

Title: The Quantum Null Energy Condition from Causality

Date: Jul 31, 2017 01:00 PM

URL: <http://pirsa.org/17070067>

Abstract: <p>IT from Qubit web seminar</p>

q nec-talk (page 1 of 94) — Edited

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THE QUANTUM NULL ENERGY CONDITION FROM CAUSALITY

based on arXiv: **1706.09432** with:

Srivatsan Balakrishnan, Zuhair Khandker, Huajia Wang

Tom Faulkner
University of Illinois at Urbana-Champaign



OUTLINE

Warnings:

$$2\pi = 1$$

- motivational QNEC
- proving the ANEC in two ways I
- introduce $f(s)$ and properties thereof
- whence $f(s)$? causality of entangling surfaces in AdS/CFT
- calculation (modular flow using defect OPE)
- generalizations and conclusions



I
THE QNEC



some local negative energy
density



THE QNEC

- Globally, energy in QFT is positive (above ground state), but local energy density can be negative
- Classical field theory, Null Energy Condition (NEC):

$$T_{uu}(y) \sim (\partial_u \phi)^2 \geq 0 \quad u = \text{light-like coordinate}$$

- Quantum mechanics: violated by quantum fluctuations, but:

$$\langle T_{uu}(y) \rangle_\psi \geq D_u^2 S_{EE}(A) \quad y \in \partial A$$

I
Bousso, Fisher, (Koeller), Leichenauer, Wall '15



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Bousso, Fisher, (Koeller), Leichenauer, Wall '15



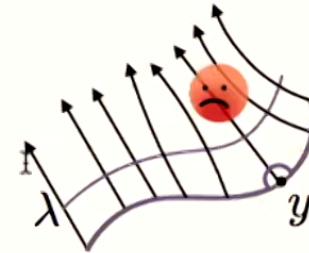


MOTIVATION I - LEARNING ABOUT QFT FROM GENERAL RELATIVITY

- gravity focuses light rays (does not anti-focus)
- Raychaudhuri's equation:

$$\frac{d\theta}{d\lambda} = -\theta^2 - \sigma^2 - G_N T_{uu} \leq 0$$

$$\theta \sim \mathcal{K}_u \sim \frac{\delta \text{Area}}{\delta x^u(y)}$$



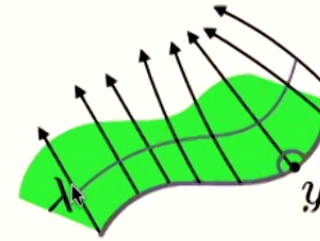
- But violated in quantum (semi-classical) gravity?



MOTIVATION I - LEARNING ABOUT QFT FROM GENERAL RELATIVITY

- Area \sim entropy \longrightarrow generalized entropy

$$S_{\text{gen}} = S_{EE}(A) + \frac{\text{Area}(\partial A)}{4G_N}$$



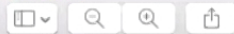
- Leads to quantum focusing conjecture (QFC):

$$\Theta = \frac{\delta}{\delta x^u(y)} (\text{Area} + 4G_N S_{EE}) \quad \frac{d\Theta(A; y)}{d\lambda} \leq 0$$

- QNEC follows from flat space limit:

$$\theta = 0 \quad \sigma = 0$$



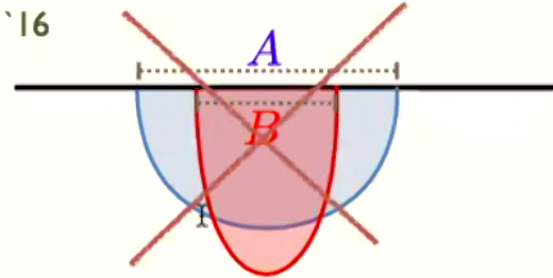


MOTIVATION II - BULK CAUSALITY AND ADS/CFT

- In AdS/CFT the QNEC follows from causal properties of the HRT surface: Koeller, Leichenauer, '16

- Entanglement Wedge Nesting

$$B \subset A \quad \mathcal{E}_B \subset \mathcal{E}_A$$



- Links many different areas of study: bulk reconstruction

modular Hamiltonians

entanglement

chaos bound

ANEC

micro causality and the bootstrap

emergence of bulk locality from QFT??

defect CFTs

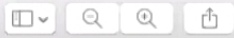




REMINDERS

- The QNEC is a conjectured property of QFTs (no gravity)
 - We will prove it in Minkowski space, although proof should extend to QFTs in curved space I
 - We will work with general QFTs with an interacting UV fixed point and $d > 2$
 - We start our story with the ANEC ...
-





TWO PATHS TO THE ANEC

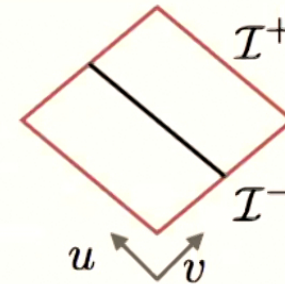
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THE ANEC

- The averaged NEC:

$$\mathcal{A}_u = \int_{-\infty}^{\infty} du' \langle T_{uu}(u', v' = 0, y) \rangle_{\psi} \geq 0$$



- Also predicted from GR (e.g. by demanding wormholes not traversable)
- Non trivial in Minkowski space ~ Hofman-Maldacena bounds

$$\text{In } d=4 \text{ CFTs: } \frac{31}{18} \geq \frac{a}{c} \geq \frac{1}{3}$$



METHOD I: MODULAR HAMILTONIANS

TF, Leigh, Parrikar, Wang, '16

$$\mathcal{H}_{tot} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}} \quad \rho_A = \text{Tr}_{\bar{A}} |\psi\rangle \langle \psi|$$

$$\text{I} \quad H_A = -\ln \rho_A$$

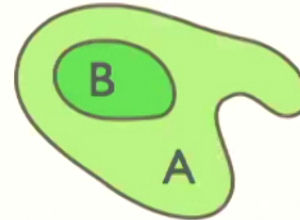
“Full” Modular Hamiltonian - better behaved in QFT:

$$K_A = H_A \otimes 1_{\bar{A}} - 1_A \otimes H_{\bar{A}}$$

Inclusion property:

$$B \subset A$$

$$K_A - K_B \geq 0$$



Proof: relative entropy monotonicity





MODULAR HAMILTONIANS

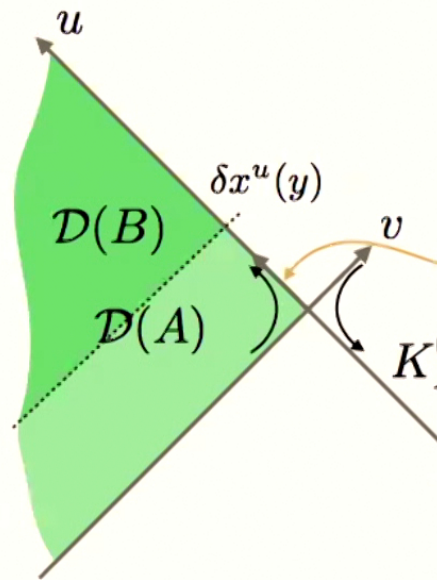
Relativistic QFT - inclusion at level of causal domains

specialize to vacuum state: $|\psi\rangle \rightarrow |0\rangle$

A: half space (Rindler) cut

$K_A^0 =$ Boost operator

B: small null deformation thereof



$$K_A^0 - K_B^0 = \int dy \int_{-\infty}^{\infty} du' \delta x^u(y) T_{uu}(u')$$



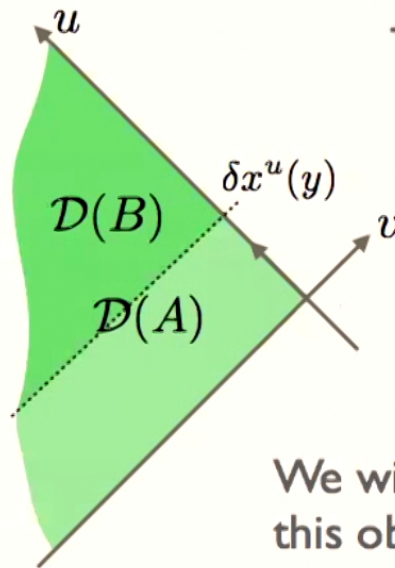


MODULAR HAMILTONIANS

Our approach was perturbative in $\delta x^u(y)$

Turns out this is an exact result Lashkari '17

Casini, Teste, Torroba '17



$$\implies [K_B^0, K_A^0] = i(K_A^0 - K_B^0)$$

Only true for vacuum:
“Half-Sided Modular Inclusions”

$$e^{-iK_B^0 s} e^{iK_A^0 s} = \mathfrak{e}^{i(1-e^{-s})P_u}$$

gens null
“translations”

$$P_u = K_B^0 - K_A^0$$

We will be studying
this object a lot, but
for non vacuum states!

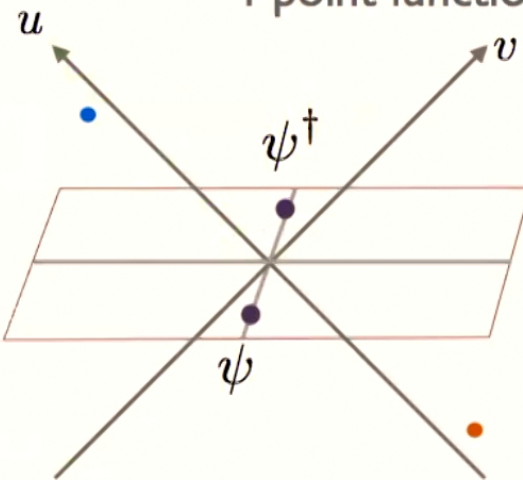


METHOD II: CAUSALITY

Hartman, Kundu, Tajdini '16

$$f(u) \propto \langle \psi | \mathcal{O}(u, v) \mathcal{O}(-u, -v) | \psi \rangle$$

4-point function - conformal bootstrap



$$\langle \psi | [\mathcal{O}, \mathcal{O}] | \psi \rangle = 0$$

Causality: space-like separated

$f(u)$ analytic in u, v



COMPUTABLE: LIGHTCONE OPE

$$f_{\mathbb{I}}(u) \propto \langle \psi | \mathcal{O}(u, v) \mathcal{O}(-u, -v) | \psi \rangle$$

$$\frac{\mathcal{O}(u, v) \mathcal{O}(-u, -v)}{\langle \mathcal{O}(u, v) \mathcal{O}(-u, -v) \rangle} \sim \sum_k u^{(\Delta_k + \ell_k)/2} v^{(\Delta_k - \ell_k)/2} \mathcal{O}_k(0)$$

	KINEMATICS	DOMINANT OPS.	EXAMPLE
Euclidean OPE:	$u \sim v$	lowest dimension Δ	$\mathcal{O}_{\text{primary}}$
Lightcone OPE:	$v \ll u$	lowest twist $\tau = \Delta - \ell$	$T_{uu} \quad \begin{matrix} \tau = d - 2 \\ \ell = 2 \end{matrix}$

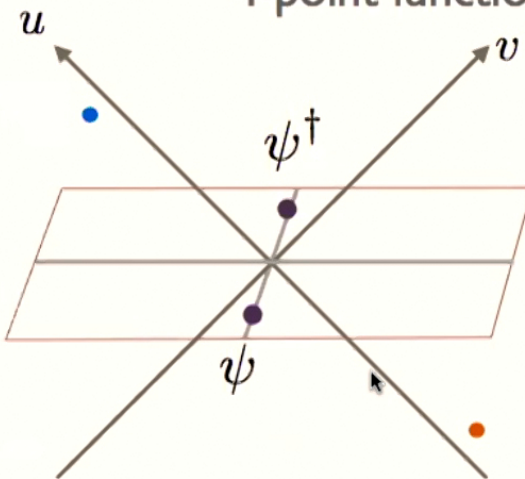


METHOD II: CAUSALITY

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4-point function - conformal bootstrap



$$\langle \psi | [\mathcal{O}, \mathcal{O}] | \psi \rangle = 0$$

Causality: space-like separated

$f(u)$ analytic in u, v

Computability: in certain limits

Positivity: bound $f(u)$ generally



COMPUTABLE: LIGHTCONE OPE

$$f(u) \propto \langle \psi | \mathcal{O}(u, v) \mathcal{O}(-u, -v) | \psi \rangle$$

$$\frac{\mathcal{O}(u, v) \mathcal{O}(-u, -v)}{\langle \mathcal{O}(u, v) \mathcal{O}(-u, -v) \rangle} \sim \sum_k u^{(\Delta_k + \ell_k)/2} v^{(\Delta_k - \ell_k)/2} \mathcal{O}_k(0)$$

	KINEMATICS	DOMINANT OPS.	EXAMPLE
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Lightcone OPE:	$v \ll u$	lowest twist $\tau = \Delta - \ell$	$T_{uu} \quad \begin{matrix} \tau = d - 2 \\ \ell = 2 \end{matrix}$

But also: $\partial_u^{\ell-2} T_{uu} \quad \ell \geq 2$



COMPUTABLE: LIGHTCONE OPE

Computable in the light cone limit: $v \rightarrow 0$ $u =$ fixed and somewhat large

Re-sum $\partial_u^{\ell-2} T_{uu}(0)$ into an integral:

$$f = 1 - C_T^{-1} (-uv)^{\tau/2} u \mathcal{A}_u + \dots$$

$$\mathcal{A}_u = \int_{-\infty}^{\infty} \langle T_{uu}(u', v=0) \rangle_{\psi} du'$$

Analyticity relates \mathcal{A}_u to $f(u)$ in a range where it cannot be computed but it can be bounded (Wedge reflection positivity)

Hartman, Kundu, Tajdini '16 $\mathcal{A}_u \geq 0$

Casini '11





HOW ARE THESE RELATED??

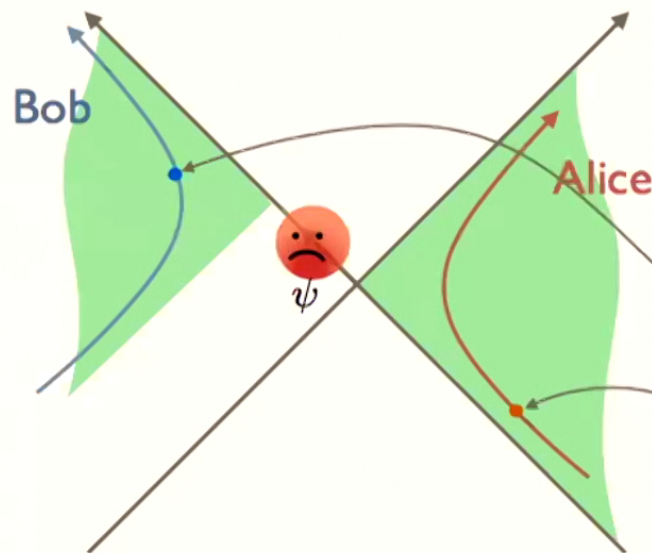
Quick answer: I have no idea

I





COMBINING THEM



$$[M_B, M_A] = 0$$

Now allow for
non-local operators

Interesting ops:

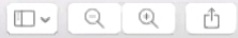
$$M_B = \rho_B^{is} \mathcal{O}_B \rho_B^{-is}$$

$$M_A = \rho_A^{is} \mathcal{O}_{\bar{A}} \rho_A^{-is}$$

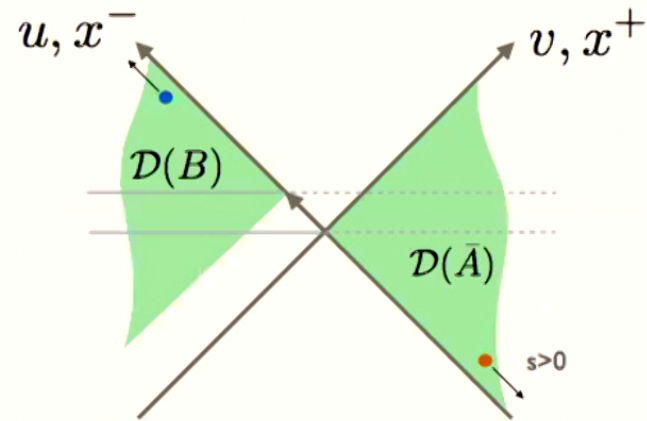
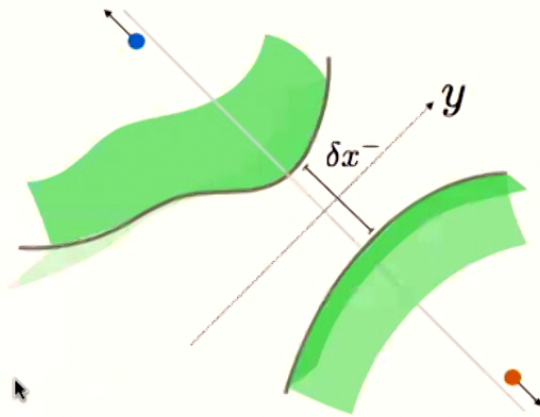
Modular flow!

Note ρ_B reduced density
matrix for: $|\psi\rangle\langle\psi|$





SETUP



Main object
of study:

$$f(s) \equiv \frac{\langle \psi | \mathcal{O}_B e^{-iK_B s} e^{iK_A s} \mathcal{O}_{\bar{A}} | \psi \rangle}{\langle \Omega | \mathcal{O}_B e^{-iK_B^0 s} e^{iK_A^0 s} \mathcal{O}_{\bar{A}} | \Omega \rangle}$$



COMMENTS $f(s) \equiv \frac{\langle \psi | \mathcal{O}_B e^{-iK_B s} e^{iK_A s} \mathcal{O}_{\bar{A}} | \psi \rangle}{\langle \Omega | \mathcal{O}_B e^{-iK_B^0 s} e^{iK_A^0 s} \mathcal{O}_{\bar{A}} | \Omega \rangle}$

- The full modular Hamiltonian annihilates: $K_A |\psi\rangle = 0$

$$e^{iK_A s} \mathcal{O}_{\bar{A}} e^{-iK_A s} |\psi\rangle = \rho_{\bar{A}}^{is} \mathcal{O}_{\bar{A}} \rho_{\bar{A}}^{-is} |\psi\rangle \quad (\text{recall } K_A = -\ln \rho_A + \ln \rho_{\bar{A}})$$

- Normalized with respect to vacuum modular flow

$$e^{-iK_B^0 s} e^{iK_A^0 s} = e^{i(1-e^{-s})P_-^0}$$

- We will now study properties of $f(s)$

- Very similar to ANEC $f(u)$ with $u \sim e^s$

see also: Bound on Chaos

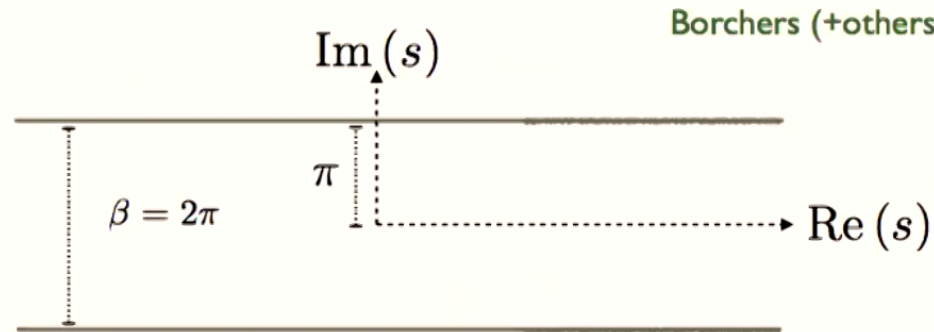
$$f(t)$$





PROPERTIES:
$$f(s) \equiv \frac{\langle \psi | \mathcal{O}_B e^{-iK_B s} e^{iK_A s} \mathcal{O}_{\bar{A}} | \psi \rangle}{\langle \Omega | \mathcal{O}_B e^{-iK_B^0 s} e^{iK_A^0 s} \mathcal{O}_{\bar{A}} | \Omega \rangle}$$

- **Causality:** modular flow \sim thermal time for non-equilibrium states \sim analog KMS condition \sim analyticity in the thermal strip:



- Requires “nice states” (cyclic and separating) \sim large entanglement \sim all reasonable states in QFT (without firewalls)



PROPERTIES:
$$f(s) \equiv \frac{\langle \psi | \mathcal{O}_B e^{-iK_B s} e^{iK_A s} \mathcal{O}_{\bar{A}} | \psi \rangle}{\langle \Omega | \mathcal{O}_B e^{-iK_B^0 s} e^{iK_A^0 s} \mathcal{O}_{\bar{A}} | \Omega \rangle}$$

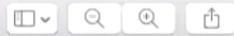
- **Computability:** working in same ANEC light-cone limit for the operator insertions: $(\pm u, \pm v) \quad v \rightarrow 0, u = \text{fixed}$

$$f(s) = 1 - C_T^{-1} e^s u (-uv)^{\frac{d-2}{2}} \mathcal{Q}_u + \dots \quad s \gg 1 \text{ (fixed)}$$

$$\mathcal{Q}_u = \int_{\partial A}^{\partial B} du' T_{uu}(u', v' = 0, y) + \left(\frac{\delta S_{EE}(\rho_A)}{\delta X^u(y)} - \frac{\delta S_{EE}(\rho_B)}{\delta X^u(y)} \right)$$

- Computed using a defect OPE (later)
 - Modular flow \sim vacuum mod flow for ops light like separated from entangling surface





PROPERTIES: $f(s) \equiv \frac{\langle \psi | \mathcal{O}_B e^{-iK_B s} e^{iK_A s} \mathcal{O}_{\bar{A}} | \psi \rangle}{\langle \Omega | \mathcal{O}_B e^{-iK_B^0 s} e^{iK_A^0 s} \mathcal{O}_{\bar{A}} | \Omega \rangle}$

- **Positivity:** use Cauchy-Schwarz to bound

$$|\langle b | a \rangle|^2 \leq \langle a | a \rangle \langle b | b \rangle$$

I

$$\text{Re} f(s) < 1$$

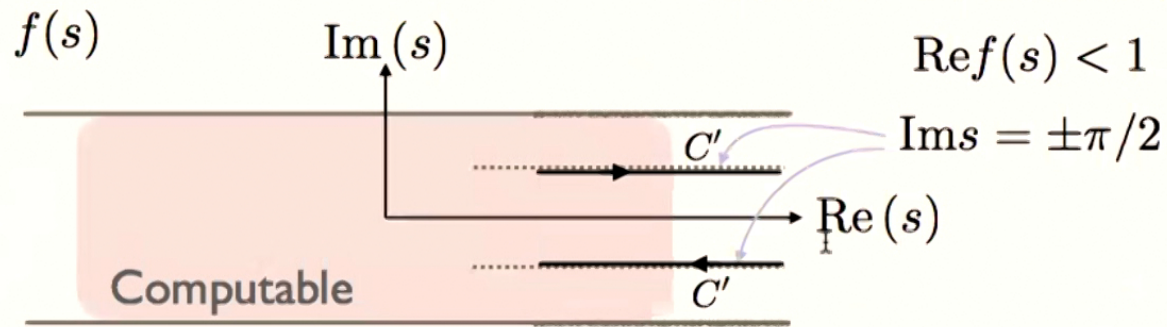
$$\text{Im} s = \pm \pi/2$$



PROPERTIES: $f(s) \equiv \frac{\langle \psi | \mathcal{O}_B e^{-iK_B s} e^{iK_A s} \mathcal{O}_{\bar{A}} | \psi \rangle}{\langle \Omega | \mathcal{O}_B e^{-iK_B^0 s} e^{iK_A^0 s} \mathcal{O}_{\bar{A}} | \Omega \rangle}$

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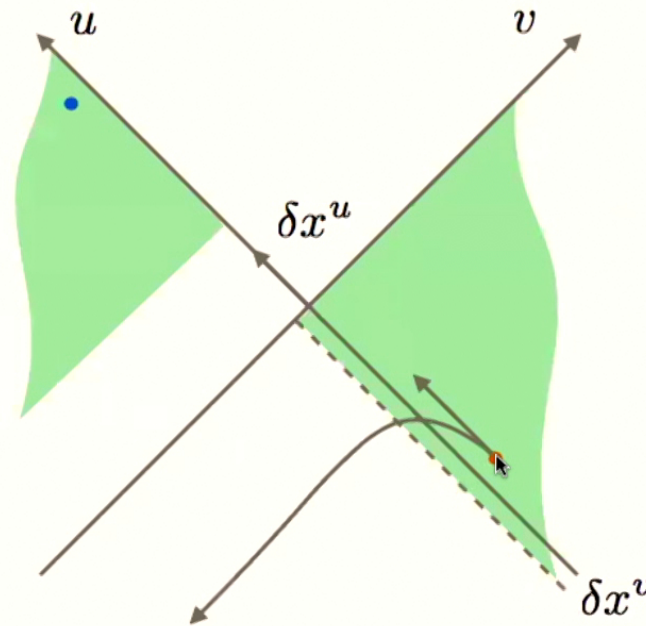


$$Q_u = \# \text{Re} \left(i \int_{C'} ds e^{-s} (1 - f(s)) \right) \geq 0$$





SOME INTUITION - CAUSALITY



Vacuum flow:

$$\langle \Omega | \mathcal{O}_B e^{-iK_B^0 s} e^{iK_A^0 s} \mathcal{O}_{\bar{A}} | \Omega \rangle$$

null translation on $\mathcal{O}_{\bar{A}}$

Slight error gets amplified
bigly. Correction:

$$\times \left(1 + e^s \frac{\delta x^v}{v} + \dots \right)$$

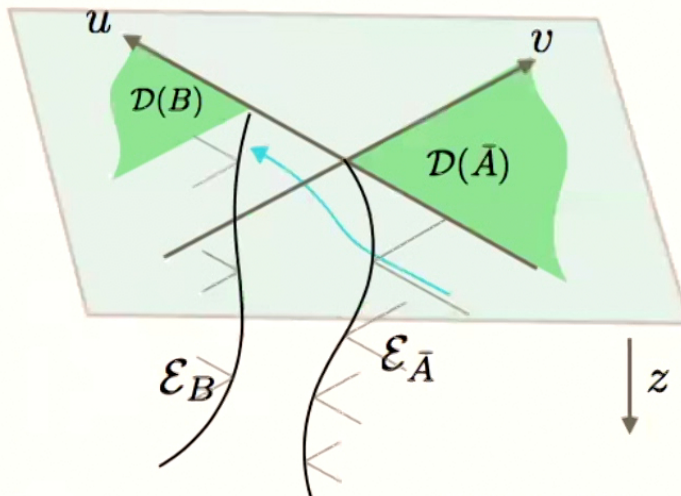
$Q_u \propto$ some delay in v ?





MORE INTUITION - ADS/CFT

Koeller, Leichenauer, '16



RT/HRT
surfaces

Jafferis et al.
Lewkowycz TF
Cotler et al.

- Entanglement wedge nesting:

$$[\mathcal{D}(B), \mathcal{D}(\bar{A})] = 0$$

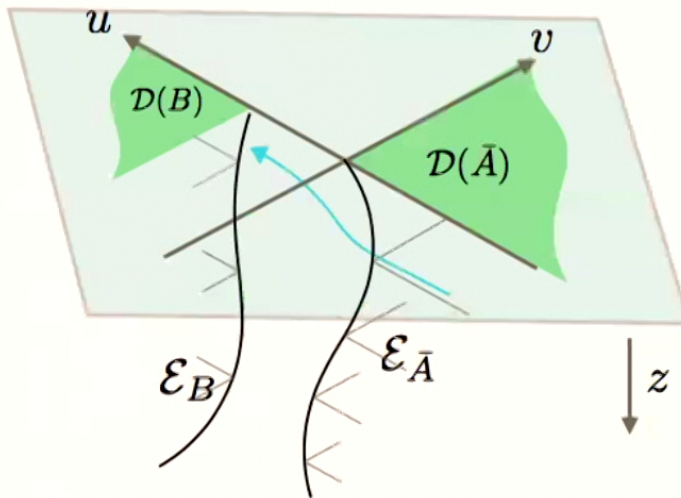
$$\implies [\mathcal{E}_B, \mathcal{E}_{\bar{A}}] = 0$$
- Try to send a null geodesic between them. Figure of merit:

$$\Delta v \propto -Q_{\partial u}$$
- Entanglement wedge reconstruction \sim related to modular flow





MORE INTUITION - ADS/CFT

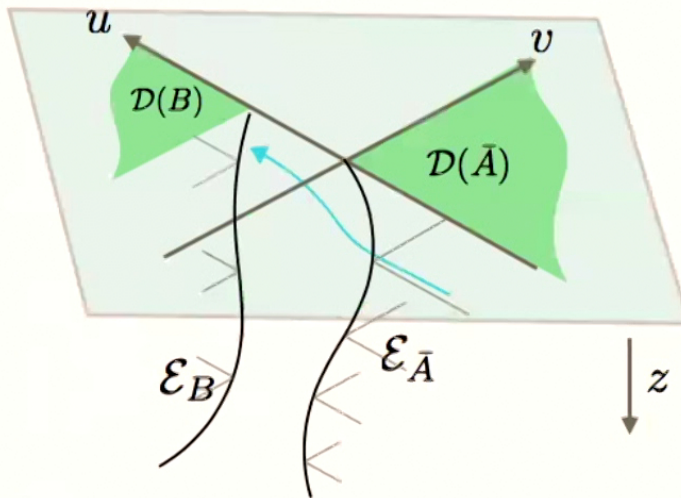


- Delay/advance from two sources:

RT/HRT
surfaces

I

MORE INTUITION - ADS/CFT



RT/HRT
surfaces

- Delay/advance from two sources:

$$\Delta v = z^2 \int_{\partial a}^{\partial b} g_{uu} du'$$

$$\Delta v = X_{A,RT}^v - X_{B,RT}^v$$

- Fefferman-Graham expansions: $z \rightarrow 0$

$$g_{uu} \propto z^{d-2} T_{uu} \quad X_{RT}^v \propto z^d \frac{\delta S_{EE}}{\delta X^u}$$





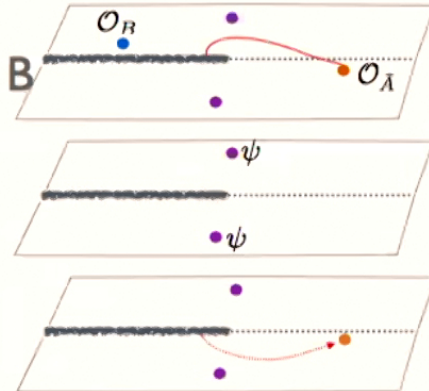
THE COMPUTATION

“You can’t compute that ...” I

REPLICA TRICK

Simpler problem:

$$\langle \psi | \mathcal{O}_B K_B \mathcal{O}_{\bar{A}} | \psi \rangle$$



- Modular Hamiltonian:

$$\lim_{n \rightarrow 1} \partial_n \rho_B^{n-1} = \ln \rho_B$$

- Path integral on n -replicated space \mathcal{M}_n $n \in \mathbb{Z}$
- n insertions of: ψ, ψ^\dagger
- matrix elements with $\mathcal{O}_B, \mathcal{O}_{\bar{A}}$
- $2n+2$ correlation function on \mathcal{M}_n

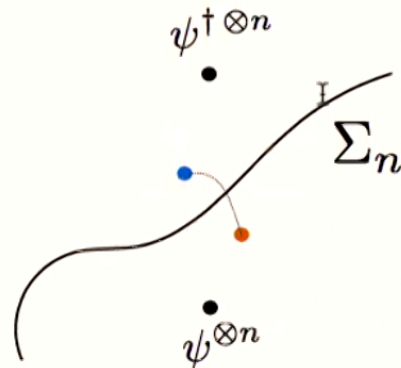
Compute $\ln \rho_B - \ln \rho_{\bar{B}}$ via simple operator monodromy





ORBIFOLD THEORY

- Alternatively view as orbifold computation: $\text{CFT}^n / \mathbb{Z}_n$



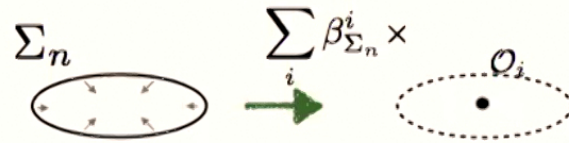
- only one stress tensor
- operators single valued
- but $\mathcal{O}_B, \mathcal{O}_{\bar{A}}$ not orbifold ops.

- Twist defect on entangling surface \sim co-dimension 2 operator

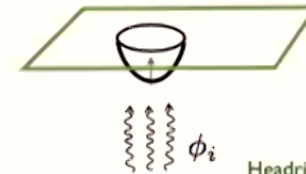




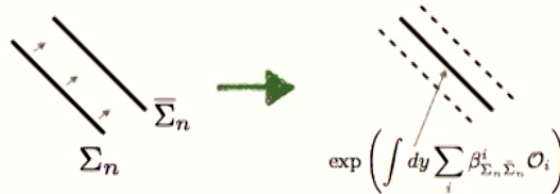
OPE METHODS FOR ENTANGLEMENT



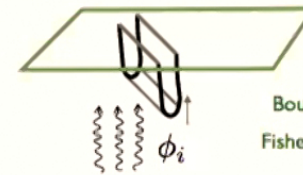
Small 'interval' limit of Mutual Information, excited state EE, etc.



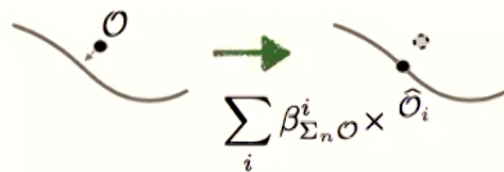
Headrick; Cardy + many more



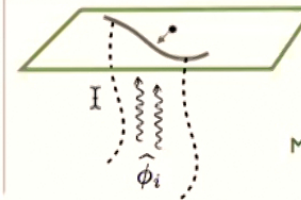
EE on light sheets (Bousso bound), vacuum modular Hamiltonians, etc.



Bousso, Casini, Fisher, Maldacena



Shape deformations of EE and Renyi entropies. Less utilized ...

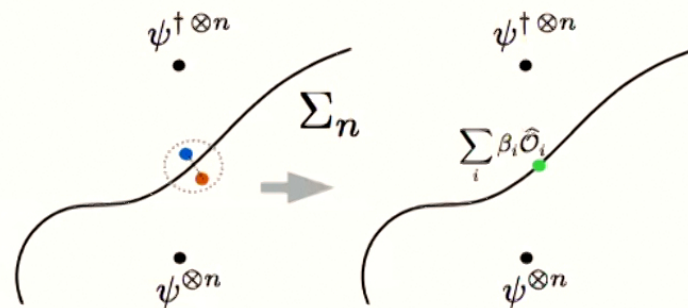


Myers, Smolkin et al.





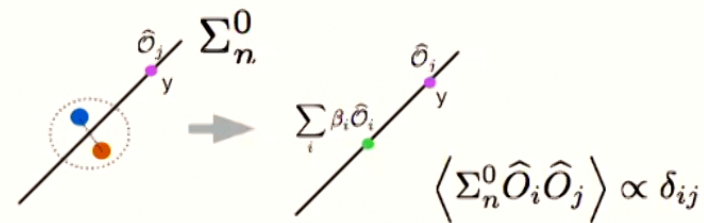
DEFECT OPE



I. What is the defect spectrum? \hat{O}_i

I
II. How to compute β_i ?

II. Computing the dOPE coefficients:



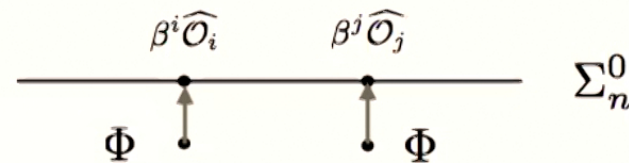
Same replacement but
in vacuum on flat defect





I. DEFECT OPERATOR SPECTRUM

- Consider a two point function in presence of Σ_n^0



$$\mathbb{H}_{d-1} \times \mathbb{S}_{\beta=2\pi n}^1$$

- Analytically continue $\langle \Sigma_n^0 \Phi \Phi \rangle$ in n (thermal correlator) TF '14
- Extract spectrum by taking limit to twist defect $\widehat{\Delta}(n)$
- Two point correlator fixed in $\lim n \rightarrow 1$ by local data of theory

$$n = 1 \quad \langle \Phi \Phi \rangle_{CFT} \quad \mathcal{O}(n-1) \quad \langle \Phi \Phi T_{\mu\nu} \rangle_{CFT}$$





THE SPECTRUM:



- Mostly interested in: $\Phi = T_{uu}$ (light-cone limit = lowest twist)
- $n=1$ then no defect - same spectrum as before (ANEC)

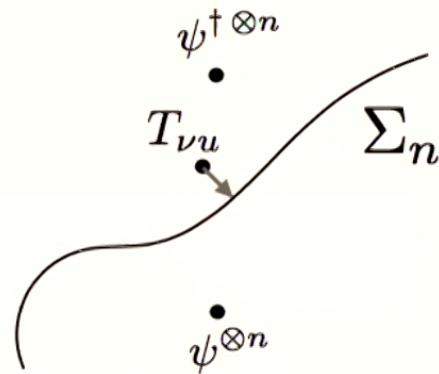
(decomposed using defect symmetries)

Operator	$n=1$	$O(n-1)$
$\partial_u^{\ell-2} T_{uu}$	$\hat{\tau} = d - 2$ $\ell \geq 2$	$\delta\hat{\tau} =$ $(n-1)\gamma_\ell$
\mathcal{D}_u	not seen	$\hat{\tau} = d - 2$ $\ell = 1$

A new operator emerges with same twist displacement op.



THE DISPLACEMENT OPERATOR



$$\nabla^\nu T_{\nu u}(x) \Sigma_n = \delta_{\Sigma_n}(u, v) \mathcal{D}_u(y) \Sigma_n$$

The displacement operator
moves around the defect

$$\lim_{n \rightarrow 1} \partial_n \langle \mathcal{D}_u \Sigma_n \rangle = \frac{\delta S_{EE}}{\delta x^-(y)}$$

At this point we have all the ingredients that go into the QNEC.
Modular flow is a similar computation (not much more difficult) ...

$$f(s) = 1 - e^s u(-uv)^{\frac{d-2}{2}} Q_u + \dots$$



THE DISPLACEMENT OPERATOR

- New operators appear away from $n=1$ for spinning operators

$$l_\Phi \geq 2 \quad \hat{\ell} = 1, \dots, l_\Phi - 1$$

- Subject to a higher spin QNEC
- Correspond to fields living on RT surface?
- One point functions of these ops \sim non-linear in the state
- Not really operators \sim limit $n \rightarrow 1$ of operators in n -replicated theory

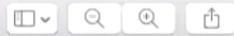




SUMMARY

	ANEC	QNEC
BOUND	$\mathcal{A}_u \geq 0$	$\mathcal{Q}_u \geq 0$
ADS/CFT	Gao-Wald Wall, Kelly '14	Entanglement Wedge Nesting Koeller, Leichenauer, '16
QFT	$\langle \psi \mathcal{O} \mathcal{O} \psi \rangle$ Hartman et al. '16	$\langle \psi \mathcal{O} e^{-isK_B} e^{isK_A} \mathcal{O} \psi \rangle$ Balakrishnan et al. '17
I BULK?	Regge limit ~ high energy scattering in bulk (stringy) ... Camanho et al.	???? $s \gg 1$ $v = \text{fixed, small}$



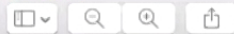


NO TIME FOR ...

- Local extrinsic curvature corrections
- From CFT to QFT
- Relation to half sided modular inclusions
- Higher spin version of the QNEC

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CONCLUSIONS SLIDE

- Causality leads to the QNEC
 - Bulk intuition important. Modular flow \sim entanglement wedge reconstruction
 - New defect OPE methods for Entanglement
 - Probe of RT/HRT near boundary of AdS
 - Modular flow promising approach to explore emergent bulk locality/causality in AdS/CFT
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