Title: Contextuality as a resource for quantum computation: the trouble with qubits

Date: Jul 27, 2017 03:30 PM

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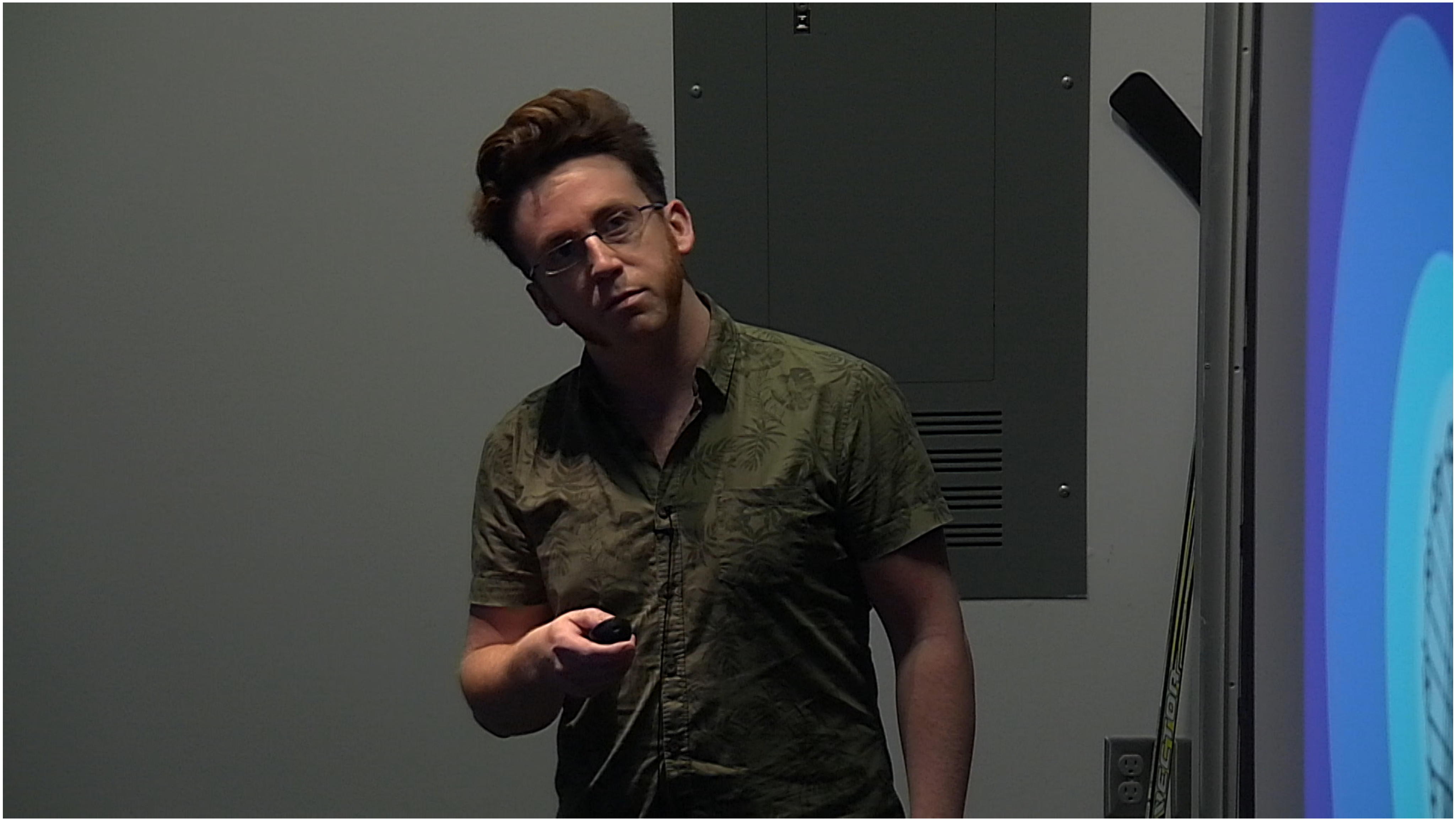
Abstract:

CONTEXTUALITY AS A RESOURCE FOR QUANTUM COMPUTATION

THE TROUBLE WITH QUBITS

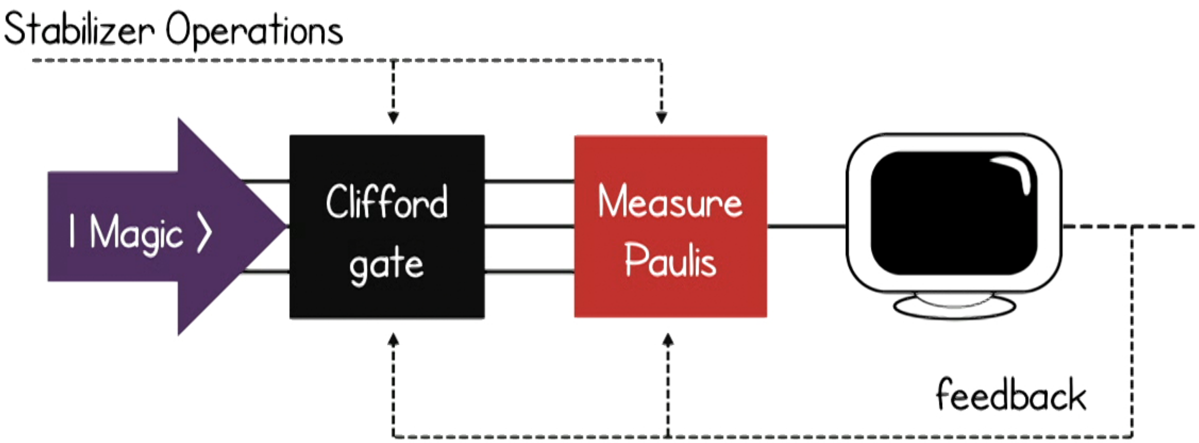
Juan Bermejo-Vega (FU Berlin)
Robert Raussendorf (UBC), Dan Browne (UCL),
Nicolas Delfosse (Station Q), Cihan Okay (UWO)

arXiv:1610.08529
PRA (2017), arXiv 1511.08506



Main Result

Contextuality is a resource for quantum computational universality in models of quantum computation with magic states **on qubits.**



Bravyi-Kitaev, PRA 2005

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Introduction

Contextuality what?

Clifford who?

Computation why?

Seriously, MAGIC? [1]

Introduction

Contextuality what?

Clifford who?

Computation why?

Seriously, MAGIC? [1]

Lucy Lawless, TV's Xena on "Xena: Warrior Princess"

"Frink: I see, alright, yes, but in episode AG04

Lucy Lawless: Wizard! "

– The Simpsons, "Treehouse of Horror X"

[1] A Wizard Did It, *TV Tropes*, http://tvtropes.org/pmwiki/pmwiki.php/Main/AWizardDidIt

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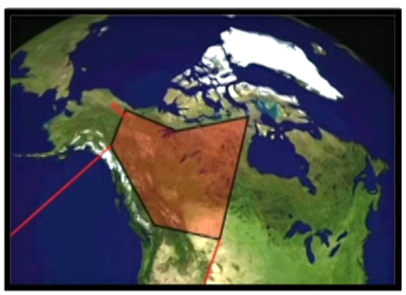
Computation Why?

Computation Why?

What powers quantum computation?



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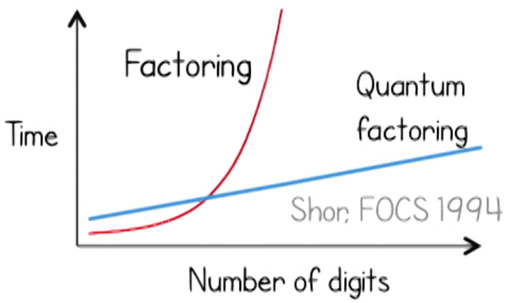
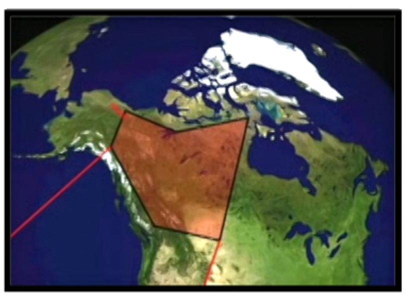
Computation Why?

What powers quantum computation?

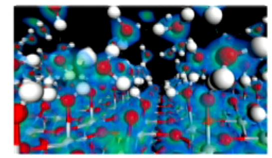




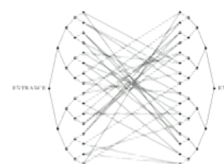
How practical is quantum computation?

Algebraic Problems

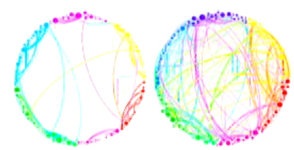
Quantum Simulation



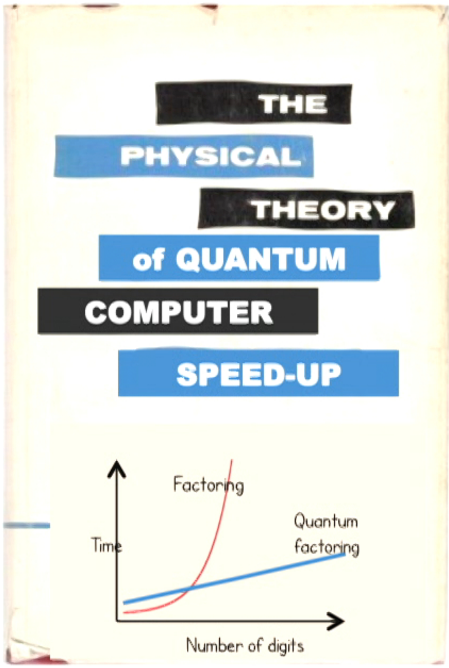
Graph Problems



Machine learning



Can we have a theory of quantum speed-ups?



Obstacles:

1. No classical analogue

6

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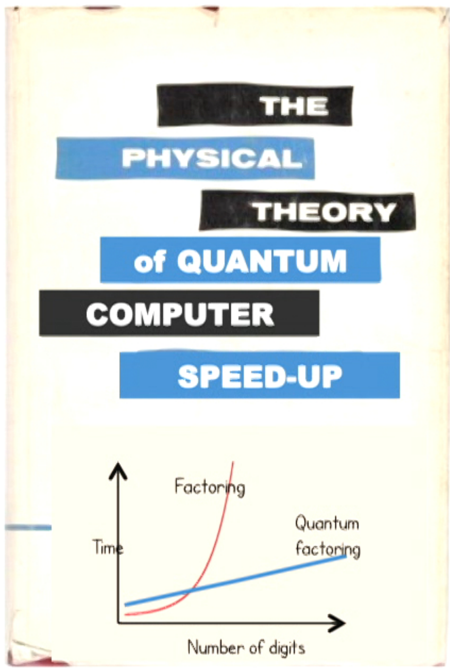


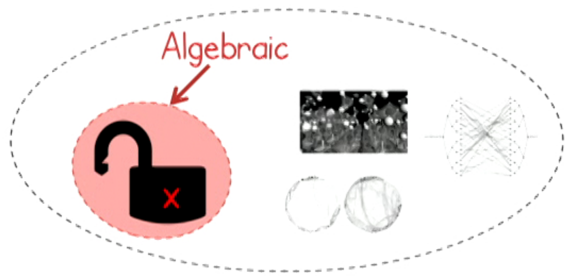
Obstacles:

1. No classical analogue
2. Emergence
3. Existence

♫♬ ♪ ♫♬ One Minute Commercial ♫♬ ♪ ♫♬

A sub-theory of quantum speed-ups

=> Normalizer Circuits and Quantum Computation, JBV, Technical University of Munich, arXiv:1611.09274



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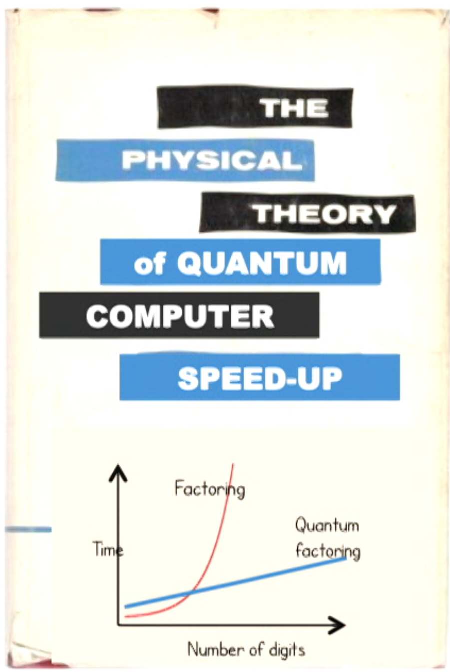

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This talk -> a more common approach:
resource theories based on contextuality

- Veitch, Ferrie, Gross, Emerson, NJP 2014, arXiv:1201.1256
- Mari, Eisert, PRL 2012, arXiv:1208.3660
- Howard, Wallman, Veitch, Emerson, Nature 2014, arXiv:1401.4174
- Delfosse, Allard Guerin, Bian, Raussendorf, PRX 2015, arXiv:1409.5170
- Raussendorf, Browne, Delfosse, Okay, Bermejo-Vega, PRA 2017, arXiv:1511.08506
- Bermejo-Vega, Delfosse, Browne, Okay, Raussendorf, arXiv:1610.08529

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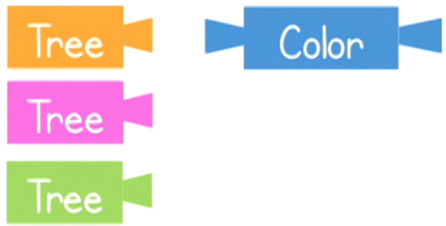
Quantum contextuality (with sharp measurements)

Quantum mechanics



Kochen, Specker, JMM 1967, Bell, RMP 1996

Quantum contextuality (with sharp measurements)

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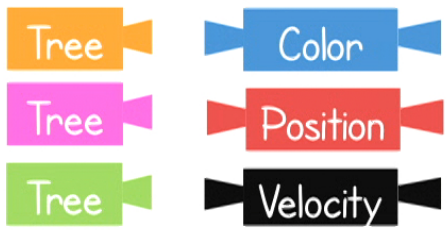

$\text{prob}(a|\text{Color}, \text{Tree}) = \text{tr}(P_a \rho)$

Kochen, Specker, JMM 1967, Bell, RMP 1996

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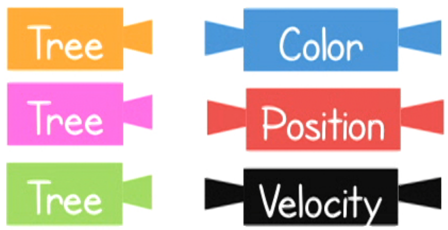
Quantum contextuality (with sharp measurements)

Quantum mechanics



prob(a|Color, Tree) = tr($P_a \rho$)

Definition: Non-contextual Hidden Variable Model

Value assignments / ontic states $\Lambda = \{\lambda_u\}_{u \in S}$

λ_1(Color) = orange
λ_2(Color) = pink

Kochen, Specker, JMM 1967, Bell, RMP 1996

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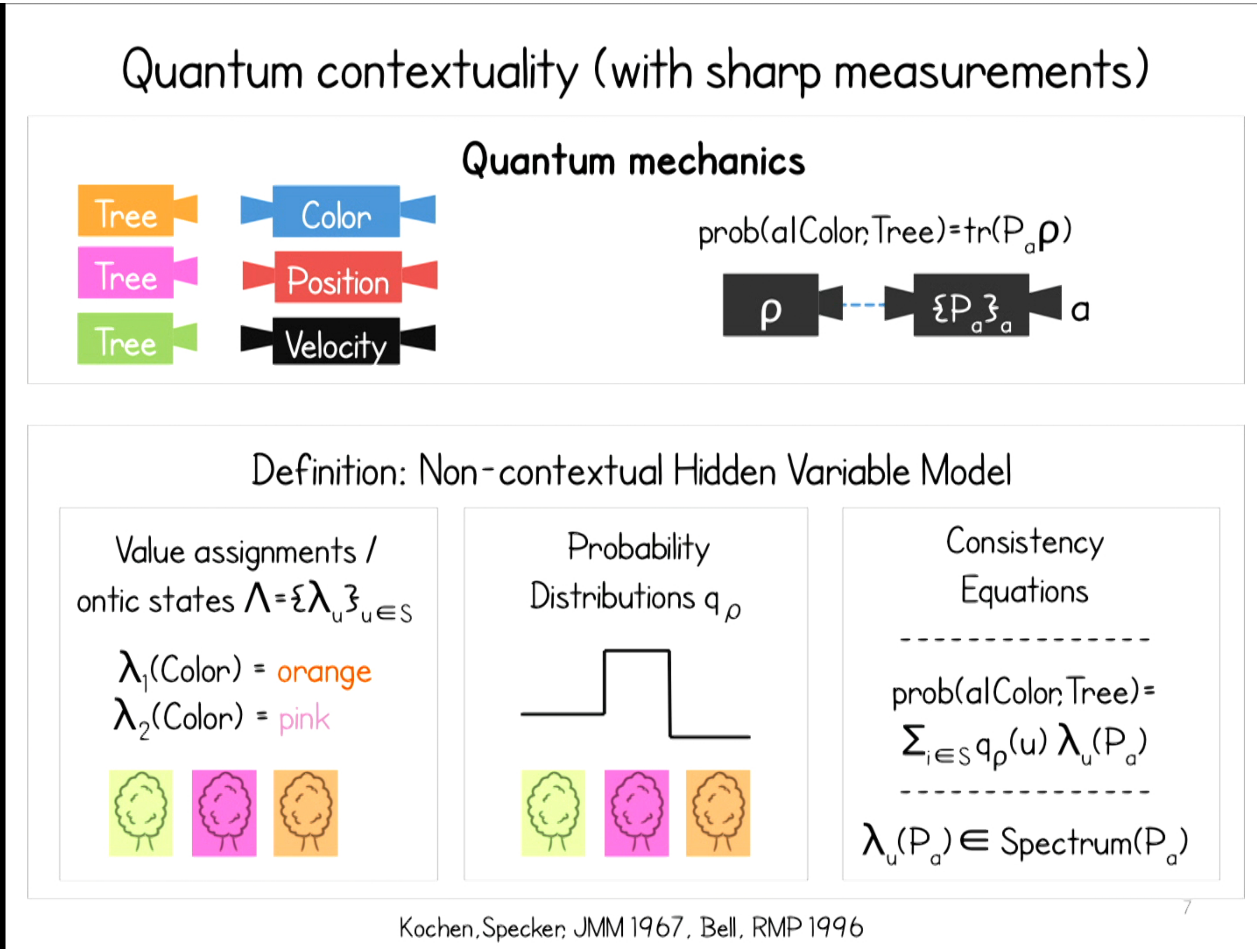
Quantum contextuality (with sharp measurements)
Quantum mechanics
Tree
Tree
Tree
Color
Position
Velocity
prob(a|Color, Tree)=tr(P_a ρ)
ρ
{P_a}_a
a
Definition: Non-contextual Hidden Variable Model
Value assignments /
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λ_1(Color) = orange
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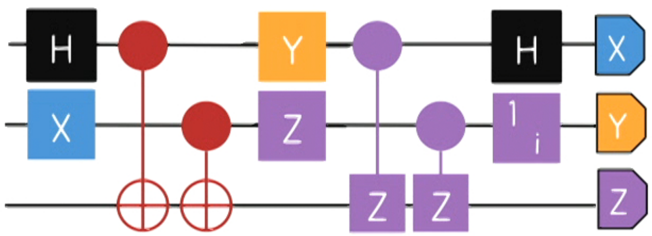
Probability
Distributions q_ρ
Consistency
Equations
prob(a|Color, Tree)=
Σ_{i∈S} q_ρ(u) λ_u(P_a)
λ_u(P_a) ∈ Spectrum(P_a)
7
Kochen, Specker, JMM 1967, Bell, RMP 1996

Clifford Who?
Quantum computation via state injection

Clifford gates



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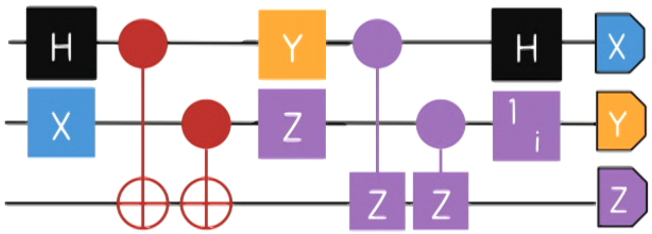
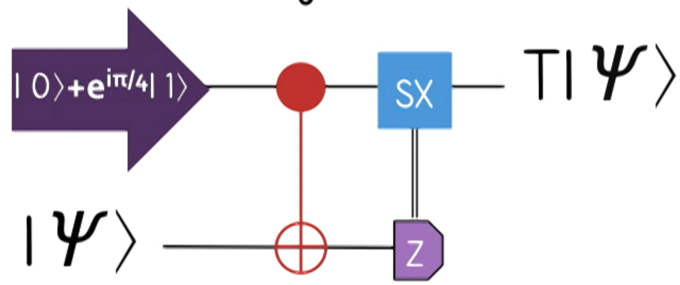
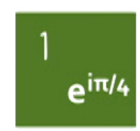
Clifford Who?
Quantum computation via state injection

Clifford gates



State injection



T = 

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Background

DiVincenzo, Peres, PRA 97 → Contextuality appears in quantum codewords

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Anders, Browne, PRL 2009
Hoban, Browne, PRL, 2011
Raussendorf, PRA 2013

} Contextuality is necessary in MBQC

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} Contextuality is necessary in QCSI with **quopits** and **rebits**

Re()

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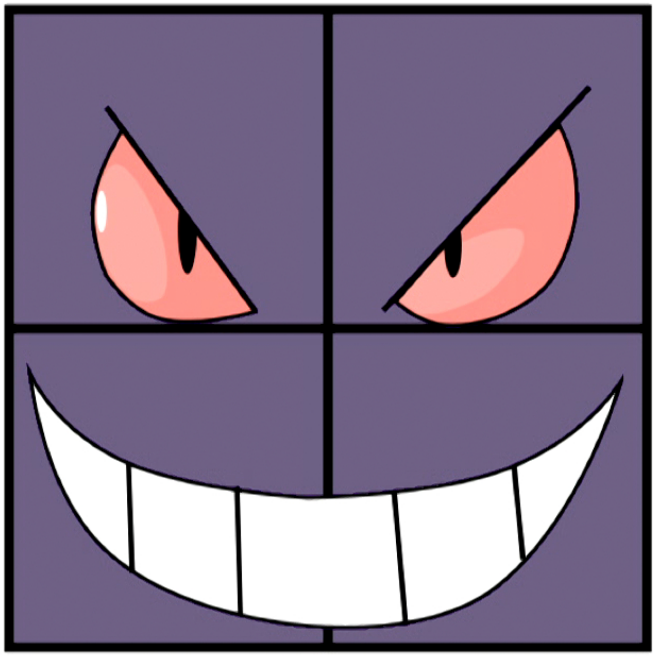
Contextuality Wars in Quantum Computation



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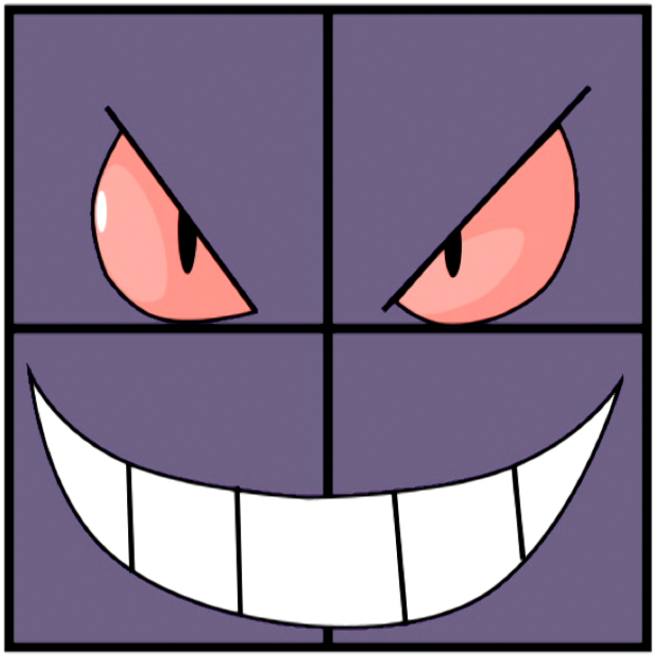
The trouble with qubits:
state independent contextuality



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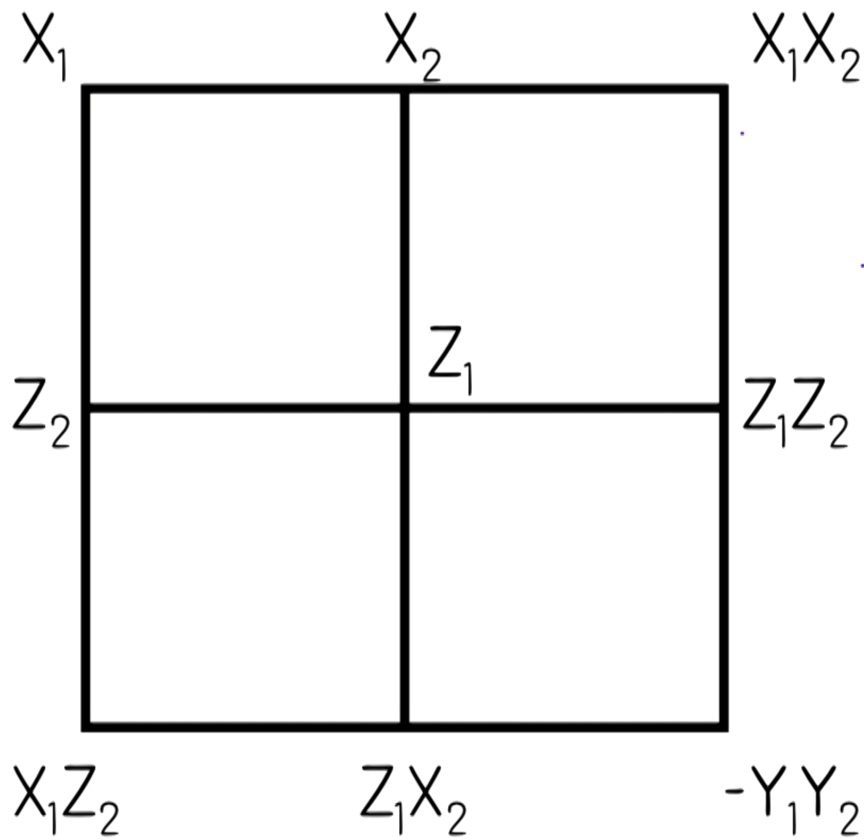

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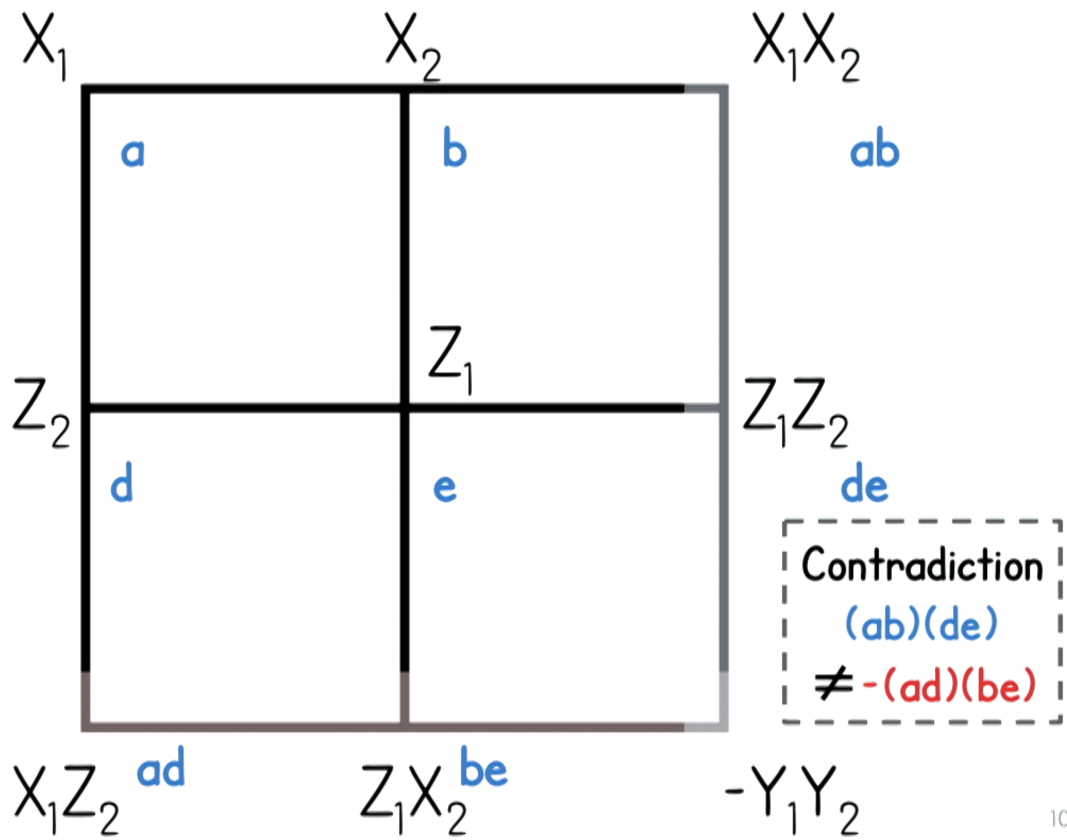

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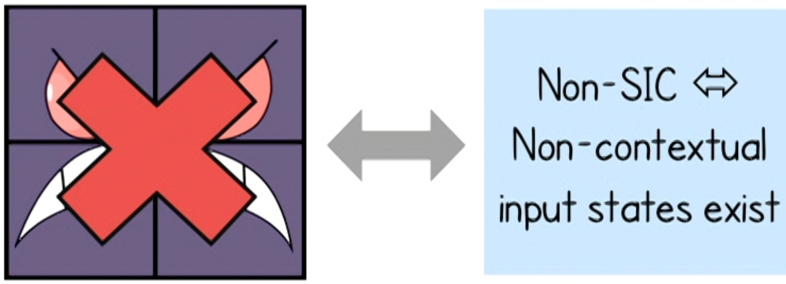
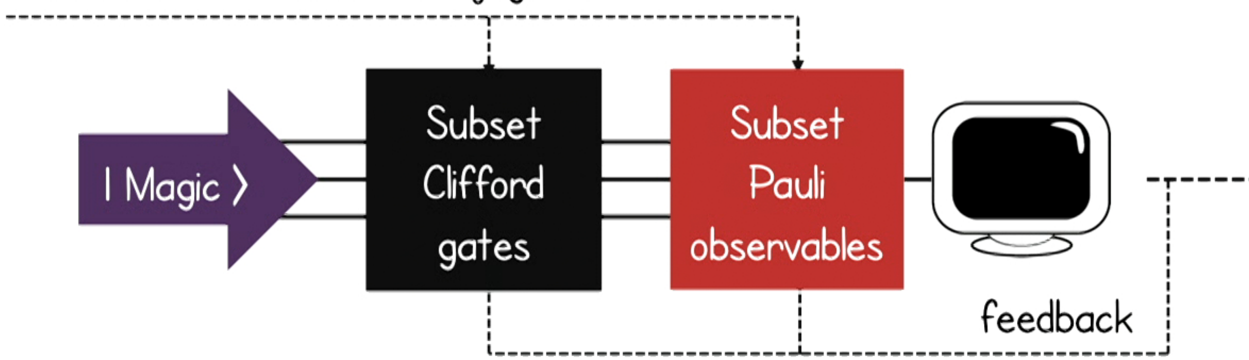
Main Result I:

Any **non-SIC** Clifford scheme of quantum computation via state injection on $n \geq 3$ qubits is universal *only if* its magic states exhibit contextuality (w.r.t. the scheme's operation)



Constraint: Clifford gates must preserve the Pauli observables under conjugation



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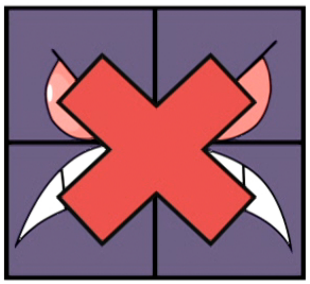
Main Result II

Universal state-injection schemes on **qubits** powered by contextual magic states (wrt the scheme's operations) **exist**



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Non-SIC ⇔ Non-contextual input states exist

+

Tomographic completeness

Main Result II

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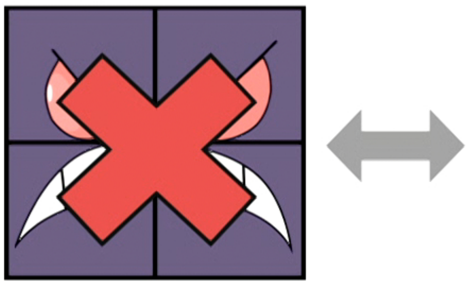
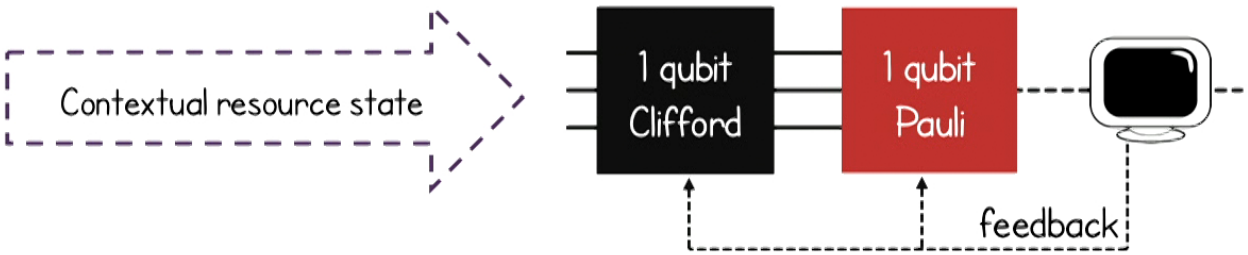


Non-SIC ⇔ Non-contextual input states exist + Tomographic completeness



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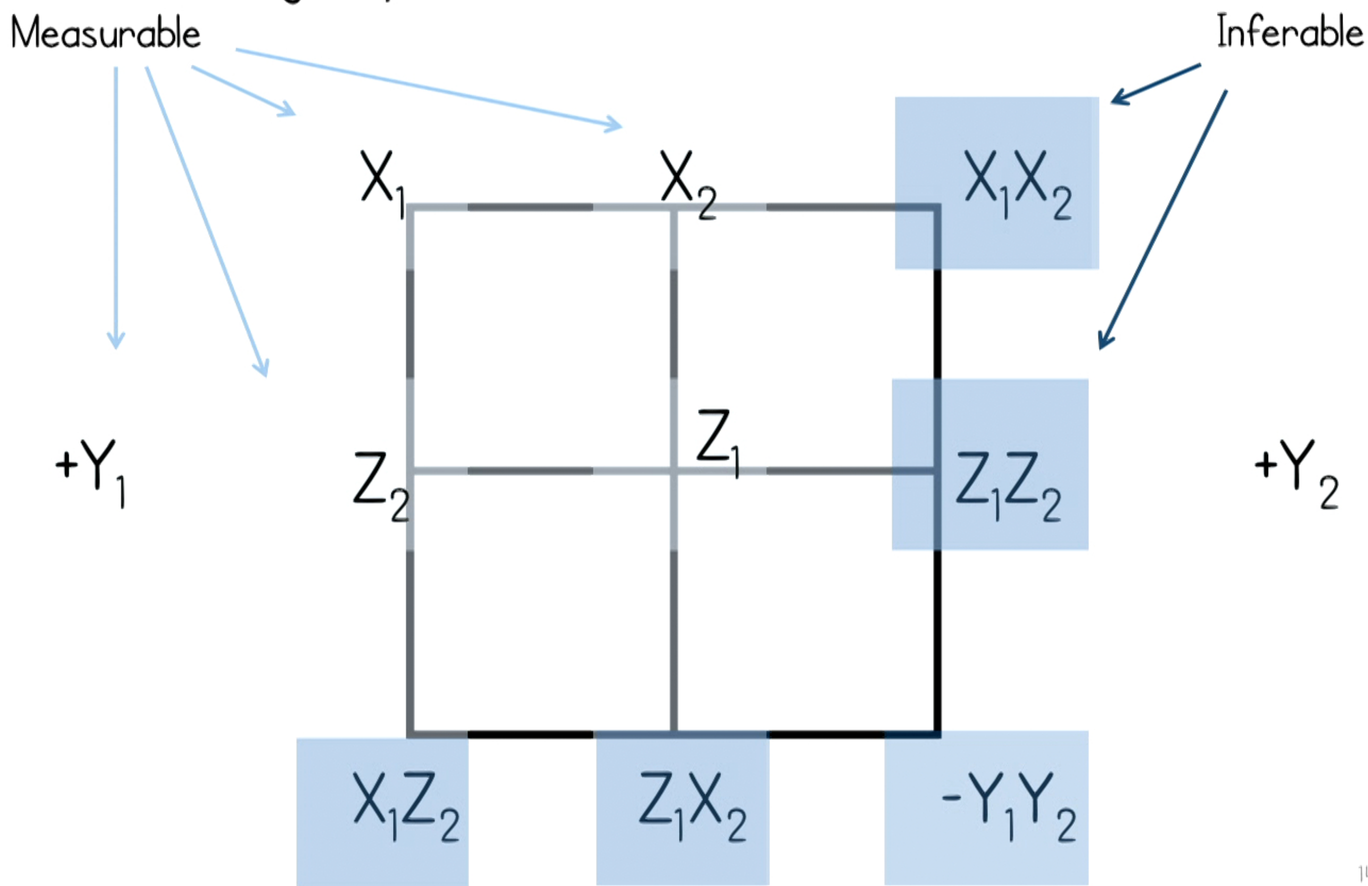
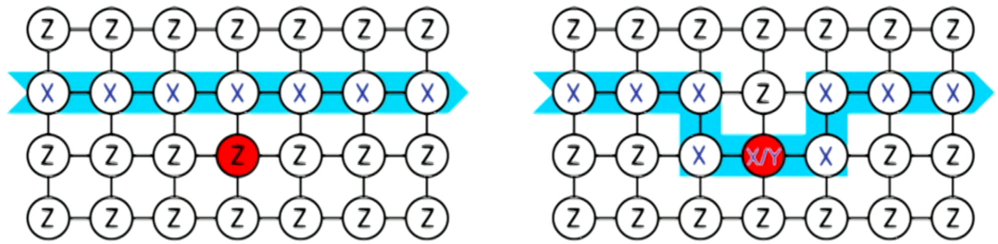
Main Result II:

Single-qubit Pauli measurements are not SIC



Main Result II

Magic resource state for 1-qubit Pauli measurements



Combine magic states
$T|+\rangle = |0\rangle + e^{i\pi/4}|1\rangle$
with regular $|+\rangle$ states.
Then apply CZs.

Basic MBQC tricks:

Left pattern = Identity gate

Right pattern = T gate

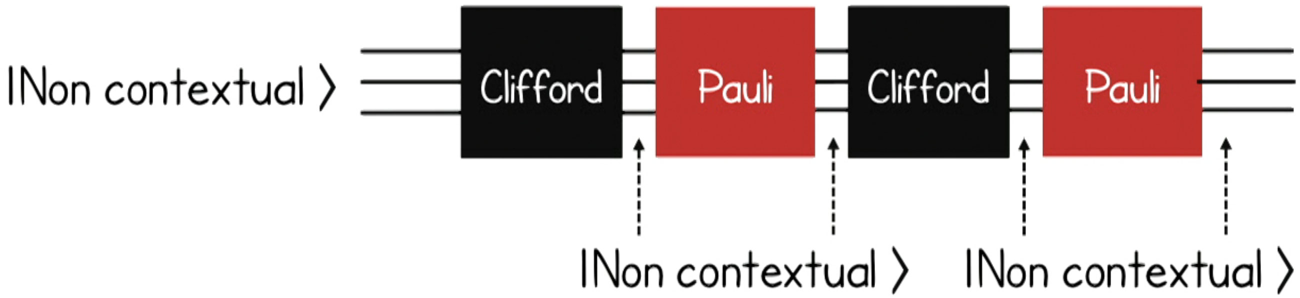
(Recall: $TXT^{\dagger} \approx X+Y$)

Raussendorf, Briegel, PRL 2001

Raussendorf, Browne, Briegel, PRA 2003

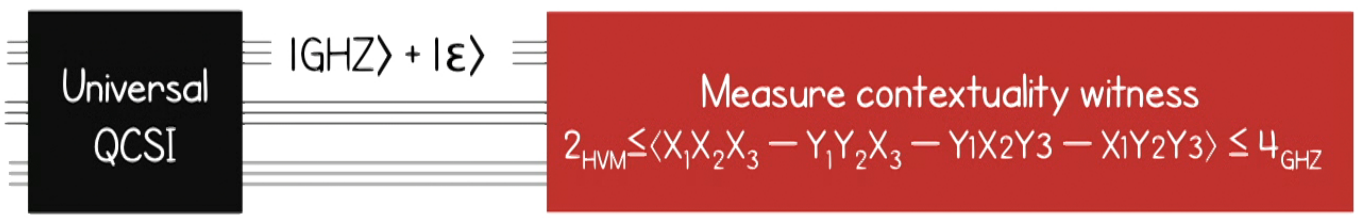
Proof of Main Result I

Non-SIC schemes cannot prepare contextual states



No universality

arXiv:1610.08529
PRA 2017, arXiv 1511.08506



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Non-contextual Hidden Variable Model

1. Identify Pauli operators with vectors of $\mathbf{Z}_2^{2n}$ -> $\sigma_a = i^{a_Z \cdot a_X} Z_1^{a_{Z_1}} \ldots Z_n^{a_{Z_n}} X_1^{a_{X_1}} \ldots X_n^{a_{X_n}}$
2. Define $[a,b] = a_Z \cdot b_X - a_Z \cdot b_X \bmod 2$
3. $a \in \mathbf{Z}_2^{2n}$ acts on value assignments λ_u -> $\lambda_{u+a}(\sigma_b) = \lambda_u(\sigma_b)\,(-1)^{[a,b]}$

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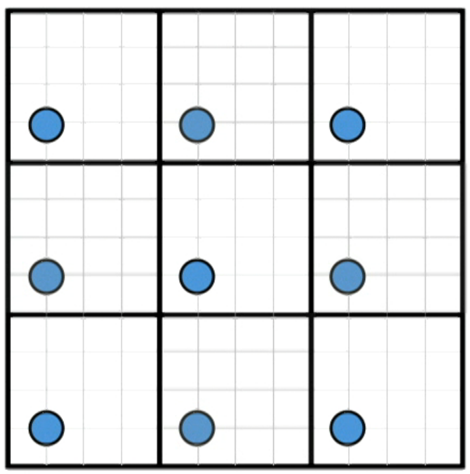
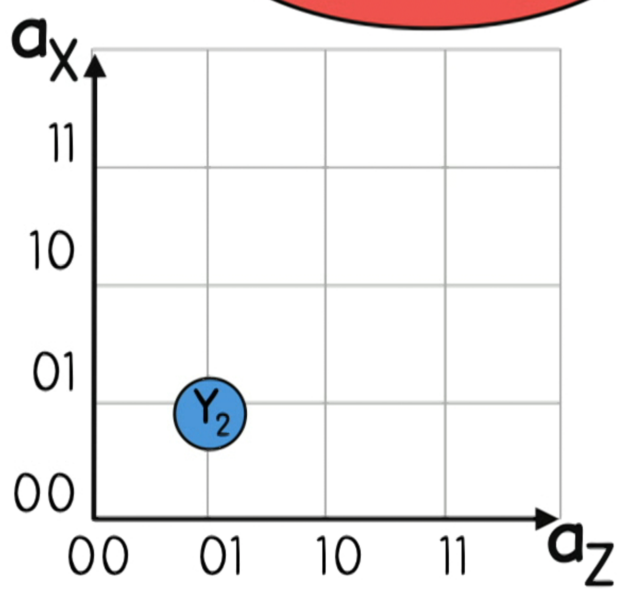
Z_2^{2n} divides $\{\lambda_u\}_u$ into equal orbits



Z_2^{2n}



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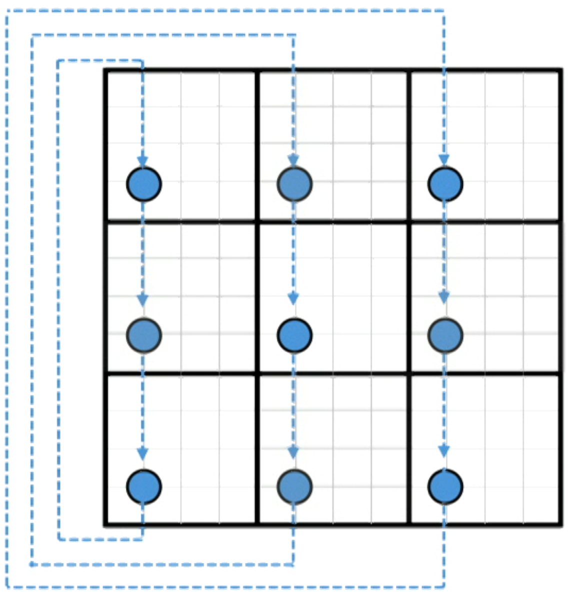
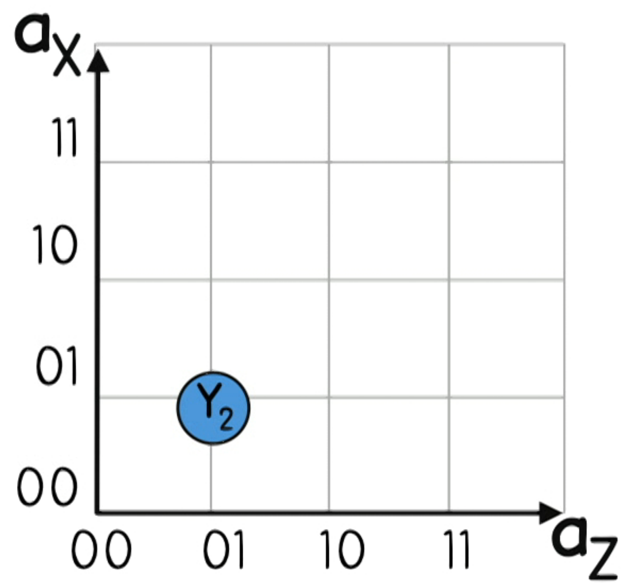
Unitary evolution



Z_2^{2n}



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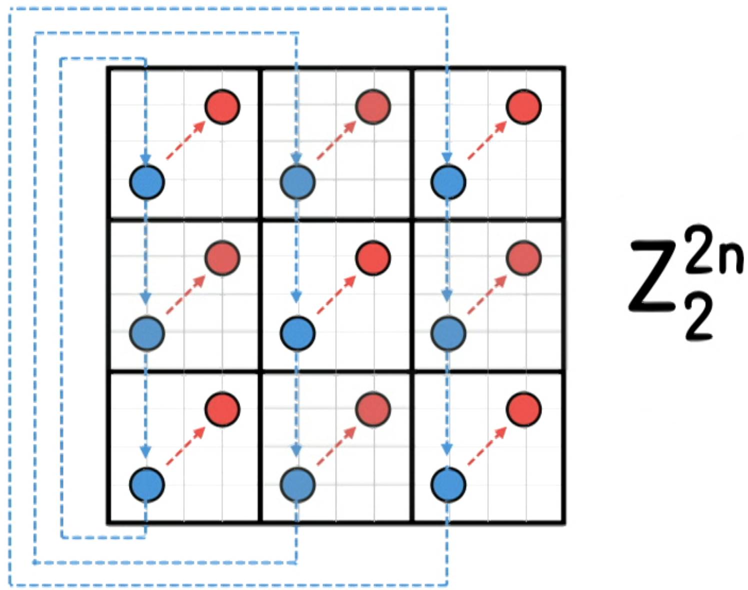
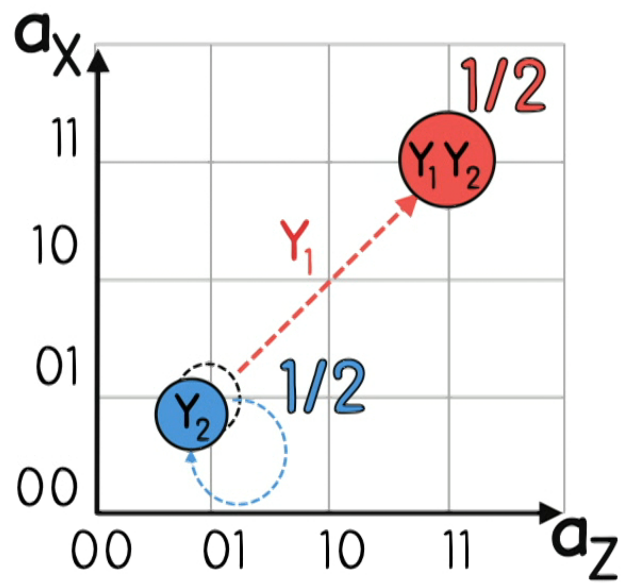


Measurement σ_a



Lawless' Principle

Algebraic constraints

$$+ \quad \lambda_u(\sigma_a\sigma_b)=\lambda_u(\sigma_a)\lambda_u(\sigma_b)$$

$$\sigma_a\sigma_b = \sigma_{a+b}$$

for commuting observables

INTUITION

1. Identify Pauli operators with vectors of $\mathbf{Z}_2^{2n}$ -> $\sigma_a = i^{a_Z \cdot a_X} Z_1^{a_{Z_1}} \ldots Z_n^{a_{Z_n}} X_1^{a_{X_1}} \ldots X_n^{a_{X_n}}$
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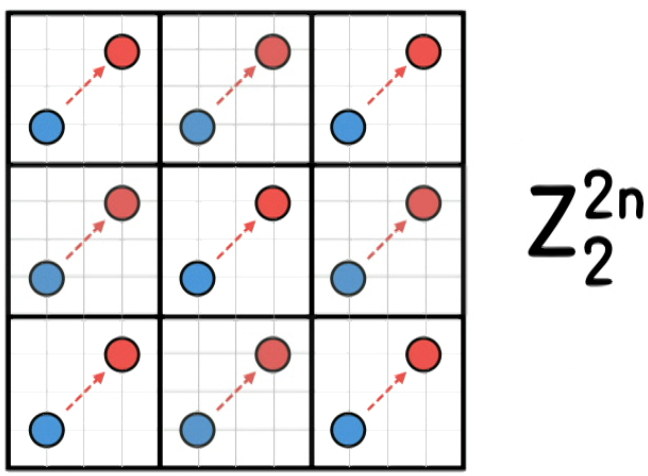

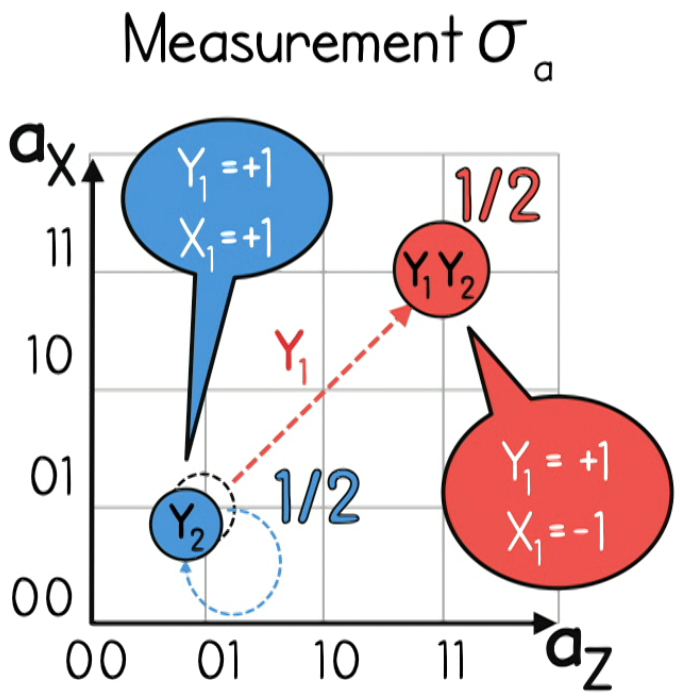

Connection with

Wigner functions

arXiv 1511.08506

Connection with Wigner Functions

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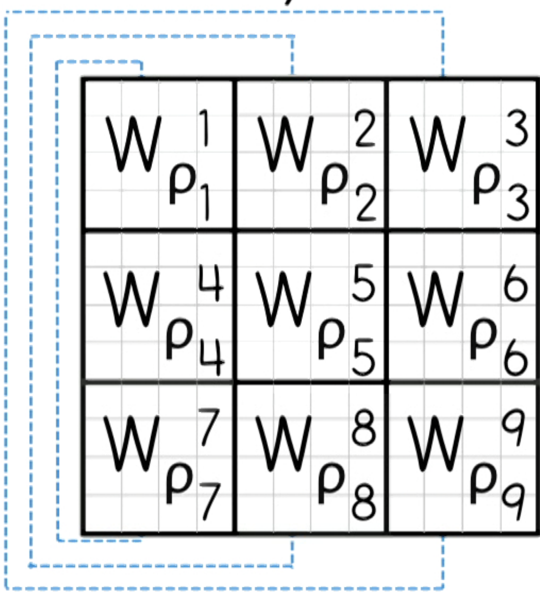


Non-contextuality ⇔

$\rho = \Sigma_a p_i \rho_i$

- λ_u^i is an orbit representative
- $W_\rho^i(u) = tr(A_u^i \rho) \geq 0$
- $A_0^i = 2^{-n} \Sigma_a \lambda_u^i(\sigma_a)\, \sigma_a$
- $A_u^i = \sigma_u A_0^i \sigma_u^\dagger$

Example: one Qubit case

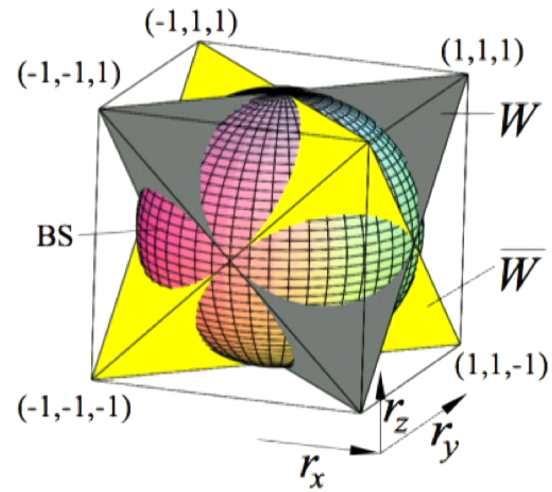
$$\sigma_a = i^{a_Z \cdot a_X} Z_1^{a_{Z_1}} \ldots Z_n^{a_{Z_n}} X_1^{a_{X_1}} \ldots X_n^{a_{X_n}} \quad \begin{cases} \lambda_{u+a}(\sigma_b) = \lambda_u(\sigma_b)\,\mathrm{sign}([\sigma_a, \sigma_b]) \\ \lambda_u(\sigma_a \sigma_b) = \lambda_u(\sigma_a)\,\lambda_u(\sigma_b) \end{cases}$$

W_ρ^1	W_ρ^2

$\lambda_u^1 : \{I, X, Y, Z\} \dashrightarrow \{+1, +1, +1, +1\}$

$\lambda_u^2 : \{I, X, Y, Z\} \dashrightarrow \{+1, +1, +1, -1\}$

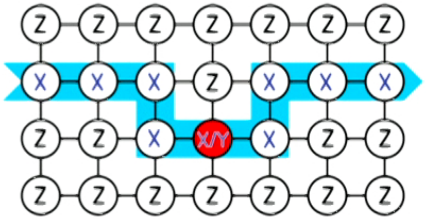
- λ_u^i is an orbit representative
- $W_\rho^i(u) = \mathrm{tr}(A_u^i \rho) \geq 0$
- $A_0^i = 2^{-n} \Sigma_a \lambda_u^i(\sigma_a)\, \sigma_a$
- $A_u^i = \sigma_u A_0^i \sigma_u^\dagger$



Galvao, PRA 05, quant-ph/0405070

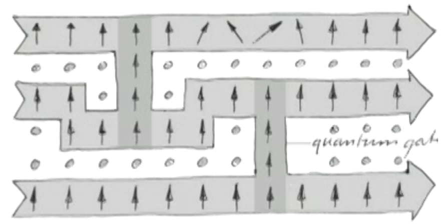
Summary

Contextuality is a resource for qubit quantum computation



arXiv:1511.08506

arXiv:1610.08529



Open Questions:

Sufficiency?

Quantifiability?

Beyond SIC-condition?

Classification?

Gottesman-Knill?

Other gate sets?

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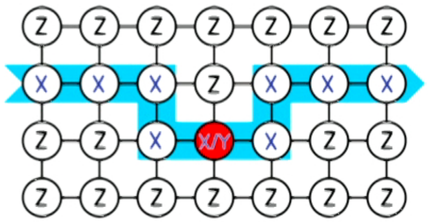

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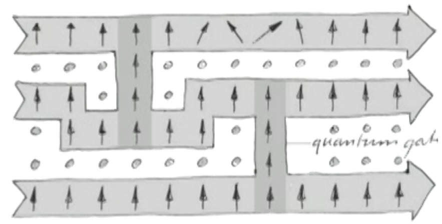

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THANKS

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