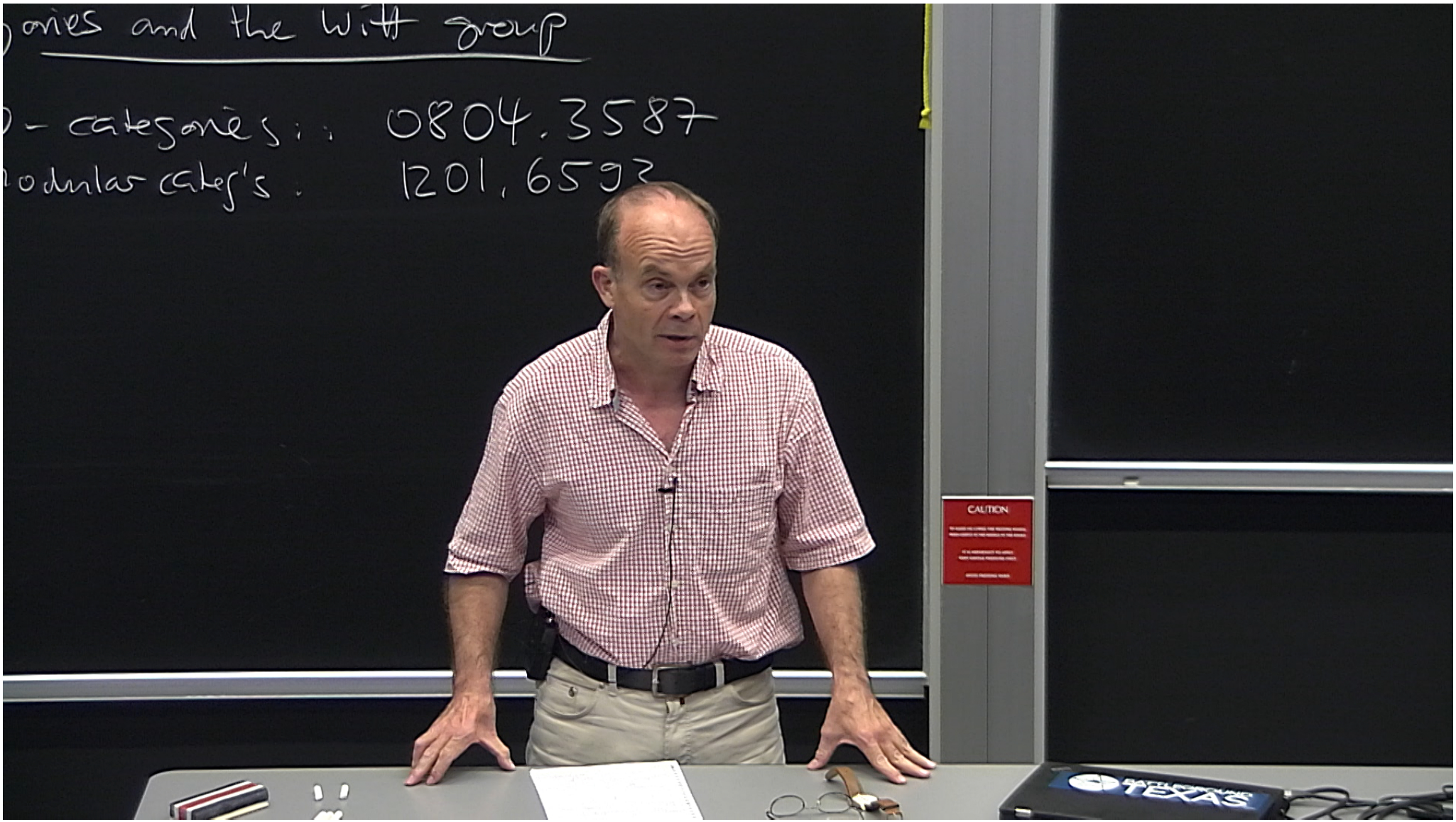
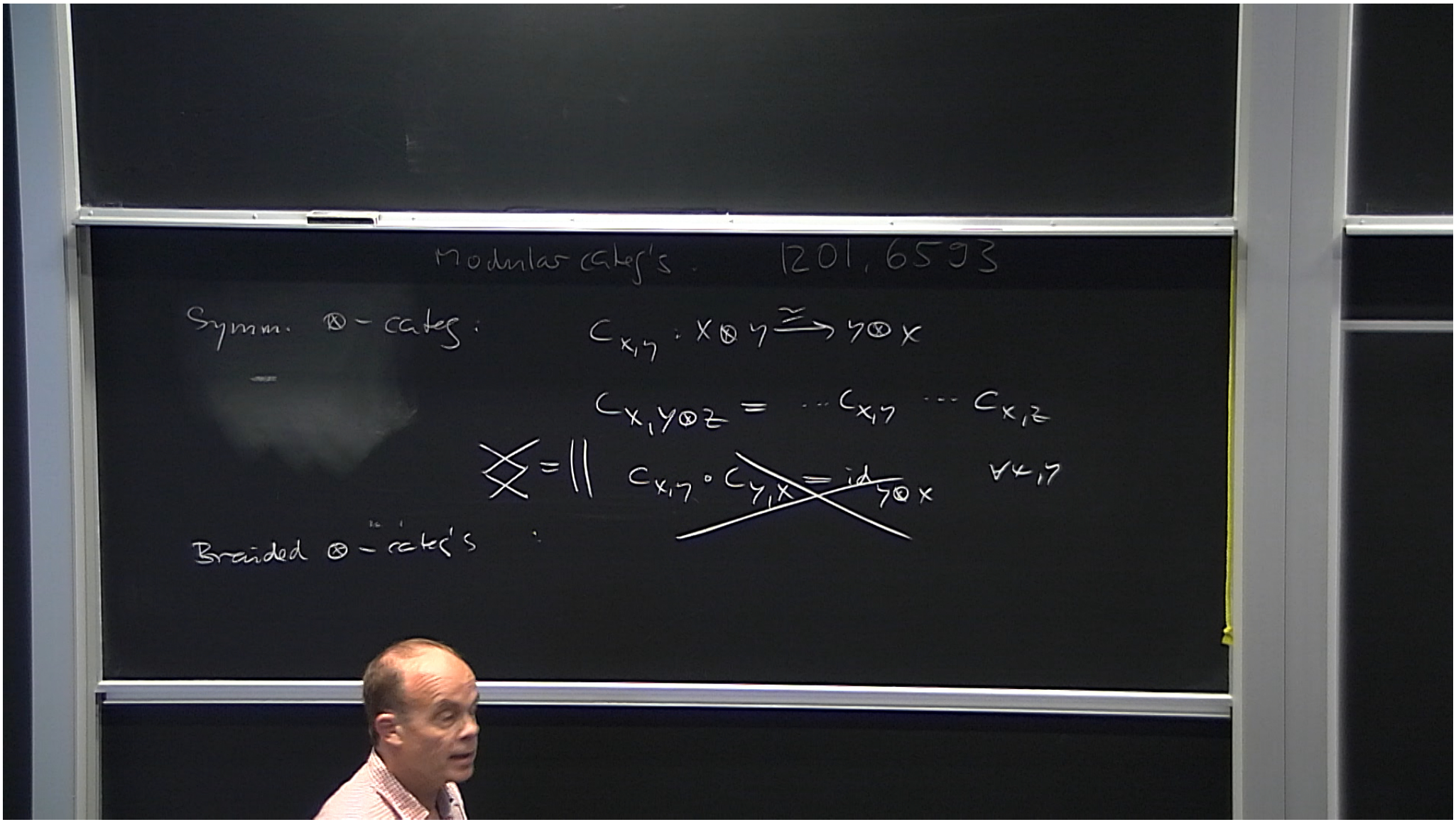
Title: Modular categories and the Witt group

Date: Jul 31, 2017 02:00 PM

URL: http://pirsa.org/17070064

Abstract: The aim of this talk is to give an introduction to modular categories, touching both basics and recent developments. I will begin with a quick reminder concerning tensor categories, in particular braided and symmetric ones, and notions like duality, fusion and spherical categories. We'll meet algebras in tensor categories, categories of modules, module categories and their connection. I will then focus on modular categories and some basic structure theory. We will consider two ways of obtaining modular categories: modularization and the Drinfeld center. (The important third one, quantum groups at root of unity, is too complicated to be discussed in any depth.) The Drinfeld center will be used to define Witt equivalence of modular categories and the Witt group. Several equivalent characterization of Witt equivalence, using module categories, will be discussed. The Witt group will be crucial for any (future) classification of modular categories, as well as for application to physics in condensed matter physics and conformal field theory.


ories and the Witt group
- categories : 0804.3587
odular categ's . 1201.6593
CAUTION


Modular categ's. 1201.6593
Symm. ⊗-categ:
$c_{X,Y} : X \otimes Y \xrightarrow{\simeq} Y \otimes X$
$c_{X,Y\otimes Z} = \cdots c_{X,Y} \cdots c_{X,Z}$
$c_{X,Y} \circ c_{Y,X} = id_{Y\otimes X}$ $\forall X,Y$
Braided ⊗-categ's

nces: $\otimes$-categories: 0804.3587

Modular categ's: 1201.6593

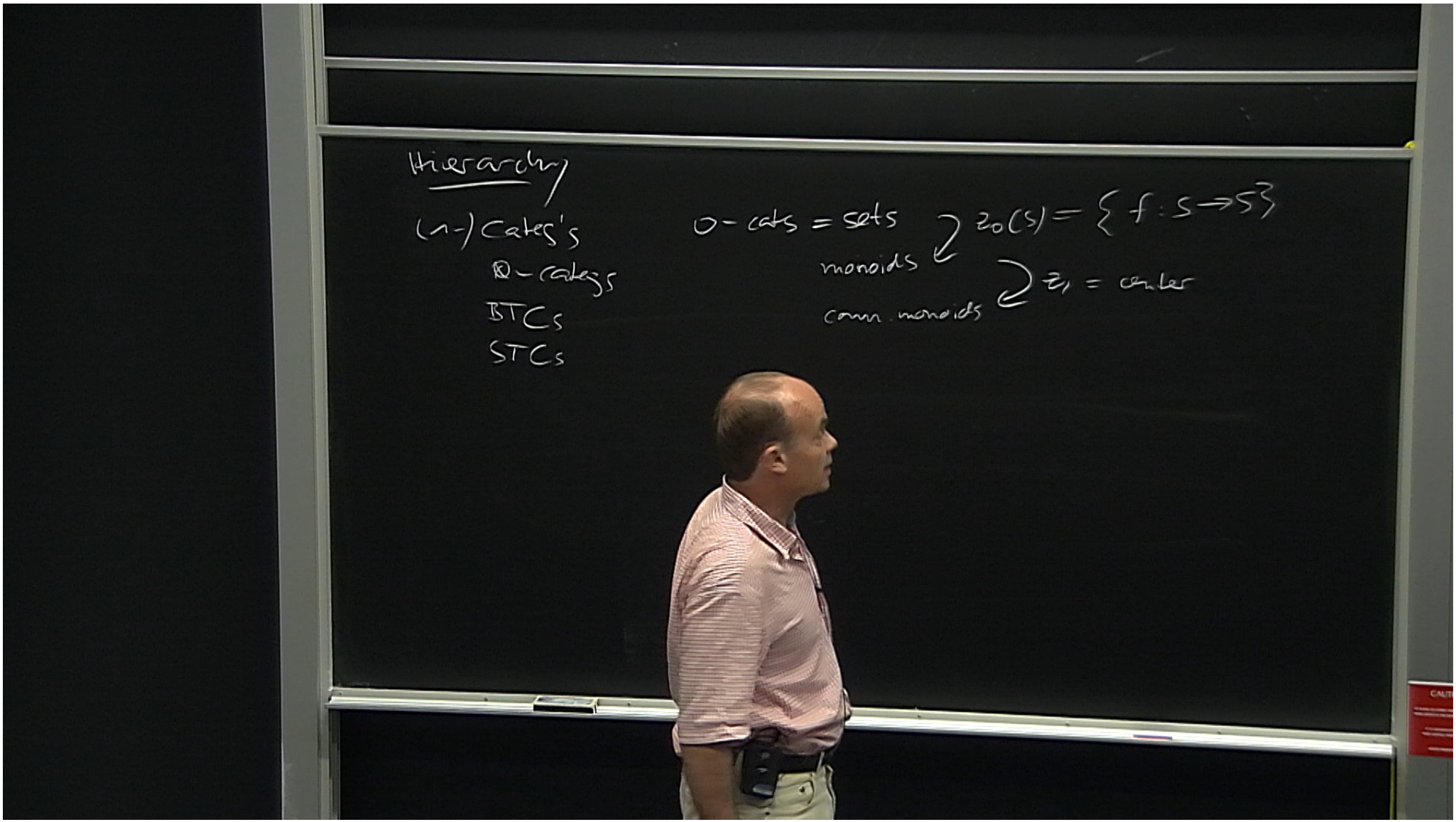
Symm. $\otimes$-categ: $c_{X,Y} : X \otimes Y \xrightarrow{\simeq} Y \otimes X$

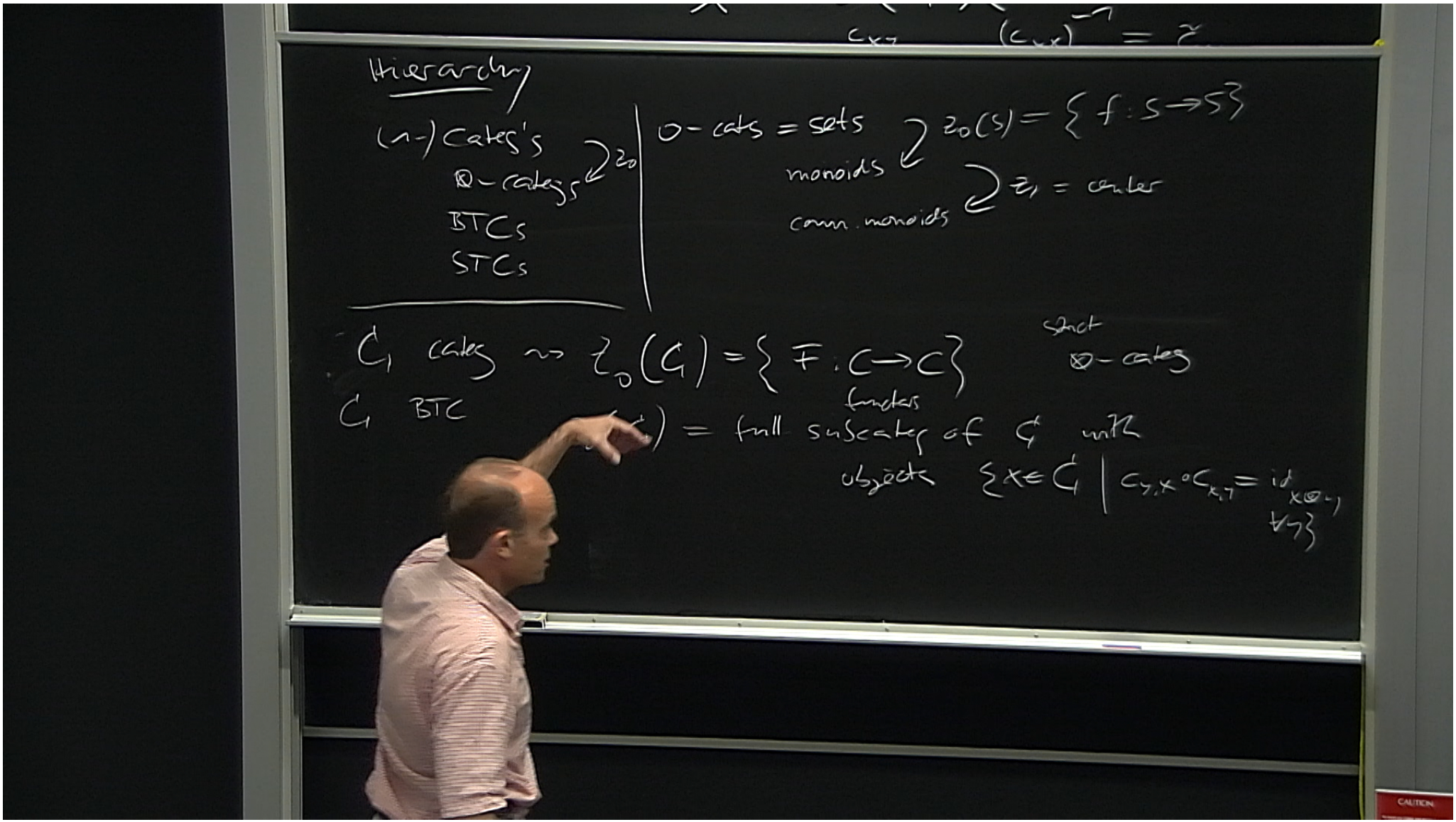
$$c_{X,Y\otimes Z} = \cdots c_{X,Y} \cdots c_{X,Z}$$

$$\times = \| \quad c_{X,Y} \circ \cancel{c_{Y,X} = id_{Y\otimes X}} \quad \forall X,Y$$

Braided $\otimes$-categ's

$$X \longrightarrow \underset{c_{X,Y}}{\times} \neq \times_{(c_{Y,X})^{-1} = \tilde{c}_{X,Y}}$$


Hierarchy
(n-)Categ's
R-categs
BTCs
STCs
0-cats = sets
monoids
comm. monoids
$Z_0(S) = \{f : S \to S\}$
Z_1 = center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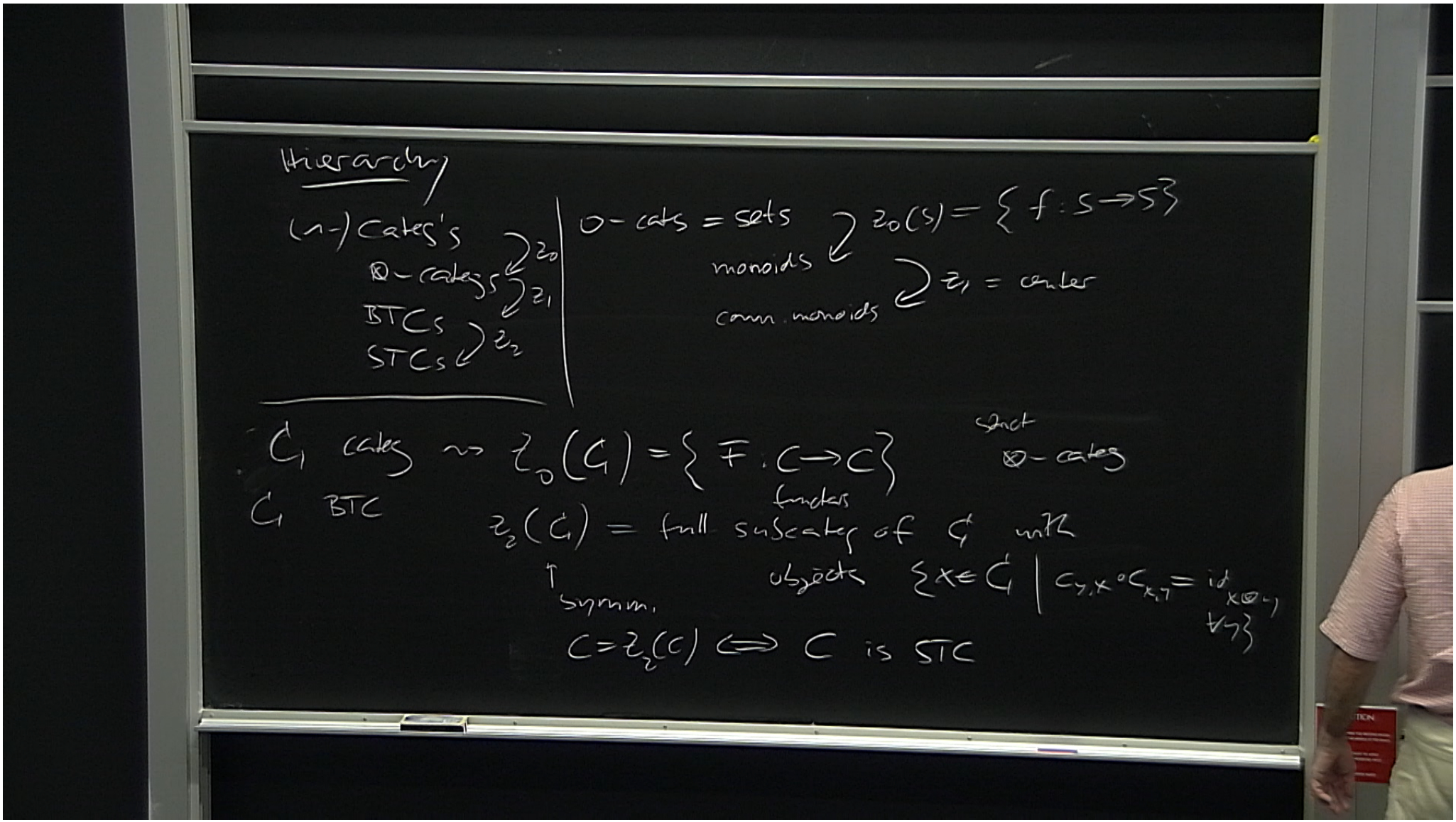
$\mathcal{C}$ categ ⇝ $z_0(\mathcal{C}) = \{ F : \mathcal{C} \to \mathcal{C} \}$
functors
strict ⊗-categ
$\mathcal{C}$ BTC
= full subcateg of $\mathcal{C}$ with
objects $\{ X \in \mathcal{C} \mid c_{Y,X} \circ c_{X,Y} = id_{X \otimes Y} \ \forall Y \}$


Hierarchy
(1-) Categ's
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BTCs
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z_0
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C categ $\rightsquigarrow$ $z_0(C) = \{ F : C \to C \}$
functors
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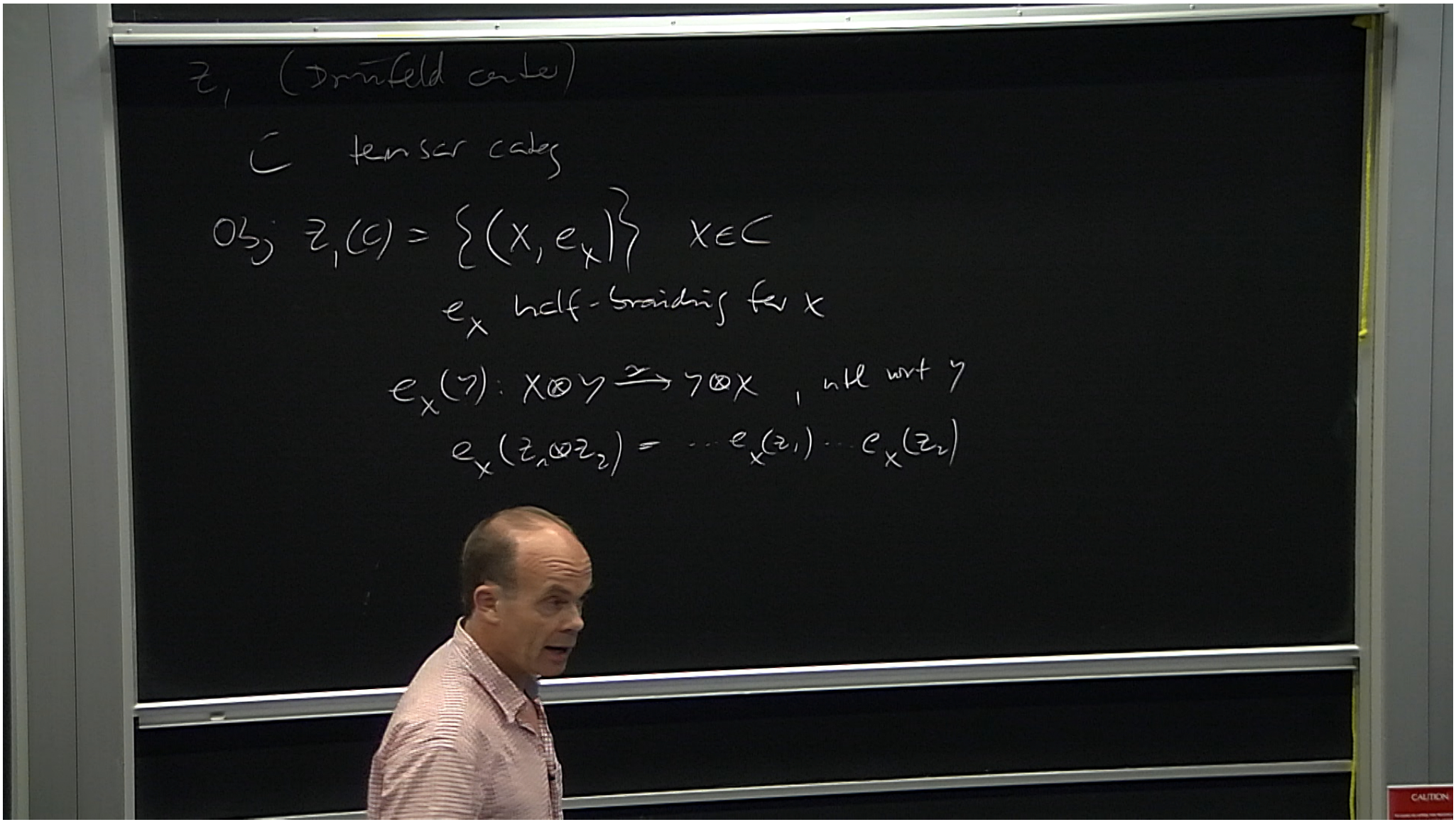
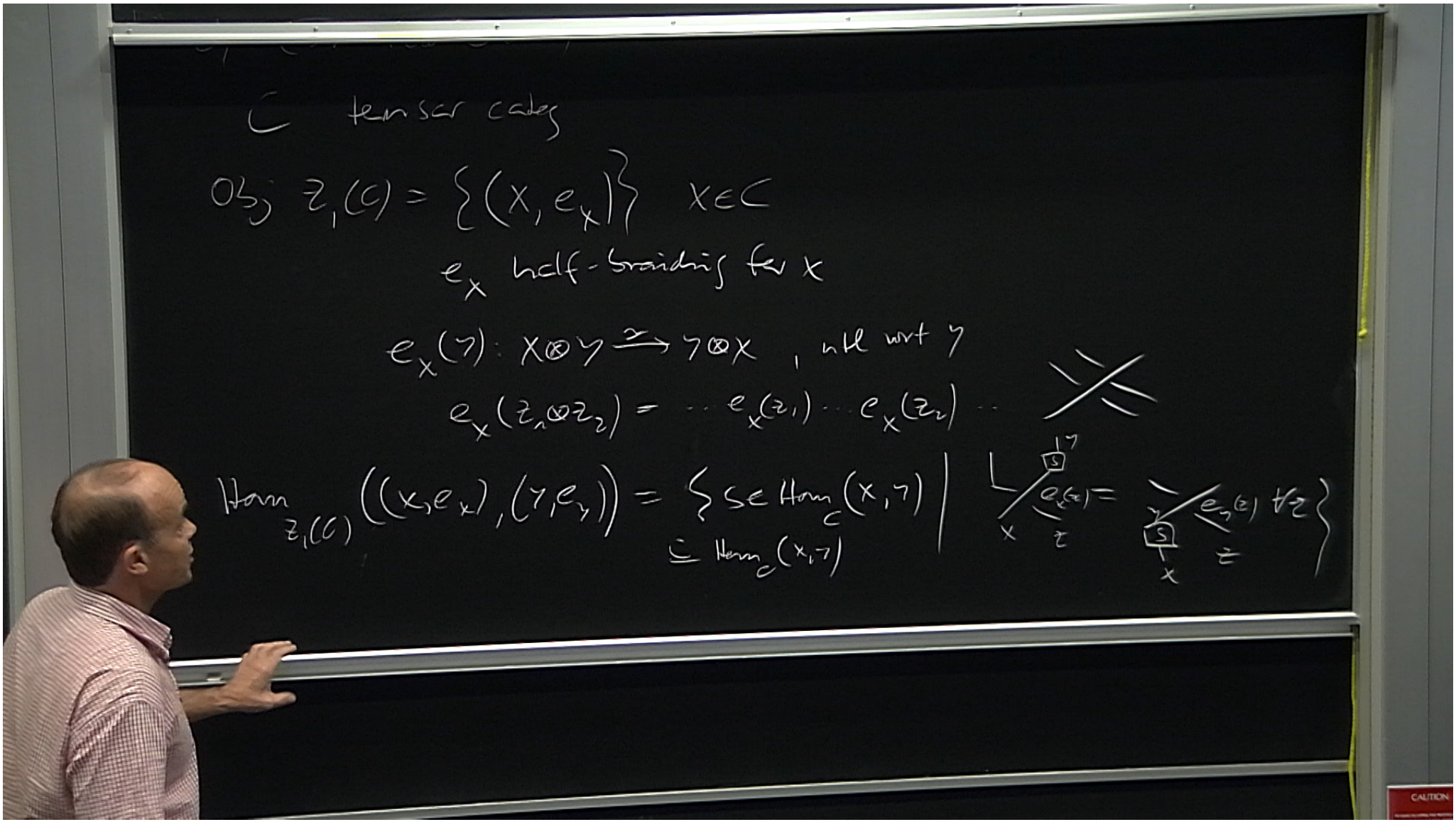
C BTC
$z_2(C)$ = full subcateg of C with
objects $\{ x \in C \mid c_{y,x} \circ c_{x,y} = id_{x \otimes y} \ \forall y \}$
symm.
$C = z_2(C) \iff C$ is STC


Z_1 (Drinfeld center)
$\mathcal{C}$ tensor categ
Obj: $Z_1(\mathcal{C}) = \{(X, e_X)\}$ $X \in \mathcal{C}$
e_X half-braiding for X
$e_X(Y): X \otimes Y \xrightarrow{\sim} Y \otimes X$, nat wrt Y
$e_X(Z_1 \otimes Z_2) = \cdots e_X(Z_1) \cdots e_X(Z_2)$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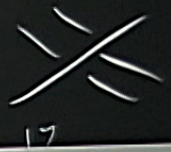
$e_X(Z_1 \otimes Z_2) = \cdots e_X(Z_1) \cdots e_X(Z_2) \cdots$
$\mathrm{Hom}_{Z_1(C)}((X, e_X), (Y, e_Y)) = \{ s \in \mathrm{Hom}_C(X, Y) \mid e_X(Z) = e_Y(Z) \ \forall Z \}$
$\subseteq \mathrm{Hom}_C(X, Y)$

$e_X(Y): X \otimes Y \longrightarrow Y \otimes X$, nat wrt Y

$e_X(z_1 \otimes z_2) = \cdots e_X(z_1) \cdots e_X(z_2) \cdots$

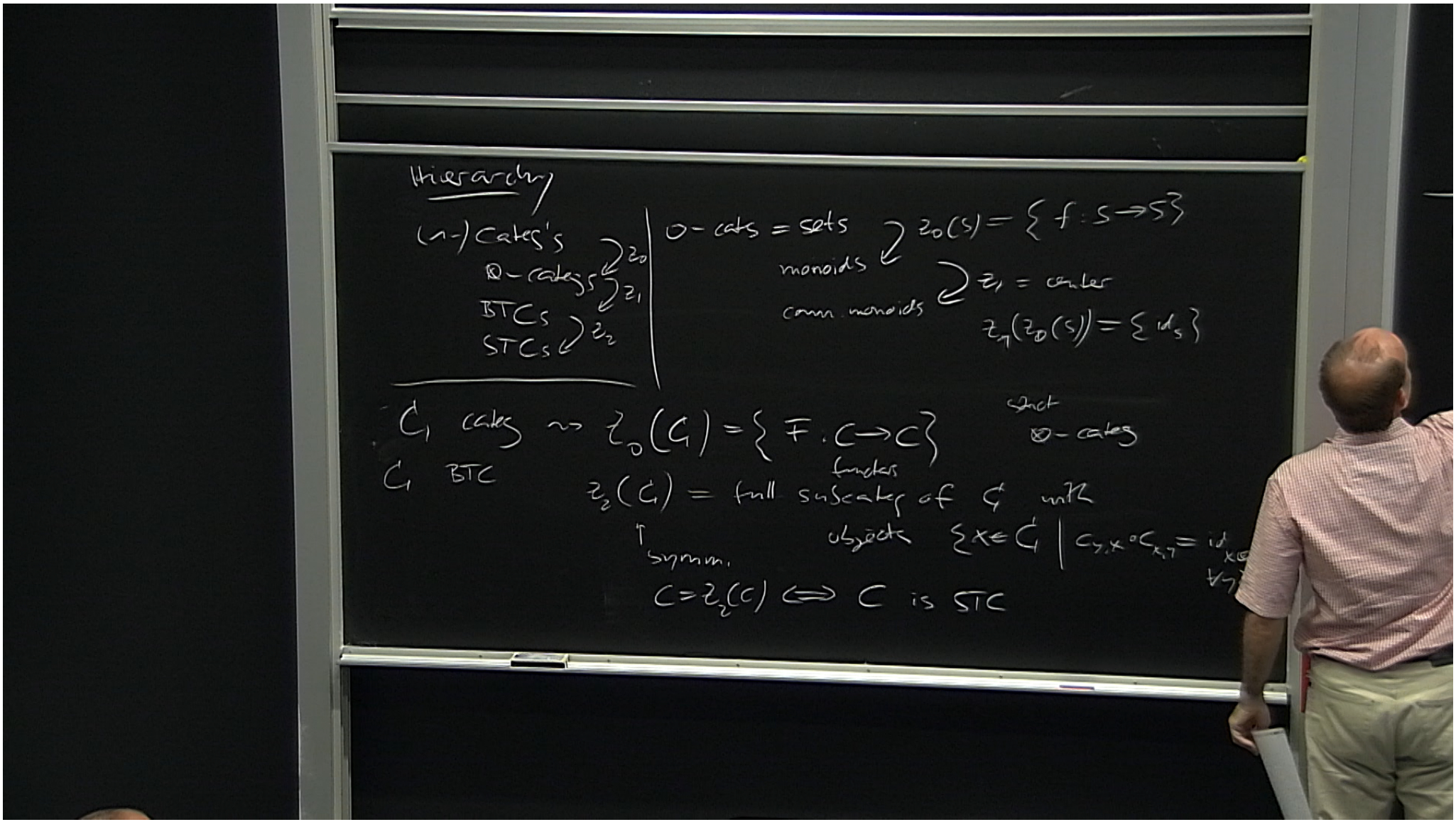


$(X, e_X) \otimes (Y, e_Y) = (X \otimes Y, e_{X \otimes Y})$ where $e_{X \otimes Y}(z) =$

$= \cdots e_X(z) \cdots e_Y(z) \cdots$

$\mathbb{1} = (1, e_1)$ where $e_1(z) = 1_z$

$c_{(X, e_X), (Y, e_Y)} := e_X(Y)$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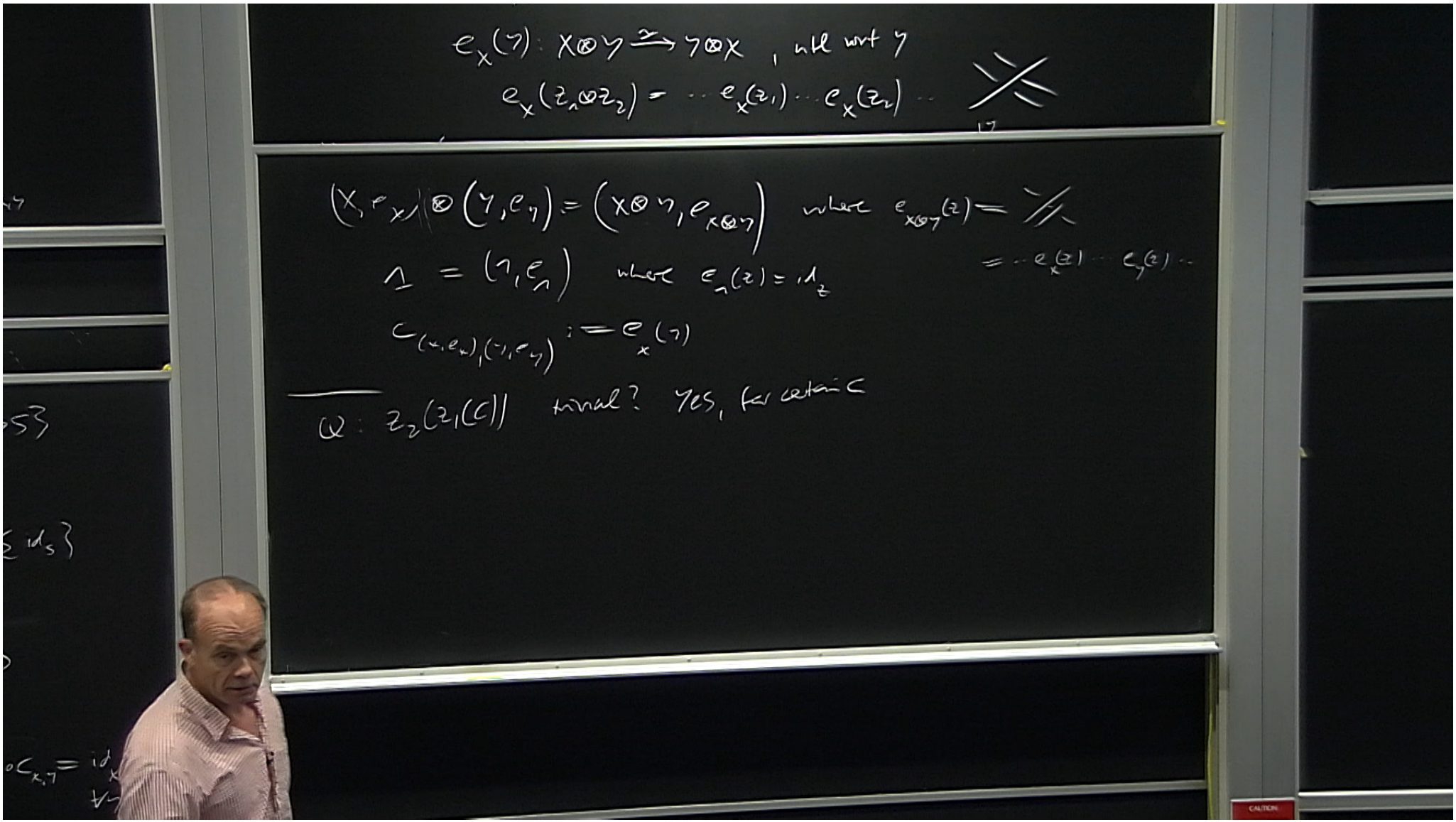
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$e_X(Z_1\otimes Z_2) = \cdots e_X(Z_1)\cdots e_X(Z_2)\cdots$
$(X, e_X)\otimes(Y, e_Y) = (X\otimes Y, e_{X\otimes Y})$ where $e_{X\otimes Y}(Z) =$
$= \cdots e_X(Z)\cdots e_Y(Z)\cdots$
$\mathbb{1} = (\mathbb{1}, e_{\mathbb{1}})$ where $e_{\mathbb{1}}(Z) = \mathrm{id}_Z$
$c_{(X,e_X),(Y,e_Y)} := e_X(Y)$
Q: $Z_2(Z_1(C))$ trivial? yes, for certain C

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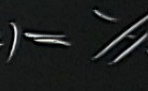
$(X, e_X) \otimes (Y, e_Y) = (X \otimes Y, e_{X \otimes Y})$ where $e_{X \otimes Y}(Z) =$ 

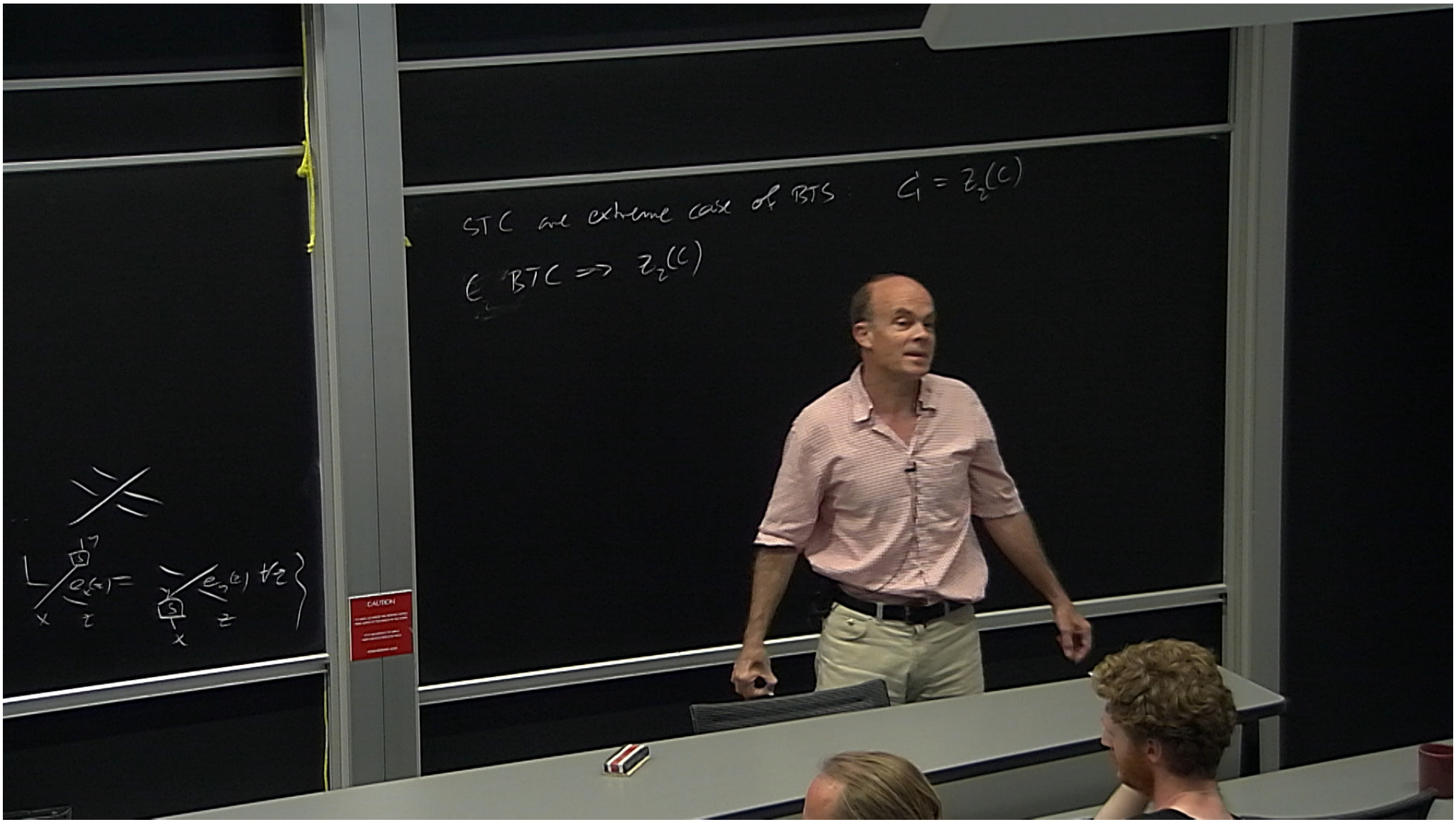
$= \cdots e_X(Z) \cdots e_Y(Z) \cdots$

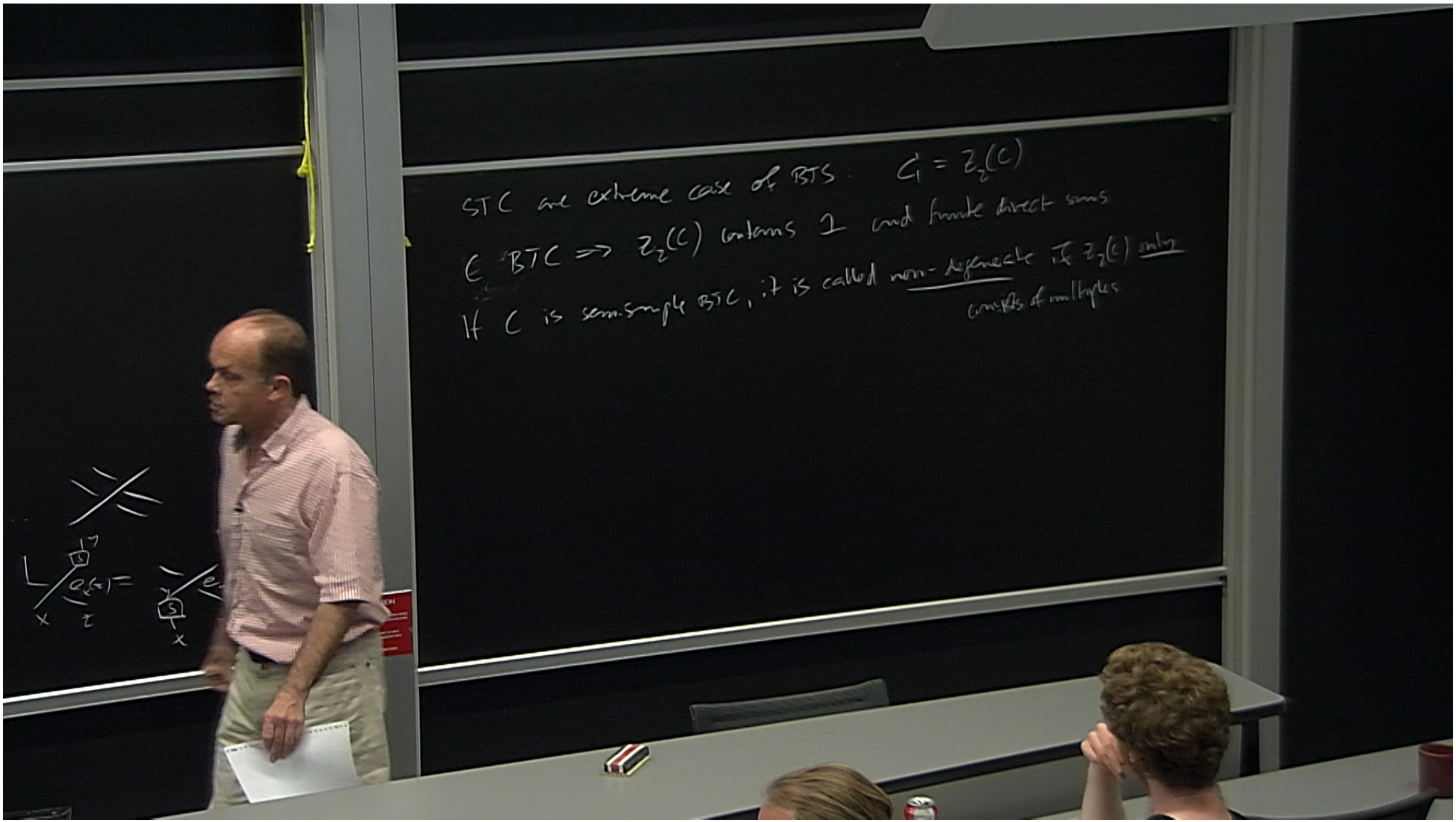
$\mathbb{1} = (\mathbb{1}, e_{\mathbb{1}})$ where $e_{\mathbb{1}}(Z) = \mathrm{id}_Z$

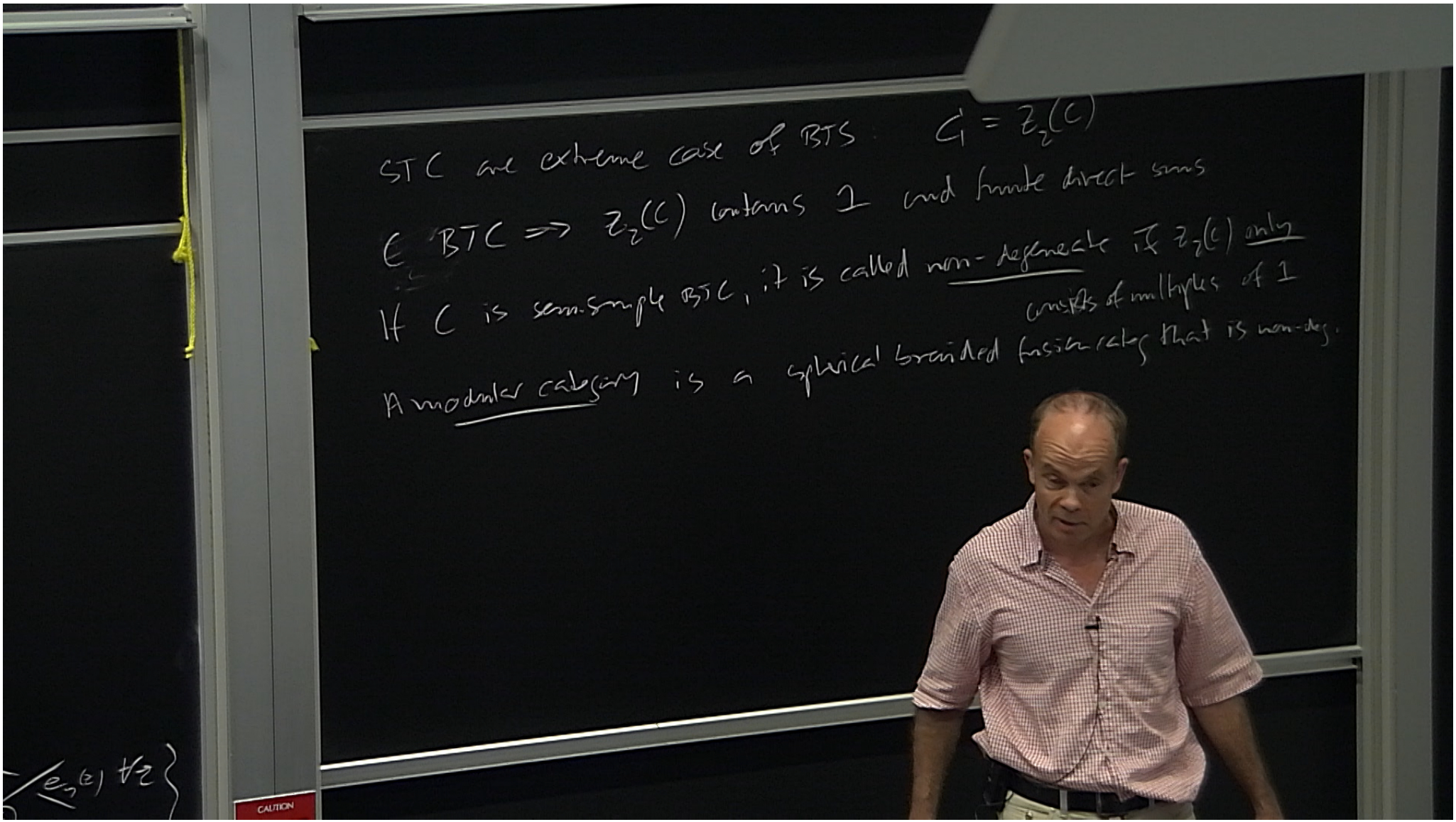
$c_{(X, e_X), (Y, e_Y)} := e_X(Y)$

Q: $Z_2(Z_1(\mathcal{C}))$ trivial? Yes, for certain $\mathcal{C}$

↑ spherical fusion category $\mathcal{C}$ with $\dim \mathcal{C} \neq 0$


STC are extreme case of BTS
$C_1 = Z_2(C)$
C_2 BTC => $Z_2(C)$


STC are extreme case of BTS
$C_1 = Z_2(C)$
$\in$ BTC $\Rightarrow$ $Z_2(C)$ contains 1 and finite direct sums
If C is semi-simple BTC, it is called non-degenerate if $Z_2(C)$ only consists of multiples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A modular category is a spherical braided fusion categ that is non-deg.

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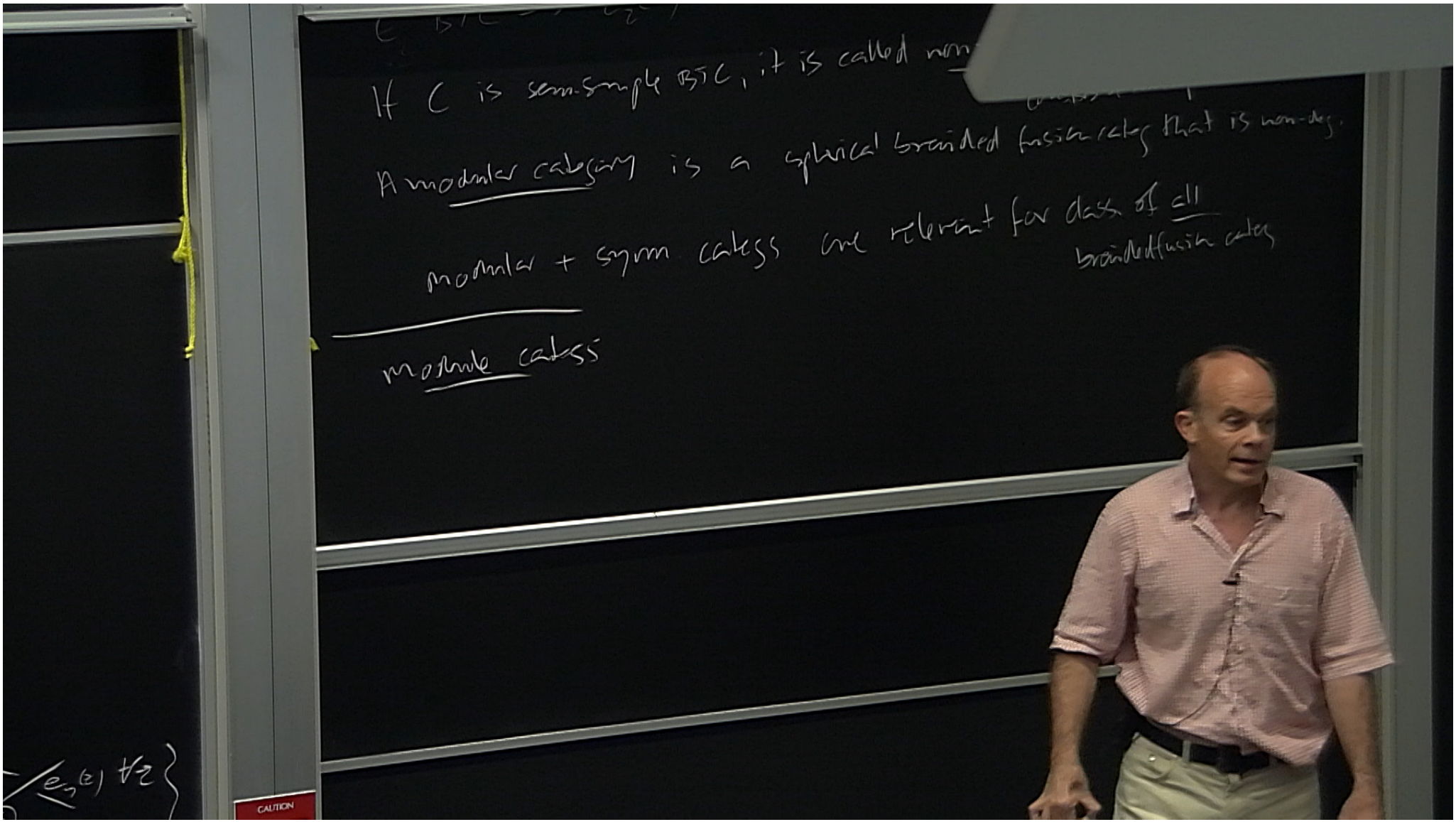
modular + symm categs are relevant for class of all braided fusion categs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modular + symm categs are relevant for class of all braided fusion categs
module categs
CAUTION

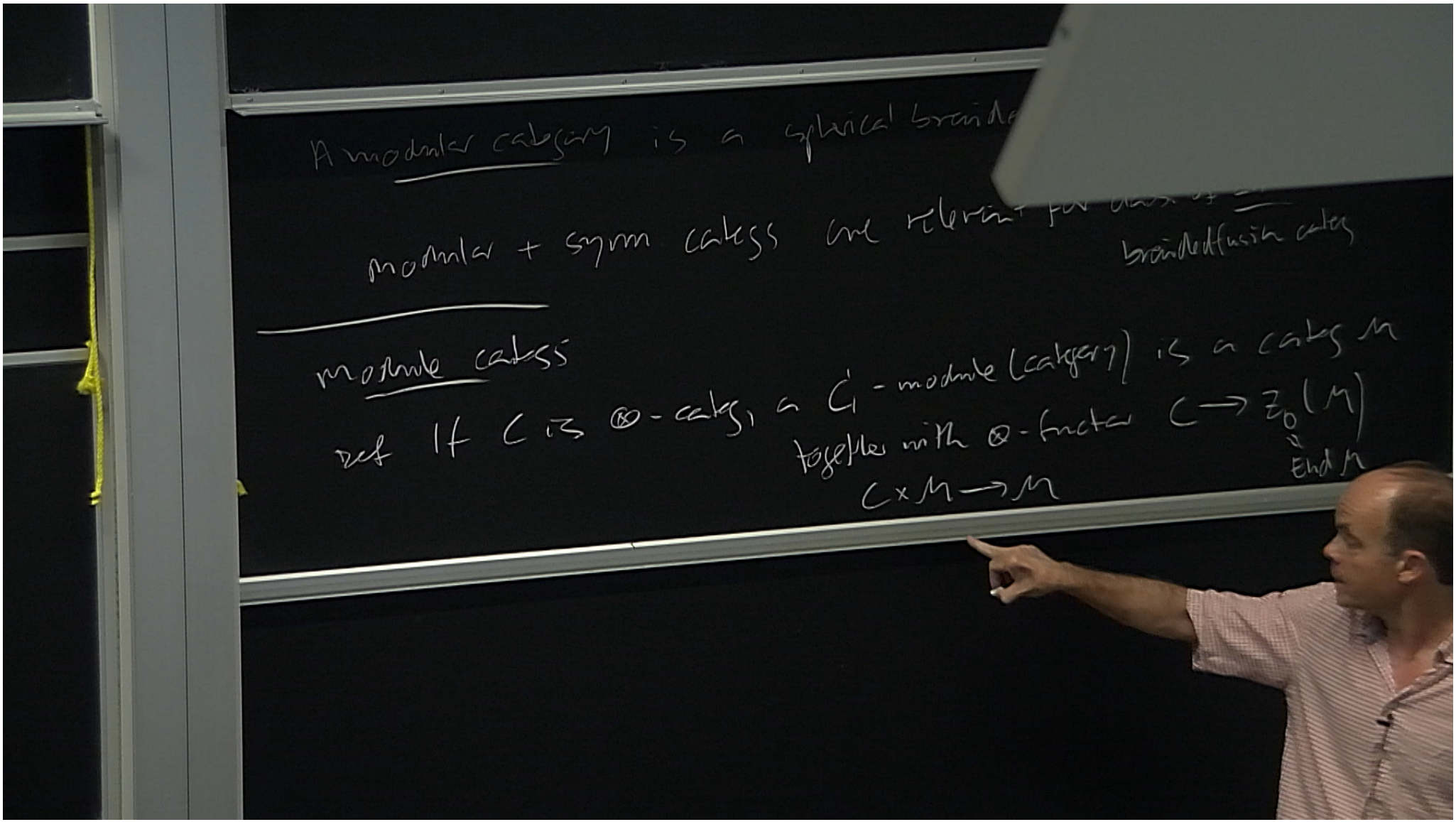
If C is semi-simple BTC, it is called non-

A modular category is a spherical braided fusion categ that is non-deg.

modular + symm. categs are relevant for class of all braided fusion categs

module categs

def If C is $\otimes$-categ, a C-module (category) is a categ M together with $\otimes$-functor $C \to Z_0(M)$


A modular category is a spherical braide
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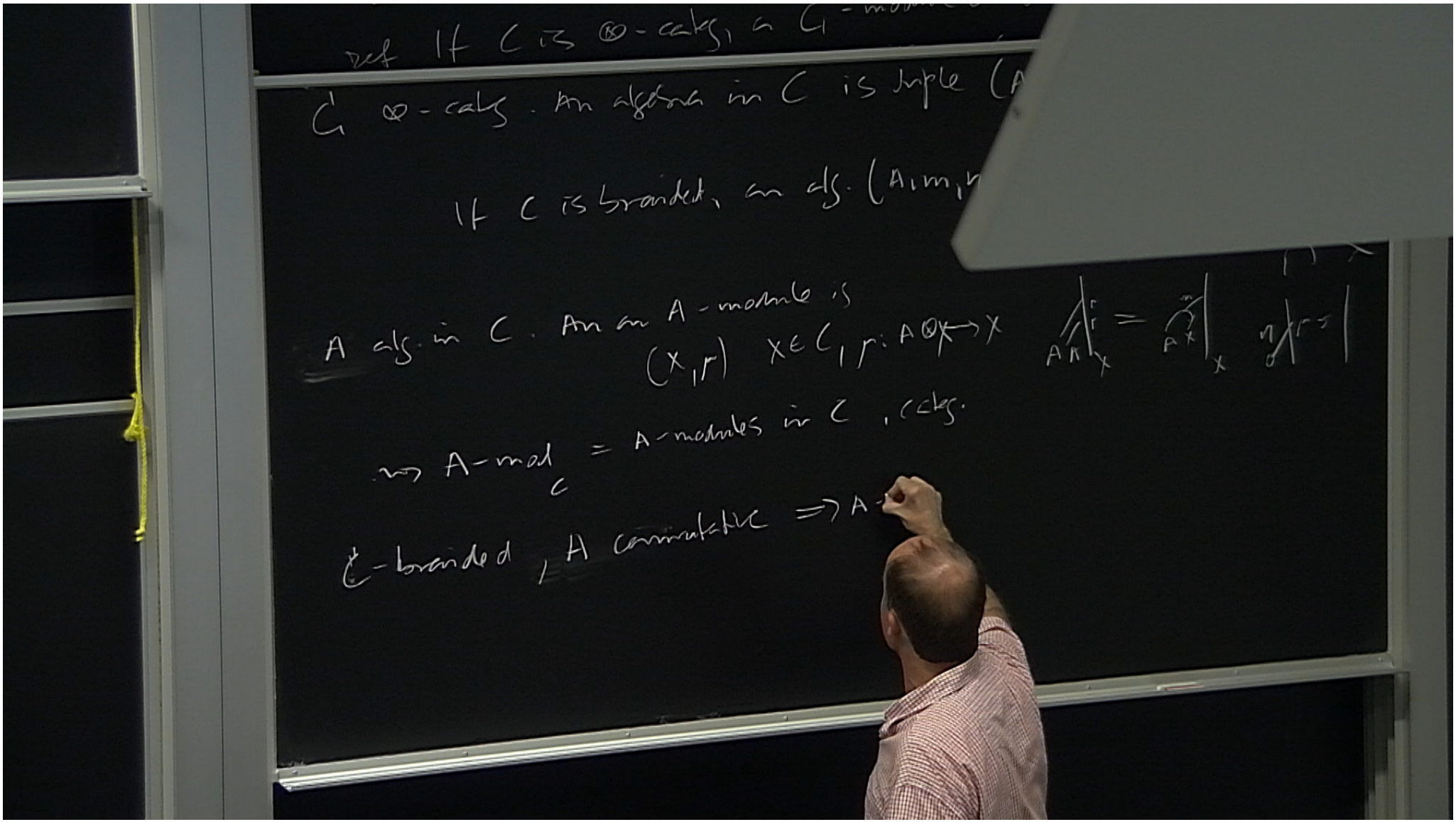
module categs
def If C is ⊗-categ, a C-module (category) is a categ M
together with ⊗-functor C → Z0(M)
End M
C × M → M


def If C is ⊗-categ, a C-
C ⊗-categ. An algebra in C is triple (A
If C is braided, an alg. (A, m,
A alg. in C. An an A-module is
(X, r) X ∈ C, r: A⊗X → X
⇝ A-mod_C = A-modules in C, ccteg.
C-braided, A commutative ⇒ A-

$\mathcal{C}$ $\otimes$-catg. An algebra in $\mathcal{C}$ is triple $(A$

If $\mathcal{C}$ is braided, an alg. $(A, m, \eta$

A alg. in $\mathcal{C}$. An an A-module is

(X, ρ) $X \in \mathcal{C}$, $\rho : A \otimes X \to X$

$\rightsquigarrow$ $A\text{-mod}_{\mathcal{C}}$ = A-modules in $\mathcal{C}$, catg.

$\mathcal{C}$-braided, A commutative $\Rightarrow$ $A\text{-mod}_{\mathcal{C}}$ is $\otimes$-catg. $(X, \rho), (X', \rho')$

$X \otimes X'$

$\mathcal{C}$ $\otimes$-categ. An algebra in $\mathcal{C}$ is triple $(A,$

If $\mathcal{C}$ is braided, an alg. (A, m, η)

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$\mathcal{C}$-braided, A commutative $\Rightarrow$ $\underline{A\text{-mod}_{\mathcal{C}}}$ is $\otimes$-categ. $(X, \rho), (X', \rho')$

$X \otimes X'$

$\mathcal{C}$ symm, " $\Rightarrow$ $A\text{-mod}_{\mathcal{C}}$ is $\underline{\text{symmetric}}$

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(X, r) $X \in \mathcal{C}$, $r: A \otimes X \to X$

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$\mathcal{C}$-braided, A commutative $\Rightarrow$ $A\text{-mod}_{\mathcal{C}}$ is $\otimes$-categ. $(X, r), (X', r')$

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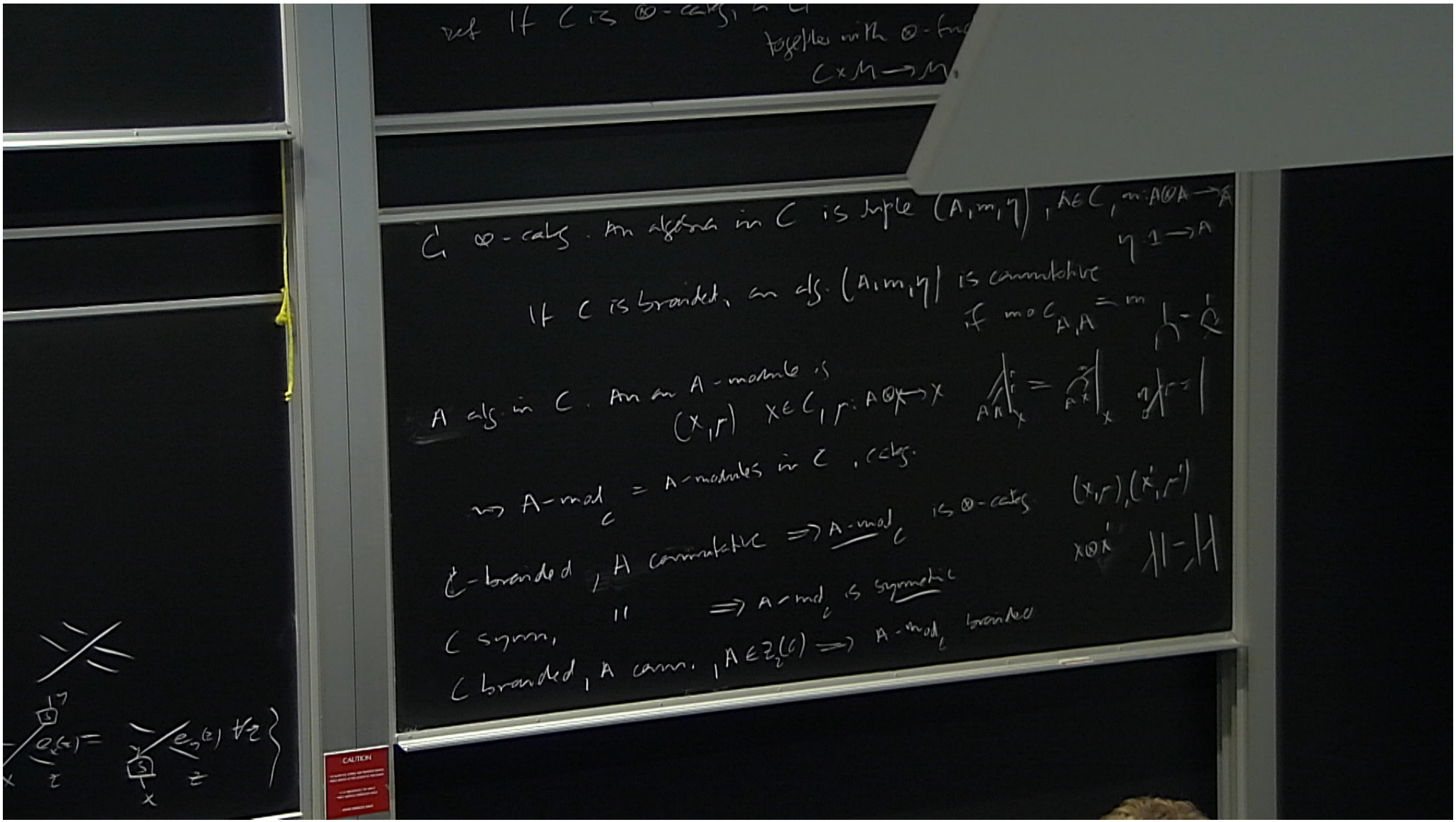
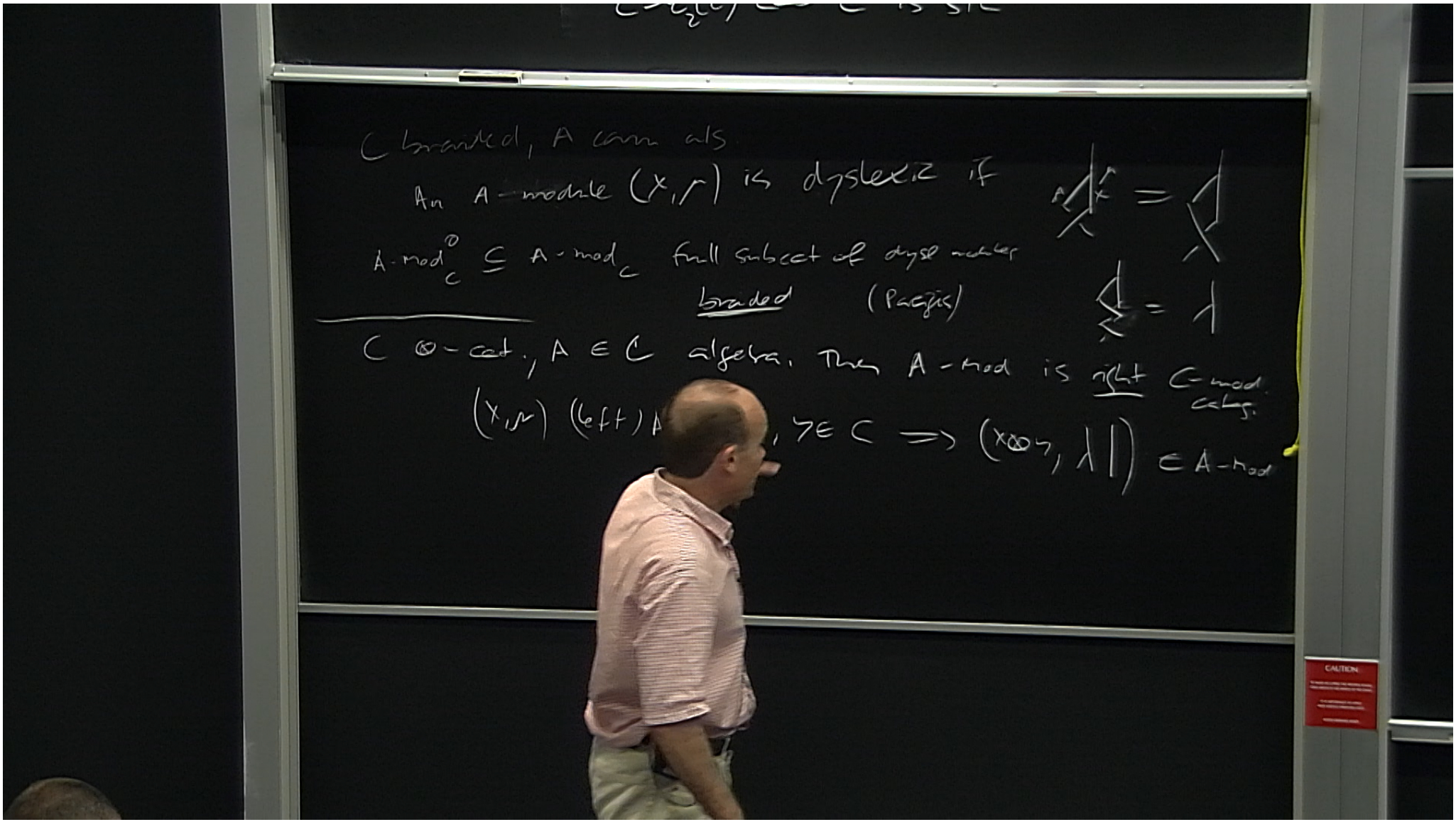
$\mathcal{C}$ symm, " $\Rightarrow$ $A\text{-mod}_{\mathcal{C}}$ is symmetric

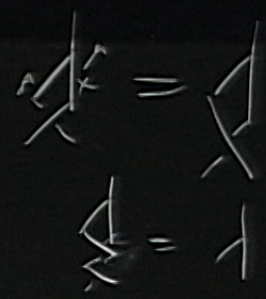
$\mathcal{C}$ braided, A comm., $A \in Z_2(\mathcal{C})$


Def If C is ⊗-categ, a C
together with ⊗-fun
C×M → M
C ⊗-categ. An algebra in C is triple (A, m, η), A∈C, m: A⊗A → A
η: 1 → A
If C is braided, an alg. (A, m, η) is commutative
if m∘c_{A,A} = m
A alg. in C. An an A-module is
(X, μ) X∈C, μ: A⊗X → X
⇝ A-mod_C = A-modules in C, categ.
C-braided, A commutative ⇒ A-mod_C is ⊗-categ
(X, μ), (X', μ')
X⊗X'
C symm, " ⇒ A-mod_C is symmetric
C braided, A comm., A∈Z_2(C) ⇒ A-mod_C braided


C braided, A comm alg.
An A-module (X, μ) is dyslexic if
A-mod°_C ⊆ A-mod_C full subcat of dyslex modules
braided (Pacific)
C ⊗-cat., A ∈ C algebra. Then A-mod is right C-mod. catg.
(X, μ) (left) A
, Y ∈ C ⇒ (X⊗Y, λ) ∈ A-mod

$\mathcal{C}$ braided, A comm alg

An A-module (X, μ) is dyslexic if



$A\text{-mod}^{\circ}_{\mathcal{C}} \subseteq A\text{-mod}_{\mathcal{C}}$ full subcat of dyslex modules

<u>braided</u> (Pareigis)

$\mathcal{C}$ $\otimes$-cat., $A \in \mathcal{C}$ algebra. Then A-mod is <u>right</u> $\mathcal{C}$-mod. catg.

(X, μ) (left) A-module, $Y \in \mathcal{C} \Rightarrow (X \otimes Y, \lambda \|) \in A\text{-mod}$

$\mathcal{C}$ braided, A comm alg.

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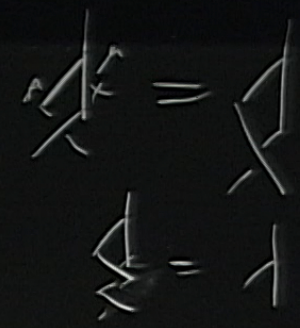

$A\text{-mod}^{\circ}_{\mathcal{C}} \subseteq A\text{-mod}_{\mathcal{C}}$ full subcat of dyslexic modules

<u>braided</u> (Pacific)

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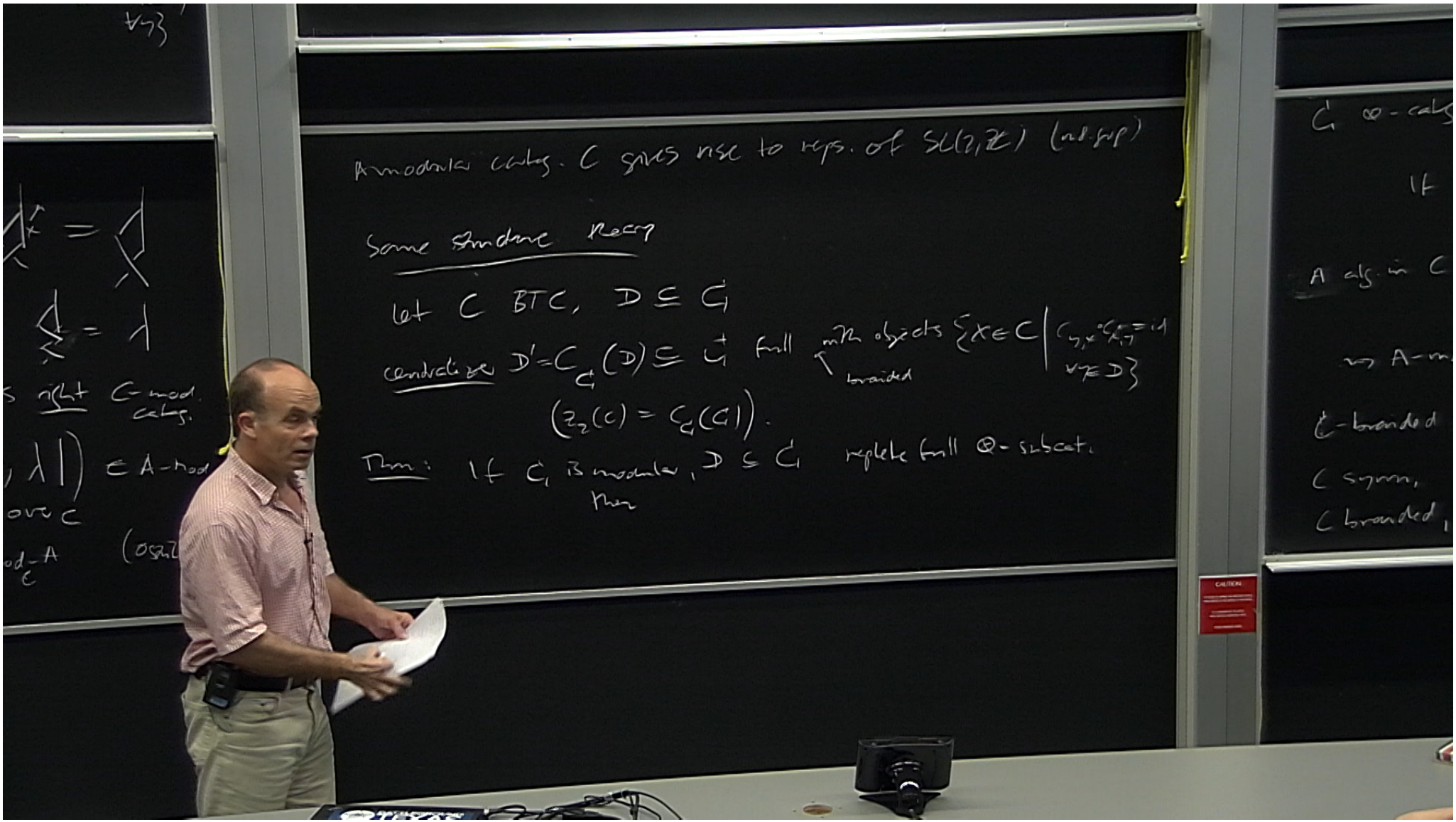
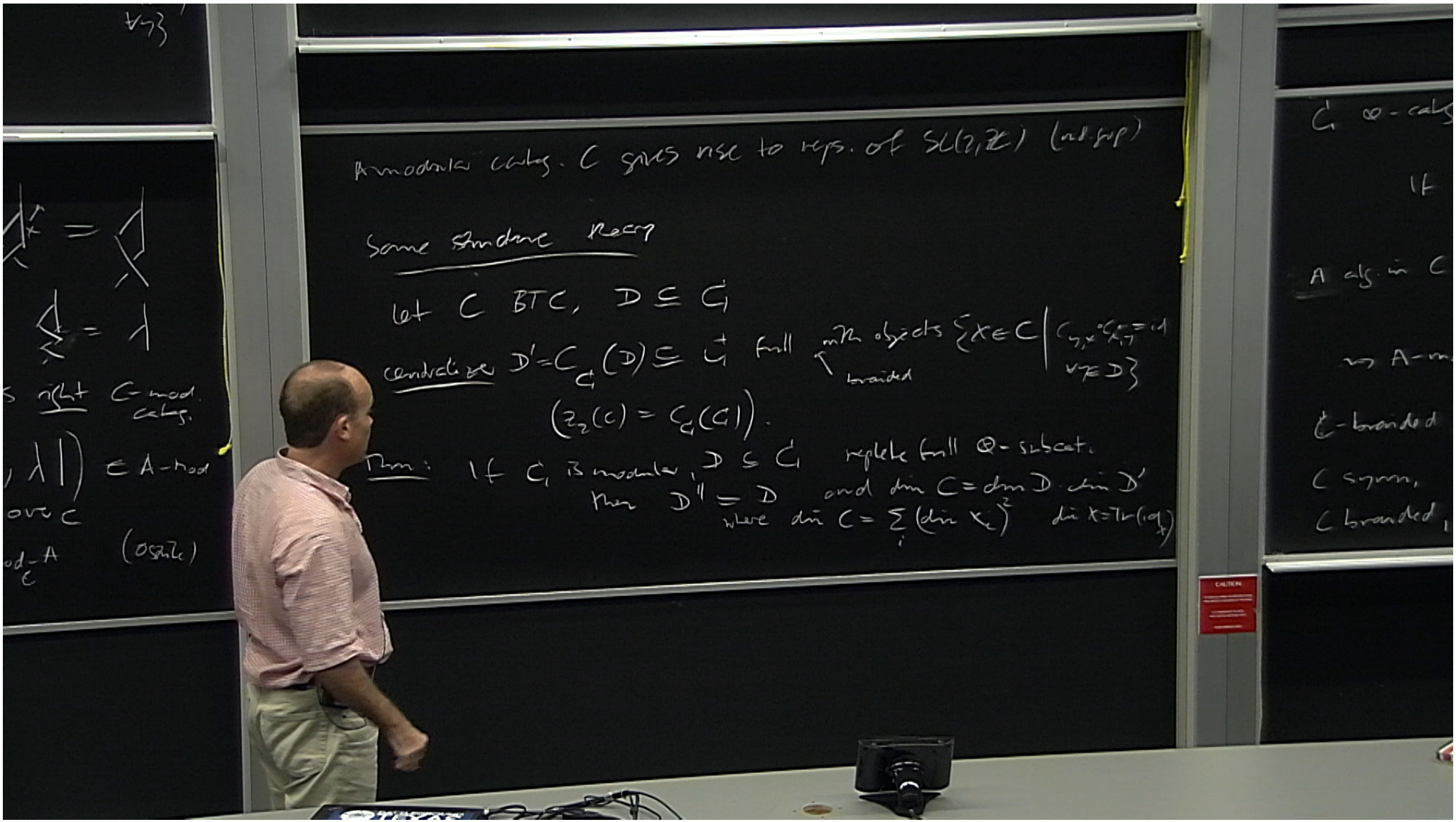
(X, μ) (left) A-module, $Y \in \mathcal{C} \Rightarrow (X \otimes Y, \lambda \|) \in A\text{-mod}$

If $\mathcal{C}$ is fusion categ, $\mathcal{M}$ fusion module cat over $\mathcal{C}$

Then $\exists$ algebra $A \in \mathcal{C}$ s.th. $\mathcal{M} = \text{mod-}A_{\mathcal{C}}$ (Ostrik)


A modular categ. C gives rise to reps. of SL(2,ℤ) (mod. grp)
Some structure theory
let C BTC, D ⊆ C
centralizer D' = C_C(D) ⊆ C full
braided
(Z_2(C) = C_C(C)).
Thm: If C is modular, D ⊆ C replete full ⊗-subcat.
then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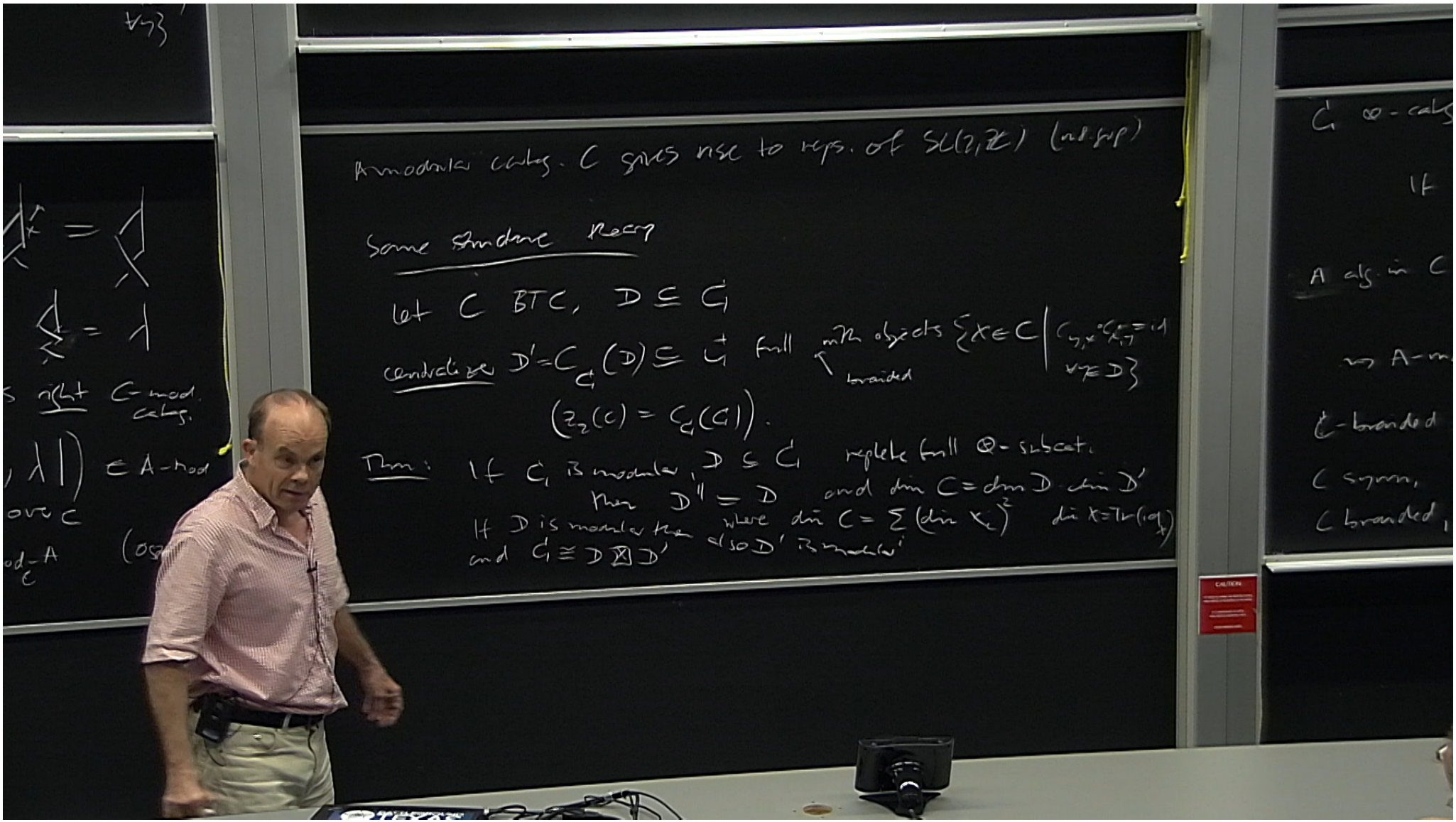
Thm: If C is modular, D ⊆ C replete full ⊗-subcat.
then D'' = D and dim C = dim D · dim D'
where dim C = Σ_i (dim X_i)^2 dim X = Tr(id_X)
right C-mod. categ.
∈ A-mod
over C
(ostrik)
C ⊗-categ
If
A alg. in C
⇝ A-m
C-braided
C symm,
C braided,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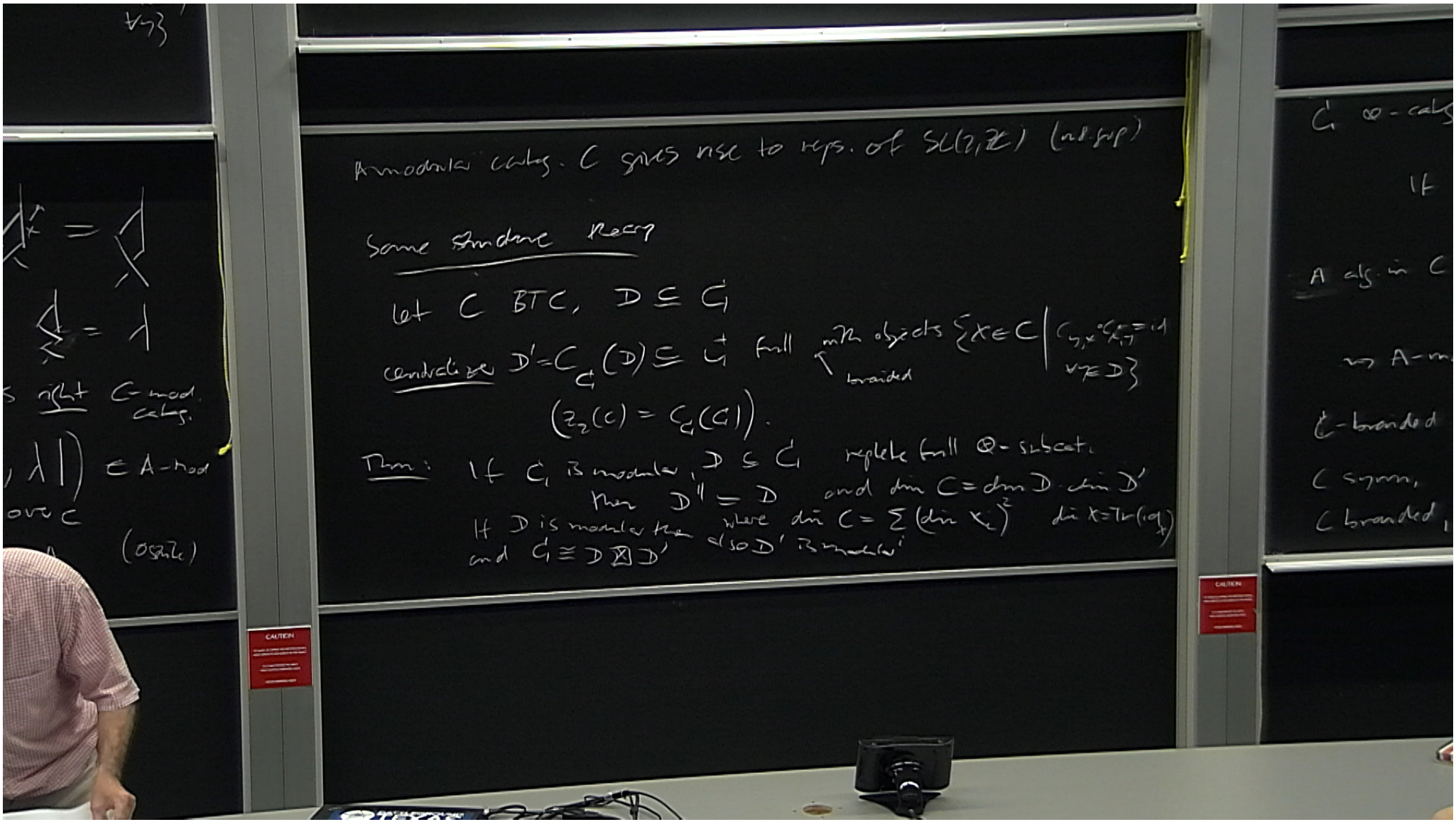
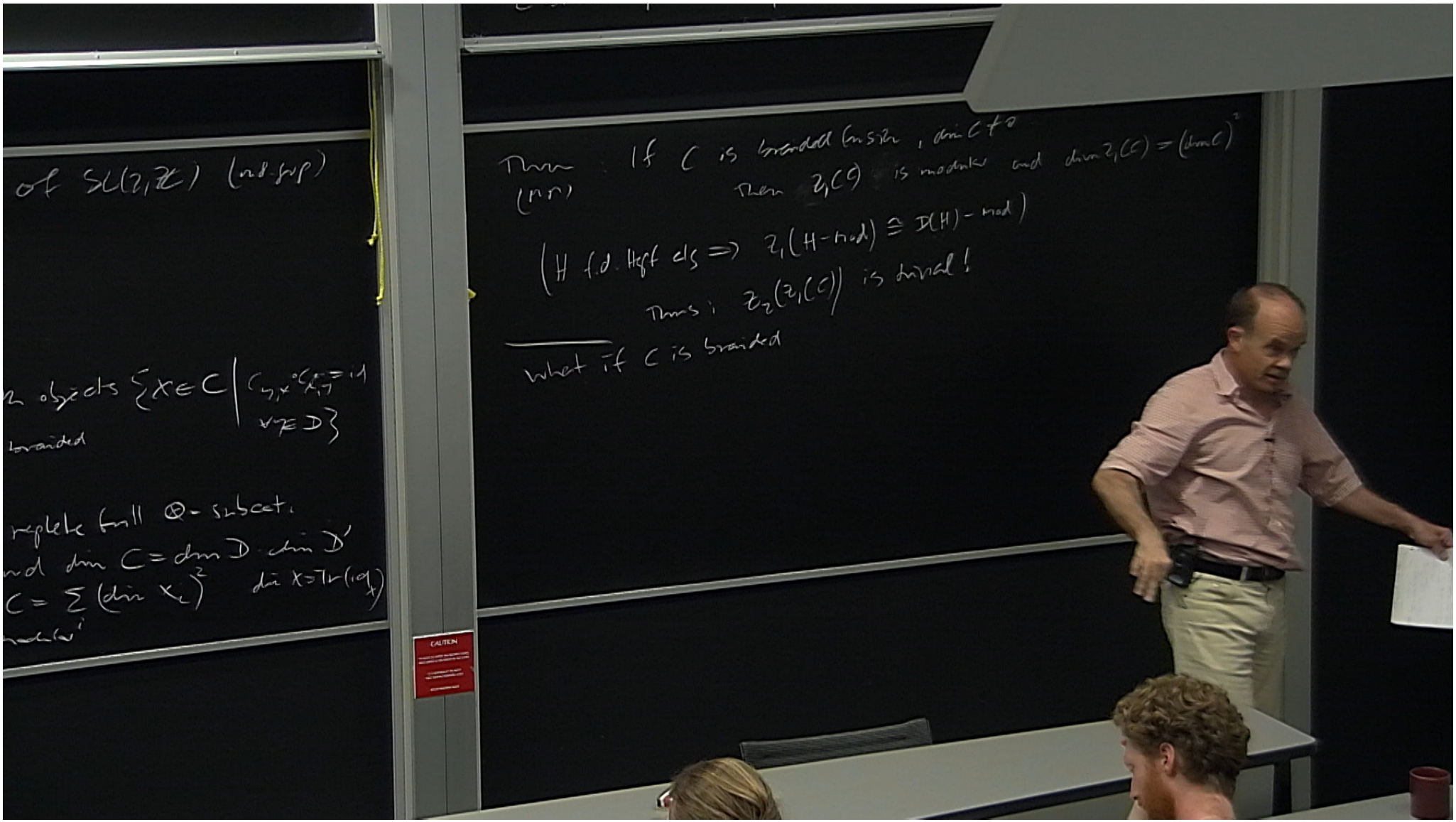
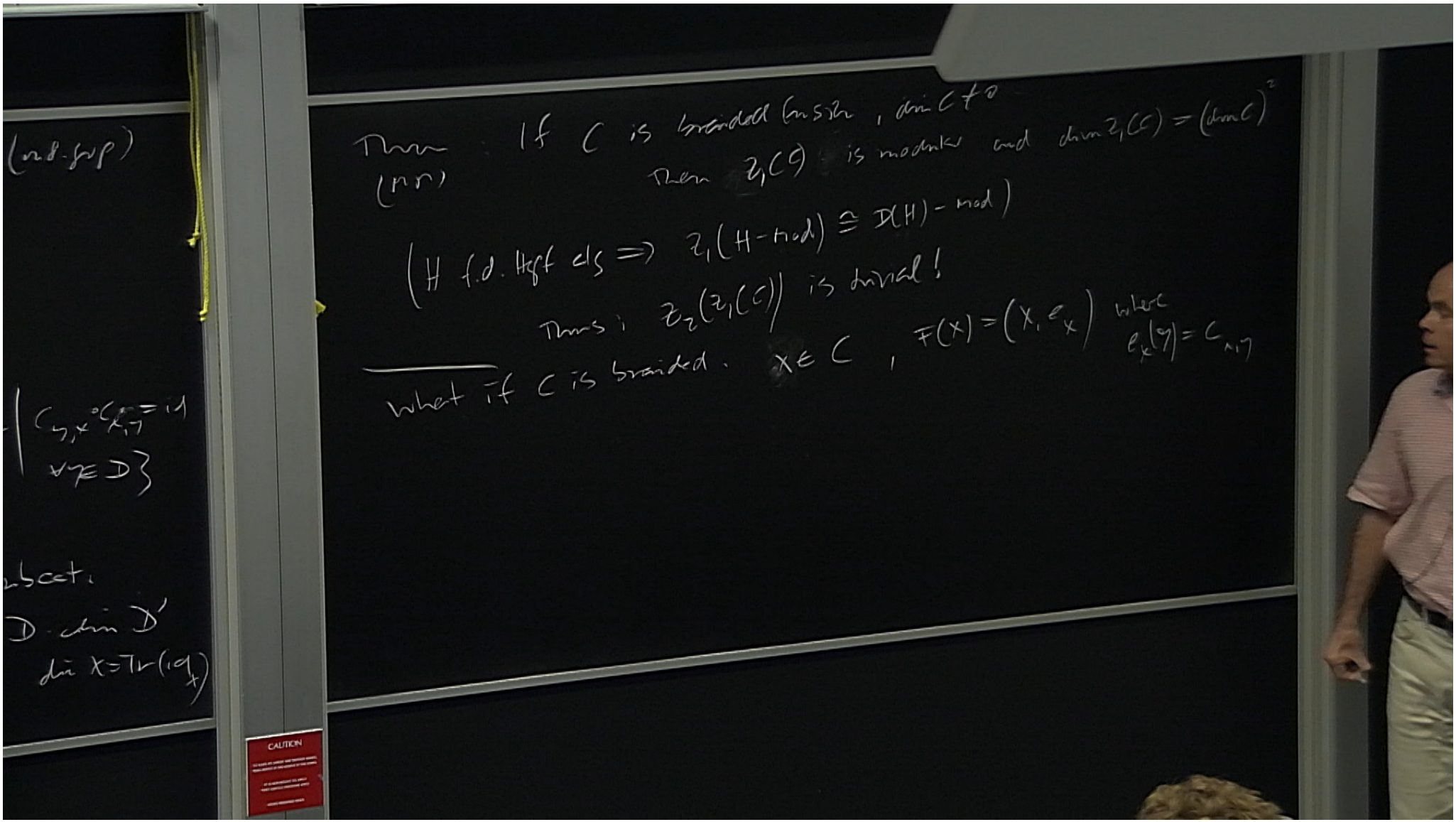
Thm: If C is modular, D ⊆ C replete full ⊗-subcat.
then D'' = D and dim C = dim D · dim D'
If D is modular then also D' is modular
and C ≅ D ⊠ D'
where dim C = Σ (dim X_i)^2
dim X = Tr(id_X)
right C-mod. categ.
A alg. in C
C-braided
C symm.
C braided


A modular categ. C gives rise to reps. of SL(2,ℤ) (mod. grp)
Some structure theory
let C BTC, D ⊆ C
centralizer D' = C_C(D) ⊆ C full mth objects {x ∈ C | c_{y,x} ∘ c_{x,y} = id ∀y ∈ D}
braided
(Z_2(C) = C_C(C)).
Thm: If C is modular, D ⊆ C replete full ⊗-subcat.
then D'' = D and dim C = dim D · dim D'
where dim C = Σ (dim x_i)^2 dim x = Tr(id_x)
If D is modular then also D' is modular
and C ≅ D ⊠ D'
right C-mod. categ.
∈ A-mod
over C
(oszile)
C ⊗-categ
If
A alg. in C
⇝ A-m
C-braided
C symm,
C braided,


of SL(2,ℤ) (mod. grp)
objects {X ∈ C | c_{y,x} ∘ c_{x,y} = id ∀y ∈ D}
braided
replete full ⊗-subcat.
and dim C = dim D · dim D'
C = Σ (dim X_i)^2
dim X = Tr(id_X)
modular
Thm (MR): If C is braided fusion, dim C ≠ 0
Then Z_1(C) is modular and dim Z_1(C) = (dim C)^2
(H f.d. Hopf alg ⇒ Z_1(H-mod) ≅ D(H)-mod)
Thus: Z_2(Z_1(C)) is trivial!
what if C is braided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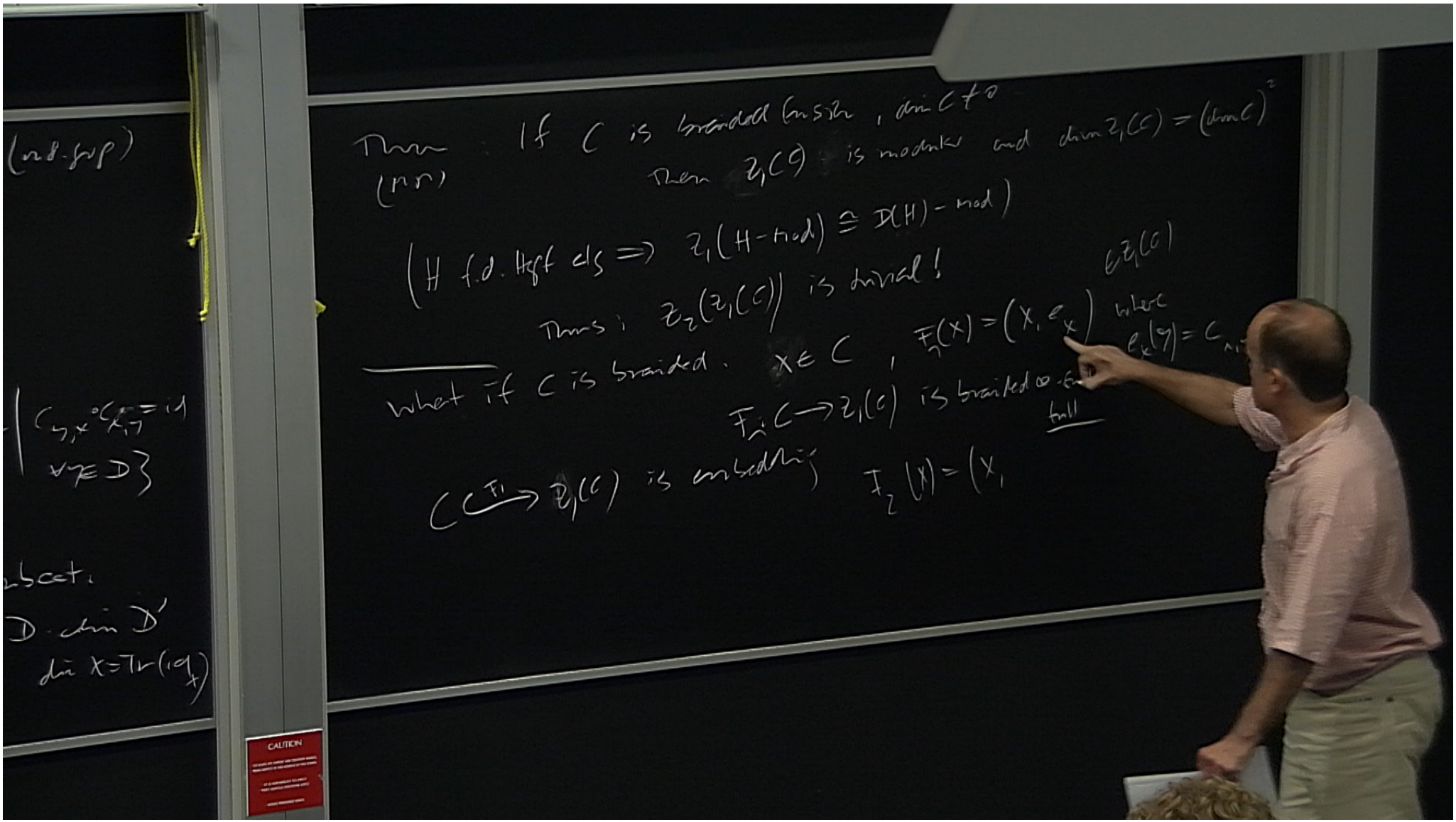
What if C is braided. X ∈ C, F(X) = (X, e_X) where e_X(Y) = c_{X,Y}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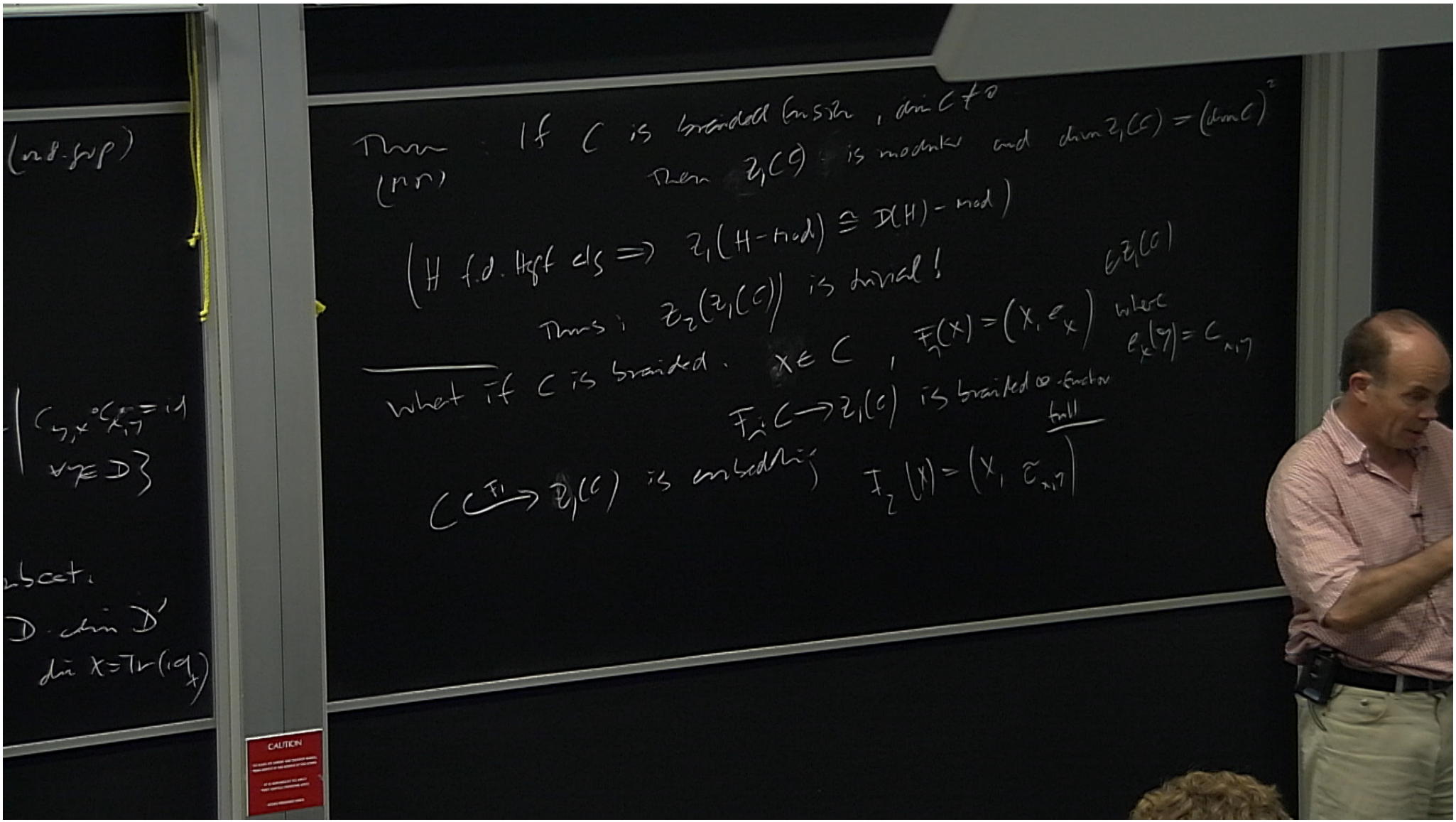
(H f.d. Hopf alg ⇒ Z1(H-mod) ≅ D(H)-mod)
Thus: Z2(Z1(C)) is trivial!
what if C is braided. X ∈ C, F1(X) = (X, e_X) ∈ Z1(C) where e_X(Y) = c_{X,Y}
F1: C → Z1(C) is braided ⊗-functor
full
C ↪ Z1(C) is embedding
F2(X) = (X,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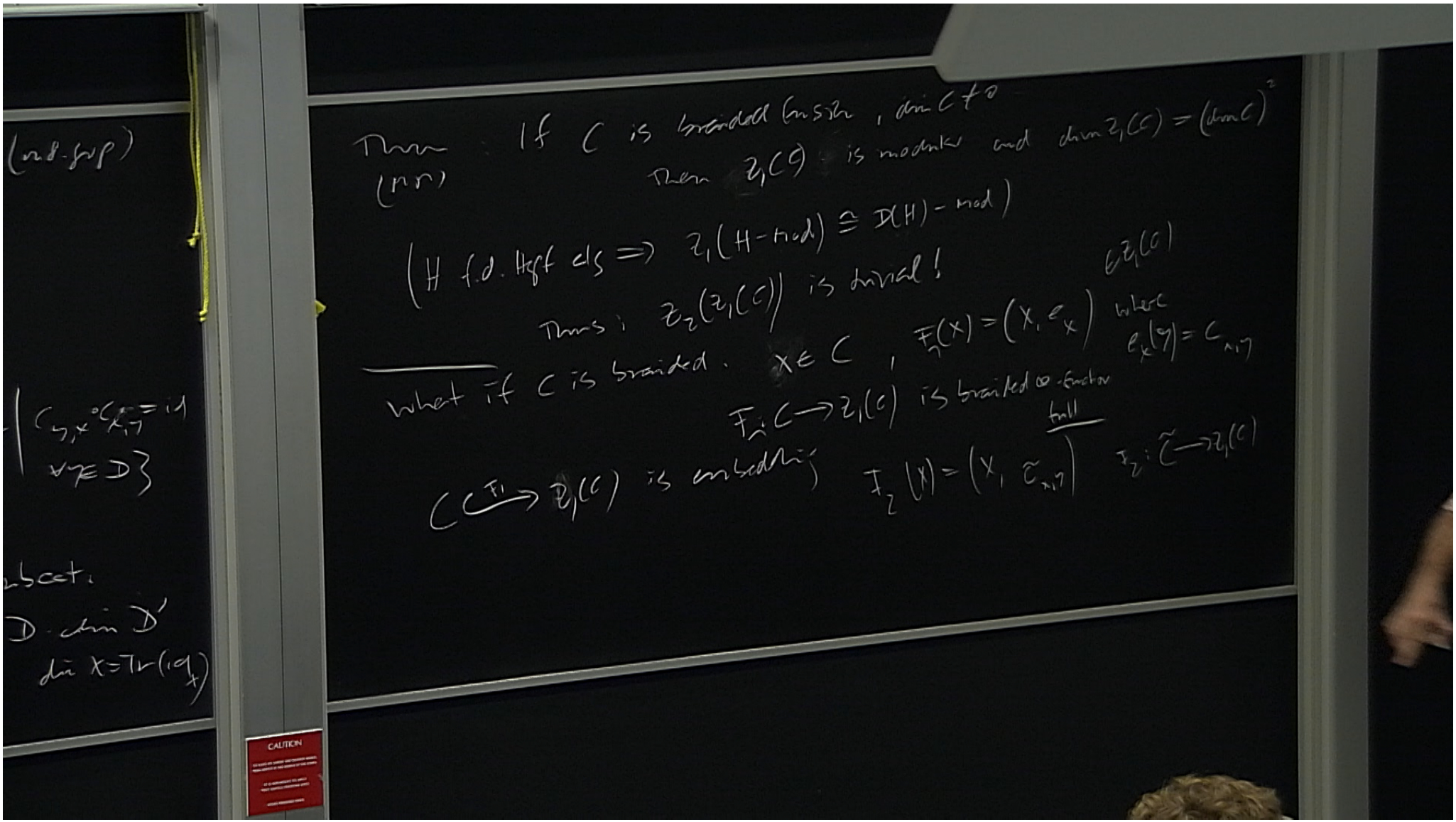
CAUTION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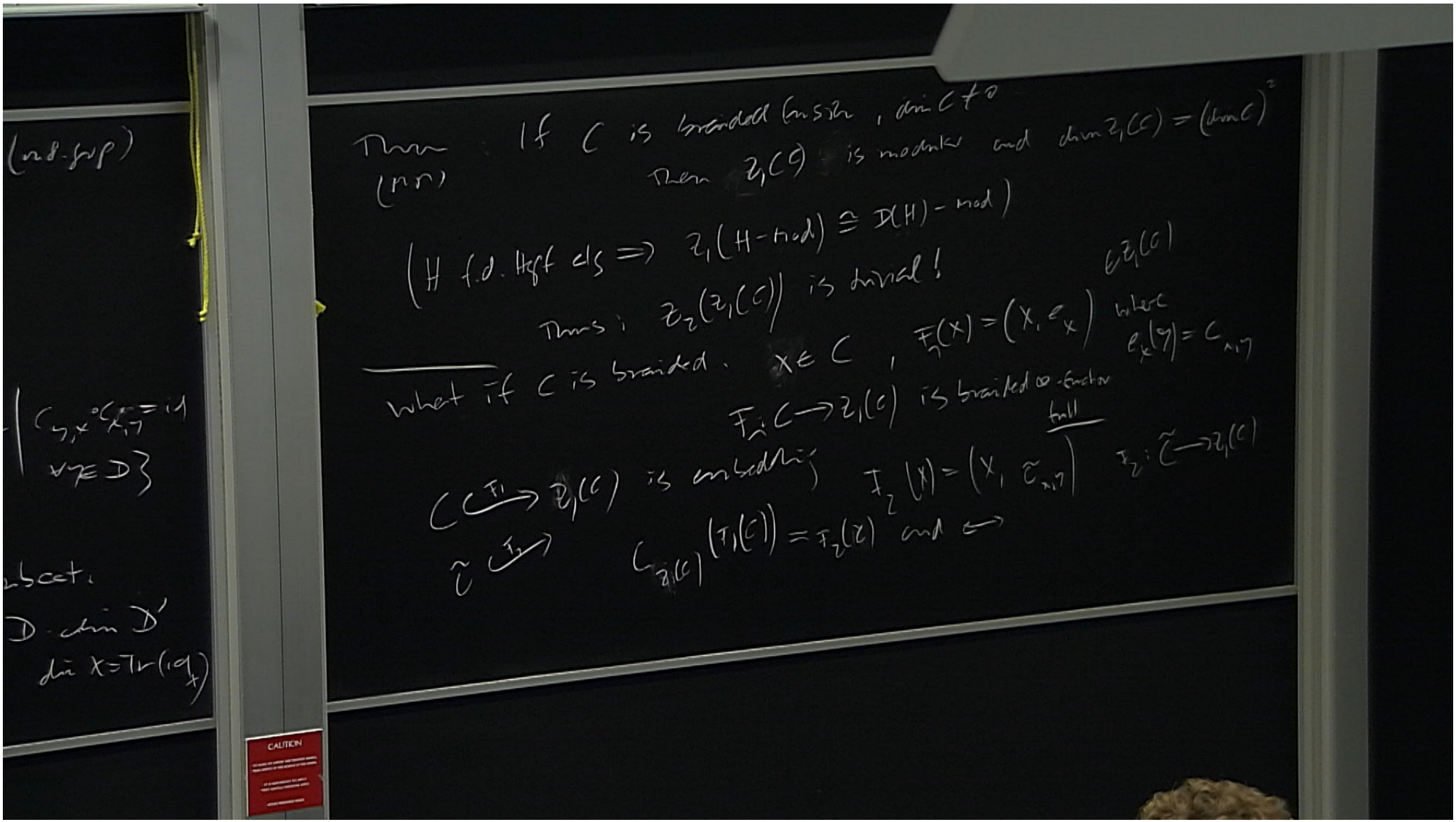
C ↪ Z1(C) is embedding
F2(X) = (X, c̃_{X,Y})
F2: C̃ → Z1(C)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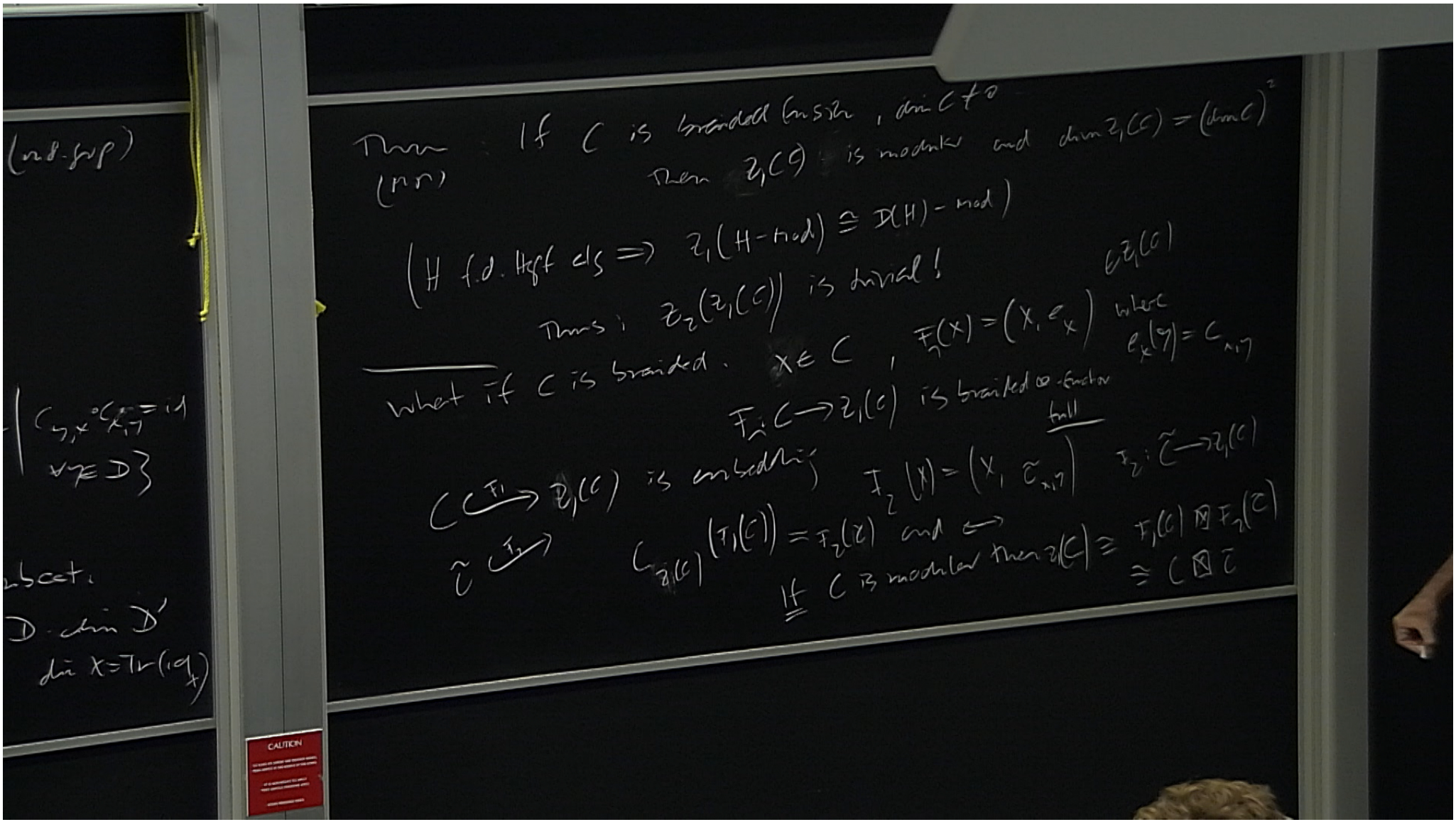
F1: C → Z1(C) is braided ⊗-functor
full
C ↪ Z1(C) is embedding
F2(X) = (X, c̃_{X,Y})
F2: C̃ → Z1(C)
C̃ ↪
C_{Z1(C)}(F1(C)) = F2(C̃) and ↔
(mod-grp)
C_{Y,X} ∘ C_{X,Y} = id
∀Y ∈ D}
D-dim D'
dim X = Tr(id_X)
CAUTION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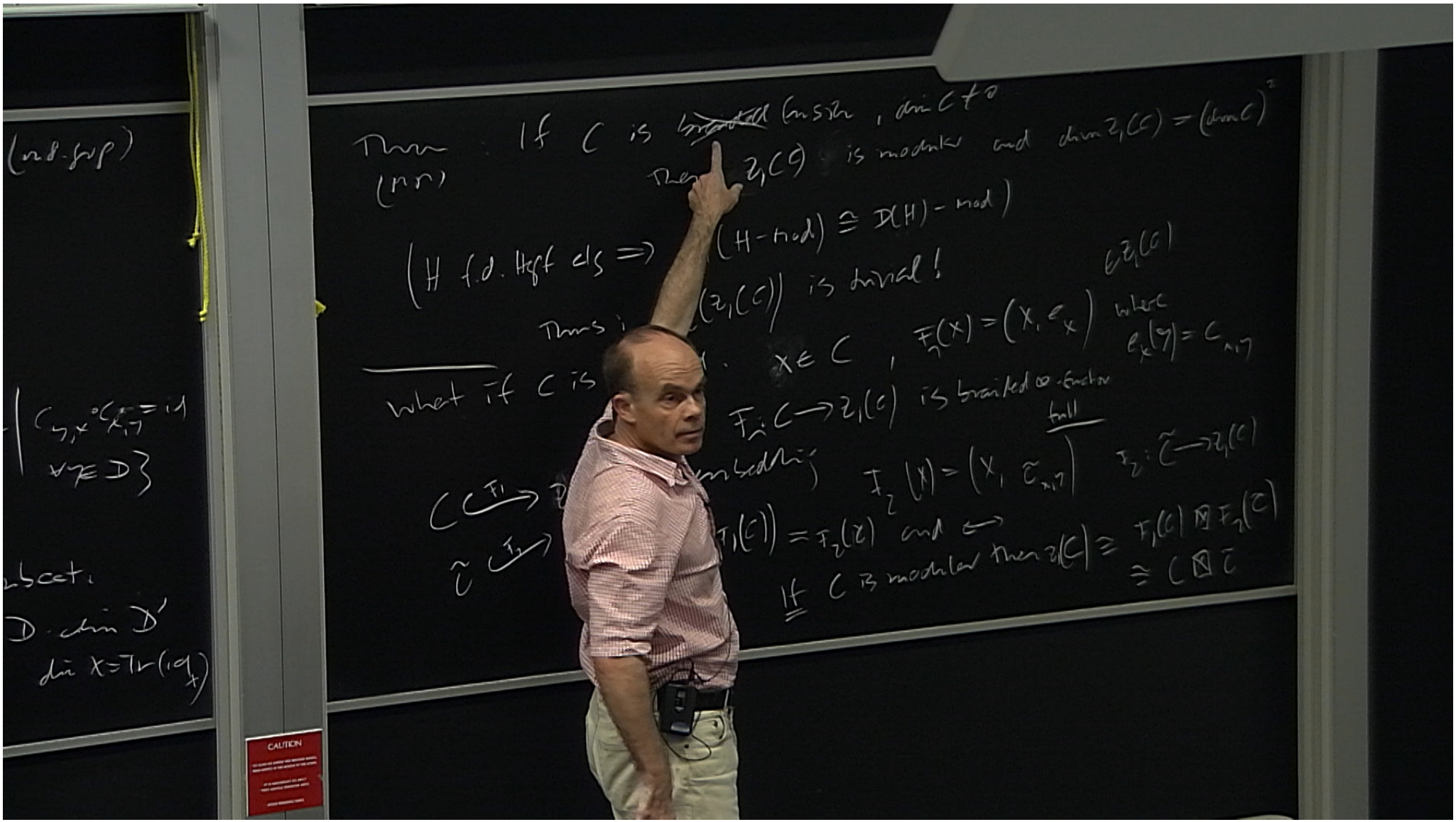
F2(X) = (X, c̃_{X,Y})
F2: C̃ → Z1(C)
C̃ ↪
C_{Z1(C)}(F1(C)) = F2(C̃) and ⟷
If C is modular then Z1(C) ≅ F1(C) ⊠ F2(C̃)
≅ C ⊠ C̃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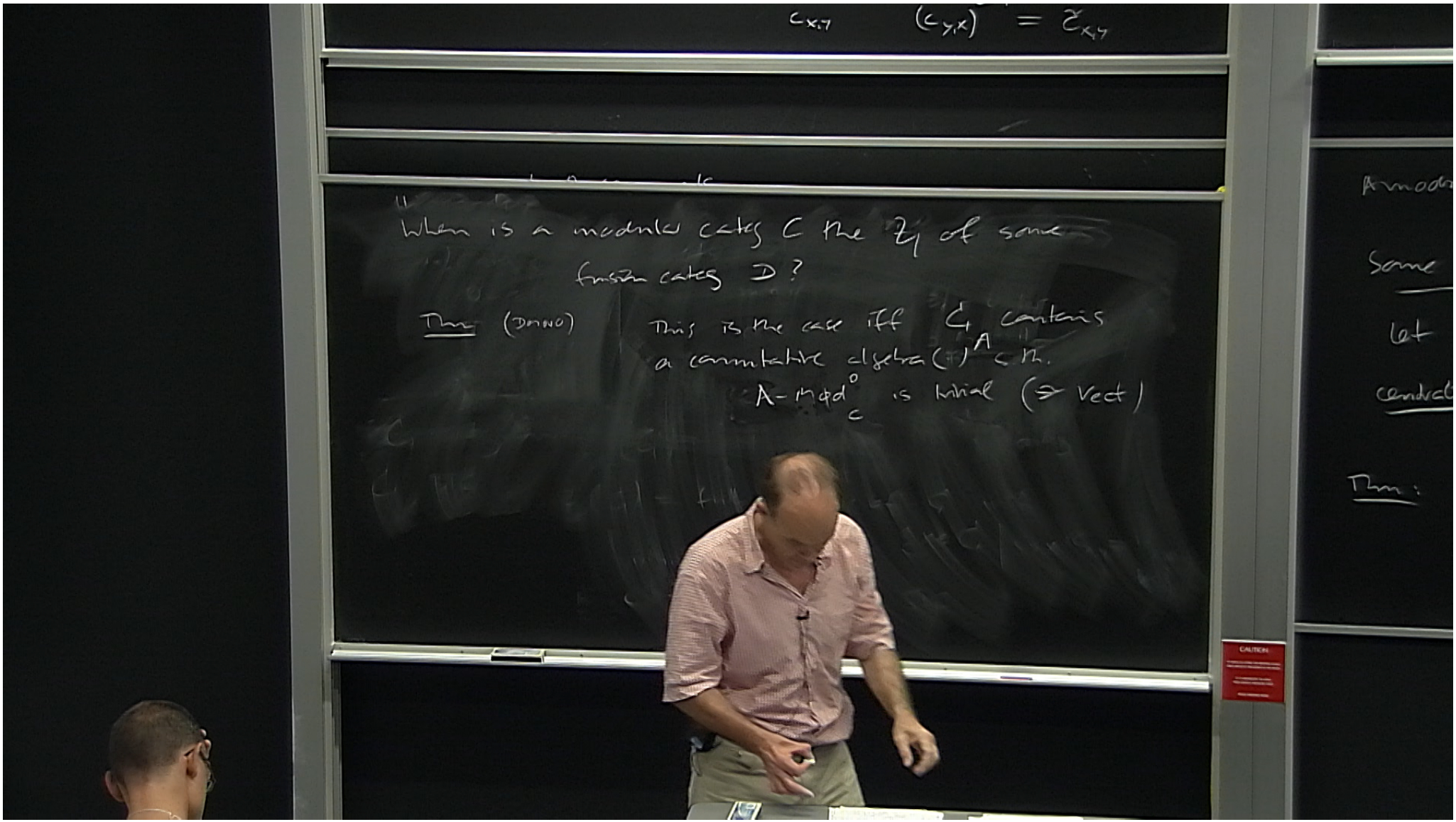
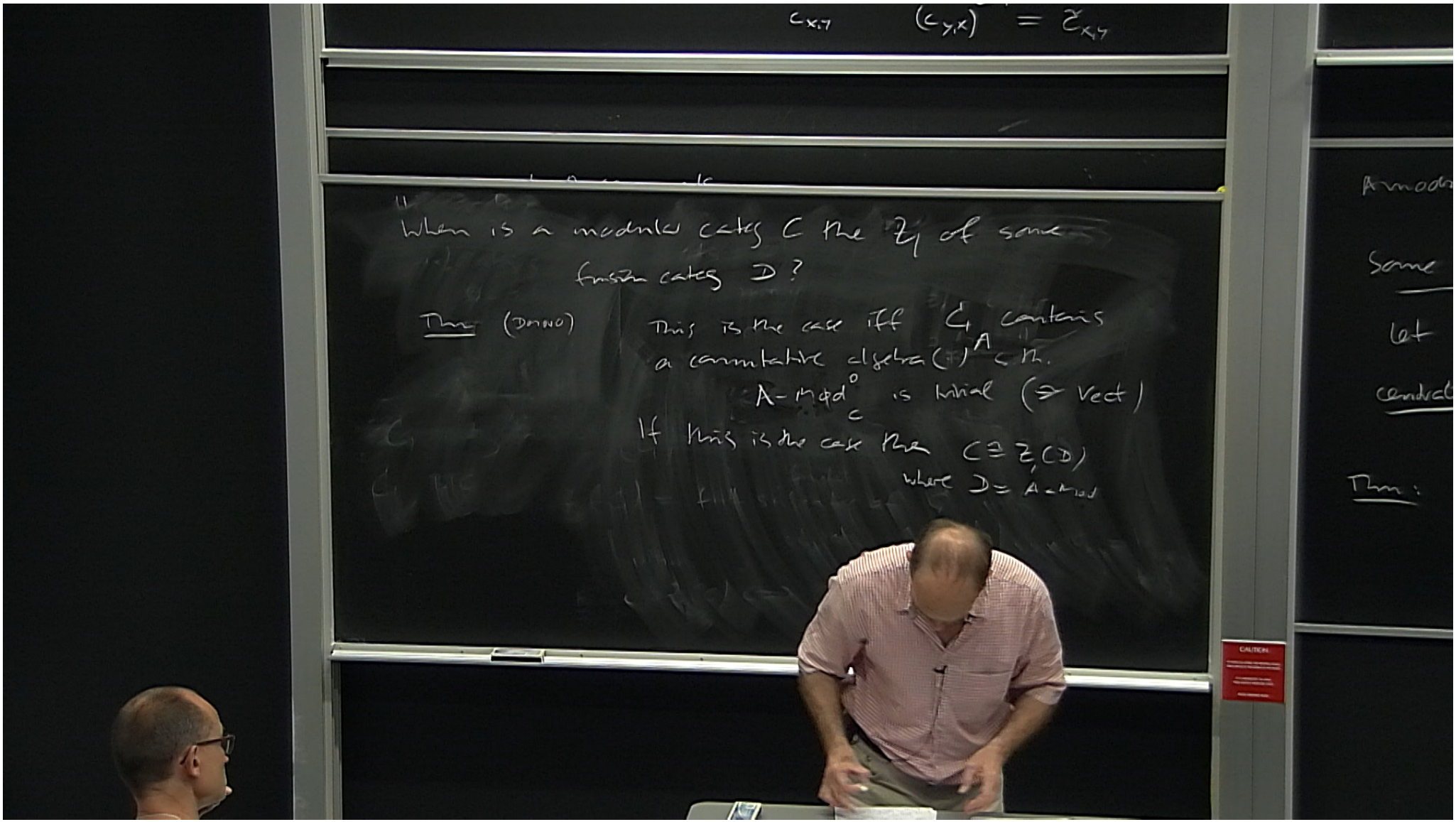
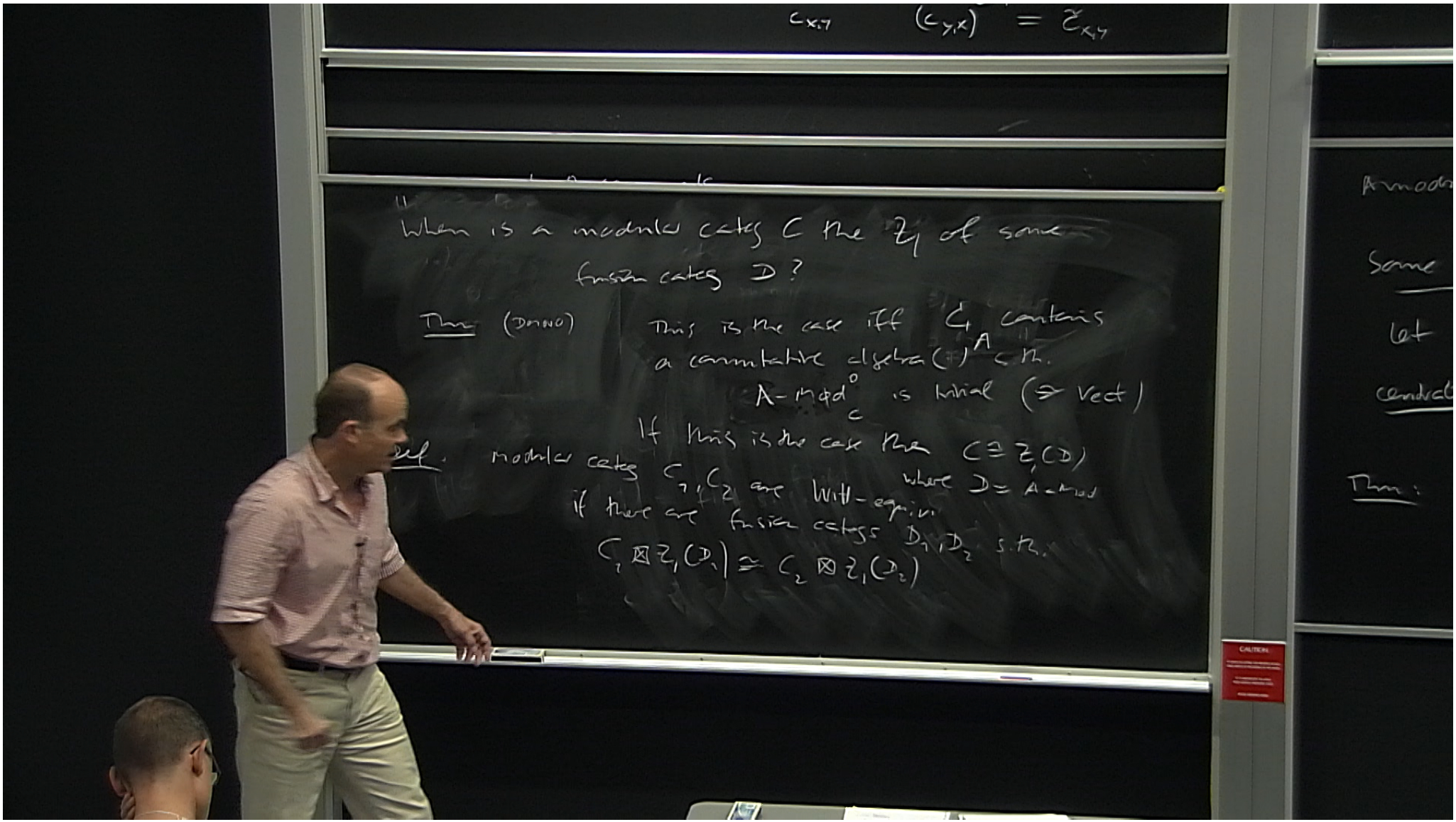
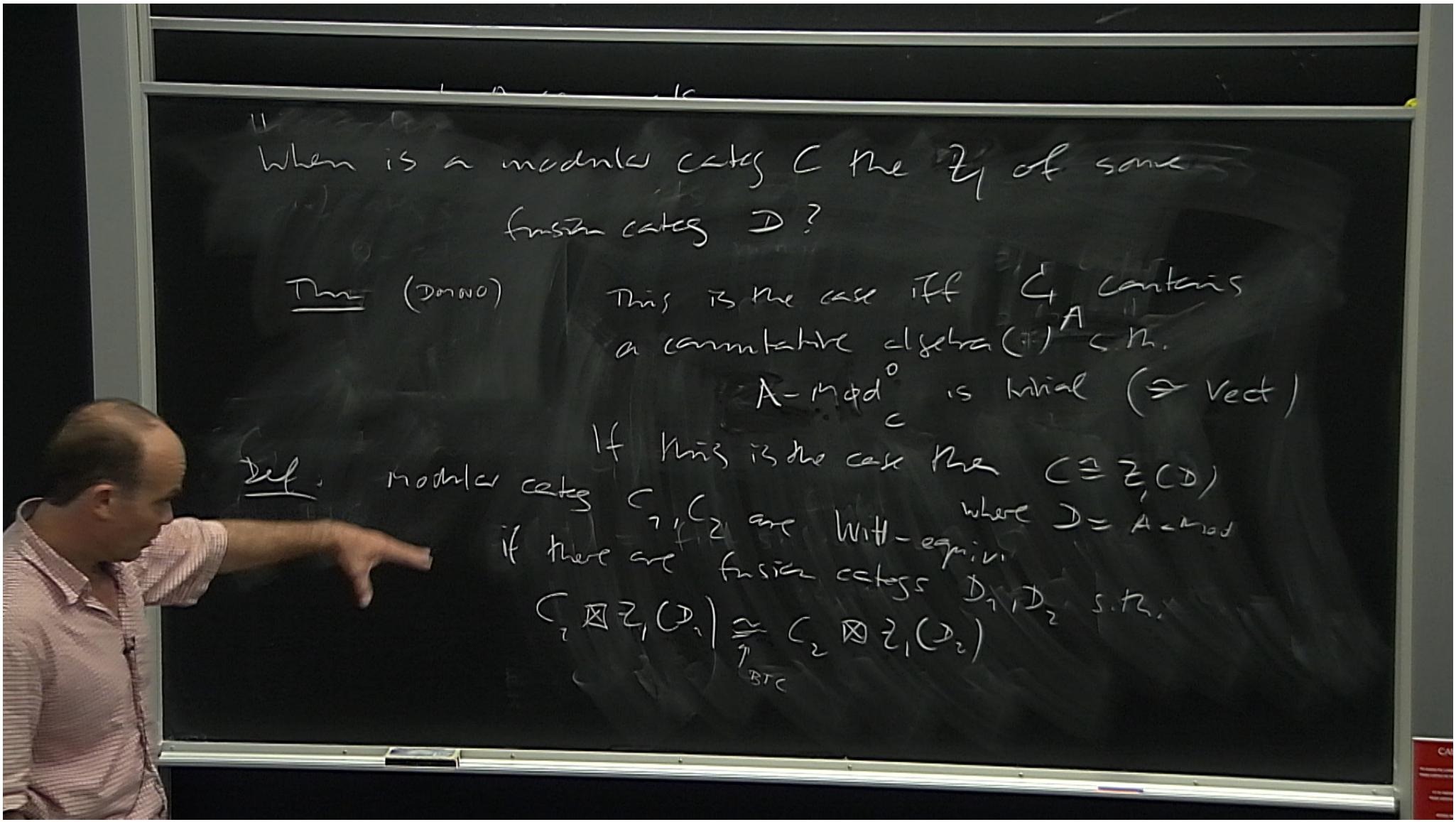
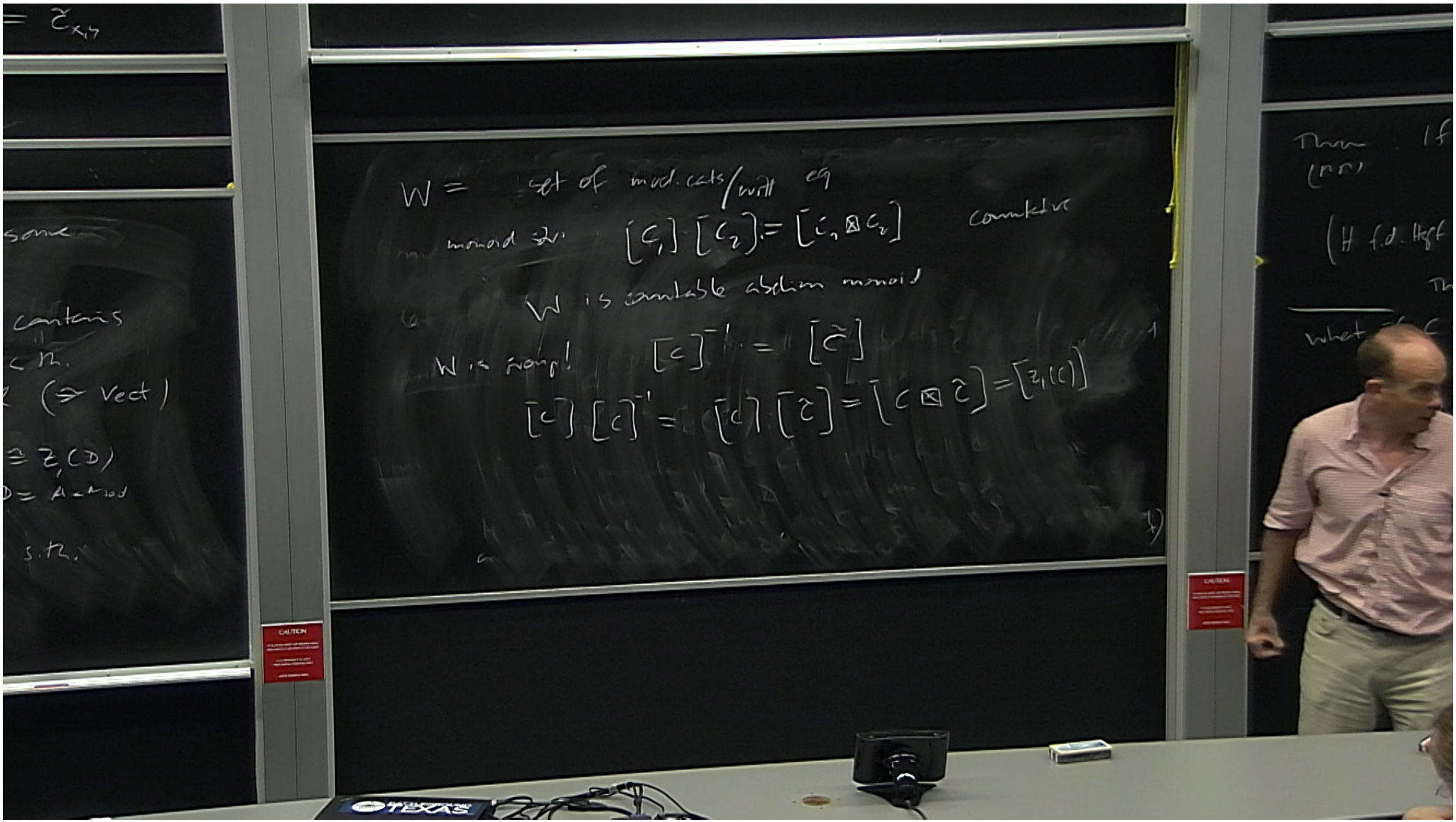
Then Z1(C) is modular and dim Z1(C) = (dim C)^2
(H f.d. Hopf alg ⇒ (H-mod) ≅ D(H)-mod)
Thus (Z1(C)) is trivial!
What if C is
x ∈ C, F1(x) = (x, ex) where ex(y) = cx,y
F1: C → Z1(C) is braided ∞-functor
full
F2(x) = (x, c̃x,y)
F2: C̃ → Z1(C)
F1(C)) = F2(C̃) and ⟷
If C is modular then Z1(C) ≅ F1(C) ⊠ F2(C̃)
≅ C ⊠ C̃
CAUTION

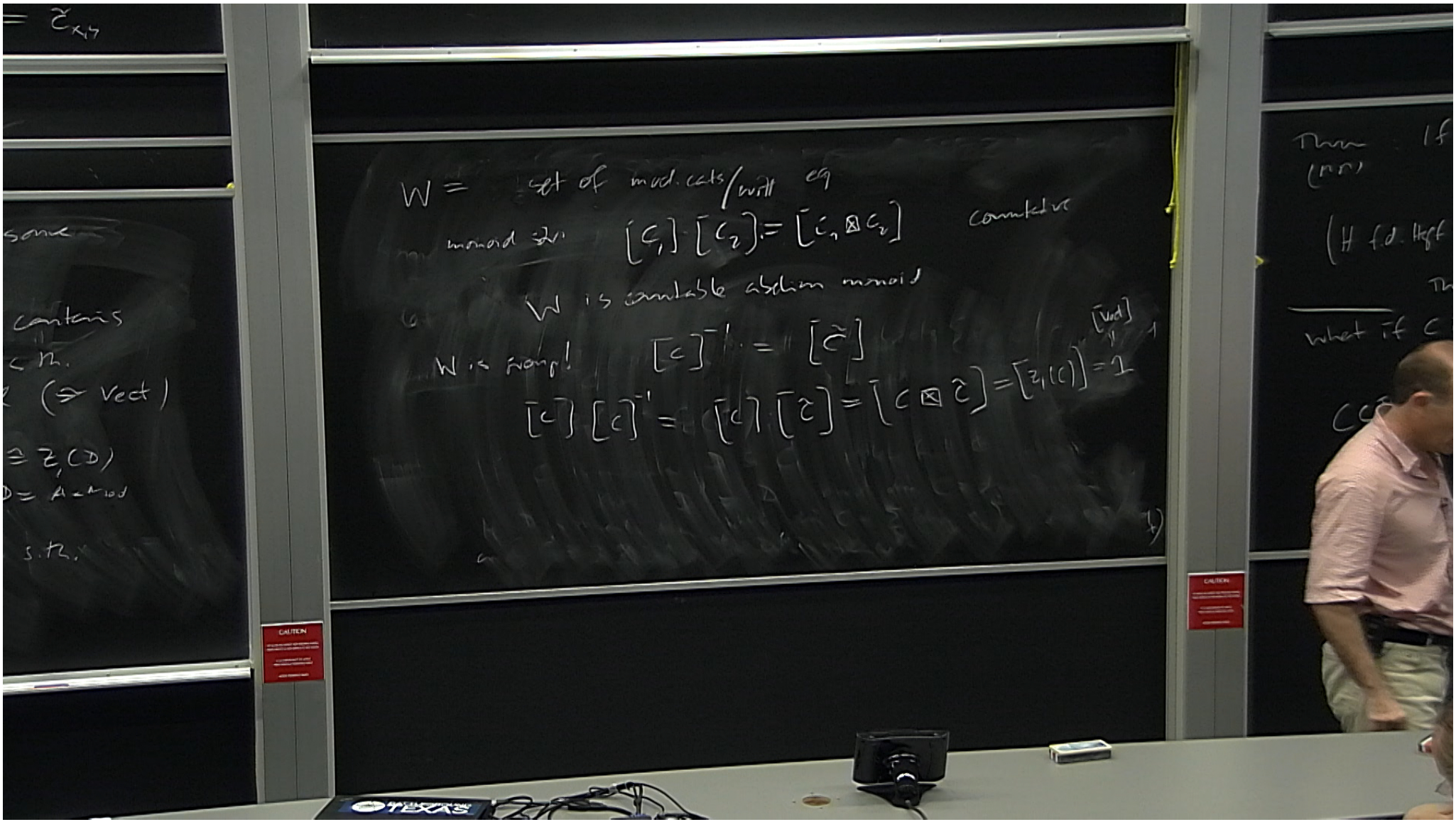

When is a modular categ C the Z of some
fusion categ D?
Thm (DMNO) This is the case iff C contains
a commutative algebra ( )^A c.th.
A-mod^0_C is trivial (≃ Vect)
Some
let
Thm:


When is a modular categ C the Z of some
fusion categ D?
Thm (Drinfeld)
This is the case iff C contains
a commutative algebra A ⊂ 1.
A-mod^0_C is trivial (≃ Vect)
If this is the case then C ≃ Z(D)
where D = A-mod


When is a modular categ C the Z1 of some
fusion categ D?
Thm (Domino) This is the case iff C contains
a commutative algebra ( )^A s.th.
A-mod^0_C is trivial (≅ Vect)
If this is the case then C ≅ Z1(D)
where D = A-mod
Def. modular categ C1, C2 are Witt-equiv.
if there are fusion categs D1, D2 s.th.
C1 ⊠ Z1(D1) ≅ C2 ⊠ Z1(D2)
Some
let
Thm:


When is a modular categ $\mathcal{C}$ the $\mathcal{Z}_1$ of some fusion categ $\mathcal{D}$?
Thm (DMNO) This is the case iff $\mathcal{C}$ contains a commutative algebra $(A) \subset \mathbb{1}$.
$A\text{-mod}^0_{\mathcal{C}}$ is trivial ($\cong$ Vect)
If this is the case then $\mathcal{C} \cong \mathcal{Z}_1(\mathcal{D})$ where $\mathcal{D} = A\text{-mod}$
Def. modular categ $\mathcal{C}_1, \mathcal{C}_2$ are Witt-equiv. if there are fusion categs $\mathcal{D}_1, \mathcal{D}_2$ s.th.
$\mathcal{C}_1 \boxtimes \mathcal{Z}_1(\mathcal{D}_1) \cong \mathcal{C}_2 \boxtimes \mathcal{Z}_1(\mathcal{D}_2)$
BTC


W = set of mod. cats/with eq
monoid str. [C1] [C2] = [C1 ⊠ C2] commutative
W is countable abelian monoid
W is group! [C]^-1 = [C̃]
[C] [C]^-1 = [C] [C̃] = [C ⊠ C̃] = [Z1(C)]
some
contains
(≅ Vect)
s.th.
Thm
(H f.d. Hopf
what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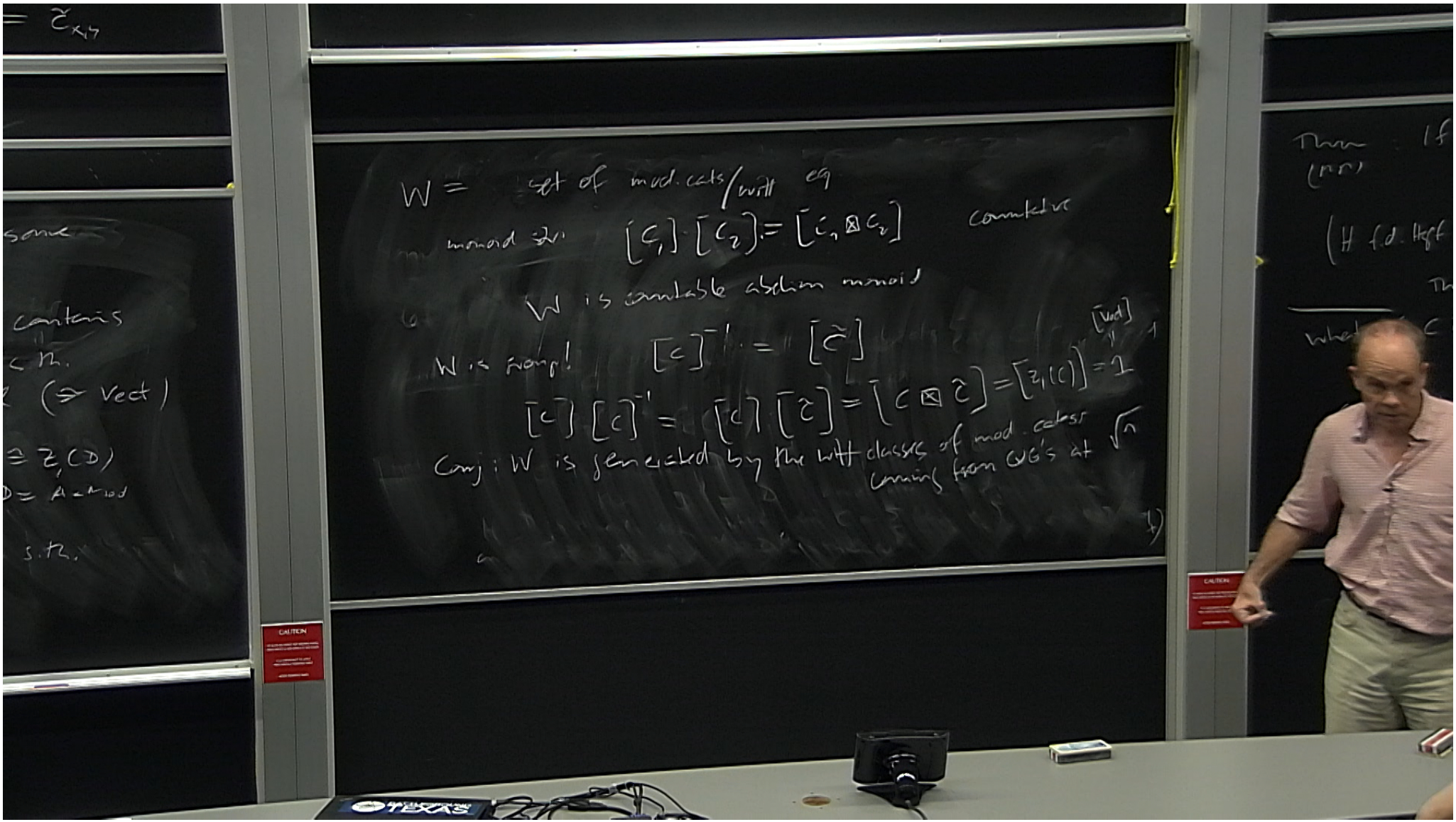
[Vect]
contains
(≅ Vect)
s.th.
Thm
what if C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[Vect]
Conj: W is generated by the Witt classes of mod. cats coming from QG's at √n
contains
(≅ Vect)
s.th.

When is a modular categ $\mathcal{C}$ the $\mathcal{Z}_1$ of some fusion categ $\mathcal{D}$?

<u>Thm</u> (Davydov) This is the case iff $\mathcal{C}$ contains a commutative algebra $(A) \in \mathcal{M}$.

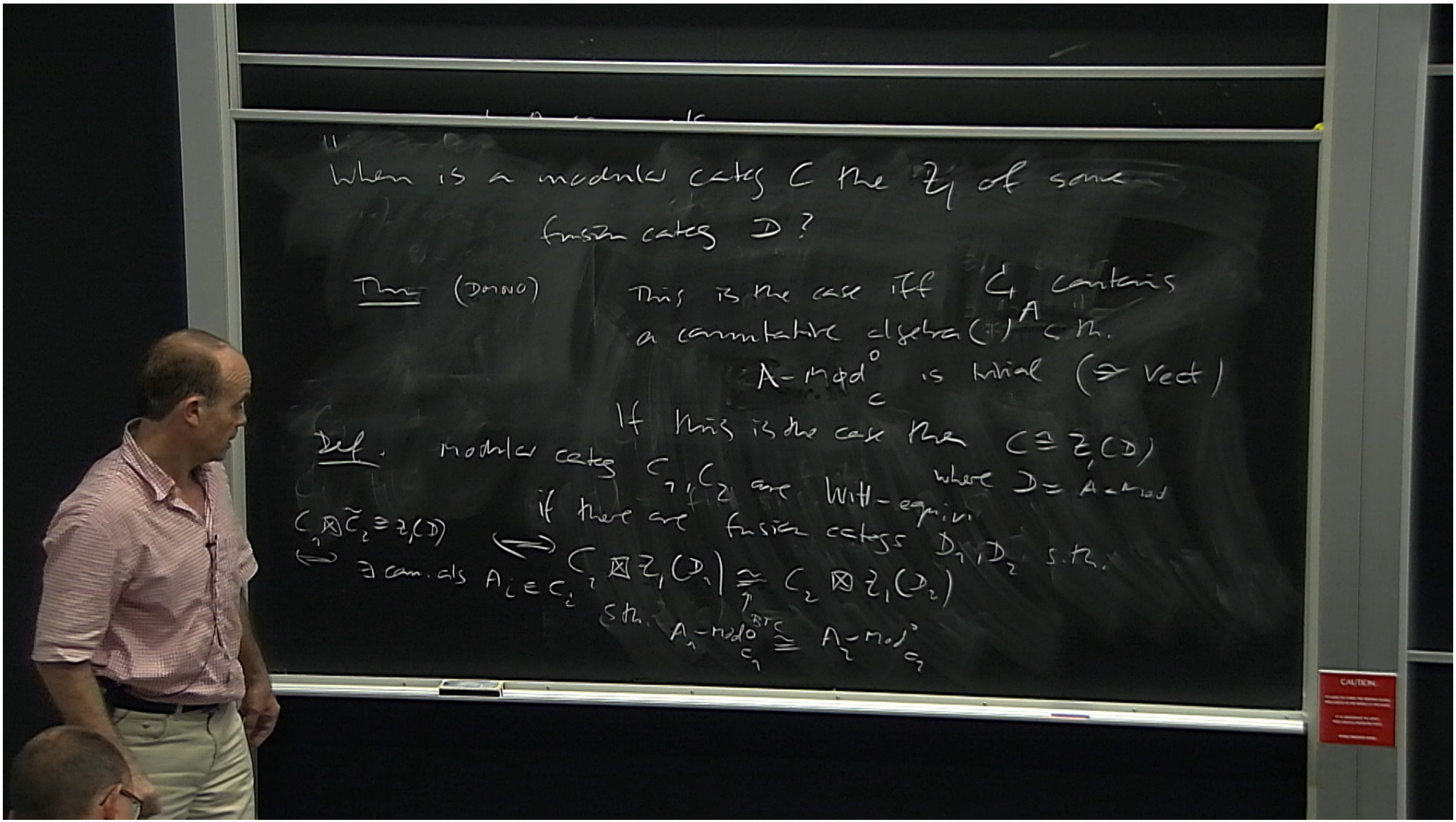
$A\text{-mod}^0_{\mathcal{C}}$ is trivial ($\cong$ Vect)

If this is the case then $\mathcal{C} \cong \mathcal{Z}_1(\mathcal{D})$ where $\mathcal{D} = A\text{-mod}$

<u>Def.</u> modular categ $\mathcal{C}_1, \mathcal{C}_2$ are Witt-equiv. if there are fusion categs $\mathcal{D}_1, \mathcal{D}_2$ s.th.

$$\mathcal{C}_1 \boxtimes \mathcal{Z}_1(\mathcal{D}_1) \underset{\text{BTC}}{\cong} \mathcal{C}_2 \boxtimes \mathcal{Z}_1(\mathcal{D}_2)$$

$\mathcal{C}_1 \boxtimes \tilde{\mathcal{C}}_2 \cong \mathcal{Z}_1(\mathcal{D})$


When is a modular categ $\mathcal{C}$ the $\mathcal{Z}_1$ of some fusion categ $\mathcal{D}$?
Thm (DMNO)
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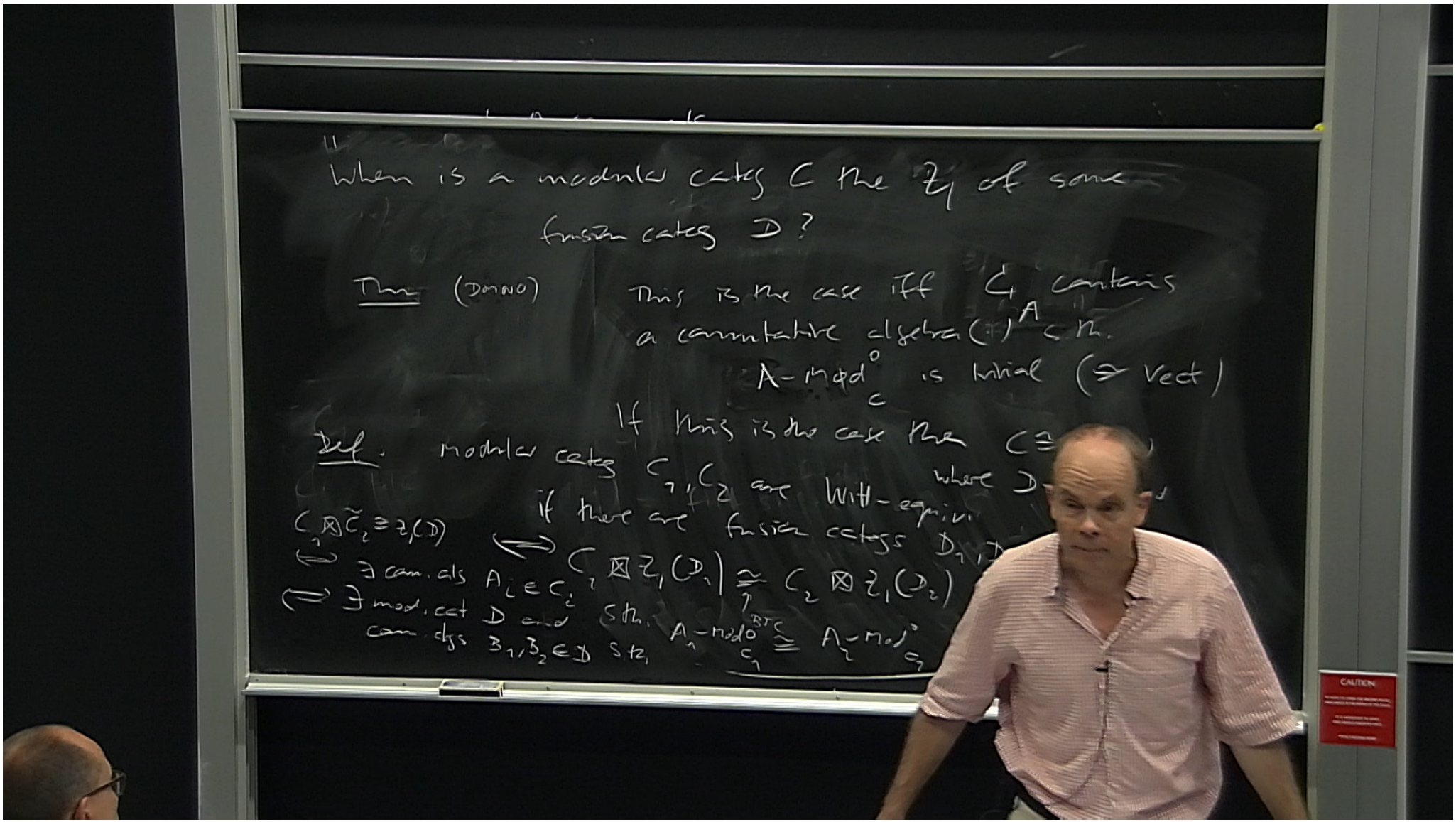
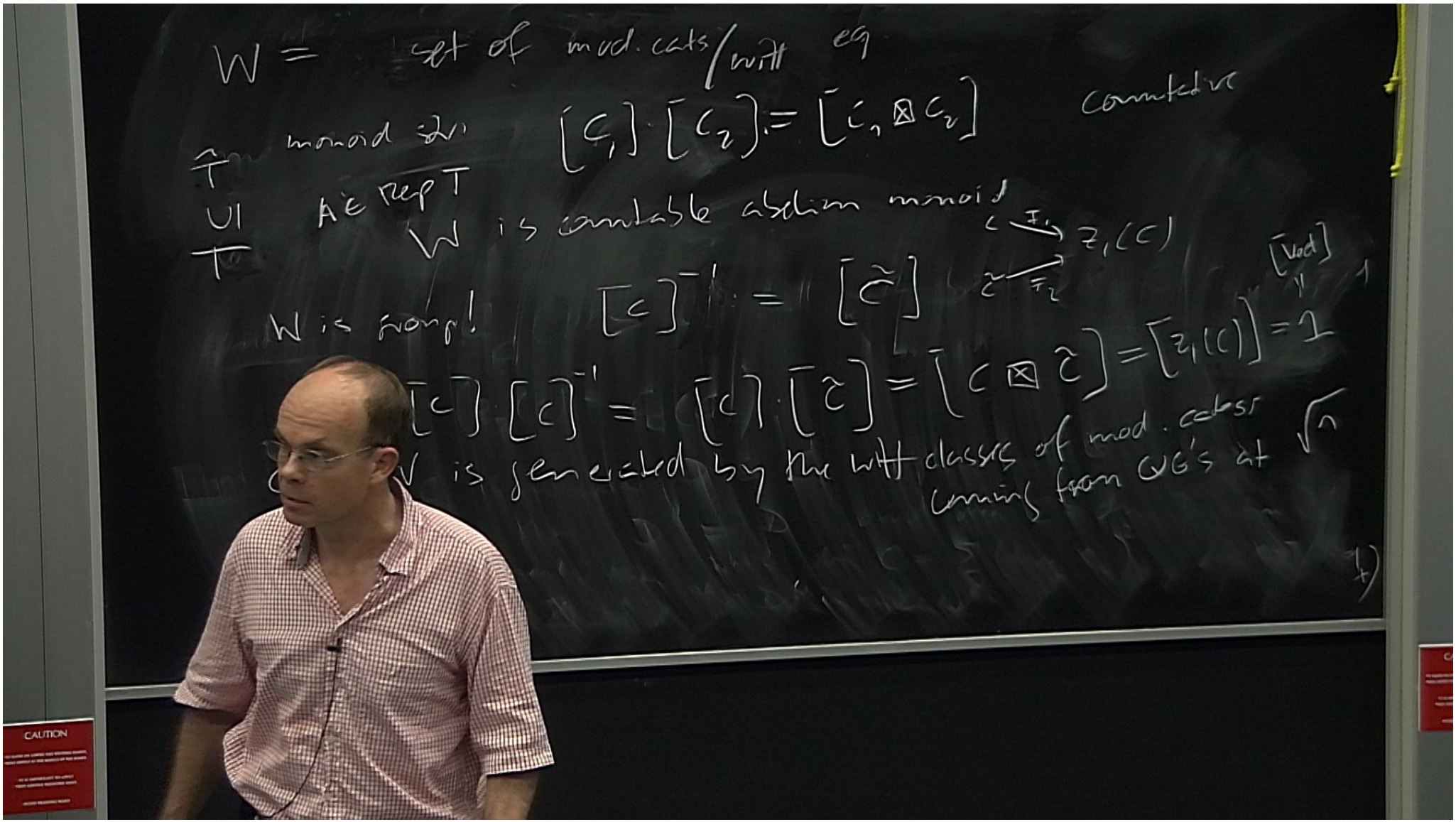
$A\text{-mod}^0_{\mathcal{C}}$ is trivial ($\cong$ Vect)
If this is the case then $\mathcal{C} \cong \mathcal{Z}_1(\mathcal{D})$ where $\mathcal{D} = A\text{-mod}$
Def. modular categ $\mathcal{C}_1, \mathcal{C}_2$ are Witt-equiv. if there are fusion categs $\mathcal{D}_1, \mathcal{D}_2$ s.th.
$\mathcal{C}_1 \boxtimes \mathcal{C}_2 \cong \mathcal{Z}_1(\mathcal{D})$
$\Longleftrightarrow$ $\mathcal{C}_1 \boxtimes \mathcal{Z}_1(\mathcal{D}_1) \cong \mathcal{C}_2 \boxtimes \mathcal{Z}_1(\mathcal{D}_2)$
$\Longleftrightarrow$ $\exists$ com. als $A_i \in \mathcal{C}_i$ s.th. $A_1\text{-mod}^0_{\mathcal{C}_1} \cong A_2\text{-mod}^0_{\mathcal{C}_2}$
BTC


When is a modular categ C the Z1 of some
fusion categ D?
Thm (DGNO)
This is the case iff C contains
a commutative algebra (1) A ⊂ 1.
A-mod^0_C is trivial (≅ Vect)
If this is the case then C ≅
Def. modular categ C1, C2 are Witt-equiv.
if there are fusion categs D1,
C1 ⊠ C̃2 ≅ Z1(D)
⟺ C1 ⊠ Z1(D1) ≅ C2 ⊠ Z1(D2)
BTC
⟺ ∃ com. als A_i ∈ C_i s.th. A1-mod^0_C1 ≅ A2-mod^0_C2
⟺ ∃ mod. cat D and
com. algs B1, B2 ∈ D s.t.
CAUTION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monoid str.
[C1] · [C2] := [C1 ⊠ C2]
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T̂
∪|
T
A ∈ Rep T
W is countable abelian monoid
W is group!
[C]^-1 := [C̃]
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[Vect]
W is generated by the Witt classes of mod. cats coming from QG's at √n