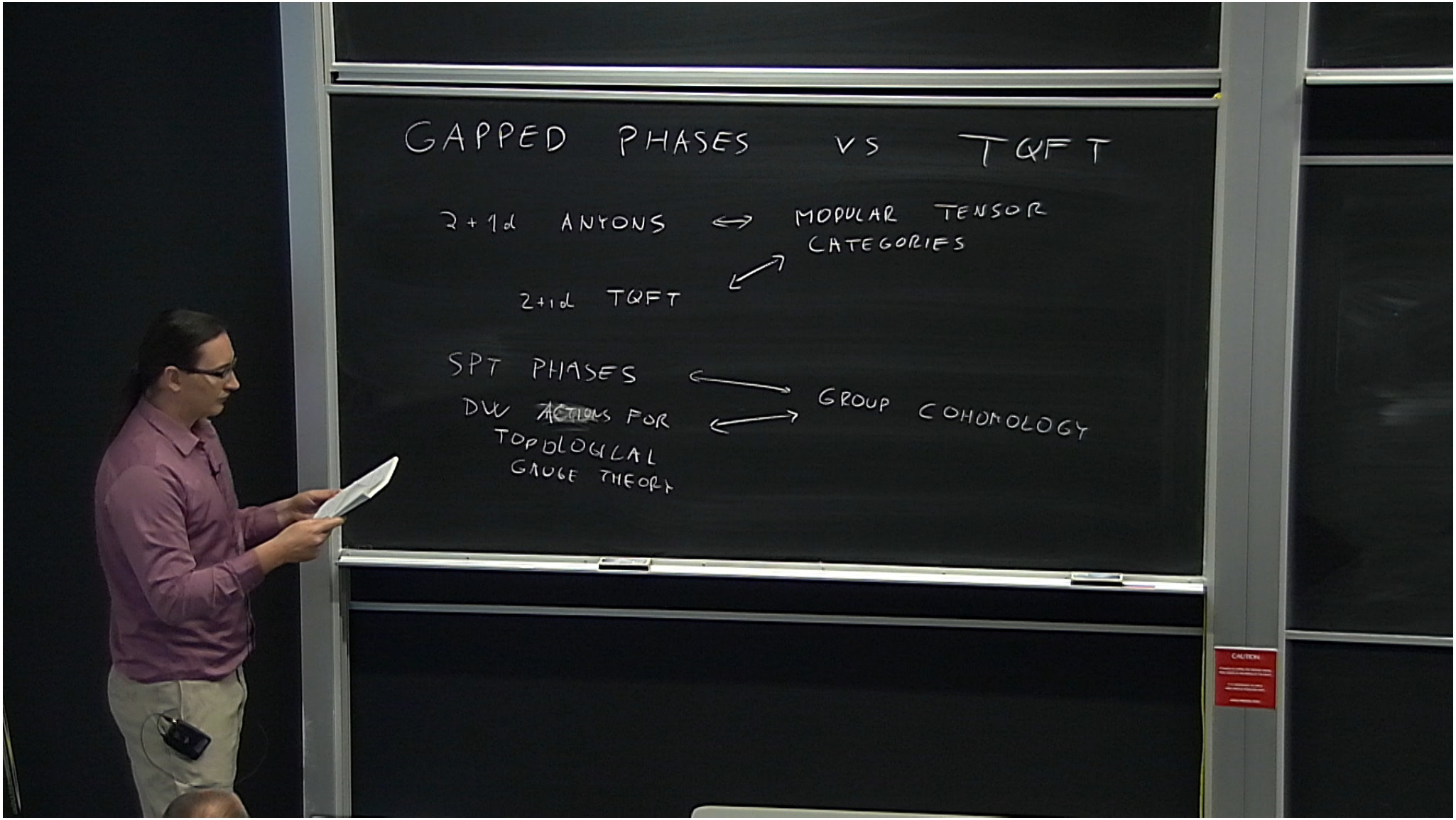
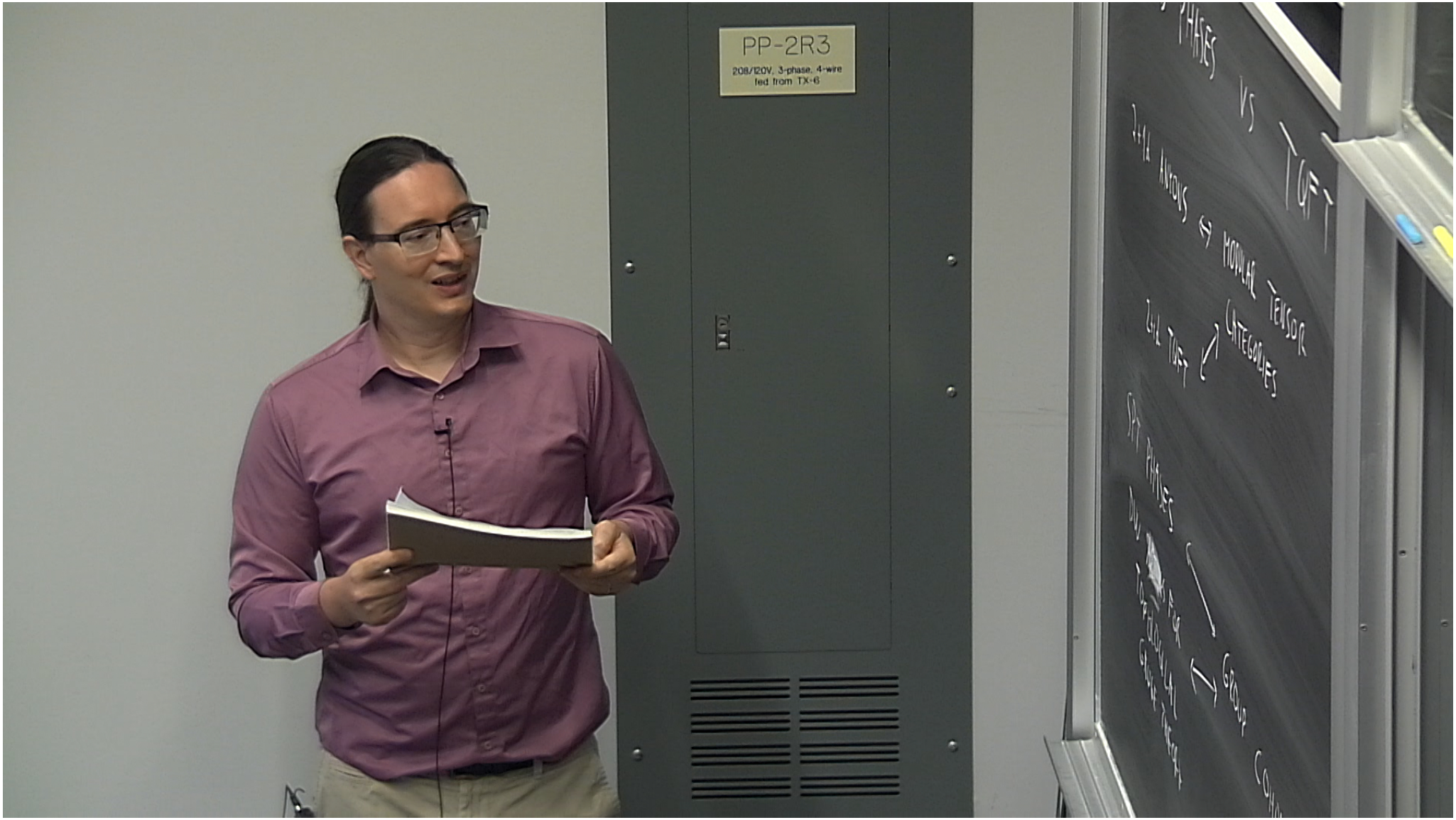
Title: Gapped phases of matter vs. Topological field theories

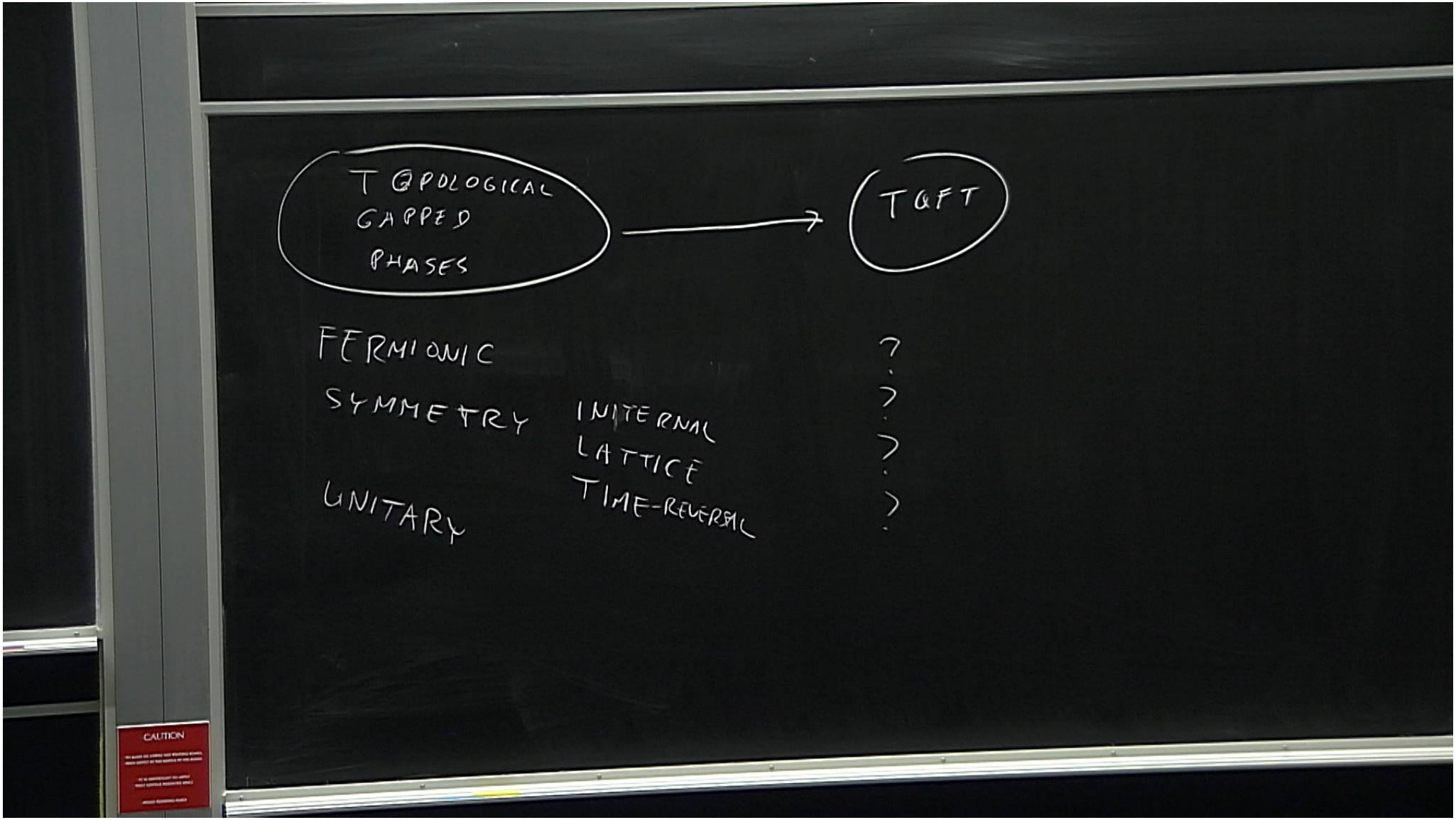
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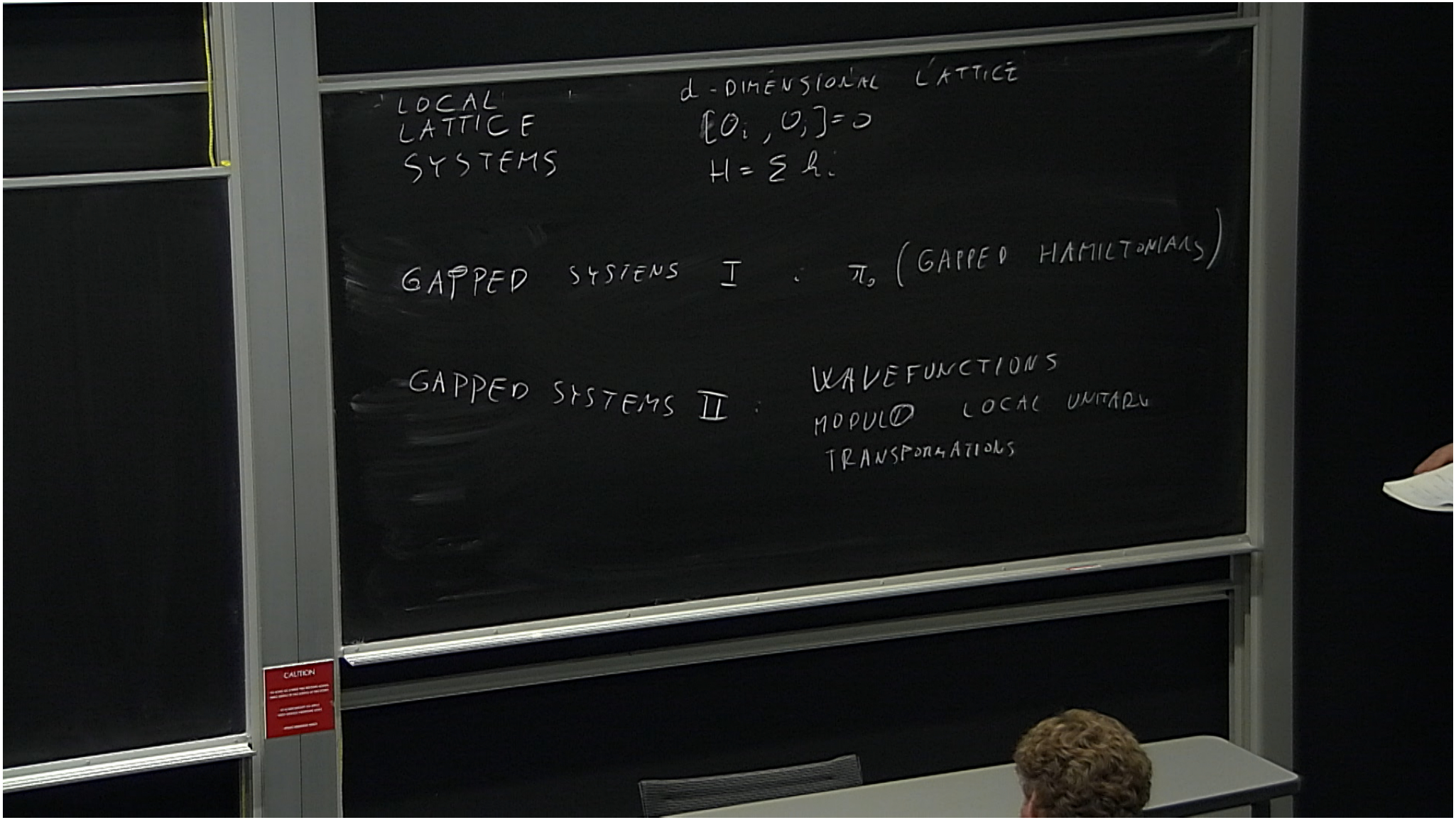
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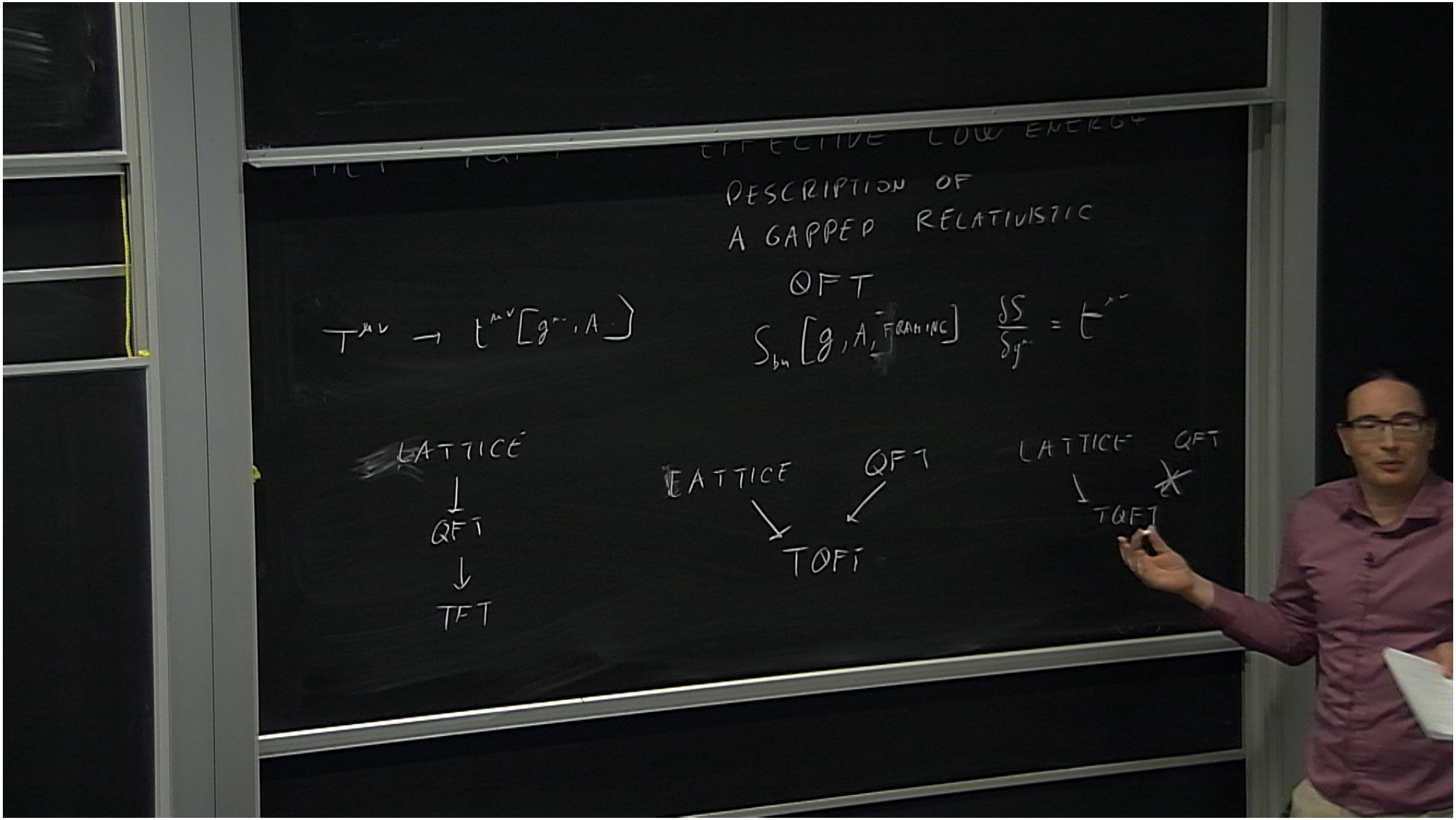
Abstract: I will discuss the relation between topological field theories and gapped phases of matter. I will propose a general formalism to define a class of TFTs which can be realized by commuting projector Hamiltonians. This allows one to apply rigorous mathematical theorems about TFTs to gapped phases of matter. I will also discuss the role of generalized cohomology theories and spectra in the classification of SPT phases.

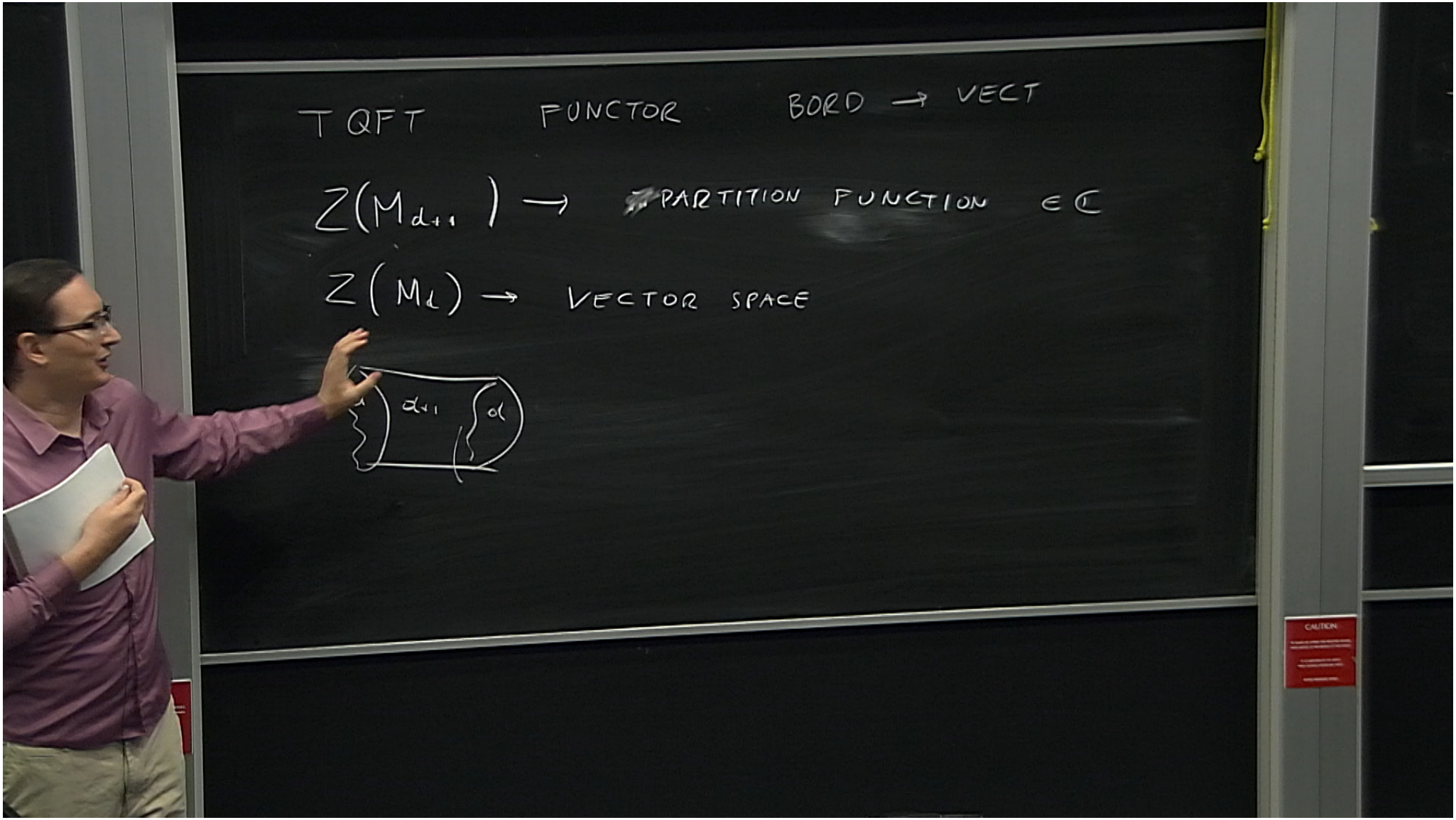

GAPPED PHASES VS TQFT
2+1d ANYONS ⟺ MODULAR TENSOR CATEGORIES
2+1d TQFT
SPT PHASES ⟺ GROUP COHOMOLOGY
DW ACTIONS FOR TOPOLOGICAL GAUGE THEORY

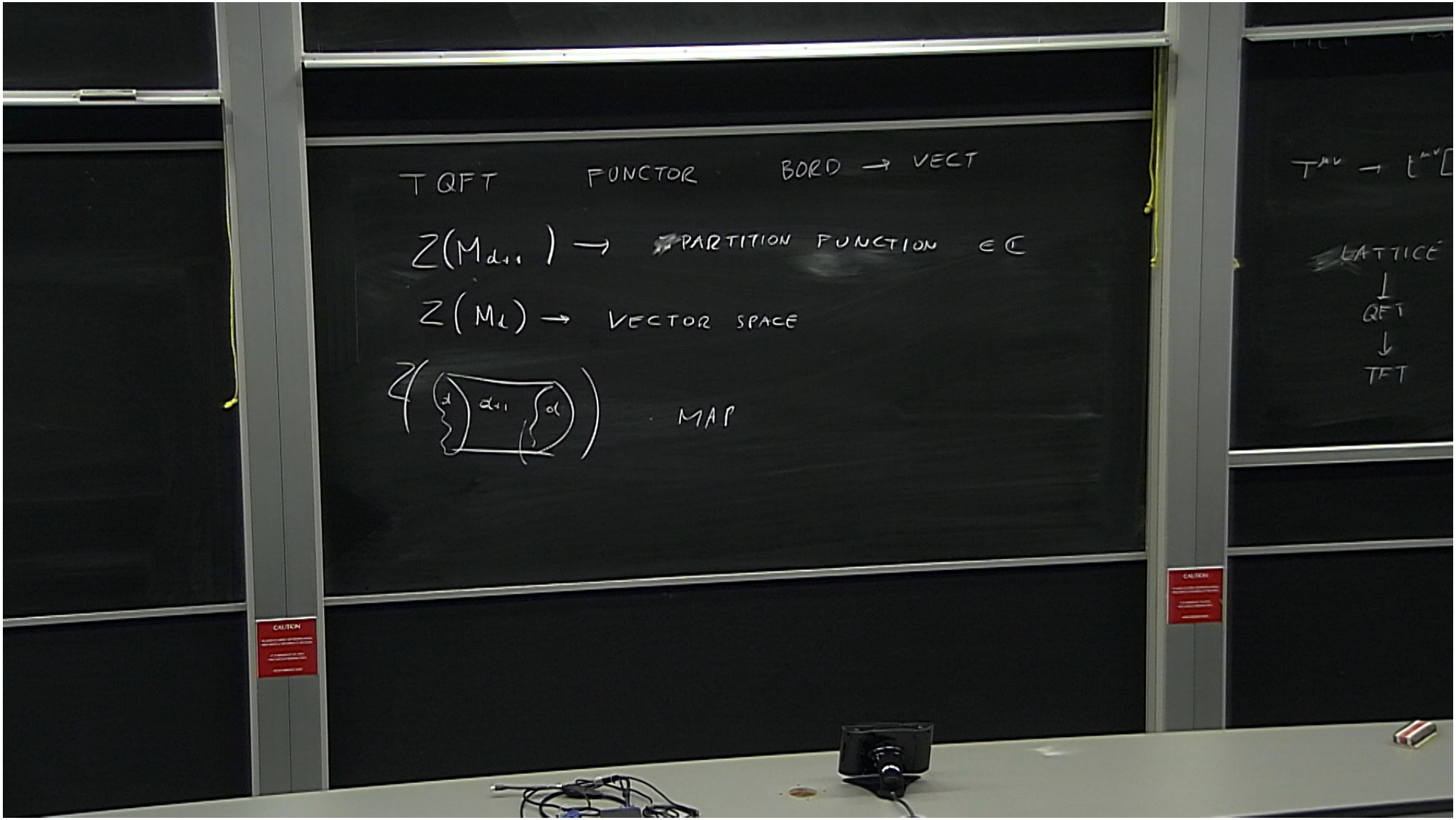

PP-2R3
208/120V, 3-phase, 4-wire
fed from TX-6
PHASES
VS
TQFT
MODULAR TENSOR
CATEGORIES
SPT PHASES
GROUP

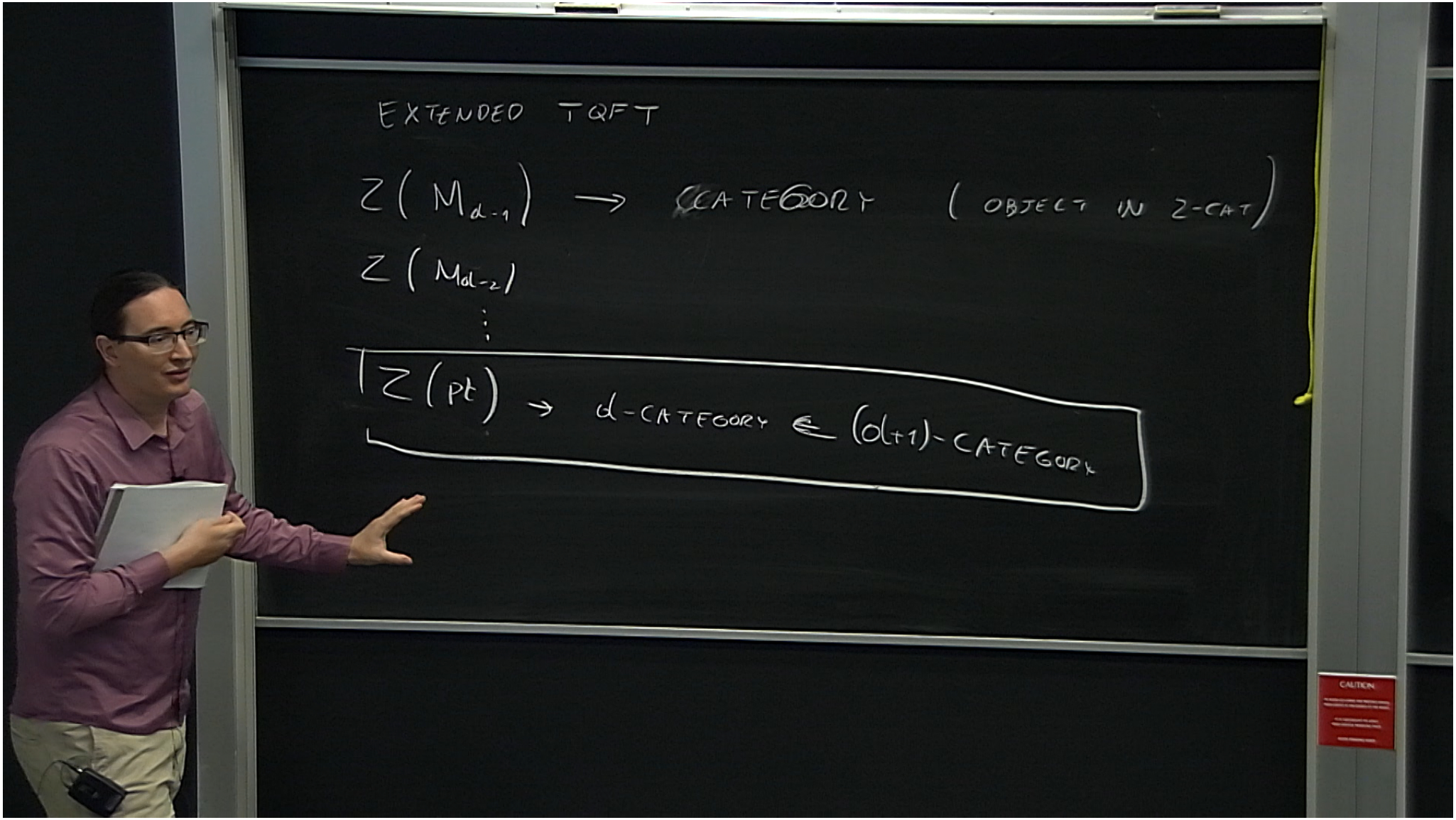

TOPOLOGICAL GAPPED PHASES
TQFT
FERMIONIC
SYMMETRY
INTERNAL
LATTICE
TIME-REVERSAL
UNITARY
? ? ? ?
CAUTION

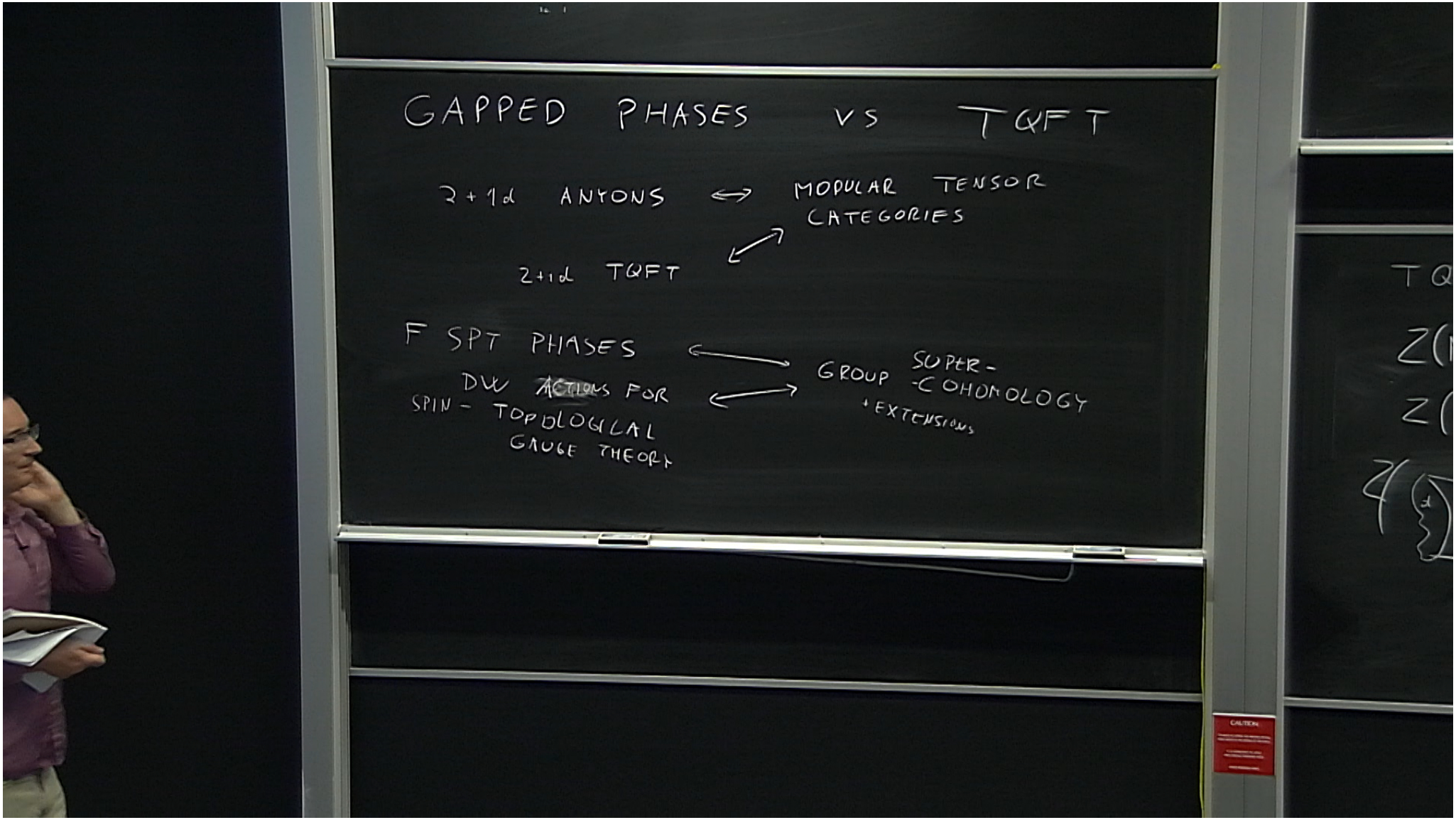

LOCAL LATTICE SYSTEMS
d-DIMENSIONAL LATTICE
$[O_i, O_j] = 0$
$H = \sum h_i$
GAPPED SYSTEMS I : π_0 (GAPPED HAMILTONIANS)
GAPPED SYSTEMS II : WAVEFUNCTIONS MODULO LOCAL UNITARY TRANSFORMATIONS
CAUTION


EFFECTIVE LOW ENERGY
DESCRIPTION OF
A GAPPED RELATIVISTIC
QFT
$T^{\mu\nu} \to t^{\mu\nu}[g^{\mu\nu}, A]$
$S_{bulk}[g, A, \text{FRAMING}]$
$\frac{\delta S}{\delta g^{\mu\nu}} = t^{\mu\nu}$
LATTICE
QFT
TFT
LATTICE
QFT
TQFT
LATTICE
QFT
TQFT

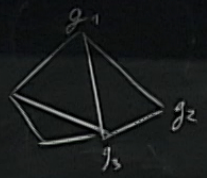

TQFT FUNCTOR BORD → VECT
$Z(M_{d+1}) \to$ PARTITION FUNCTION $\in \mathbb{C}$
$Z(M_d) \to$ VECTOR SPACE
$d+1$
d


TQFT FUNCTOR BORD → VECT
$Z(M_{d+1}) \rightarrow$ PARTITION FUNCTION $\in \mathbb{C}$
$Z(M_d) \rightarrow$ VECTOR SPACE
MAP
LATTICE
QFT
TFT

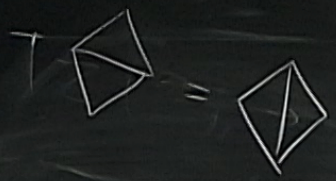

EXTENDED TQFT
Z(M_{d-1}) → CATEGORY (OBJECT IN 2-CAT)
Z(M_{d-2})
Z(pt) → d-CATEGORY ∈ (d+1)-CATEGORY


GAPPED PHASES VS TQFT
2+1d ANYONS ⟺ MODULAR TENSOR CATEGORIES
2+1d TQFT
F SPT PHASES ⟺ GROUP SUPER-COHOMOLOGY + EXTENSIONS
DW ACTIONS FOR SPIN-TOPOLOGICAL GAUGE THEORY

GAUGE THEORY



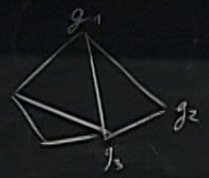
$$Z = \frac{1}{|G|^{\#(V)}} \sum_{g_i} \prod_S \alpha(g_{s(1)}, \ldots, g_{s(d+1)})$$



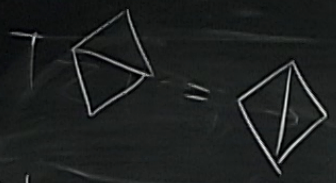
$$\delta\alpha = 1$$

$$\alpha \to \alpha \cdot \delta\lambda$$

GAUGE THEORY



$$Z = \frac{1}{|G|^{\#(\gamma)}} \sum_{g_\cdot} \prod_S \alpha(g_{s(1)}, \dots g_{s(d+1)})$$



$$\delta\alpha = 1$$

$$\alpha \to \alpha \cdot \delta\lambda$$

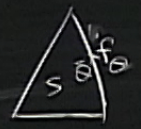
$$[\alpha] \in H^{d+1}(G, U(1))$$

2-cell $\quad L_g \otimes L_{g'} = L_{gg'}$

ASSOCIATOR α

TQFT FUNCTOR BORD → VECT

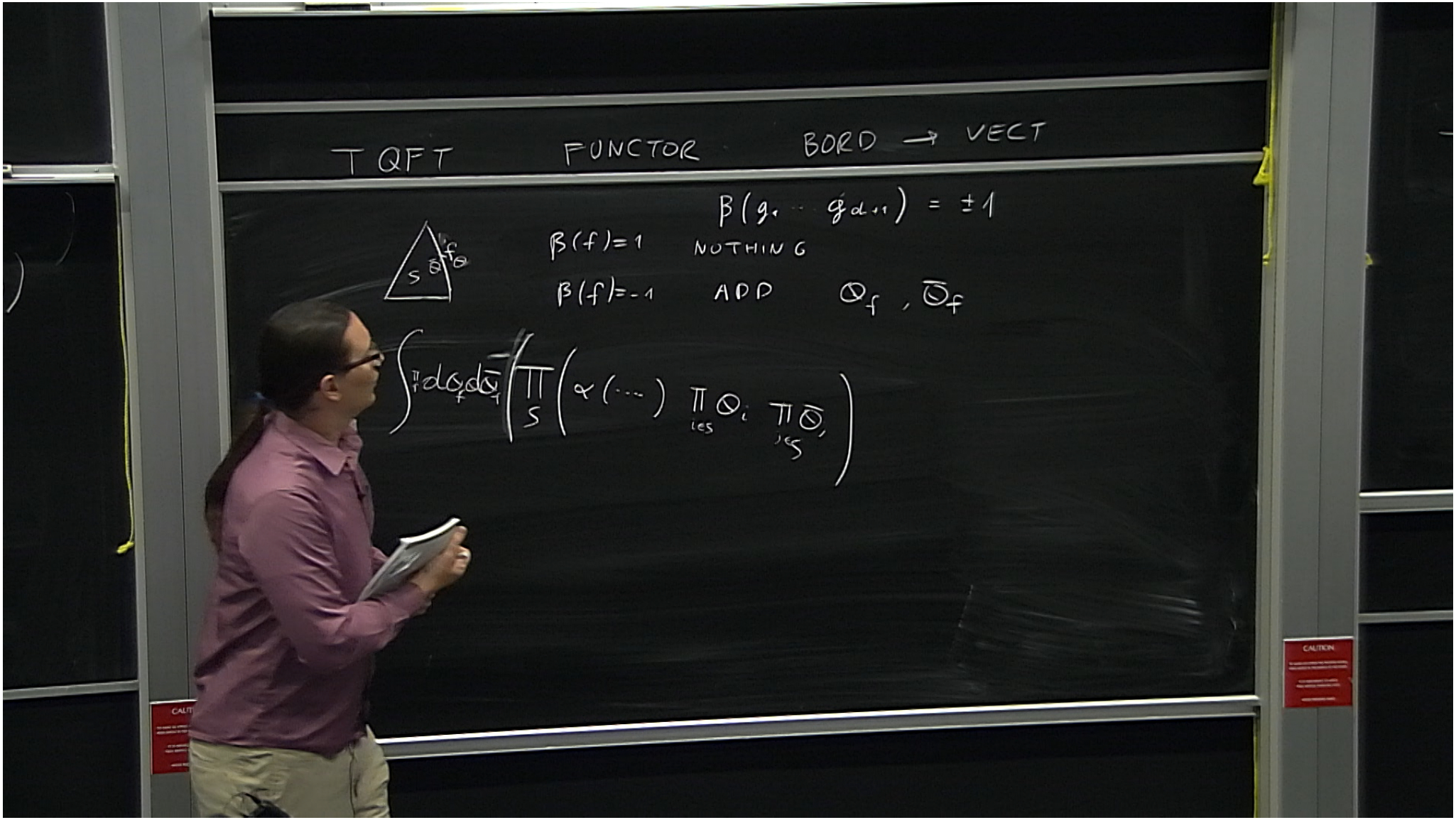
$\beta(g_1 \cdots g_{d+1}) = \pm 1$



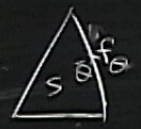
$\beta(f) = 1$ NOTHING

$\beta(f) = -1$ ADD θ_f, $\bar{\theta}_f$

$$\int \prod_i d\theta_f d\bar{\theta}_f \left(\prod_s \left(\alpha(\cdots) \prod_{i \in s} \theta_i \prod_{i \in s} \bar{\theta}_i \right) \right)$$


TQFT FUNCTOR BORD → VECT
$\beta(g_1 \cdots g_{d+1}) = \pm 1$
$\beta(f) = 1$ NOTHING
$\beta(f) = -1$ ADD $\mathcal{O}_f$, $\bar{\mathcal{O}}_f$
$\int \prod_i d\alpha_f d\bar{\alpha}_f \left(\prod_S \left(\alpha(\cdots) \prod_{i \in S} \mathcal{O}_i \prod_{i \in S} \bar{\mathcal{O}}_i \right) \right)$

TQFT FUNCTOR BORD → VECT



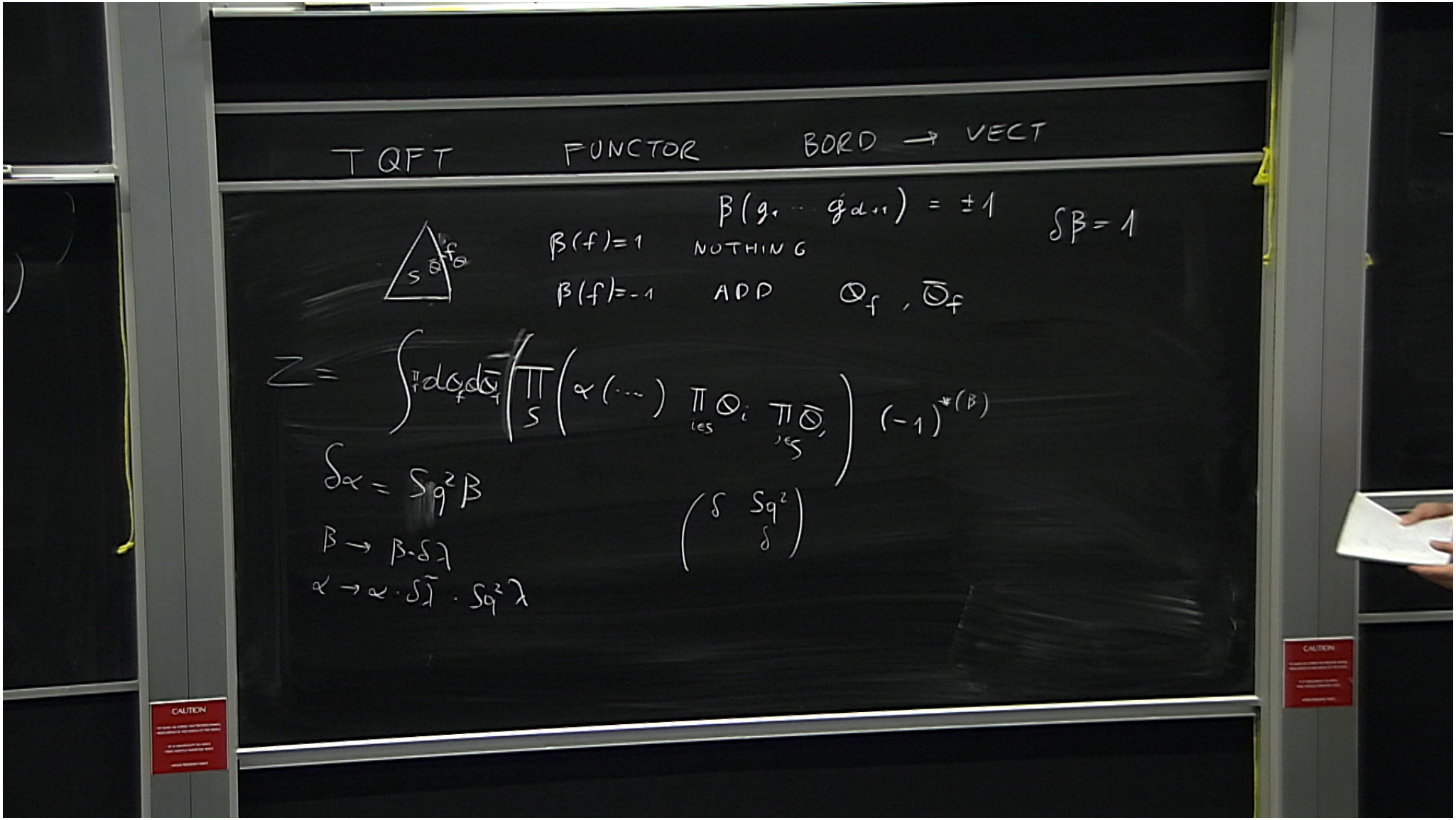
$\beta(g_1 \cdots g_{d+1}) = \pm 1$ $\quad \delta\beta = 1$

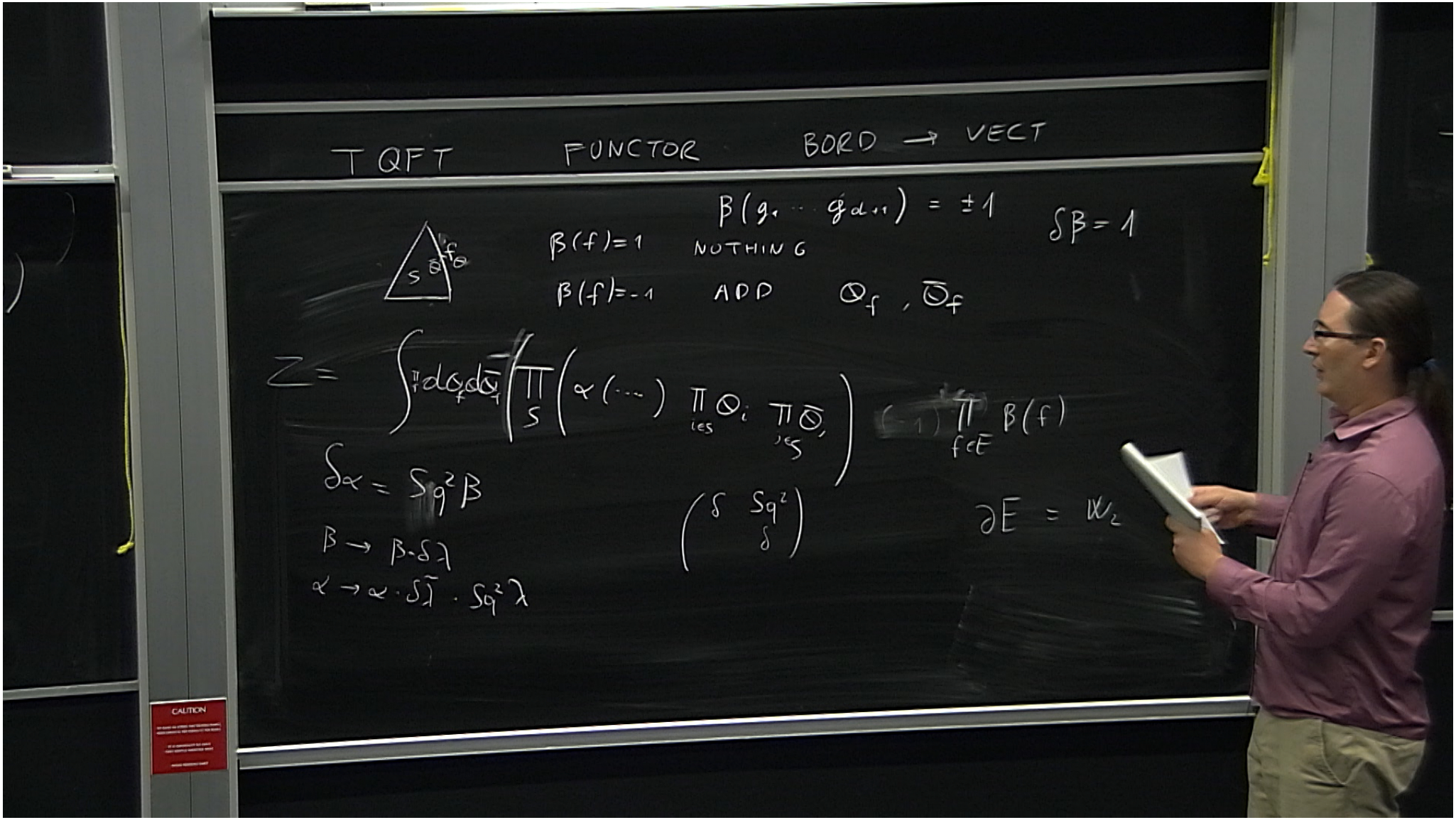
$\beta(f) = 1$ NOTHING

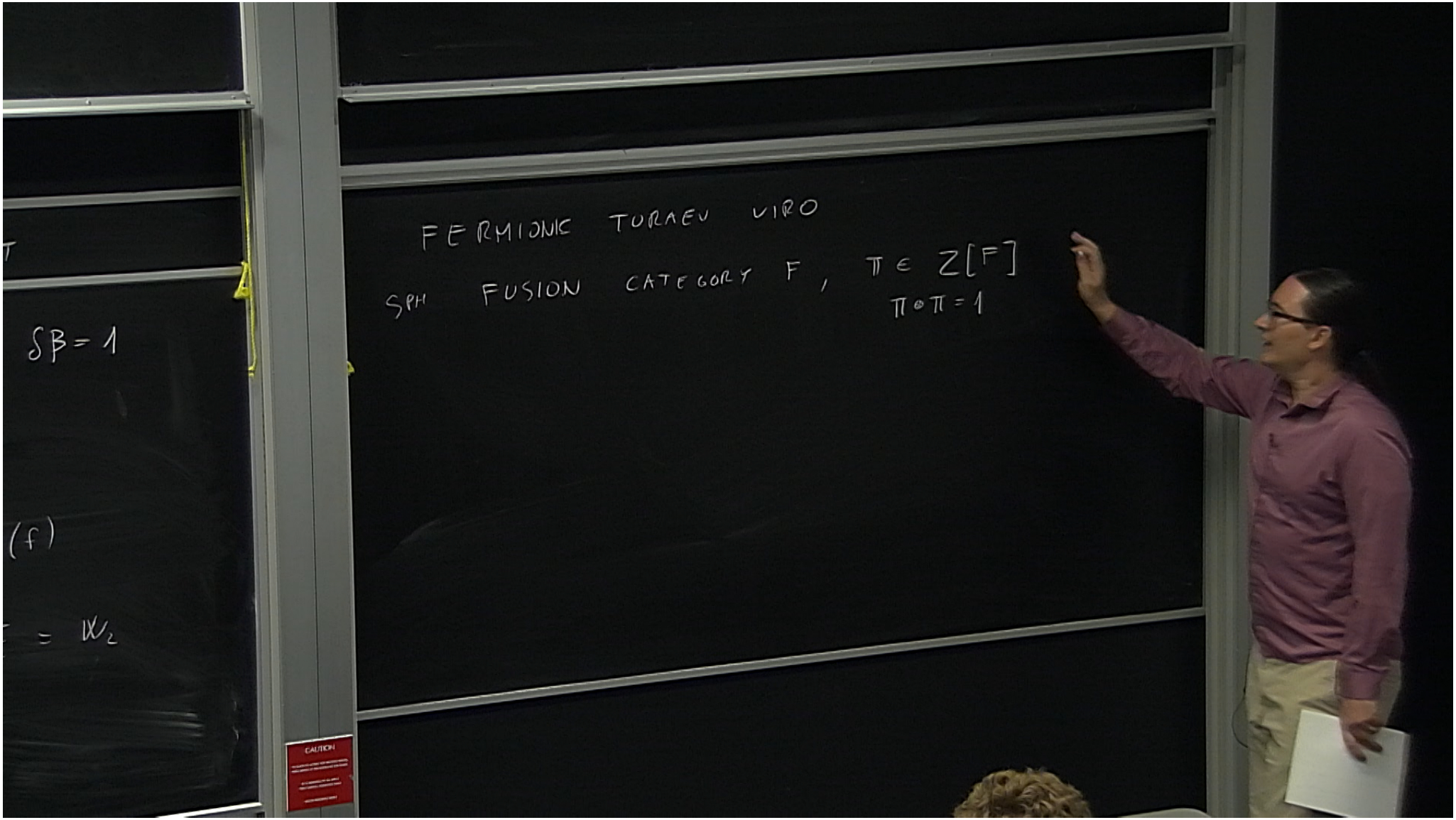
$\beta(f) = -1$ ADD $\mathcal{O}_f$, $\bar{\mathcal{O}}_f$

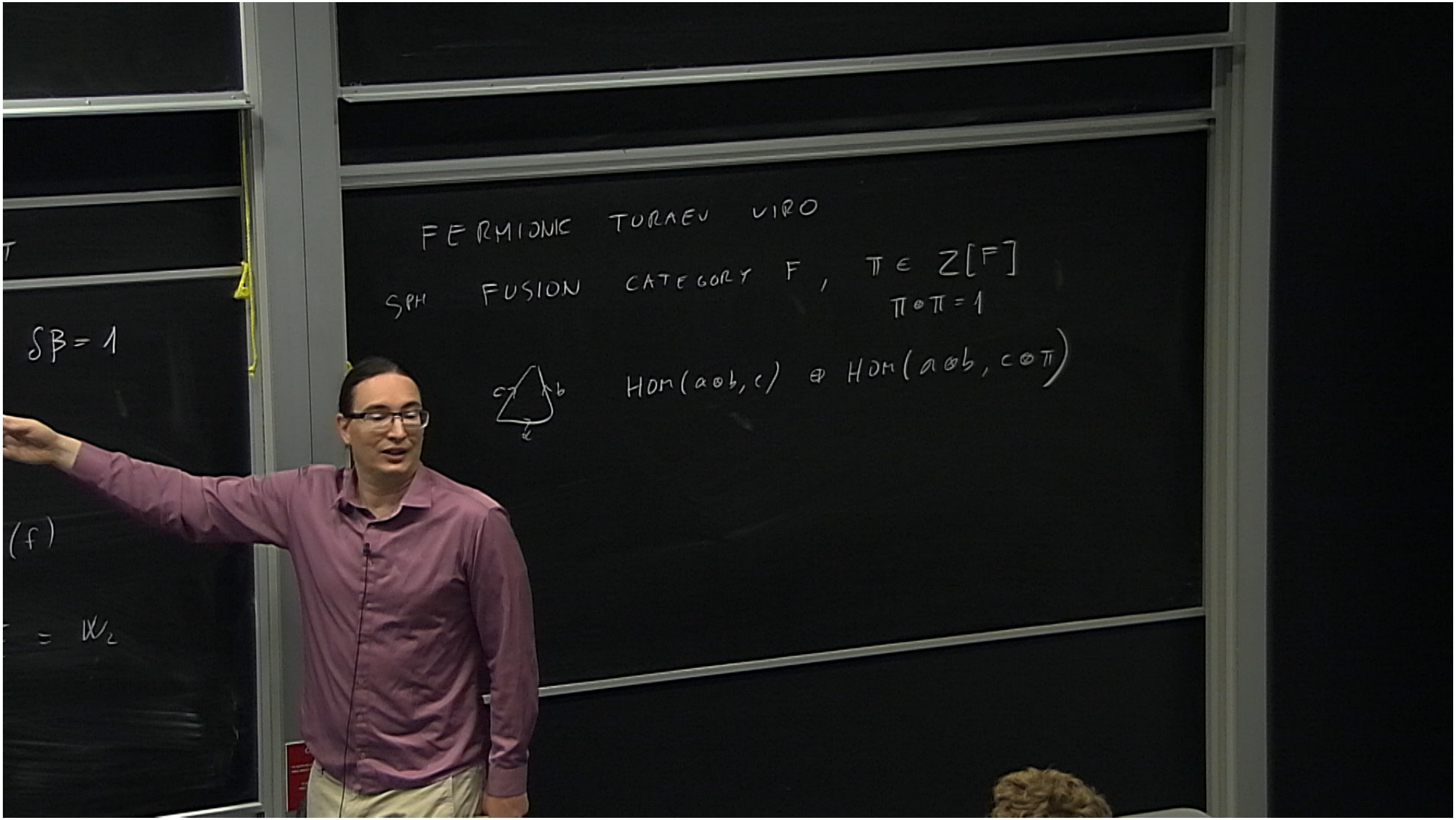
$$Z = \int \prod_f d\alpha_f d\bar{\alpha}_f \left(\prod_S \left(\alpha(\cdots) \prod_{i \in S} \mathcal{O}_i \prod_{i \in S} \bar{\mathcal{O}}_i \right) (-1)^{*(\beta)} \right)$$

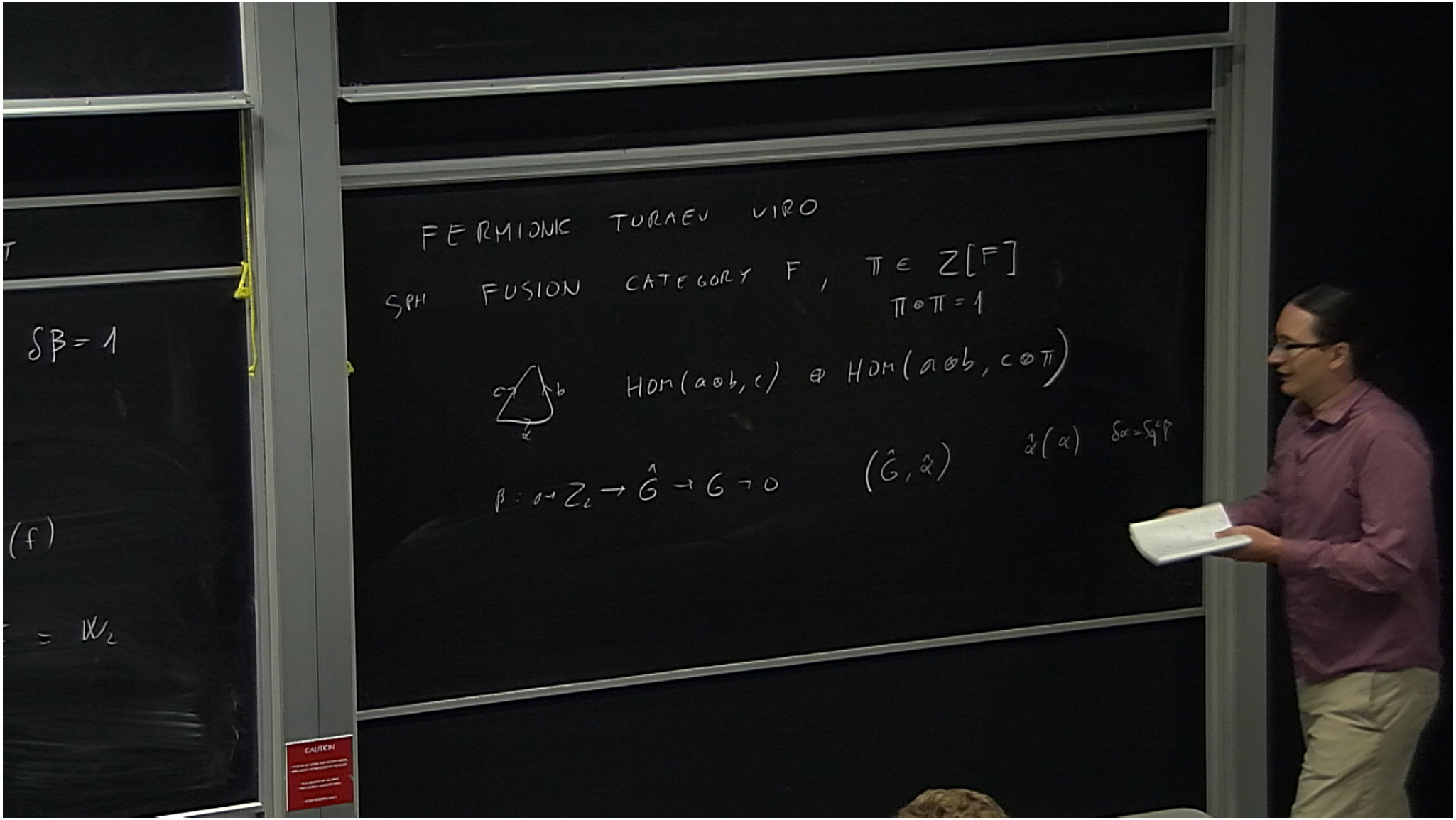
$\delta\alpha = S_g \beta$

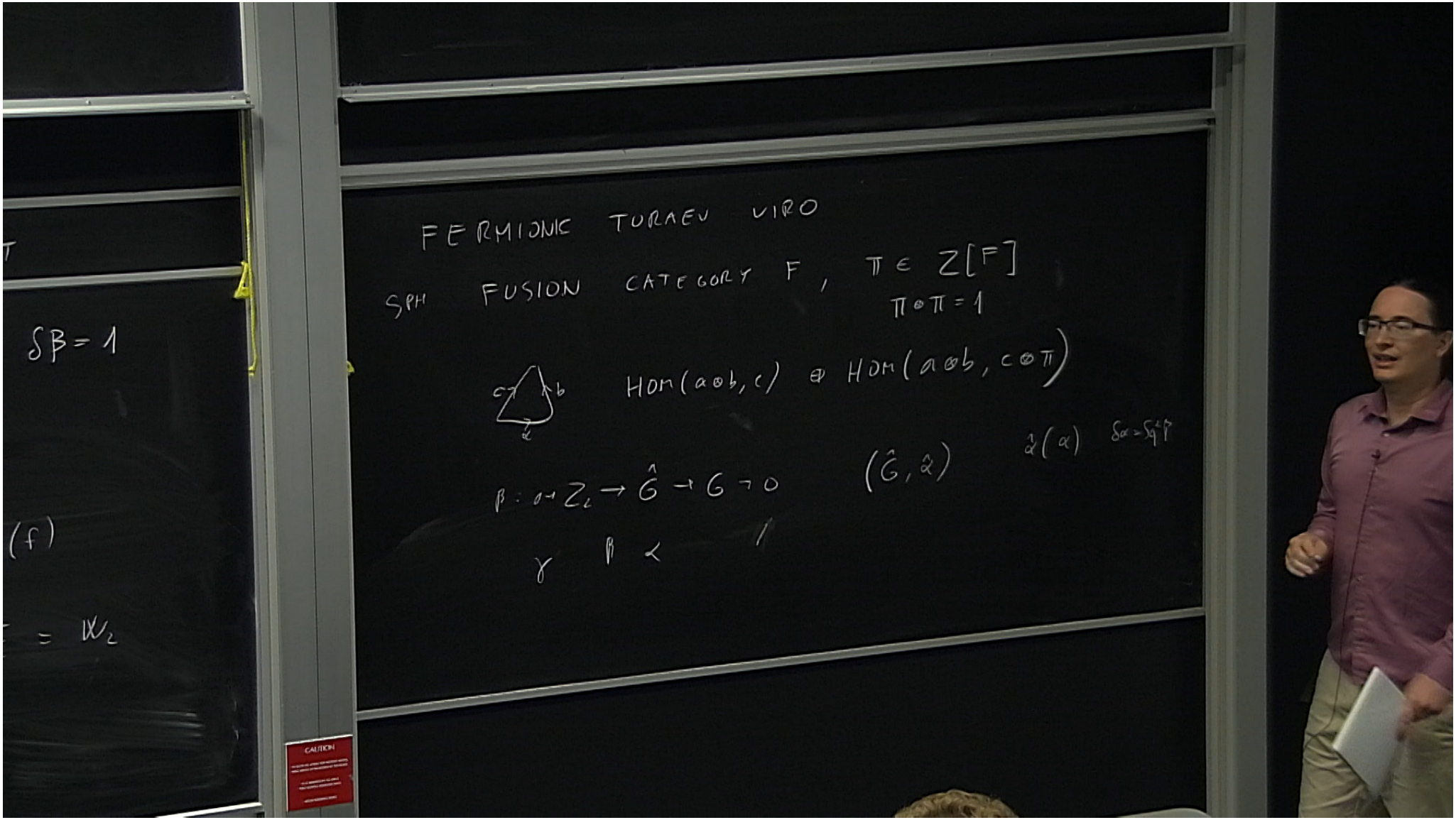

TQFT FUNCTOR BORD → VECT
$\beta(g_1 \cdots g_{d+1}) = \pm 1$
$\delta\beta = 1$
$\beta(f) = 1$ NOTHING
$\beta(f) = -1$ ADD $\mathcal{O}_f$, $\bar{\mathcal{O}}_f$
$Z = \int \prod_f d\alpha_f d\bar{\alpha}_f \left(\prod_s \left(\alpha(\cdots) \prod_{\text{les}} \mathcal{O}_i \prod_{\text{'es}} \bar{\mathcal{O}}_i \right) \right) (-1)^{*(\beta)}$
$\delta\alpha = \delta q^2 \beta$
$\beta \to \beta \cdot \delta\gamma$
$\alpha \to \alpha \cdot \delta\tilde{\gamma} \cdot \delta q^2 \lambda$
$\begin{pmatrix} \delta & \delta q^2 \\ & \delta \end{pmatrix}$

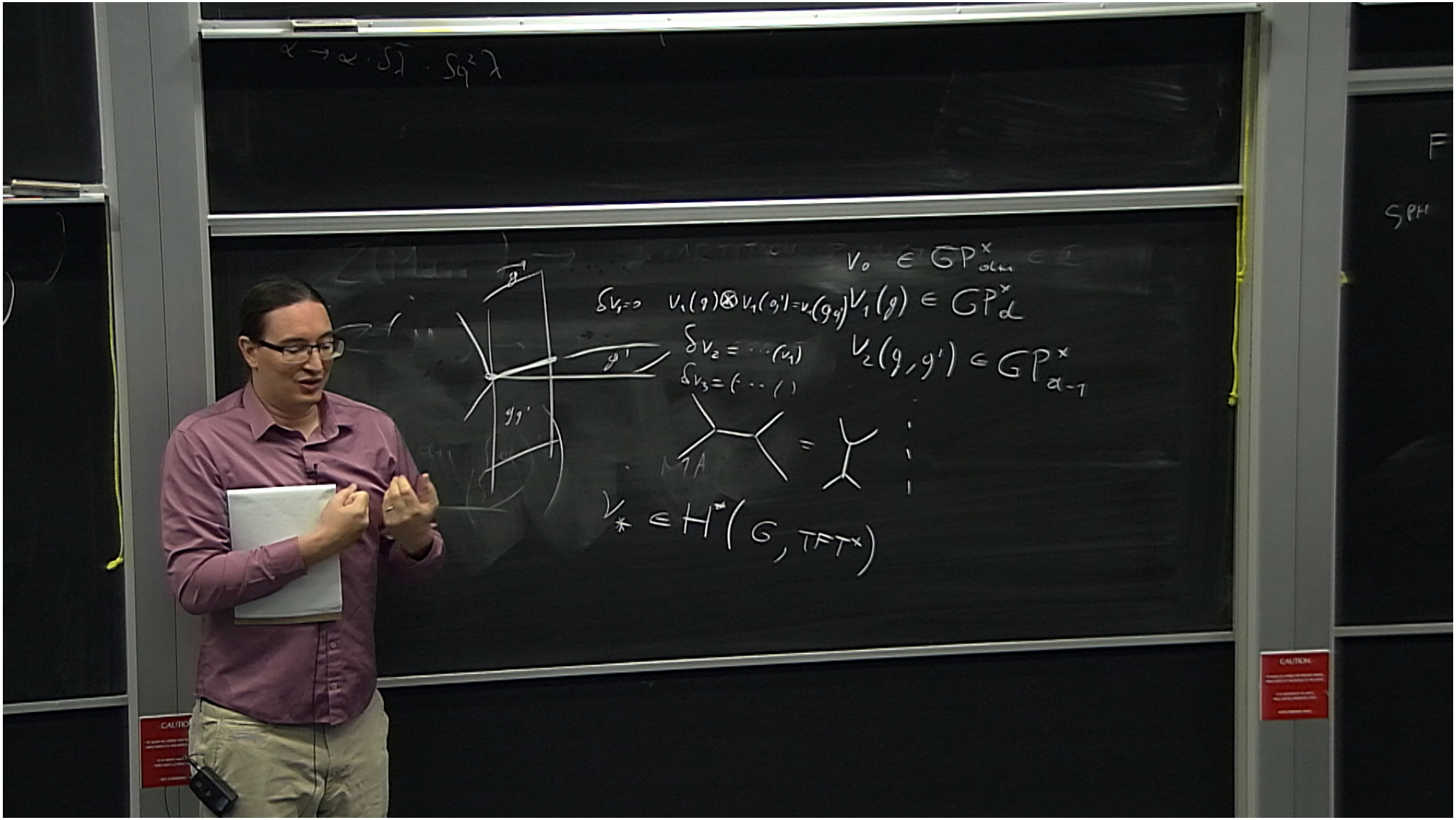

TQFT FUNCTOR BORD → VECT
$\beta(g_1 \cdots g_{d+1}) = \pm 1$
$\delta\beta = 1$
$\beta(f) = 1$ NOTHING
$\beta(f) = -1$ ADD $\mathcal{O}_f$, $\bar{\mathcal{O}}_f$
$Z = \int \prod_f d\alpha_f d\bar{\alpha}_f \left(\prod_S \left(\alpha(\cdots) \prod_{i \in S} \mathcal{O}_i \prod_{j \in S} \bar{\mathcal{O}}_j \right) \right) \prod_{f \in E} \beta(f)$
$\delta\alpha = \delta q^2 \beta$
$\beta \to \beta \cdot \delta\gamma$
$\alpha \to \alpha \cdot \delta\bar{\gamma} \cdot \delta q^2 \gamma$
$\begin{pmatrix} \delta & \delta q^2 \\ & \delta \end{pmatrix}$
$\partial E = W_2$


FERMIONIC TORAEV VIRO
SPH FUSION CATEGORY F, $\pi \in Z[F]$
$\pi \otimes \pi = 1$
$S\beta = 1$
(f)
CAUTION


FERMIONIC TORAEV VIRO
SPH FUSION CATEGORY F, π ∈ Z[F]
π⊗π = 1
Hom(a⊗b, c) ≅ Hom(a⊗b, c⊗π)
SB = 1
(f)


FERMIONIC TORAEV VIRO
SPH FUSION CATEGORY F, $\pi \in Z[F]$
$\pi \otimes \pi = 1$
$\mathrm{Hom}(a \otimes b, c) \cong \mathrm{Hom}(a \otimes b, c \otimes \pi)$
$\beta : 0 \to Z_2 \to \hat{G} \to G \to 0$
$(\hat{G}, \hat{\alpha})$
$\hat{\alpha}(\alpha)$
$S\beta = 1$
(f)
CAUTION


FERMIONIC TORAEV VIRO
SPH FUSION CATEGORY F, $\pi \in Z[F]$
$\pi \otimes \pi = 1$
$\mathrm{HOM}(a \otimes b, c) \oplus \mathrm{HOM}(a \otimes b, c \otimes \pi)$
$\beta : 0 \to Z_2 \to \hat{G} \to G \to 0$
$(\hat{G}, \hat{\alpha})$
$\hat{\alpha}(\alpha)$
$S\beta = 1$
(f)
CAUTION


$V_0 \in GP^{\times}_{\text{alm}}$
$\delta V_1 = 0$ $V_1(g) \otimes V_1(g') = V_2(g,g') V_1(gg') \in GP^{\times}_{d}$
$\delta V_2 = \cdots (V_1)$ $V_2(g,g') \in GP^{\times}_{d-1}$
$\delta V_3 = (\cdots)$
$V_* \in H^*(G, TFT^{\times})$