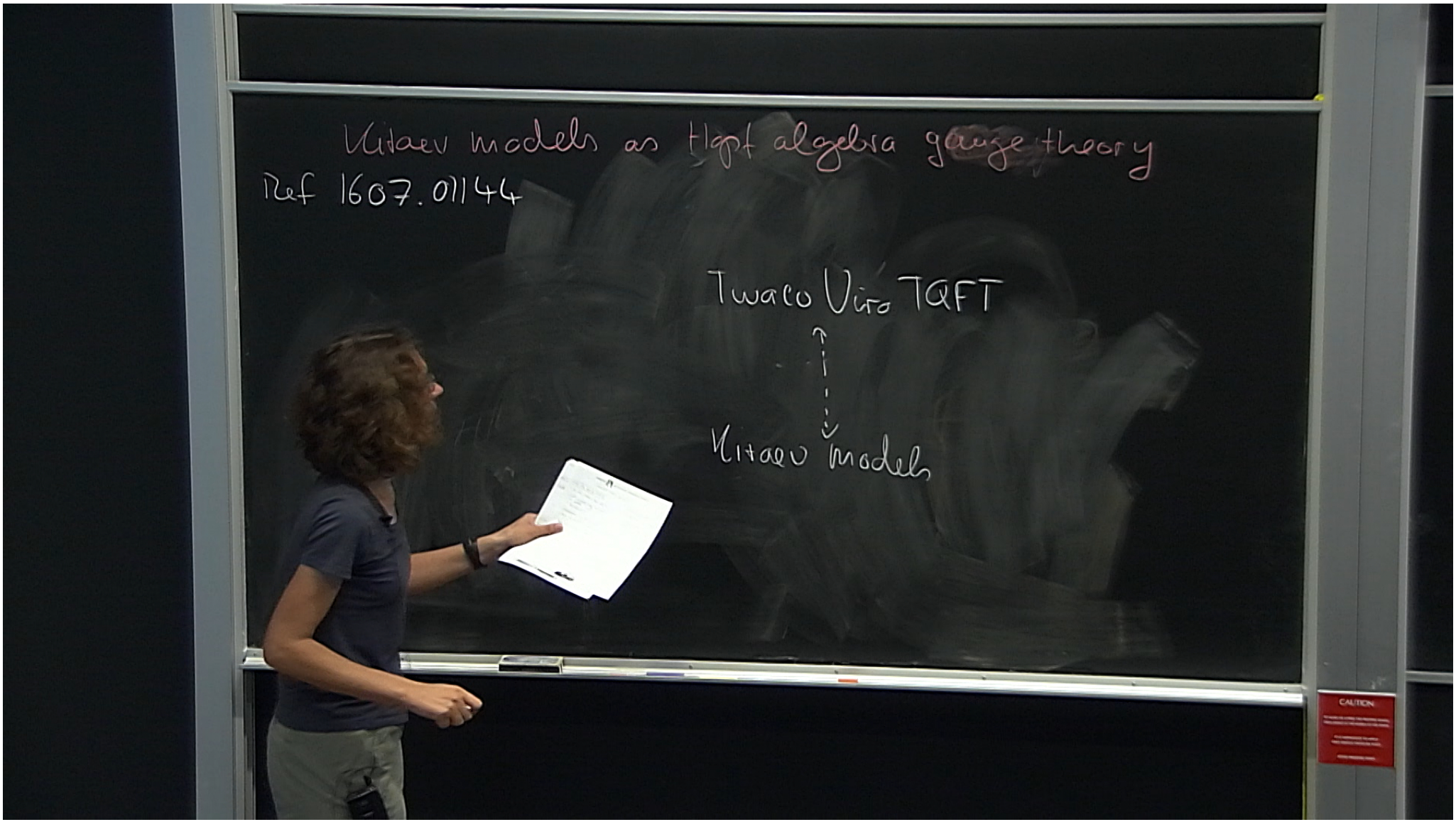
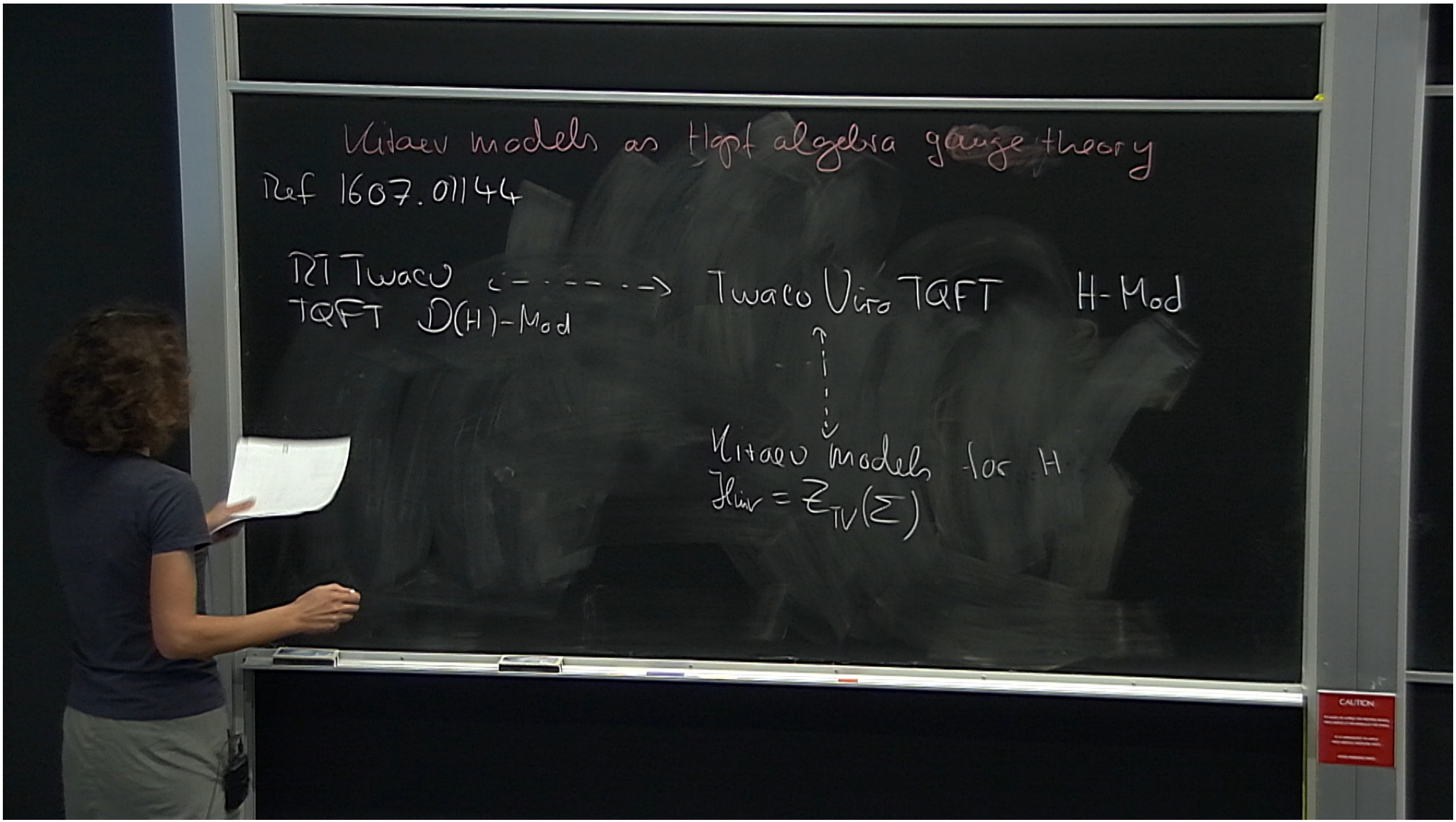
Title: Kitaev lattice models as a Hopf algebra gauge theory

Date: Aug 01, 2017 11:00 AM

URL: http://pirsa.org/17080002

Abstract: We show that Kitaev's lattice model for a finite-dimensional semisimple Hopf algebra H is equivalent to the combinatorial quantisation of Chern-Simons theory for the Drinfeld double D(H). As a result, Kitaev models are a special case of combinatorial quantization of Chern-Simons theory by Alekseev, Grosse and Schomerus. This equivalence is an analogue of the relation between Turaev-Viro and Reshetikhin-Turaev TQFTs and relates them to the quantisation of moduli spaces of flat connections.
We show that the topological invariants of the two models, the algebra of operators acting on the protected space of the Kitaev model and the quantum moduli algebra from the combinatorial quantisation formalism, are isomorphic. This is established in a gauge theoretical picture, in which both models appear as Hopf algebra valued lattice gauge theories.


Kitaev models as Hopf algebra gauge theory
Ref 1607.01144
Twaco Vito TQFT
Kitaev models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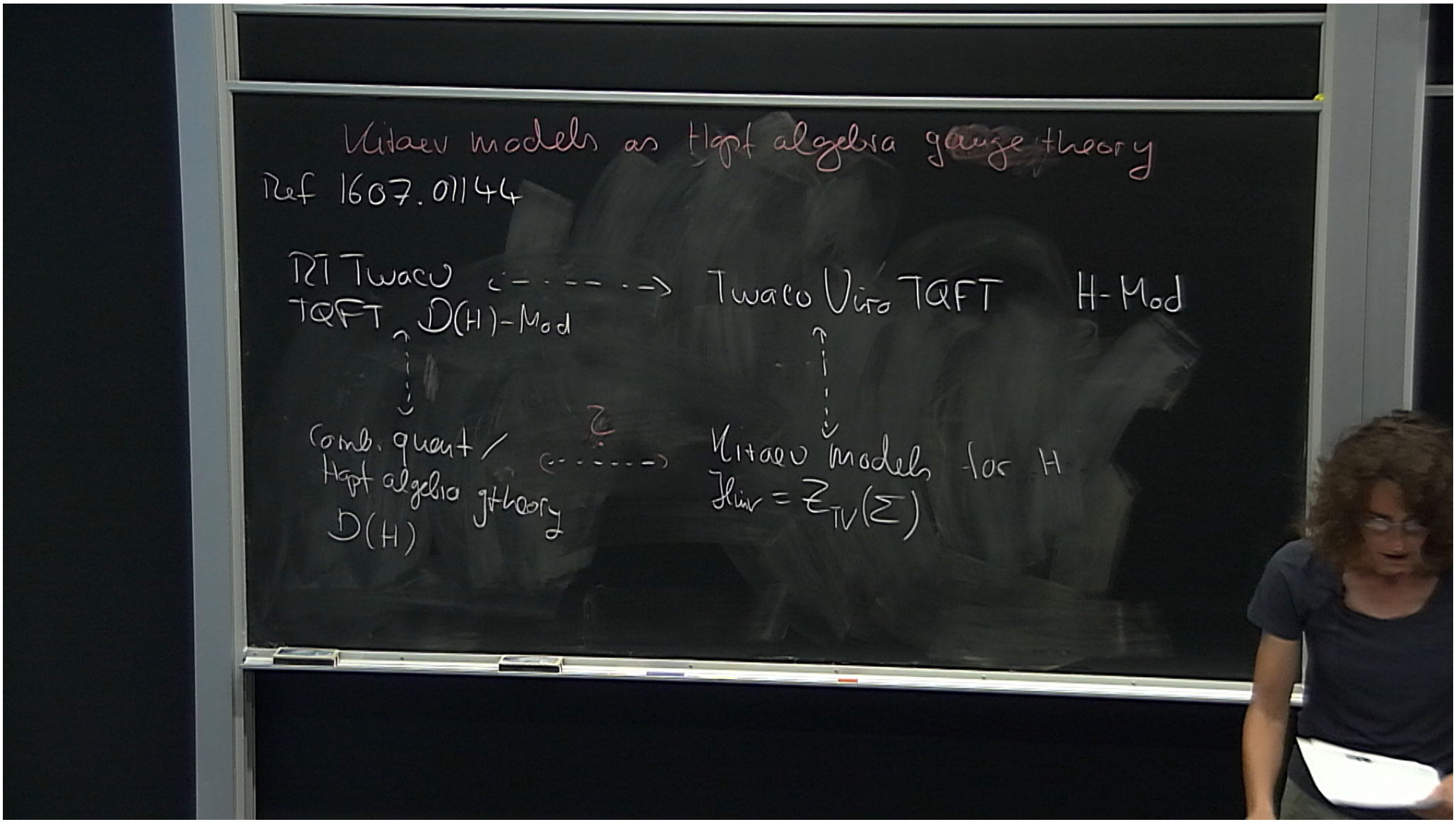
H-Mod
Kitaev models for H
Hinv = Z_TV(Σ)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Comb. quant /
Hopf algebra gtheory
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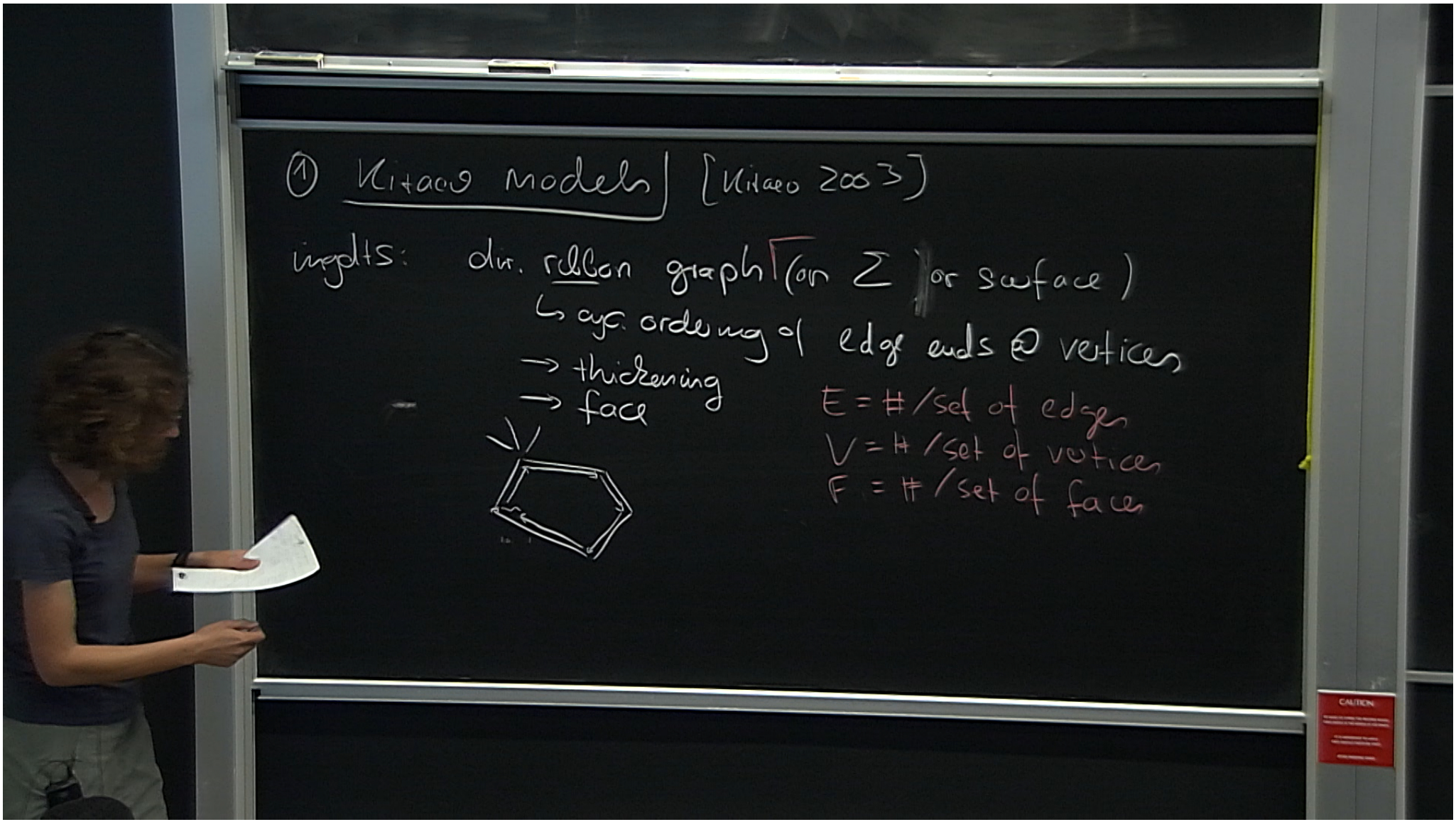
① Kitaev models [Kitaev 2003]

ingdts: dir. ribbon graph (on $\Sigma$ or surface)

↳ cyc. ordering of edge ends @ vertices

→ thickening

→ face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E = # / set of edges
V = # / set of vertices
F = # / set of faces

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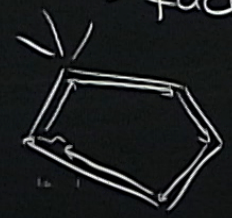

$E = \#$ / set of edges

$V = \#$ / set of vertices

$F = \#$ / set of faces

2) fin-dim semisimple Hopf algebra $H$

ext. Hilbert space: $\mathcal{H}_{ext} = H^{\otimes E}$

algebra of $\Delta$-operators: $e \in E \to L^{h \in H}_{e\pm}$, $T^{\alpha \in H^*}_{e\pm} : H^{\otimes E} \to H^{\otimes E}$

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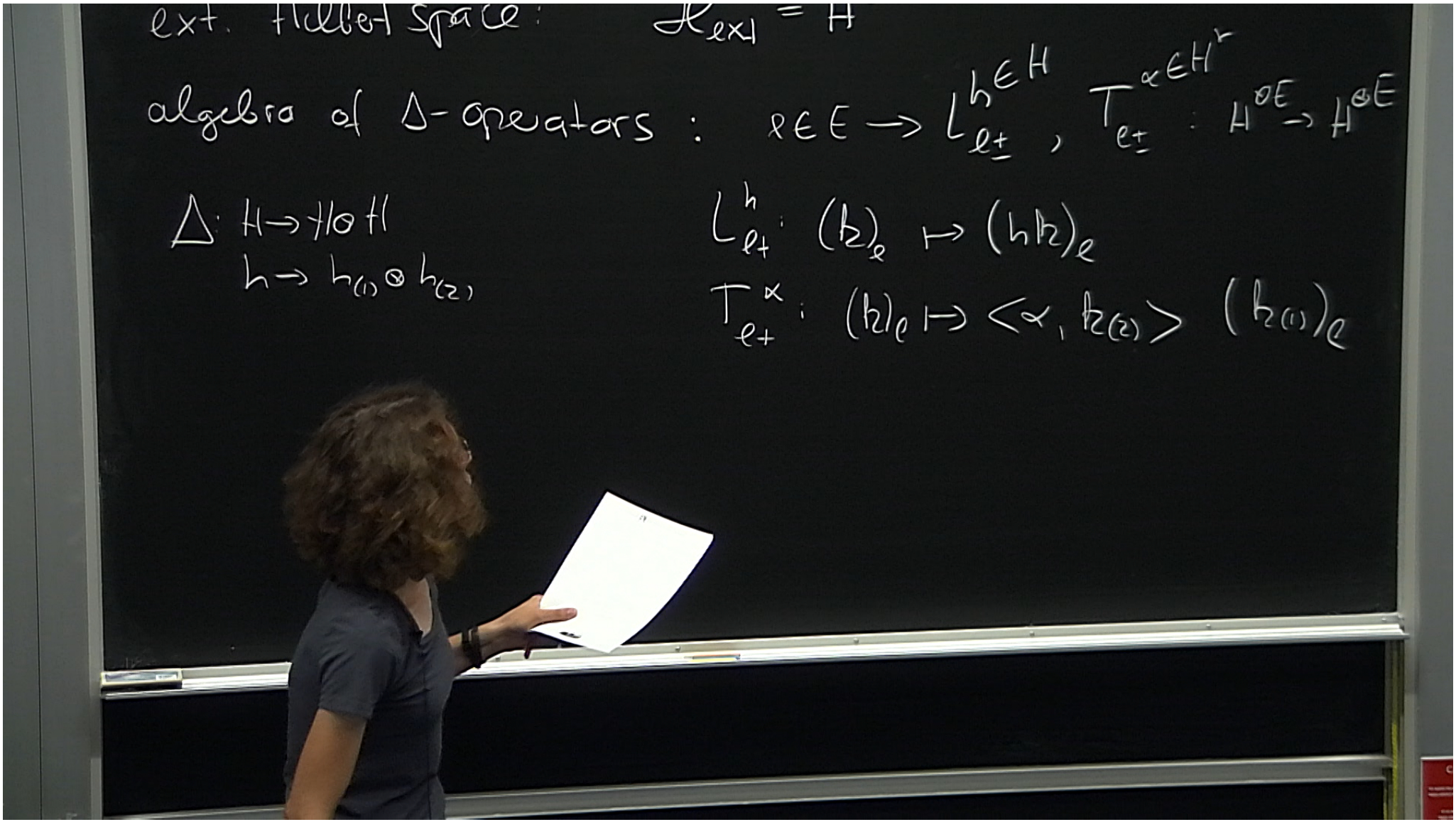
$$L^{h}_{e+} : (k)_e \mapsto (hk)_e$$

$$T^{\alpha}_{e+} : (k)_e \mapsto \langle \alpha, k_{(2)} \rangle \, (k_{(1)})_e$$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$\Delta : H \to H \otimes H$
$h \to h_{(1)} \otimes h_{(2)}$
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$L^h_{e-} = S \circ L^{S(h)}_{e+} \circ S : (k)_e \to (kh)_e$

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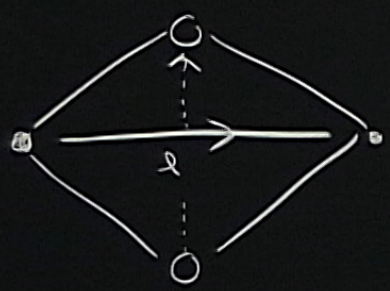


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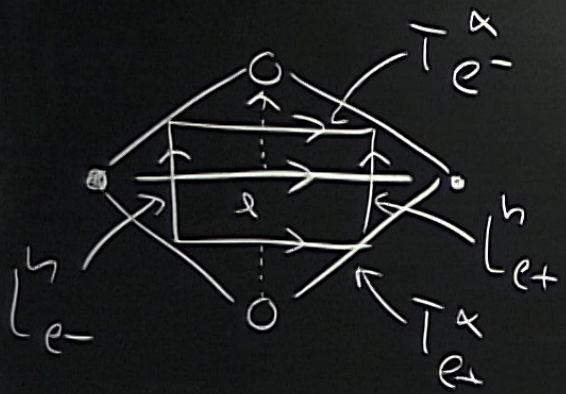


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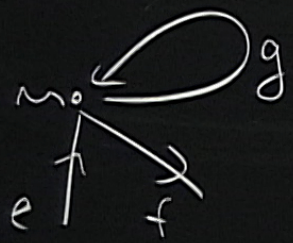
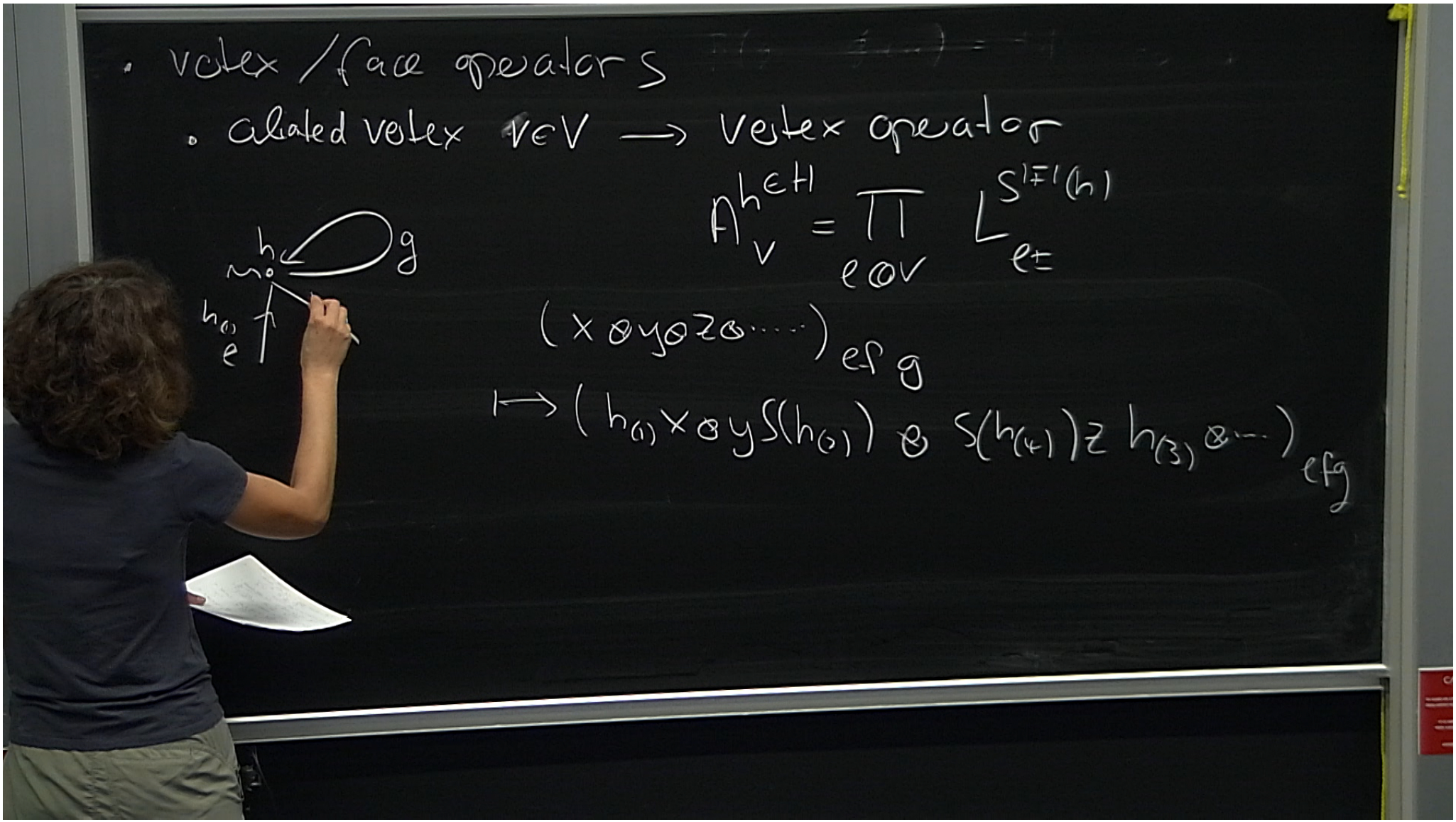
- vertex / face operators
  - oriented vertex $v \in V \longrightarrow$ vertex operator

$$A_v^{h^{\epsilon+1}} = \prod_{e \triangleleft v} L_{e^\pm}^{S^{\mp 1}(h)}$$




$$(x \otimes y \otimes z \otimes \cdots)_{e \triangleleft g}$$

$$\longmapsto (h_{(1)} x \otimes y S(h_{(2)}) \otimes S(h_{(4)}) z h_{(3)} \otimes \cdots)_{e \triangleleft g}$$


• vertex / face operators
• oriented vertex $v \in V$ → vertex operator
$A_v^{h^{\epsilon+1}} = \prod_{e \in v} L_{e\pm}^{S^{\mp 1}(h)}$
$(x \otimes y \otimes z \otimes \cdots)_{e \cap g}$
$\mapsto (h_{(1)} x \otimes y S(h_{(2)}) \otimes S(h_{(4)}) z h_{(3)} \otimes \cdots)_{e \cap g}$
h
g
$h_{(1)}$
e

- vertex / face operators
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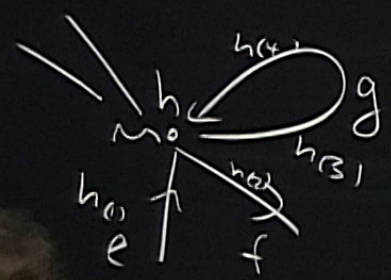


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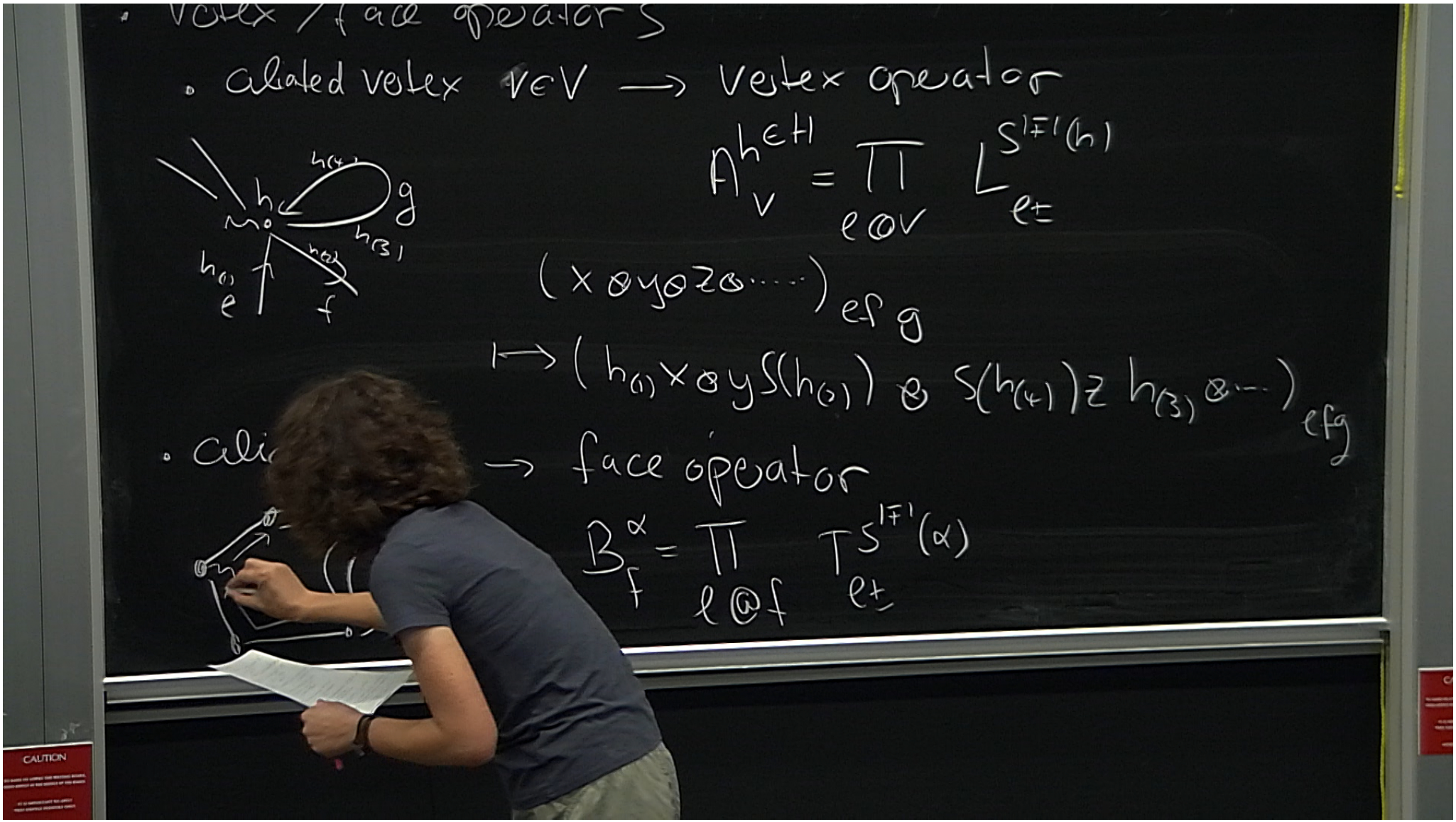
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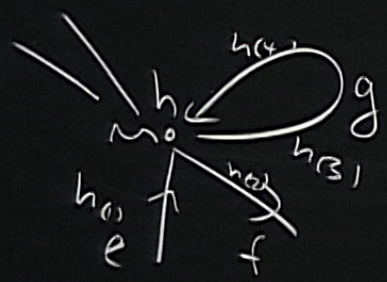
$A_v^{h\in H} = \prod_{e\otimes v} L_{e\pm}^{S^{|F|}(h)}$
$(x\otimes y\otimes z\otimes\cdots)_{efg}$
$\mapsto (h_{(1)}x\otimes y S(h_{(2)})\otimes S(h_{(4)})z\, h_{(3)}\otimes\cdots)_{efg}$
• ali
→ face operator
$B_f^{\alpha} = \prod_{e\otimes f} T_{e\pm}^{S^{|F|}(\alpha)}$
h
g
$h_{(1)}$
$h_{(2)}$
$h_{(3)}$
$h_{(4)}$
e
f
CAUTION

- vertex / face operators

- oriented vertex $v \in V \longrightarrow$ vertex operator




$$A_v^{h \in H} = \prod_{e \circledcirc v} L_{e\pm}^{S^{|F|}(h)}$$

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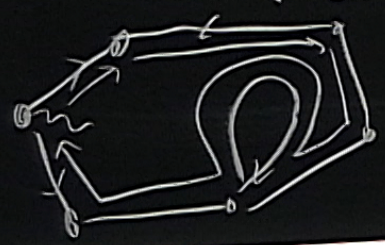
- oriented face $\longrightarrow$ face operator



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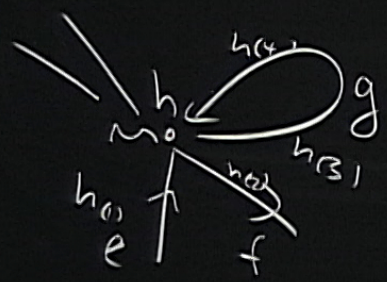


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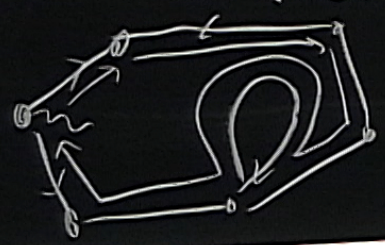

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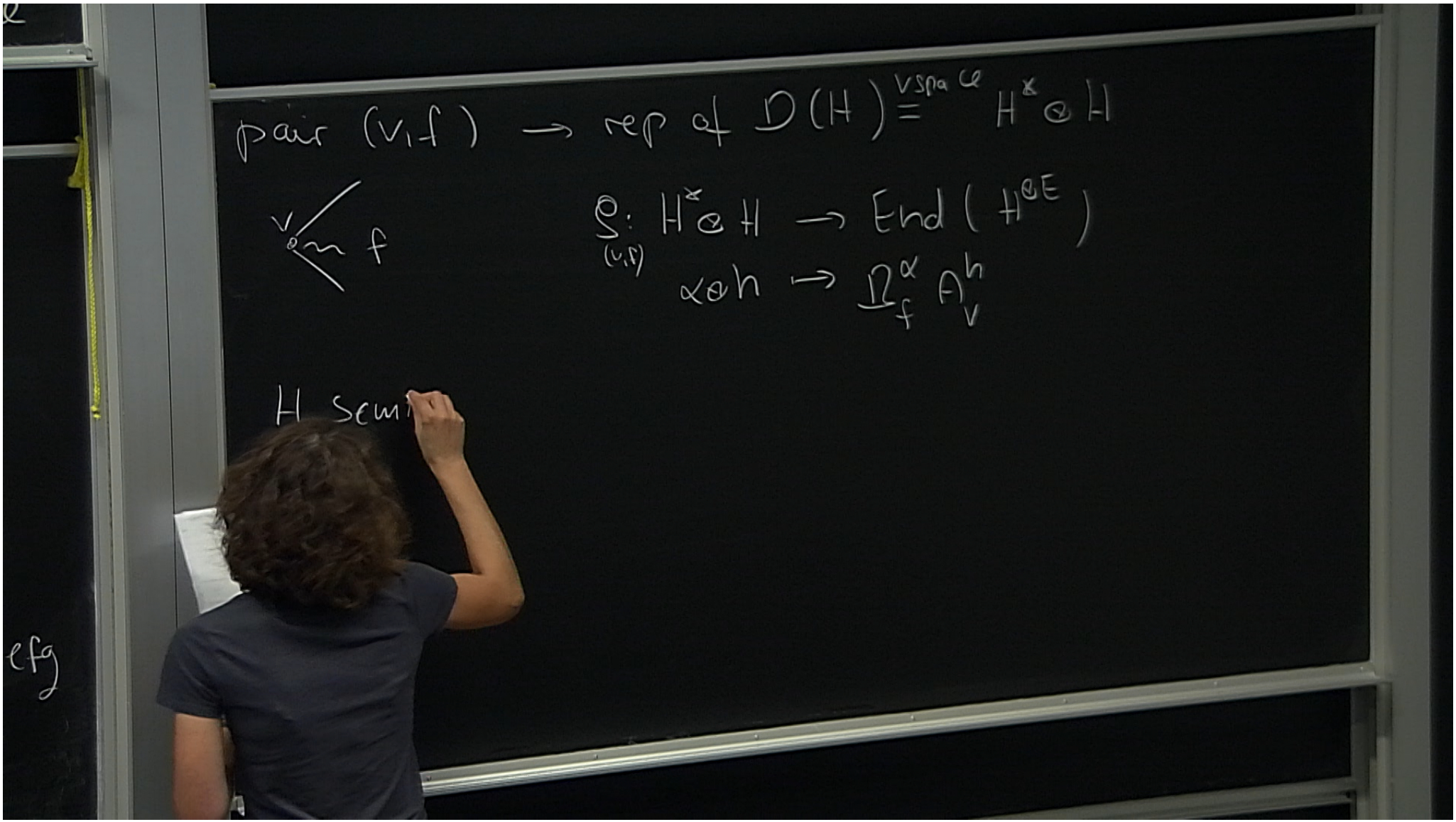
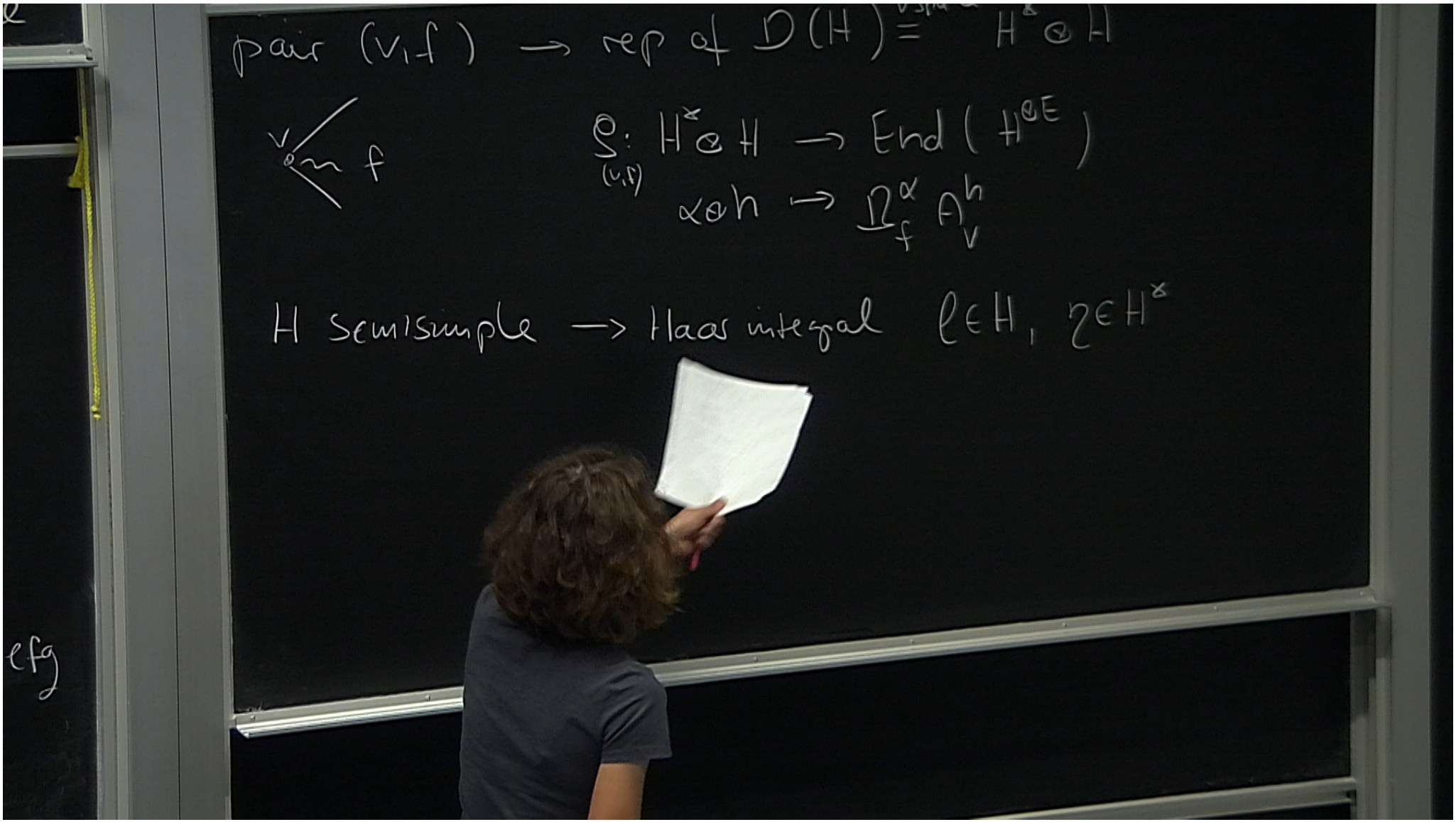

pair (v, f) → rep of D(H) ≅ (vspace) H* ⊗ H
v
f
ρ_(v,f): H* ⊗ H → End(H^⊗E)
α ⊗ h → Ω_f^α A_v^h
H semi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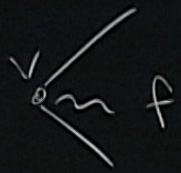
H semisimple → Haar integral ℓ ∈ H, η ∈ H*

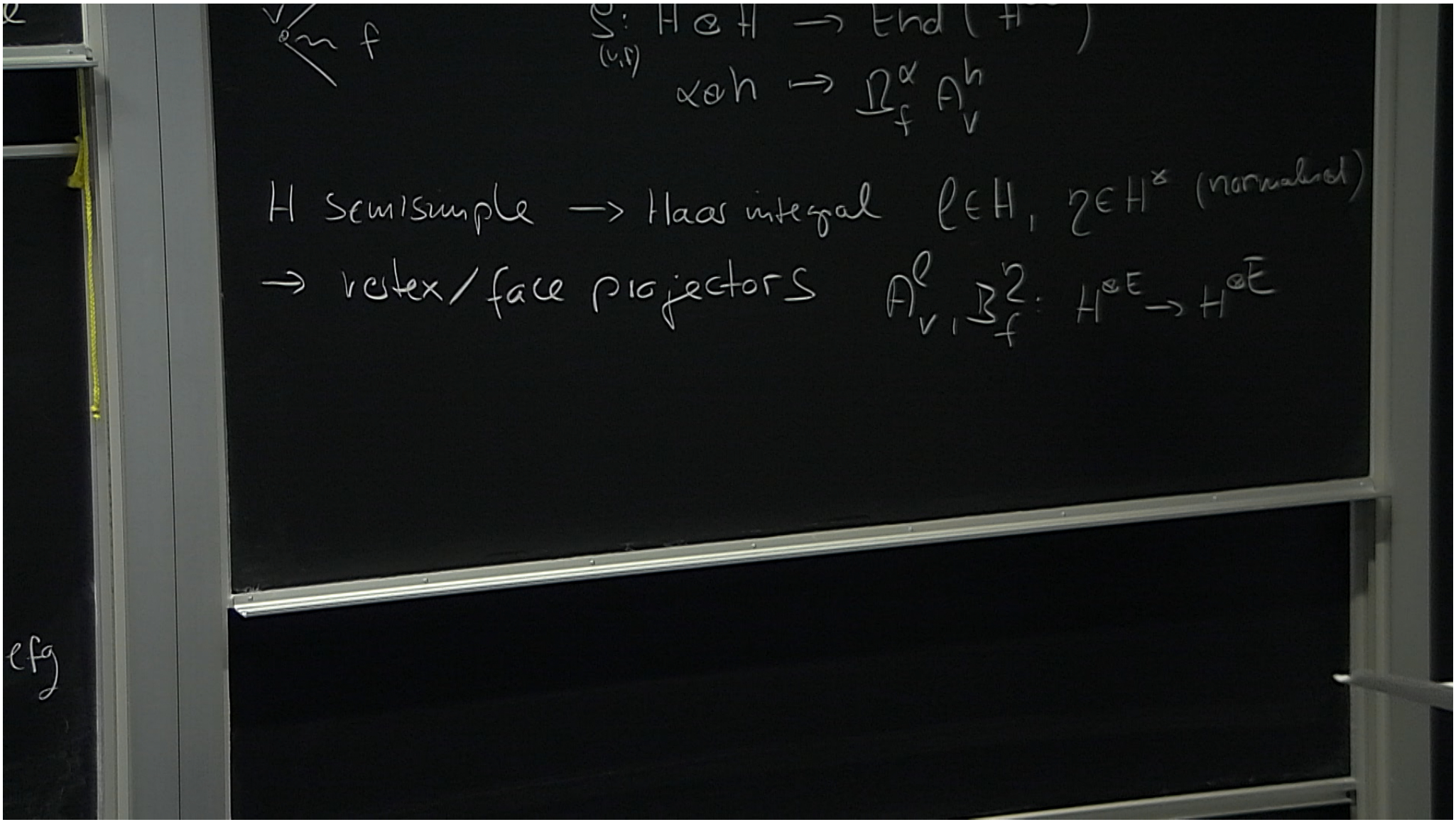



$$\underset{(v,f)}{\rho}: H^* \otimes H \to \mathrm{End}(H^{\otimes E})$$

$$\alpha \otimes h \mapsto \underline{\Omega}_f^{\alpha} A_v^h$$

$H$ semisimple $\to$ Haar integral $\ell \in H$, $\eta \in H^*$

$\to$ vertex/face projectors $A_v^{\ell}, B_f^{\eta}: H^{\otimes E} \to H^{\otimes E}$

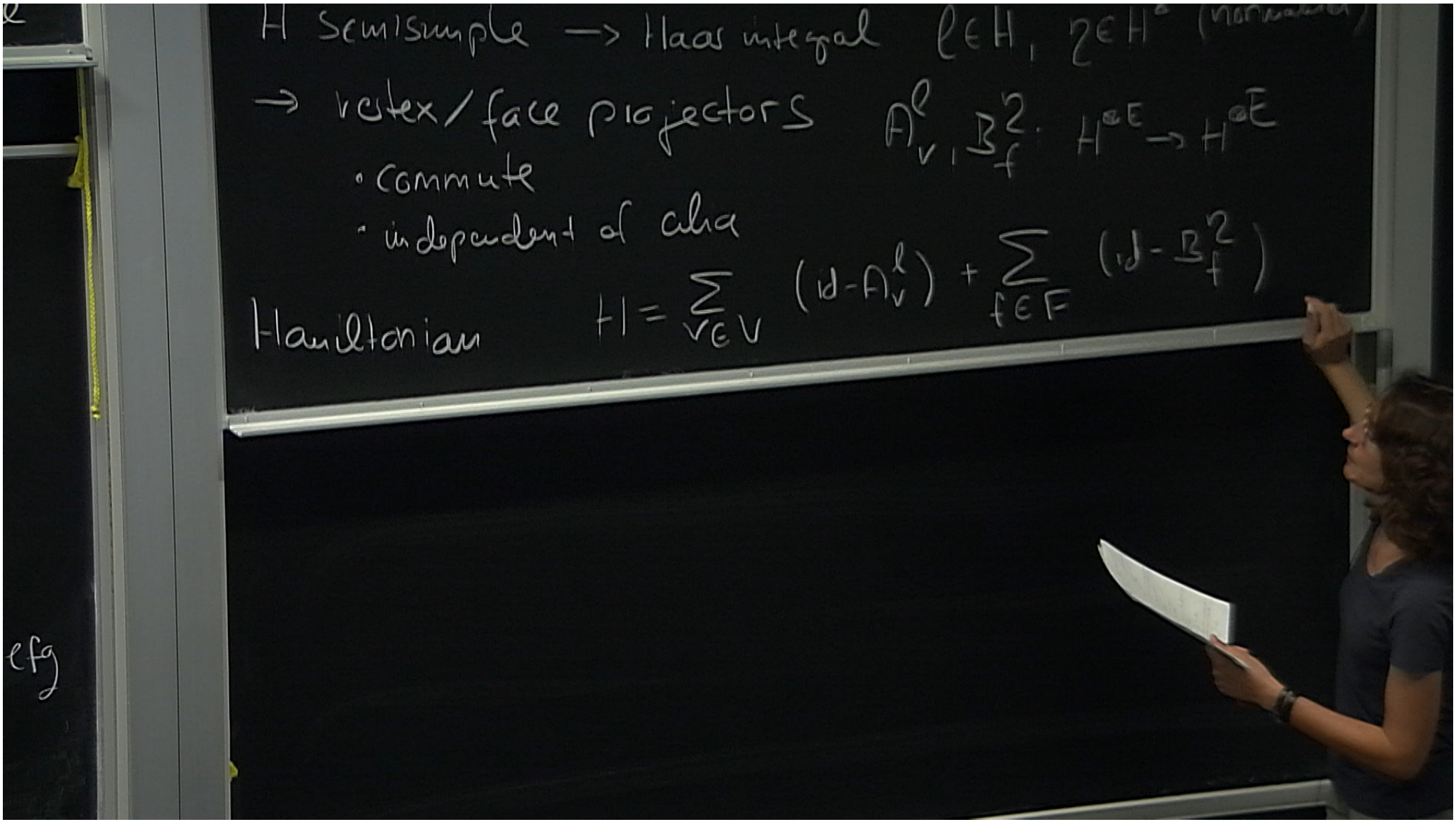

f
$\rho_{(v,f)}: H \otimes H \to \mathrm{End}(H^{\otimes E})$
$\alpha \otimes h \to \Omega_f^\alpha A_v^h$
H semisimple → Haar integral $\ell \in H$, $\eta \in H^*$ (normalized)
→ vertex/face projectors $A_v^\ell, B_f^\eta : H^{\otimes E} \to H^{\otimes E}$
efg

$\alpha \otimes h \to \Omega_f^\alpha A_v^h$

$H$ semisimple $\to$ Haar integral $\ell \in H$, $\lambda \in H^*$ (normalized)

$\to$ vertex/face projectors $A_v^\ell, B_f^\lambda : H^{\otimes E} \to H^{\otimes E}$

- commute
- independent of choice


H semisimple → Haar integral $\ell \in H$, $\Lambda \in H^*$ (normalized)
→ vertex/face projectors $A_v^\ell, B_f^\Lambda : H^{\otimes E} \to H^{\otimes E}$
• commute
• independent of choice
Hamiltonian $H = \sum_{v \in V} (\mathrm{id} - A_v^\ell) + \sum_{f \in F} (\mathrm{id} - B_f^\Lambda)$

ground state: $\mathcal{H}_{inv} = \{ x \in H^{\otimes E} \mid A_v^{\ell} x = B_f^2 x = x \}$

efg

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top. invariant: depends only on $\Sigma_\Gamma$ ← glue disk to faces

efg

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efg

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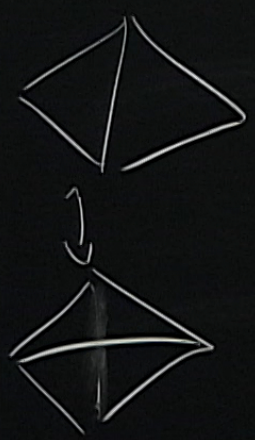

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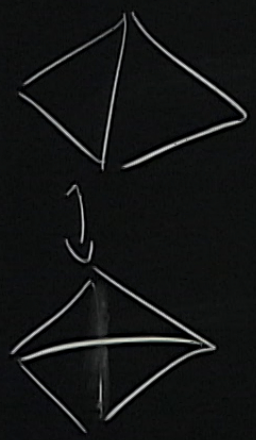
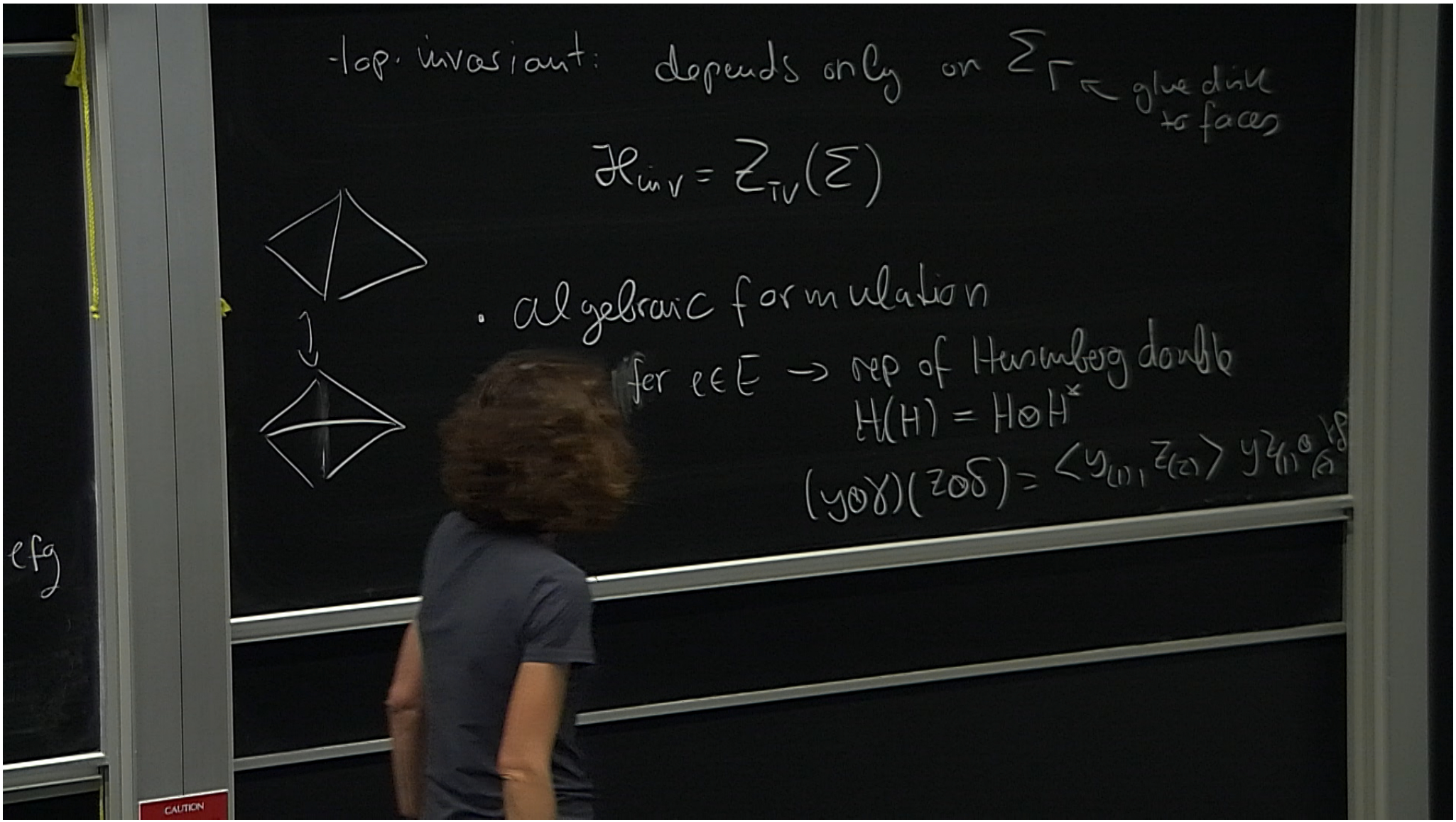


- algebraic formulation

$\Delta$-ops for $e \in E$ → rep of Heisenberg double

$$\mathcal{H}(H) = H \otimes H^*$$

efg


top. invariant: depends only on $\Sigma_\Gamma$ ← glue disk to faces
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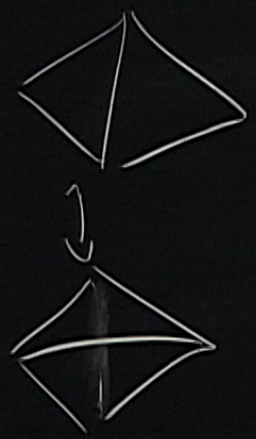
• algebraic formulation
for $e \in E$ → rep of Heisenberg double
$H(H) = H \otimes H^*$
$(y \otimes \gamma)(z \otimes \delta) = \langle y_{(1)}, z_{(2)} \rangle \, y z_{(1)} \otimes \gamma \triangleright \delta$
efg
CAUTION

top. invariant: depends only on $\Sigma_\Gamma$ ← glue disk to faces

$$\mathcal{H}_{inv} = Z_{TV}(\Sigma)$$



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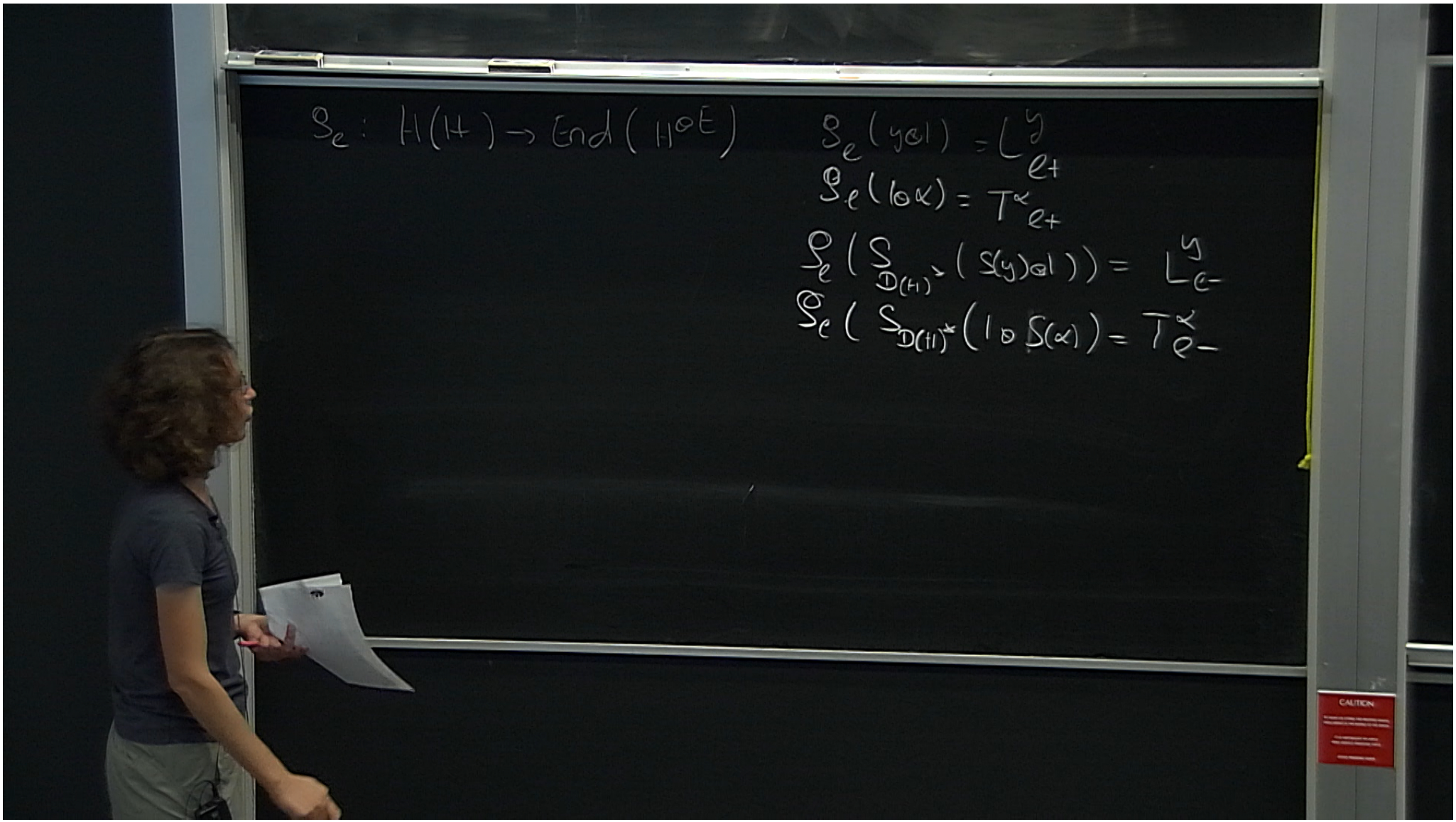
efg


$\rho_e : H(H) \to \mathrm{End}(H \otimes E)$
$\rho_e(y \otimes 1) = L^y_{e+}$
$\rho_e(1 \otimes \alpha) = T^\alpha_{e+}$
$\rho_e(S_{D(H)^*}(S(y) \otimes 1)) = L^y_{e-}$
$\rho_e(S_{D(H)^*}(1 \otimes S(\alpha)) = T^\alpha_{e-}$

$\rho_e : H(H) \to \mathrm{End}(H^{\otimes E})$

algebra of $\Delta$-op $S$

$\cong H(H)^{\otimes E} \cong \mathrm{End}_{\mathbb{C}}(H^{\otimes E})$

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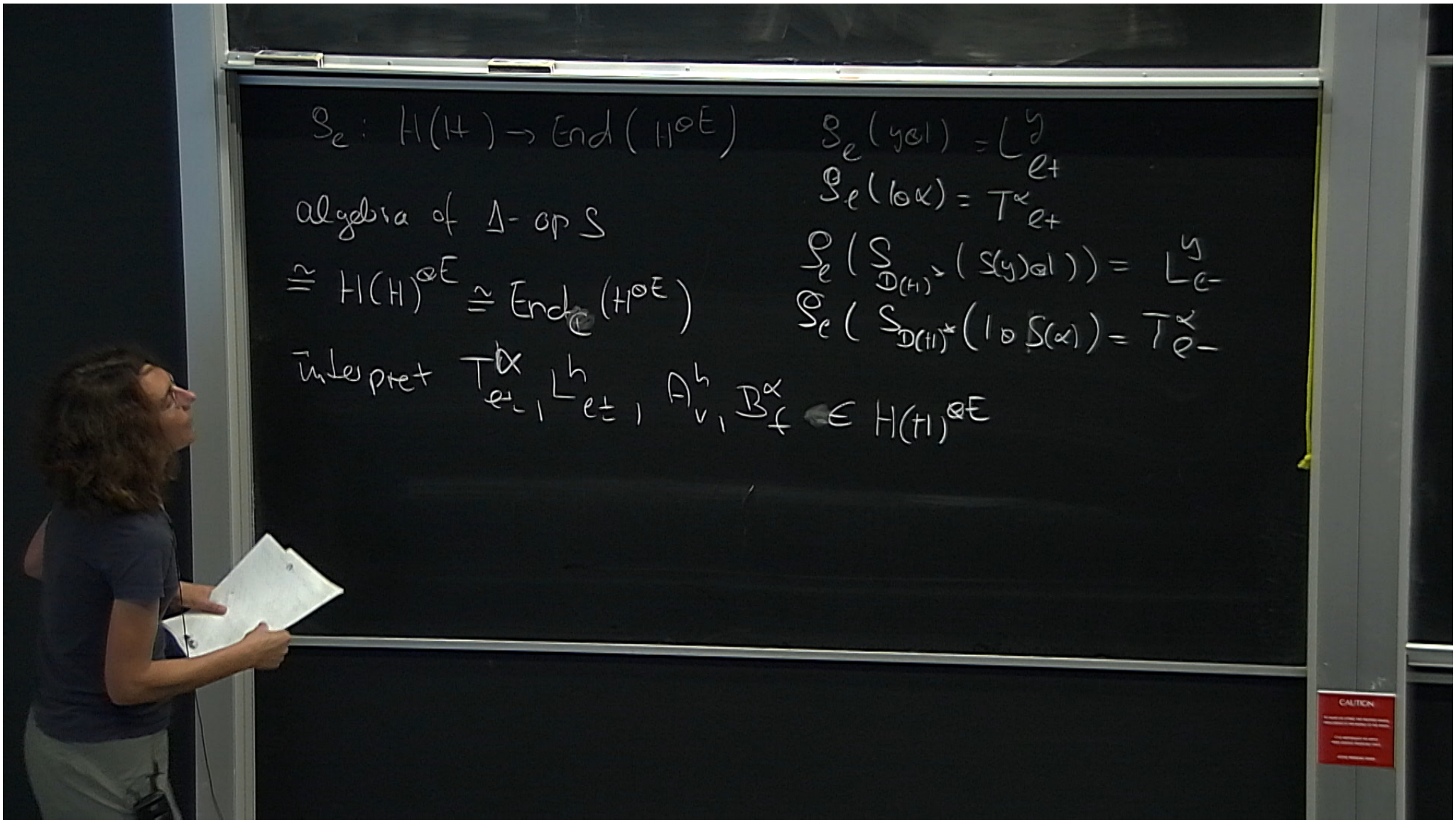
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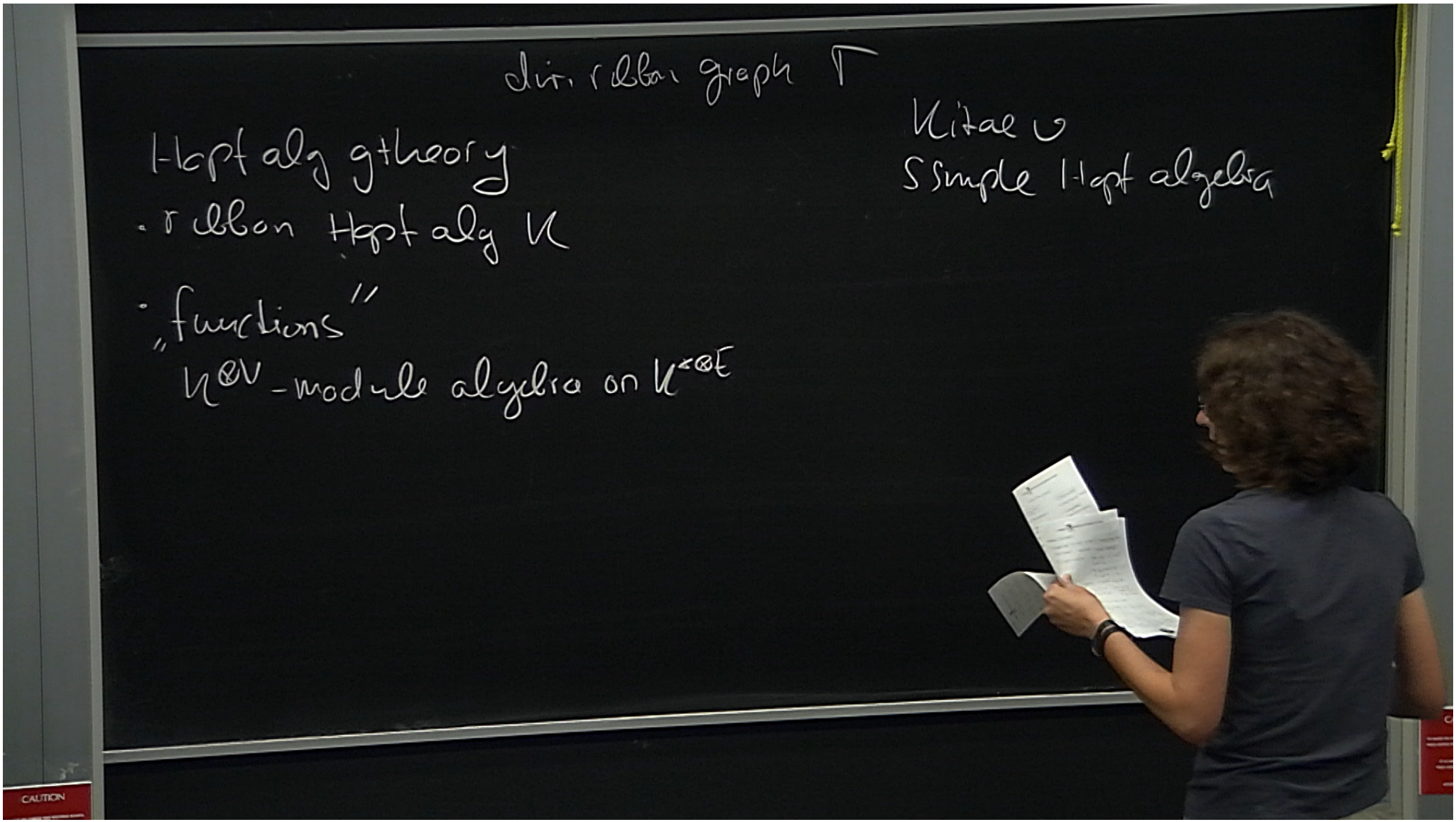
algebra of $\Delta$-op $S$
$\cong H(H)^{\otimes E} \cong \mathrm{End}_{\mathbb{C}}(H^{\otimes E})$
$\rho_e(S_{D(H)^*}(S(y) \otimes 1)) = L^y_{e-}$
$\rho_e(S_{D(H)^*}(1 \otimes S(\alpha))) = T^\alpha_{e-}$
interpret $T^\alpha_{e\pm}, L^h_{e\pm}, A^h_v, B^\alpha_f \in H(H)^{\otimes E}$


dir. ribbon graph Γ
Hopf alg theory
· ribbon Hopf alg K
· „functions"
K^{⊗V}-module algebra on K^{×⊗E}
Kitaev
S simple Hopf algebra

dir. ribbon graph $\Gamma$

Hopf alg theory

- ribbon Hopf alg $K$
- „functions"
  $K^{\otimes V}$-module algebra on $K^{*\otimes E}$
- „observables"
  $\mathcal{A}^\vee_{inv} = \{ x \in K^{*\otimes E} \mid x \Delta h = \varepsilon(h) x \ \forall h \in K^{\otimes V} \}$
- quantum moduli, $Q_{inv} = \{ x \in \mathcal{A}^\vee_{inv} \mid Hol_f(\alpha) x = x \varepsilon(\alpha) \}$

Kitaev

- S simple Hopf algebra
- „functions": $\Delta$-ops
  $H(H)^{\otimes E}$

dir. ribbon graph $\Gamma$

H-Hopf alg theory

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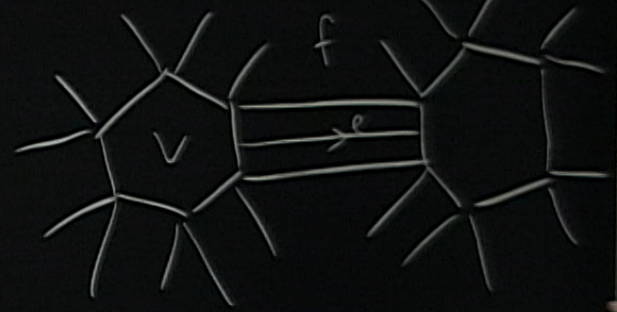
Kitaev

- simple H-Hopf algebra
- „functions": $\Delta$-ops
  $H(H)^{\otimes E}$
- ?
- ? $End_{\pi}(\mathcal{H}_{inv})$

② Kitaev models & Hopf algebra gauge theory

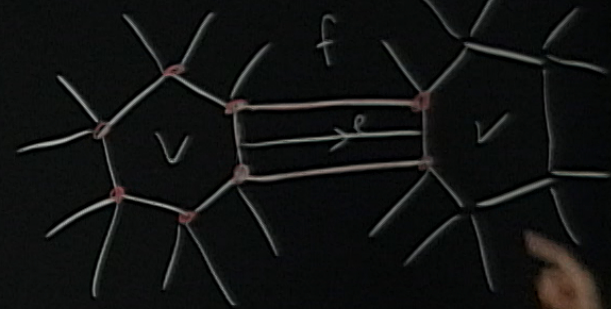
idea: thickened ribbon graph $\Sigma$

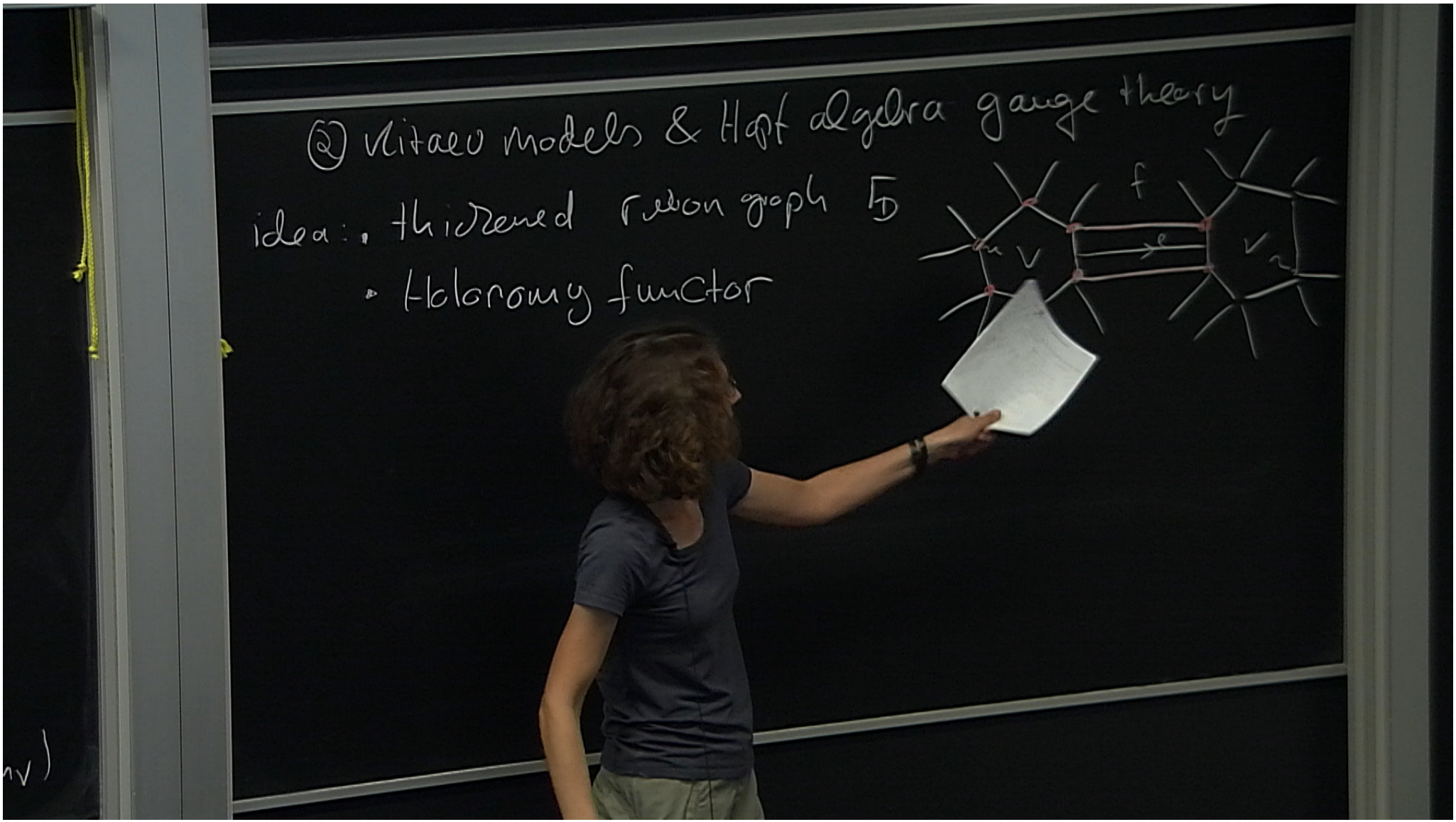


② Kitaev models & Hopf algebra gauge theory

idea: • thickened ribbon graph $\Sigma$

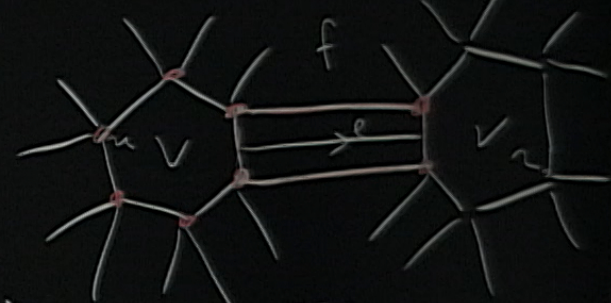
• Holonomy functor




② Kitaev models & Hopf algebra gauge theory
idea: • thickened ribbon graph
• Holonomy functor
f
e
v

② Kitaev models & Hopf algebra gauge theory

idea: • thickened ribbon graph $\Gamma_D$



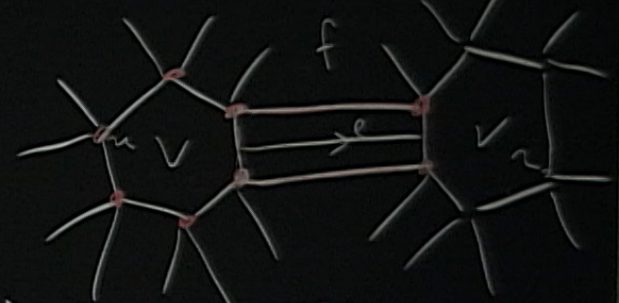

• Holonomy functor

$$\mathrm{Hol}: \mathcal{G}(\Gamma_D) \to \mathrm{Hom}_{\mathbb{C}}(H \otimes H^*, (H \otimes H^*) \otimes E)$$

path gpd

② Kitaev models & Hopf algebra gauge theory

idea: • thickened ribbon graph $\Gamma_D$



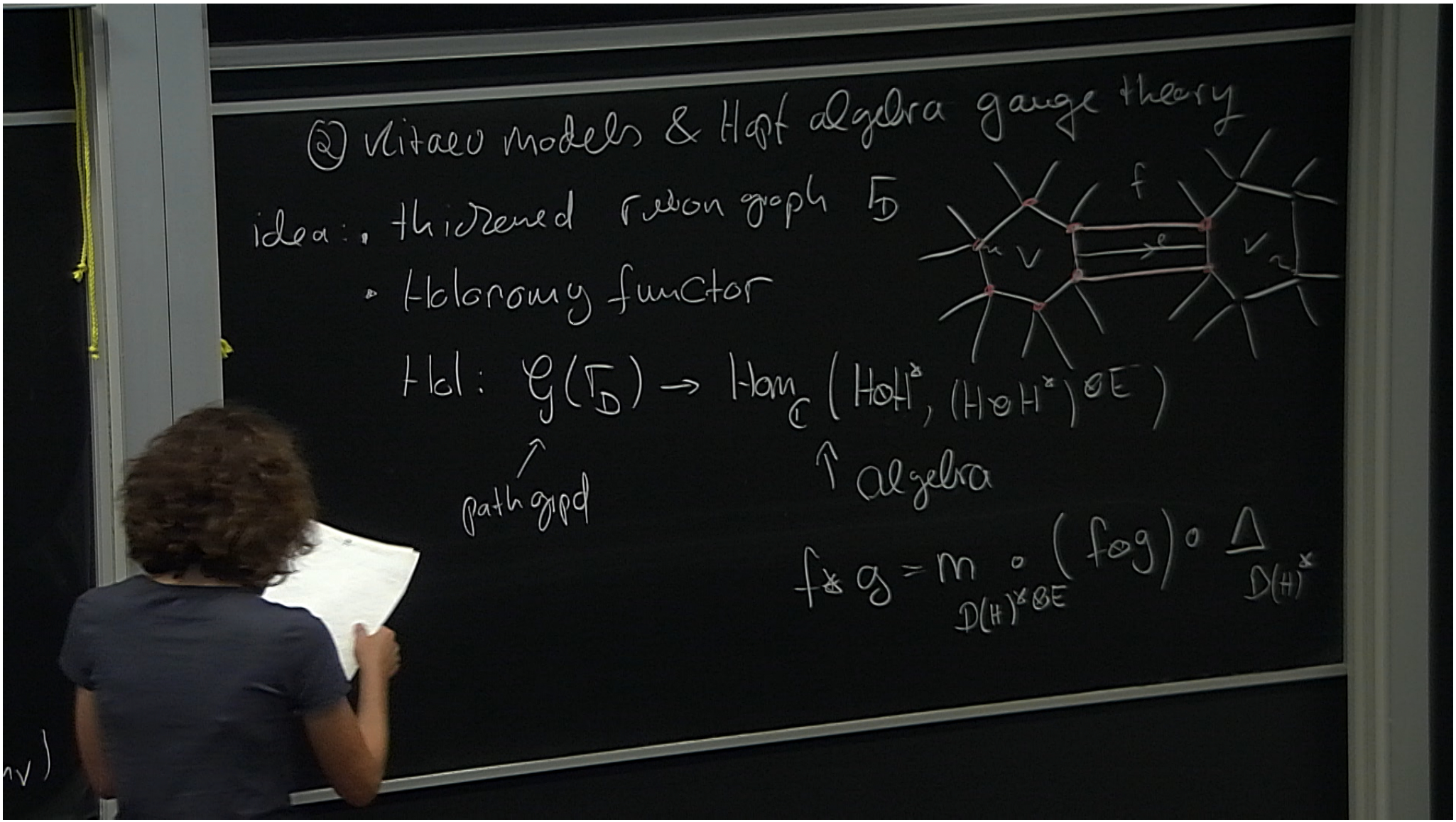

• Holonomy functor

$$\mathrm{Hol}: \mathcal{G}(\Gamma_D) \to \mathrm{Hom}_{\mathbb{C}}(H \otimes H^*, (H \otimes H^*) \otimes E)$$

path gpd

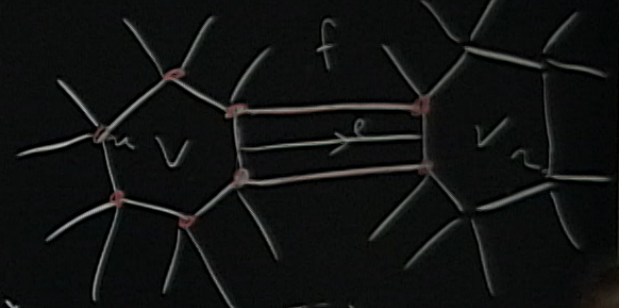
algebra

$$f * g = m \circ (f \otimes g) \circ \Delta$$


② Kitaev models & Hopf algebra gauge theory
idea: • thickened ribbon graph $\Gamma_D$
• Holonomy functor
$Hol: \mathcal{G}(\Gamma_D) \to Hom_{\mathbb{C}}(Hol H^*, (H \otimes H^*) \otimes E)$
path gpd
algebra
$f * g = m_{D(H)^* \otimes E} \circ (f \otimes g) \circ \Delta_{D(H)^*}$
f
e
v
v

② Kitaev models & Hopf algebra gauge theory

idea: • thickened ribbon graph $\Gamma_D$




• Holonomy functor

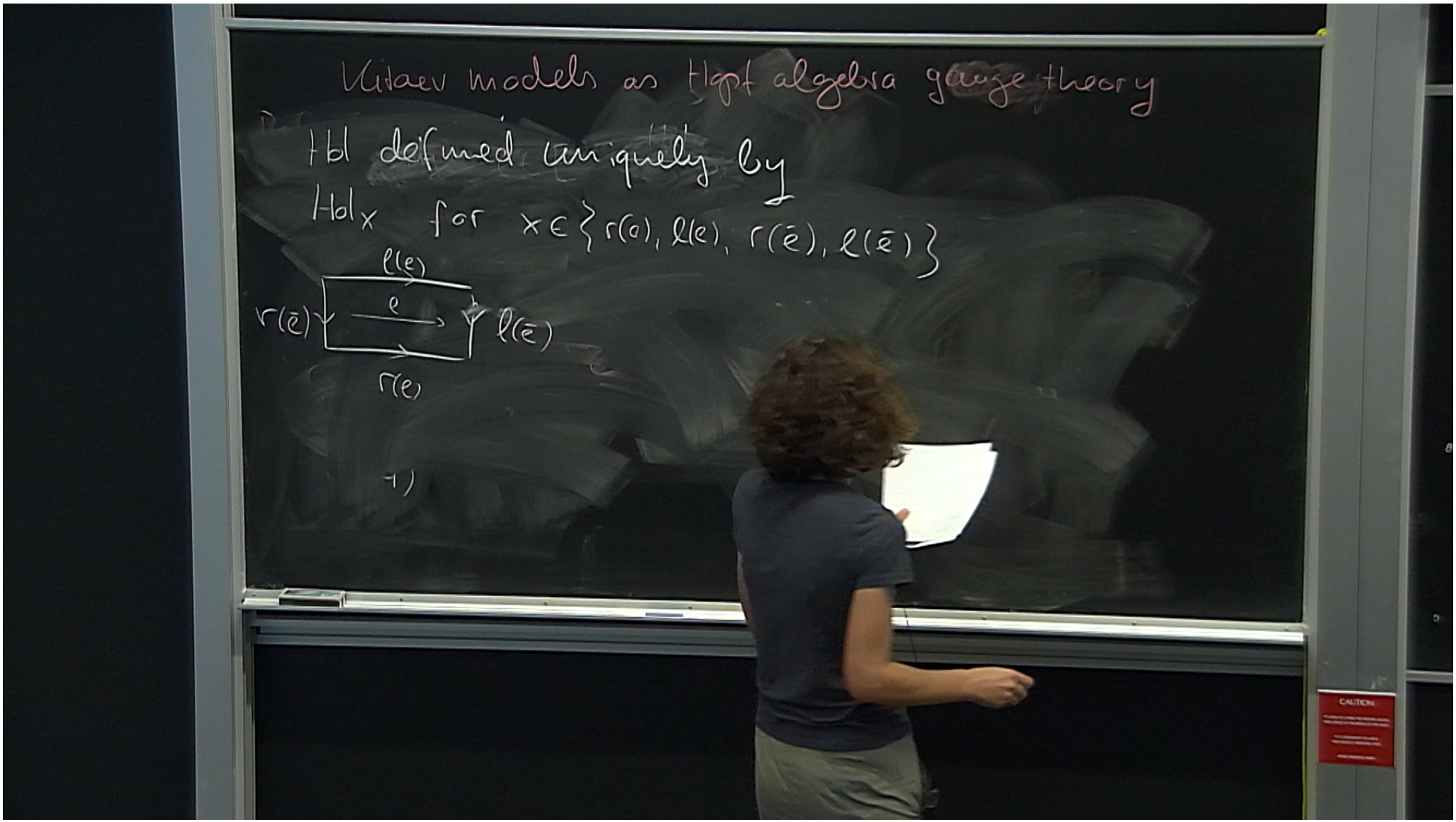
$$\mathrm{Hol}: \mathcal{G}(\Gamma_D) \to \mathrm{Hom}_{\mathbb{C}}(H \otimes H^*, (H \otimes H^*) \otimes E)$$

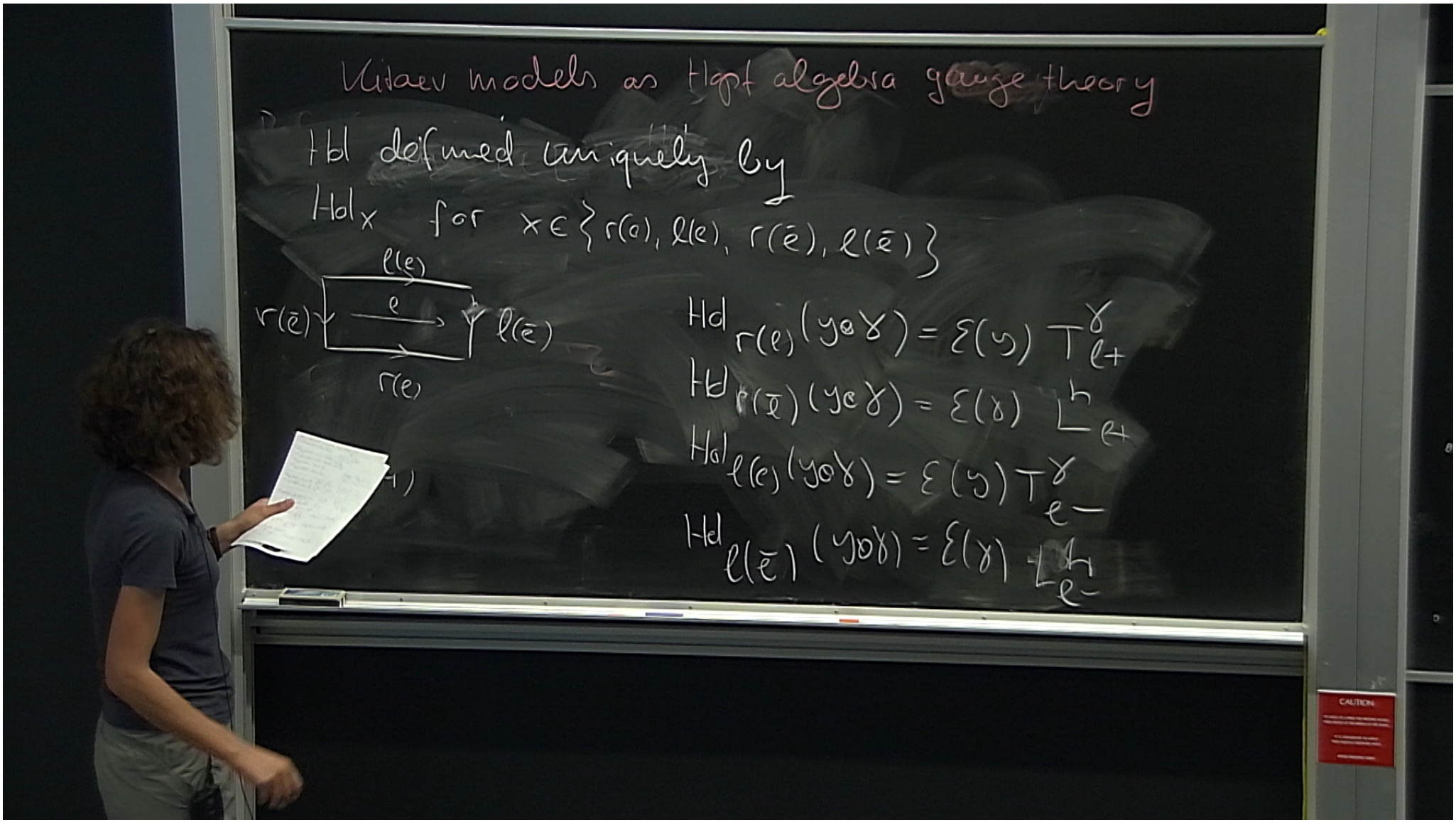
path gpd

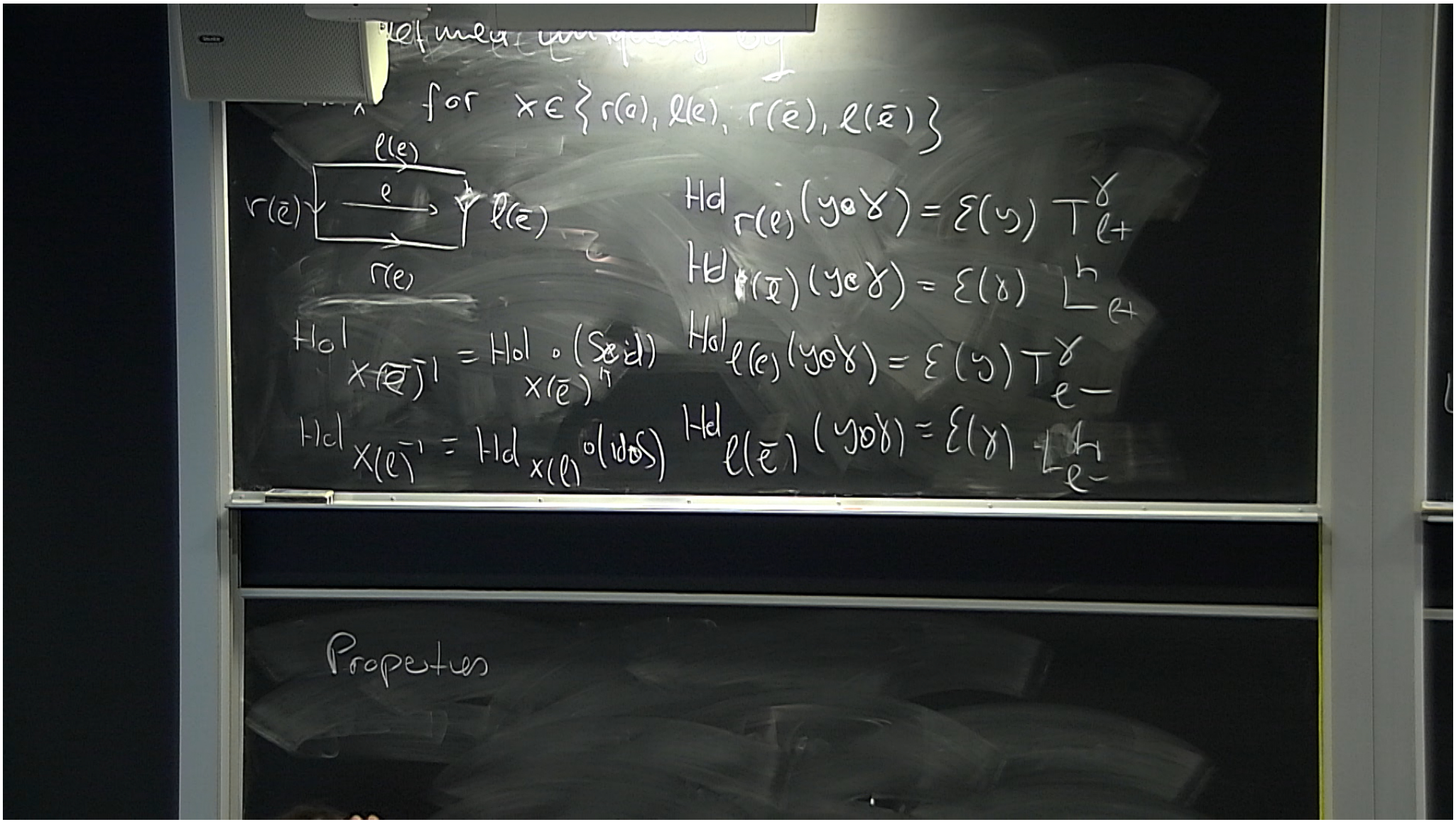
algebra

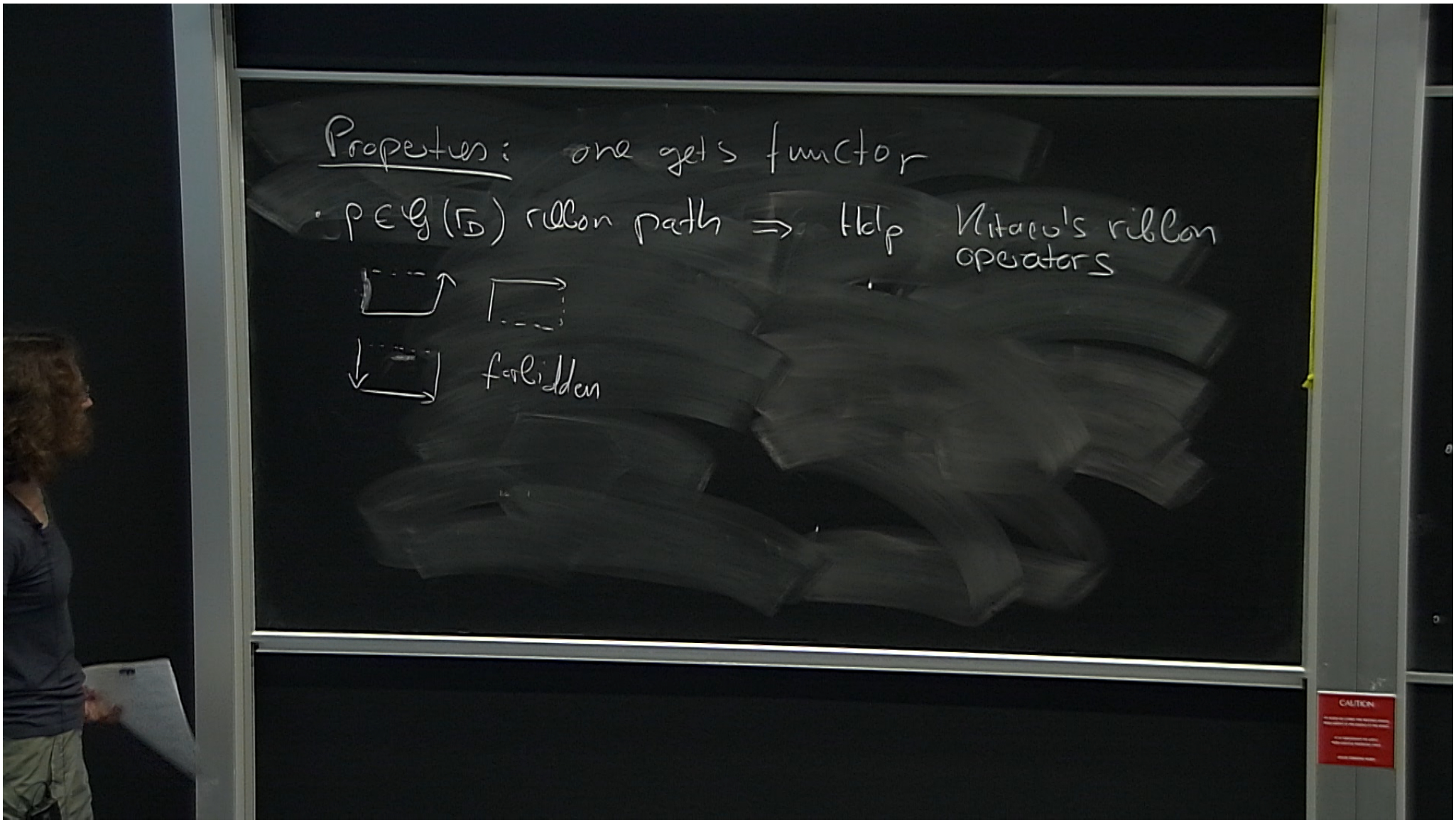
$$\mathrm{Hol}_{\phi_v} = 1 \cdot \varepsilon \qquad f * g = m_{D(H)^* \otimes E} \circ (f \otimes g) \circ \Delta_{D(H)^*}$$

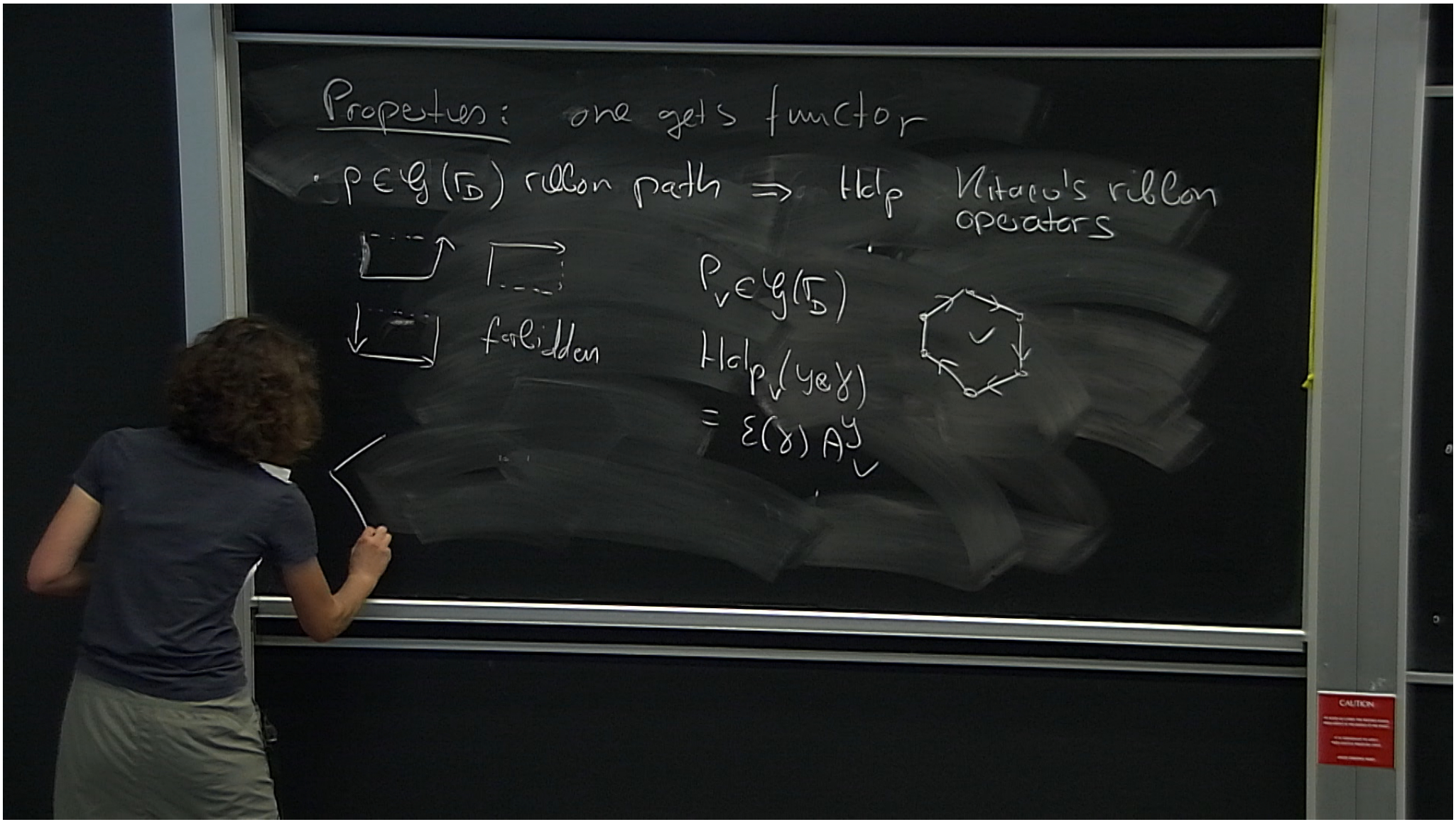
$$\mathrm{Hol}_{p \circ q} = \mathrm{Hol}_p * \mathrm{Hol}_q$$


Kitaev models as Hopf algebra gauge theory
Hbl defined uniquely by
Hbl_x for x ∈ {r(e), l(e), r(ē), l(ē)}
l(e)
r(ē)
e
l(ē)
r(e)


Kitaev models as Hopf algebra gauge theory
Hol defined uniquely by
Hol_x for x ∈ {r(e), l(e), r(ē), l(ē)}
l(e)
r(ē)
e
l(ē)
r(e)
Hol_r(e)(y⊗γ) = ε(y) T^γ_{e+}
Hol_r(ē)(y⊗γ) = ε(γ) L^h_{e+}
Hol_l(e)(y⊗γ) = ε(y) T^γ_{e−}
Hol_l(ē)(y⊗γ) = ε(γ) L^h_{e−}

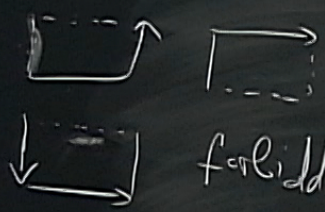

for x ∈ {r(e), ℓ(e), r(ē), ℓ(ē)}
ℓ(e)
r(ē)
e
ℓ(ē)
r(e)
Hd r(e)(y⊗γ) = ε(y) T^γ_{ℓ+}
Hd r(ē)(y⊗γ) = ε(γ) L^h_{ℓ+}
Hd ℓ(e)(y⊗γ) = ε(y) T^γ_{ℓ−}
Hd ℓ(ē)(y⊗γ) = ε(γ) L^h_{ℓ−}
Hd x(ē)^{−1} = Hd x(ē) ∘ (S⊗id)
Hd x(ℓ)^{−1} = Hd x(ℓ) ∘ (id⊗S)
Properties


Properties: one gets functor
· $p \in \mathcal{G}(\Gamma_D)$ ribbon path $\Rightarrow$ $Hol_p$ Kitaev's ribbon operators
forbidden


Properties: one gets functor
· p ∈ 𝒫(Γ_D) ribbon path ⇒ Hol_p Kitaev's ribbon operators
forbidden
P_v ∈ 𝒫(Γ_D)
Hol_{P_v}(y ⊗ γ)
= ε(γ) A^y_v

<u>Properties:</u> one gets functor

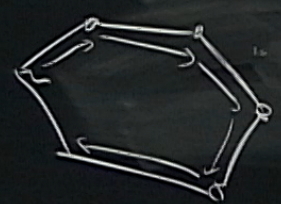
- $p \in \mathcal{G}(\Gamma_D)$ ribbon path $\Rightarrow$ $\mathrm{Hol}_p$ Kitaev's ribbon operators



forbidden

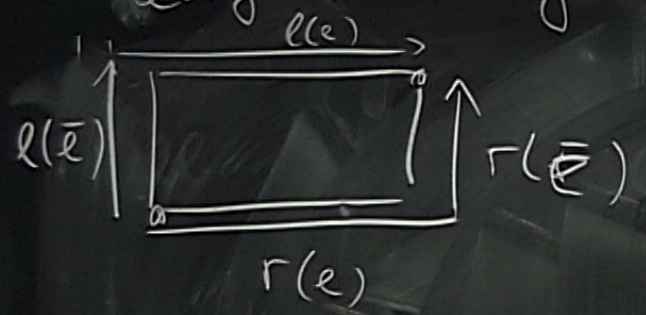
$p_v \in \mathcal{G}(\Gamma_D)$

$\mathrm{Hol}_{p_v}(y \otimes \gamma) = \varepsilon(\gamma) A_v^y$



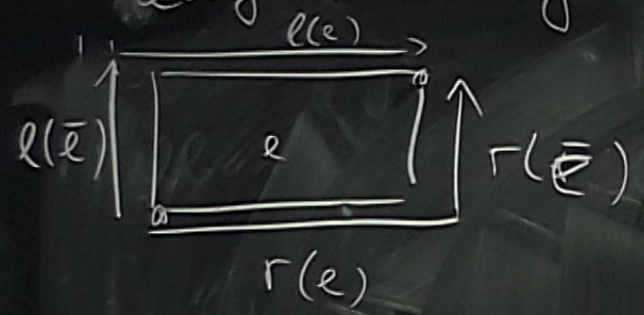
$p_f \in \mathcal{G}(\Gamma_D)$: $\mathrm{Hol}_{p_f}(y \otimes \gamma) = \varepsilon(y) B_f^\gamma$

edge rectangles




$$\mathrm{Hol}_{r(\bar{e})\circ r(e)}(y\otimes\gamma)$$
$$= \mathrm{Hol}_{\ell(e)\circ\ell(\bar{e})}(y\otimes\gamma) = (y\otimes\gamma)_e$$

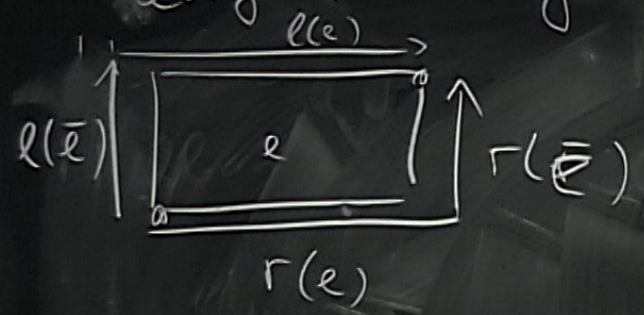
edge rectangles




$$\mathrm{Hol}_{r(\bar{e})\circ r(e)}(y\otimes\gamma) = \mathrm{Hol}_{\ell(e)\circ\ell(\bar{e})}(y\otimes\gamma) = (y\otimes\gamma)_e$$

$\longrightarrow$ flatness cond. of edges

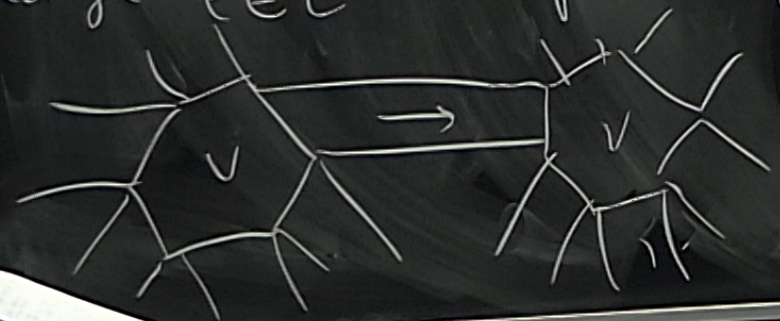
edge rectangles

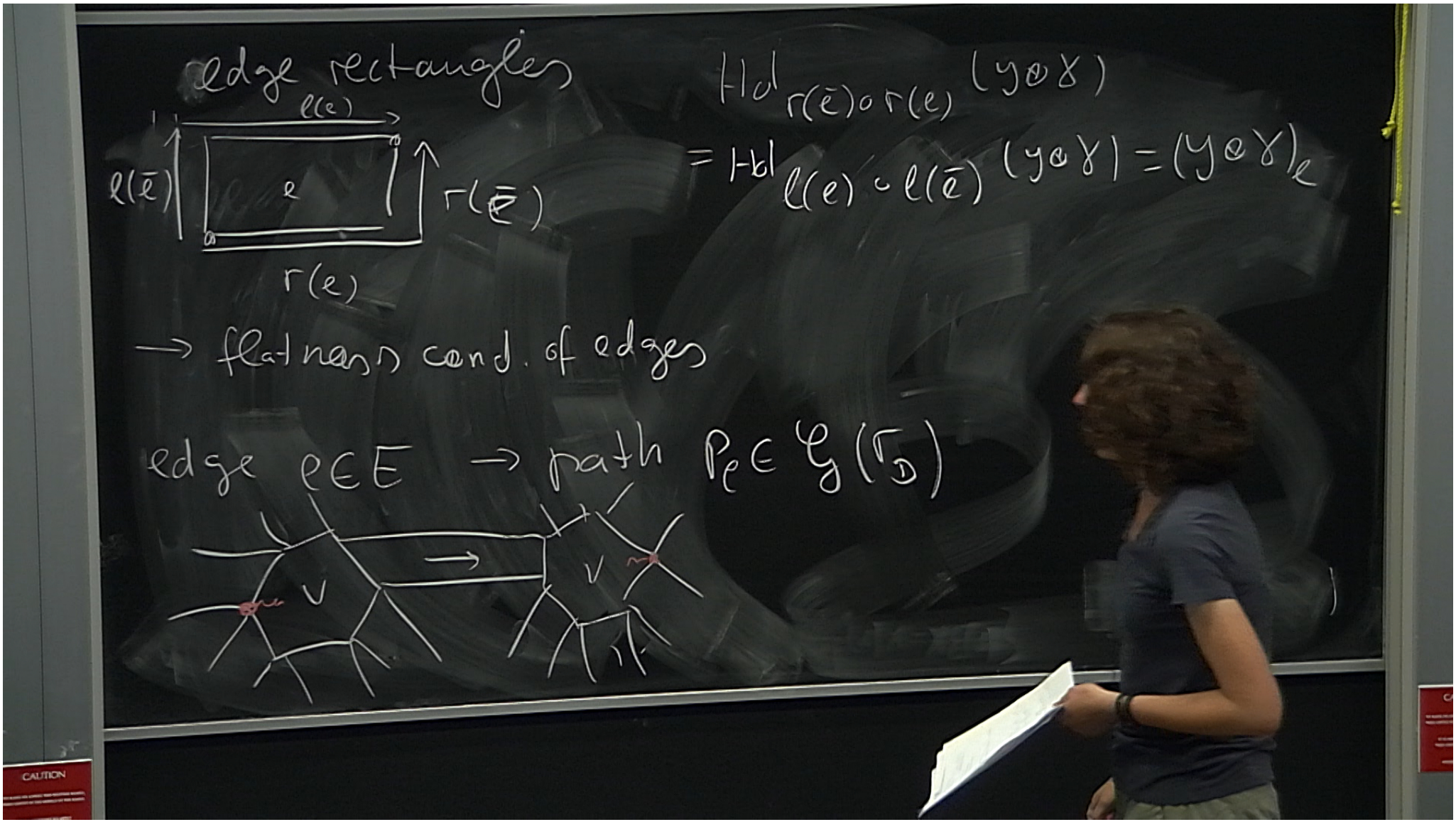


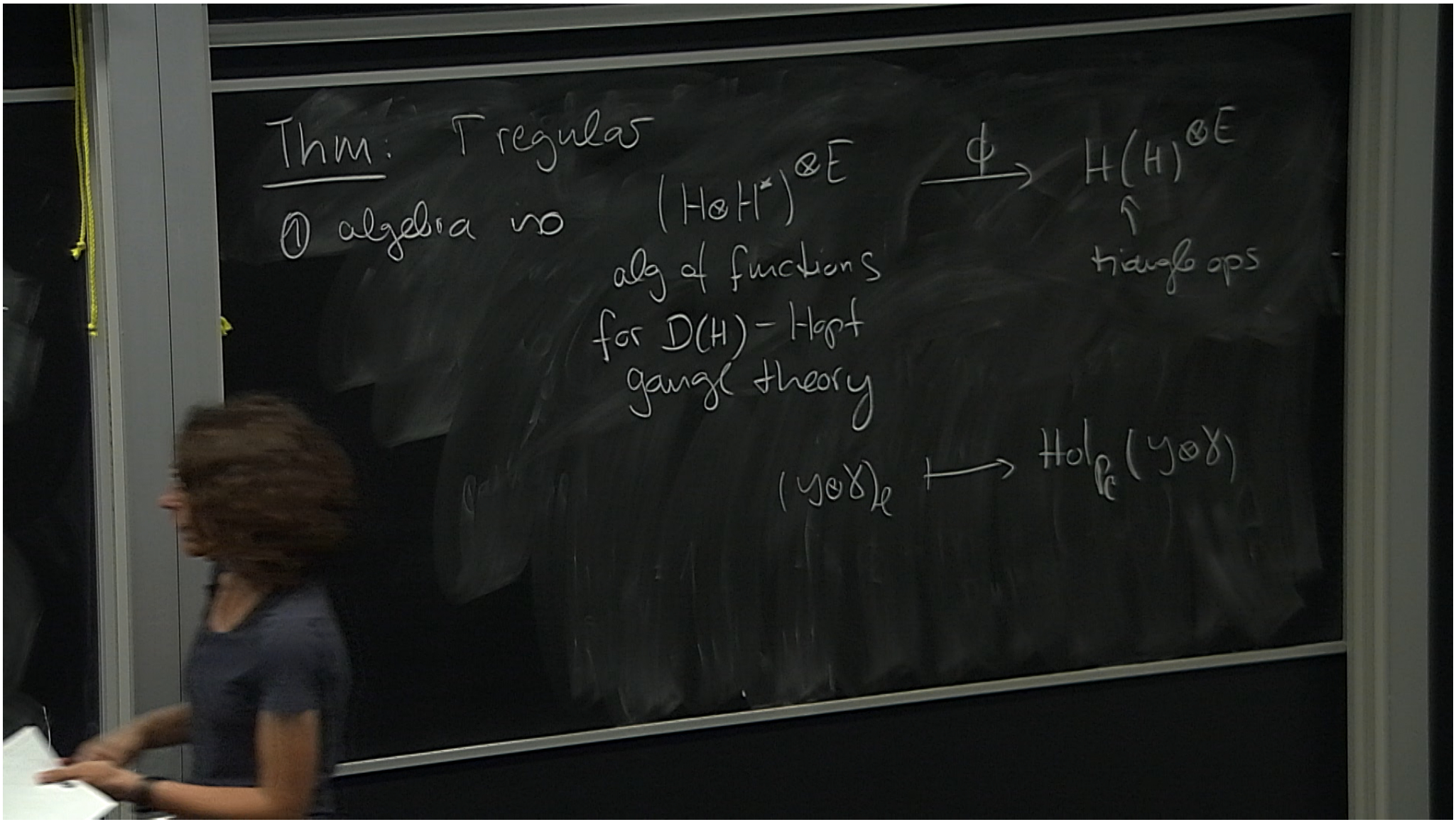

$$\mathrm{Hol}_{r(\bar{e})\circ r(e)}(y\otimes\gamma) = \mathrm{Hol}_{\ell(e)\circ\ell(\bar{e})}(y\otimes\gamma) = (y\otimes\gamma)_e$$

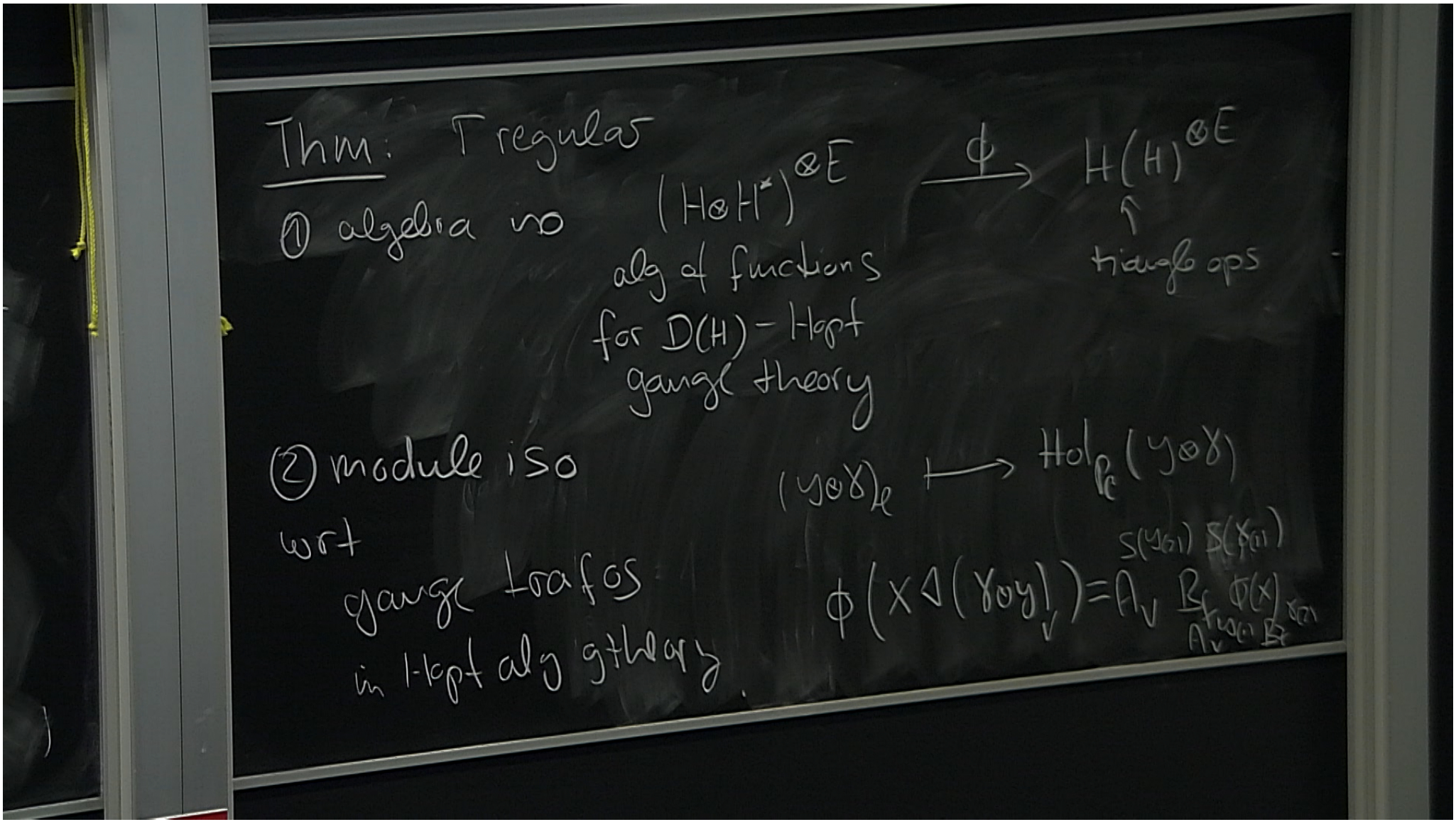
$\rightarrow$ flatness cond. of edges

edge $e\in E$ $\rightarrow$ path $P_e \in \mathcal{G}(\Gamma_D)$




edge rectangles
$\ell(e)$
$\ell(\bar{e})$
$e$
$r(\bar{e})$
$r(e)$
$\mathrm{Hol}_{r(\bar{e})\circ r(e)}(y\otimes\gamma)$
$= \mathrm{Hol}_{\ell(e)\circ\ell(\bar{e})}(y\otimes\gamma) = (y\otimes\gamma)_e$
$\rightarrow$ flatness cond. of edges
edge $e\in E$ $\rightarrow$ path $P_e \in \mathcal{G}(\Gamma_D)$
$v$
$v$


Thm: Γ regular
① algebra iso
$(H \otimes H^*)^{\otimes E} \xrightarrow{\phi} H(H)^{\otimes E}$
alg of functions
for D(H) - Hopf
gauge theory
triangle ops
$(y \otimes \gamma)_\ell \longmapsto \mathrm{Hol}_{P_\ell}(y \otimes \gamma)$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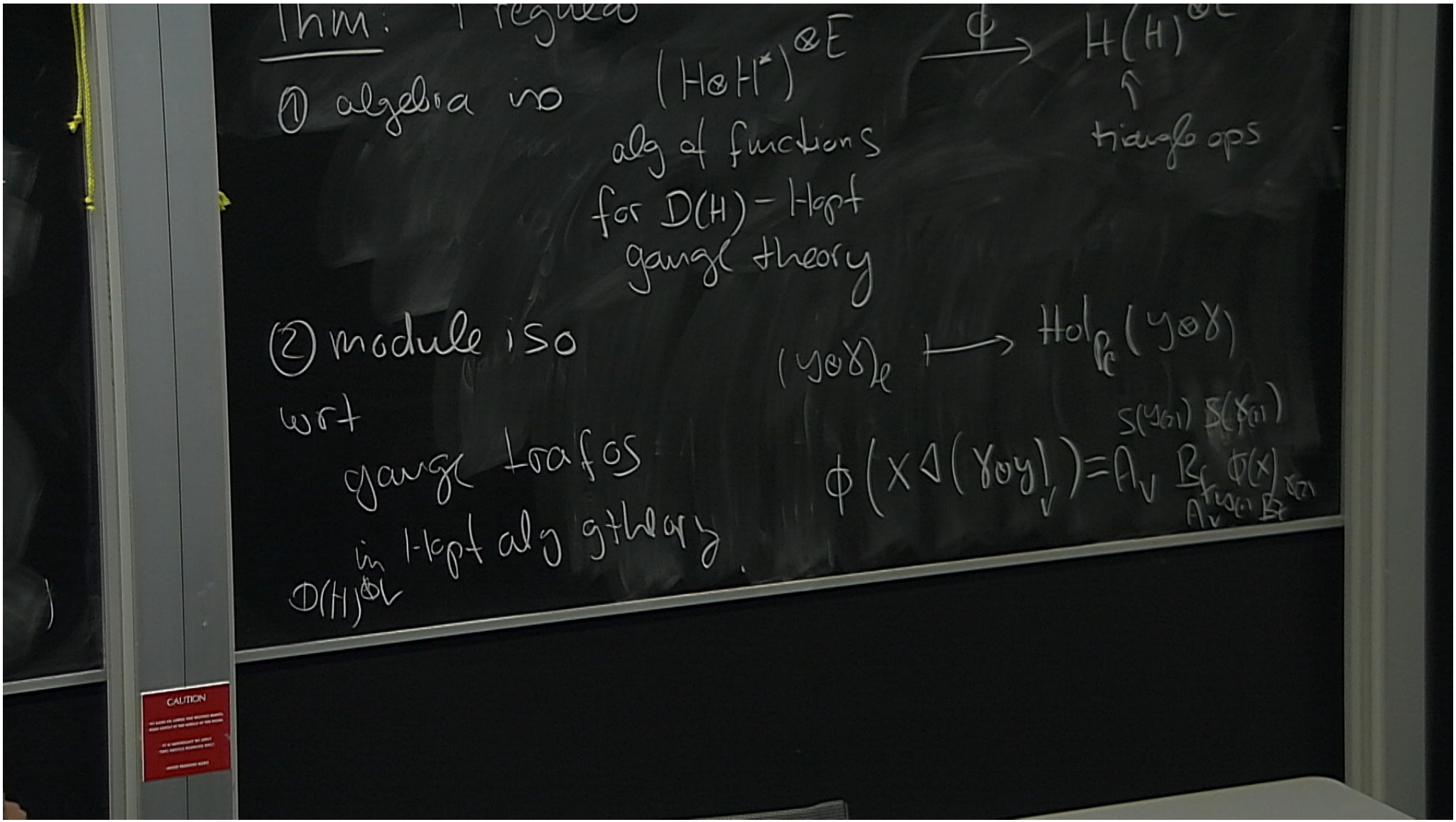
② module iso
wrt
gauge trafos
in Hopf alg gtheory
$(y \otimes \gamma)_e \longmapsto \mathrm{Hol}_{P_e}(y \otimes \gamma)$
$\phi(X \triangleleft (\gamma \otimes y)_v) = A_v B_f \phi(X)$


Thm: I regular
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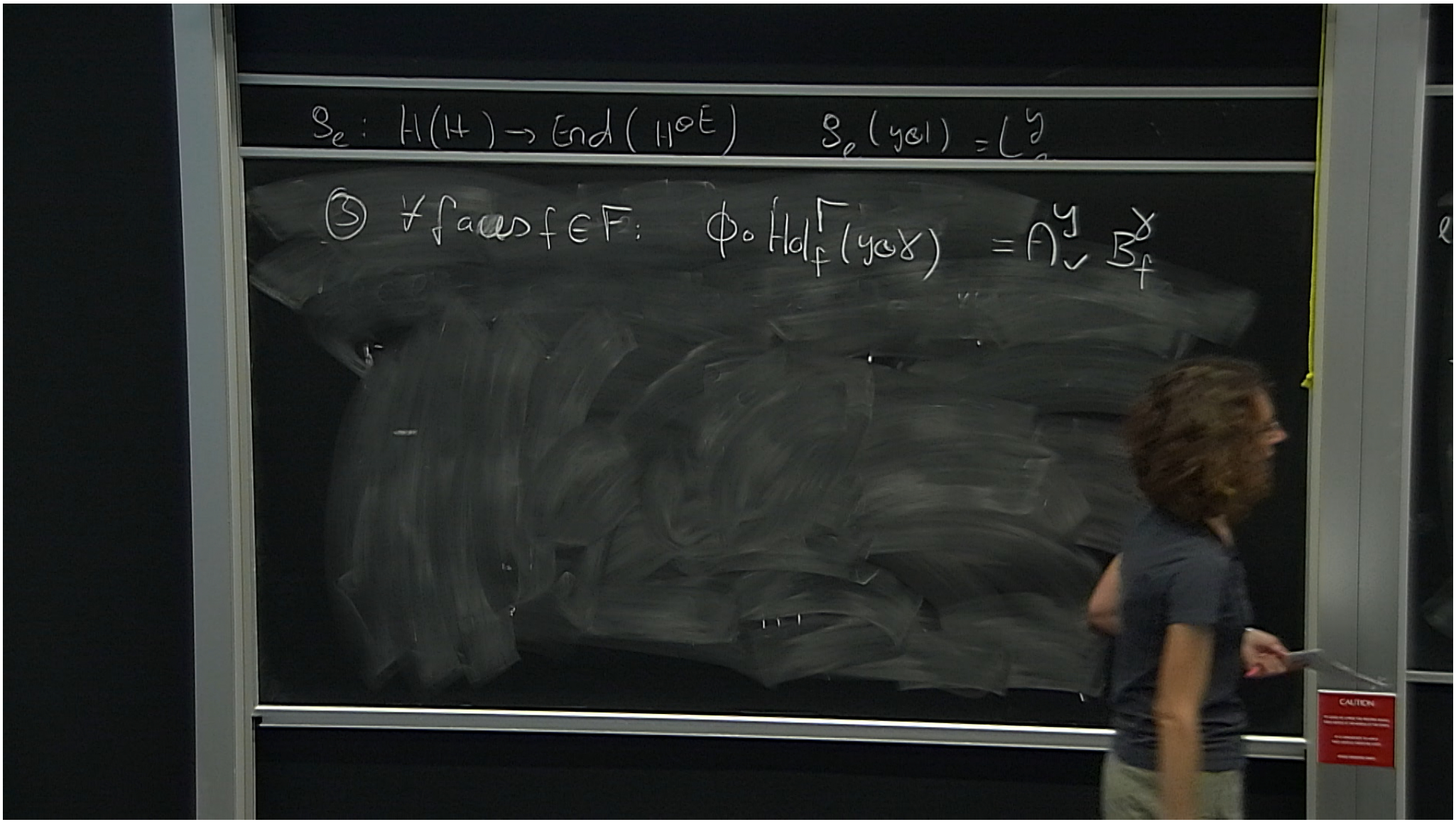
$D(H)^{\otimes L}$
$(y \otimes \gamma)_\ell \longmapsto Hol_{P_\ell}(y \otimes \gamma)$
$\phi(x \triangleleft (\gamma \otimes y)_v) = A_v \, B_f \, \phi(x)$
CAUTION


$\rho_e : H(H) \to \mathrm{End}(H^{\otimes E})$ $\rho_e(y \otimes 1) = L^y$
③ $\forall$ faces $f \in F$: $\phi \circ \mathrm{Hol}_f^{\Gamma}(y \otimes \gamma) = A^y_{\vee} B^{\gamma}_f$

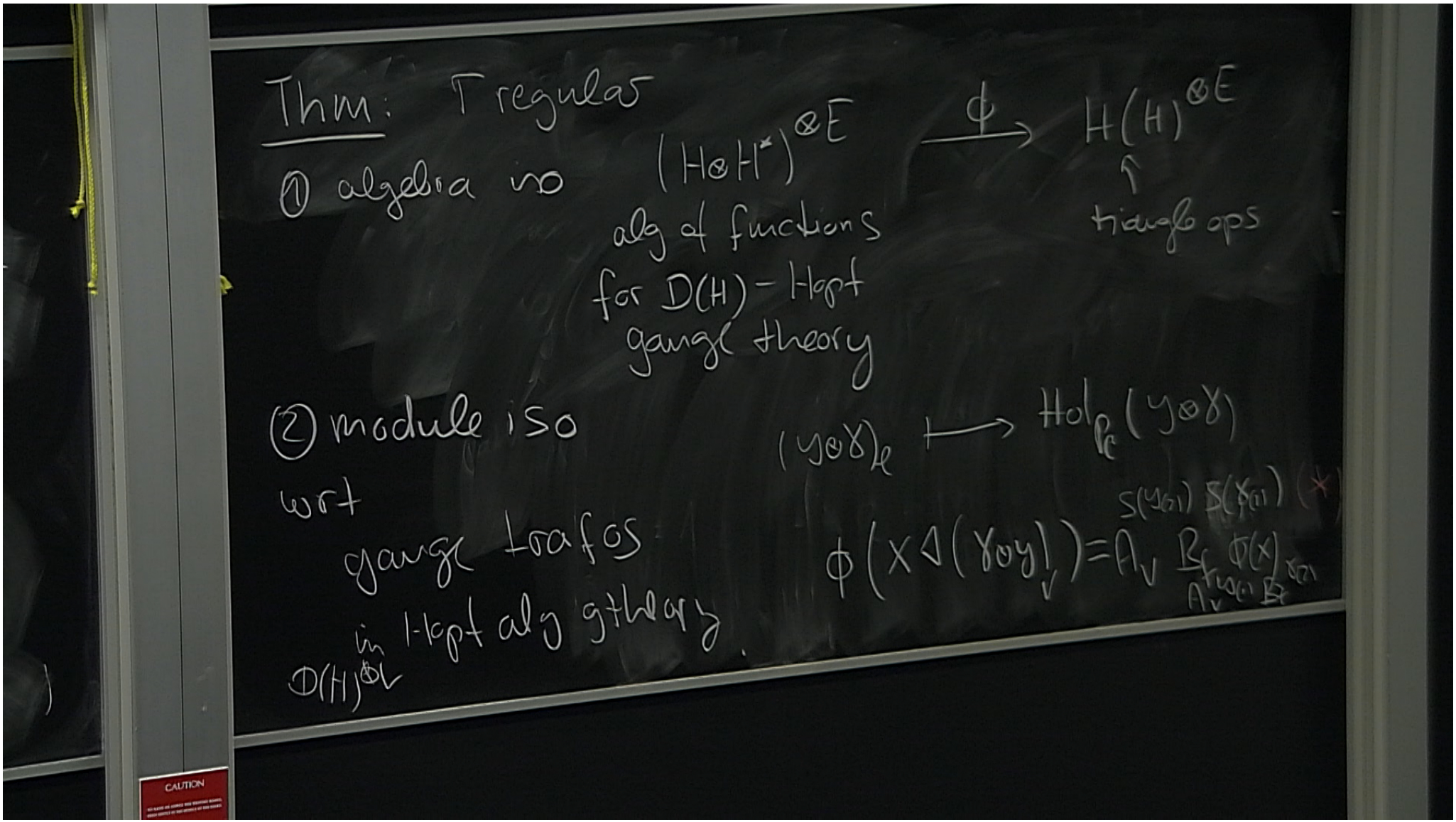
$\rho_e : H(H) \to \mathrm{End}(H^{\otimes E})$ $\quad \rho_e(y \otimes 1) = L^y$

(5) $\forall$ faces $f \in F$: $\phi \circ \mathrm{Hol}_f^{\Gamma}(y \otimes \gamma) = A^y_{\vee} B^{\gamma}_f$

$\sim$ compatible with flatness

<u>Consequence</u> Kitaev model $\overset{\phi}{\cong}$ Hopf alg gtheory

obs algebra $\mathcal{A}^*_{m_V}$ $\cong$ $\Delta$-ops invariant under $*$)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D(H)^⊗L
(y⊗γ)_ℓ ⟼ Hol_{P_ℓ}(y⊗γ)
φ(x ◁ (γ⊗y)_v) = A_v B_f φ(x)
S(y(2)) S(γ(2)) (*)
A_v^{y(1)} B_f^{γ(2)}

$\rho_e : H(H) \to \mathrm{End}(H^{\otimes E})$ $\quad \rho_e(y \otimes 1) = L^y$

(5) $\forall$ faces $f \in F$: $\phi \circ \mathrm{Hol}_f^{\Gamma}(y \otimes \gamma) = A^y_{\vee} B^{\gamma}_f$

$\sim$ compatible with flatness

<u>Consequence</u>: Kitaev model $\overset{\phi}{\cong}$ Hopf alg gtheory

obs algebra $\mathcal{A}^*_{m_V}$ $\cong$ $\Delta$-ops invariant under $*$)

qu moduli space

...

$\rho_e : H(H) \to \mathrm{End}(H^{\otimes E})$ $\quad \rho_e(y \otimes 1) = L^y_e$

(5) $\forall$ faces $f \in F$: $\phi \circ \mathrm{Hol}^{\Gamma}_f(y \otimes \gamma) = A^y_{\vee} B^{\gamma}_f$

$\sim$ compatible with flatness

<u>Consequence</u> Kitaev model $\overset{\sim}{=}$ Hopf alg gtheory

obs algebra $\mathcal{A}^*_{mv}$ $\overset{\phi}{\cong}$ $\Delta$-ops invariant under $\triangleright$

qu moduli space $Q^*_{mv}$ $\overset{\cong}{\phi}$ $\mathrm{End}_{\mathbb{F}}(\mathcal{H}_{mv})$

...