

Title: The Hopf C*-algebraic quantum double models - symmetries beyond group theory

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Abstract: I will give an introduction to the Kitaev quantum double models for Hopf C*-algebras. To this end I will introduce a graphical tensor-network notation to represent the algebraic objects and axioms. Using this notation I will then present the vertex- and plaquette symmetries of the model and discuss their interaction and the excitation structure they give rise to.

The Hopf C*-algebraic quantum double models - symmetries beyond group theory

Andreas Bauer

Dahlem Center of Complex Quantum Systems - FU Berlin

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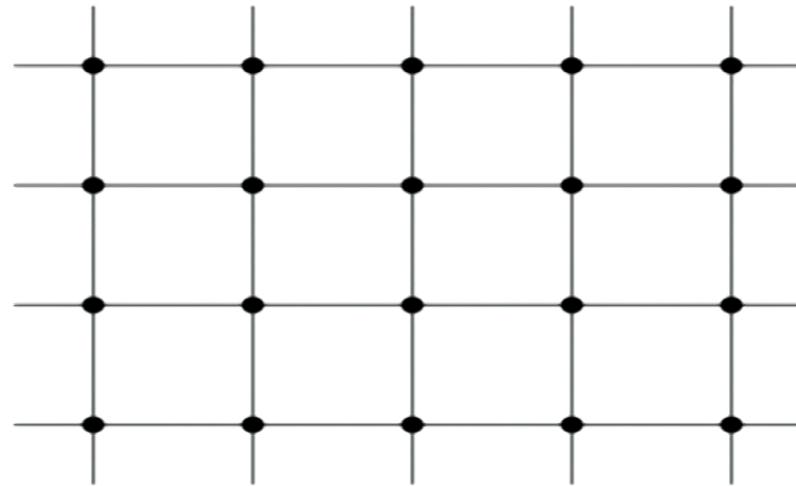


Outline

- ▶ String-net picture of the toric code
- ▶ Hopf C*-algebras in tensor network notation
- ▶ The plaquette and vertex symmetries
- ▶ Excitations and quantum double algebra
- ▶ String- and ribbon operators

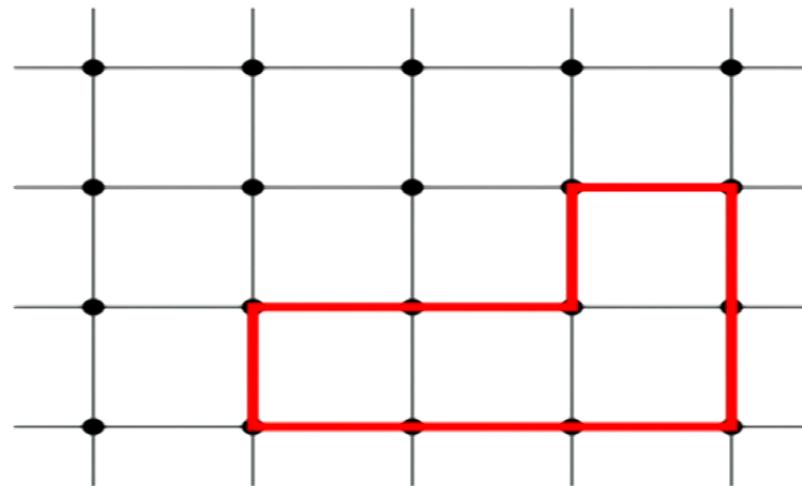
String-net picture of the toric code

- *Groundstate:* superposition of all closed loop configurations



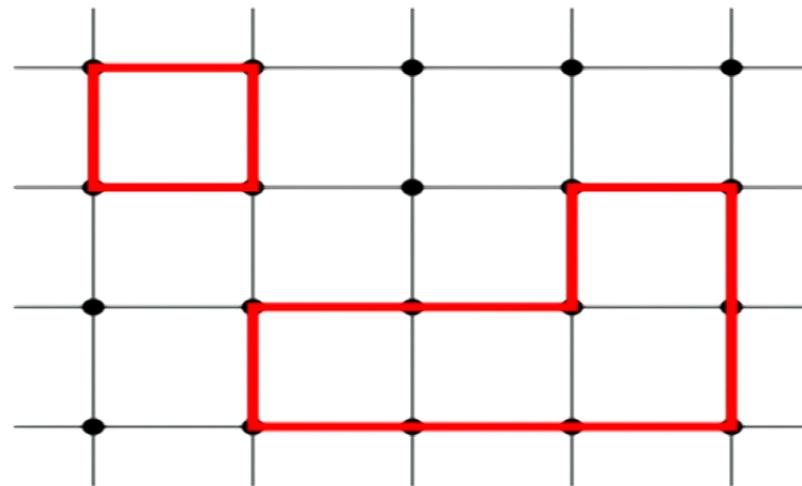
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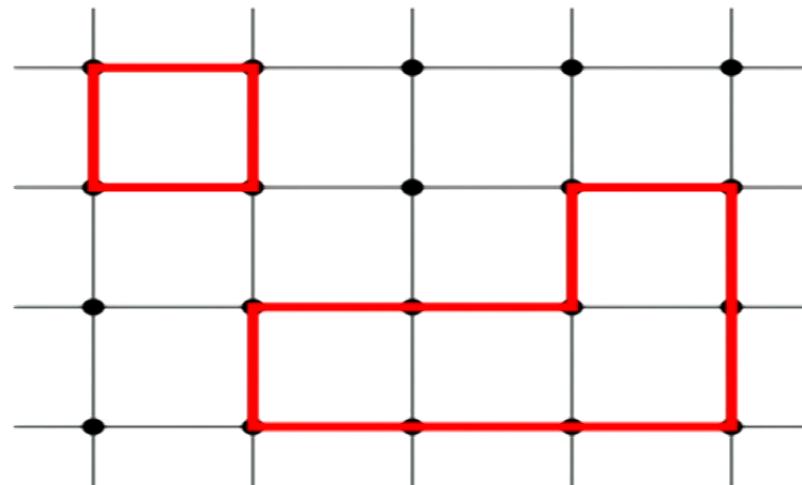
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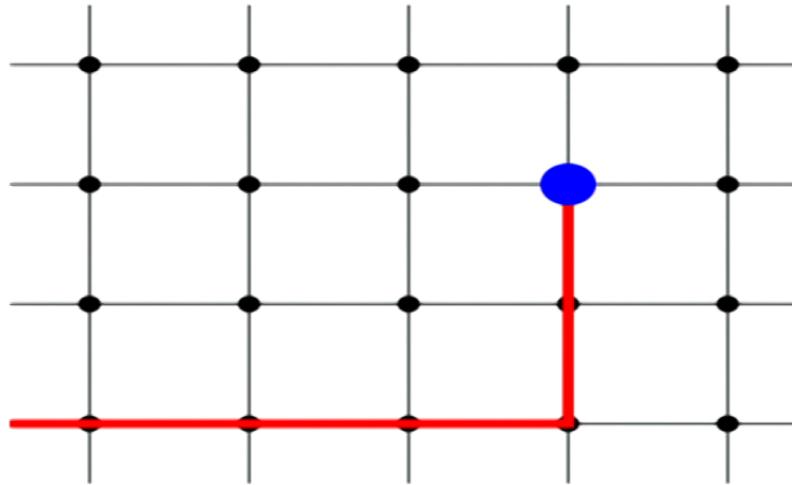
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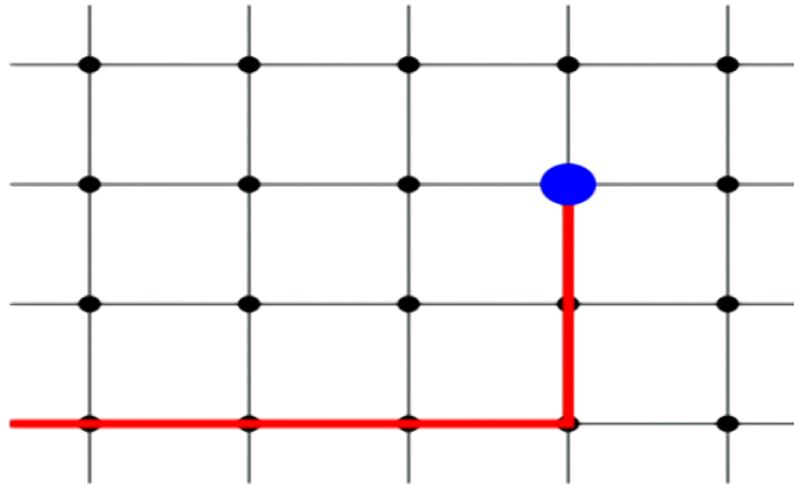
String-net picture of the toric code

- *Groundstate:* superposition of all closed loop configurations
- *Excitations:* **electric:** loop ends



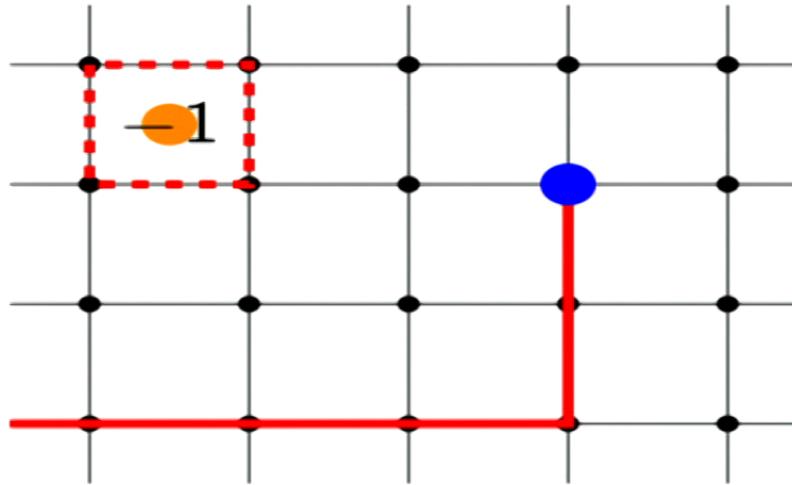
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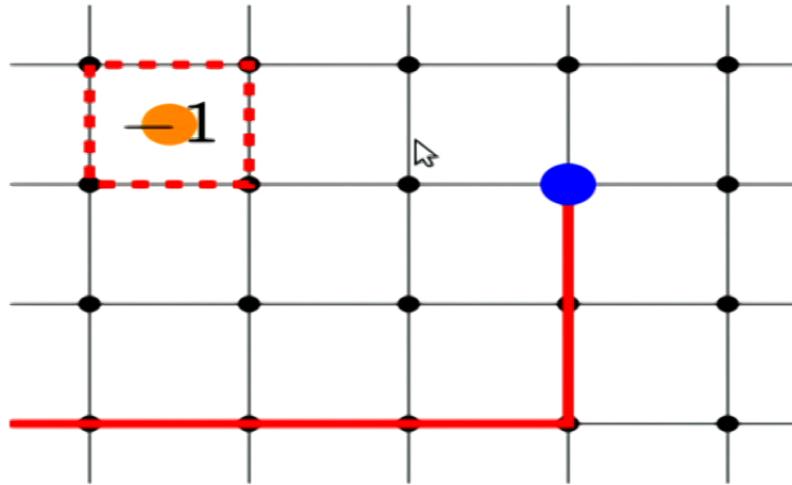
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- ▶ *Groundstate:* superposition of all closed loop configurations
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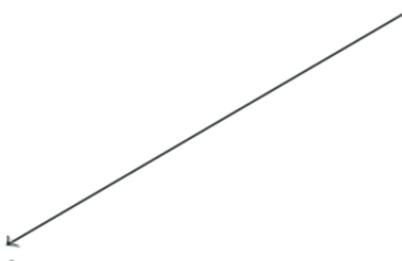
Generalisations

Toric code: \mathbb{Z}_2

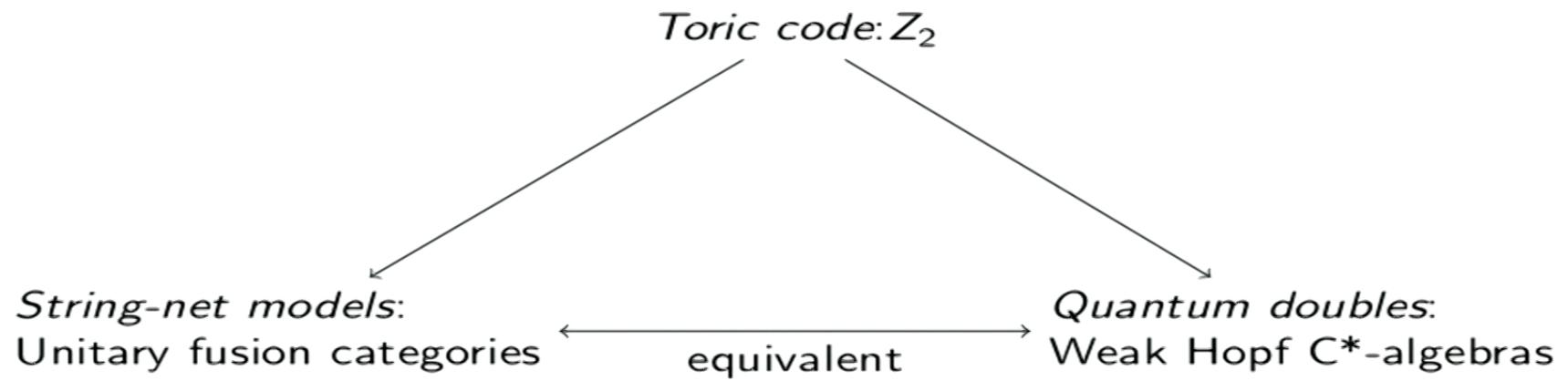
Generalisations

Toric code: Z_2

String-net models:
Unitary fusion categories



Generalisations



Hopf C*-algebras – objects

$$\eta : \mathbb{C}^1 \rightarrow \mathbb{C}^H$$



$$\mu : \mathbb{C}^{H \times H} \rightarrow \mathbb{C}^H$$



Hopf C*-algebras – objects

$$\eta : \mathbb{C}^1 \rightarrow \mathbb{C}^H$$



$$\mu : \mathbb{C}^{H \times H} \rightarrow \mathbb{C}^H$$



$$\epsilon : \mathbb{C}^H \rightarrow \mathbb{C}^1$$



$$\Delta : \mathbb{C}^H \rightarrow \mathbb{C}^{H \times H}$$



$$S : \mathbb{C}^H \rightarrow \mathbb{C}^H$$



Hopf C*-algebras – objects

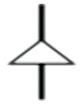
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$$\mu : \mathbb{C}^{H \times H} \rightarrow \mathbb{C}^H$$



$$*: \mathbb{C}^H \rightarrow \mathbb{C}^H$$



$$\epsilon : \mathbb{C}^H \rightarrow \mathbb{C}^1$$



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Hopf C*-algebras – axioms

$$\epsilon \circ \mu = \epsilon \otimes \epsilon$$



Hopf C*-algebras – axioms

$$\epsilon \circ \mu = \epsilon \otimes \epsilon$$

$$\Delta \circ \eta = \eta \otimes \eta$$

$$\epsilon \circ \eta = 1$$

$$\begin{aligned} \text{Diagram 1: } & \text{A black dot at the top is connected by two lines to a white circle at the bottom. This is followed by an equals sign, then two separate black dots at the top, each connected by a single line to a white circle at the bottom.} \\ \text{Diagram 2: } & \text{A black dot at the top is connected by two lines to a white circle at the bottom. This is followed by an equals sign, then two separate white circles at the bottom, each connected by a single line to a black dot at the top.} \\ \text{Diagram 3: } & \text{A black dot at the top is connected by a single line to a white circle at the bottom.} \end{aligned}$$

Hopf C*-algebras – axioms

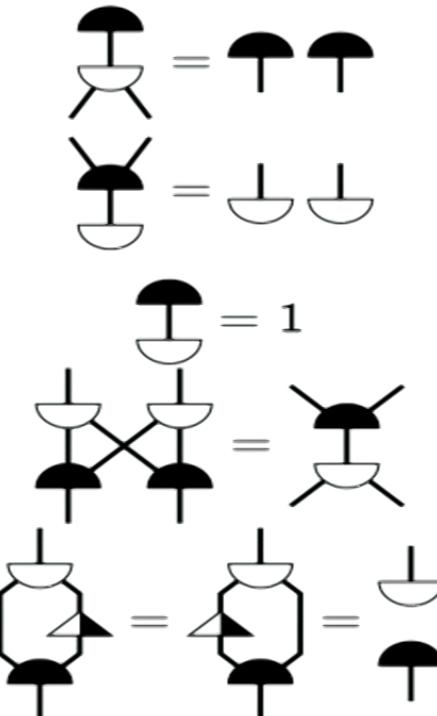
$$\epsilon \circ \mu = \epsilon \otimes \epsilon$$

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$$\epsilon \circ \eta = 1$$

$$(\mu \otimes \mu) \circ (\mathbb{1} \otimes \text{swap} \otimes \mathbb{1}) \\ \circ (\Delta \otimes \Delta) = \Delta \circ \mu$$

$$\mu \circ (\mathbb{1} \otimes S) \circ \Delta \\ = \mu \circ (S \otimes \mathbb{1}) \circ \Delta = \eta \circ \epsilon$$



Hopf C*-algebras – axioms

$$\epsilon \circ \mu = \epsilon \otimes \epsilon$$

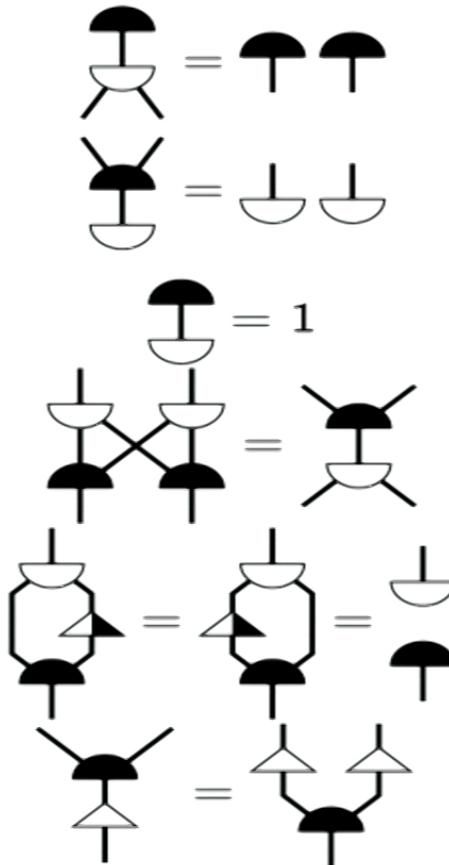
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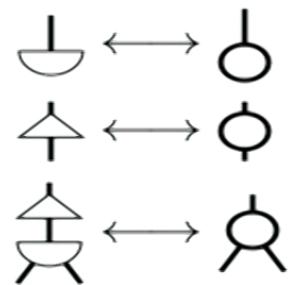
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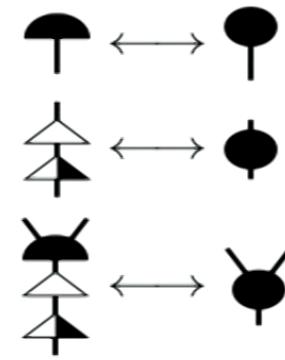
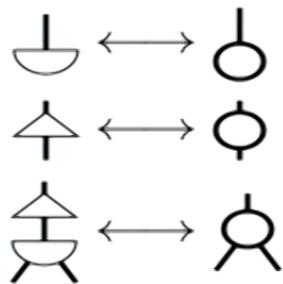
$$\Delta \circ * = (* \otimes *) \circ \Delta$$



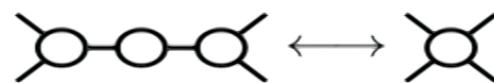
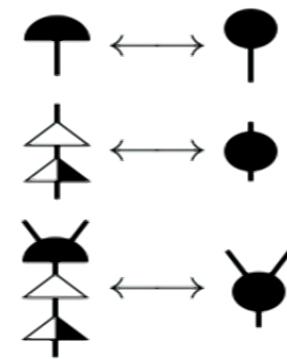
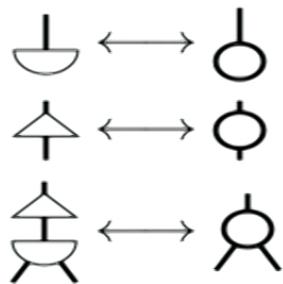
Hopf C*-algebras – cyclic form



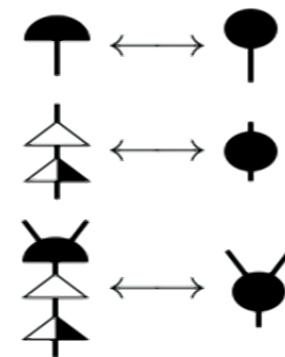
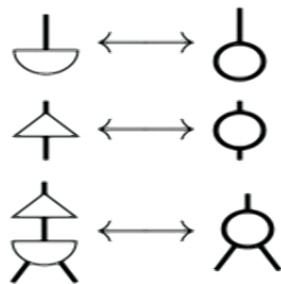
Hopf C*-algebras – cyclic form



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Hopf C*-algebras – cyclic form



Hopf C*-algebras as symmetries



symmetry operations



particles

Hopf C*-algebras as symmetries

	symmetry operations		particles
	trivial symmetry opera- tion for all particles		trivial particle for all symmetry opera- tions

Hopf C*-algebras as symmetries

\circ	symmetry operations	\bullet	particles
$\circ-$	trivial symmetry operation for all particles	$\bullet-$	trivial particle for all symmetry operations
$- \circ$	inverse adjoint symmetry operation adjoint particle	$- \bullet$	adjoint anti-particle adjoint symmetry operations

Hopf C*-algebras as symmetries

	symmetry operations		particles
	trivial symmetry operation for all particles		trivial particle for all symmetry operations
	inverse adjoint symmetry operation adjoint particle		adjoint anti-particle adjoint symmetry operations
	concatenation of symmetry operations copying of a particle		fusion of particles copying a symmetry

Hopf C*-algebras – cyclic form



$$\begin{array}{c} e \quad d \\ \diagdown \quad \diagup \\ \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \\ a \quad b \quad c \end{array} = \begin{array}{c} e \\ \diagup \quad \diagdown \\ a \quad b \quad c \quad d \end{array}$$

Hopf C*-algebras – cyclic form

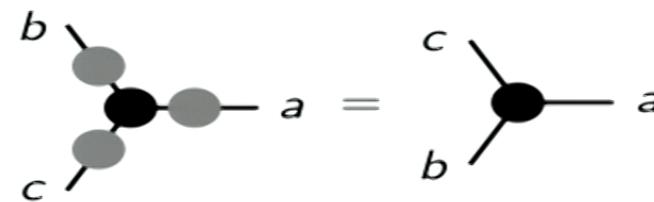
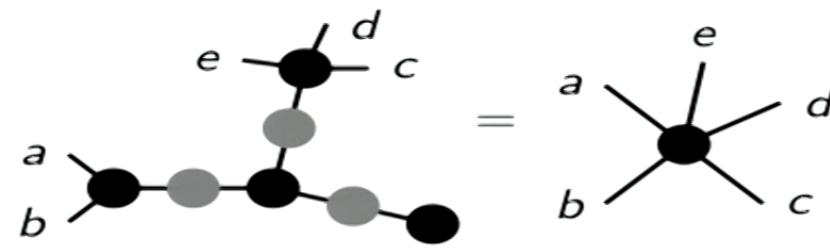
►

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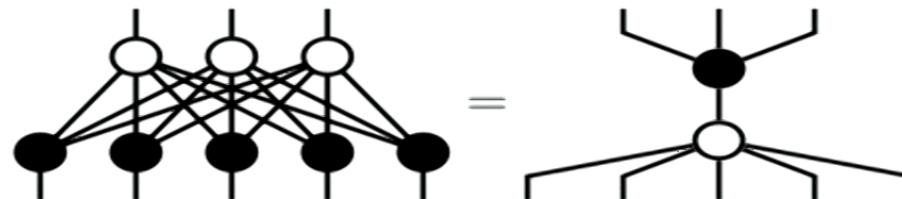
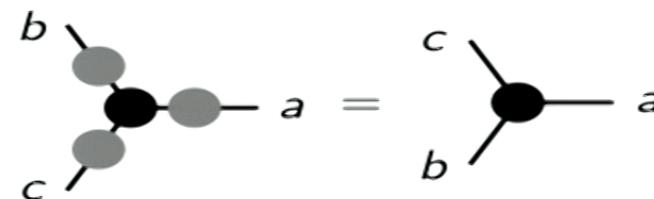
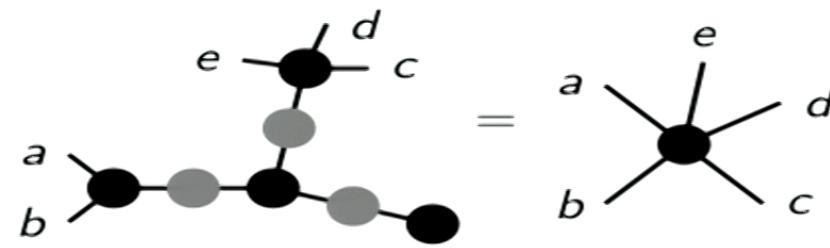
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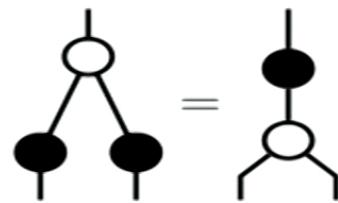
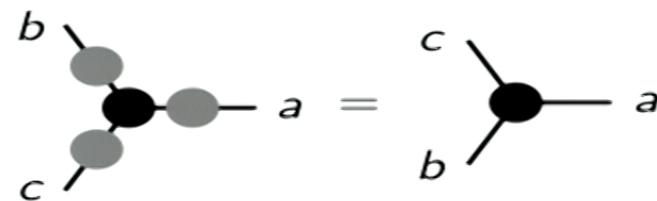
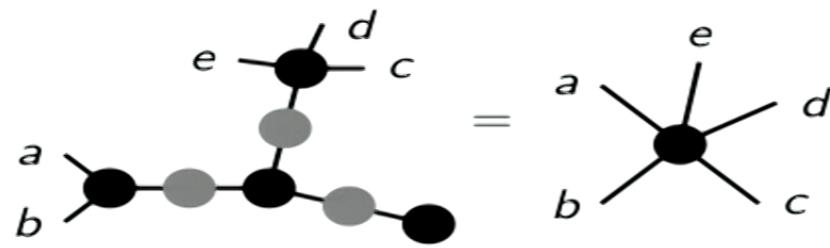
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Hopf C*-algebras – cyclic form



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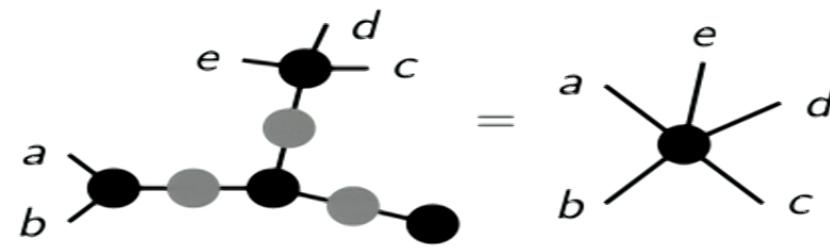


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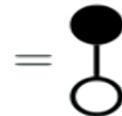
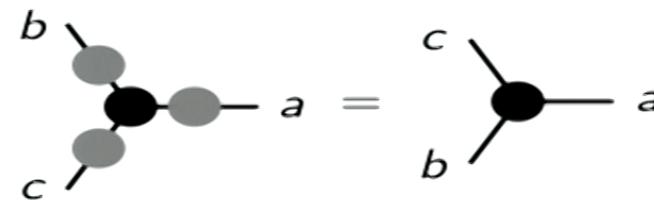
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Hopf C*-algebras – cyclic form

►



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Hopf C*-algebras as quantum groups

- ▶ Every group defines Hopf C*-algebra

$$\begin{array}{c} c \quad b \\ \diagup \quad \diagdown \\ d - \bullet - a \\ \diagdown \quad \diagup \\ e \end{array} = \delta_{a,b,c,d,e,\dots}$$

$$\begin{array}{c} c \quad b \\ \diagup \quad \diagdown \\ d - \circ - a \\ \diagdown \quad \diagup \\ e \end{array} = \delta_{abcde\dots,1}$$

Hopf C*-algebras as quantum groups

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- ▶ Hopf C*-algebra axioms can be seen as linearisation of group axioms

Hopf C*-algebras – representations

- ▶ Representation: homomorphism to matrix algebra

$$\boxed{R} : \quad \begin{array}{c} \text{Diagram showing two boxes labeled } R \text{ connected by a horizontal line, with vertical lines extending upwards from each box.} \\ = \\ \text{Diagram showing a single box labeled } R \text{ with a circular connection point above it, and vertical lines extending downwards from the box.} \end{array}$$

Hopf C*-algebras – representations

- ▶ Representation: homomorphism to matrix algebra

$$\boxed{R} : \quad \begin{array}{c} \text{---} \\ | \\ \boxed{R} \\ | \\ \text{---} \end{array} \quad \begin{array}{c} \text{---} \\ | \\ \boxed{R} \\ \backslash \diagup \\ \backslash \diagdown \\ \text{---} \end{array} \quad = \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \\ \diagup \quad \diagdown \\ \text{---} \end{array} \quad \boxed{R}$$

- ▶ Tensor product of representations via co-algebra

$$\boxed{R_1 \otimes R_2} \quad \begin{array}{c} \text{---} \\ | \\ (a, b) \end{array} \quad \begin{array}{c} \text{---} \\ | \\ (a', b') \end{array} \quad = \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \\ \diagup \quad \diagdown \\ a \quad a' \end{array} \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \\ \diagup \quad \diagdown \\ b \quad b' \end{array}$$

Hopf C*-algebras – representations

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- ▶ Left- and right-regular representation:



Local spin systems with commuting symmetries

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 $\mathcal{G} := \bigotimes_{s \in S} G_s$

Local spin systems with commuting symmetries

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- ▶ State space is representation of global Hopf C*-algebra $\mathcal{G} := \bigotimes_{s \in S} G_s$
- ▶ Decompose state space into *irreducible subspaces* under action of \mathcal{G} :

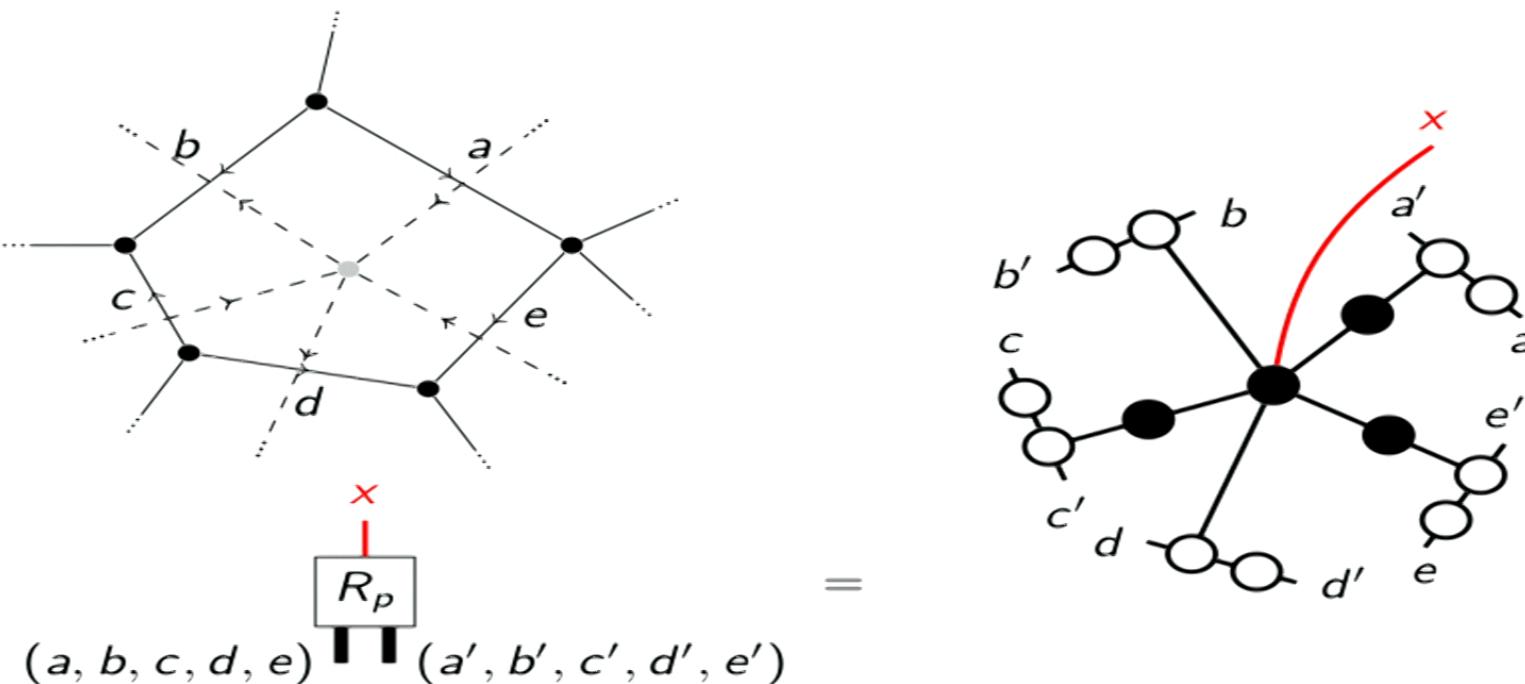
$$H = \bigoplus_{\alpha, m} V^{\alpha, m}, \text{ where } \alpha = (\alpha_1, \dots, \alpha_S) \text{ and } m = 1, \dots, M_\alpha$$

Vertex- and plaquette symmetries – definition

- ▶ Spins are associated to the edges of a two dimensional lattice, local state space is vector space of Hopf C*-algebra G

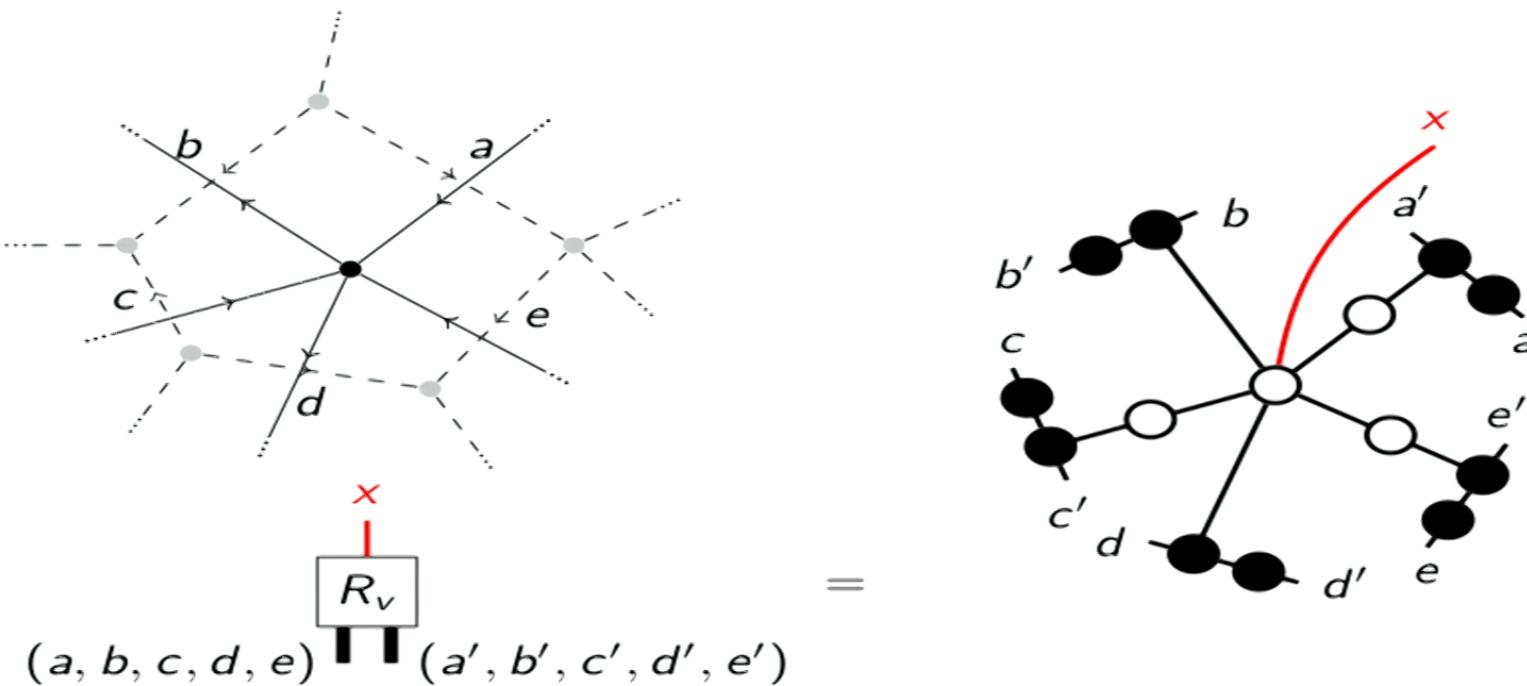
Vertex- and plaquette symmetries – definition

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- ▶ Plaquette symmetry: representation of Hopf C*-algebra G



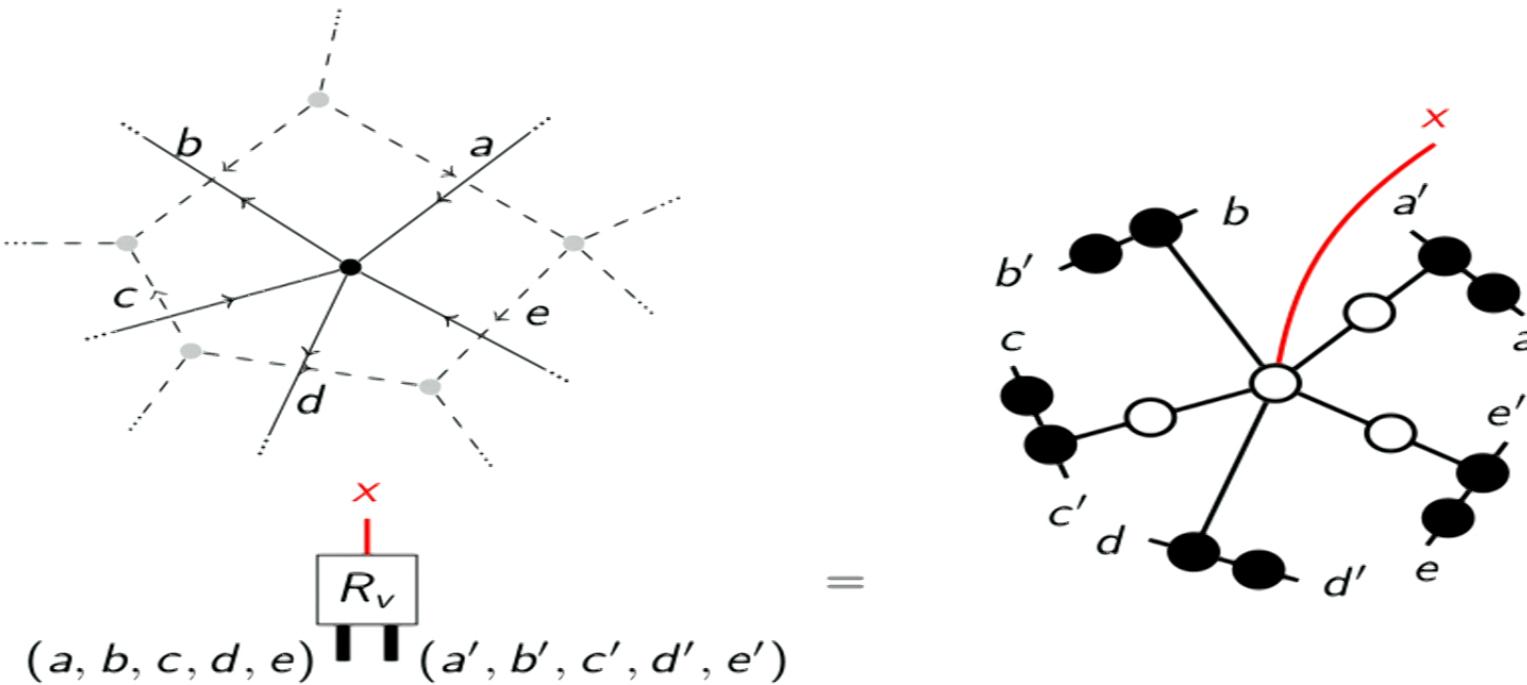
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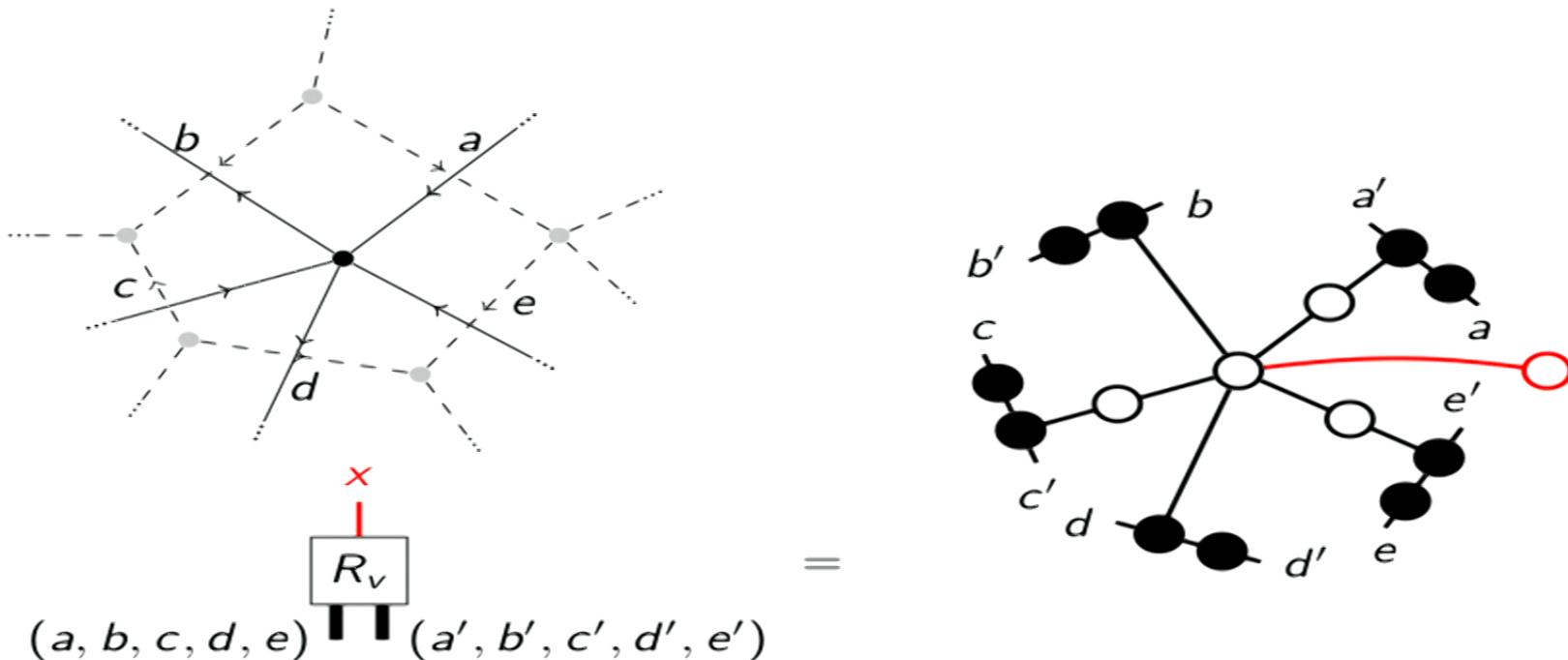
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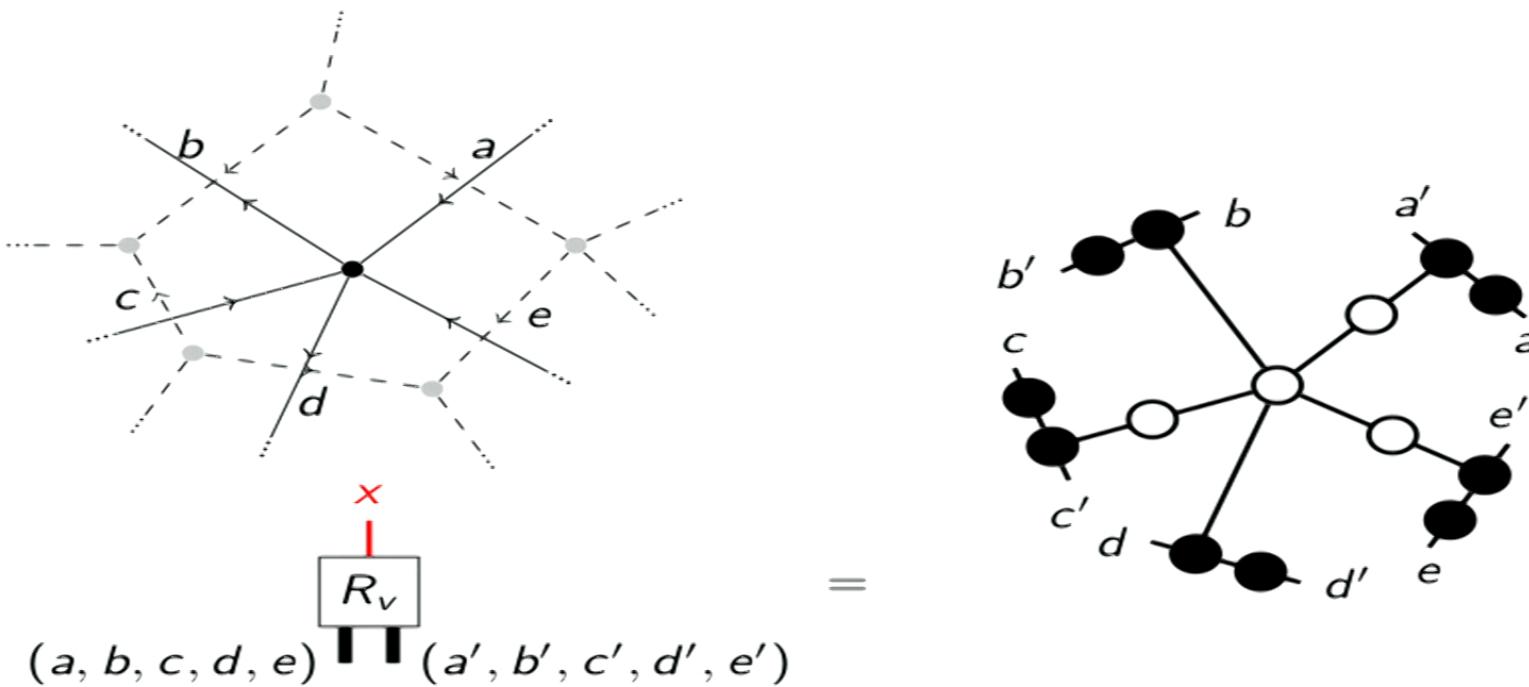
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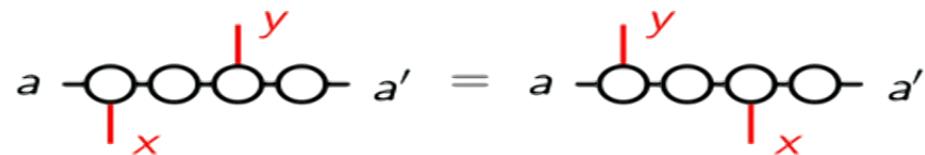
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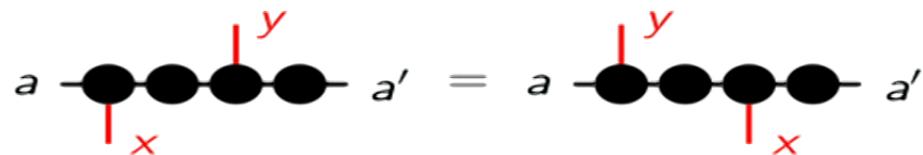
Vertex- and plaquette symmetries – commutation properties

- Vertex-vertex and plaquette-plaquette pairs commute

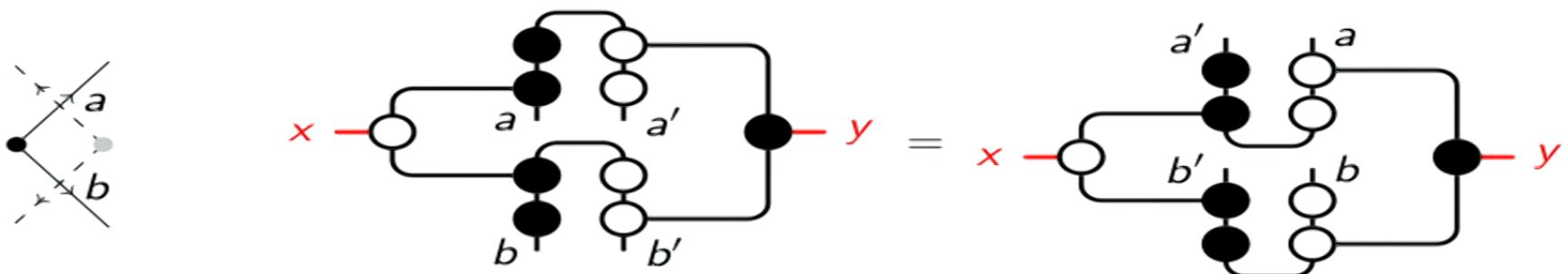


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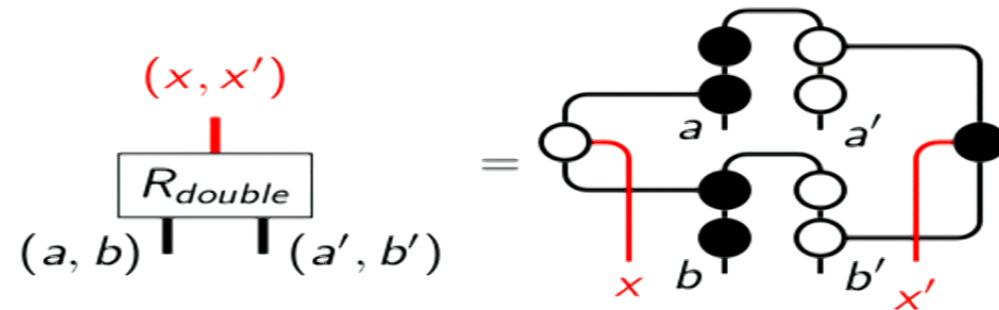


- Neighboring plaquette-vertex pairs commute depending on location of red index



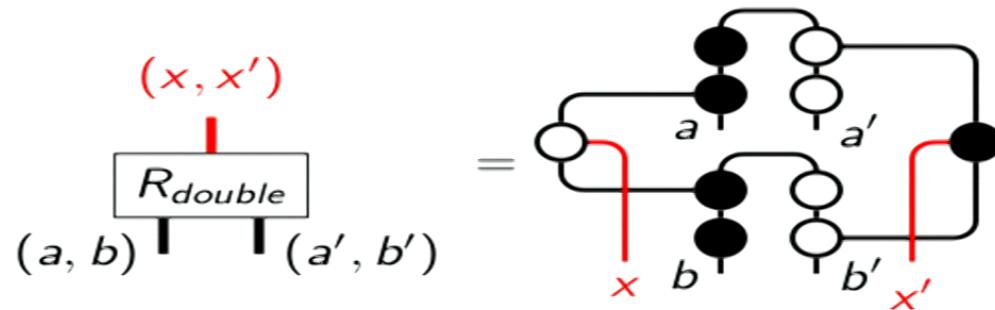
Plaquette and vertex term – together

- ▶ Consider product of plaquette- and vertex term



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- ▶ Do not commute $\Rightarrow R_{double}$ is not a representation of the tensor product of G and its dual

Plaquette and vertex term – together

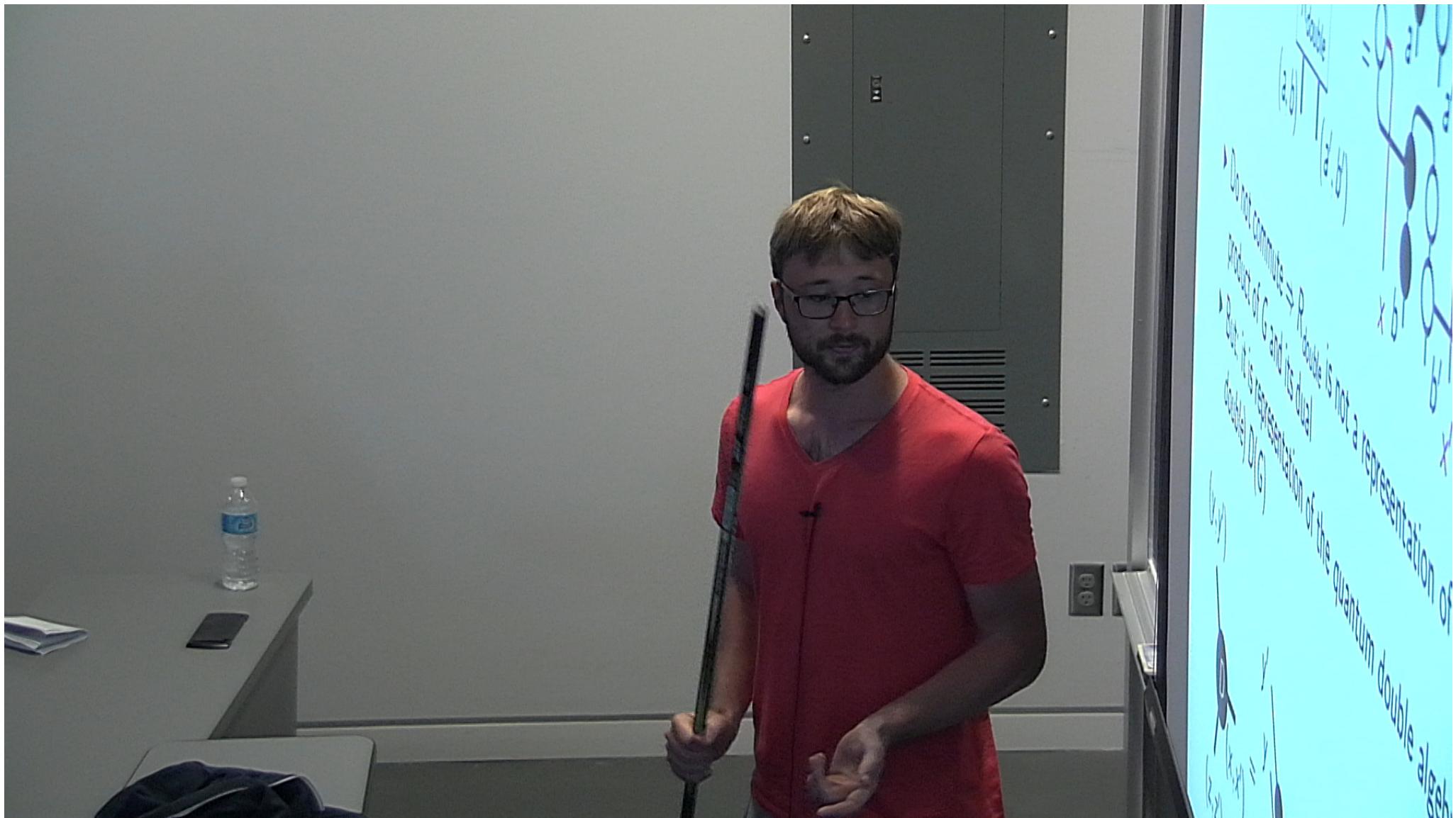
- Consider product of plaquette- and vertex term

$$\begin{array}{c}
 (x, x') \\
 | \\
 R_{\text{double}} \\
 | \\
 (a, b) \quad (a', b')
 \end{array}
 = \begin{array}{c}
 \text{Diagram showing the product of a plaquette term } R_{\text{double}} \text{ and a vertex term.} \\
 \text{The plaquette term } R_{\text{double}} \text{ is represented by a rectangle with vertices } (a, b), (a', b'), (x, x'), \text{ and top edge } (x, x'). \\
 \text{The vertex term is a complex network of nodes (black and white circles) and lines connecting them.} \\
 \text{Red lines and labels } x, x', a, a', b, b' \text{ indicate specific components or paths within the vertex term.}
 \end{array}$$

- Do not commute $\Rightarrow R_{\text{double}}$ is not a representation of the tensor product of G and its dual
- But: it is representation of the *quantum double algebra* (or *Drinfeld double*) $D(G)$

$$\begin{array}{c}
 (y, y') \\
 | \\
 D \\
 | \\
 (x, x') \\
 | \\
 (z, z')
 \end{array}
 = \begin{array}{c}
 \text{Diagram showing the product of a double algebra term } D \text{ and a vertex term.} \\
 \text{The double algebra term } D \text{ is represented by a circle with vertices } (y, y'), (x, x'), (z, z'), \text{ and top edge } (y, y'). \\
 \text{The vertex term is a complex network of nodes (black and white circles) and lines connecting them.} \\
 \text{Red lines and labels } y, y', x, x', z, z' \text{ indicate specific components or paths within the vertex term.}
 \end{array}$$

$$\begin{array}{c}
 (y, y') \quad (x, x') \\
 | \quad | \\
 D \quad D \\
 | \quad | \\
 (y, y') \quad (x, x') = \begin{array}{c}
 \text{Diagram showing the properties of the double algebra term } D. \\
 \text{The first part shows } (y, y') \text{ and } (x, x') \text{ connected to } D. \\
 \text{The second part shows } D \text{ connected to } (y, y') \text{ and } (x, x'). \\
 \text{The third part shows } D \text{ connected to } (x, x') \text{ and } (y, y'). \\
 \text{Red lines and labels } y, y', x, x' \text{ indicate specific components or paths within the vertex term.}
 \end{array}
 \end{array}$$



The quantum double algebra

- Quantum double algebra is bi-crossed product:

$$\begin{array}{c} (y, y') \\ \diagdown \quad \diagup \\ \textcircled{D} \\ \diagup \quad \diagdown \\ (z, z') \end{array} \quad (x, x') = \begin{array}{c} y' \\ \diagup \quad \diagdown \\ \textcircled{y} \quad \textcircled{z} \\ \diagdown \quad \diagup \\ \textcircled{z'} \quad \textcircled{z} \\ \diagup \quad \diagdown \\ \textcircled{x} \quad \textcircled{x'} \end{array}$$
$$(y, y') \quad \textcircled{D} \quad (x, x') = \begin{array}{c} y' \\ \diagup \quad \diagdown \\ \textcircled{y} \quad \textcircled{y} \\ \diagup \quad \diagdown \\ \textcircled{y} \quad \textcircled{y} \\ \diagup \quad \diagdown \\ \textcircled{x} \quad \textcircled{x'} \end{array}$$
$$\textcircled{D} \quad (x, x') = \begin{array}{c} \textcircled{x'} \\ \diagup \quad \diagdown \\ \textcircled{x} \quad \textcircled{x} \end{array}$$

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$$(y, y') \quad \textcircled{D} \quad (x, x') = \begin{array}{c} y' \\ \diagup \quad \diagdown \\ \textcircled{y} \quad \textcircled{y} \\ \diagup \quad \diagdown \\ \textcircled{y} \quad \textcircled{y} \\ \diagup \quad \diagdown \\ \textcircled{x'} \quad \textcircled{x} \end{array}$$
$$\textcircled{D} \quad (x, x') = \begin{array}{c} \textcircled{x'} \\ \diagup \quad \diagdown \\ \textcircled{x} \end{array}$$

The quantum double algebra

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$$\begin{array}{c} (y, y') \\ \diagdown \quad \diagup \\ \textcircled{D} \\ \diagup \quad \diagdown \\ (z, z') \end{array} \quad (x, x') = \begin{array}{c} y' \\ \diagup \quad \diagdown \\ \textcircled{y} \quad \textcircled{z} \\ \diagdown \quad \diagup \\ \textcircled{z'} \quad \textcircled{z} \\ \diagup \quad \diagdown \\ \textcircled{x} \quad \textcircled{x'} \end{array}$$
$$(y, y') \quad \textcircled{D} \quad (x, x') = \begin{array}{c} y' \\ \diagup \quad \diagdown \\ \textcircled{y} \quad \textcircled{y} \\ \diagup \quad \diagdown \\ \textcircled{y} \quad \textcircled{y} \\ \diagup \quad \diagdown \\ \textcircled{x} \quad \textcircled{x'} \end{array}$$
$$\textcircled{D} \quad (x, x') = \begin{array}{c} \textcircled{x'} \\ \diagup \quad \diagdown \\ \textcircled{x} \quad \textcircled{x} \end{array}$$

- Quasi-triangular \Rightarrow admits braiding

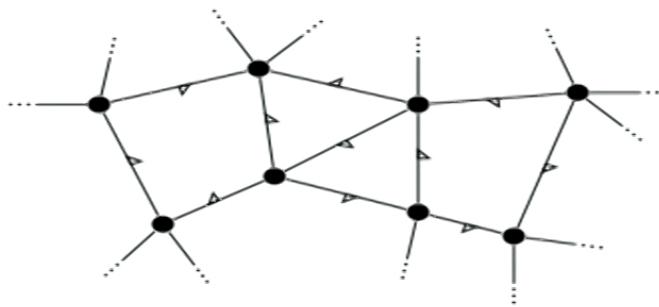
The quantum double algebra

- Quantum double algebra is bi-crossed product:

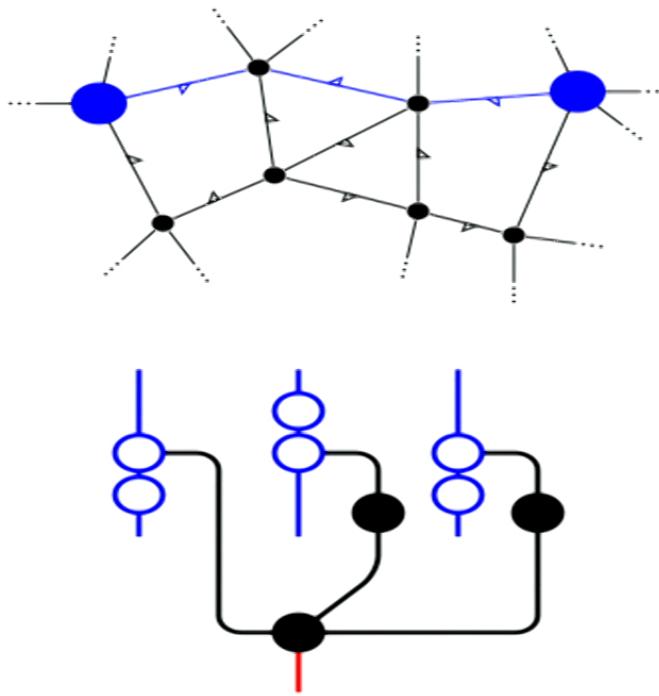
$$\begin{array}{c} (y, y') \\ \diagdown \quad \diagup \\ \textcircled{D} \\ \diagup \quad \diagdown \\ (z, z') \end{array} \quad (x, x') = \begin{array}{c} y' \\ \diagup \quad \diagdown \\ \textcircled{y} \quad \textcircled{z} \\ \diagdown \quad \diagup \\ \textcircled{z'} \quad \textcircled{z} \\ \diagup \quad \diagdown \\ \textcircled{x'} \quad \textcircled{x} \end{array}$$
$$(y, y') \quad \textcircled{D} \quad (x, x') = \begin{array}{c} y' \\ \diagup \quad \diagdown \\ \textcircled{y} \quad \textcircled{y} \\ \diagup \quad \diagdown \\ \textcircled{y} \quad \textcircled{y} \\ \diagup \quad \diagdown \\ \textcircled{x'} \quad \textcircled{x} \end{array}$$
$$\textcircled{D} \quad (x, x') = \begin{array}{c} \textcircled{x'} \\ \diagup \quad \diagdown \\ \textcircled{x} \end{array}$$

- Quasi-triangular \Rightarrow admits braiding

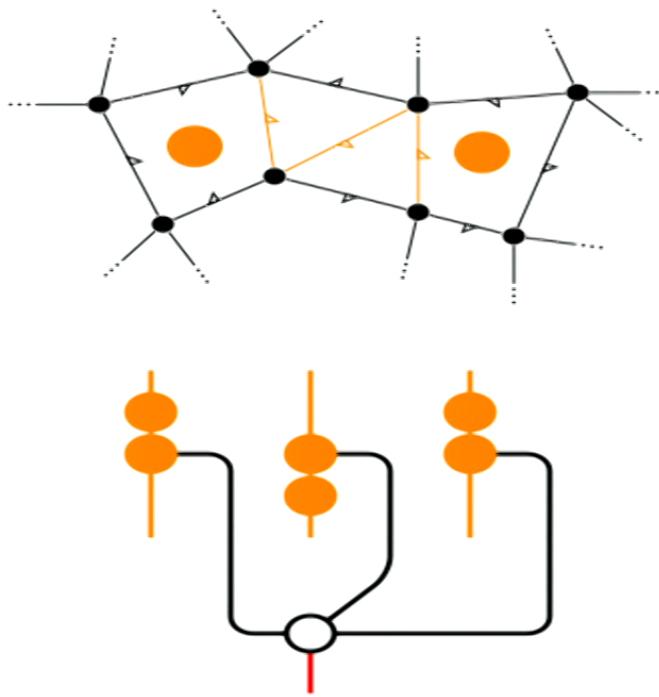
Open string and ribbon operators



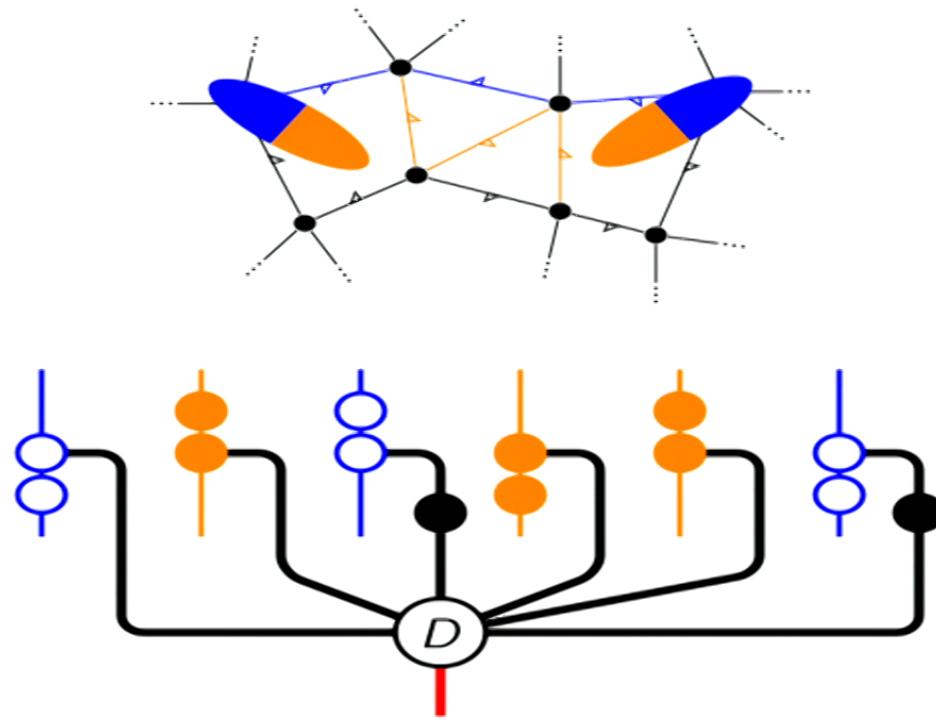
Open string and ribbon operators



Open string and ribbon operators

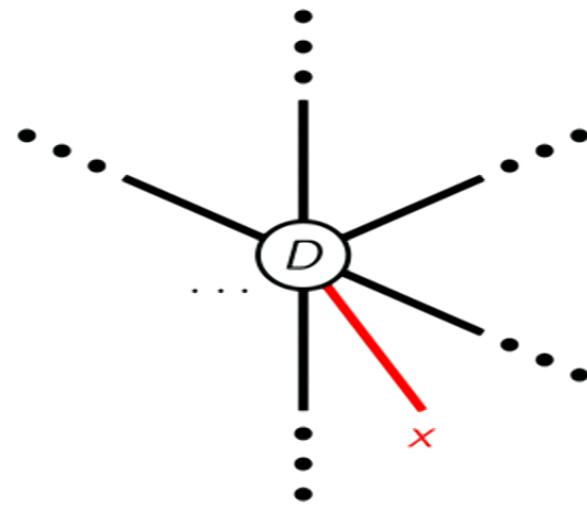


Open string and ribbon operators



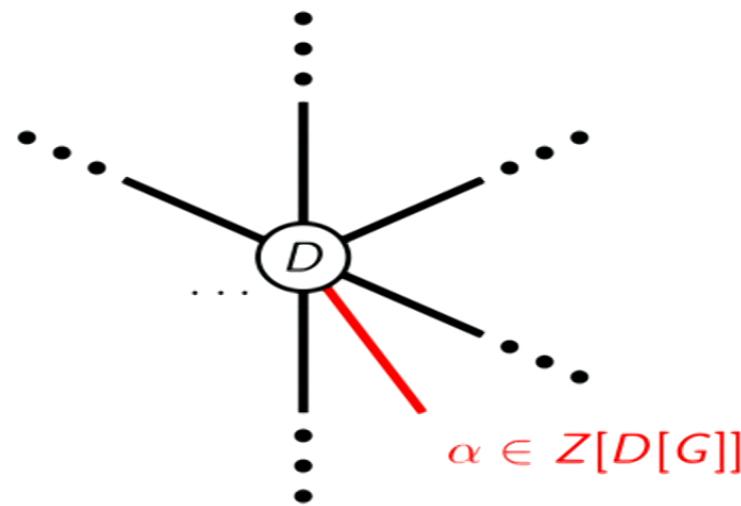
Closed string- and ribbon operators

- ▶ Closed ribbon operator should be nowhere visible



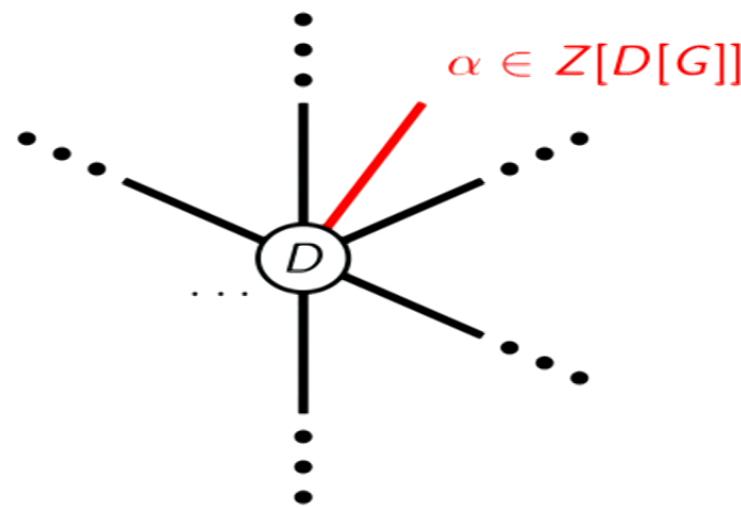
Closed string- and ribbon operators

- ▶ Closed ribbon operator should be nowhere visible
- ▶ ⇒ Put element of the center \Rightarrow commutes through



Closed string- and ribbon operators

- ▶ Closed ribbon operator should be nowhere visible
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Thanks for your attention!

