Title: Nonlocality and contextuality as fine-tuning

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Abstract:

Nonlocality and contextuality as fine-tuning

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Violations of Bell inequalities and noncontextuality inequalities as fine-tuning

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Failure of Bell-local models and noncontextual models to reproduce correlations in no-disturbance phenomena as non-existence of classical causal models satisfying no-fine-tuning

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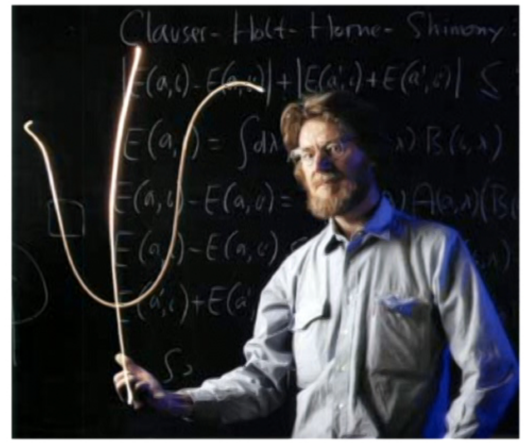
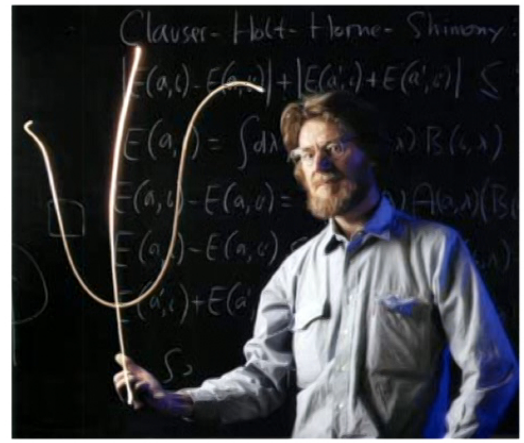
"For me then this is the real problem with quantum theory: the apparently essential conflict between any sharp formulation [of quantum theory] and fundamental relativity…

It may be that a real synthesis of quantum and relativity theories requires not just technical developments but radical conceptual renewal."

J.S. Bell (1986)

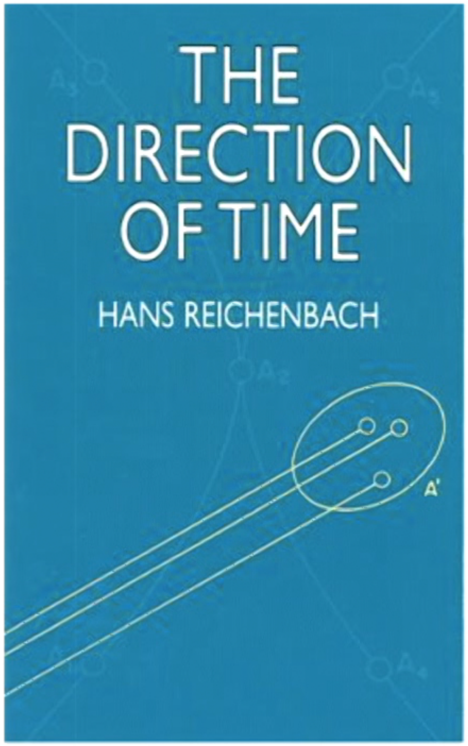


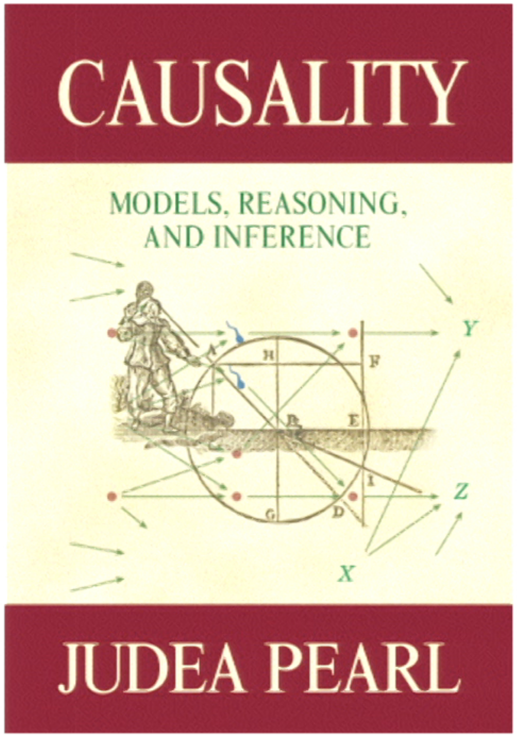
"Do we then have to fall back on "no signalling faster than light" as the expression of the fundamental causal structure of contemporary theoretical physics? That is hard for me to accept. For one thing we have **lost the idea that correlations can be explained, or at least this idea awaits reformulation**".



– J.S. Bell, "La Nouvelle Cuisine" (1990)

How do we explain correlations?



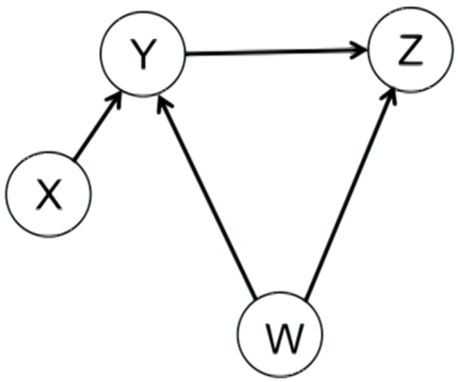


Reichenbach's Principle of Common Cause

If two events are correlated, either one is a cause of the other, or they share a common cause, such that conditioned on the common cause, the events become uncorrelated.

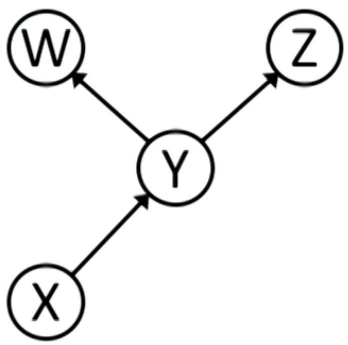
- Fundamental in many areas of science, from medical research to machine learning.
- BUT: leads to Bell's theorem and contradiction with quantum theory.

(Classical) causal networks



- Causal structure → **Directed Acyclic Graph (DAG)**
  - **Nodes:** random variables
  - **Arrows:** causal links
  - ***Parents*** of Z: set of direct causes of Z (here $\mathrm{Pa}(Z)=\{Y,W\}$)
  - ***Ancestors*** of Z: set of causes of Z (here $\mathrm{An}(Z)=\{X,Y,W\}$)
  - ***Descendants*** of Z: set of effects of Z, i.e. nodes for which Z is an ancestor. (here $\mathrm{De}(Z)=\{\}$)
  - ***Non-descendants*** of Z: (here $\mathrm{Nd}(Z)=\{X,Y,W\}$).

Classical causal networks



- A DAG *G* encodes causal constraints on probability distributions.
- For example:

$$P(Z|Y,X) = P(Z|Y)$$

- *Y screens off Z* from *X.*

Classical causal networks

- Conditional independences:

$$(X \perp Y|Z) \Leftrightarrow P(X|Y,Z) = P(X|Z)$$

- Satisfy the semi-graphoid axioms:

  - Symmetry $(X\perp Y|Z) \Leftrightarrow (Y\perp X|Z)$
  - Decomposition $(X\perp YW|Z) \Rightarrow (X\perp Y|Z)$
  - Weak Union $(X\perp YW|Z) \Rightarrow (X\perp Y|ZW)$
  - Contraction $(X\perp Y|Z)\ \&\ (X\perp W|ZY) \Rightarrow (X\perp YW|Z)$

Classical causal networks

- The conditional independences implied by a graph are encoded in the following principle:

Causal Markov Condition:

A variable is independent of its non-effects given its direct causes

- In the genealogical language, a variable is independent of its non-descendants given its parents.

$$(X \perp \mathrm{Nd}(X)|\mathrm{Pa}(X))$$

Classical causal networks

- The conditional independences implied by a graph are encoded in the following principle:

Causal Markov Condition:

A variable is independent of its non-effects given its direct causes

Causal Markov Condition ➔ Reichenbach's Principle

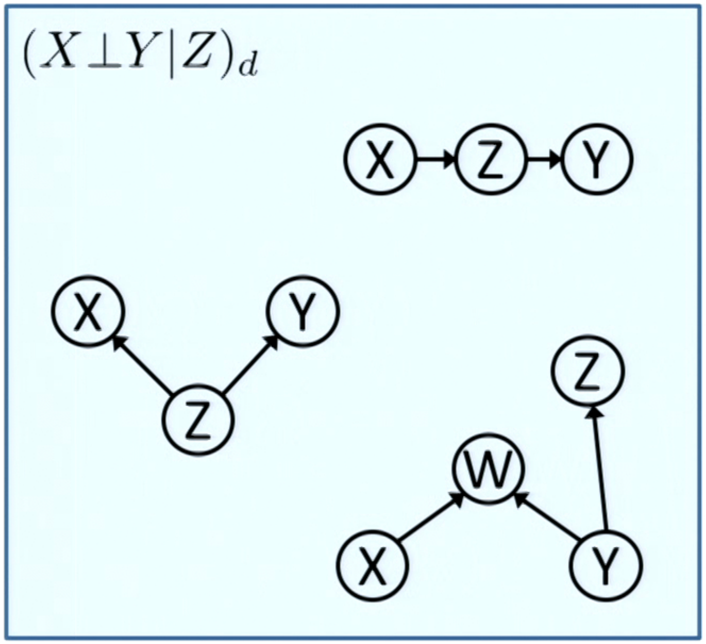
Classical causal networks

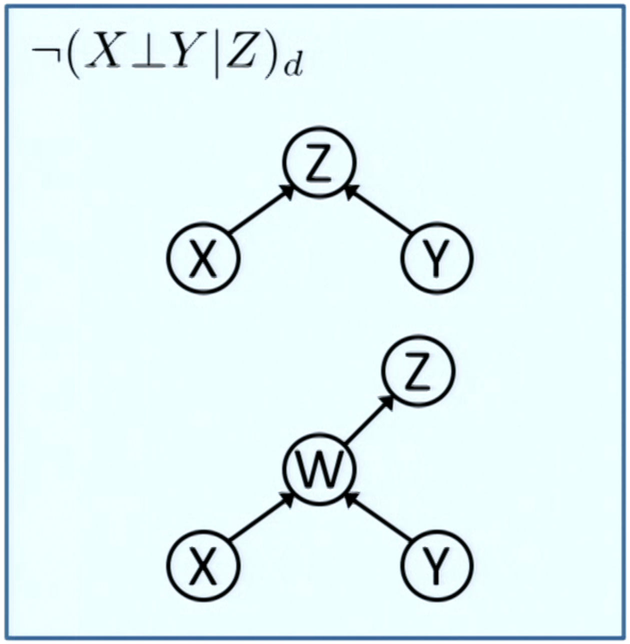
- Consider a graph *G* on *n* variables $X_1, \ldots X_n$
- The Causal Markov Condition is equivalent to the requirement that every probability distribution *P compatible with G* can be factorised as

$$P(X_1, ..., X_n) = \prod_{i=1}^{n} P(X_i|\mathrm{Pa}(X_i))$$

Classical causal networks

- Two sets of variables *X* and *Y* are **d-separated** given a set *Z* *iff* *Z* *blocks* all paths from *X* to *Y*





Classical causal networks

- **d-separation** is a *sound* and *complete* criterion for conditional independence:

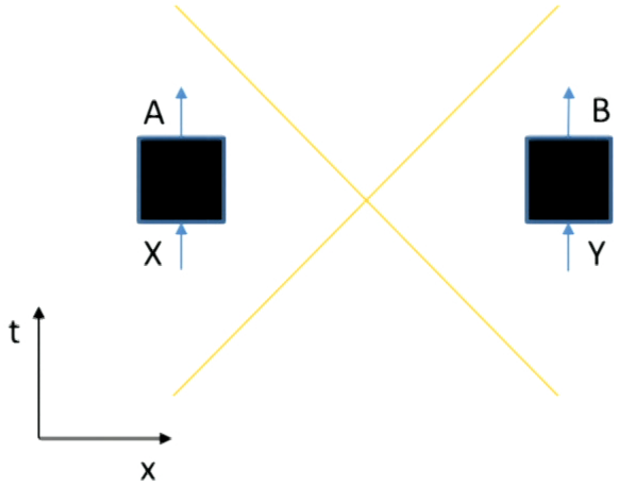
- *Sound*: for all *P* compatible with a graph *G:*

$$(X \perp Y|Z)_d \Rightarrow (X \perp Y|Z)$$

- *Complete:* if for all *P* compatible with *G,* $(X \perp Y|Z)$, then

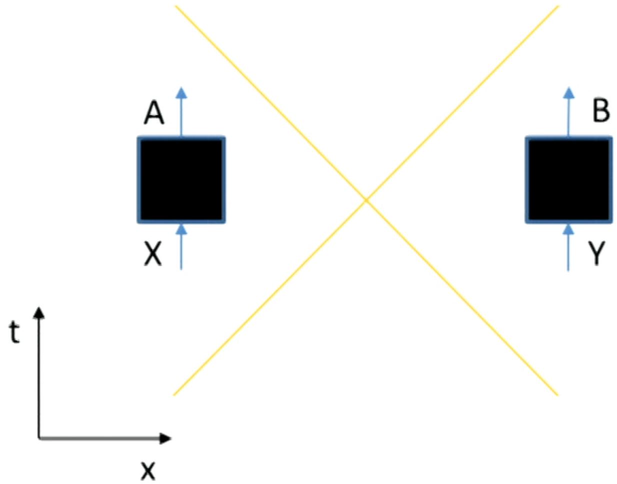
$$(X \perp Y|Z)_d$$

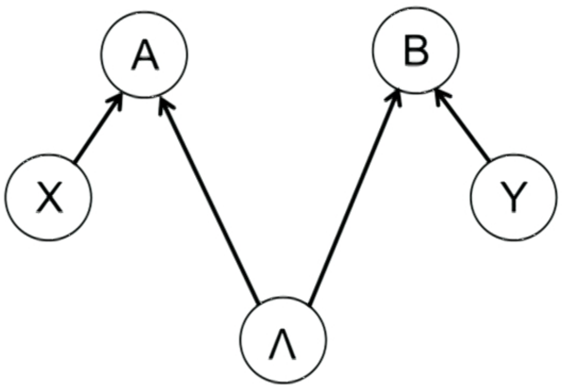
Causal networks and Bell's theorem



→ Bell inequalities → Contradictions with Quantum Mechanics

Causal networks and Bell's theorem

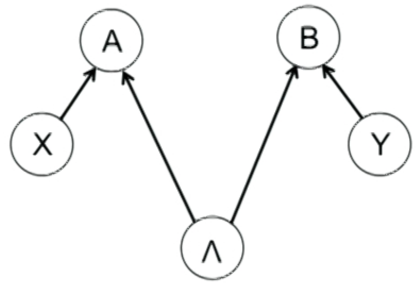




- Relativistic causality
- Free choice
- Reichenbach's Principle

→ Bell inequalities → Contradictions with Quantum Mechanics

Causal networks and Bell's theorem



- Reichenbach's Principle implies

$$P(A,B,X,Y,\Lambda) = P(A|X,\Lambda)P(B|Y,\Lambda)P(\Lambda)P(X)P(Y)$$

$$\Rightarrow P(A,B,\Lambda|X,Y) \equiv P(A,B,X,Y,\Lambda)/P(X,Y)$$
$$= P(A|X,\Lambda)P(B|Y,\Lambda)P(\Lambda)$$

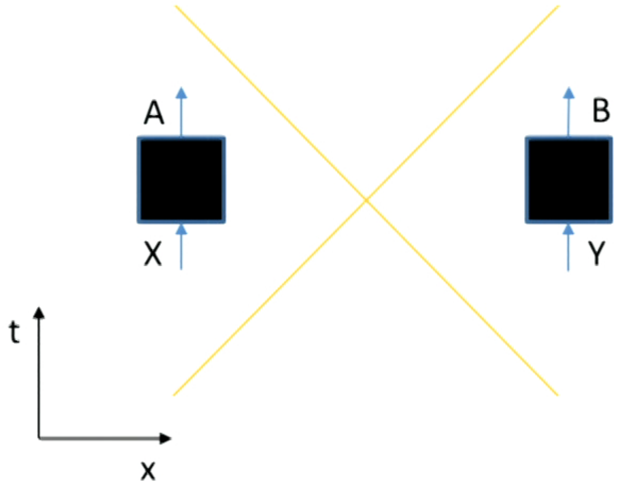
- Sum over the latent variable

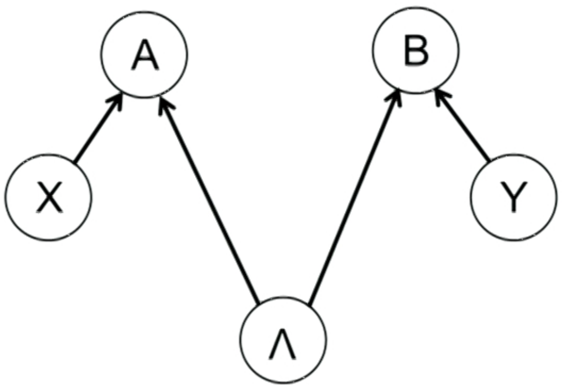
$$P(A,B|X,Y) = \sum_{\Lambda} P(A|X,\Lambda)P(B|Y,\Lambda)P(\Lambda)$$

→ Factorisable (Local Hidden Variable model)

→ Bell inequalities → Contradiction with quantum correlations

Causal networks and Bell's theorem





- Relativistic causality
- Free choice
- Reichenbach's Principle

→ Bell inequalities → Contradictions with Quantum Mechanics

Breaking Reichenbach

- We can break*

  - REICHENBACH'S PRINCIPLE (1956): If two events A and B are correlated, and neither is a cause of the other, then they have a common cause C, such that conditioning on C eliminates the correlation.

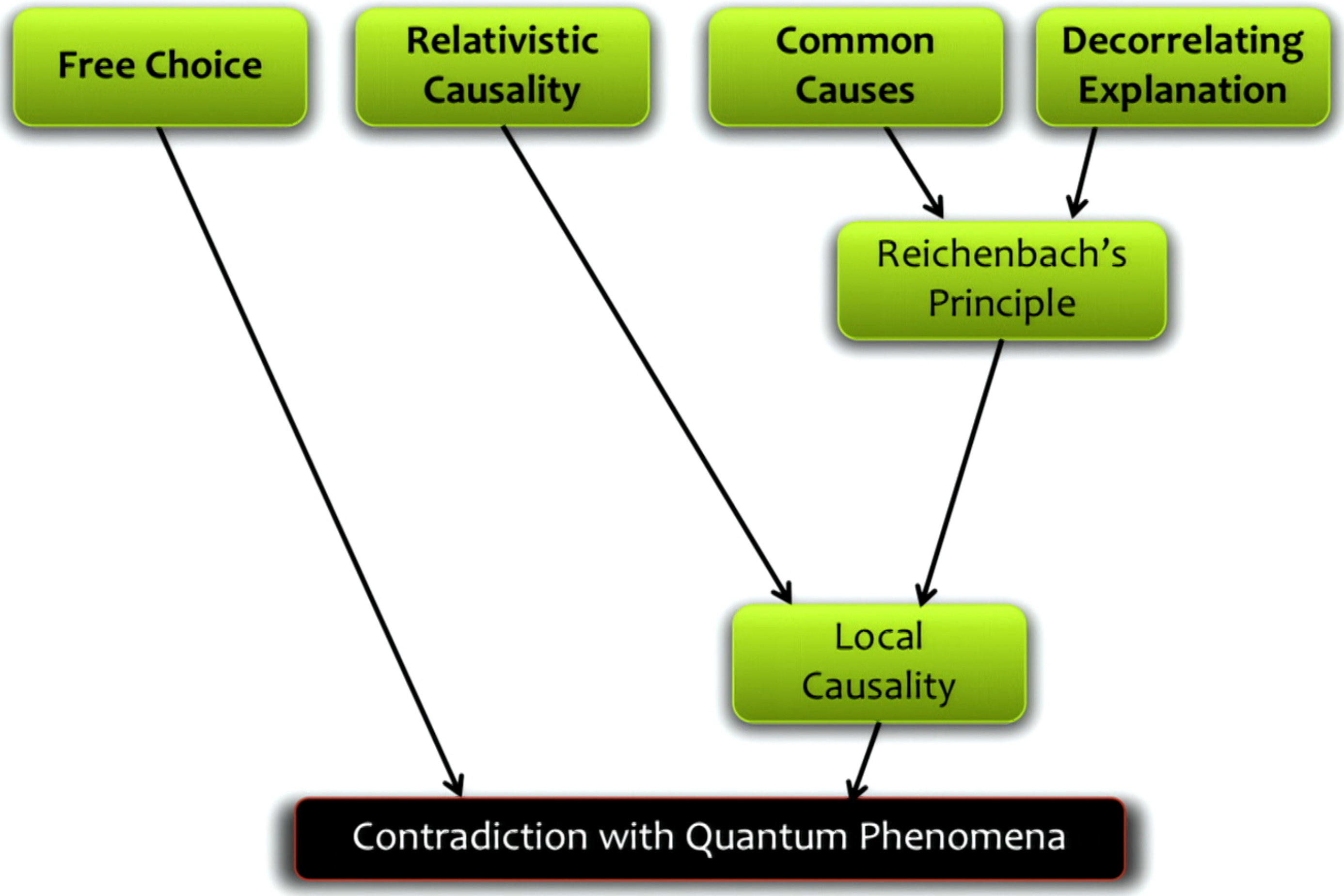
  into

  - PRINCIPLE of COMMON CAUSE: If two events A and B are correlated, and neither is a cause of the other, then they have a common cause that explains the correlation.

  plus

  - PRINCIPLE of DECORRELATING EXPLANATION: A cause C, common to events A and B, explains a correlation between them only if conditioning on C eliminates the correlation.

* Cavalcanti and Lal, J. Phys. A 47, 424018 (2014)



Eric Cavalcanti, Natal, November 2016

Wiseman and Cavalcanti, arXiv:1503.06413

Quantum Causal Networks

Henson, Lal and Pusey (HLP), New J. Phys. 16, 113043 (2014)

Pienaar and Brukner (PB), New J. Phys. 17, 073020 (2015)

Chaves, Majenz and Gross, Nat. Commun. 6, 5766 (2015)

Costa, Shrapnel, New J. Phys. 18 063032 (2016)

Allen, Barrett, Horsman, Lee and Spekkens, arxiv:1609.09487 (2016)

- Generalisations of causal networks to the quantum case
- Provide (steps towards) a causal explanation of quantum correlations → resolution of the ”easy problem” of Bell?

How about contextuality?

- Bell nonlocality is a special case of KS-contextuality.
- But in general contextuality scenarios, causal structure doesn't seem to play the conceptual role it does in Bell.
- Different approach: show that *all possible causal structures* that allow for contextuality violate a causal principle: no-fine-tuning.

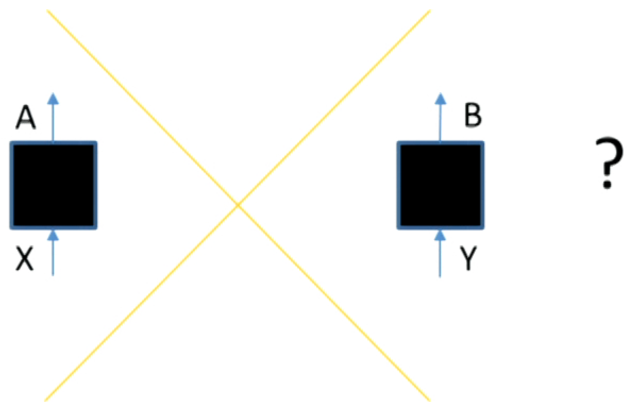
- **No fine-tuning (NFT):** Every conditional independence between variables must arise as a consequence of the causal graph and not due to special choices of causal-statistical parameters.

$$(X \perp Y | Z) \Rightarrow (X \perp Y | Z)_d$$

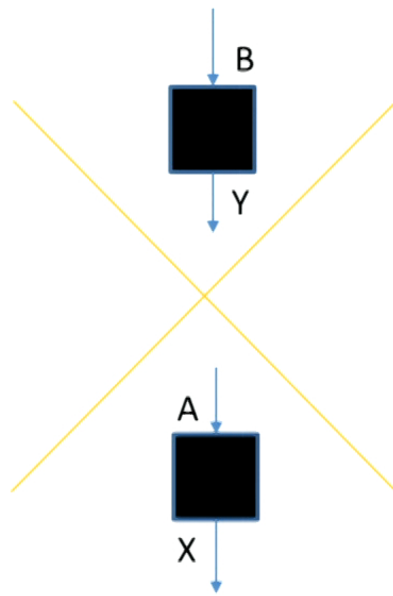
Why no-fine-tuning?

- Occam's razor
- Leibniz's principle: one should not postulate ontological/causal explanations that are not apparent in the phenomena
- E.g.: if we can't signal faster-than-light, don't postulate faster-than-light causation.

Finely tuned Bells



Finely tuned Bells



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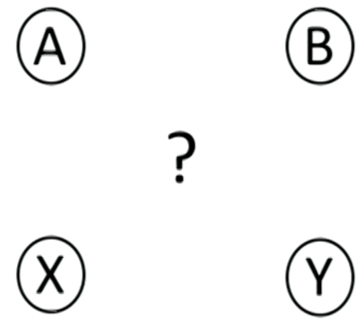
Finely tuned Bells

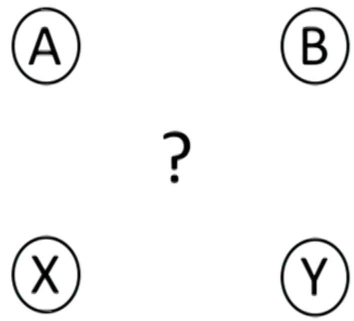


- Choice independence (CI): $(X \perp Y)$
- Local setting dependence (LSD): $\neg(X \perp A), \neg(Y \perp B)$
- No-signalling (NS): $(A \perp Y|X), (B \perp |X|Y)$

- CI + LSD + NS + NFT → BI

Wood and Spekkens, NJP **17**, 33002 (2015)

Removing assumptions



- ~~Choice independence (CI): $(X\perp Y)$~~
- ~~Local setting dependence (LSD): $\neg(X\perp A), \neg(Y\perp B)$~~
- No-signalling (NS): $(A\perp Y|X), (B\perp|X|Y)$

- NS +NFT → BI

Cavalcanti, arXiv:1705.05961 [quant-ph] (2017)

- ***Result 1:*** *No classical causal model can reproduce violations of Bell inequalities without fine-tuning.*
  - *No extra assumption related to free choice*
  - → *Allows generalisation to KS-noncontextuality*

- ***Result 1:*** *No classical causal model can reproduce violations of Bell inequalities without fine-tuning.*
  - *No extra assumption related to free choice*

  → *Allows generalisation to KS-noncontextuality*

- ***Result 2:*** *No classical causal model can reproduce violations of Kochen-Specker inequalities without fine-tuning.*

KS-noncontextuality from no-fine-tuning

- Set of measurements: $m \in \mathcal{M}$
- Set of measurement outcomes: $\mathcal{O}_m = \mathcal{O} \;\; \forall m$
- Measurement settings: $X_1, X_2, ..., X_n$

  $x_i \in \mathcal{M}$
- Outcomes: $A_1, ..., A_n$

  $a_i \in \mathcal{O}$
- Phenomenon: $\mathcal{P}(A_1, ..., A_n | X_1, ..., X_n)$

- Contexts:

$$\mathcal{C}_{\mathcal{M}} \subset \mathbb{P}(\mathcal{M})$$

$$m_1, m_2 \text{ compatible} \Leftrightarrow \{m_1, m_2\} \in \mathcal{C}_{\mathcal{M}}$$

- Contextuality scenario: in every run:

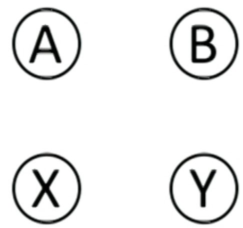
$$\{x_1, ..., x_n\} \in \mathcal{C}_{\mathcal{M}}$$

- Bell scenario: a contextuality scenario where

$$\mathcal{M} = \mathcal{M}_1 \cup \mathcal{M}_2 \cup ... \cup \mathcal{M}_n$$

$$\mathcal{M}_i \cap \mathcal{M}_j = \{\ \} \ \ \forall i \neq j$$

$$x_1 \in \mathcal{M}_1, \ x_2 \in \mathcal{M}_2, ..., x_n \in \mathcal{M}_n$$

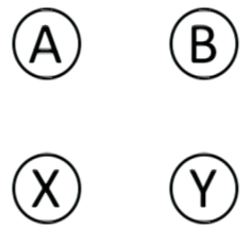


- A phenomenon satisfies **No-disturbance** iff

$$\mathcal{P}(A|X,Y) = \mathcal{P}(A|X) \quad \mathcal{P}(B|X,Y) = \mathcal{P}(B|Y)$$

for all values for which the conditionals are defined.

(In Bell scenarios, no-disturbance = no-signalling)



- A (classical) **causal model** Γ for a phenomenon consists of:

$$\exists\, \Lambda \ , G \text{ on } \{A, B, X, Y, \Lambda\} \text{ and } P \text{ compatible with } G \text{ s.t.}$$

$$\mathcal{P}(A, B, X, Y) = \sum_{\Lambda} P(A, B, X, Y, \Lambda)$$

- (**No fine-tuning**): A causal model Γ is said to satisfy *no fine-tuning* or be *faithful* relative to a phenomenon $\mathcal{P}$ iff every conditional independence in $\mathcal{P}$ corresponds to d-separation in the graph *G of* Γ.

$$(X \perp Y | Z) \Rightarrow (X \perp Y | Z)_d$$

- A causal model satisfies **factorisability** iff:

$$P(A,B|X,Y) = \sum_{\Lambda} P(A|X,\Lambda)P(B|Y,\Lambda)P(\Lambda)$$

- A causal model for a Bell scenario is **Bell-local** iff it is factorisable.

- A causal model for a contextuality scenario satisfies **KS-noncontextuality** iff it is factorisable and deterministic, i.e.

$$P(A|X,\Lambda),\ P(B|Y,\Lambda) \in \{0,1\}$$

- **Fine's theorem:** A phenomenon has a factorisable model iff it has a KS-noncontextual model.

Results

- **Theorem 1**:

  No-fine-tuning + no-disturbance → KS-noncontextuality

- **Corollary 1**:

  No-fine-tuning + no-signalling → Bell-locality

→ Every causal model that reproduces the violation of a KS-inequality in a no-disturbance phenomenon requires fine-tuning.

→ There exist quantum phenomena involving single systems that cannot be reproduced by any classical causal model without fine-tuning.

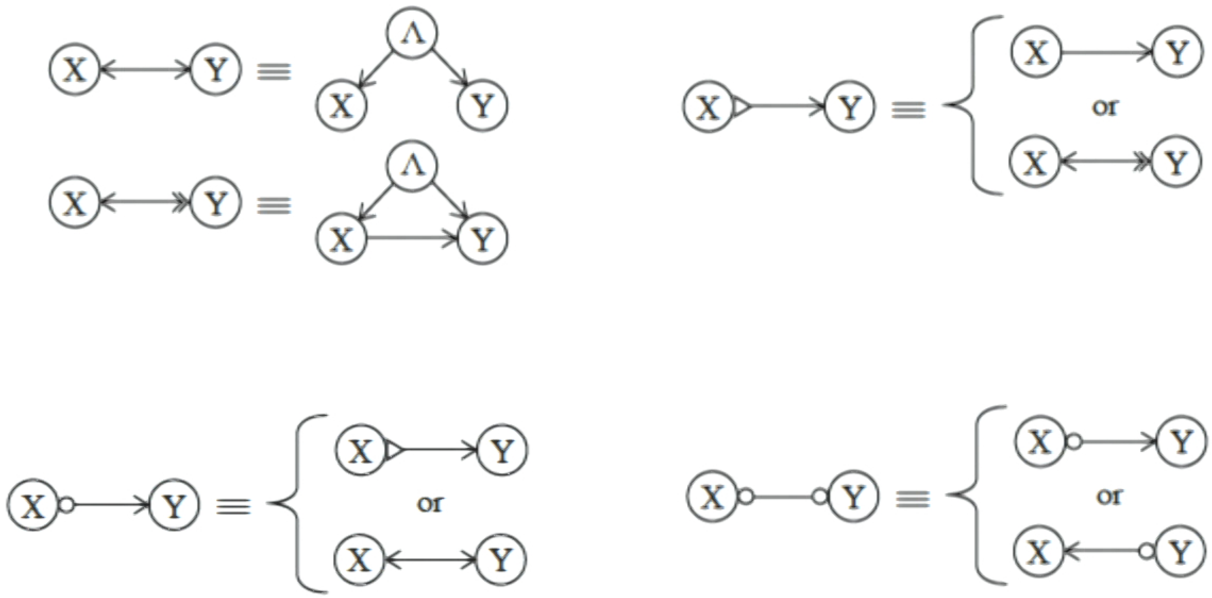
Sketch of proof

- No-fine-tuning: $(X \perp Y | Z) \Rightarrow (X \perp Y | Z)_d$
- No-signalling: $(A \perp Y | X), (B \perp X | Y)$

$$\Rightarrow \ (A \perp Y | X)_d, (B \perp X | Y)_d$$

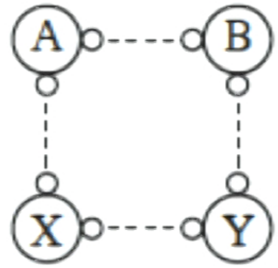
- Exclude all fine-tuned graphs
- Show that all remaining graphs lead to factorisability → KS-noncontextuality

Longer sketch of proof

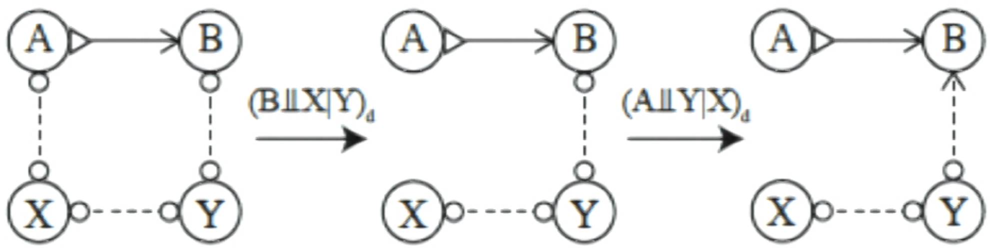


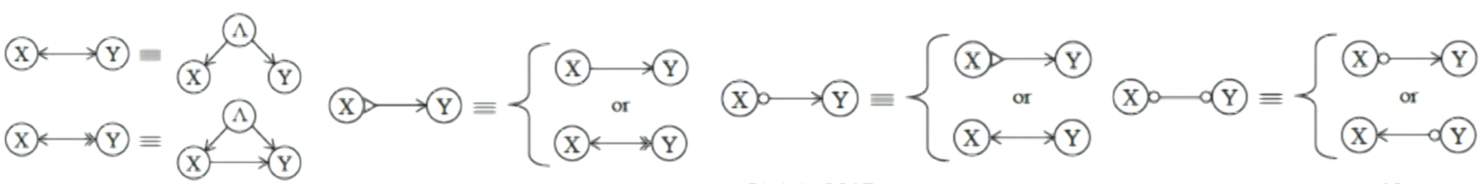
- Step 1

$$(A \perp Y | X)_d, (B \perp X | Y)_d \quad \Rightarrow$$

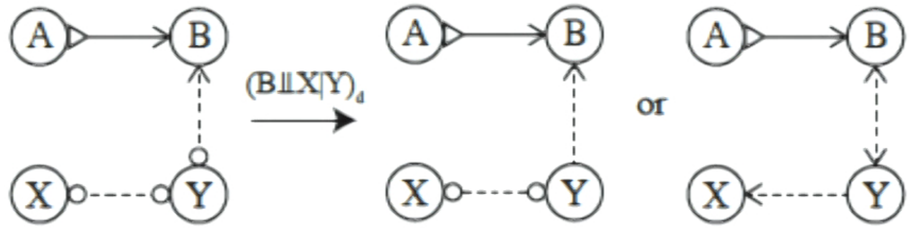


- Step 2a



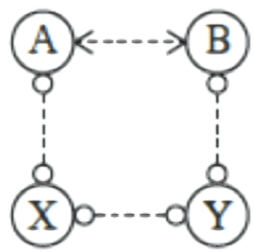


- Step 2b

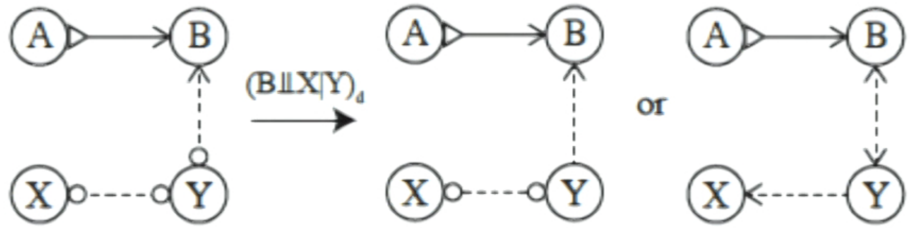


→Factorisability

- Remaining DAGs:



- Step 2b



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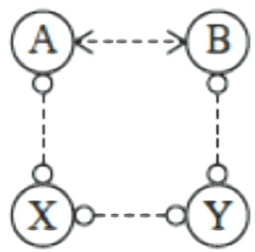
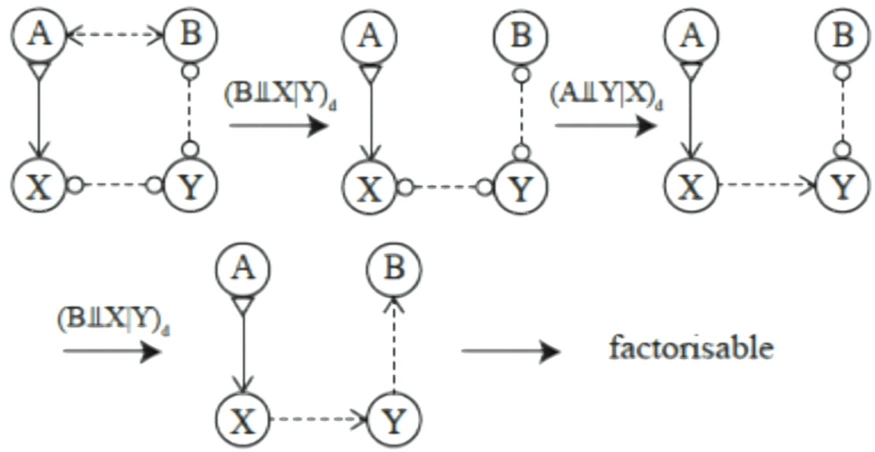
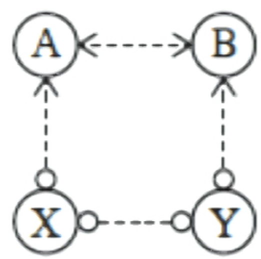


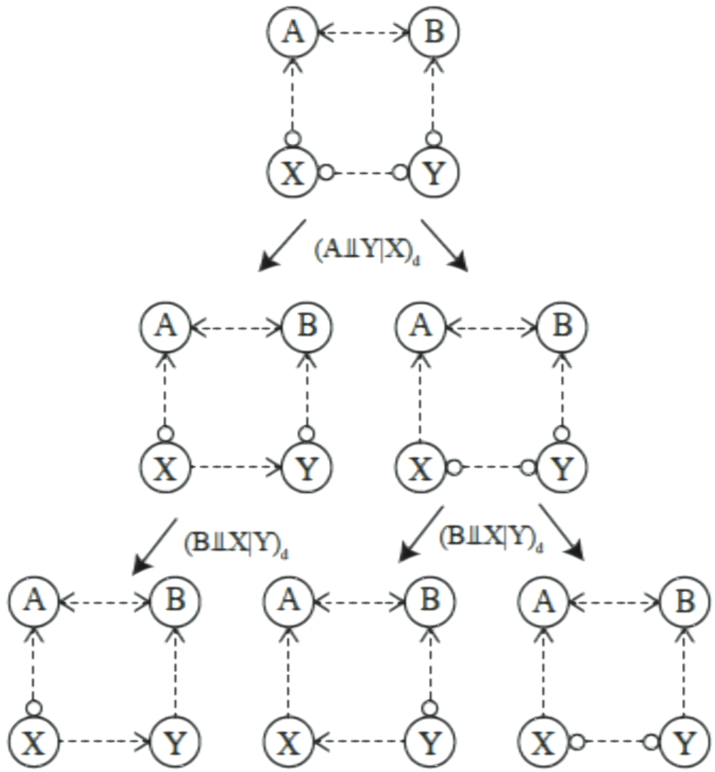
- Step 3



- Remaining DAGs:

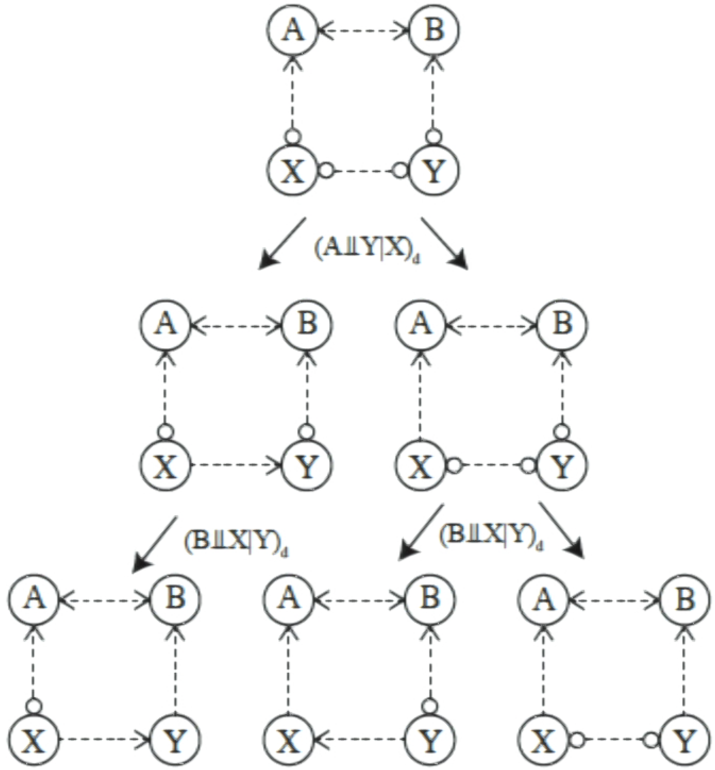


- Step 4



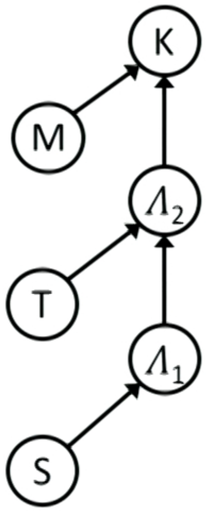
→ All remaining DAGs factorisable

- Step 4



→ All remaining DAGs factorisable

How about Spekkens-contextuality?



- Ontological causal model:

$$\mathcal{P}(K|S,T,M) = \sum_{\Lambda_1,\Lambda_2} P(K|M,\Lambda_2)P(\Lambda_2|T,\Lambda_1)P(\Lambda_1|S)$$

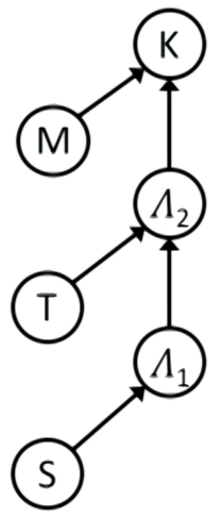
- Equivalence classes:

$$S \sim S' \Leftrightarrow \mathcal{P}(K|STM) = \mathcal{P}(K|S'TM)\ \forall K,T,M$$
$$T \sim T' \Leftrightarrow \mathcal{P}(K|STM) = \mathcal{P}(K|ST'M)\ \forall K,S,M$$
$$M \sim M' \Leftrightarrow \mathcal{P}(K|STM) = \mathcal{P}(K|STM')\ \forall K,S,T$$

- Non-contextuality:

$$S \sim S' \Rightarrow P(\Lambda_1|S) = P(\Lambda_1|S')$$
$$T \sim T' \Rightarrow P(\Lambda_2|T,\Lambda_1) = P(\Lambda_2|T',\Lambda_1)$$
$$M \sim M' \Rightarrow P(K|M,\Lambda_2) = P(K|M',\Lambda_2)$$

How about Spekkens-contextuality?



- Ontological causal model:

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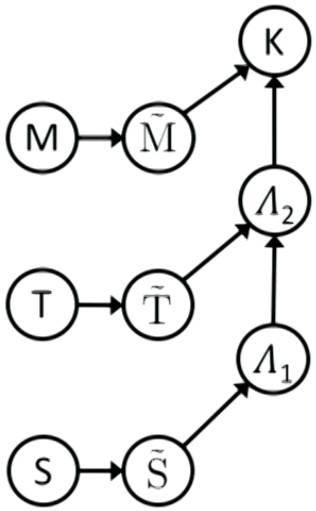
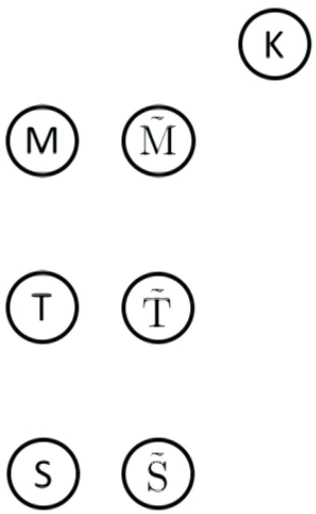
$$M \sim M' \Rightarrow P(K|M,\Lambda_2) = P(K|M',\Lambda_2)$$



Spekkens-NC ontological model:

$$\mathcal{P}(K|S,T,M) = \sum_{\Lambda_1,\Lambda_2} P(K|\tilde{M},\Lambda_2)P(\Lambda_2|\tilde{T},\Lambda_1)P(\Lambda_1|\tilde{S})$$

Spekkens-NC from no-fine-tuning?



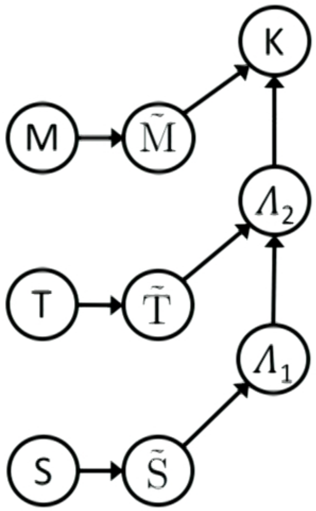
- Equivalence classes → conditional independences

$$\mathcal{P}(K|S,\tilde{S},T,M) = \mathcal{P}(K|\tilde{S},T,M)$$
$$\mathcal{P}(K|S,T,\tilde{T},M) = \mathcal{P}(K|S,\tilde{T},M)$$
$$\mathcal{P}(K|S,T,M,\tilde{M}) = \mathcal{P}(K|S,T,\tilde{M})$$

- From No-fine-tuning

$$(K \perp S|\tilde{S},T,M)_d$$
$$(K \perp T|S,\tilde{T},M)_d$$
$$(K \perp M|S,T,\tilde{M})_d$$

Spekkens-NC from no-fine-tuning



- Conjecture: all non-fine-tuned graphs are compatible with a Spekkens-NC model:

$$\mathcal{P}(K|S,T,M) = \sum_{\Lambda_1,\Lambda_2} P(K|\tilde{M},\Lambda_2)P(\Lambda_2|\tilde{T},\Lambda_1)P(\Lambda_1|\tilde{S})$$

Wrapping up

- **Non-contextuality and Bell-locality** both arise from the requirement of **no-fine-tuning** on classical causal models;
- Unifies KS-NC and Bell-nonlocality as violations of classical causality;
- **No assumption of determinism** is needed;
- **Theory-independent** derivation;

Wrapping up

- Questions for further research:
  - Complete proof for Spekkens-NC
  - More settings per context?
  - Fine-tuning as "resource waste" → insight into quantum advantage?
  - Experimentally robust generalisation? (e.g. strengths of causal connection)

Wrapping up

- (Very plausible) conjecture: ”reasonable” quantum causal models do *not* require fine-tuning

→ Satisfactory causal explanation of contextual correlations?

→ But can they be the basis for an ontological model?

The Bell still rings

"*More importantly*, the **"no signalling" notion rests on concepts which are desperately vague** . . . The assertion that "we cannot signal faster than light" immediately provokes the question: "Who do we think *we* are?" - J.S. Bell , "La Nouvelle Cuisine" (1990)

- Quantum causal models are intrinsically operational
  - Causality as an emergent phenomenon?
    E. G. Cavalcanti, J. Phys. Conf. Ser. 701, 12002 (2016).