Title: Signal processing in precision measurements: a primer for theorists

Date: Aug 22, 2017 03:00 PM

URL: http://pirsa.org/17080026

Abstract:


CAUTION

Outline

- Force and Displacement Measurements
  - Brownian Motion and Thermal Noise
  - Laser shot noise
  - Measurement backaction
  - Optical cooling/feedback
- Spin measurements
  - Atom shot noise (clocks, magnetometers, accelerometers, gyros)

Thermal Noise

- System in contact with a heat bath

→ random motion in harmonic oscillator, Random voltage (Johnson noise)

- Random motion reduced if system is cooled
- Limits ability to detect a signal

3

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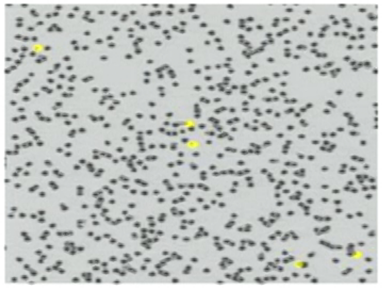
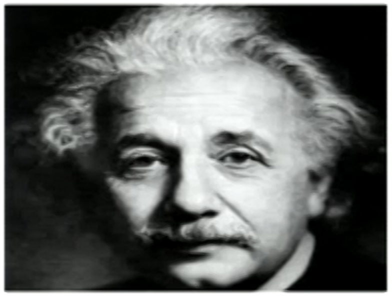
Background



- Brown (1827) pollen motion in fluid
- Einstein (1905) establishes theory of Brownian motion, depends on viscosity



- Johnson/Nyquist 1926

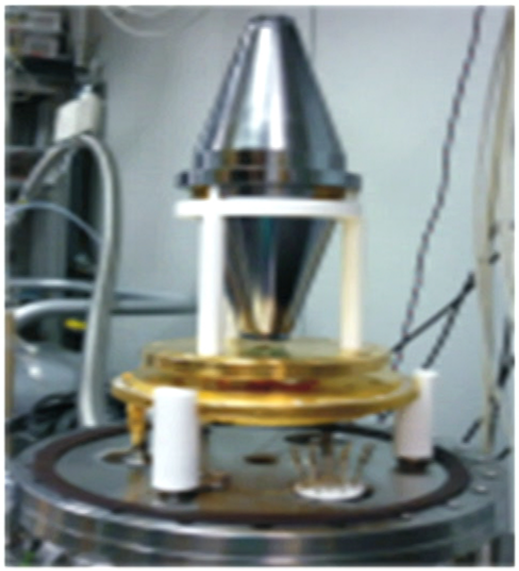
  Voltage noise in resistors
- Callen, Welton (1951)

  Relates random motion to dissipation

  -> Fluctuation-Dissipation Theorem

Thermal Brownian noise in atomic clocks

- Brownian noise in high-reflectivity optical coatings – limits clock precision

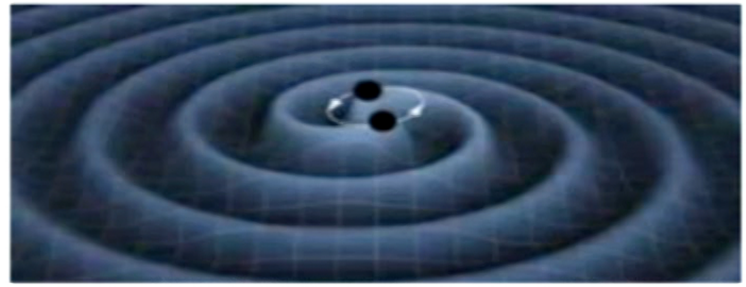


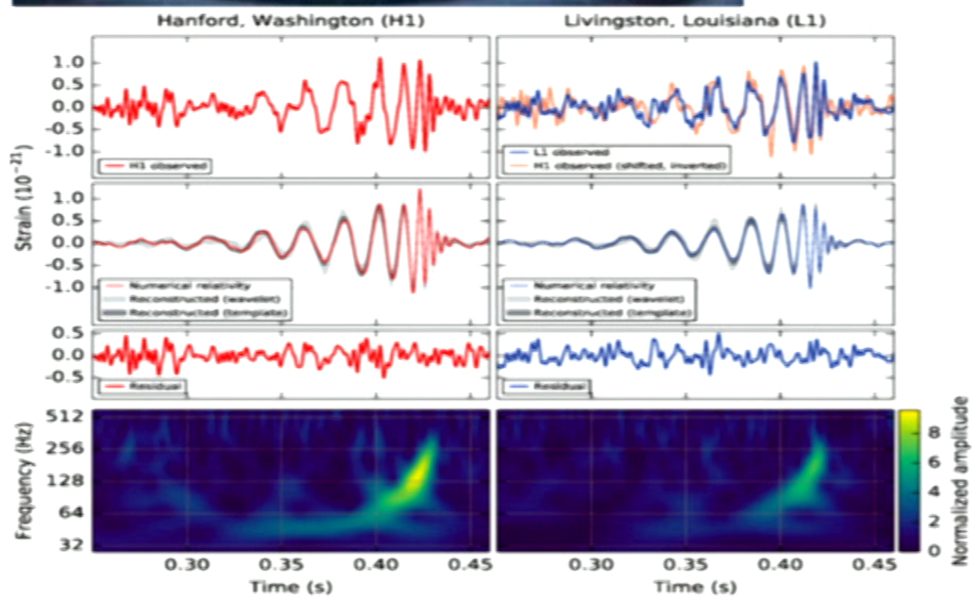
Jun Ye group

Ultra-stable fabry-perot resonator

Length standard affected by thermal motion in thin optical coatings

Gravitational waves





B. P. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration) Phys. Rev. Lett. **116**, 061102 (2016).

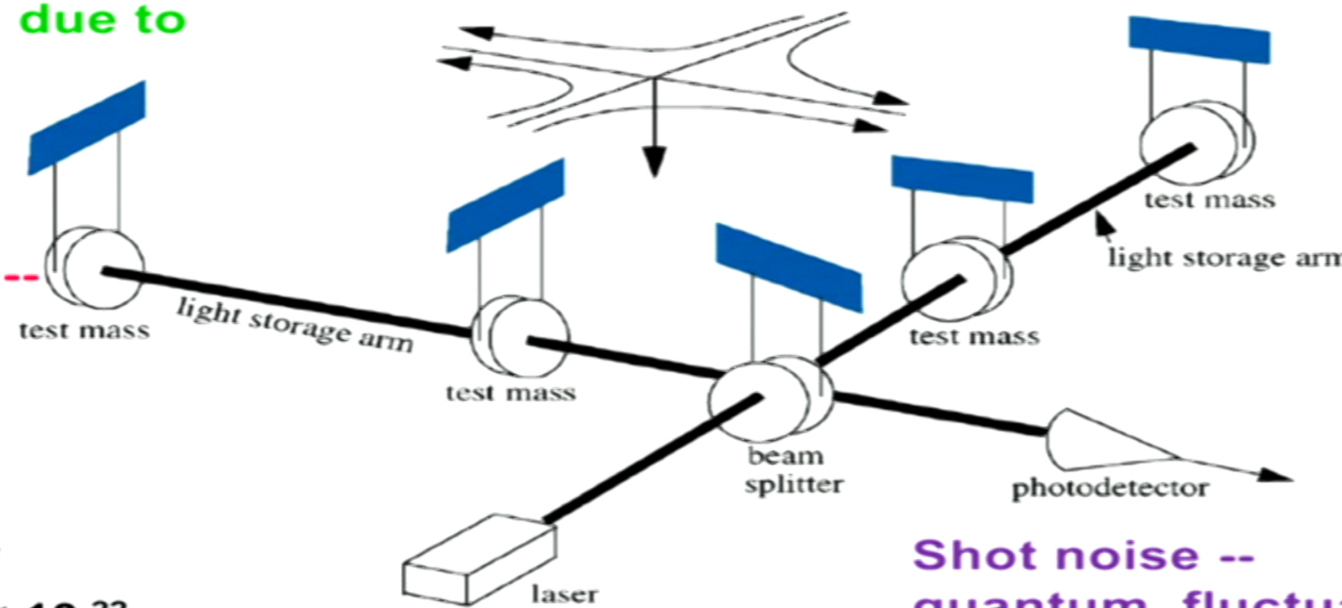
Thermal brownian noise in interferometers

P.R. Saulson, PRD, 42, 2437 (1990)

Interferometer detectors

Seismic motion -- ground motion due to natural and anthropogenic sources

Thermal noise -- vibrations due to finite temperature



$$h = \Delta L / L$$

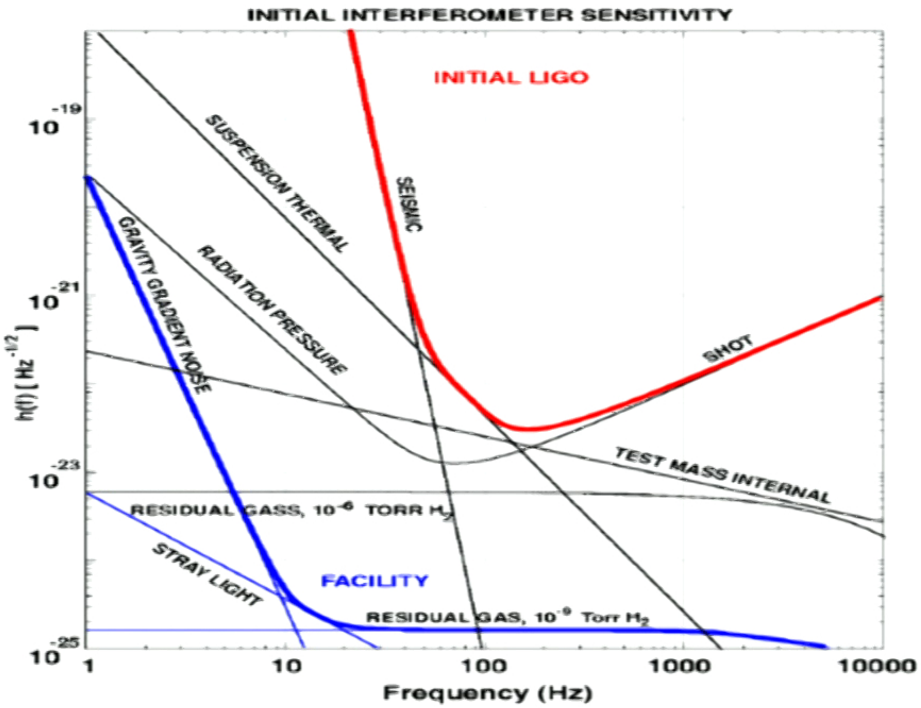
want to get $h \leq 10^{-22}$;
can build L = 4 km;
must measure
$\Delta L = h L \leq 4\times10^{-19}$ m

Shot noise -- quantum fluctuations in the number of photons detected

AJW, LIGO SURF, 6/16/06

A. Weinstein, notes caltech.edu/laac/undergraduate_resources.shtml

Sensitivity of LIGO



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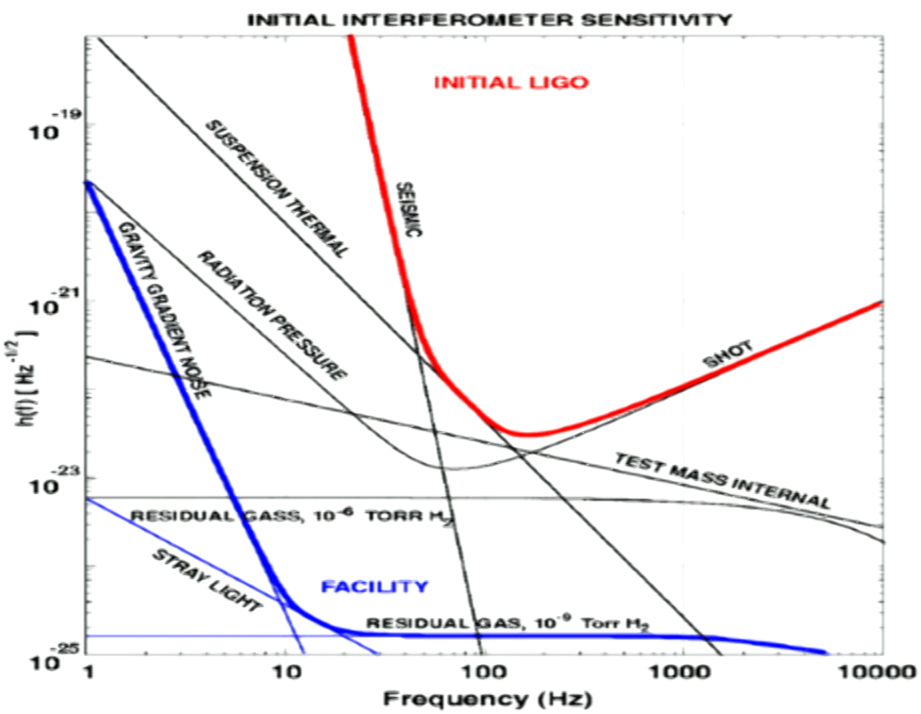

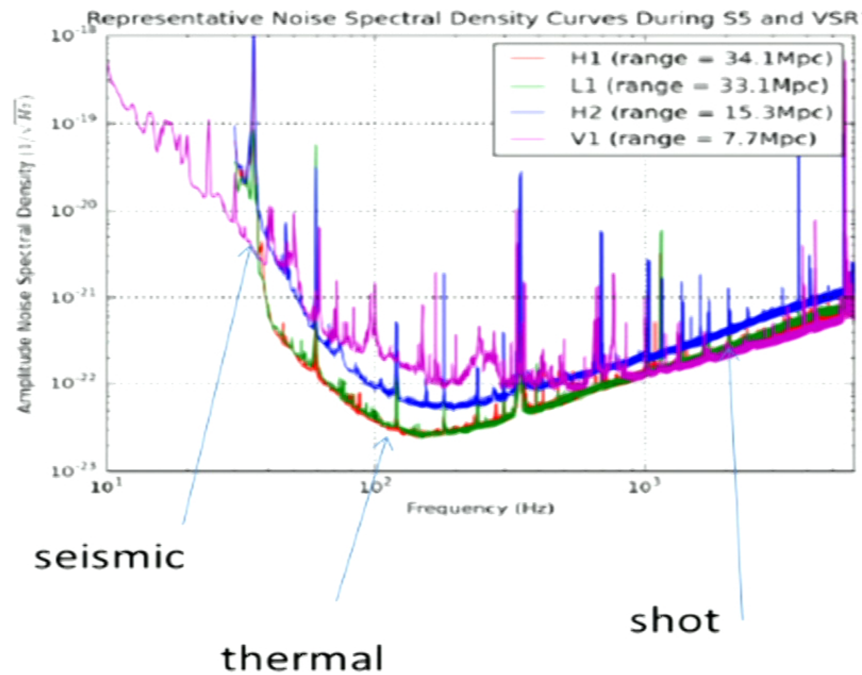

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LIGO-T0900499

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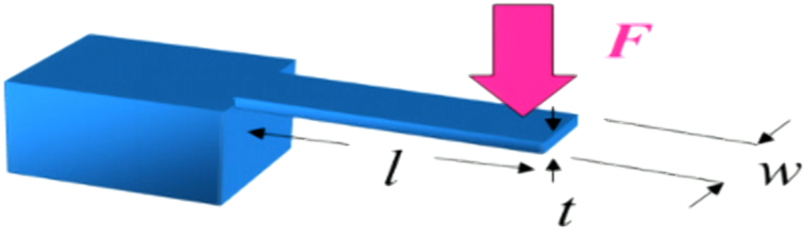
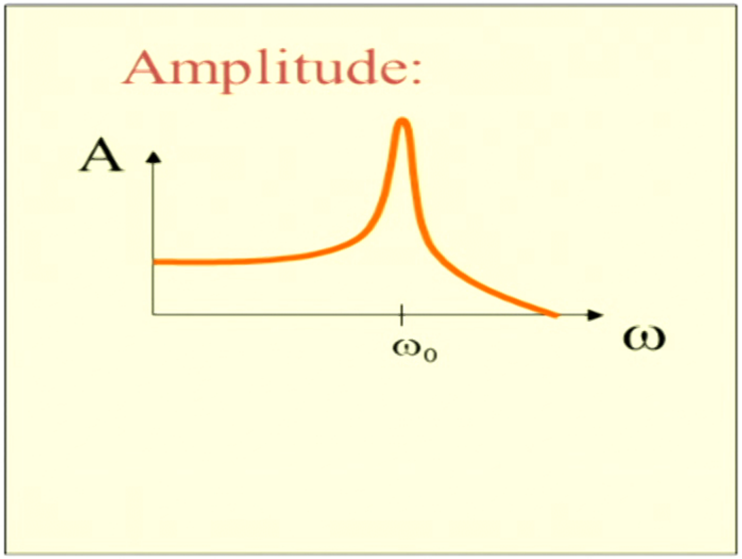
Resonant force detection

- Cantilever is like a spring:

$$F = -Kx$$

$$\omega_0 = \sqrt{\frac{K}{m}}$$



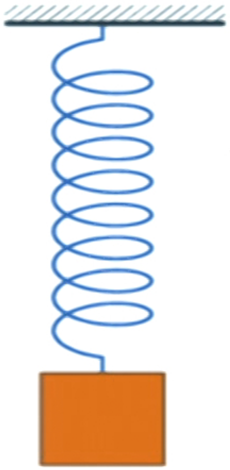


$$A_{(\omega=0)} = \frac{F}{k}$$ Constant force

$$A_{(\omega=\omega_0)} = \frac{F}{k}Q$$ Driving force on resonance of cantilever ω_0

Q can be very large >100,000

Dissipation

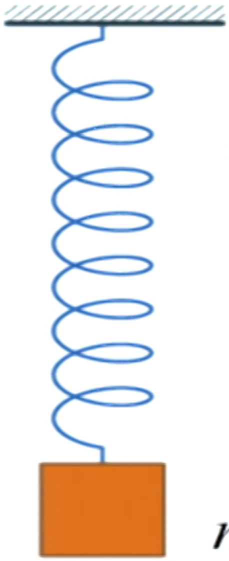


$$E = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2$$ Energy stored in oscillation

$$Q \equiv 2\pi \frac{E}{\delta E}$$ Energy stored in oscillation / energy dissipated in 1 cycle

13

Dissipation



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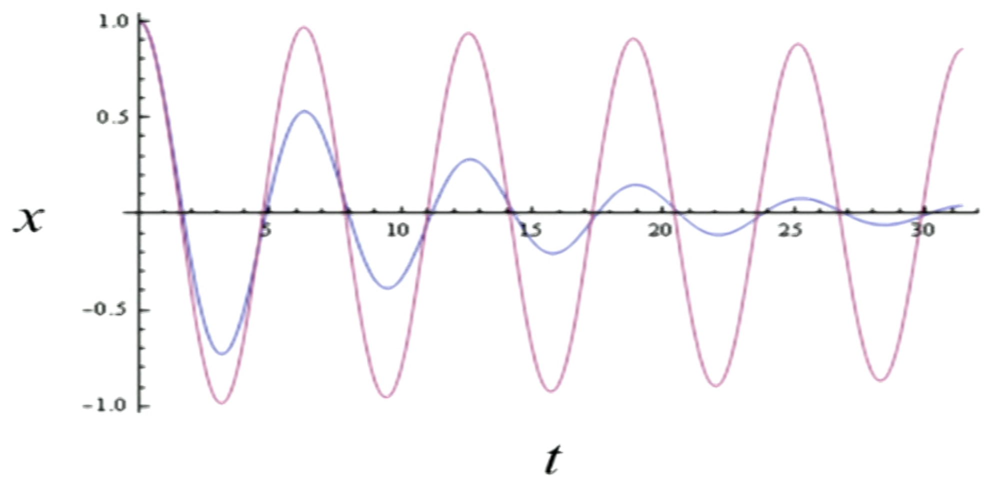
Energy stored in oscillation / energy dissipated in 1 cycle

$$m\ddot{x} = -kx - \gamma\dot{x}$$

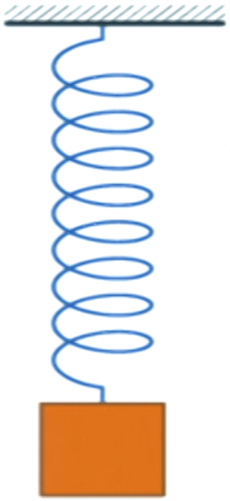
$$Q \cong \frac{\sqrt{mk}}{\gamma}$$



Plays crucial role in force detection

14

Dissipation



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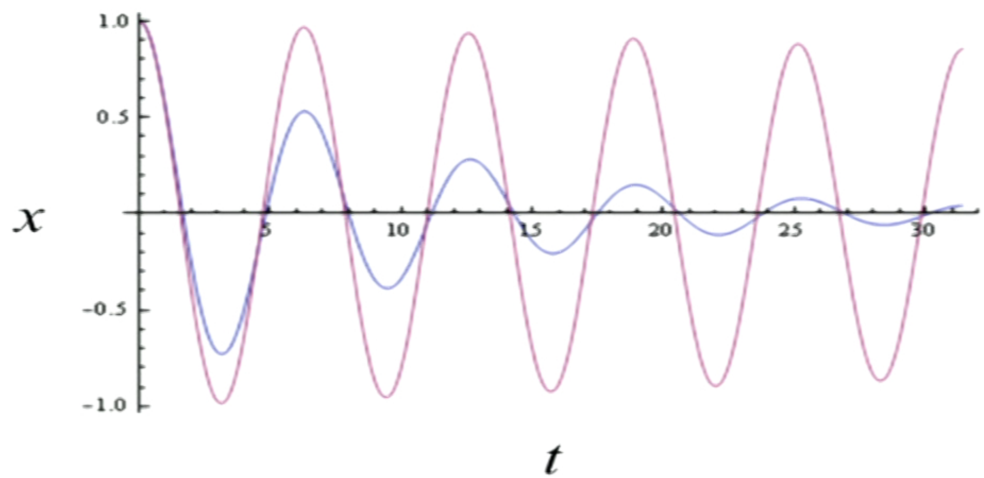

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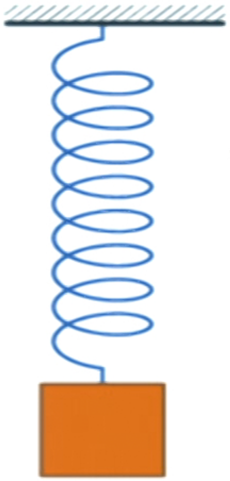

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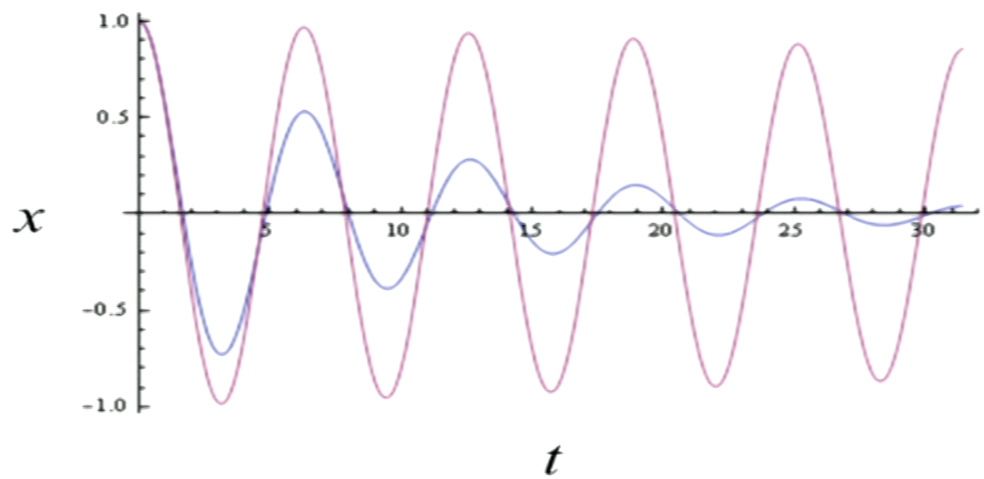

Plays crucial role in force detection

14

Cantilever response

$$\ddot{x} + \omega_0^2 x + \gamma\dot{x} = \frac{F(t)}{m}$$

For a harmonic driving force: $F(t) = f_0 e^{-i\omega t}$

$$-\omega^2 \ddot{x} + \omega_0^2 x - i\gamma\omega x = \frac{f_0}{m}$$

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Susceptibility:

$$\chi(\omega) = \frac{x(\omega)}{f(\omega)} = \frac{1}{m}\left(\frac{1}{-\omega^2 + \omega_0^2 - i\omega\gamma}\right)$$

$$\mathrm{DC}: \omega \to 0, x = \frac{f_0}{k}$$

$$\lim_{\omega \to \infty} \chi(\omega) = -\frac{1}{m\omega^2}$$

Cantilever response

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Steady state *x(t)* in presence of driving force

$$f_0 \cos(\omega t)$$

$$\begin{aligned} x(t) &= \mathrm{Re}\left[\chi(\omega) f_0 e^{-i\omega t}\right] \\ &= \mathrm{Re}[|\chi(\omega)| f_0 e^{-i\omega t} e^{i\phi}] \\ &= f_0 |\chi(\omega)| \cos(\omega t - \phi(\omega)) \end{aligned}$$

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$$|\chi(\omega)| = \frac{1}{m} \frac{1}{\sqrt{(\omega^2 - \omega_0^2)^2 + \omega^2\gamma^2}}$$

$$\tan\phi(\omega) = \frac{\omega\gamma}{\omega_0^2 - \omega^2}$$

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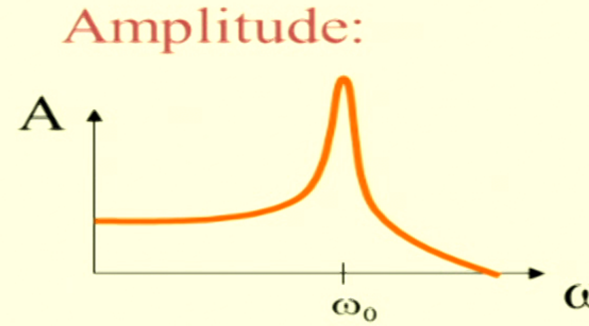

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Dissipation

$$\frac{dW}{dt} = F(t)\dot{x}(t)$$

Steady-state dissipated power:

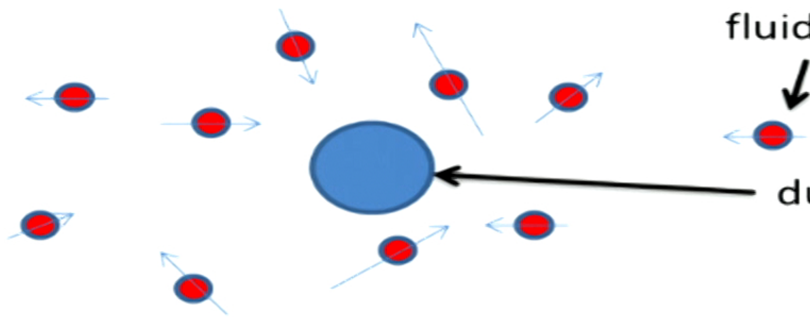
$$P = \frac{1}{2}\omega f_0^2 \left|\chi(\omega)\right| \sin\phi(\omega) = \frac{1}{2} f_0^2 \omega \,\mathrm{Im}[\chi(\omega)]$$

$$\mathrm{Im}[\chi(\omega)] = \frac{1}{m}\frac{\omega\gamma}{(\omega^2-\omega_0^2)^2+(\omega\gamma)^2}$$

On resonance: $$P = \frac{1}{2} f_0^2 \frac{1}{m\gamma}$$

Fundamental limitation: thermal noise

Brownian motion – random "kicks" given to particle due to thermal bath



- Random "kicks" are given to cantilever due to finite T of oscillator

$$\frac{1}{2}k\langle x^2\rangle = \frac{1}{2}k_B T \quad \Longrightarrow \quad F_{\min} = \left(\frac{4kk_B Tb}{Q\omega_0}\right)^{1/2}$$

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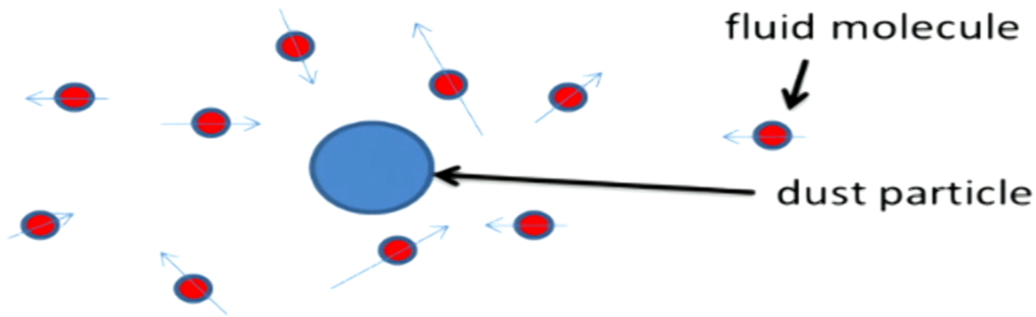

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Brownian thermal noise

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$$\overline{F} = 0$$

$$\overline{F^2} \neq 0$$

F(t) is random Langevin force,

x(t) varies at ω_0

Amplitude and phase varies on time scale $1/\gamma$

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Amplitude and phase varies on time scale $1/\gamma$

Heating rate:

$$\frac{d}{dt} < n > = -\gamma(< n > - \overline{n}_{th})$$

$$< n > (t) = \overline{n}_{th}(1 - \exp[-t / \gamma])$$

Near ground state thermalization occurs at rate: $\overline{n_{th}}\gamma = \frac{k_B T}{\hbar Q}$

Displacement spectral density

For a time series x(t):

$$\tilde{x}(\omega) = \frac{1}{\sqrt{\tau}} \int_0^{\tau} x(t) e^{i\omega t} dt$$

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$$S_{xx}(\omega) \equiv \int_{-\infty}^{+\infty} \langle x(t) x(0) \rangle \, e^{i\omega t} \, dt \, .$$

By Weiner-Khinchin Theorem
Assuming x is a stationary random process

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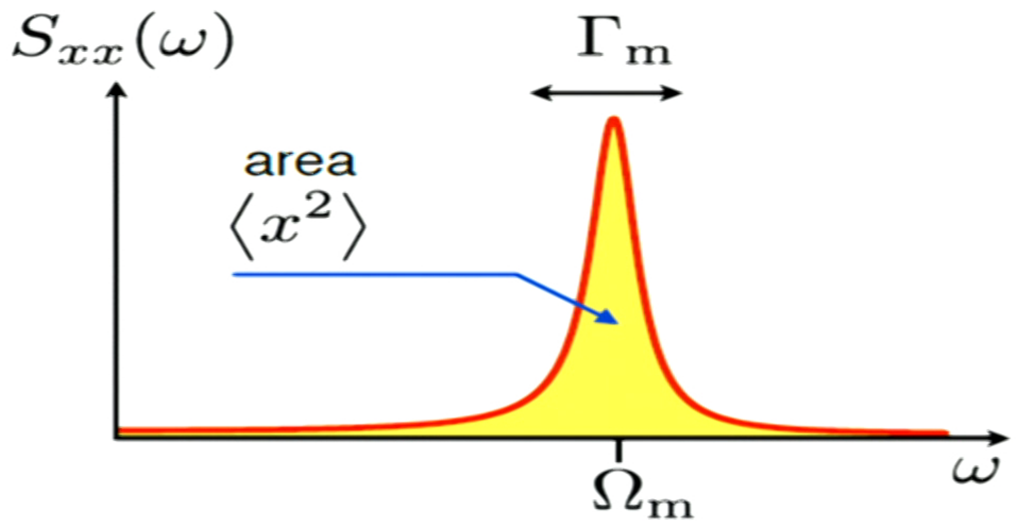

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$$\int_{-\infty}^{+\infty} S_{xx}(\omega) \frac{d\omega}{2\pi} = \langle x^2 \rangle$$

23

Fundamental limitation: thermal noise

- White noise background due to finite T of oscillator

$$S_X(\omega) = |\chi(\omega)|^2 S_F$$

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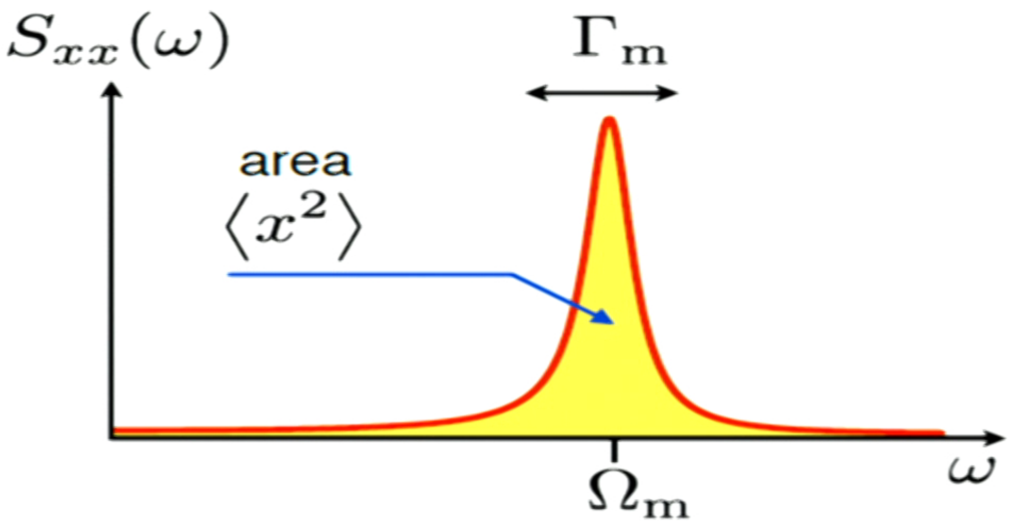

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Fundamental limitation: thermal noise

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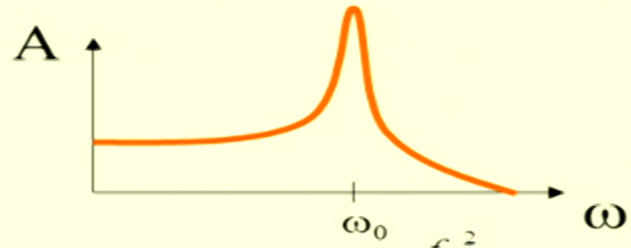
$$S_F^{1/2} = \left(\frac{2}{\pi Q f_0} \right)^{1/2} k x_{rms}$$



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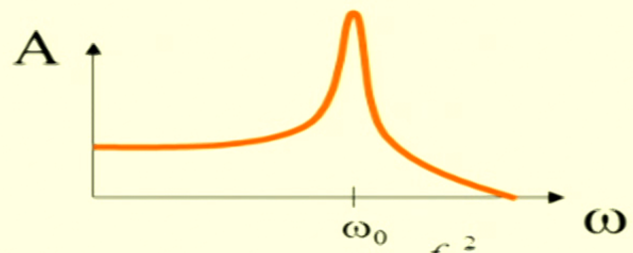

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$$\frac{1}{2} k \langle x^2 \rangle = \frac{1}{2} k_B T \Rightarrow S_F^{1/2} = \left(\frac{4 k k_B T}{Q \omega_0} \right)^{1/2}$$

$$F_{min} = S_F^{1/2} B^{1/2} \Rightarrow F_{\min} = \left(\frac{4 k k_B T b}{Q \omega_0} \right)^{1/2}$$

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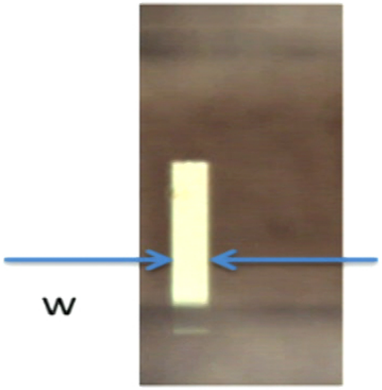
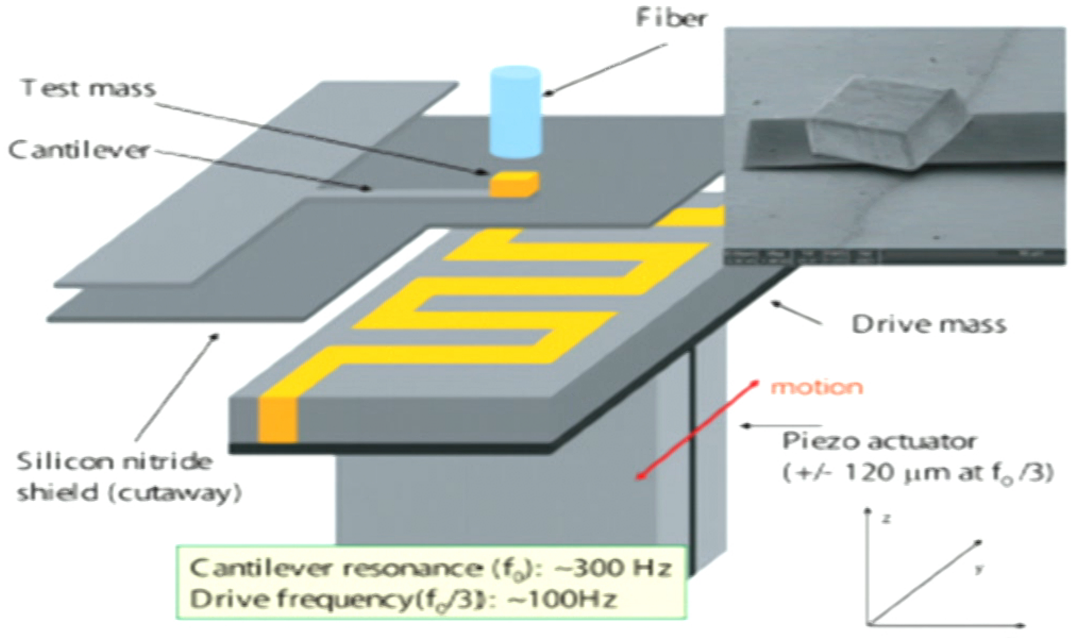
Stanford cantilever experiment



w= 50 μm
l= 250 μm
t=0.3 μm

Silicon Cantilevers:
$F_{min} \sim 10 \times 10^{-18}$ N/√Hz
at 4 K at Q=10^5



Best Yukawa constraints at ~ 10 um range:

A.A. Geraci, S.J. Smullin, D. M. Weld, J. Chiaverini, and A. Kapitulnik, *Phys. Rev. D 78*, 022002 (2008).

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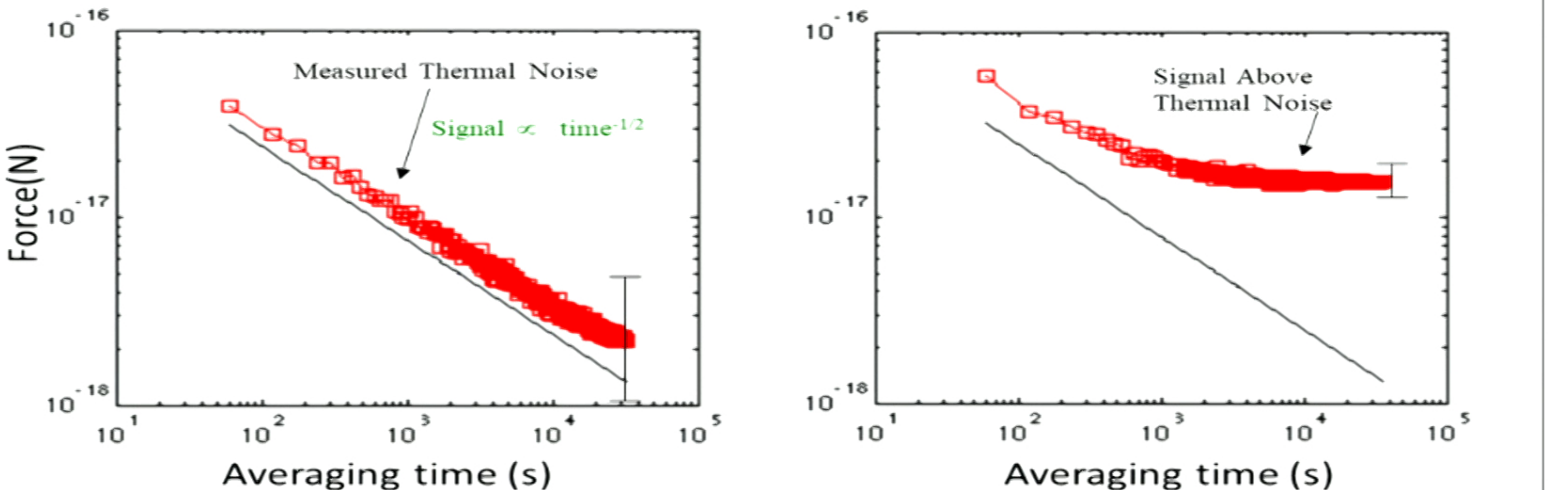
Averaging Data



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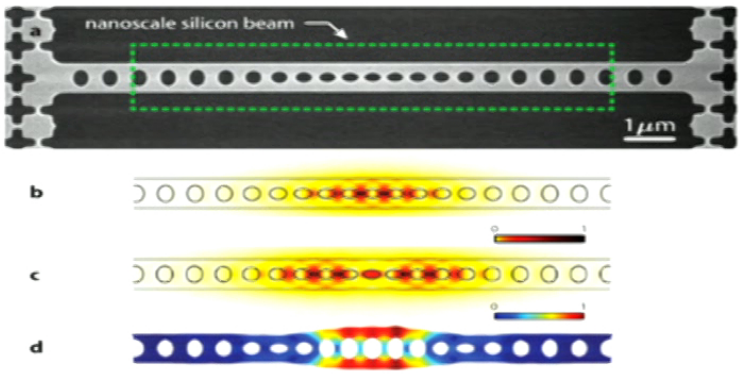
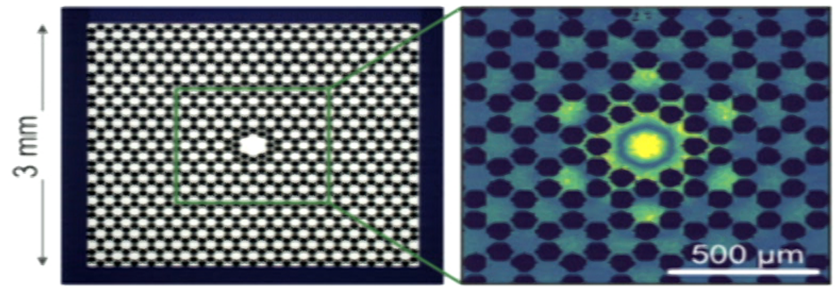
Advances in cryogenic nano-oscillators

Significantly improved sensitivities (higher frequencies)





Schliesser group, Copenhagen

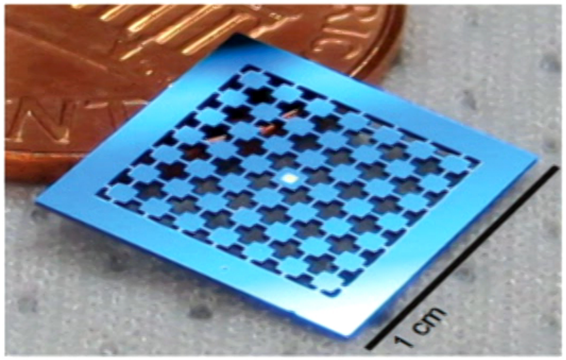
Painter group, Caltech

Si:
freq=5 GHz
$Q_m = 5\text{X}10^{10}$
mass =136 fg
T=60 mK

Also nanotubes:

J. Moser, J. Guttinger, A. Eichler, M. J. Esplandiu, D. E. Liu, M. I. Dykman, and A. Bachtold, Nat. Nanotechnol. **8**, 493 (2013).

$\sim 10\ \text{zN}/\sqrt{\text{Hz}}$



Regal group, JILA

SiN:
freq=1.5 MHz
$Q_m = 2\text{X}10^{8}$
mass=10 ng
T=30 mK

Fluctuation-Dissipation theorem

- In equilibrium, thermal fluctuations are related to dissipation:

$$S_F^{1/2} = \left(\frac{4kk_B T}{Q\omega_0} \right)^{1/2}$$

$$S_F = 4 k_B T \, m\Gamma$$
$$\Gamma = \omega_0 / Q$$

e.g. Johnson noise in a resistor: $S_V = 4 k_B T R$

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$$\Gamma = \omega_0 / Q$$

e.g. Johnson noise in a resistor: $S_V = 4 \, k_B T \, R$

FDT: $S_X = 2k_B T \dfrac{\mathrm{Im}[\chi(\omega)]}{\omega}$ $\quad \mathrm{Im}[\chi(\omega)] = \dfrac{1}{m} \dfrac{\omega\gamma}{(\omega^2 - \omega_0^2)^2 + (\omega\gamma)^2}$

$$S_X(\omega) = |\chi(\omega)|^2 S_F$$

Quantum limit: T→0

- Zero-point fluctuations

$$S_X = 2k_B T \frac{\mathrm{Im}[\chi(\omega)]}{\omega} \quad \rightarrow \quad \hbar \coth\left(\frac{\hbar\omega}{2k_B T}\right)\mathrm{Im}[\chi(\omega)]$$

$$x_{\mathrm{ZPF}} = \sqrt{\frac{\hbar}{2m_{\mathrm{eff}}\Omega_{\mathrm{m}}}}$$

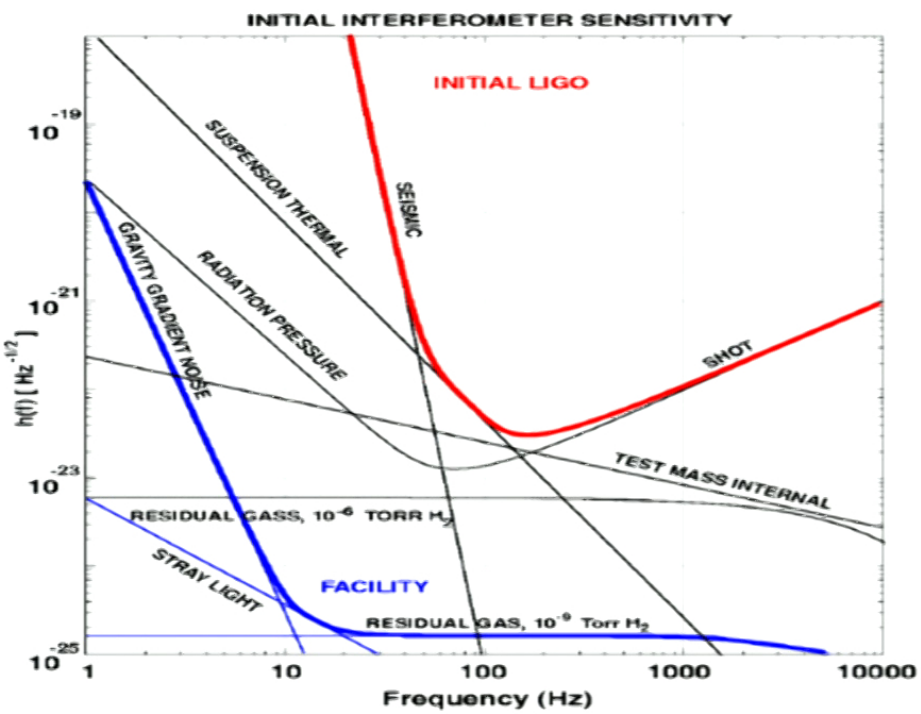
Shot noise

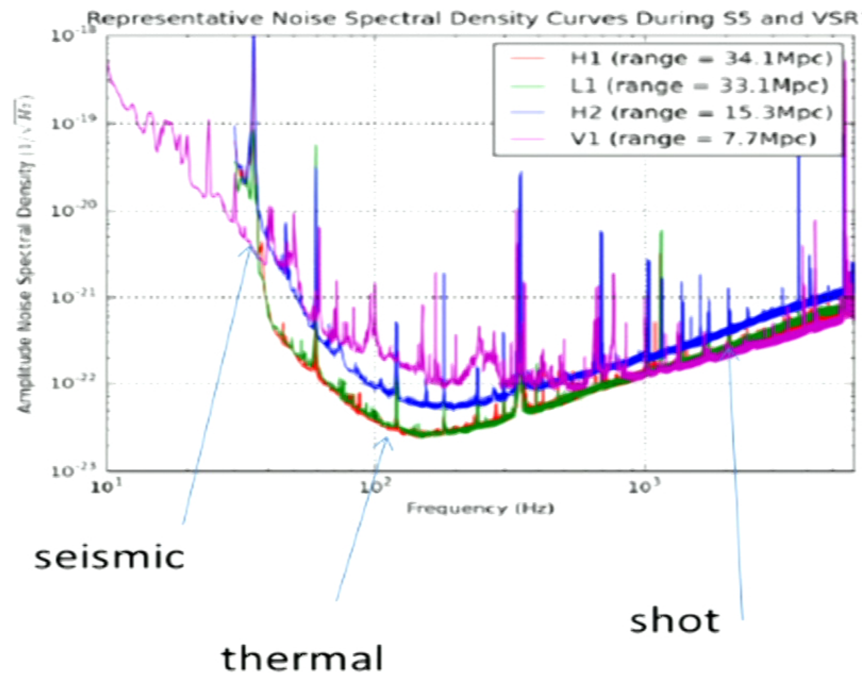
- Apart from Brownian thermal noise, still need to resolve displacement of oscillator!

Shot noise

- Apart from Brownian thermal noise, still need to resolve displacement of oscillator!
- Consider optical detection
- Discrete nature of photons imposes measurement imprecision

Sensitivity of LIGO





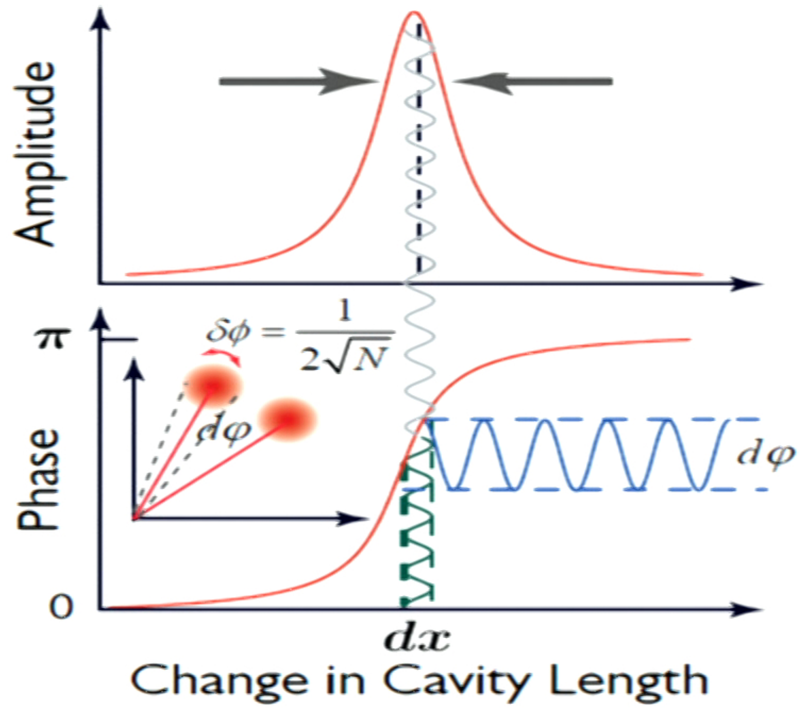
Alan Weinstein, notes
caltech.edu/laac/undergraduate_resources.shtml

LIGO-T0900499

Shot-noise limited measurement

- Optical cavity readout

$$\delta\phi \approx \frac{1}{\sqrt{N}}$$



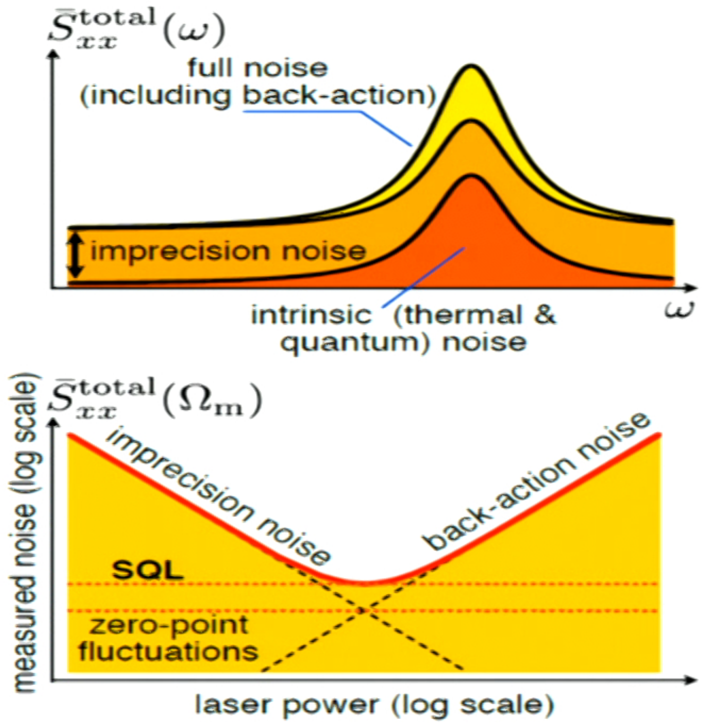
M. Aspelmeyer, T. Kippenberg, F. Marquardt, Arxiv: 1303.0377 (2013).

Radiation pressure shot noise

- Cannot increase N arbitrarily
- As we measure position more precisely, momentum uncertainty affects position at later time by Heisenberg
- Backaction $\Delta x_{rp} \propto \sqrt{N}$

C. Caves, PRL, 45, 75 (1990).

Standard quantum limit



$$\bar{S}_{\text{xx}}^{\text{total}}(\omega) = \bar{S}_{\text{xx}}^{\text{th}}(\omega) + \bar{S}_{\text{xx}}^{\text{imp}}(\omega) + \bar{S}_{\text{FF}}(\omega)\,|\chi_{xx}(\omega)|^2$$

thermal — Shot noise — Radiation Pressure shot noise

$$\bar{S}_{\text{xx}}^{\text{imp}}(\omega) \cdot \bar{S}_{\text{FF}}(\omega) \geq \frac{\hbar^2}{4}$$

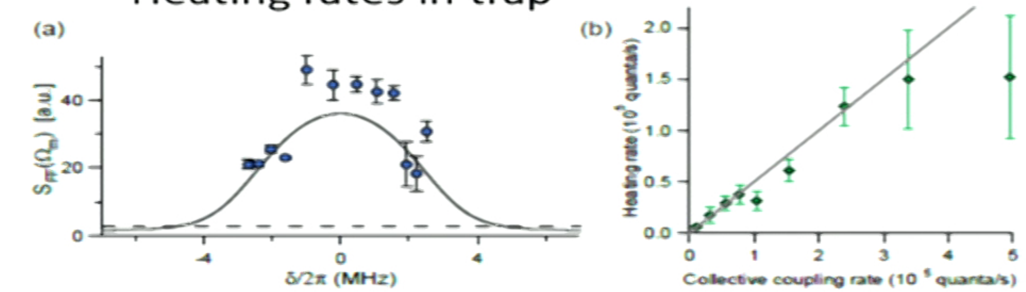
M. Aspelmeyer, T. Kippenberg, F. Marquardt, Arxiv: 1303.0377 (2013).

Experimental measurement of radiation pressure back action

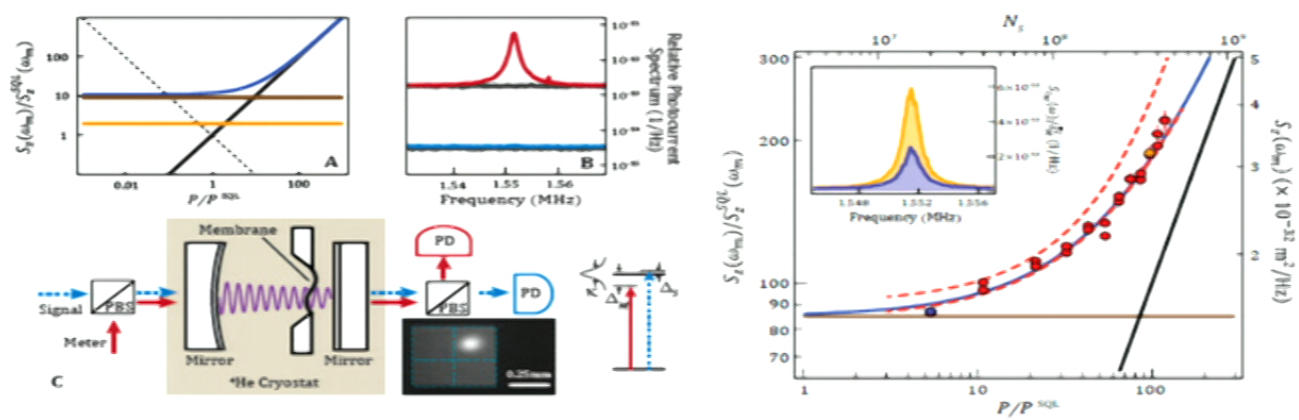
- Atoms

Murch, K. W.,et.al.,
Nature Phys. **4**, 561 (2008).

Heating rates in trap



- Membranes



Purdy et. al, Science 339, 801 (2013)

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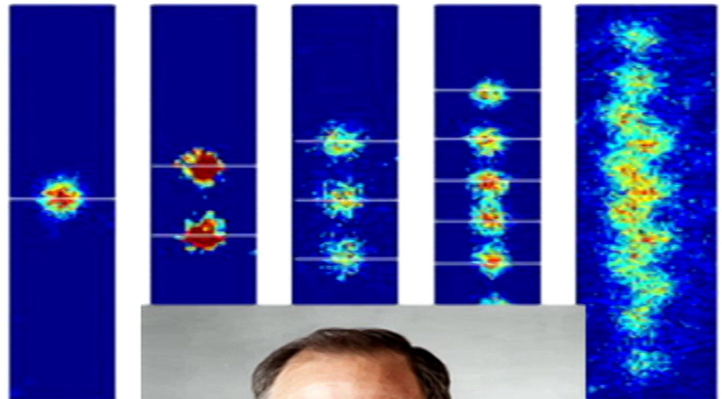
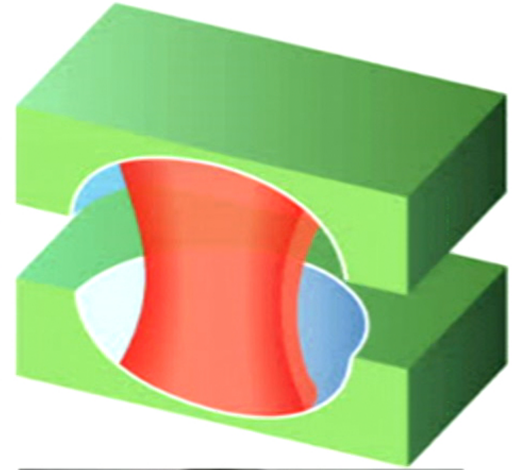
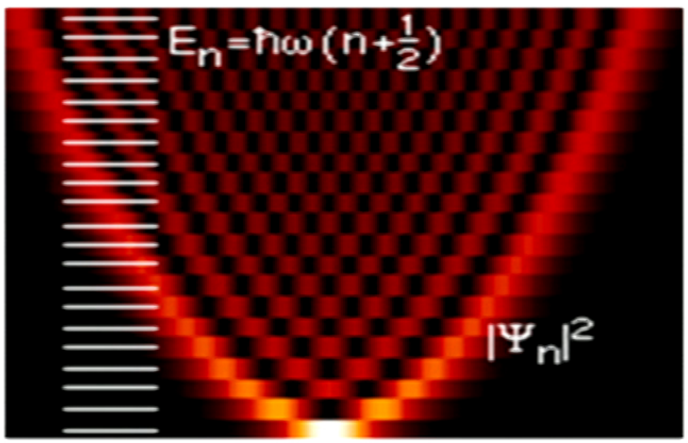
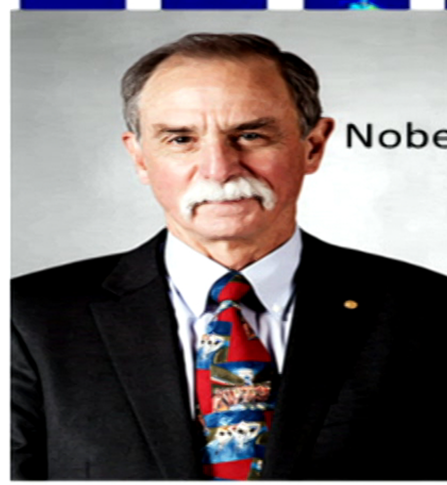
Quantum Regime





High fidelity quantum control:
Internal states $|\uparrow\rangle, |\downarrow\rangle$
motional states
Long coherence times





Nobel Prize 2012

"for ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems"

https://commons.wikimedia.org/wiki
/File:QHarmonicOscillator.png#/media/File:QHarmonicOscillator.png

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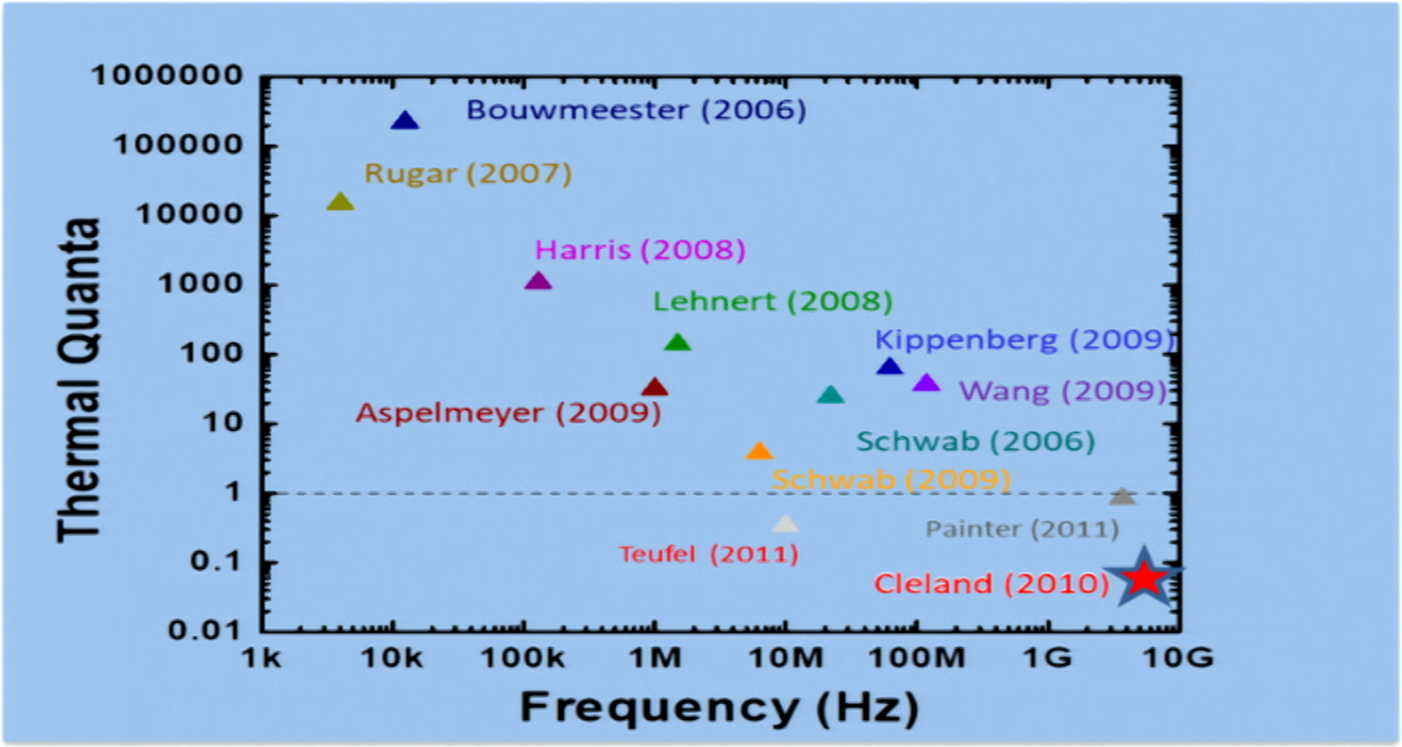
Quantum Regime

Ground state cooling of solid-state mechanical resonators

- Cryogenic cooling
- Feedback cooling
- Passive back-action cooling



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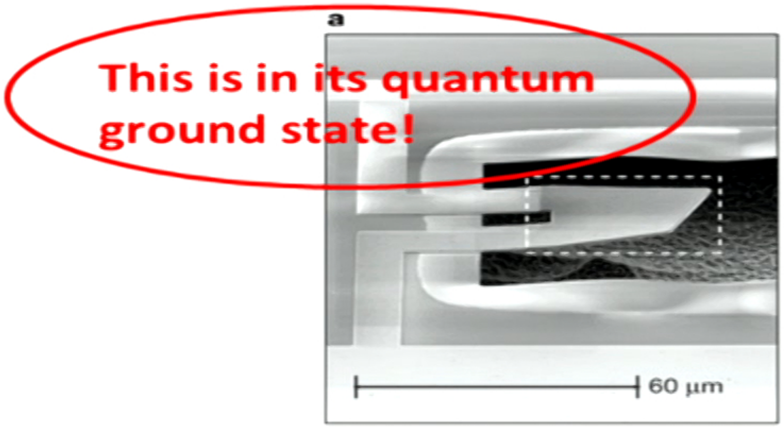
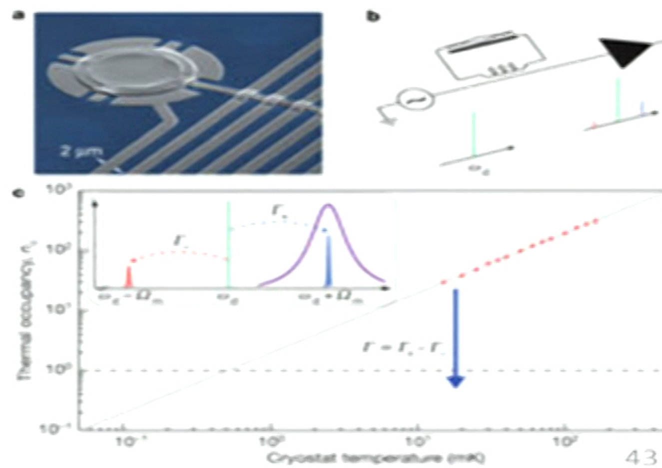
Quantum "Mechanics"



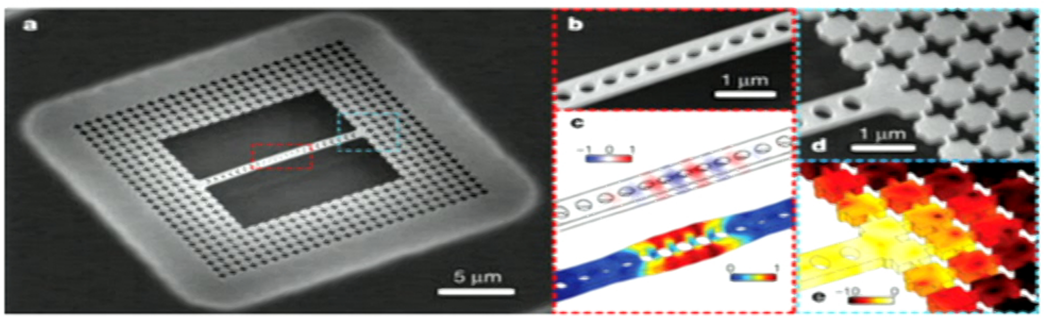
Quantum ground state and single-phonon control of a mechanical resonator
A. D. O'Connell et.al.
Nature 464, 697 (2010).

$$k_B T << \hbar\omega$$

Sideband cooling of micromechanical motion to the quantum ground state
J. D. Teufel,[1] *et.al.* Nature 475, 359 (2011).



Laser cooling of a nanomechanical oscillator into its quantum ground state
Jasper Chan,[1] *et.al.* Nature 478, 89–92(2011)



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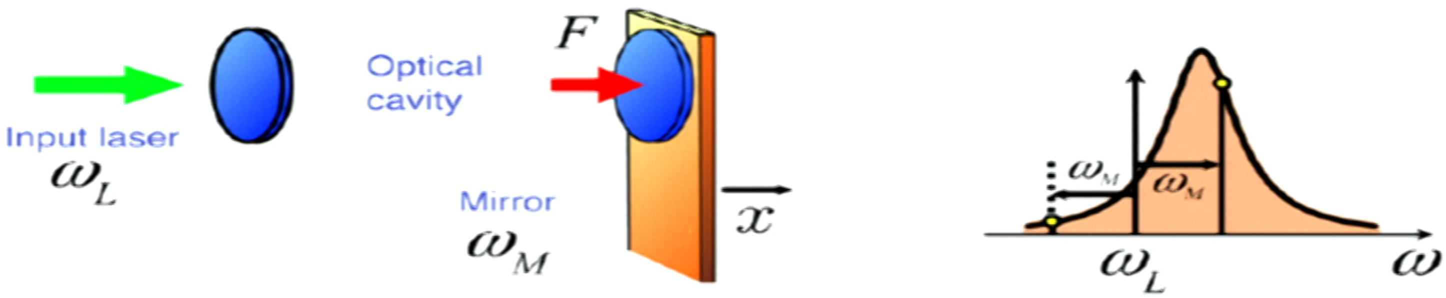
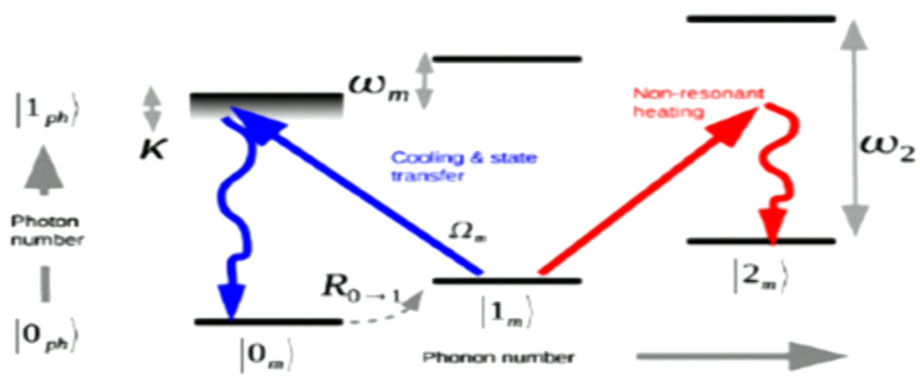
Technique for reaching quantum regime: Cavity-Cooling

$$-2\hbar g_2 k_2 \hat{x}\hat{a}_2^+\hat{a}_2 = -\hat{F}\hat{x}$$



F. Marquardt and S. Girvin, *Physics* 2009



D.E. Chang *et. al.*, PNAS 2009 44

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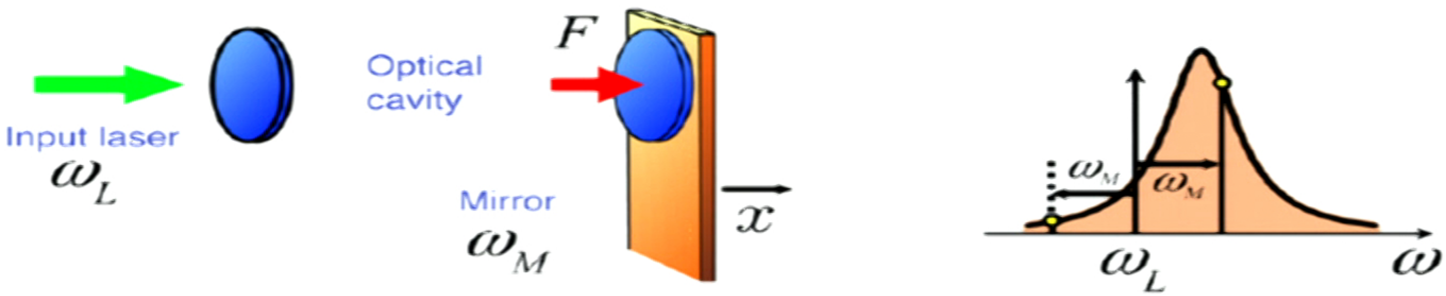
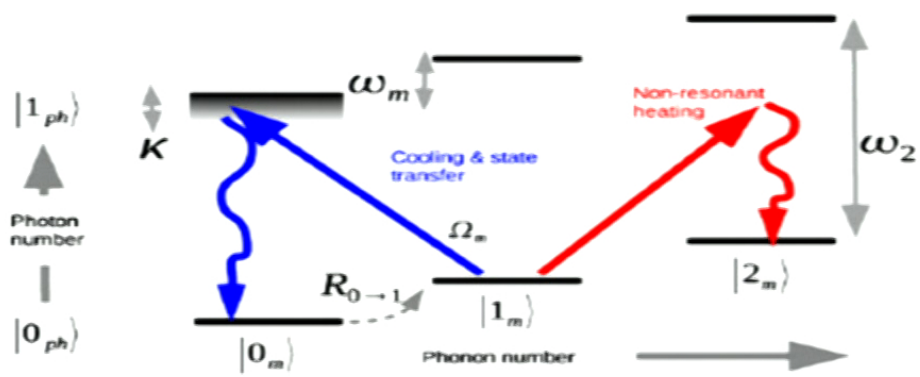


F. Marquardt and S. Girvin, *Physics* 2009



D.E. Chang *et. al.*, PNAS 2009 44

Laser-cooled force sensitivity

$$F_{\min} = \left(\frac{4kk_B T b}{Q\omega_0} \right)^{1/2} \quad \Rightarrow \quad F_{\min} = \left(\frac{4kk_B T_{\text{eff}} b}{Q_{\text{eff}}\omega_0} \right)^{1/2}$$

Damping: $$Q_{\text{eff}} = \frac{\omega_0}{\Gamma_{\text{opt}} + \Gamma_{\text{m}}} \approx Q \frac{\Gamma_{\text{m}}}{\Gamma_{\text{opt}}}$$

Cooling: $$\frac{k_B T_{\text{eff}}}{\hbar\omega_0} \approx n_f \approx \frac{\Gamma_m}{\Gamma_{\text{opt}}} n_i$$

Laser cooled force sensitivity

$$F_{\min} = \left(\frac{4kk_B T b}{Q\omega_0} \right)^{1/2} \rightarrow F_{\min} = \left(\frac{4kk_B T_{\text{eff}} b}{Q_{\text{eff}}\omega_0} \right)^{1/2}$$

Damping: $$Q_{\text{eff}} = \frac{\omega_0}{\Gamma_{\text{opt}} + \Gamma_{\text{m}}} \approx Q \frac{\Gamma_{\text{m}}}{\Gamma_{\text{opt}}}$$

Cooling: $$\frac{k_B T_{\text{eff}}}{\hbar\omega_0} \approx n_f \approx \frac{\Gamma_m}{\Gamma_{\text{opt}}} n_i + \frac{A_+}{\Gamma_{\text{opt}}}$$

Limit from laser cooling

Resolved sideband regime

$$\bar{n}_{\min} = \left(\frac{\kappa}{4\Omega_{\text{m}}} \right)^2 < 1$$

Environmental decoupling

- Essential for preserving quantum coherence in QI applications
- Large mechanical Q factors of CM mode
- Low thermalization rate:

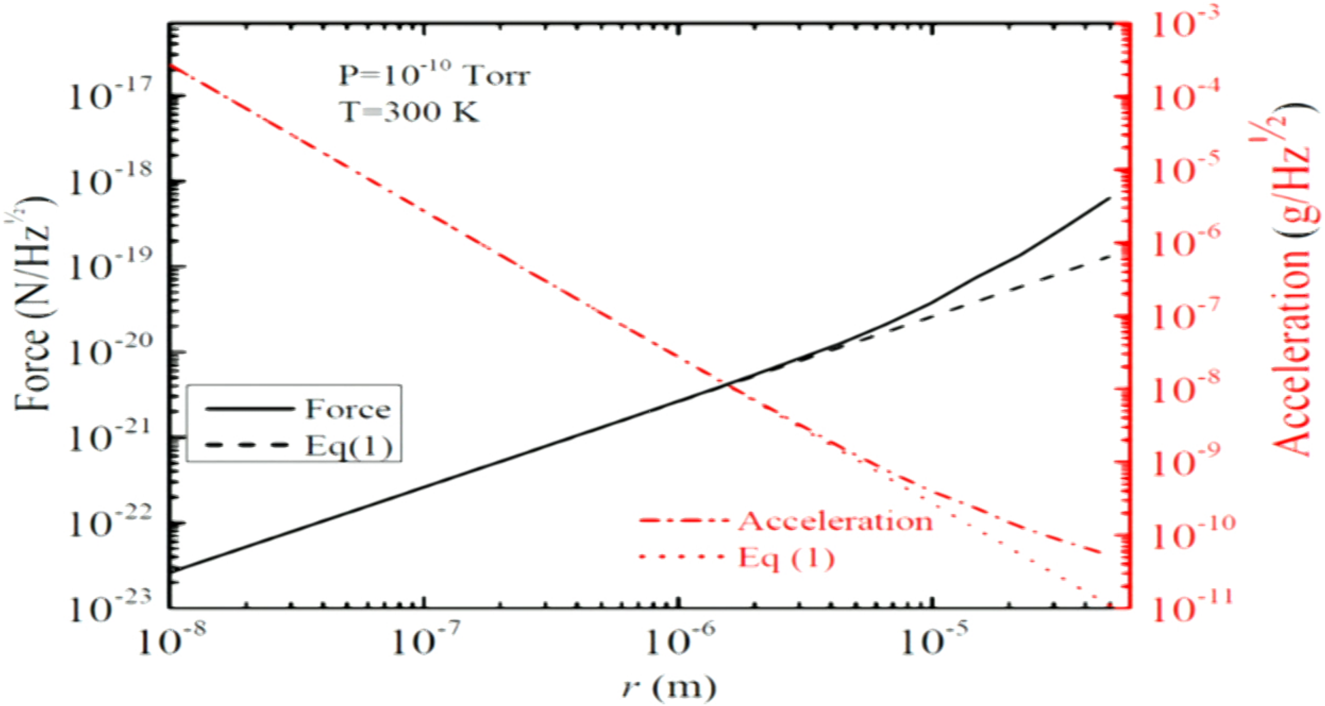
$$\Gamma_{th} = \frac{k_B T}{\hbar Q} = n_m \Gamma_m$$

→Excellent candidate for ground state cooling

$$\frac{n_i}{n_f} < Q$$

Projected sensitivity

$$F_{min} = (4 k_B T \gamma m)^{1/2} \quad (1)$$



Z. Yin, A. Geraci, T. Li, Int. J. Mod. Phys. B 27,1330018 (2013).

Summary

- Fluctuation Dissipation theorem - useful to understand measurement limitations

→ Thermal noise in optical coatings

Clocks, LIGO

→ Force sensing limits

→ Mechanical systems in quantum regime

Ground state cooled oscillators

Radiation pressure shot noise, Adv Ligo?

Levitated optomechanical systems?

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