Title: Contextuality and Temporal Correlations in Quantum Mechanics

Date: Jul 28, 2017 10:30 AM

URL: http://pirsa.org/17070056

Abstract:

Contextuality and Temporal Quantum Correlations

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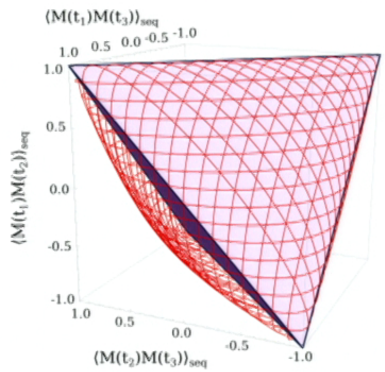

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Overview

- Motivation: Contextuality and sequential measurements
- Implementations with nanomechanical oscillators
- Simulating temporal quantum correlations
- Structure of temporal correlations

The Kochen-Specker theorem

The Kochen-Specker theorem

Quantum mechanics cannot be explained
by non-contextual hidden variable models.

What does non-contextuality mean?

Compatibility and Noncontextuality

Compatibility

Two measurements A and B are compatible ($A \sim B$) if they can be measured simultaneously or in any order without disturbance.

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Non-contextuality

Assume that $A \sim B$ and $A \sim C$. A theory is non-contextual, if it assigns to A a value $v(A)$ independently whether B or C is measured jointly with A.

The Peres Mermin square

Consider a four level system (two qubits) and the observables:

$$
\begin{aligned}
A &= \sigma_z \otimes \mathbb{1}, & B &= \mathbb{1} \otimes \sigma_z, & C &= \sigma_z \otimes \sigma_z, \\
a &= \mathbb{1} \otimes \sigma_x, & b &= \sigma_x \otimes \mathbb{1}, & c &= \sigma_x \otimes \sigma_x, \\
\alpha &= \sigma_z \otimes \sigma_x, & \beta &= \sigma_x \otimes \sigma_z, & \gamma &= \sigma_y \otimes \sigma_y.
\end{aligned}
$$

- The observables in each row (R_i) and column (C_j) commute and are compatible.

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$$

- The observables in each row (R_i) and column (C_j) commute and are compatible.
- If we assign to each of them a value $v = \pm 1$ independently of the row or column, we have

$$\prod_{i=1}^{3} R_i C_i = +1$$

- In QM: $C_3 = Cc\gamma = -\mathbb{1}$, hence $\prod_{i=1}^{3} R_i C_i = -1$.

A. Peres, PLA 151, 107 (1990); D. Mermin, PRL 65, 3373 (1990).

A testable inequality

Question

Can we translate this into an experimentally testable inequality?

Answer

Consider sequences of measurements. Then, for non-contextual models

$$\begin{aligned}\langle \mathcal{X}_{\mathrm{MP}} \rangle &= \langle A_1 B_2 C_3 \rangle + \langle a_1 b_2 c_3 \rangle + \langle \alpha_1 \beta_2 \gamma_3 \rangle \\ &\quad + \langle A_1 a_2 \alpha_3 \rangle + \langle B_1 b_2 \beta_3 \rangle - \langle C_1 c_2 \gamma_3 \rangle \\ &= \langle R_1 \rangle + \langle R_2 \rangle + \langle R_3 \rangle + \langle C_1 \rangle + \langle C_2 \rangle - \langle C_3 \rangle \leq 4.\end{aligned}$$

Here, $\langle A_1 B_2 C_3 \rangle$ means the product of the values, when the sequence $A_1 B_2 C_3$ is measured on a single instance of a state.

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Here, $\langle A_1 B_2 C_3 \rangle$ means the product of the values, when the sequence $A_1 B_2 C_3$ is measured on a single instance of a state.
In QM:

$$\langle \mathcal{X}_{\mathrm{MP}} \rangle = 6$$

for *any* quantum state (in contrast to a Bell inequality violation).

A. Cabello, PRL 101, 210401 (2008).

Challenges

Assume that $A \sim B$ and $A \sim C$. A theory is non-contextual, if it assigns to A a value $v(A)$ independently whether B or C is measured jointly with A.

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Bell

"... there is no a priori reason to believe that the results for $|\phi_3\rangle\langle\phi_3|$ should be the same. The result of an observation may reasonably depend not only on the state of the system (including hidden variables), but also on the complete disposition of the apparatus"

J.S. Bell, Rev. Mod. Phys. 38 227 (1966).

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Peres

"Suppose that we measure A first and only a later time decide whether to measure B or C or none of them. How can the outcome of the measurement A depend on this future decision?"

A. Peres, A. Ron, in "Microphysical Reality and Quantum Formalism" Kluwer, 1998.

Underlying Assumptions

Measurements

- Measurements: Classical procedures applied to some physical system α, resulting in an outcome ± 1 and a post-measurement system $\omega^{\pm}$.
- Each measurement has to be implemented in each context by the same device (same detector/laser/gate sequence/PhD student ...)
- Repeatability needs to be checked.
- Compatibility needs to be checked.

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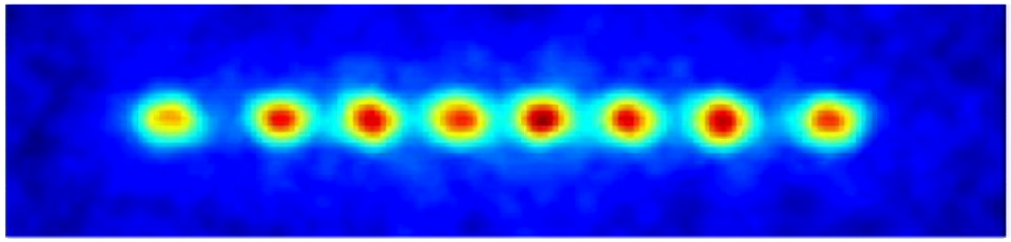
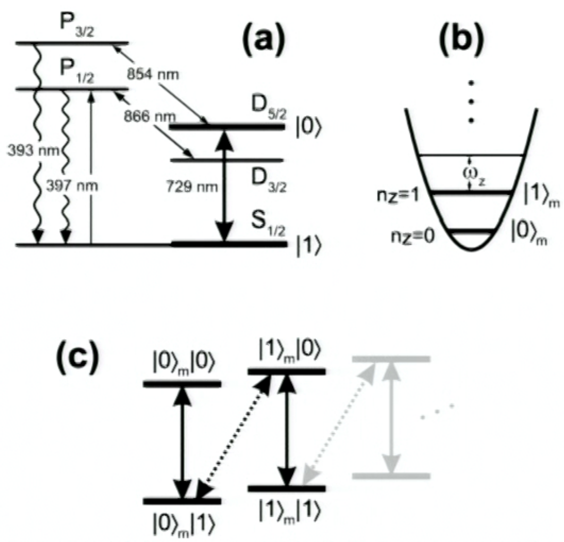
Deriving the bound

- The KS inequality contains nine measurements $A, B, ..., \gamma$.
- For a fixed HV λ each measurement has a deterministic assignment.
- The assignments are independent of the context.

Ion traps





- Typical ions: $^{9}Be^{+}$, $^{40}Ca^{+}$, $^{43}Ca^{+}$.
- Single ion manipulation with focused lasers.
- Interaction of the ions: motion of the ion crystal, or collective interaction with laser.

H. Häffner, C. Roos, and R. Blatt, Phys. Rep. 469, 155-203 (2008).

Non-demolition measurements

Aim: We want to measure the inequality from the Mermin-Peres square

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Measurements like $\sigma_z \otimes \sigma_z$ cannot be implemented by measuring σ_z on each ion, as coherences (like $|00\rangle + |11\rangle$) would be destroyed.

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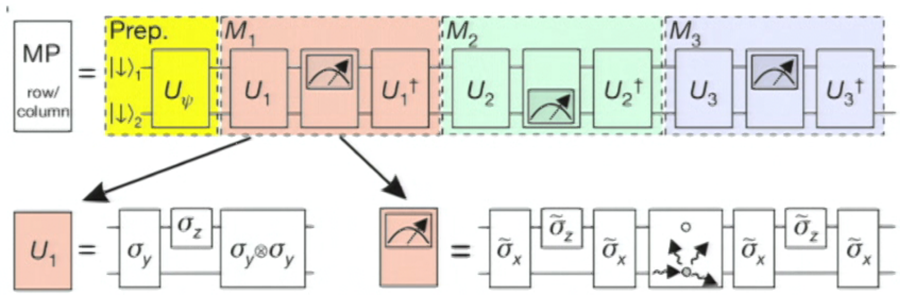
Measurements like $\sigma_z \otimes \sigma_z$ cannot be implemented by measuring σ_z on each ion, as coherences (like $|00\rangle + |11\rangle$) would be destroyed.

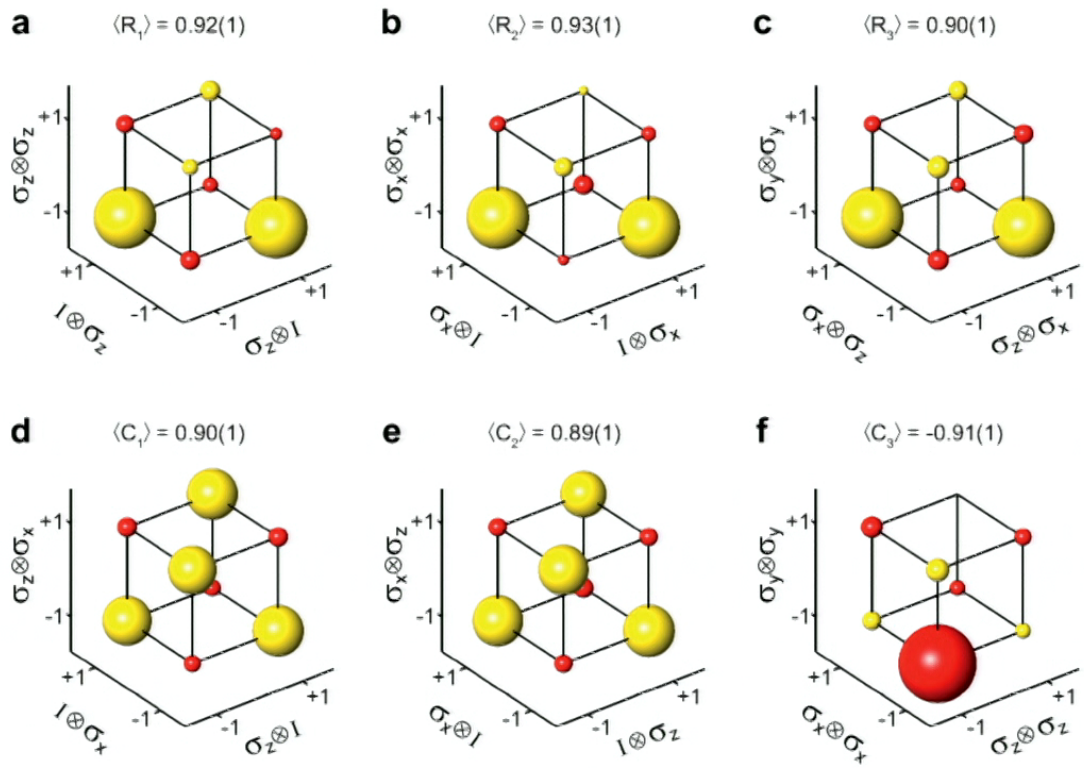
Solution

Write $\sigma_z \otimes \sigma_z = U_{nl}[\mathbb{1} \otimes \sigma_z]U_{nl}^\dagger$ and read out only one ion.



This needs 6 nonlocal gates for R_3 or C_3.

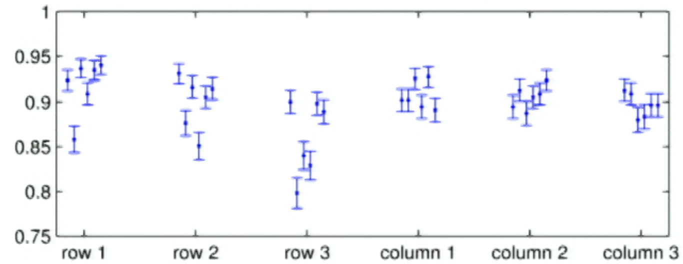
Results for the singlet state



In summary a total value of $\langle \mathcal{X}_{\mathrm{MP}} \rangle = 5.46(4) > 4$.

Compatibility and repeatability

- The order does not really matter, on average $\langle \mathcal{X}_{\mathrm{MP}} \rangle = 5.38$



- Repeatability:

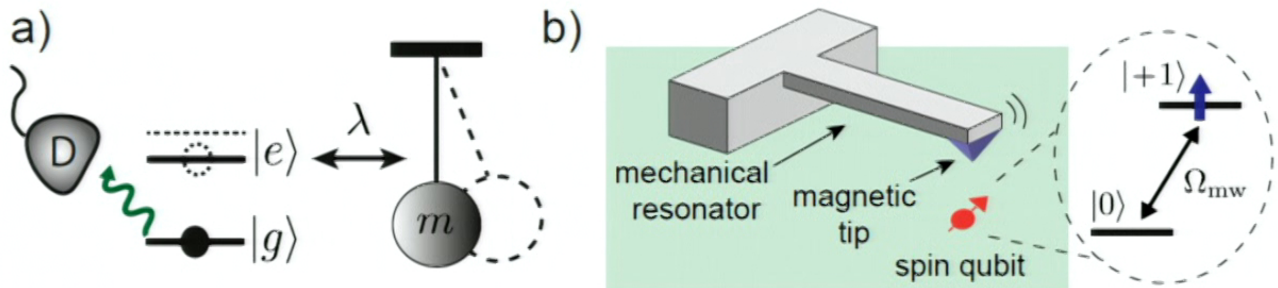
$$\begin{aligned} p(++|AA) &\in [0.95; 0.99] \\ p(+???+|AAAAA) &\in [0.9; 0.97] \end{aligned}$$

- Compatibility:

$$\begin{aligned} p(+?+|ABA) &\in [0.91; 0.95] \\ p(+??+|ABCA) &\approx 0.9 \end{aligned}$$

O. Gühne et al, PRA 2010

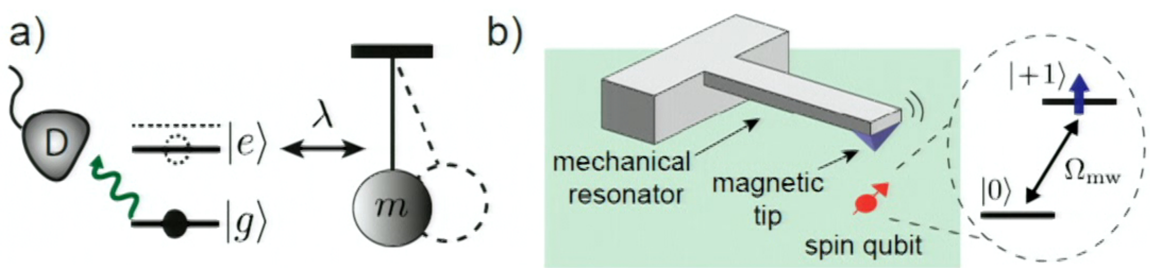
Peres Mermin squares in various dimensions



PM square for arbitrary dimensions

Questions

- Can one generalize the PM square to arbitrary dimensions?
- What are the appropriate measurements?



With this scheme, one can measure modular variables.

A. Asadian et al., PRL 2014

Modular Variables

- Consider a harmonic oscillator. The displacement is defined as

$$\mathcal{D}(\alpha) = \exp\{i(pX - qP)\}$$

with $\alpha = q + ip$.

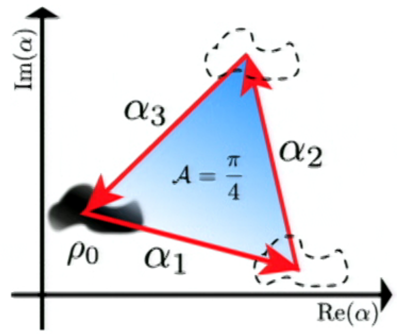
- Modular variables are the real and imaginary part

$$\mathcal{D}_R(\alpha) = \cos(pX - qP); \quad \mathcal{D}_I(\alpha) = \sin(pX - qP)$$

- Important relation:

$$\mathcal{D}(\alpha)\mathcal{D}(\beta) = e^{2i\mathrm{Im}(\alpha\beta^*)}\mathcal{D}(\beta)\mathcal{D}(\alpha)$$

$\Rightarrow$ Arbitrary commutation relations can be realized.



PM square for infinite dimensions

We can find a representation of all the nine variables on two oscillators with $\alpha_1 + \alpha_2 + \alpha_3 = 0$:

$$\begin{array}{ccc} A_{11} = \mathcal{D}_{a_1}(-\alpha_1) & A_{12} = \mathcal{D}_{a_2}(-\alpha_1) & A_{13} = \mathcal{D}_{a_1}(\alpha_1)\mathcal{D}_{a_2}(\alpha_1) \\ A_{21} = \mathcal{D}_{a_2}(-\alpha_2) & A_{22} = \mathcal{D}_{a_1}(-\alpha_2) & A_{23} = \mathcal{D}_{a_1}(\alpha_2)\mathcal{D}_{a_2}(\alpha_2) \\ A_{31} = \mathcal{D}_{a_1}(\alpha_1)\mathcal{D}_{a_2}(\alpha_2) & A_{32} = \mathcal{D}_{a_1}(\alpha_2)\mathcal{D}_{a_2}(\alpha_1) & A_{33} = \mathcal{D}_{a_1}(\alpha_3)\mathcal{D}_{a_2}(\alpha_3) \end{array}$$

Then, we consider the real part of each column and row

$$C_i = Re(A_{1i}A_{2i}A_{3i}), \quad R_j = Re(A_{j1}A_{j2}A_{j3})$$

and measure it via the modular variables. QM predicts:

$$\langle \chi_{PM} \rangle = R_1 + R_2 + R_3 + C_1 + C_2 - C_3 = 6$$

The classical bound

Problem

- It is known that for complex numbers on the unit circle:

$$\langle \chi_{PM} \rangle \leq 3\sqrt{3} \approx 5.19$$

A. Plastino et al, PRA 2010

- But HV models do not fulfil that $Re(A_{ij})^2 + Im(A_{ij})^2 = 1$

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- It is known that for complex numbers on the unit circle:

$$\langle \chi_{PM} \rangle \leq 3\sqrt{3} \approx 5.19$$

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- But HV models do not fulfil that $Re(A_{ij})^2 + Im(A_{ij})^2 = 1$
- Define a new quantity

$$\langle \chi_{PM}^{new} \rangle = \langle \chi_{PM} \rangle - \lambda \sum_{ij} [Re(A_{ij})^2 + Im(A_{ij})^2 - 1]$$

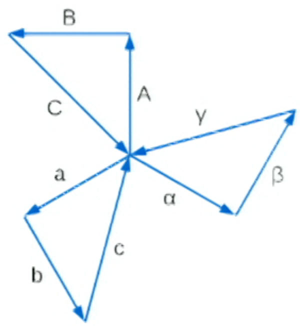
Then, for $\lambda > 4$ the bound $\langle \chi_{PM}^{new} \rangle \leq 3\sqrt{3}$ holds again.

A. Asadian et al, PRL 2015

Further Results

Single modes

- One cannot implement the MPS with a single mode.
- But note that two modes may be implemented in a single nanomechanical oscillator.



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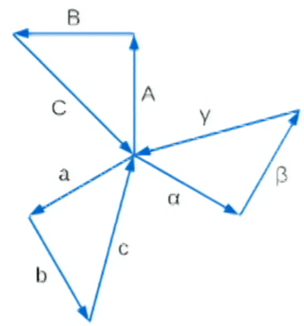


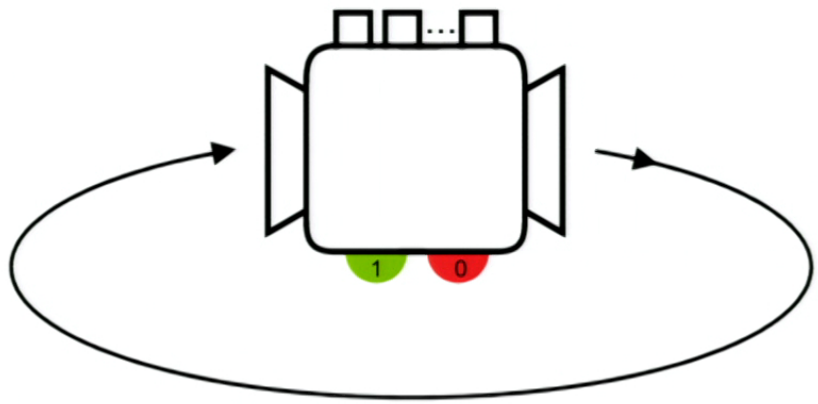
Arbitrary dimensions

- Define X and P via the discrete Fourier transform.
- Define displacements and modular variables
- If d is even, also the desired commutation relation can be realized.

$\Rightarrow$ One arrives at an PM square for (nearly) arbitrary dimensions.

A. Asadian et al, PRL 2015

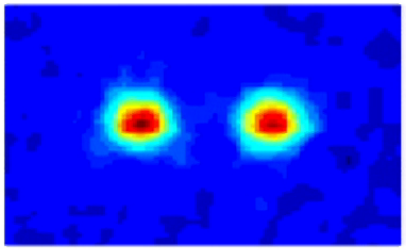
Classical simulation of Kochen-Specker experiments



Initial questions

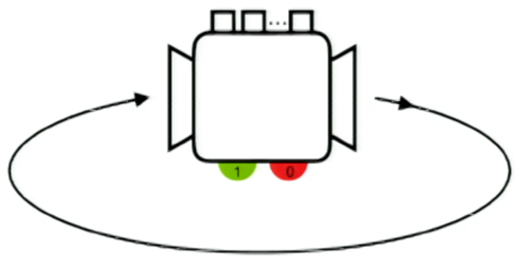
In an experiment, it was found that:

$$\langle \mathcal{X}_{\mathrm{MP}} \rangle = 5.46(4) > 4.$$



- Is this value surprising?
- Could it be explained by a classical mechanism?
- For instance, if the system remembers the measurements made?
- What memory is required for that?

Mathematical formulation



- We have an infinite sequence of questions

$$\overleftrightarrow{\mathcal{Q}} = \{..., Q_{t-1}, Q_t, Q_{t+1}, ...\}$$

- We obtain an infinite sequence of answers

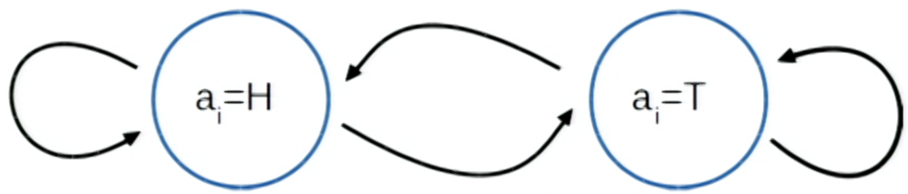
$$\overleftrightarrow{\mathcal{A}} = \{..., A_{t-1}, A_t, A_{t+1}, ...\}$$

- The questions are chosen randomly from the nine measurements, the answers obey the conditions of the Peres-Mermin square.

Simulating simple time series

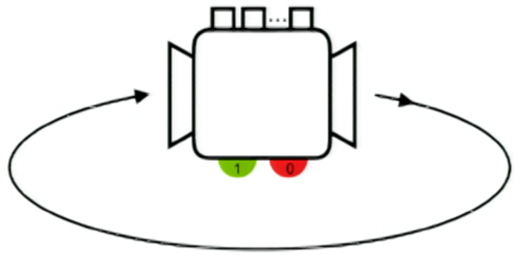
Hidden Markov models

- Only one question is allowed $\Rightarrow$ observe random sequence $\overleftrightarrow{\mathcal{A}}$
- A HMM has internal states $i \in \{1, ..., N\}$
- For any state there is an output probability $A = \{a_{ki}\} = P(A_k|i)$
- In addition, there are transition probabilities $U = \{u_{ij}\} = P(i \mapsto j|i)$.
- Finally, one has an initial state distribution $\pi_i = P(i)$



L. Rabiner, B Juang, IEEE ASSP Magazine, 1986

Deterministic processes: Mealy machines



Ingredients

- A memory that can be in states $i \in \{1, ..., k\}$
- For any memory state there is a table A_i with the answers to the possible questions.
- For any memory state there is a table U_i, describing the update rules for the memory, depending on the question.

How many memory states are needed to simulate a given process?

A simple Mealy machine

1

Q	a	b
A	+	-
U	1	2

2

Q	a	b
A	-	-
U	3	2

3

Q	a	b
A	-	+
U	1	1

Q = (a, a, b, b, a, b, a, b, a, a, ...)
S = (1, 1, 1, 2, 2, 3, 1, 1, 2, 3, ...)
A = (+, +, - , -, - , +, +, -, - , -, ...)

ε

ε-machines

- Consider only one question.
- One can split the answers in past and future

$$\overleftarrow{A} = \{..., A_{-3}, A_{-2}, A_{-1}\}$$
$$\overrightarrow{A} = \{A_0, A_1, A_2, ...\}$$

- Two pasts are equivalent, if they predict the same future:

$$\overleftarrow{a} \sim \overleftarrow{a}' \Leftrightarrow P(\overrightarrow{A}|\overleftarrow{a}) = P(\overrightarrow{A}|\overleftarrow{a}')$$

- The equivalence classes define the causal states s. The output for a given causal state defines transitions between them.
- The statistical complexity is the entropy of the distribution of the $S = \{s\}$. This is the memory required for the simulation.

J. P. Crutchfield, Physica D 75, 11 (1994).

ε

ε-transducer

- We consider questions and answers as a single variable:

$$\overleftrightarrow{\mathcal{Z}} = (\overleftrightarrow{\mathcal{Q}}, \overleftrightarrow{\mathcal{A}})$$

- Define equivalence relations for the past outcomes:

$$\overleftarrow{z} \sim \overleftarrow{z}' \Leftrightarrow P(\overrightarrow{\mathcal{A}} | \overrightarrow{\mathcal{Q}}, \overleftarrow{z}) = P(\overrightarrow{\mathcal{A}} | \overrightarrow{\mathcal{Q}}, \overleftarrow{z}')$$

- This defines causal states and the corresponding statistical complexity.

N. Barnett, J. P. Crutchfield, J. Stat. Phys. 161, 404 (2015).

HMM and ε

Properties

- ε-machines are special HMM.
- ε-machines are unifilar: The output defines the transition.
- For an ε-machine the state contains no oracular information (information about the future that is not contained in the past)
- In other words: $H(S, \overrightarrow{\mathcal{A}} | \overleftarrow{\mathcal{A}}) = 0$.

J.P. Crutchfield et al., arXiv:1007.5354

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Example:Biased flip of a coin

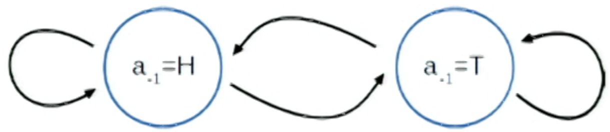
Consider a coin that flips with a certain probability:

$$P(a_1 = T | a_0 = H) = \frac{1}{2} - \epsilon, \qquad P(a_1 = H | a_0 = T) = \frac{1}{2} - \epsilon$$

Simple example

Simulation with an ε-machine

- The causal states are defined by the last output.
- Both causal states are equally probable.
- 1 bit of complexity/memory.



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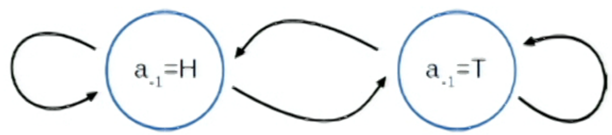
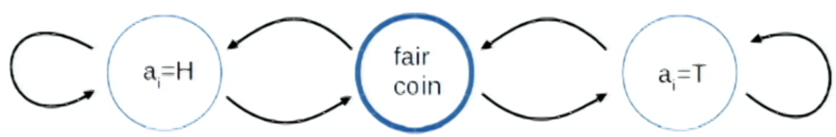


Simulation with HMM

- Three states: fair coin, two completely biased coins.
- If ϵ is small: less than one bit.



Perfect correlations & Mealy machines

Correlations to be simulated

- Measurements can be repeated: $v(A_1|AA) = v(A_2|AA)$ etc.
- The machine reproduces all six Mermin-Peres predictions.
- The observables are compatible in a single sequence: $v(A_1|ABCA) = v(A_4|ABCA)$ etc.

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Correlations to be simulated

- Measurements can be repeated: $v(A_1|AA) = v(A_2|AA)$ etc.
- The machine reproduces all six Mermin-Peres predictions.
- The observables are compatible in a single sequence: $v(A_1|ABCA) = v(A_4|ABCA)$ etc.
- The observables fulfil other compatibility constraints, e.g. $v(A_1|ACaA) = v(A_4|ACaA)$.
- ...
- The machine reproduces all quantum predictions.

$$\begin{bmatrix} A & B & C \\ a & b & c \\ \alpha & \beta & \gamma \end{bmatrix}$$

Four-state Mealy machine

- Consider the following four A_i and the update tables U_i:

$$\begin{bmatrix} - & - & + \\ - & - & + \\ + & + & + \end{bmatrix} \begin{bmatrix} + & - & + \\ + & - & - \\ + & + & + \end{bmatrix} \begin{bmatrix} - & + & + \\ - & - & + \\ + & - & - \end{bmatrix} \begin{bmatrix} + & + & + \\ + & - & - \\ + & - & - \end{bmatrix}$$

$$U_1 : \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 2 \end{bmatrix} \; U_2 : \begin{bmatrix} 4 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \; U_3 : \begin{bmatrix} 1 & 4 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \; U_4 : \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

- This machine predicts: $C^+_{(1)} c^+_{(1)} \gamma^-_{(3)} C^+_{(3)} B^+_{(3)} A^+_{(4)} c^-_{(4)} A^+_{(2)} \cdots$

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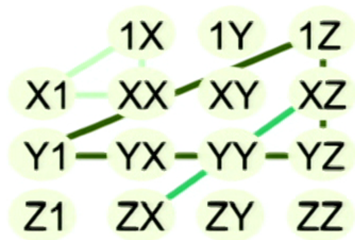
- This machine predicts: $C^+_{(1)} c^+_{(1)} \gamma^-_{(3)} C^+_{(3)} B^+_{(3)} A^+_{(4)} c^-_{(4)} A^+_{(2)} \cdots$
- It reproduces all deterministic predictions of QM for the PM square.
- Machines with three states cannot do this, so it is optimal.
- Can all two-qubit effects be simulated with two bits of memory?

M. Kleinmann et al., New J. Phys. 13, 113011 (2011), see also P. Blasiak, Ann. Phys. 353, 326 (2015) G. Fagundes, et al., arXiv:1611.07515.

Improving the Peres-Mermin square

Question

Are there KS inequalities for qubits with a higher violation?



Using all Pauli matrices one can find a correlation with $\langle \chi_{MP} \rangle = 15$ (in QM), but for noncontextual theories: $\langle \chi_{MP} \rangle \leq 9$.

A. Cabello, Phys. Rev. A 82, 032110 (2010)

Improving the Peres-Mermin square

Question

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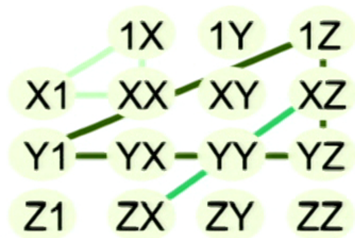

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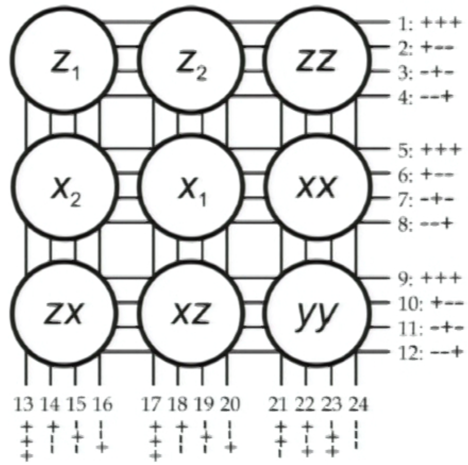
A. Cabello, Phys. Rev. A 82, 032110 (2010)

Theorem

Simulating this extended PM square for two qubits requires more than two classical bits as a memory.

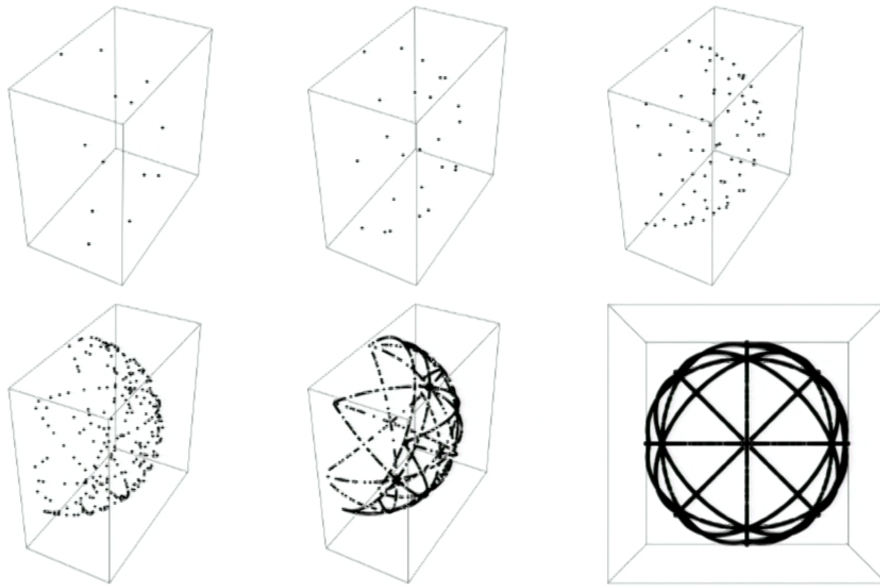
Simulating quantum predictions: ε-transducer



- Measuring a sequence or row projects the system in one of 24 quantum states.
- These are the causal states of the ε-transducer.

A. Cabello, M. Gu, O Gühne, in preparation

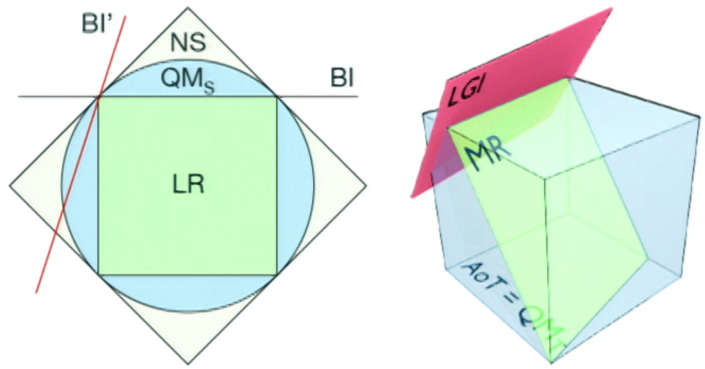
Simulating quantum predictions: ε-transducer



- From the Yu-Oh measurements, one obtains an infinite set of quantum states.
- Does an ε-transducer require an infinite amount of memory?

A. Cabello, M. Gu, O Gühne, in preparation

General temporal correlations



General temporal correlations

Question

Can we characterize all the probabilities coming from sequential quantum measurements?

Properties

- Consider sequences of length k and the set of all probabilities

$$p(x, y, z, \dots | X, Y, Z, \dots).$$

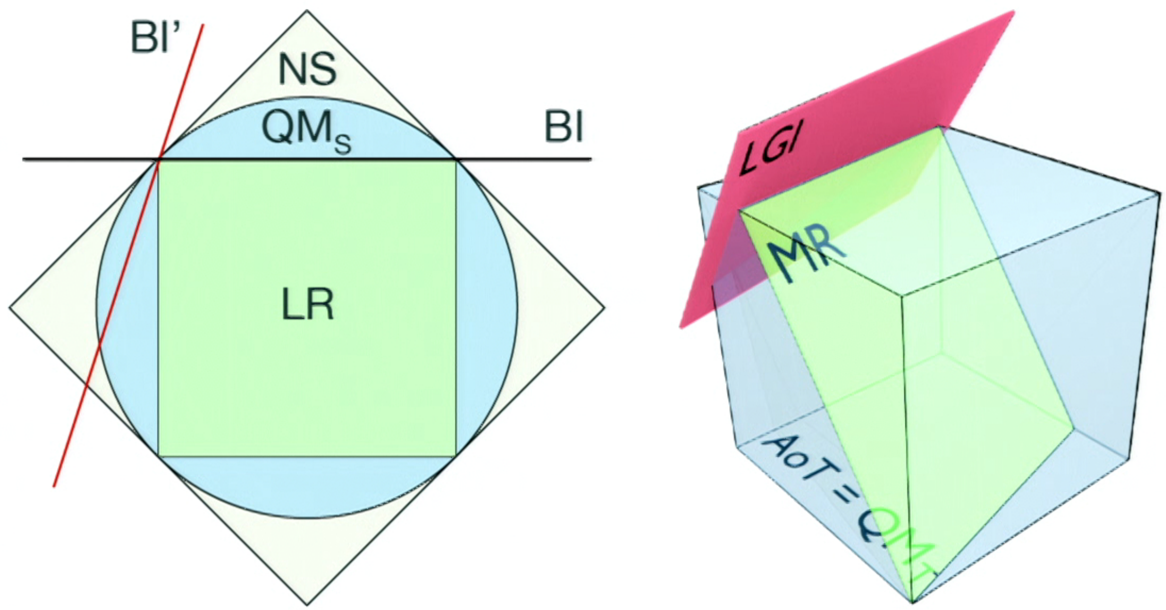
- The probabilities have to obey the arrow of time (AoT):

$$p(a, \cdot | AB) = p(a, \cdot | AA)$$

- How does this set look?
  What are the quantum mechanically allowed probabilities?

L. Clemente, J. Kofler, Phys. Rev. Lett. 116, 150401 (2016)

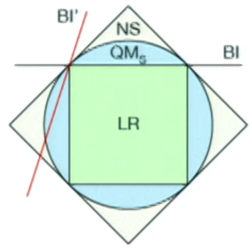
General temporal correlations

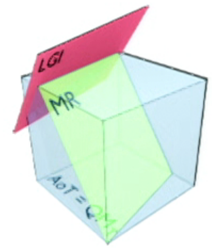


For spatial correlations, this is a well studied set.

Picture from L. Clemente, J. Kofler, Phys. Rev. Lett. 116, 150401 (2016)

Results





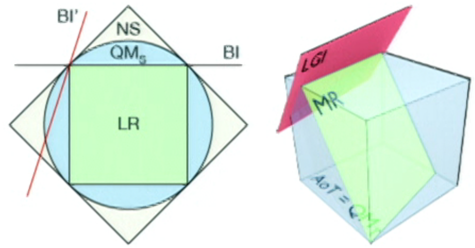
Result 1

The extremal points of the temporal correlation polytope are exactly the deterministic assignments that obey AoT.

Result 2

All extremal points can origin from QM, but they may require general measurements and high-dimensional systems

Results



Result 1

The extremal points of the temporal correlation polytope are exactly the deterministic assignments that obey AoT.

Result 2

All extremal points can origin from QM, but they may require general measurements and high-dimensional systems

Result 3

Some simple extremal points cannot origin from two-dimensional systems.

J. Hoffman, MSc thesis, T. Fritz, NJP 2010, L. Clemente et al., PRL 2016

Dimension witnesses

- Consider two measurements with two outcomes and then:

$$T = p(+-|AA) + p(++|AB) + p(-+|BA) + p(-+|BB)$$

- We can reach $T = 4$ with a deterministic AoT assignment.
- For qubits: If $p(+-|AA) = p(-+|BB) = 1$ the measurements must be projective.
  But then $p(++|AB) = p(-+|BA) = 1$ cannot be reached.
- We have the inequality

$$T \overset{2D}{\leq} 3.18623 \overset{3D}{\leq} 4$$

- This may be tested experimentally ...

J. Hoffmann et al., in preparation.

Conclusion

Homework for the weekend

Design an experiment, which can be used to test both notions of contextuality at the same time.