Title: Blandford-Znajek process without plasma

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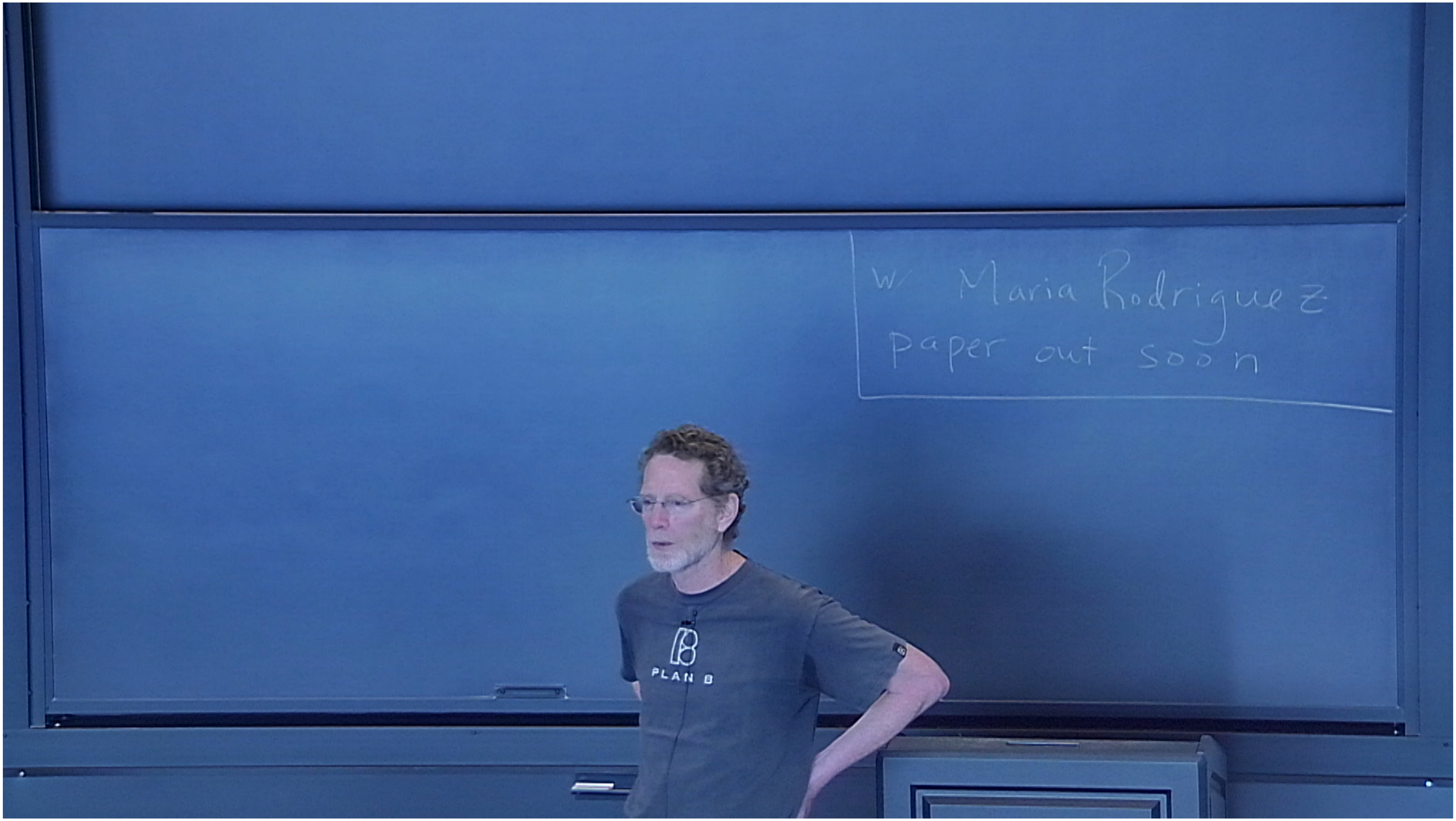
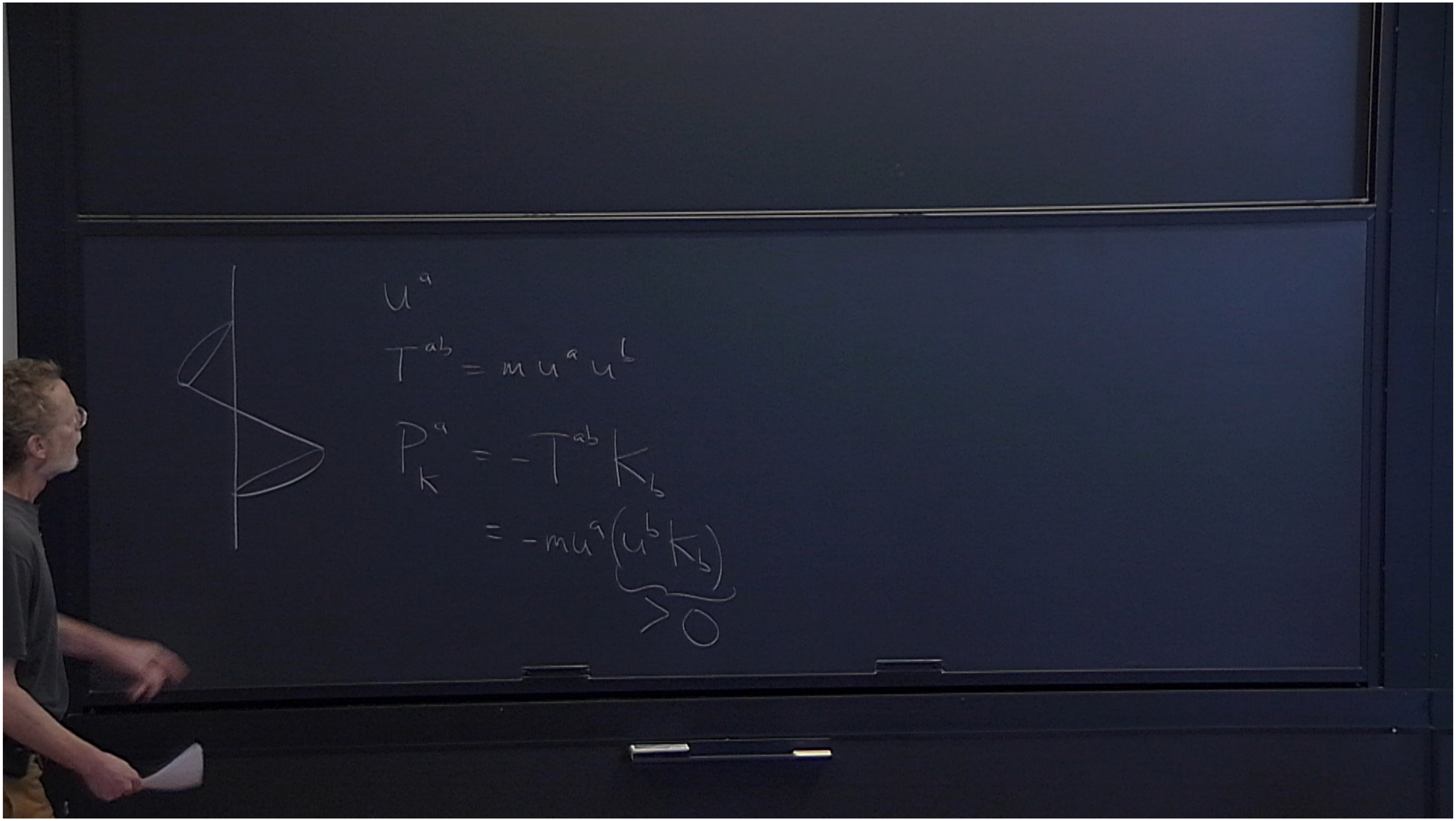
URL: http://pirsa.org/17080014

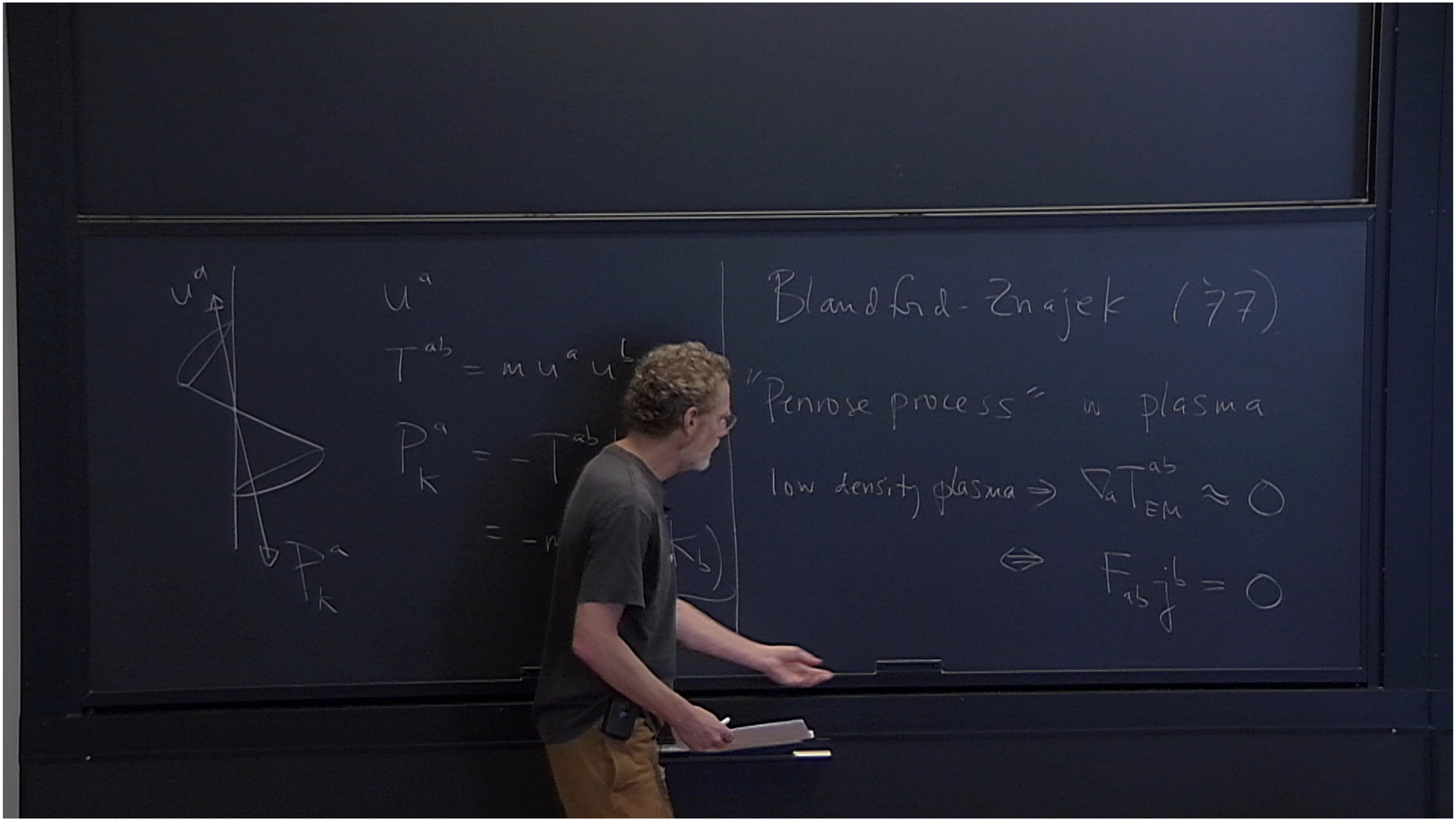
Abstract: <p>In 1977, Blandford and Znajek discovered a process by which a spinning </p>

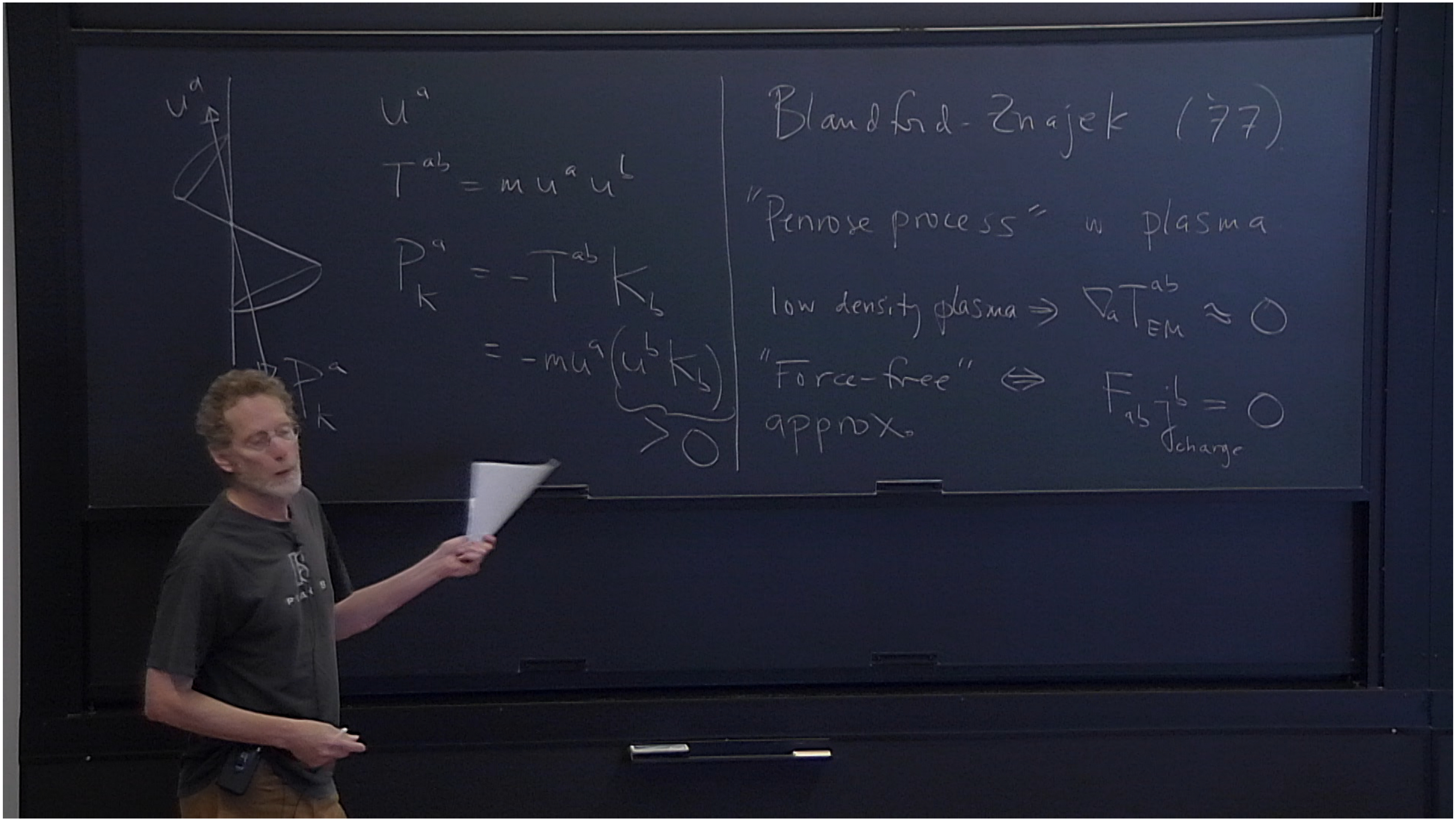
<p>black hole can transfer rotational energy to a force-free plasma, offering a possible mechanism for energy and jet emissions from quasars and other astrophysical sources. This Blandford-Znajek (BZ) mechanism is a Penrose process, which exploits the presence of an ergosphere supporting negative energy states, and it involves currents of electrical charge sourcing the toroidal magnetic field component of the emitted Poynting flux. </p>

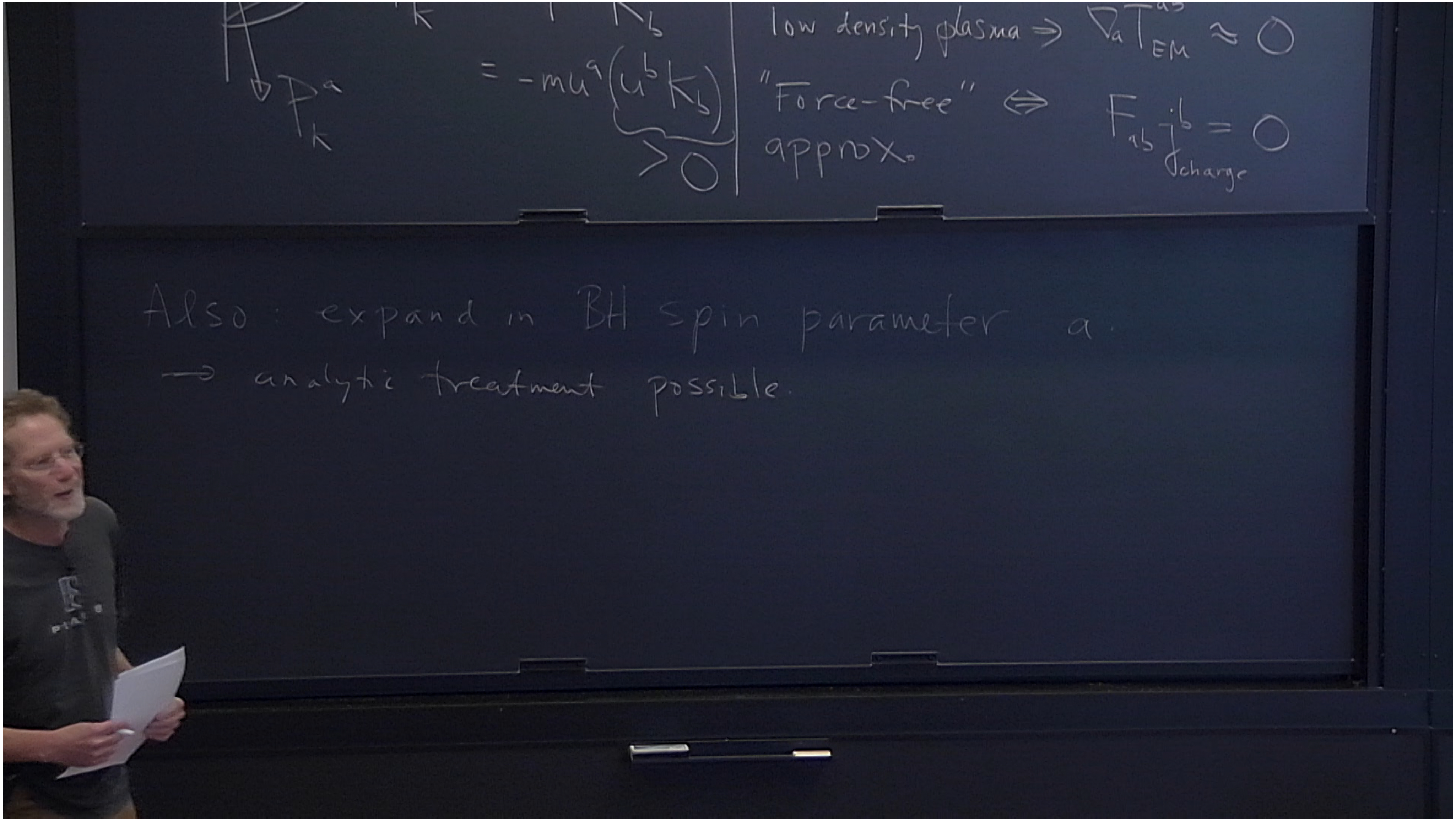
<p>In this talk, I will discuss a version of the BZ process requiring only vacuum electromagnetic fields outside the black hole. The setting is somewhat artificial, since it assumes the black hole is cylindrical rather than spheroidal, or that the black hole lives in 2 spatial dimensions, but it is nevertheless of theoretical interest. The radiation power and horizon regularity relations are identical to those of the BZ mechanism with plasma, and the solution can be given in simple, closed form for a wide class of metrics, so it helps to illuminate the nature of the original mechanism. For asymptotically Anti-de Sitter black holes it presumably has an interesting dual CFT description, but we haven't quite yet figured out what that is.</p>

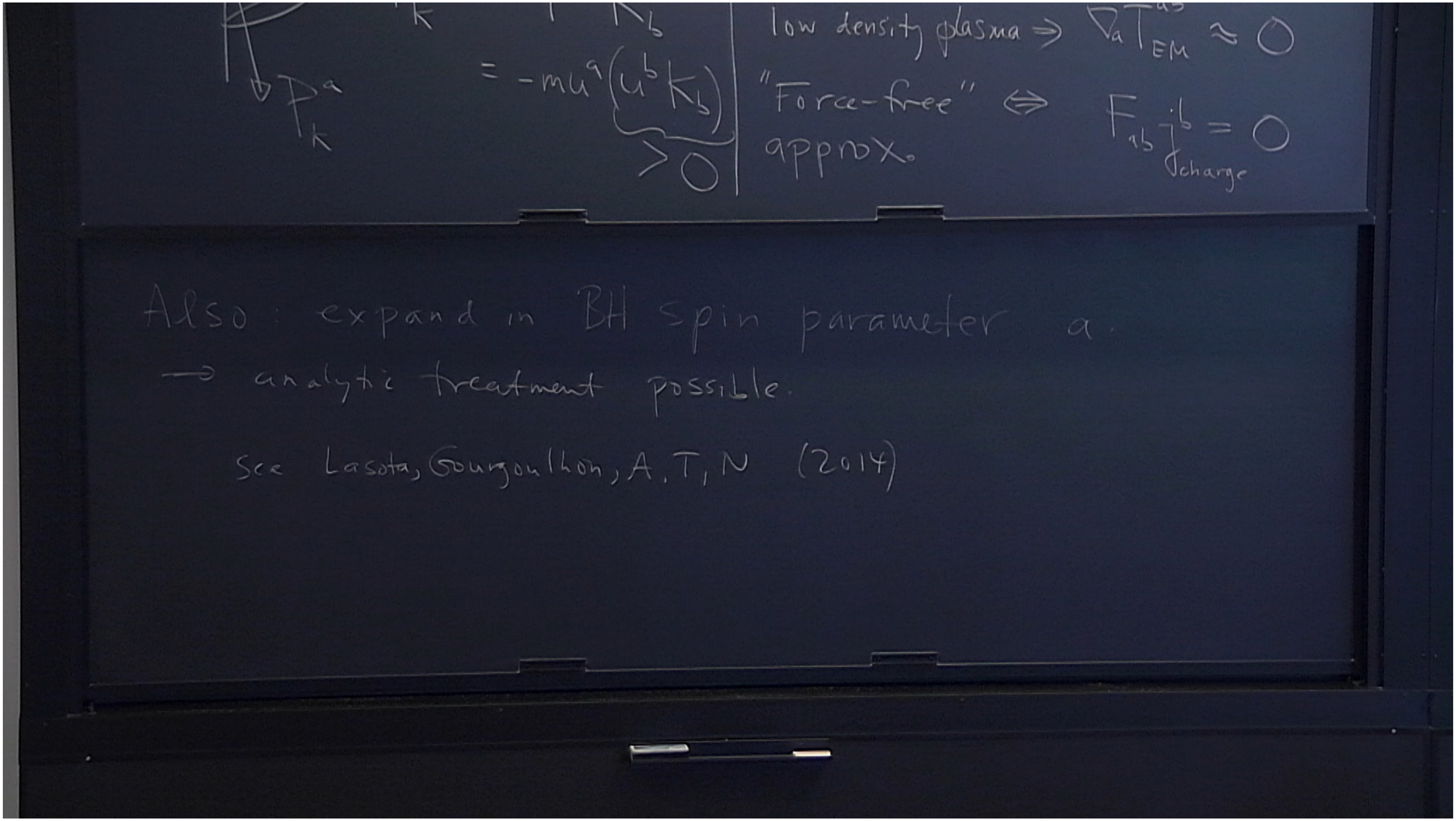

w/ Maria Rodriguez
paper out soon
PLAN B

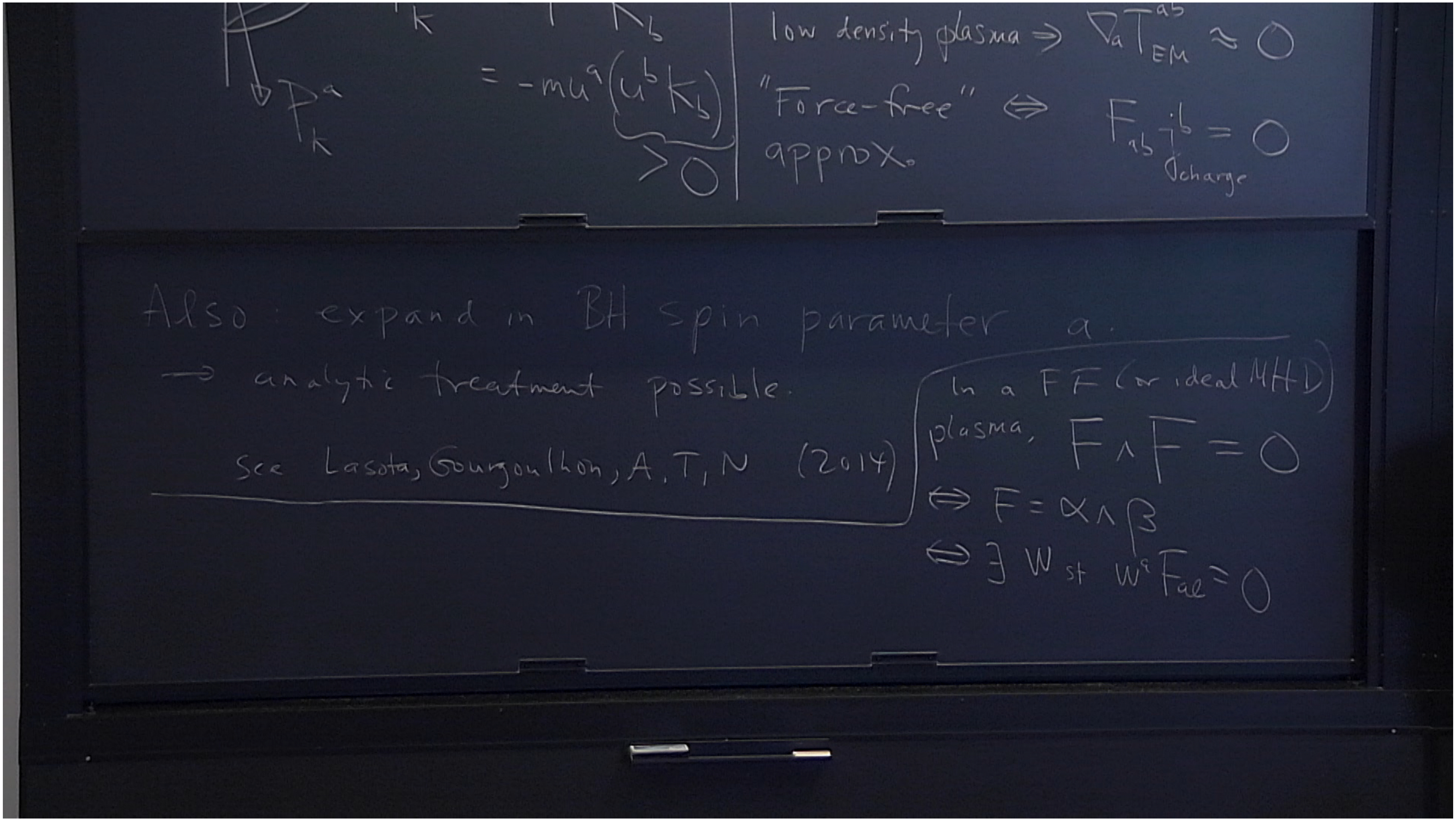

u^a
$T^{ab} = m u^a u^b$
$P_K^a = -T^{ab} K_b$
$= -m u^a (u^b K_b)$
> 0

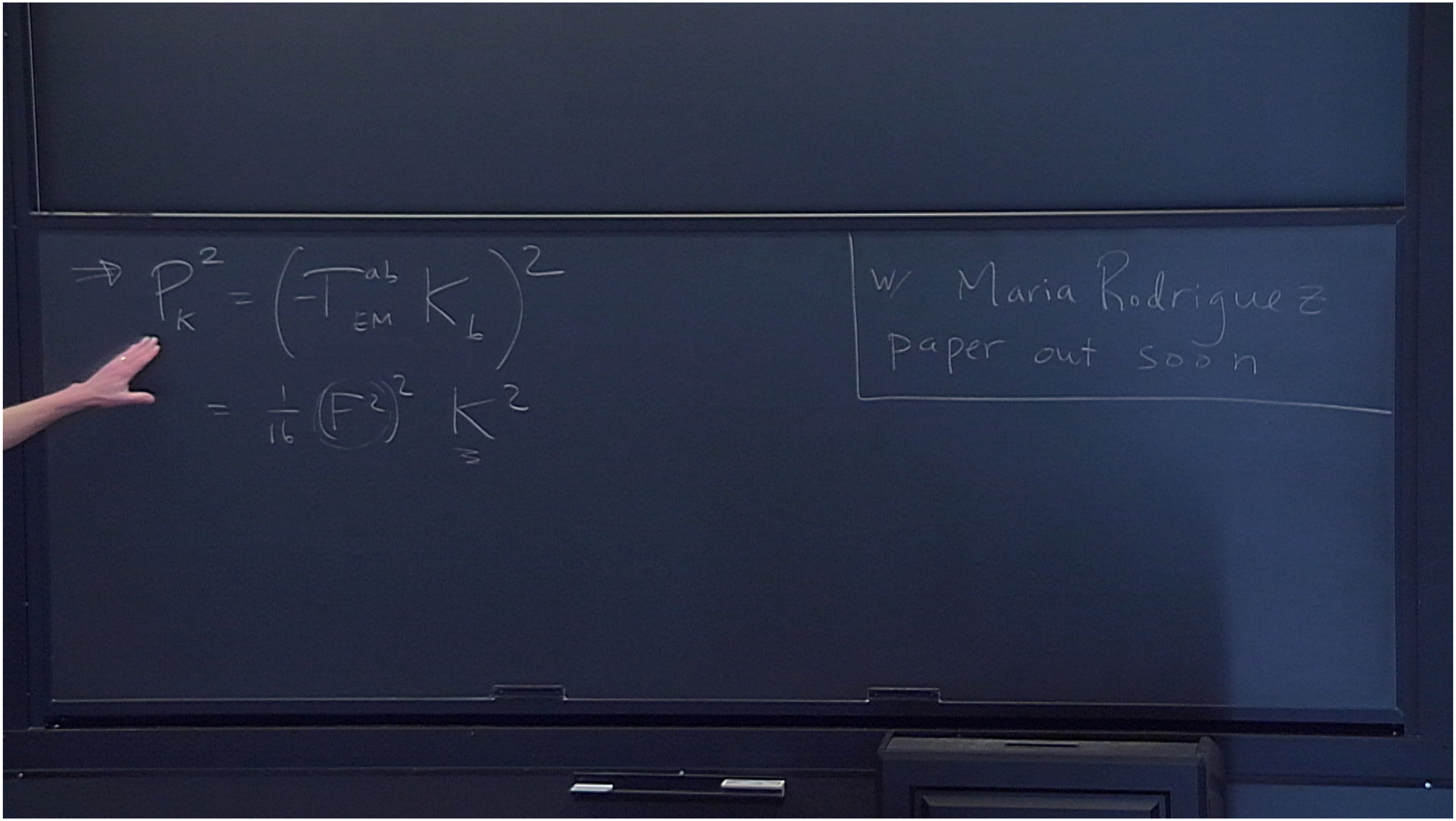

Blandford-Znajek ('77)
"Penrose process" in plasma
low density plasma ⇒ $\nabla_a T^{ab}_{EM} \approx 0$
⇔ $F_{ab} j^b = 0$
u^a
$T^{ab} = m u^a u^b$
P^a_k


u^a
P_k^a
u^a
$T^{ab} = m u^a u^b$
$P_k^a = -T^{ab} K_b$
$= -m u^a (u^b K_b)$
> 0
Blandford-Znajek ('77)
"Penrose process" in plasma
low density plasma $\Rightarrow \nabla_a T^{ab}_{EM} \approx 0$
"Force-free" $\Leftrightarrow F_{ab} j^b = 0$
approx.
j_{charge}


low density plasma ⇒ $\nabla_a T^{ab}_{EM} \approx 0$
"Force-free" ⇔ $F_{ab} j^b_{charge} = 0$
approx.
$= -mu^a(u^b K_b)$
> 0
Also: expand in BH spin parameter a.
→ analytic treatment possible.


= -mu^a (u^b K_b) > 0
low density plasma ⇒ ∇_a T_EM^ab ≈ 0
"Force-free" ⇔ F_ab j^b = 0
approx.
j_charge
Also: expand in BH spin parameter a.
→ analytic treatment possible.
See Lasota, Gourgoulhon, A, T, N (2014)


= -mu^a (u^b k_b)
> 0
low density plasma ⇒ ∇_a T^ab_EM ≈ 0
"Force-free" ⇔ F_ab j^b = 0
approx.
j_charge
Also: expand in BH spin parameter a.
→ analytic treatment possible.
See Lasota, Gourgoulhon, A, T, N (2014)
In a FF (or ideal MHD) plasma, F ∧ F = 0
⇔ F = α ∧ β
⇔ ∃ W st W^a F_ae = 0


$\Rightarrow P_K^2 = \left(-T_{EM}^{ab} K_b\right)^2$
$= \frac{1}{16} (F^2)^2 K^2$
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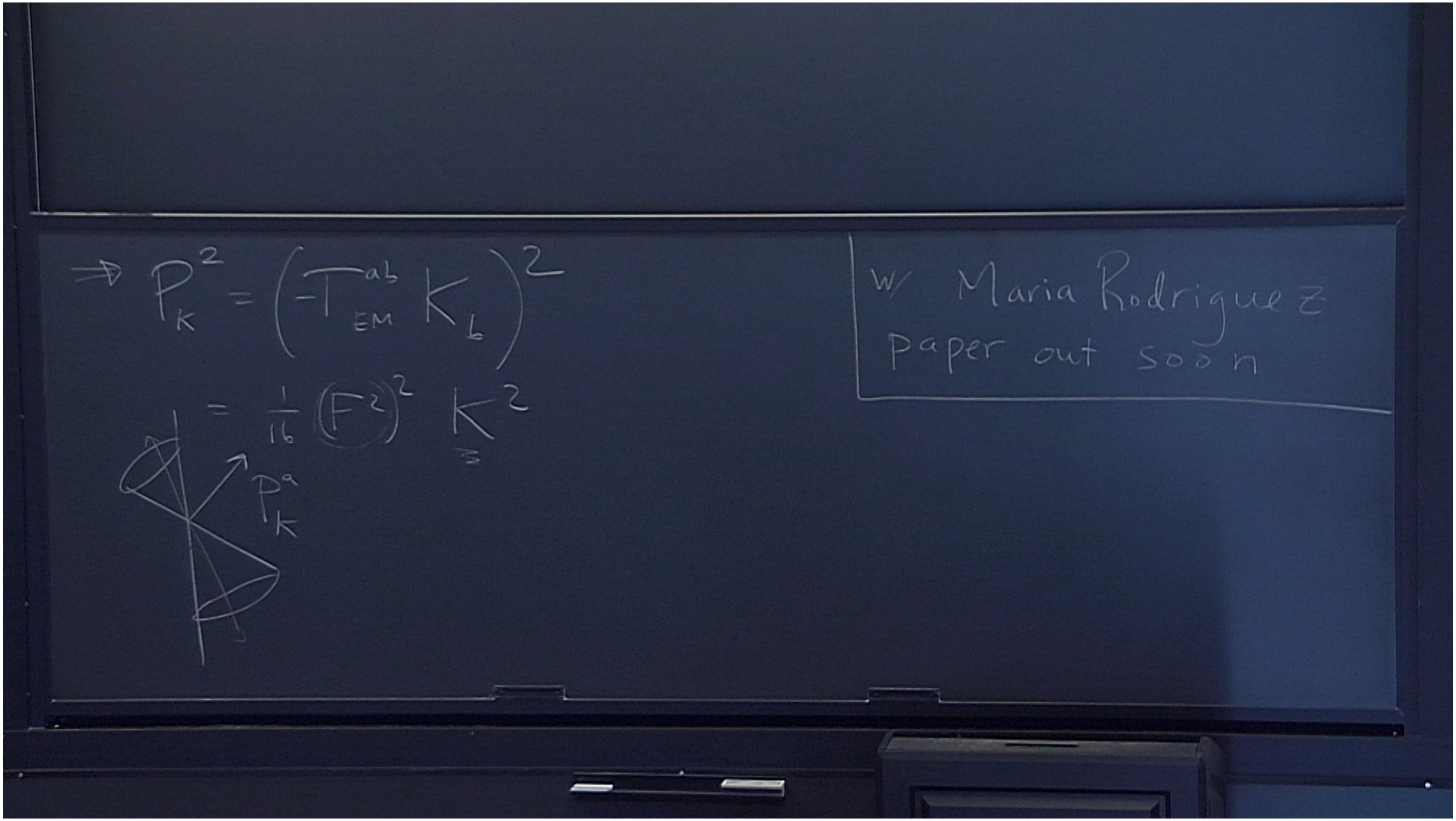
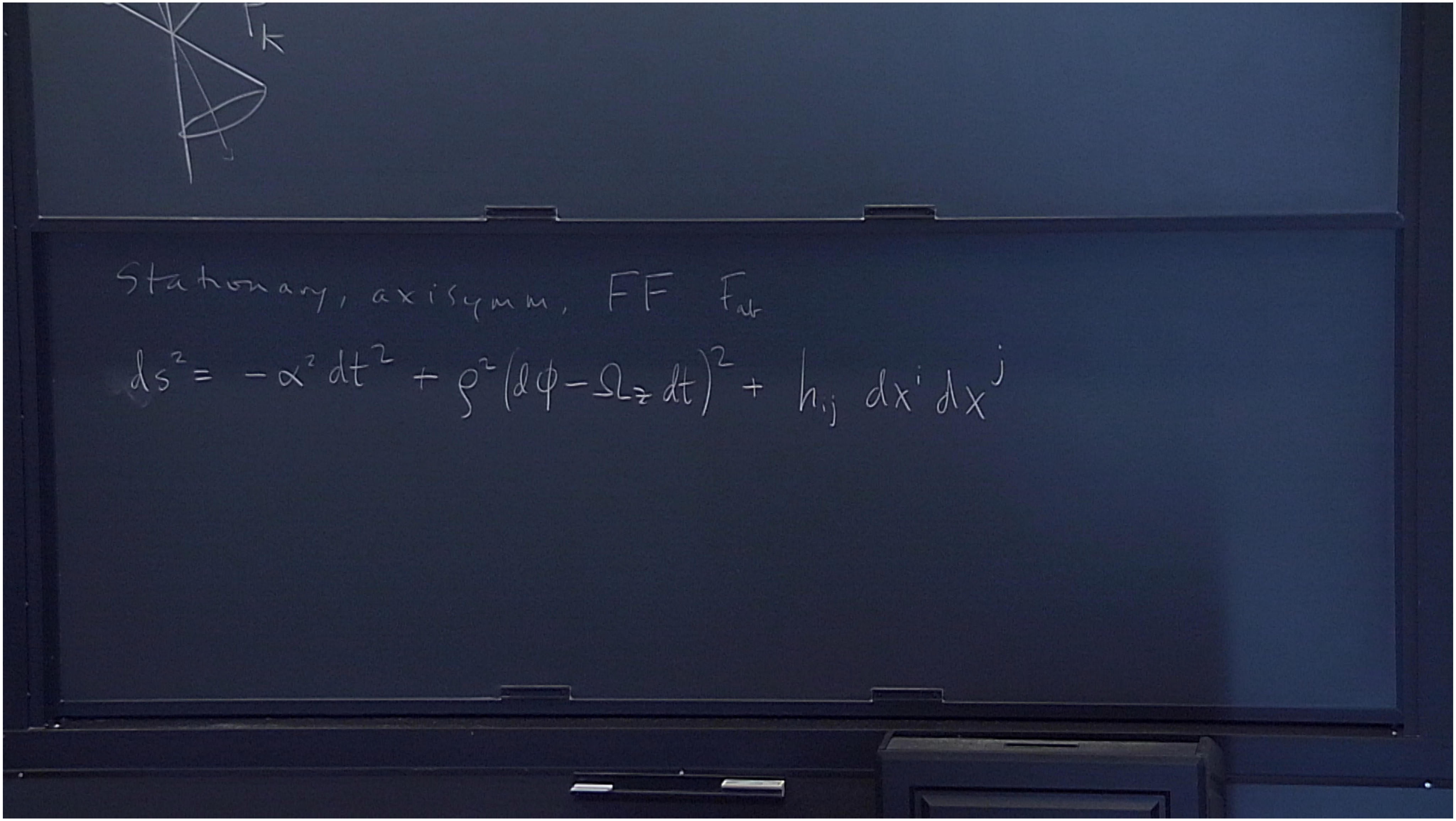
paper out soon



$\Rightarrow P_K^2 = \left(-T_{EM}^{ab} K_b\right)^2$
$= \frac{1}{16} (F^2)^2 K^2$
P_K^a
w/ Maria Rodriguez
paper out soon


Stationary, axisymm, FF F_{ab}
$ds^2 = -\alpha^2 dt^2 + \rho^2 (d\phi - \Omega_z dt)^2 + h_{ij} dx^i dx^j$

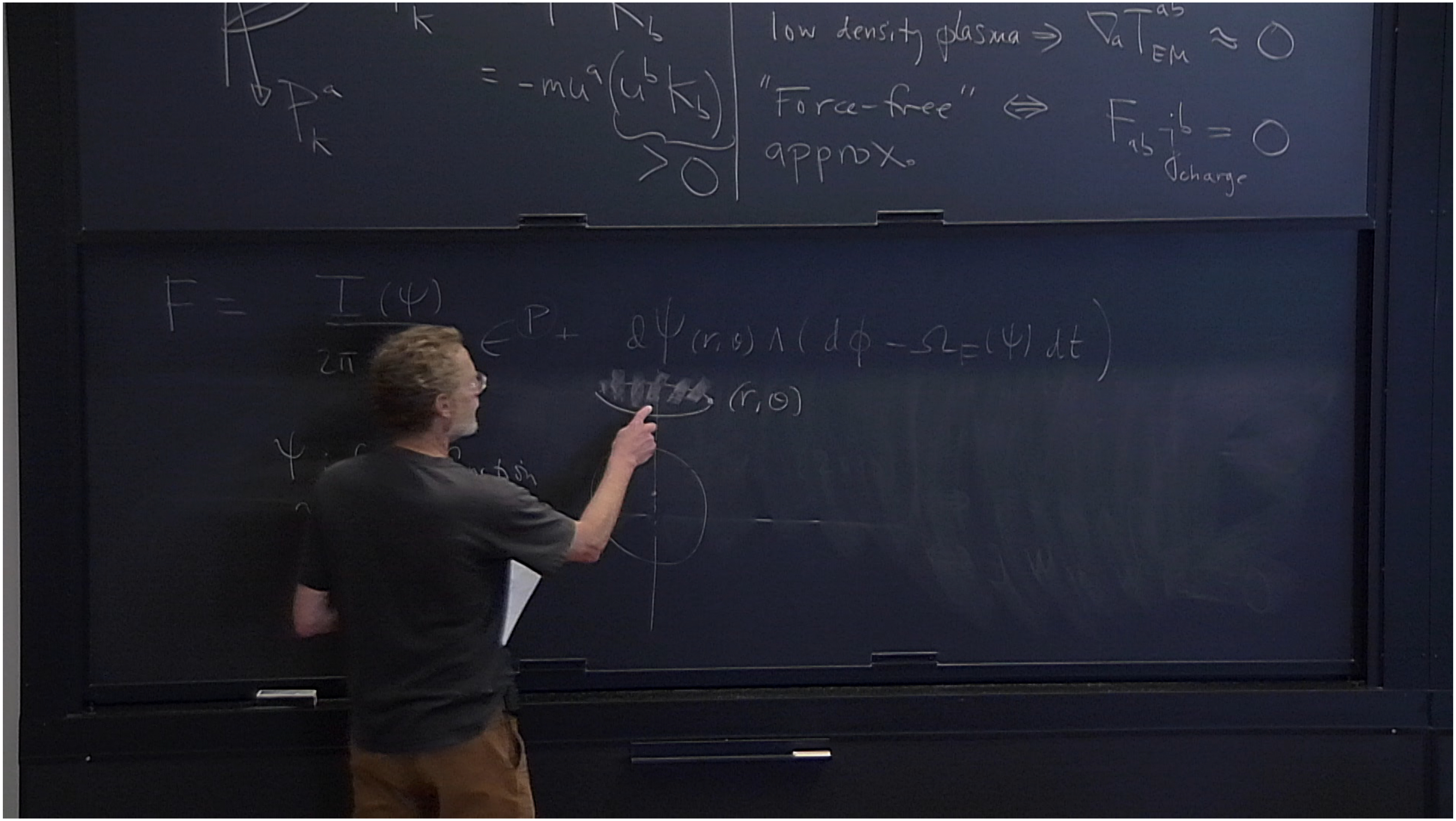
Stationary, axisymm, FF F_{ab}

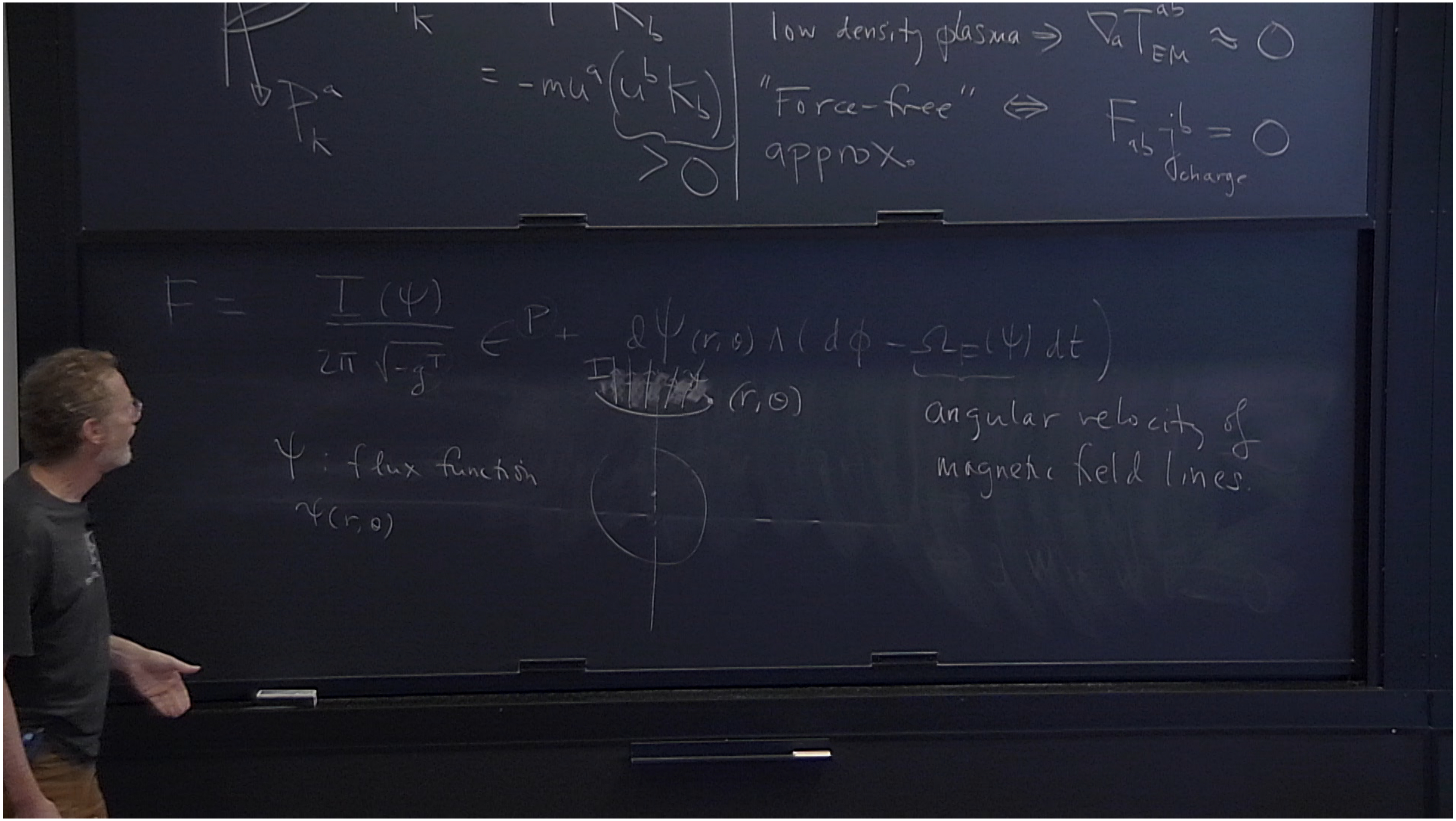
$$ds^2 = \underbrace{-\alpha^2 dt^2 + \rho^2(d\phi - \Omega_z dt)^2}_{\text{toroidal}} + \underbrace{h_{ij}\, dx^i dx^j}_{\text{poloidal}}$$

$\alpha = \alpha(r,\theta)$ $\qquad x^i = r, \theta$

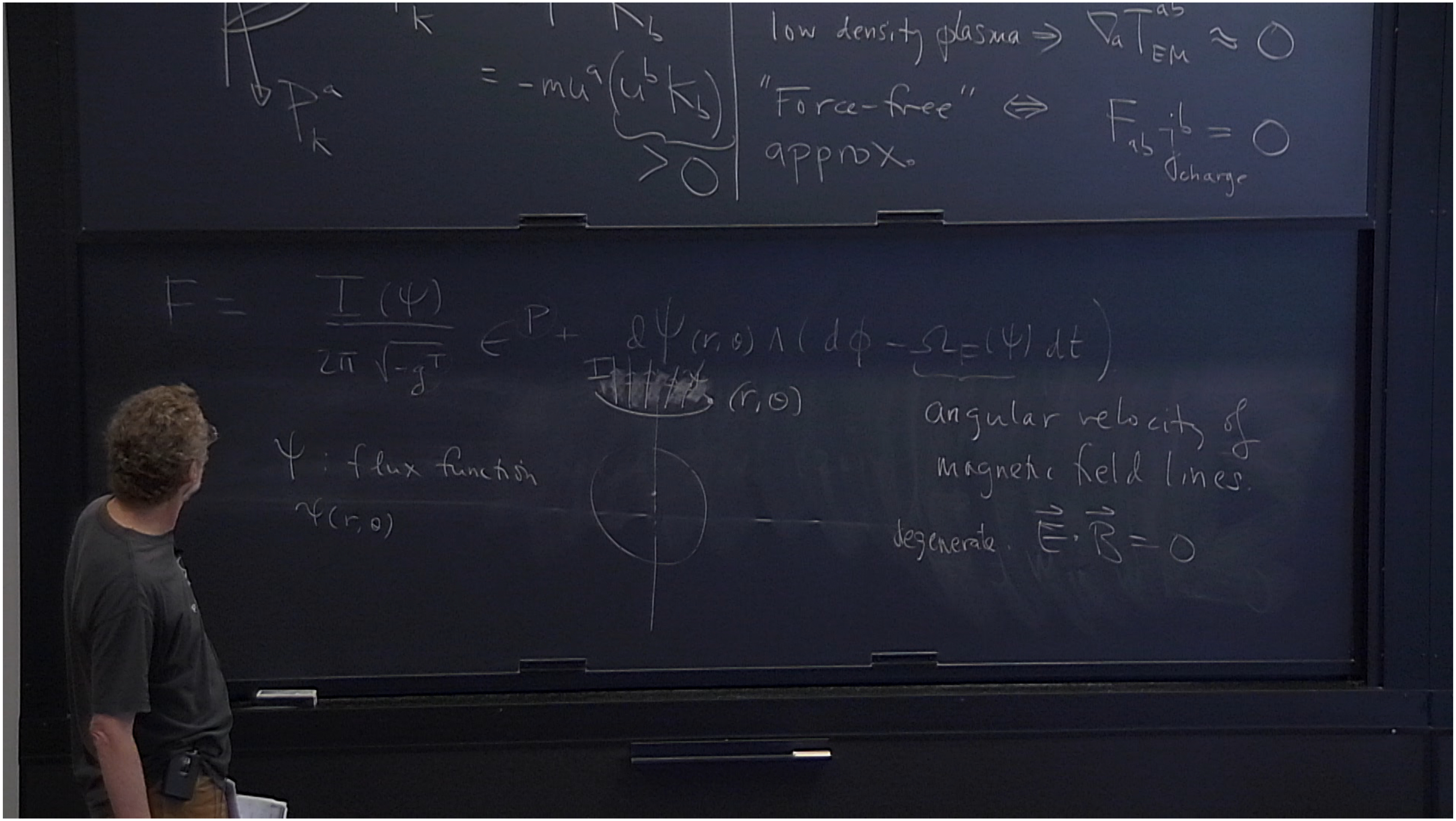
$\rho = \rho(r,\theta)$

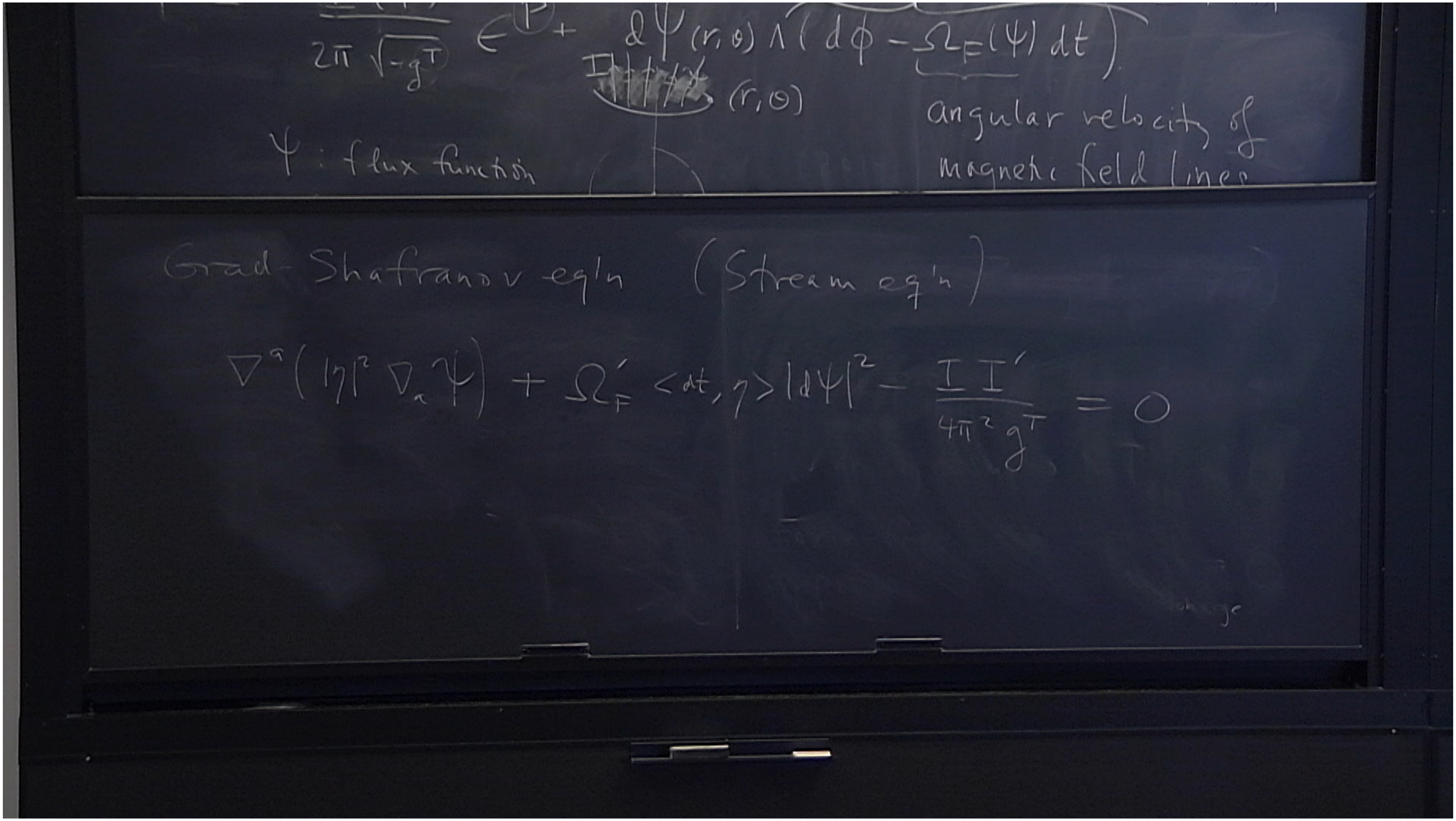
$\Omega_z = \Omega_z(r,\theta)$

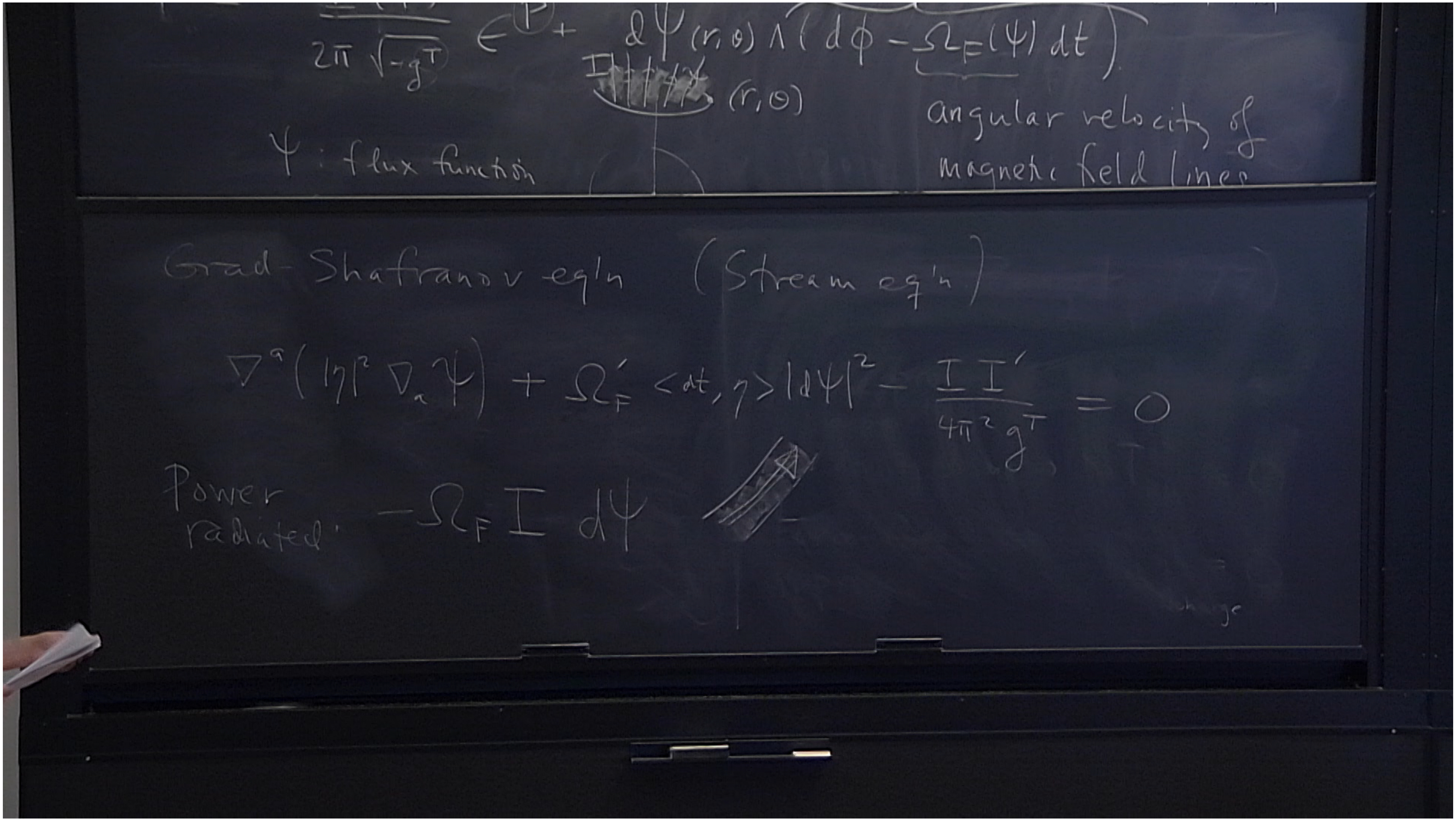

low density plasma ⇒ ∇a T^ab EM ≈ 0
"Force-free" ⇔ Fab j^b = 0
approx.
jcharge
= -mu^a(u^b kb)
>0
(r, θ)



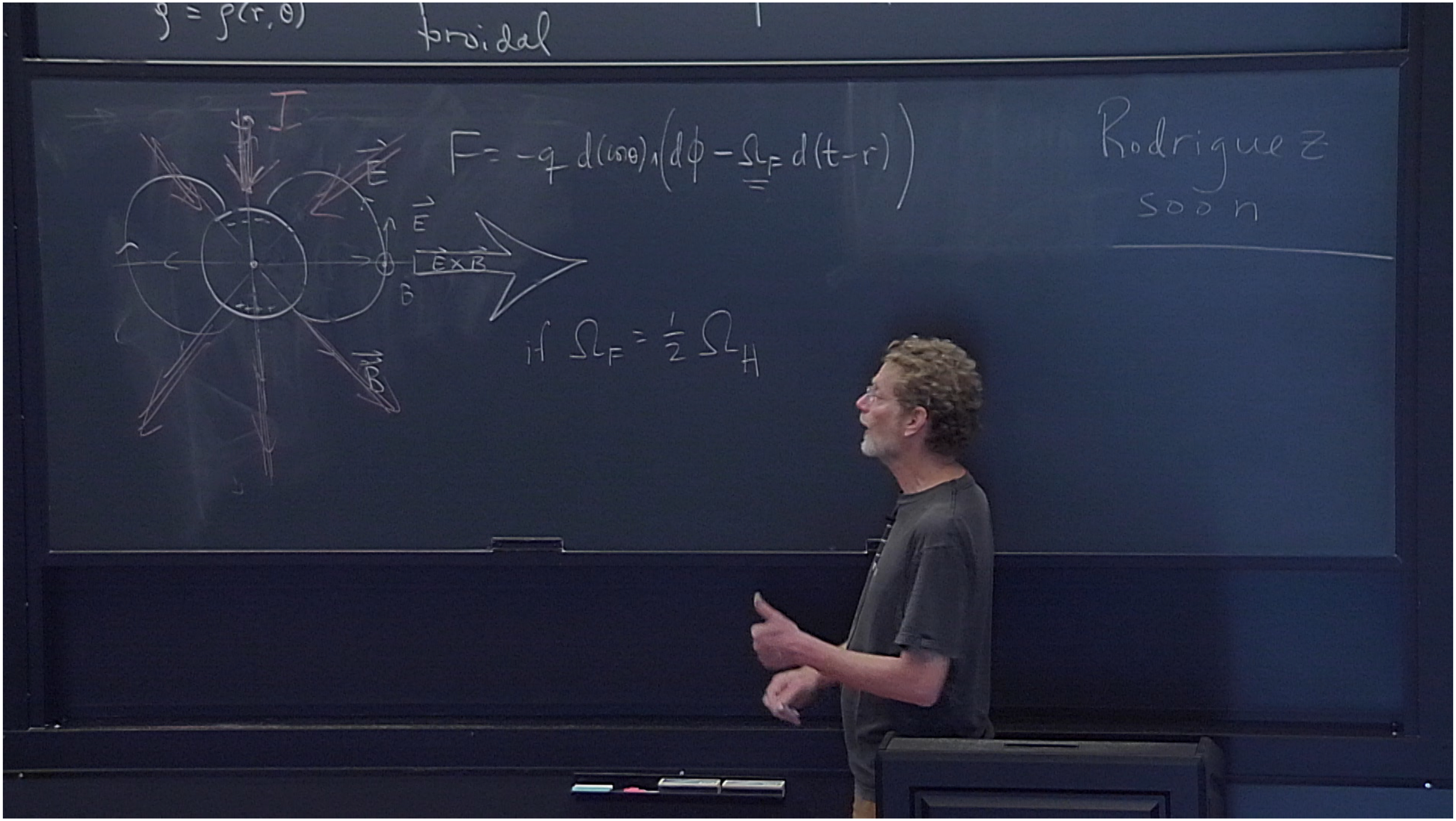
low density plasma ⇒ ∇a T^ab EM ≈ 0
"Force-free" ⇔ Fab j^b = 0
approx.
j charge
= -mu^a(u^b kb)
> 0
F = I(ψ) / 2π √-g^T ε^P+ dψ(r,θ) ∧ (dφ - ΩF(ψ) dt)
(r,θ)
angular velocity of magnetic field lines.
ψ : flux function
ψ(r,θ)

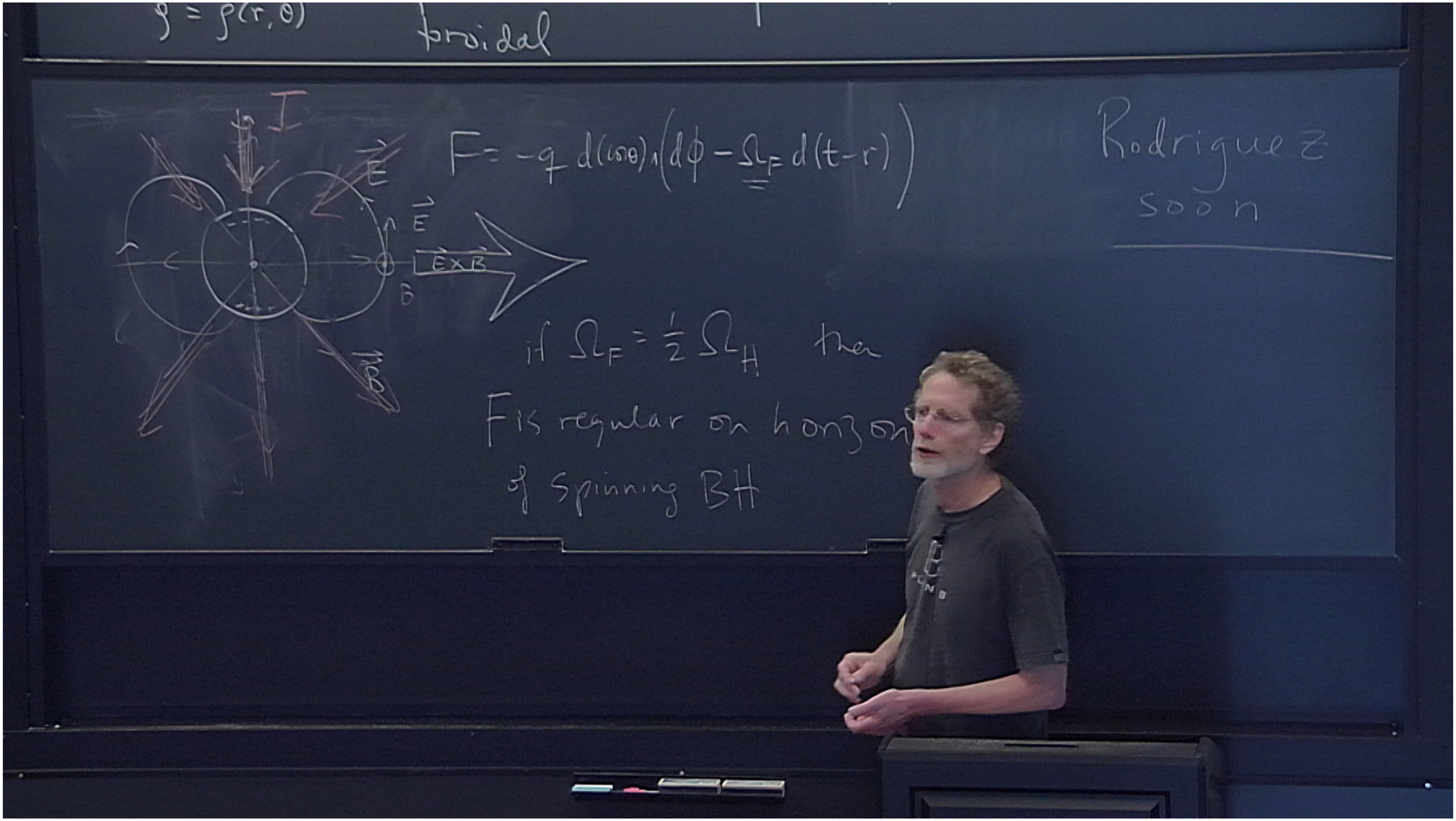

low density plasma ⇒ ∇a T^ab EM ≈ 0
"Force-free" ⇔ Fab j^b = 0
approx.
jcharge
= -mu^a (u^b k_b)
> 0
F = I(Ψ) / 2π √-g^T ∈^P_+ dΨ(r,θ) ∧ (dφ − Ω_F(Ψ) dt)
(r,θ)
angular velocity of
magnetic field lines.
Ψ : flux function
Ψ(r,θ)
degenerate. E · B = 0

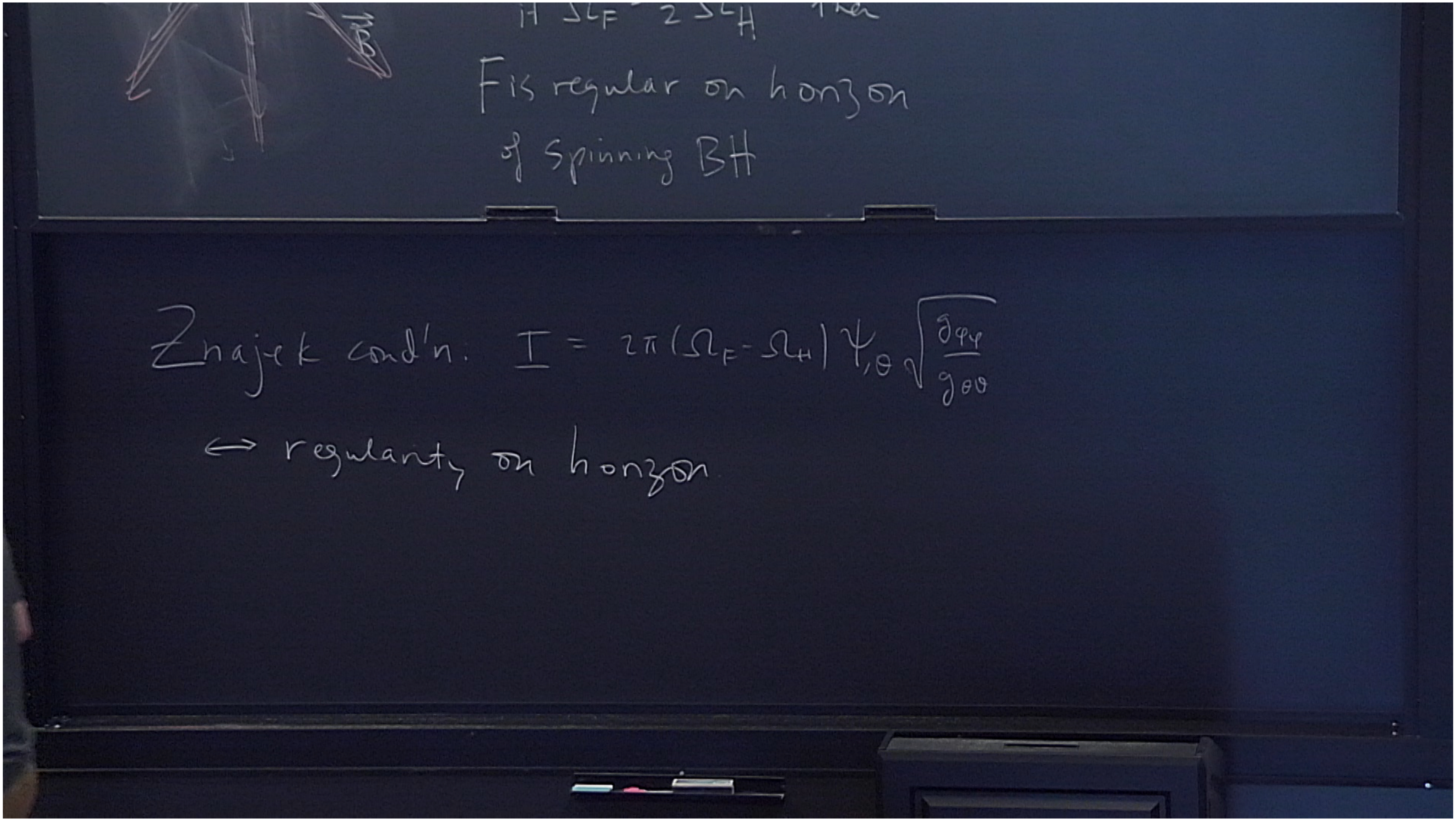

$2\pi\sqrt{-g^T}$
$d\psi(r,\theta) \wedge (d\phi - \Omega_F(\psi)\,dt)$
(r,θ)
angular velocity of magnetic field lines
ψ : flux function
Grad-Shafranov eq'n (Stream eq'n)
$\nabla^a(|\eta|^2 \nabla_a \psi) + \Omega_F' \langle dt, \eta\rangle |d\psi|^2 - \frac{I I'}{4\pi^2 g^T} = 0$

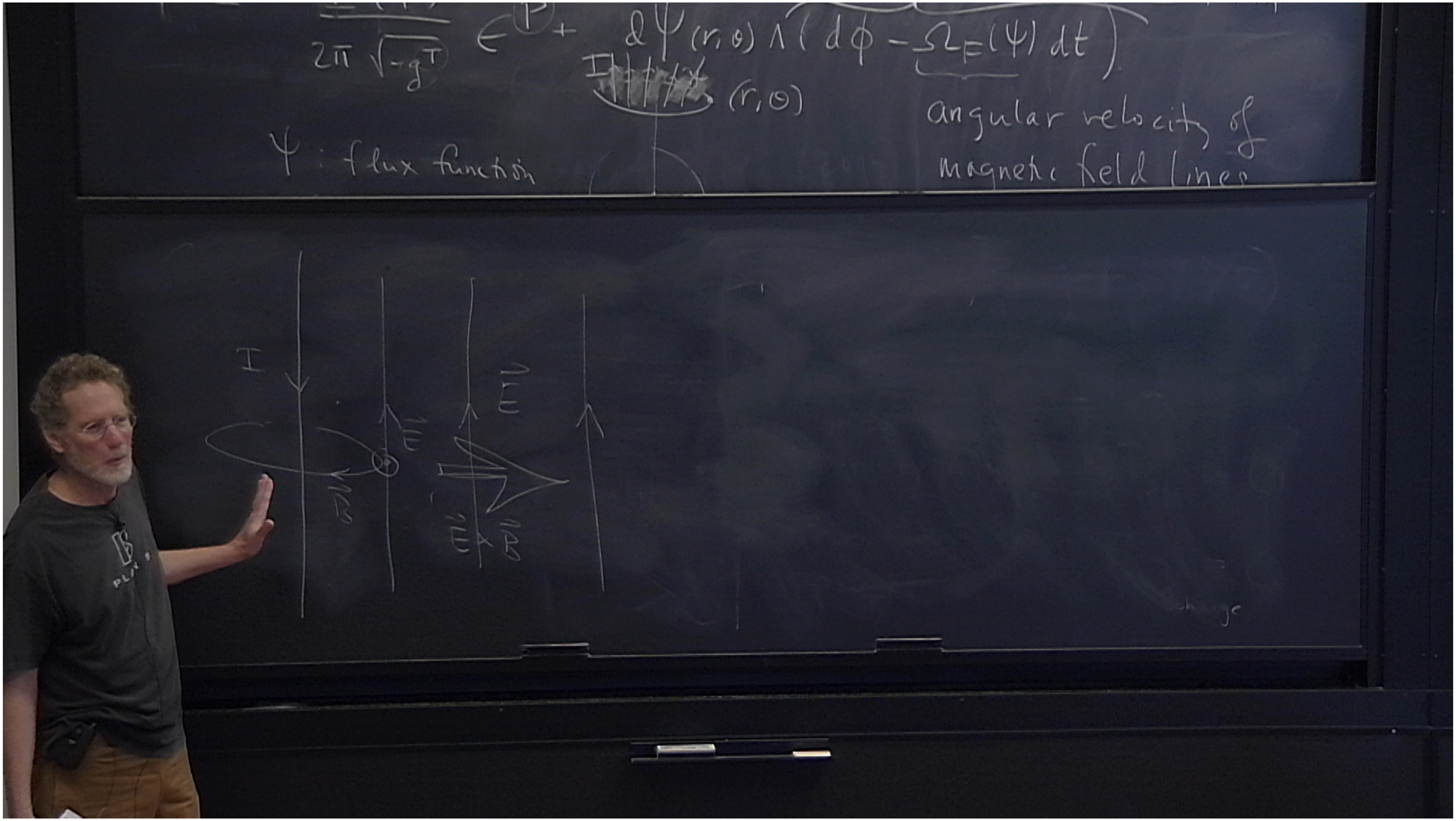


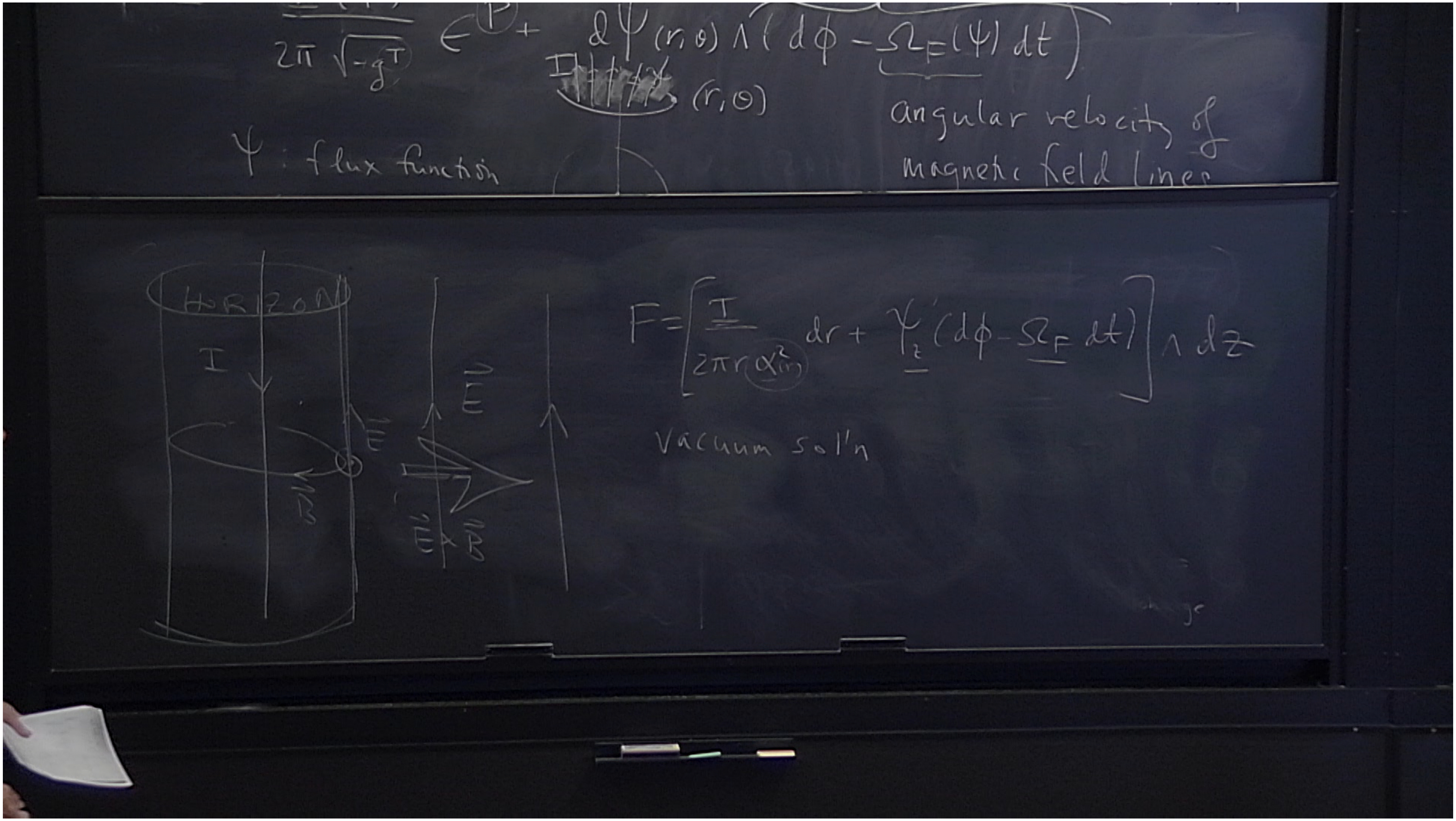
$2\pi\sqrt{-g^T}$
$d\psi(r,\theta) \wedge (d\phi - \Omega_F(\psi)\,dt)$
(r,θ)
ψ : flux function
angular velocity of magnetic field lines
Grad-Shafranov eq'n (Stream eq'n)
$\nabla^a(|\eta|^2 \nabla_a \psi) + \Omega_F' \langle dt, \eta\rangle |d\psi|^2 - \frac{I I'}{4\pi^2 g^T} = 0$
Power radiated: $-\Omega_F I \, d\psi$

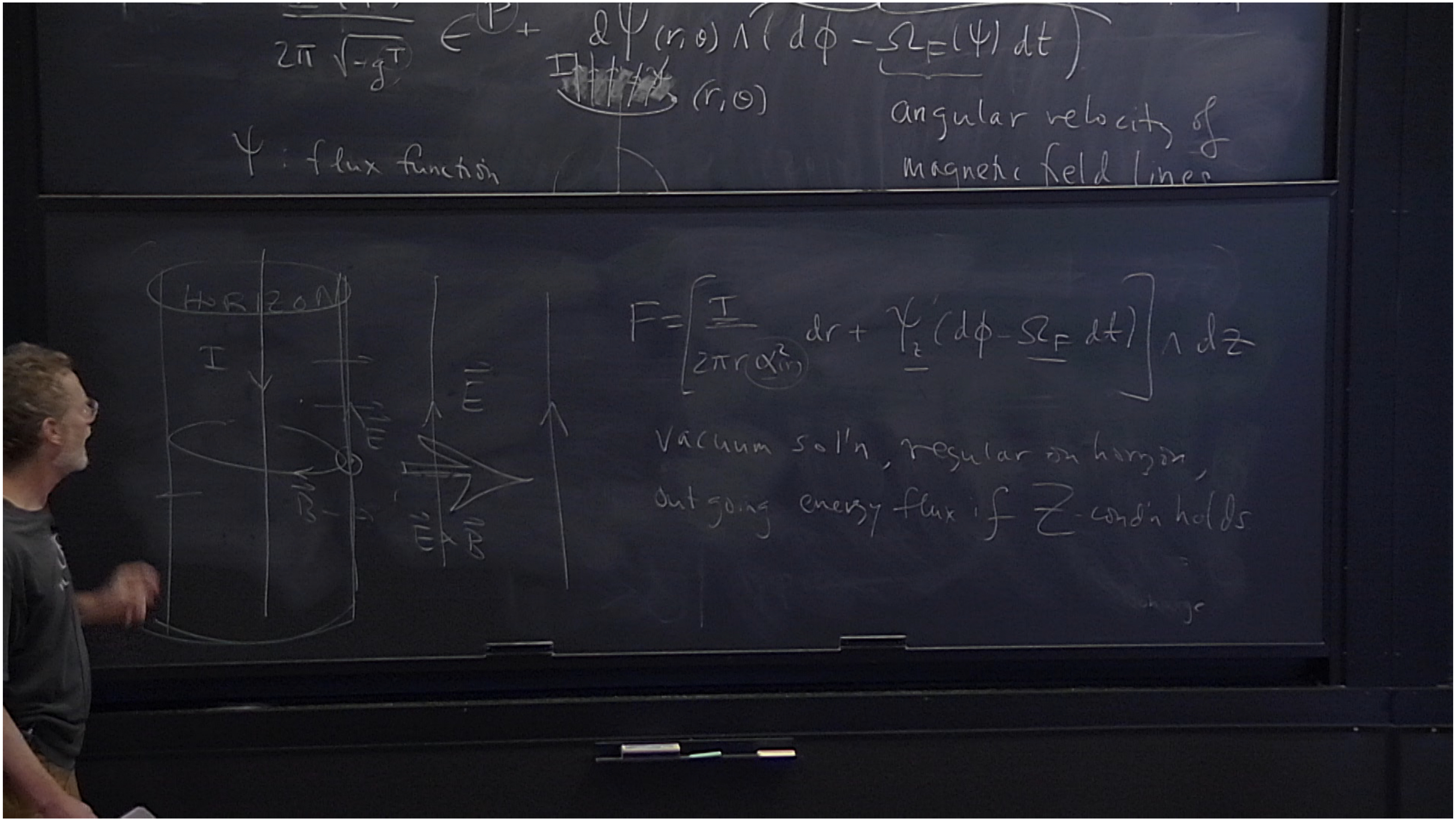

Rodriguez
soon

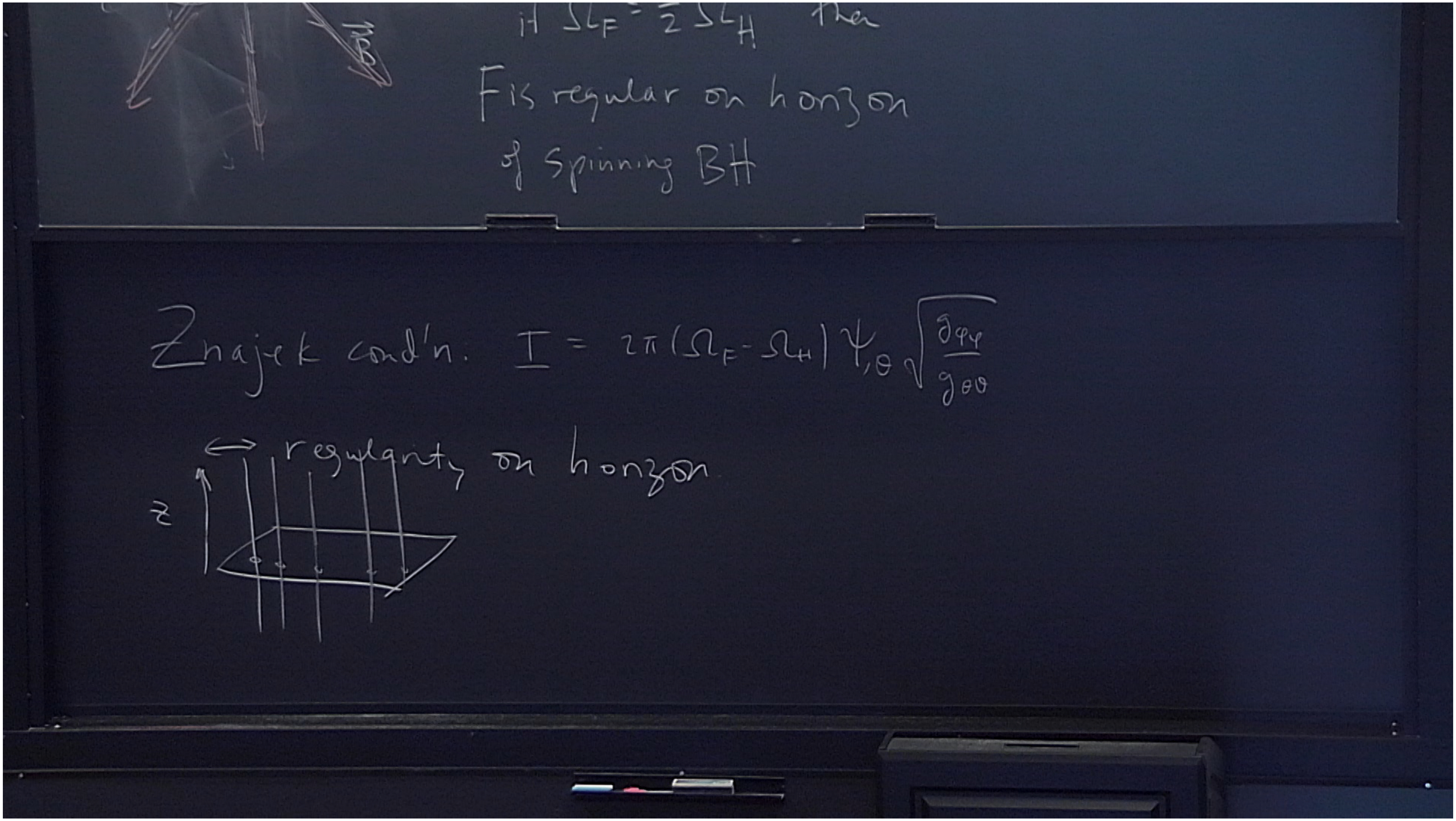

$\rho = \rho(r, \theta)$
toroidal
I
$\vec{E}$
$\vec{E}$
$\vec{E} \times \vec{B}$
B
$\vec{B}$
$F = -q\, d(\cos\theta) \wedge \left(d\phi - \Omega_F\, d(t-r)\right)$
Rodriguez
soon
if $\Omega_F = \frac{1}{2}\Omega_H$ then
F is regular on horizon
of spinning BH

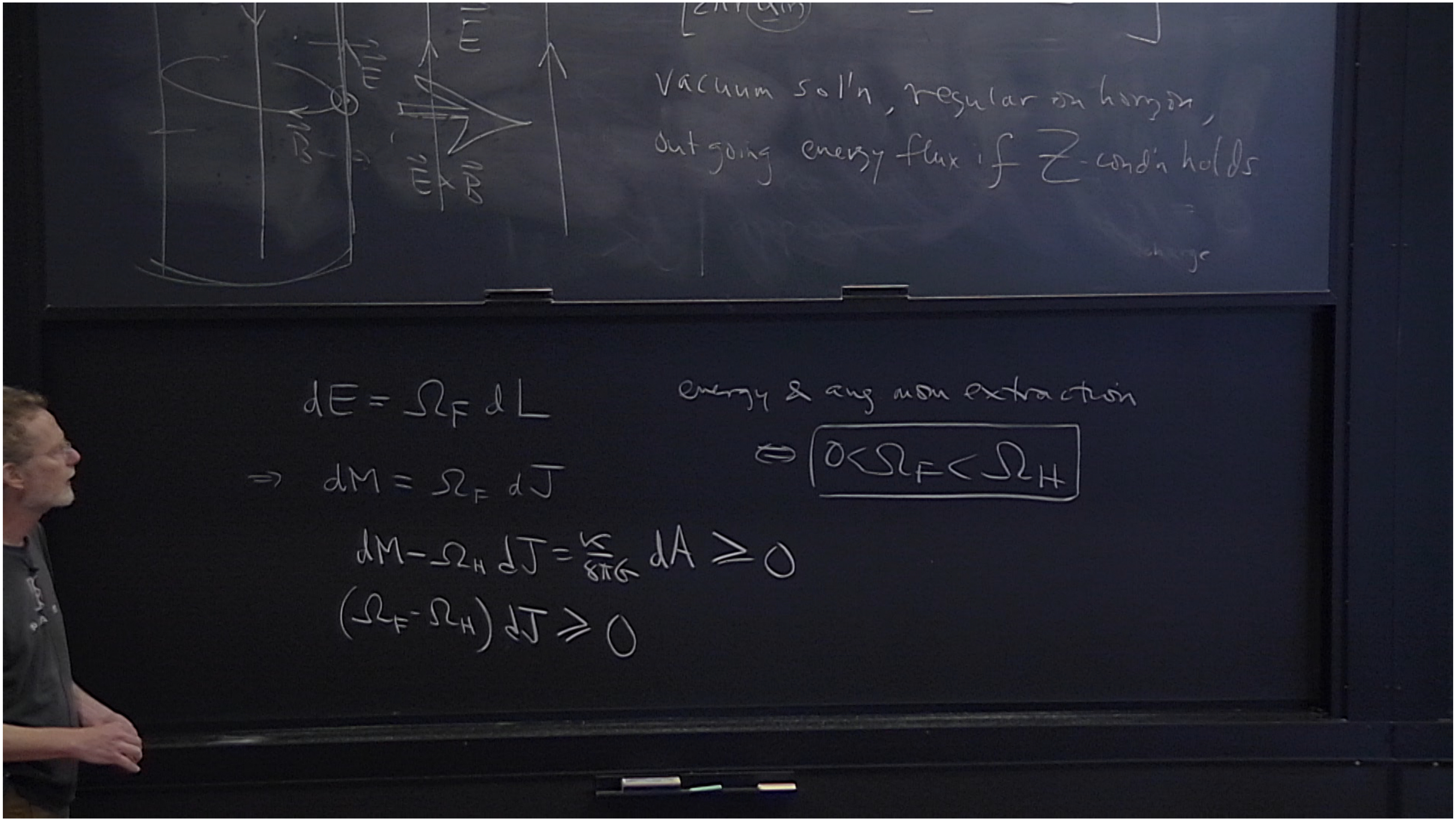

F is regular on horizon
of spinning BH
Znajek cond'n: $I = 2\pi(\Omega_F - \Omega_H)\,\psi_{,\theta}\sqrt{\frac{g_{\varphi\varphi}}{g_{\theta\theta}}}$
⟷ regularity on horizon

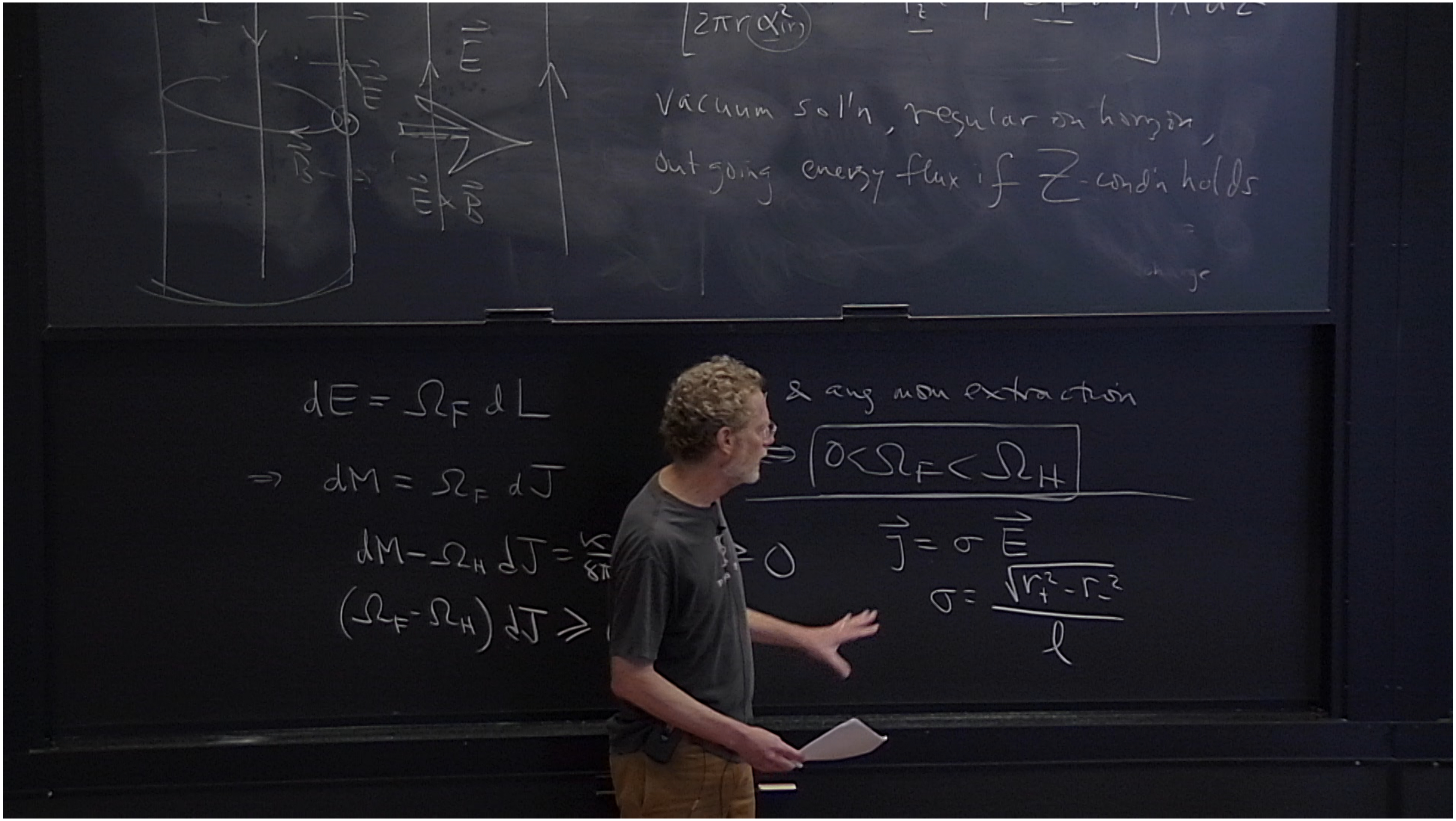

2π √-g^T
dΨ(r,θ) ∧ (dφ − Ω_F(Ψ) dt)
(r,θ)
angular velocity of
magnetic field lines
Ψ : flux function
I
E
E
B
E × B


2π √-g^T ε
dψ(r,θ) ∧ (dφ − Ω_F(ψ) dt)
(r,θ)
angular velocity of
magnetic field lines
ψ : flux function
HORIZON
I
E
B
E × B
F = [I / (2πr α²(r)) dr + ψ_z′ (dφ − Ω_F dt)] ∧ dz
Vacuum sol'n


2π √-g^T
dψ(r,θ) ∧ (dφ − Ω_F(ψ) dt)
(r,θ)
angular velocity of magnetic field lines
ψ : flux function
HORIZON
I
E
B
E × B
F = [I / (2πr α²(r)) dr + ψ'_z (dφ − Ω_F dt)] ∧ dz
Vacuum sol'n, regular on horizon,
outgoing energy flux if Z-cond'n holds


if $\Omega_F = \frac{1}{2}\Omega_H$ then
F is regular on horizon
of spinning BH
Znajek cond'n: $I = 2\pi(\Omega_F - \Omega_H)\,\psi_{,\theta}\sqrt{\frac{g_{\varphi\varphi}}{g_{\theta\theta}}}$
$\Leftrightarrow$ regularity on horizon
z
B


Vacuum sol'n, regular on horizon,
outgoing energy flux if Z-cond'n holds
$\vec{E}$
$\vec{B}$
$\vec{E} \times \vec{B}$
energy & ang mom extraction
$\Leftrightarrow \boxed{0 < \Omega_F < \Omega_H}$
$dE = \Omega_F \, dL$
$\Rightarrow \quad dM = \Omega_F \, dJ$
$dM - \Omega_H \, dJ = \frac{\kappa}{8\pi G} dA \geq 0$
$(\Omega_F - \Omega_H) \, dJ \geq 0$


Vacuum sol'n, regular on horizon,
outgoing energy flux if Z-cond'n holds
dE = Ω_F dL
⇒ dM = Ω_F dJ
& ang mom extraction
⇒ 0 < Ω_F < Ω_H
(Ω_F - Ω_H) dJ ≥
J = σ E
σ = √(r_+² - r_-²) / ℓ