Title: Contextuality, the PBR theorem and their effects on simulation of quantum systems

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Abstract: This talk will be about constraints on any model which reproduces the qubit stabilizer sub-theory. We show that the minimum number of classical bits required to specify the state of an n-qubit system must scale as ~ n(n-3)/2 in any model that does not contradict the predictions of the quantum stabilizer sub-theory. The Gottesman-Knill algorithm, which is a strong simulation algorithm is in fact, very close to this bound as it scales at ~n(2n+1). This is a result of state-independent contextuality which puts a lower bound on the minimum number of states a model requires in order to reproduce the statistics of the qubit stabilizer sub-theory.

Contextuality, PBR and their effect on the simulation of quantum systems

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The main result

The minimum number of classical bits required to specify the state of an n-qubit system in any model that reproduces stabilizer statistics is

$$\frac{n}{2}(n-1)$$

Overview

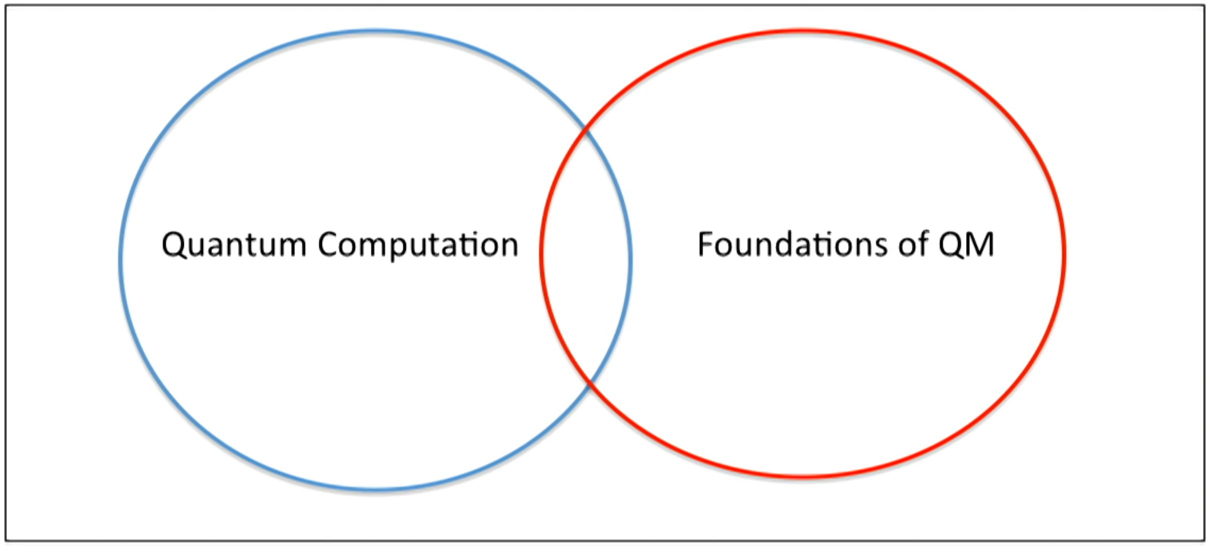
- Why should you care?
- How is it related to contextuality?
- How did we do it?
- What now?

What does it mean to simulate quantum statistics?

	P_1	P_2	.....	P_n
M_1^1				
M_1^2				
$\vdots$				
M_m^l				

$$\Pr(k \mid P, M)$$

Why should one care about simulation of Quantum systems?



Context

- Stabilizer sub-theory: Fault tolerant quantum computation
- Universal quantum computation: injecting "magic" states into stabilizer circuits

Qudits:

- magic states ⟺ Contextuality
- Non-negative Wigner functions ⟹ efficient classical sampling

Qubits: simulability

- state-independent contextuality
- Contextuality a computational resource?
- No efficient classical sampling

What we show

Qubits:

- The explicit effect of state-independent contextuality on size of the state-space of model
- Qubit stabilizer sub-theory is efficiently simulatable because the number of quantum states grows nicely
- A sampling algorithm cannot do much better than Gottesman-Knill

n-Qubit Stabilizer sub-theory

- Measurements: n-qubit Pauli Observables

- Preparations: eigenstates of n-qubit Pauli operators

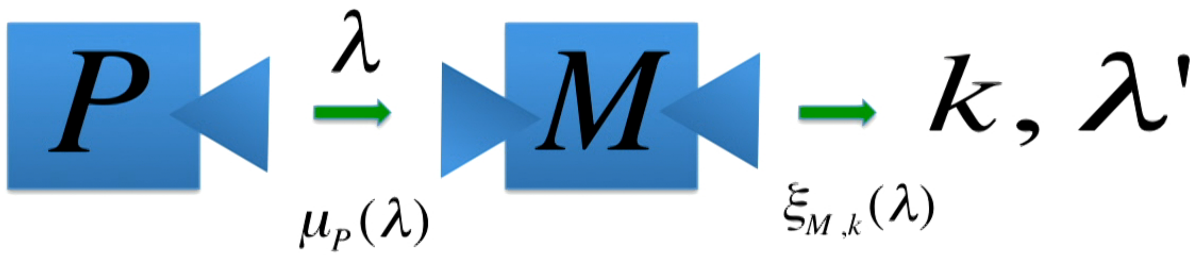
- Transformations: Clifford Unitaries

Ontological Models

- State of the system $\lambda \epsilon \Lambda$
- $\Pr(\lambda|P) = \mu_P(\lambda)$
- $\Pr(k|M,\lambda) = \xi_{k,M}(\lambda)$

Reproduce quantum predictions:

$$\Pr(k|M,P) = \sum_{\Lambda} \mu_P(\lambda)\, \xi_{k,M}(\lambda) = Tr(\Pi_k \rho)$$


P
λ
$\mu_P(\lambda)$
M
$\xi_{M,k}(\lambda)$
k, λ'

Perfectly distinguishable preparation procedures cannot have ontic overlap

$$Supp(P_\rho) \cap Supp(P_\sigma) = \varnothing, \quad Tr(\rho\sigma) = 0$$

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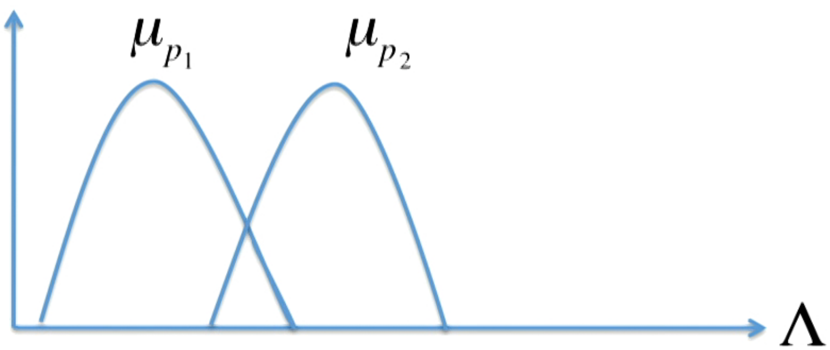
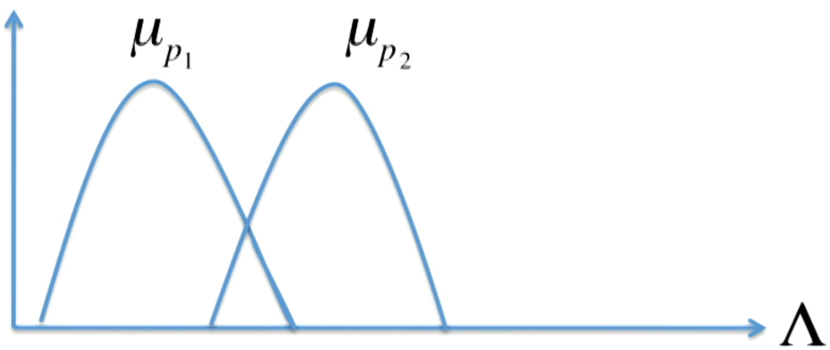
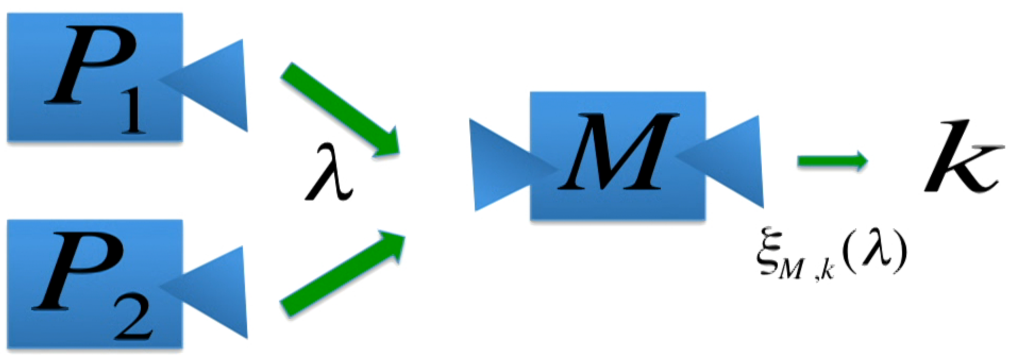


Perfectly distinguishable preparation procedures cannot have ontic overlap

$$Supp(P_\rho) \cap Supp(P_\sigma) = \emptyset, \quad Tr(\rho\sigma) = 0$$





The state of the system can be described after a non-demolition measurement

$$\rho \rightarrow \rho'$$

$$\lambda \in Supp(P_{\rho}) \rightarrow \lambda' \in Supp(P_{\rho'})$$

Two requirements:

1. Experimentally distinguishable states have disjoint support:

$$Supp(P_{\rho_i}) \cap Supp\left(P_{\rho_j}\right) = \emptyset, \quad Tr(\rho_i \rho_j) = 0$$

2. The state of the system can be described even after a measurement:

$$\rho \to \rho'$$

$$\lambda \in Supp(\rho) \to \lambda' \in Supp(\rho')$$

PBR

$$\bigcap_{PBR} Supp(\rho_i) = \emptyset$$

Proof:

$$\rho_1 = \{XI, IX, XX\} \qquad \rho_1' = \{YY, -ZZ, XX\}$$

$$\rho_2 = \{ZI, IZ, ZZ\} \quad \xrightarrow{YY=1} \quad \rho_2' = \{YY, ZZ, -XX\}$$

$$\rho_3 = \{XI, IZ, XZ\} \qquad \rho_3' = \{YY, XZ, ZX\}$$

$$\rho_4 = \{ZI, IX, ZX\} \qquad \rho_4' = \{YY, XZ, ZX\}$$

Contextuality restricts overlap between states

	ρ_3	ρ_4	
ρ_1	X_1	X_2	XX
ρ_2	Z_2	Z_1	ZZ
	XZ	ZX	YY

Result applies to sets equivalent to PBR set

$$Def : s = \{\rho_i\}, h = \{\sigma_i\}, s \sim h \; iff \; \exists C \; s.t. \; C^+ \rho_i C = \sigma_i$$

$$\bigcap_{e(PBR)} Supp(\rho_i) = \varnothing$$

Proof:

$$C^+ \begin{matrix} \rho_1 = \{XI, IX, XX\} \\ \rho_2 = \{ZI, IZ, ZZ\} \\ \rho_3 = \{XI, IZ, XZ\} \\ \rho_4 = \{ZI, IX, ZX\} \end{matrix} C \xrightarrow{C^+(YY)C = -1} C^+ \begin{matrix} \rho_1' = \{-YY, ZZ, XX\} \\ \rho_2' = \{-YY, ZZ, XX\} \\ \rho_3' = \{-YY, XZ, -ZX\} \\ \rho_4' = \{-YY, -XZ, ZX\} \end{matrix} C$$

Other PBR like sets with empty overlap

$$e\{\langle ZI,IZ\rangle,\langle XI,IX\rangle,\langle XI,IY\rangle,\langle YI,IZ\rangle\}$$

$$e\{\langle ZI,IZ\rangle,\langle XI,IX\rangle,\langle XI,IY\rangle,\langle YI,IY\rangle\}$$

$$e\{\langle ZI,IZ\rangle,\langle XI,IX\rangle,\langle XI,IY\rangle,\langle XX,ZY\rangle\}$$

All sets can be used to construct proofs of contextuality

Other sets with empty overlap

For a system of 2 qubits,

$$\bigcap_{s} Supp(\rho_i) = \varnothing, \forall |s| > 5$$

Proof:

One cannot construct any set of states with more than 5 states, such that one of its subsets of 4 is not PBR like.

n-qubits

For a system of n qubits,

$$\bigcap_{s} Supp(\rho_i) = \varnothing, \forall |s| > 3^{n-2}5$$

Proof: On the board (If I have time)

n-qubits

$$\bigcap_s Supp(\rho_i) = \varnothing, \forall |s| > 3^{n-2}5$$

This implies that any ontic state can be in support of at most $3^{n-2}5$ stabilizer states (preparation procedures corresponding to $3^{n-2}5$ stabilizer states) .

Min no. ontic states required = (no.of stabilizer states) / (max no. of states the ontic state can be in the supp of)

$$\min |\Lambda| = \frac{|stab|}{\max |s|}$$

n-qubits

$$\min |\Lambda| \sim 2^{\frac{n^2}{2} - \frac{1}{2}n}$$

Minimum number of classical bits required to specify ontic state:

$$\sim \frac{1}{2} n(n-1)$$

Gottesman-Knill simulation:

$$n(2n+1)$$

Answers to questions about contextuality and qubit stabilizers

Q: What is the effect of the presence of contextuality in the qubit sub-theory on simulation?

A: No model can do much better than Gottesman-Knill
→ min. information required for any model is asymptotically $\sim n^2$

Q: How is it different from the qudit sub-theory?

A: The absence of contextuality allows a sampling algorithm to do better than Gottesman-Knill.

Wigner function $\sim n$

Contextuality: an explicit link to classical simulation

- Can this approach be applied to other sub-theories?
- Can we develop a measure of contextuality that has a direct link to simulability?

Contextuality: an explicit link to classical simulation

Definition 3.1.1 *A non-contextual value assignment for a set of observables* $O = \{O_i | i = 1, ..n\}$ *is a function* $\nu : O \to R$ *such that* $\nu(O_i)$ *is an eigenvalue of the hermitian operator describing* O_j *and* $\nu(O_i O_j) = \nu(O_i)\nu(O_j)$ *if* O_i *and* O_j *commute.*

Kochen-Specker proof ➔ No non-contextual value assignment possible

Contextuality: an explicit link to classical simulation

Theorem: The eigenstates of a set of observables that do not allow a non-contextual value assignment cannot have an ontic overlap

- The largest set of quantum states that can be simulated by a single ontic state is the largest set that does not allow a proof of contextuality
- Min. size of ontic space bounded by the size of the largest set of states that does not allow a proof of contextuality

Summary

- A link between contextuality in qubit stabilizer sub-theory
- A bound on the size of the state space of any model that reproduces qubit- stabilizer statistics
- Can this approach be applied to other quantum sub-theories?