

Title: A physical picture for quantum contextuality

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Abstract:

A PHYSICAL PICTURE FOR QUANTUM CONTEXTUALITY

Giulio Chiribella

*Department of Computer Science, The University of Hong Kong,
CIFAR-Azrieli Global Scholars Program*

Contextuality: Conceptual Issues, Operational Signatures, and Applications
PI, July 24-28 2017

CIFAR
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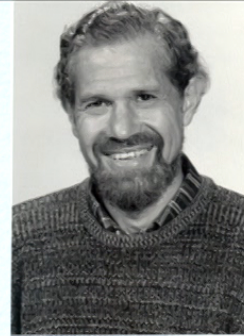
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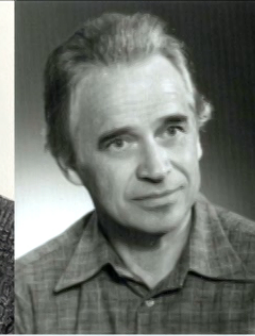
RECRUITMENT
PROGRAM OF GLOBAL EXPERTS

CONTEXTUALITY

Quantum physics is strikingly different from classical physics.



S. Kochen



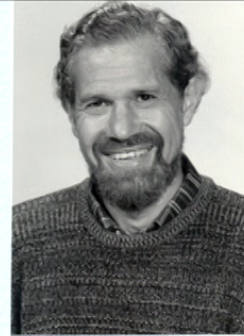
E. Specker

One of the most fundamental non-classical features is **contextuality**
(Kochen and Specker, 1967)

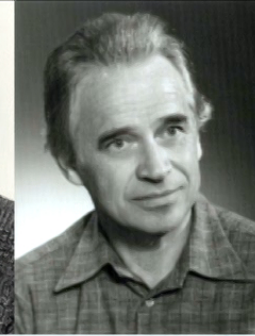
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Also a **resource** for quantum information processing

→ important to understand how contextuality works

A PHYSICAL PICTURE?

Quantum contextuality raises two fundamental questions:

- 1) Why is Nature contextual?
- 2) Why is Nature contextual *in the particular way* dictated by quantum theory?

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- 1) Why is Nature contextual?
- 2) Why is Nature contextual *in the particular way* dictated by quantum theory?

Both questions cannot be addressed *within* quantum theory.

They require

- a **framework**, describing possible alternatives to QT
- a **set of physical principles**, determining the “amount of contextuality” seen in Nature

STATE OF THE ART

Framework: general probabilistic theories (GPTs)

Principles: QT *can* be derived from various sets of principles
e.g.

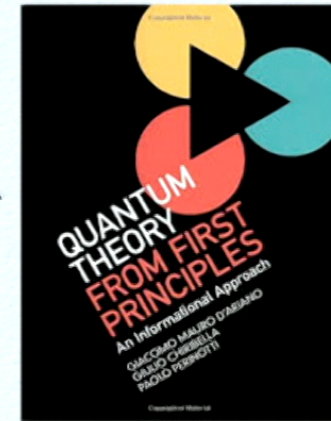
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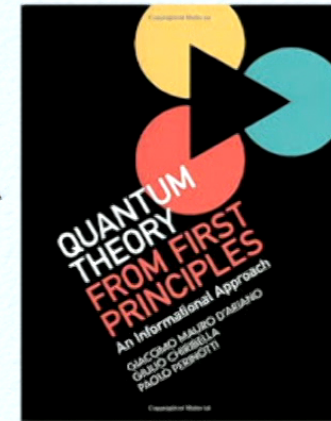
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What is missing?



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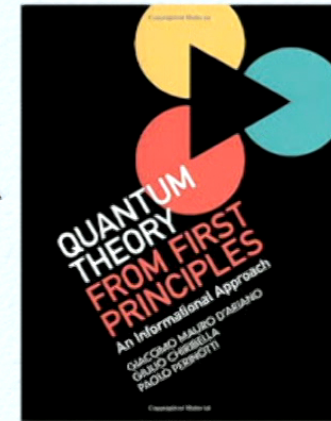
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What is missing?

A **direct** and **compelling** explanation of contextuality



THIS TALK

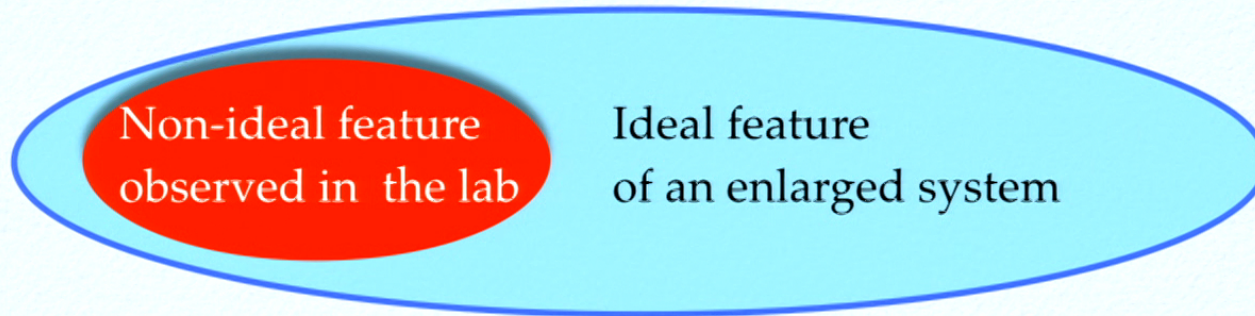
In this talk, I will give:

- 1) a **definition** of contextuality in GPTs
- 2) a **principle** that bounds contextuality in every GPT
- 3) a **physical picture** about the origin of quantum contextuality

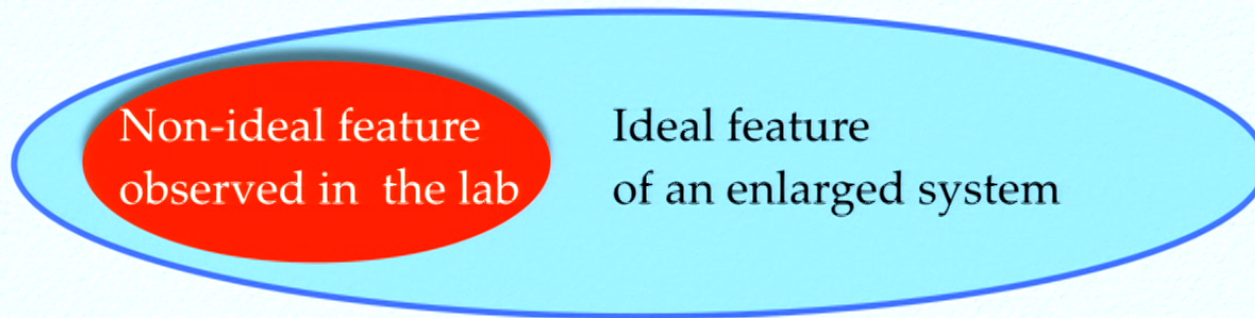
KEY IDEA: THE “PURIFICATION PHILOSOPHY”

Non-ideal feature
observed in the lab

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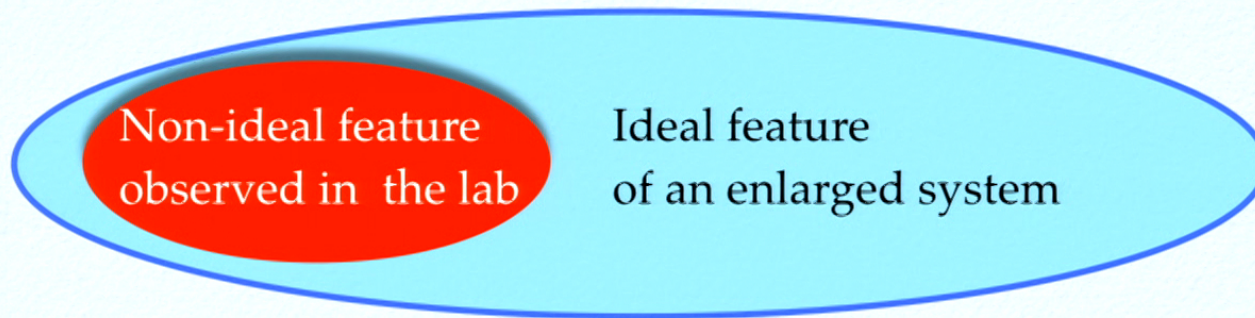


KEY IDEA: THE “PURIFICATION PHILOSOPHY”



	Non-ideal	Ideal
States	Mixed	Pure
Evolutions	Irreversible	Reversible
Measurements	Unsharp	Sharp

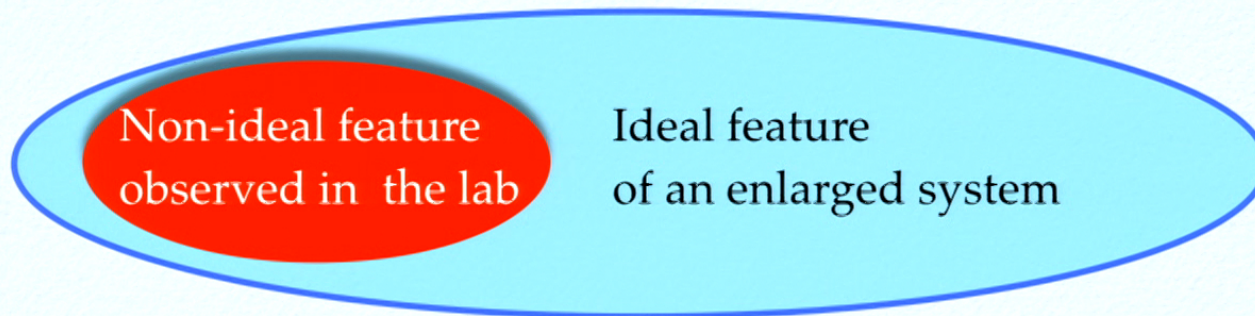
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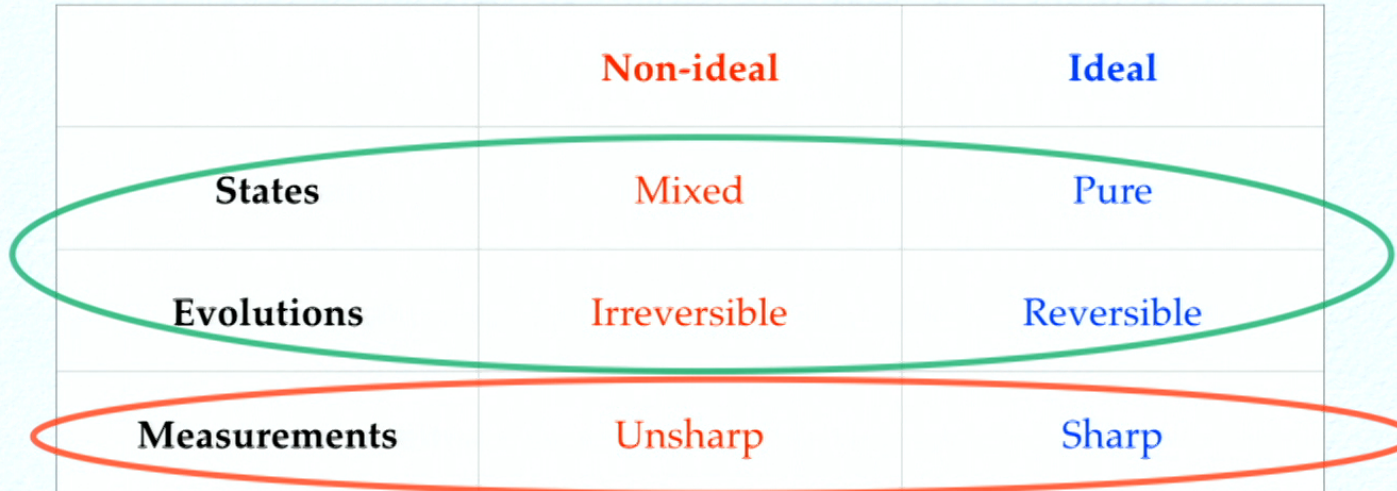


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 1404.3348

A PICTURE FOR CONTEXTUALITY

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“Purification brings [contextuality] in,
Sharpness cuts it down”
from Chiribella-Yuan, 1404.3348

HEURISTICS

Purification

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Different notions / frameworks.

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Probability distribution $p(y | x)$ “Nature’s response strategy”

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Probability distribution $p(y | x)$ “Nature’s response strategy”

KS contextuality: Nature’s response strategy $p(y | x)$
is incompatible with the existence of
“predetermined answers”

GRAPH-THEORETIC FRAMEWORK

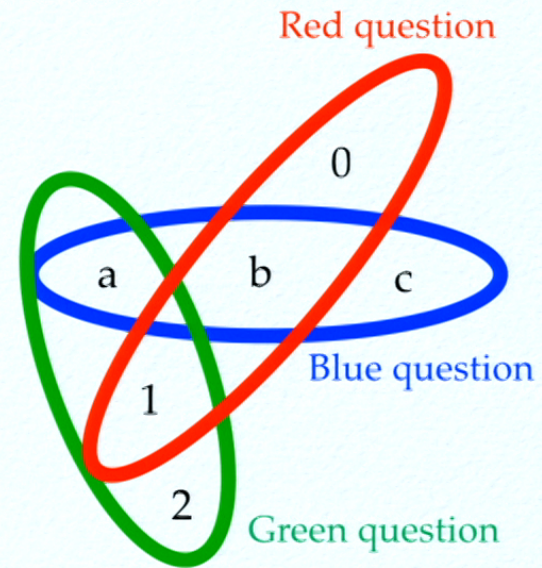
Question x = set

Answer y = element in that set

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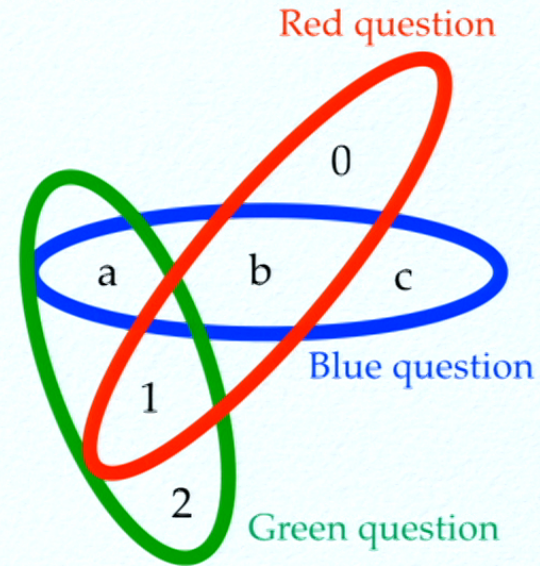
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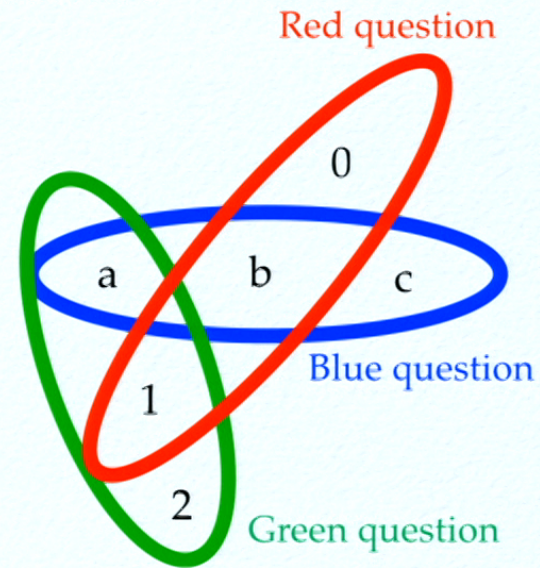
$$p(y|x) = p(y|x') \quad \forall y \in x \cap x'$$

Definition:

response is compatible with predetermined answers if

$$p(y|x) = \sum_{\lambda} p(\lambda) \delta_{y, f(x, \lambda)}$$

$$f(x, \lambda) \in x' \implies f(x', \lambda) = f(x, \lambda)$$



QUANTIFYING CONTEXTUALITY

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Given a set of allowed strategies $p(y|x)$
the amount of contextuality of that set is quantified by
the maximum of the payoff over $p(y|x)$

THREE IMPORTANT VALUES

ω_C = maximum payoff if answers are predetermined

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$$p(y|x) = \left[P_y^{(x)} \rho \right] \quad \begin{array}{l} \rho \text{ density matrix} \\ \text{(of arbitrary dimension)} \end{array}$$

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ω_G = maximum payoff over all $p(y|x)$

Key observation: $\omega_C \leq \omega_Q \leq \omega_G$

Problem: find physical principles that determine ω_Q

THE BIG CAVEAT

PROJECTIVE, PROJECTIVE, PROJECTIVE!

The restriction to **projective** measurements is **essential**
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Ana Belen Sainz, PhD thesis, 2013

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If we allow arbitrary quantum measurements (POVMs), the maximum payoff is

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→ **it is the restriction to projective measurements that makes the problem non-trivial!**

Ana Belen Sainz, PhD thesis, 2013

THREE RELATED ISSUES

ISSUE 1: WHY SHOULD WE CARE ABOUT PROJECTIVE MEASUREMENTS?

After all, they are **suboptimal** for many tasks
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But what about *us*?

Is there any *operational* reason to care
about projective measurements?

ISSUE 2: WHAT IS A PROJECTIVE MEASUREMENT OUTSIDE QUANTUM THEORY?

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But *what type*?

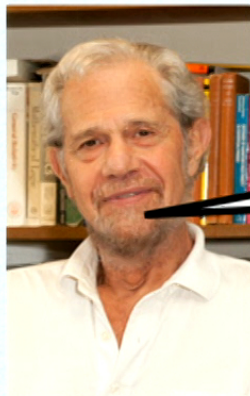
What is the definition of “projective measurement” in GPTs?

ISSUE 3: WHAT COUNTS AS A COMPELLING PHYSICAL PRINCIPLE?

A fundamental principle for KS contextuality is the Exclusivity principle (E-principle):

“if every two events in a set are mutually exclusive,
then all events in the set are mutually exclusive”

(Specker 1960, “Orthocoherence”, Cabello 2012,
Henson 2012)



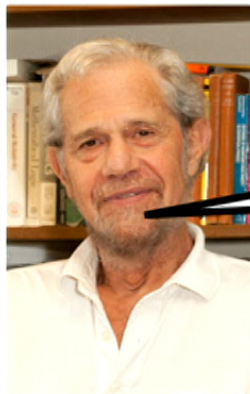
S. Kochen

Ernst and I spent many hours discussing the principle.

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The difficulty lays in trying to justify it
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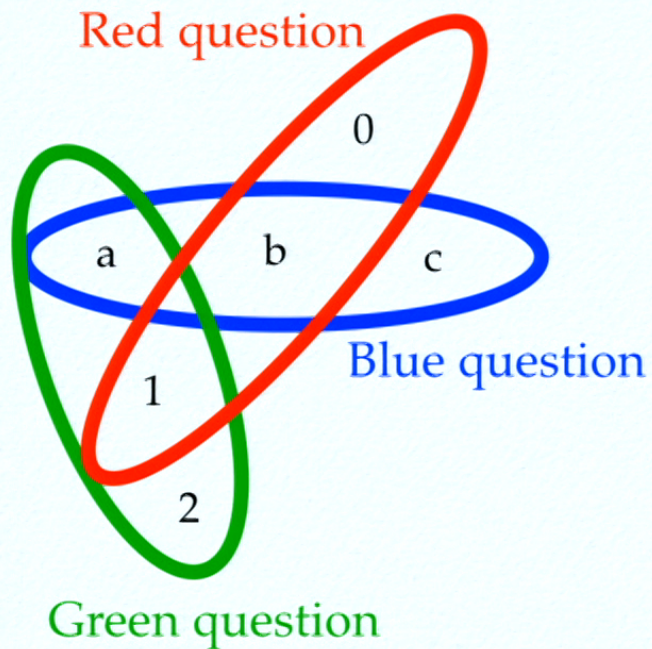
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in A. Cabello,
arXiv:1212.1756



WHY SHOULD *THIS* BE A FUNDAMENTAL LAW OF NATURE?



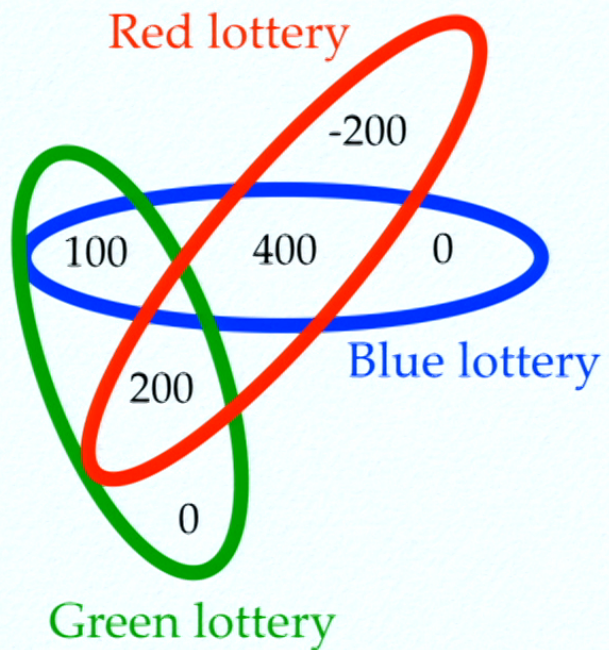
The pairs
 $\{a,b\}$, $\{a,1\}$, and $\{b,1\}$
are mutually exclusive

The exclusivity principle
demands

$$p(a) + p(b) + p(1) \leq 1$$

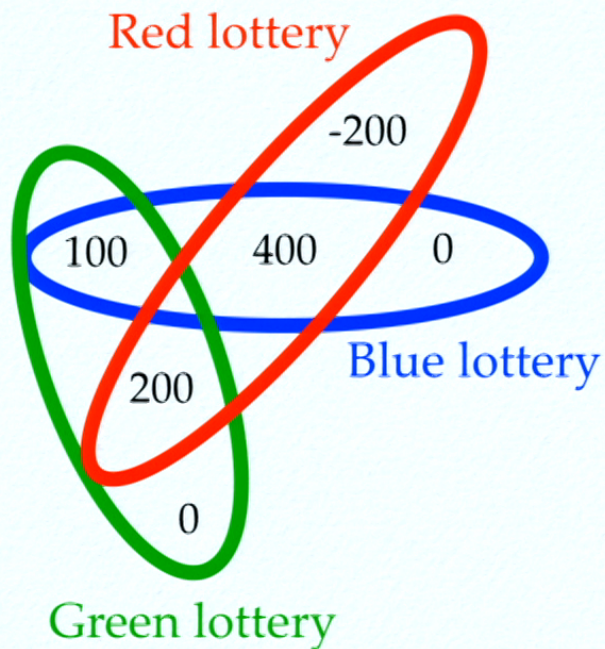
EXAMPLE, C'TD

Three lotteries, you can buy a ticket for one of them.



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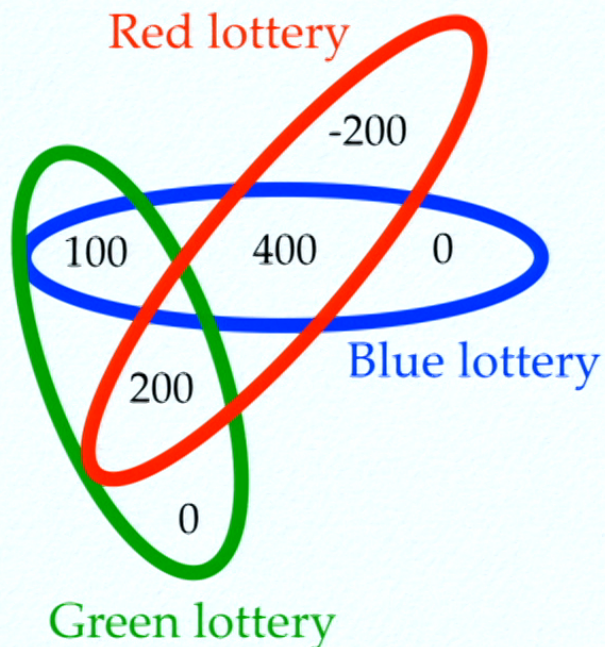
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But it is
**in contradiction with
the E-principle.**

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To find a justification,
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To find a justification, we need to remember that, in QT, the E-principle is a property of **projective measurements**.

To justify the E-principle in general physical theories, we must study **projective measurements in GPTs**.


DEFINITION OF PROJECTIVE MEASUREMENTS IN GPTS

Chiribella-Yuan, 1404.3348

INGREDIENT 1: REPEATABILITY

MEASUREMENTS IN GPTS

- Demolition measurement $\{m_i\}_{i \in X}$

graphical representation: 

MEASUREMENTS IN GPTS

- **Demolition measurement** $\{m_i\}_{i \in X}$

graphical representation: 

- **Non-demolition measurement** $\{\mathcal{M}_i\}_{i \in X}$

graphical representation: 

REPEATABLE MEASUREMENTS

Measurement $\{m_i\}_{i \in X}$ is **repeatable** if exists $\{\mathcal{M}_i\}_{i \in X}$ such that

$$\text{A} \text{---} \boxed{\mathcal{M}_i} \text{---} \text{A} \text{---} \text{D}_{m_i} = \text{A} \text{---} \text{D}_{m_i} \quad \forall i$$

In short: same result when the measurement is performed multiple times.

INGREDIENT 2: MINIMAL DISTURBANCE

NON-DISTURBANCE CONDITION

$\{m_i\}_{i \in X}$ does not disturb $\{n_j\}_{j \in Y}$ if exists $\{\mathcal{M}_i\}_{i \in X}$
such that

$$\sum_{i \in X} \text{A} \text{---} \boxed{\mathcal{M}_i} \text{---} \text{A} \text{---} \text{D} n_j = \text{A} \text{---} \text{D} n_j \quad \forall j$$

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If a measurement does not disturb another measurement,
then the two are *compatible* (i.e. *jointly measurable*)

MINIMALLY-DISTURBING MEASUREMENTS

A measurement is **minimally-disturbing**
if it **disturbs the smallest possible set of measurements**,
i.e.
the set of measurements with which it is incompatible.

In short: doing the measurement now does not
preclude the possibility
to extract the refined information later.

SHARP MEASUREMENTS

A measurement is **sharp** if it can be realized by a test that is **repeatable** and **minimally disturbing**.

(in quantum theory: sharp = projective)

DEFINING KS CONTEXTUALITY IN GPTS

A DEFINITION OF KS CONTEXTUALITY IN GPTS

Given a contextuality game,
define ω_T
as the maximum payoff achievable with strategies
of the form

$$p(y|x) = \text{---} \rho \text{---} \overset{A}{\text{---}} m_y^{(x)} \text{---}$$


where $\{m_y^{(x)}\}_{y \in x}$ is a **sharp measurement**

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where $\{m_y^{(x)}\}_{y \in x}$ is a **sharp measurement**

The theory T is **contextual** if $\omega_T > \omega_C$
for some contextuality game.

OTHER DEFINITIONS OF SHARP MEASUREMENTS

Other definitions of sharp measurement have been proposed in the literature:

- Measurements consisting of extremal effects
(Holevo 1985, Cabello-Severini-Winter 2010)
- Measurements that distinguish maximal sets
of distinguishable pure states
(Hardy 2001)
- Coarse-graining of pure orthogonal measurements
(Barnum-Masanes-Ududec 2015, Chiribella Yuan 2016)

A
PHYSICAL REASON
FOR
THE
EXCLUSIVITY
PRINCIPLE

LESS INFORMATION, MORE SHARPNESS

Axiom LIMS:

Coarse-graining a sharp measurement
yields another sharp measurement.

Let us put ourselves in the standard situation
where the physical theory is **causal***

meaning that the probabilities of outcomes at time t do not
depend on the settings at time $t' > t$

**Chiribella-D'Ariano-Perinotti, PRA 2010.*

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Example:

$$\{m_1, m_2, m_3\} \text{ sharp} \implies \{m_1, m_2 + m_3\} \text{ sharp}$$

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In fact, LIMS could be even included in the *definition* of sharp measurement.

DERIVING THE E-PRINCIPLE

Theorem (Chiribella-Yuan 2014):
Every theory satisfying LIMS
satisfies the E-principle.

THE E-PRINCIPLE HIERARCHY

The E-principle can be lifted to a hierarchy of more and more restrictive conditions

Define $\mathbf{x} := (x_1, x_2, \dots, x_L)$

$\mathbf{y} := (y_1, y_2, \dots, y_L)$

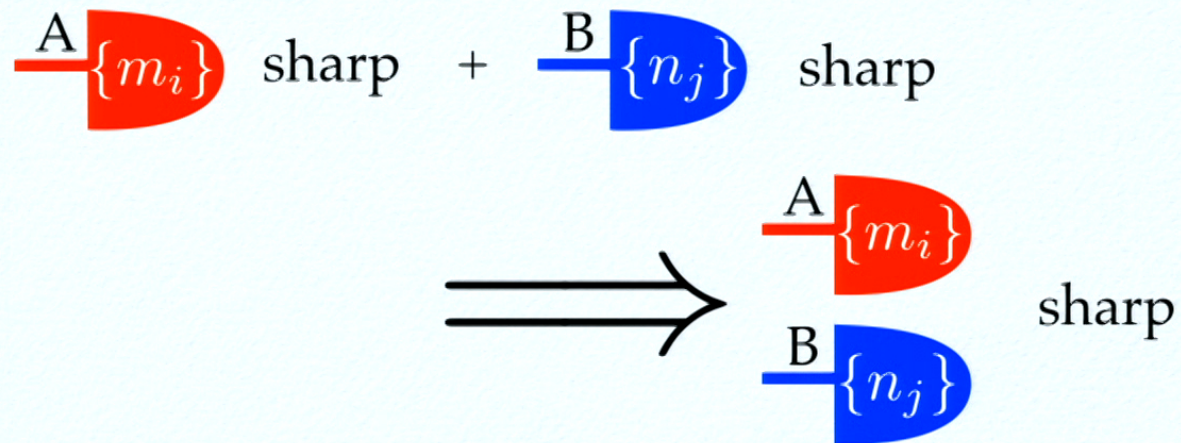
$$p_L(\mathbf{y}|\mathbf{x}) := p(y_1|x_1) p(y_2|x_2) \dots p(y_L|x_L)$$

The L-th level of the hierarchy is defined by applying the E-principle to $p_L(\mathbf{y}|\mathbf{x})$
(with a suitable definition of exclusive events)

LOCALITY OF SHARP MEASUREMENTS

Axiom LSM

Two sharp measurements performed in parallel
are a sharp measurement for the composite system



DERIVING THE E-PRINCIPLE HIERARCHY

Theorem (Chiribella-Yuan 2014):

Every theory satisfying LIMS and LSM
satisfies all levels of the E-principle hierarchy.

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The first levels of the hierarchy already determine famous quantum bounds like Klyachko's inequality (Cabello-Severini-Winter 2014) or the Tsirelson bound (Cabello 2015)

FROM
SHARP
MEASUREMENTS
TO
ARBITRARY MEASUREMENTS

LOCAL ORTHOGONALITY

Local Orthogonality (LO) is a multipartite principle introduced in the study of non-locality (Fritz *et al*, 2013).

LO is similar to the E-principle,
except that it applies to *all measurements*
not only to sharp measurements.

LOCAL ORTHOGONALITY

Local Orthogonality (LO) is a multipartite principle introduced in the study of non-locality (Fritz *et al*, 2013).

LO is similar to the E-principle, except that it applies to *all measurements not only to sharp measurements*.

Still, also LO can also be reduced to the properties of sharp measurements.

FUNDAMENTAL SHARPNESS

Axiom FS:

Every unsharp measurement arises from a sharp measurement performed jointly on the system and an environment

$$\text{A} \text{---} m_i = \text{A} \text{---} M_i \text{---} \rho_E \text{---} E \quad \forall i \in X$$

$\{M_i\}_{i \in X}$ sharp

DERIVING LOCAL ORTHOGONALITY

Theorem (Chiribella Yuan 2014):

Every theory satisfying LIMS, LSM, and FS
satisfies Local Orthogonality
(at all levels of their hierarchy)

BACK TO THE “PURIFICATION PHILOSOPHY”

FUNDAMENTAL SHARPNESS

Axiom FS:

Every unsharp measurement arises from a sharp measurement performed jointly on the system and an environment

$$\text{A} \text{---} \text{red semi-circle } m_i = \text{A} \text{---} \text{blue semi-circle } M_i \text{ with } \text{green semi-circle } \rho_E \text{ below it} \quad \forall i \in X$$

$\{M_i\}_{i \in X}$ sharp

BACK TO THE “PURIFICATION PHILOSOPHY”

THE MOST IDEAL MEASUREMENT

Suppose that there exists a sharp measurement
that is also *purity-preserving*

i.e.

a measurement that sends pure states to pure states
when performed locally on a composite system.

Then...

Theorem (Chiribella 2017):

Every theory satisfying

Purification and One Ideal Measurement satisfies

- Fundamental Sharpness
- Locality of Sharp Measurements
- Less information, More Sharpness
- All alternative definitions of sharp measurements coincide

Theorem (Chiribella 2017):

Every theory satisfying

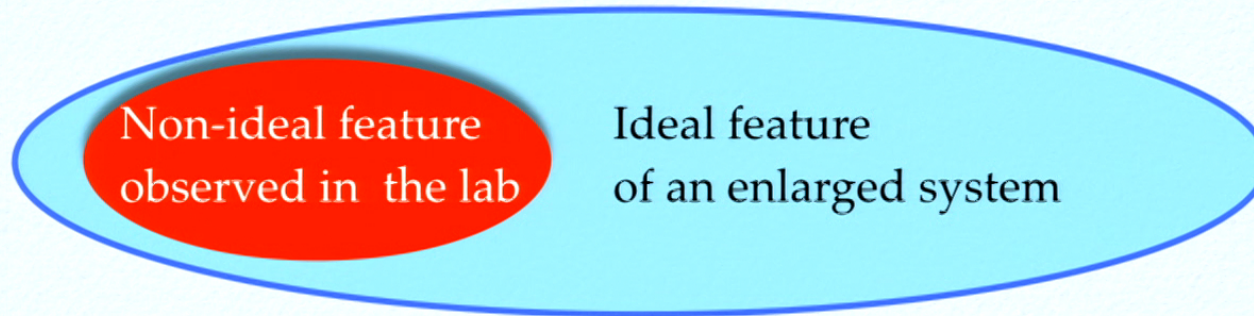
Purification and One Ideal Measurement satisfies

- Fundamental Sharpness
- Locality of Sharp Measurements
- Less information, More Sharpness
- All alternative definitions of sharp measurements coincide

(Informed) Conjecture:

Purification and One Ideal Measurement
determine exactly the quantum boundary of
contextuality and non-locality.

A PICTURE FOR CONTEXTUALITY



*Purification brings contextuality (and non-locality) in,
Sharpness cuts it down*

TAKE-HOME MESSAGES

- 1) To talk about KS contextuality in GPTs
one has to define sharp measurements first
- 2) The E-principle and Local Orthogonality
arise from the properties of sharp measurements
- 3) Conjecture: Purification and One Ideal Measurement
define the quantum set.

Framework + one alternative derivation of the E-principle:

Chiribella and Yuan,

Bridging the gap between general probabilistic theories and the device-independent framework for nonlocality and contextuality,
Information and Computation, **250**, 15-49 (2016)

Sharp measurements in GPTs:

Chiribella and Yuan,

Measurement sharpness cuts nonlocality and contextuality in every physical theory,
arXiv:1404.3348