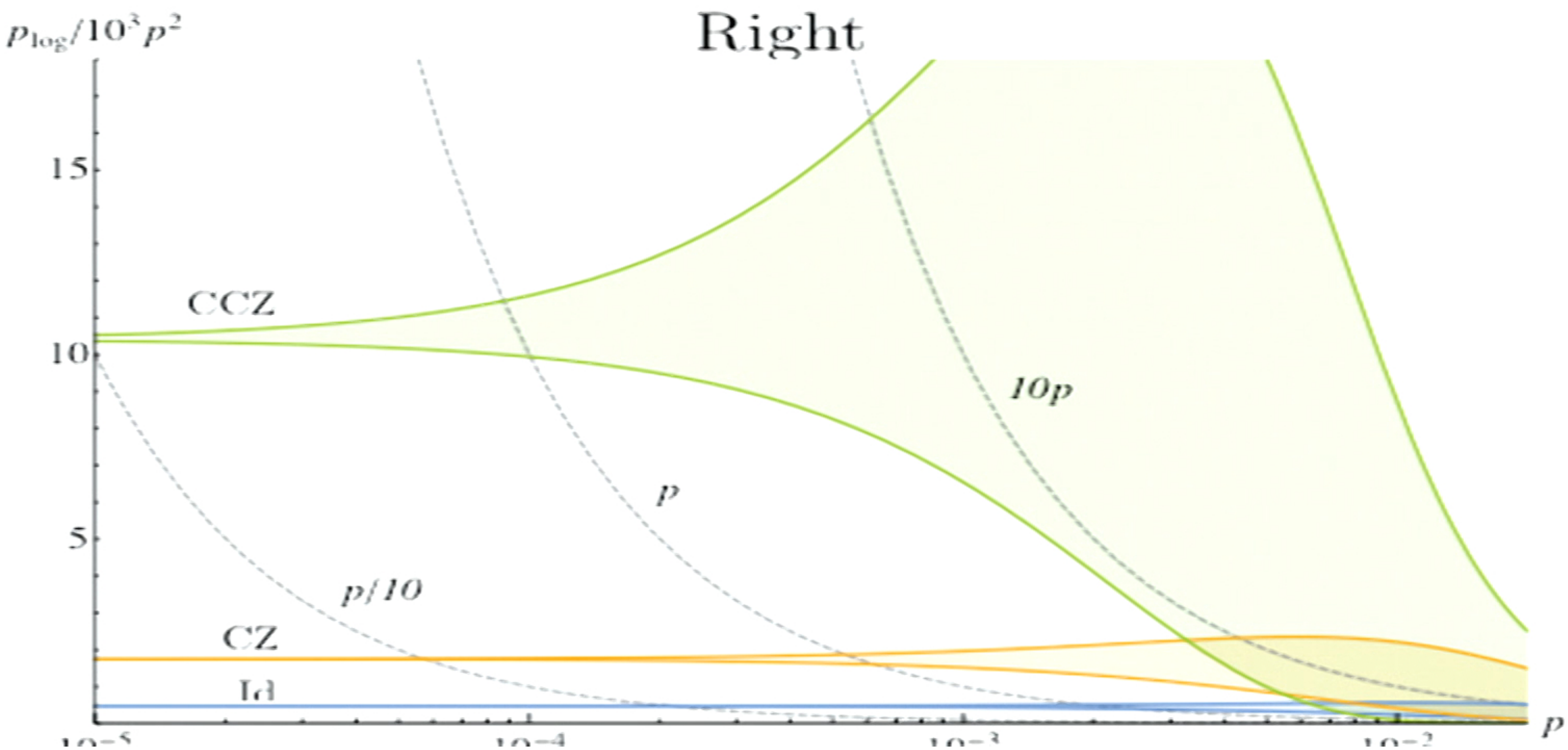


exREC pseudothresholds

Left: $p_{CCZ} = p_{CX} = p_1 = p$

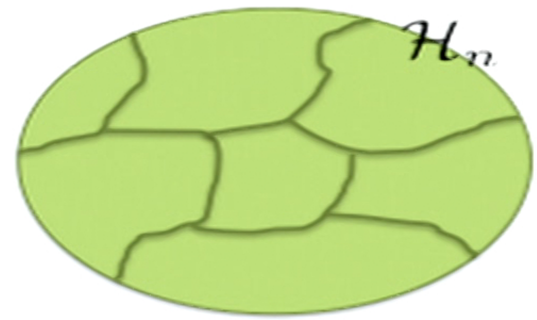
Right: $p_{CCZ}/10 = p_{CX} = 10p_1 = p$

Gate	p	$p/10^{2-q}$
I & H	4.1×10^{-4}	1.9×10^{-4}
CNOT	1.4×10^{-4}	5.3×10^{-4}
CCZ	8.2×10^{-5}	6.1×10^{-4}



Bounds on transversal gates (in prep. w/ Tomas, Alex)

Definitions: for $[n, k]$ code with stabilizer $\mathcal{S}$



$$\mathcal{L} = \mathcal{N}(\mathcal{S}) \backslash \mathcal{S} - \mathcal{S} = \{g\mathcal{S} : g \in \mathcal{N}(\mathcal{S}) - \mathcal{S}\}$$

For $G \in \mathcal{L}$, $d(G) := \min\{|\mathrm{supp}(g)| : g \in G\}$

$$d = d_\downarrow := \min_{G \in \mathcal{L}} d(G)$$

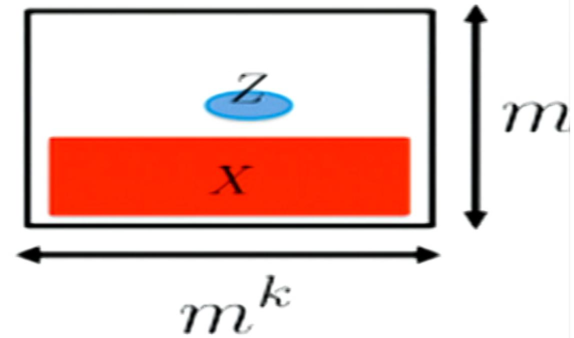
$$d_\uparrow = \max_{G \in \mathcal{L}} d(G)$$

Theorem: Consider a codeblock of an $[n, k, d]$ code with $d > 1$. Then $\exists C < 1$ s.t.

$$d_\uparrow < d_\downarrow / C^{M-1} \qquad (1)$$

implies transversal gates are in the M^{th}-level of the Clifford hierarchy.

Main result: A depth-1 (i.e. transversal) circuit for C^kZ on $m \times m^k$ Z-gauge Bacon-Shor codes



Lemma 1: Round-robin logical gates
arXiv:1603.03948

Corollary 1: A very efficient scheme for fault-tolerant computing at low distance, esp. $d = 3$

Lemma 2: Non-transversal fault-tolerance
Lemma 3: Error-correction with clifford recovery

} $3 \times 9 \rightarrow 3 \times 3$ for CCZ

2nd Main result: transversal gates on any stabilizer code are limited to the Clifford hierarchy.

Corollary 2: asymmetry in logical operators is necessary to have transversal gates high in the hierarchy.

Matching pseudothresholds $(m \times m)$ logical identity