Title: Noncontextuality: how we should define it, why it is natural, and what to do about its failure

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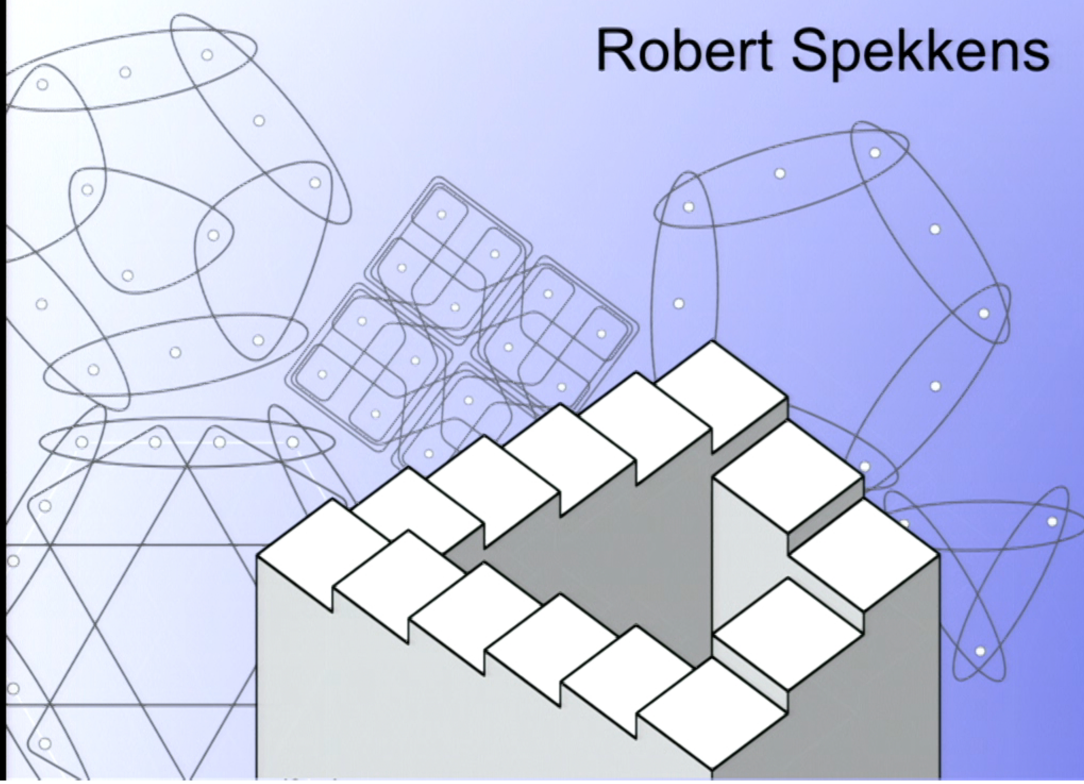
Abstract:

Noncontextuality: how we should define it, why it is natural, and what to do about its failure

Robert Spekkens



Contextuality: Conceptual Issues, Operational Signature, and applications
July 24, 2017

What we want in a notion of nonclassicality

Subject to direct experimental test	Constitutes a resource	Applicable to a broad range of physical scenarios

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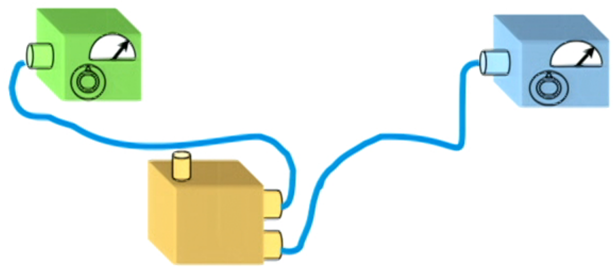
	Subject to direct experimental test	Constitutes a resource	Applicable to a broad range of physical scenarios
Failure to admit a locally causal model	✓	✓	✗

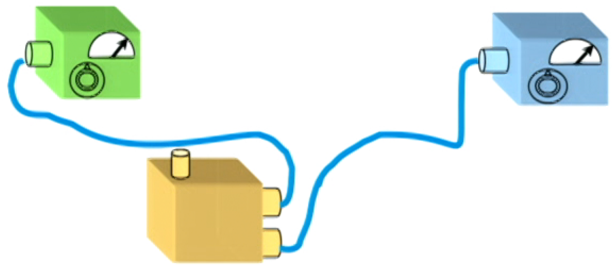
What is needed to witness the failure of local causality



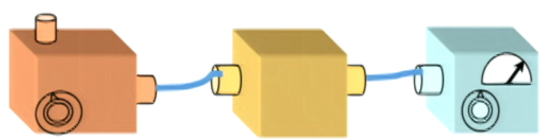
What we want in a notion of nonclassicality

	Subject to direct experimental test	Constitutes a resource	Applicable to a broad range of physical scenarios
Failure to admit a locally causal model	✓	✓	✗
Failure to admit a noncontextual model			✓

What is needed to witness the failure of local causality



What is needed to witness the failure of noncontextuality



What we want in a notion of nonclassicality

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Failure to admit a locally causal model	✓	✓	✗
Failure to admit a noncontextual model	?	?	✓

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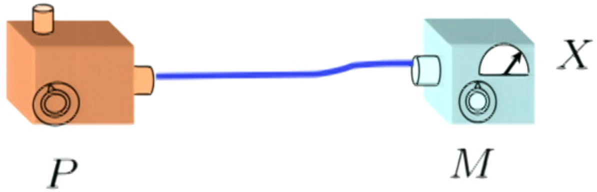
What we want in a notion of nonclassicality

	Subject to direct experimental test	Constitutes a resource	Applicable to a broad range of physical scenarios
Failure to admit a locally causal model	✓	✓	✗
Failure to admit a noncontextual model	✓	✓	✓

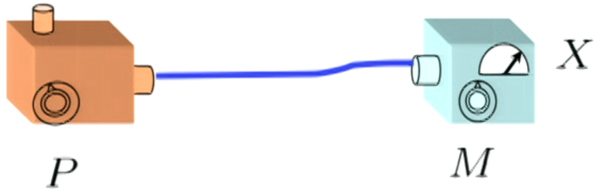
Defining the principle of noncontextuality

Operational theory



$$p(X|M,P)$$

Operational theory



$$p(X|M,P)$$

Ontological model of an operational theory

$\lambda \in \Lambda$ Ontic state space

causally mediates between P and M

Operational theory

$$p(X|M,P)$$

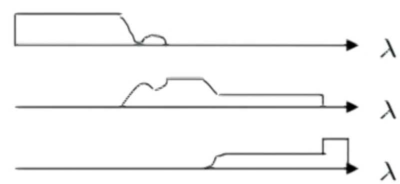
Ontological model of an operational theory

$\lambda \in \Lambda$ Ontic state space

causally mediates between P and M

$\mathsf{P} \leftrightarrow \mu(\lambda|\mathsf{P})$

$\mathsf{M} \leftrightarrow \xi(X|\mathsf{M},\lambda)$



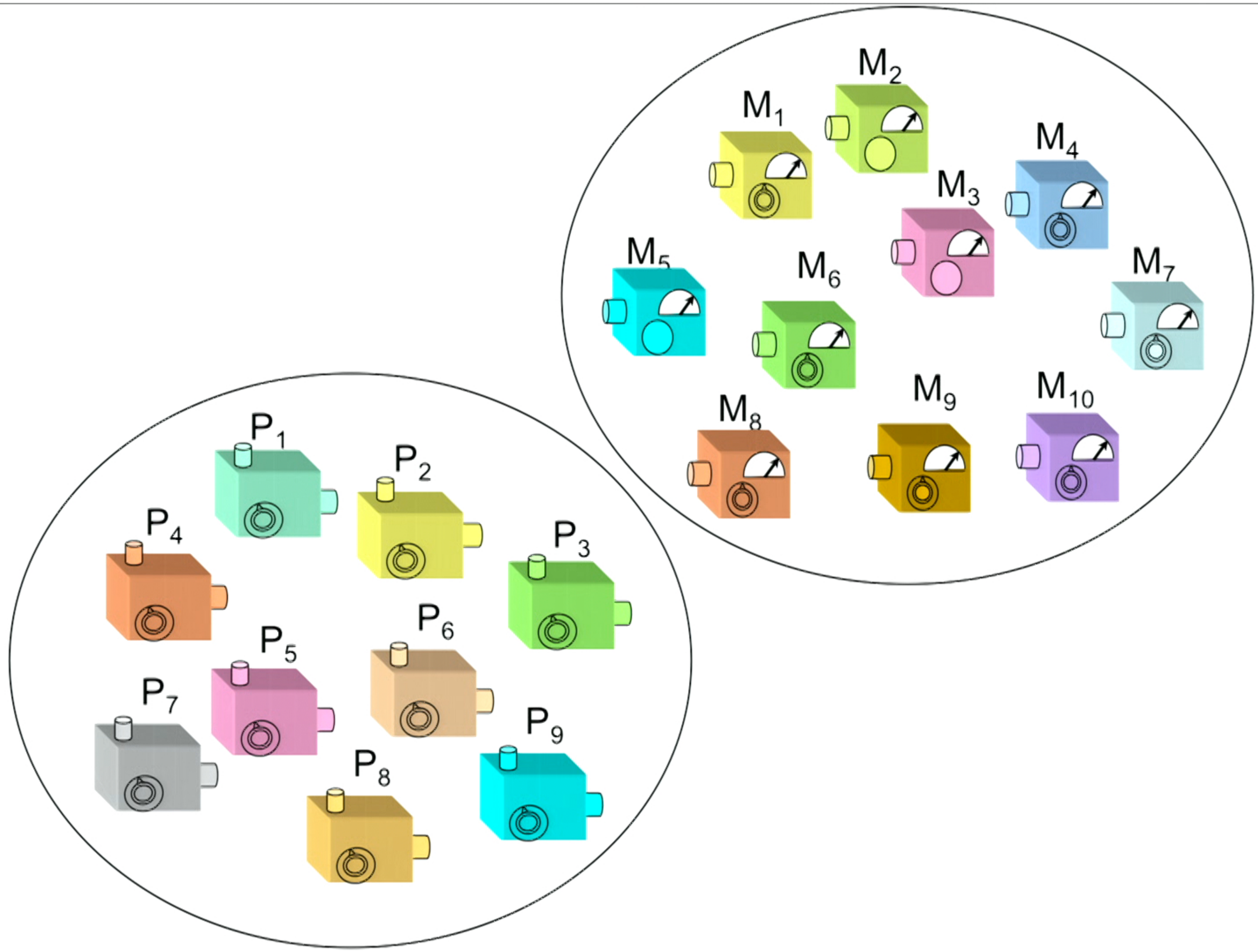
$$p(X|M,P) = \sum_\lambda \xi(X|M,\lambda)\,\mu(\lambda|P)$$

An ontological model of an operational theory is noncontextual if

Operational equivalence of two experimental procedures → Equivalent representations in the ontological model

RWS, Phys. Rev. A 71, 052108 (2005)

Preparation Noncontextuality



M_1
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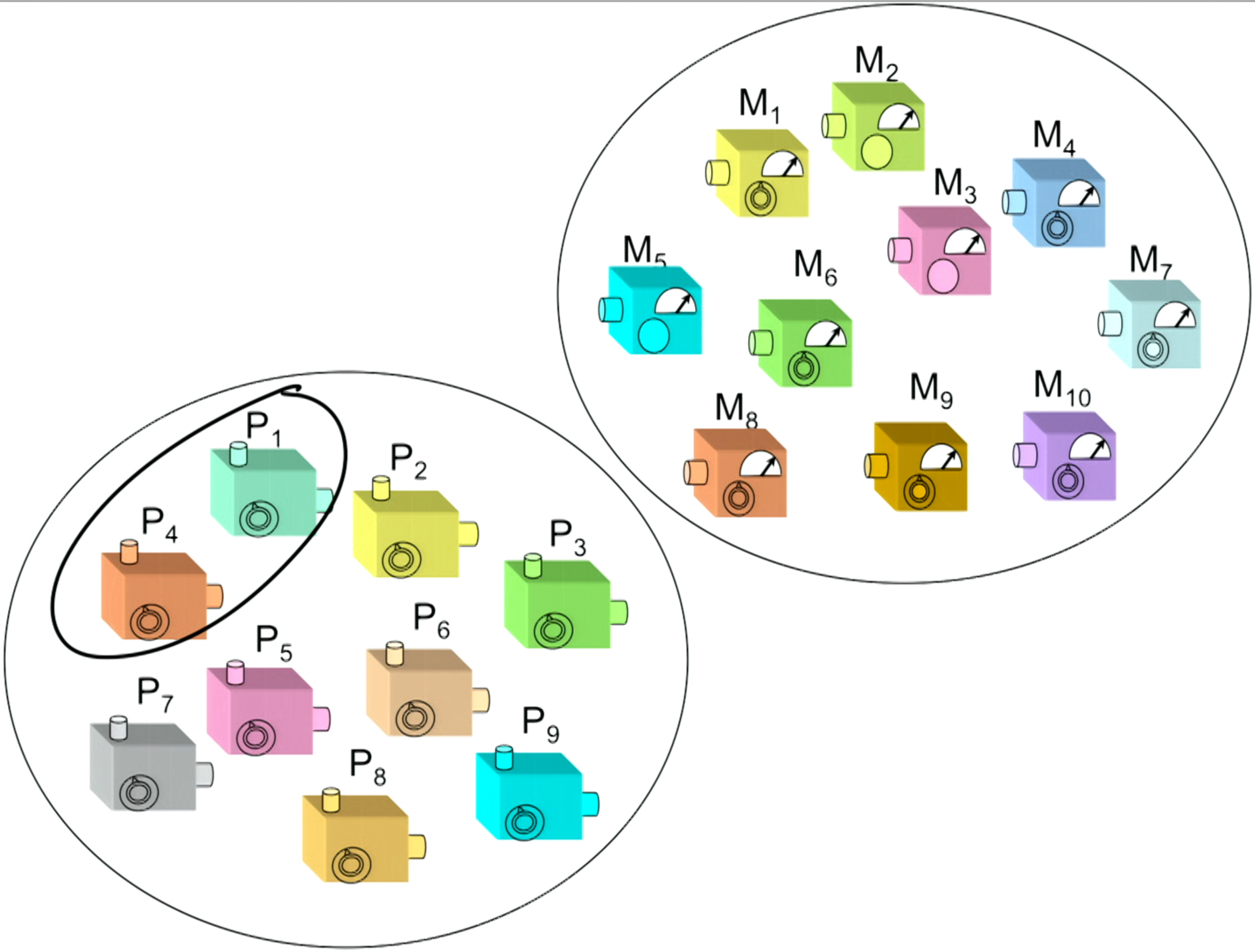

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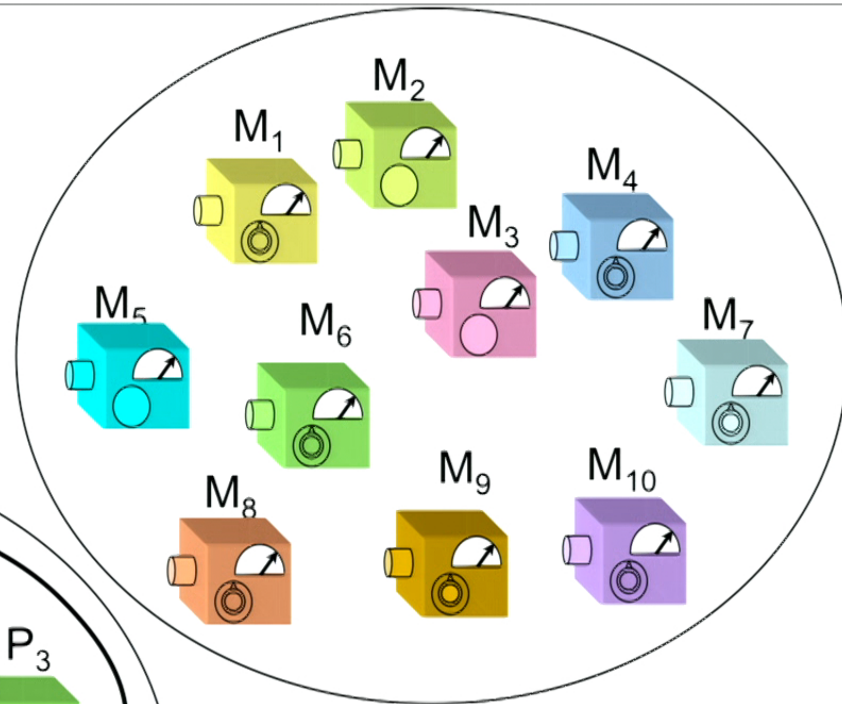
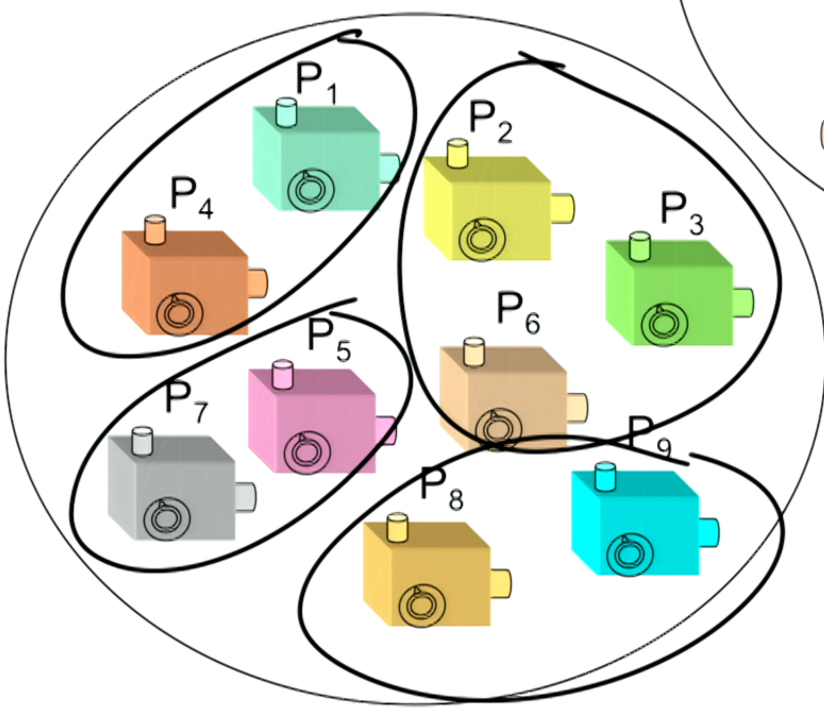
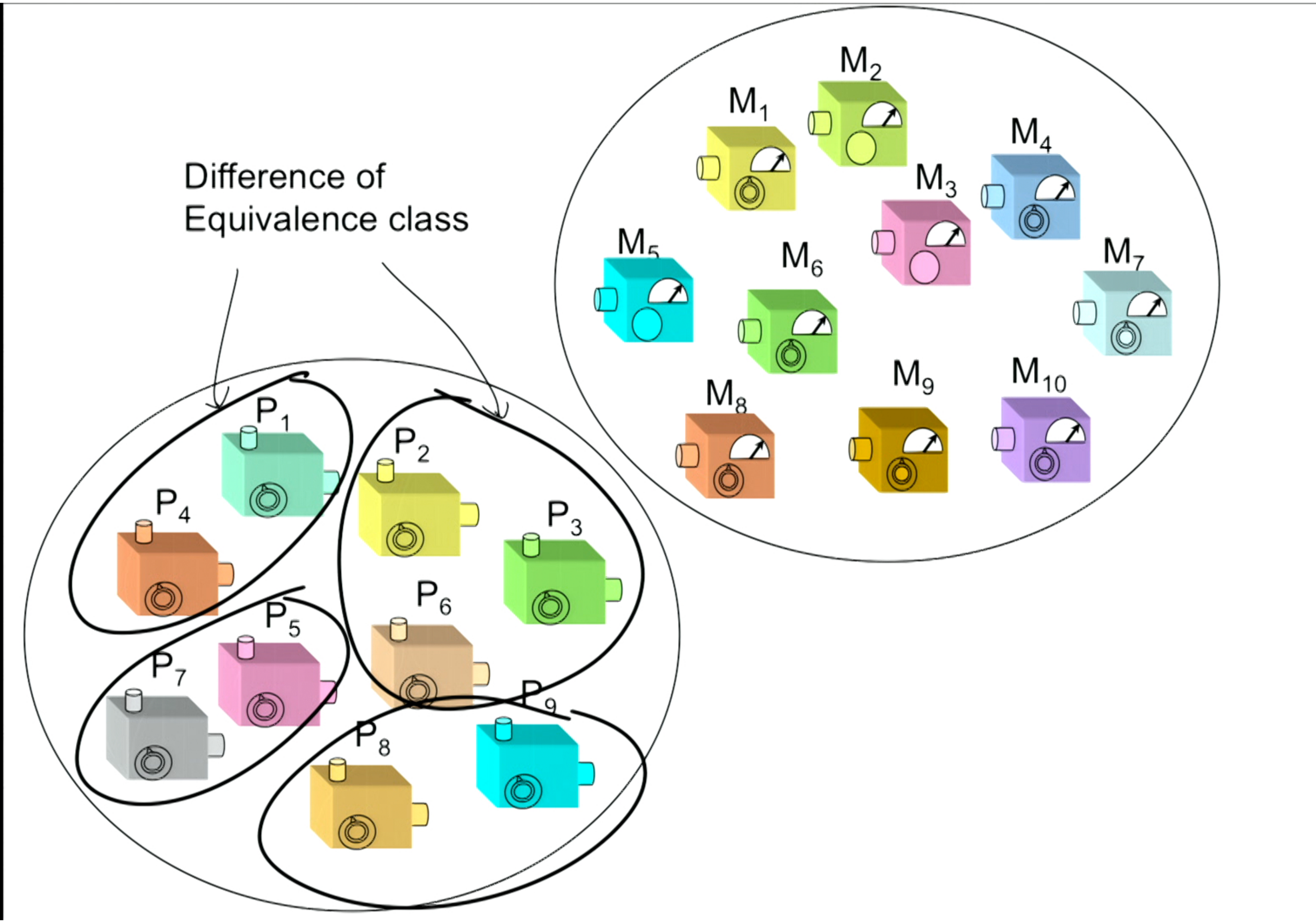
Operational equivalence classes of preparations

$$\mathsf{P} \simeq \mathsf{P}'$$

$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M})$$







Difference of
Equivalence class
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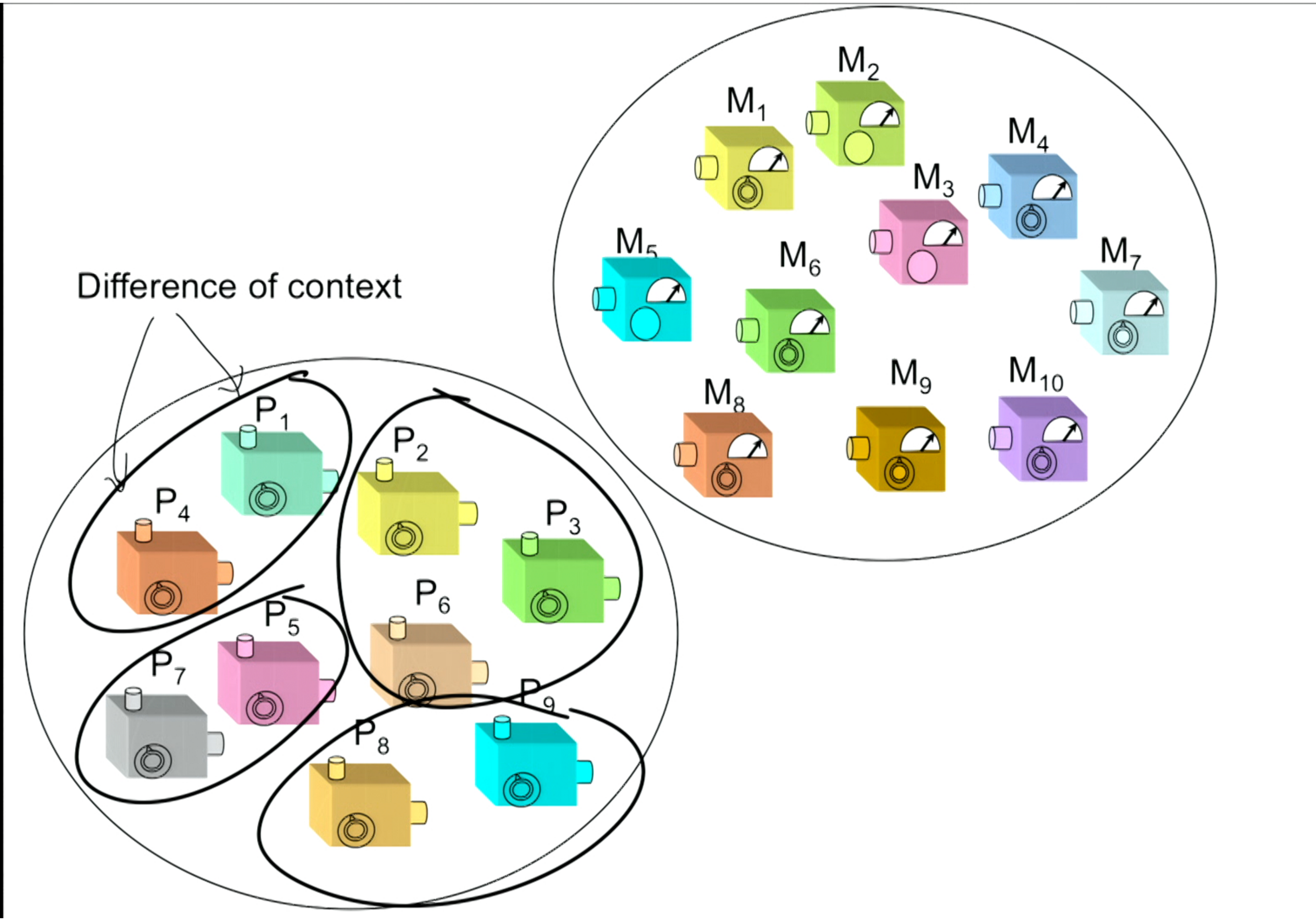
Difference of context
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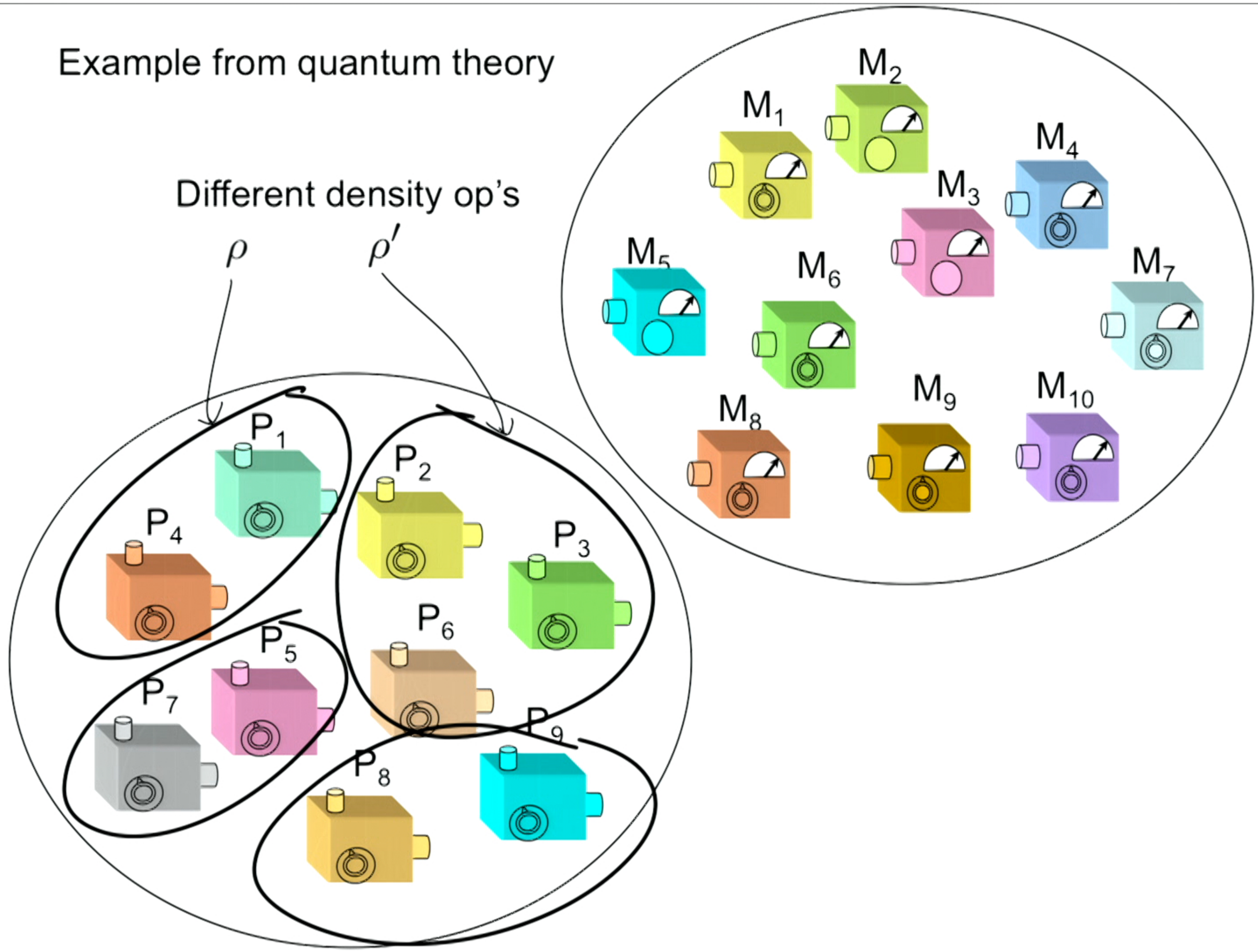
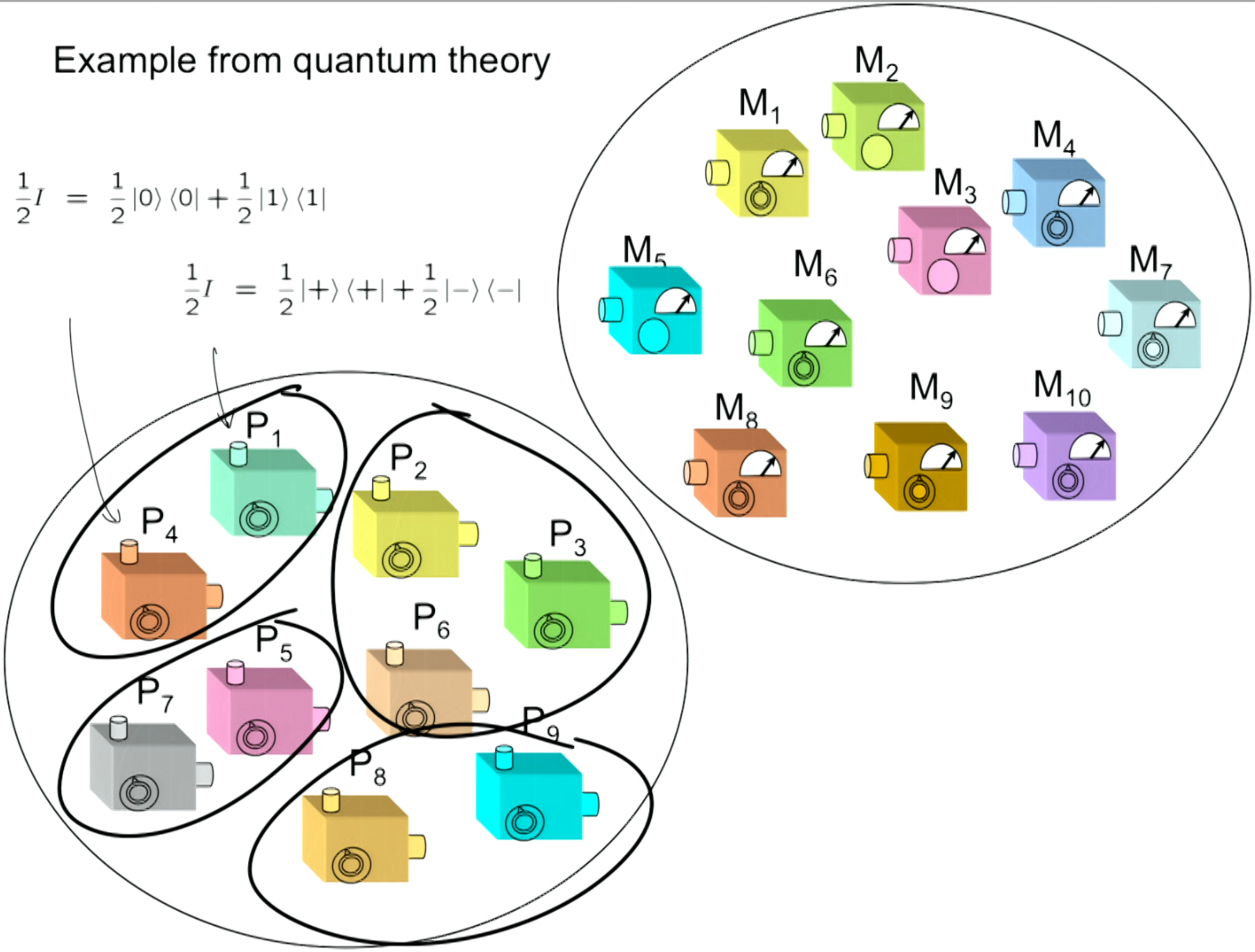
Example from quantum theory



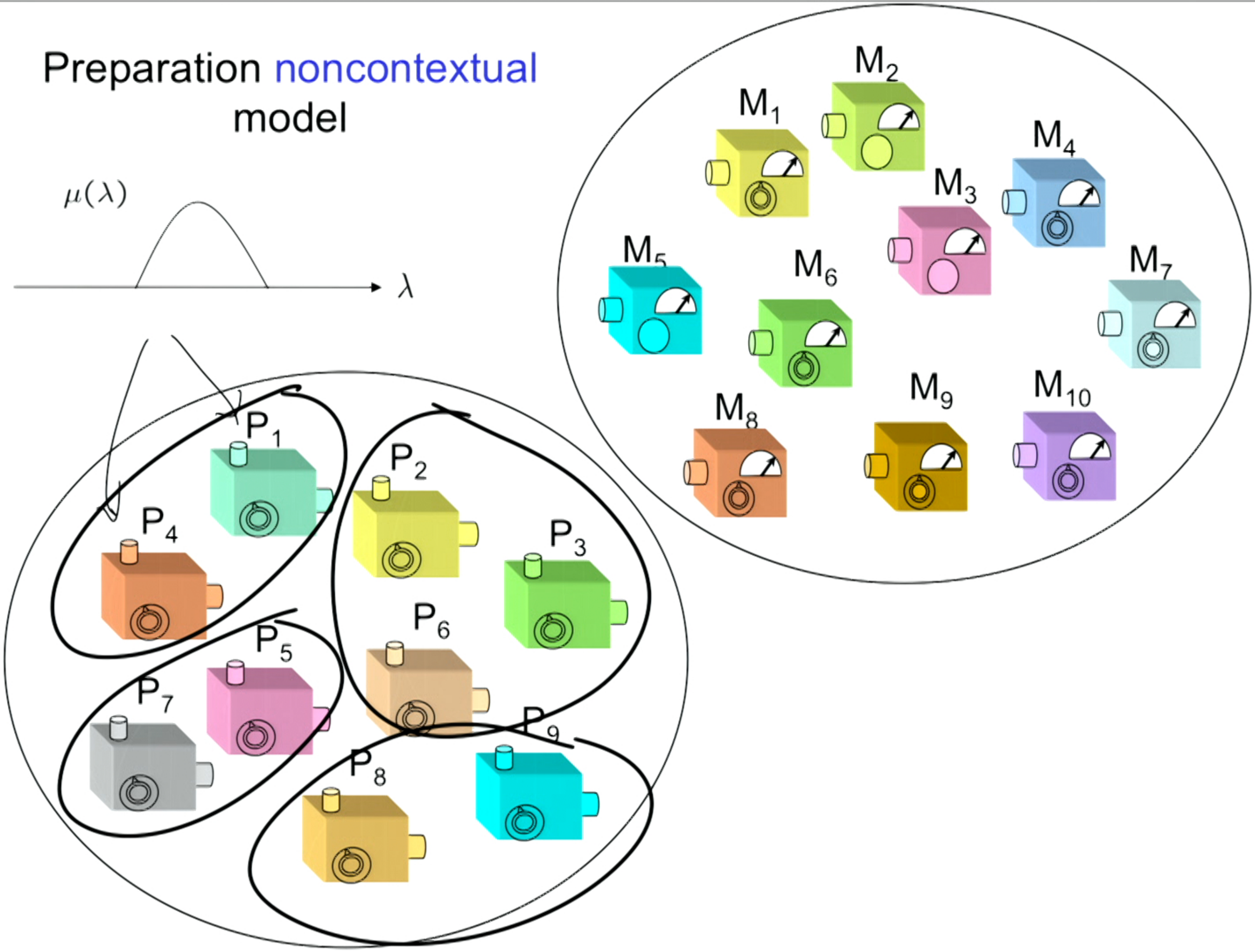
Example from quantum theory

$$\frac{1}{2}I = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

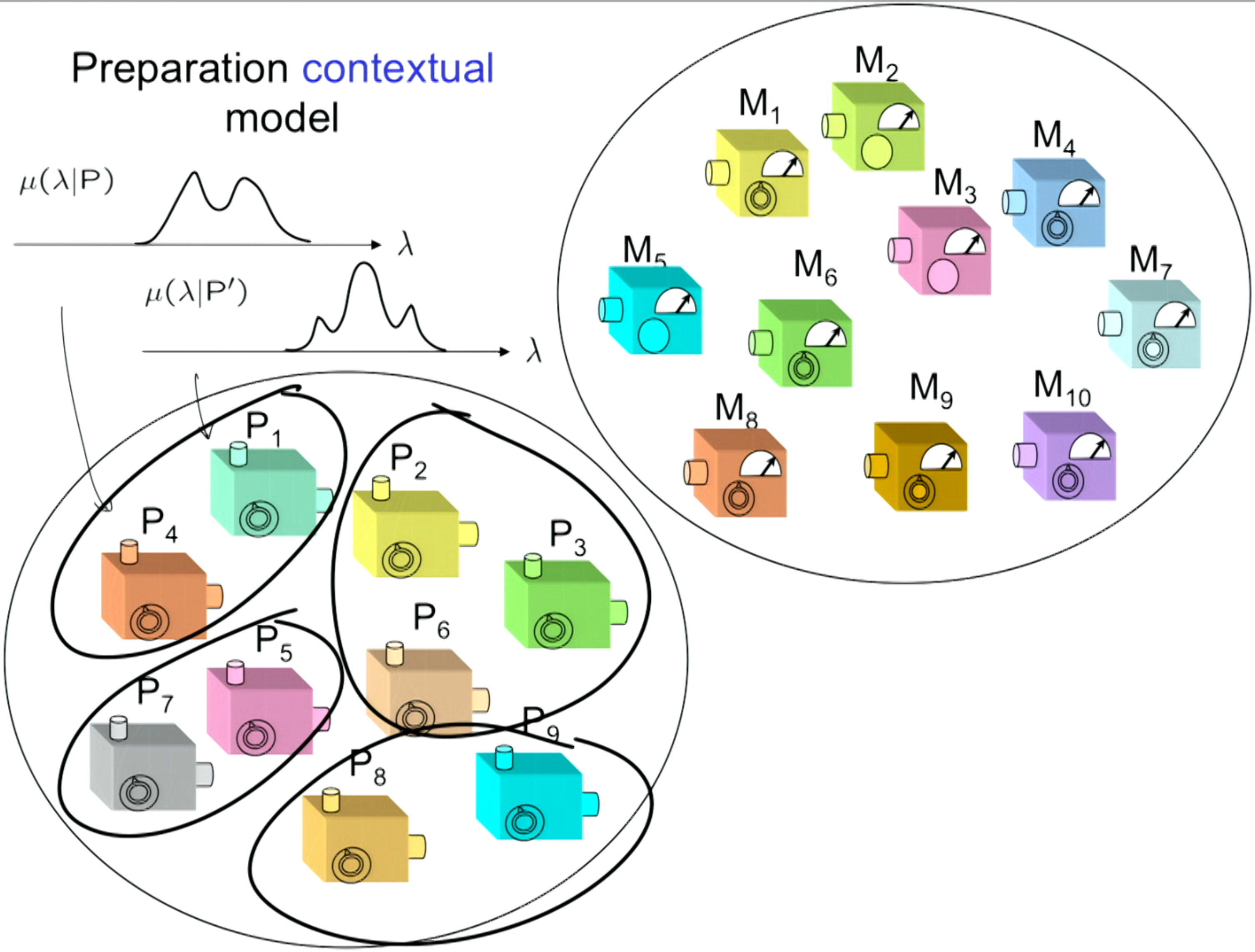
$$\frac{1}{2}I = \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$



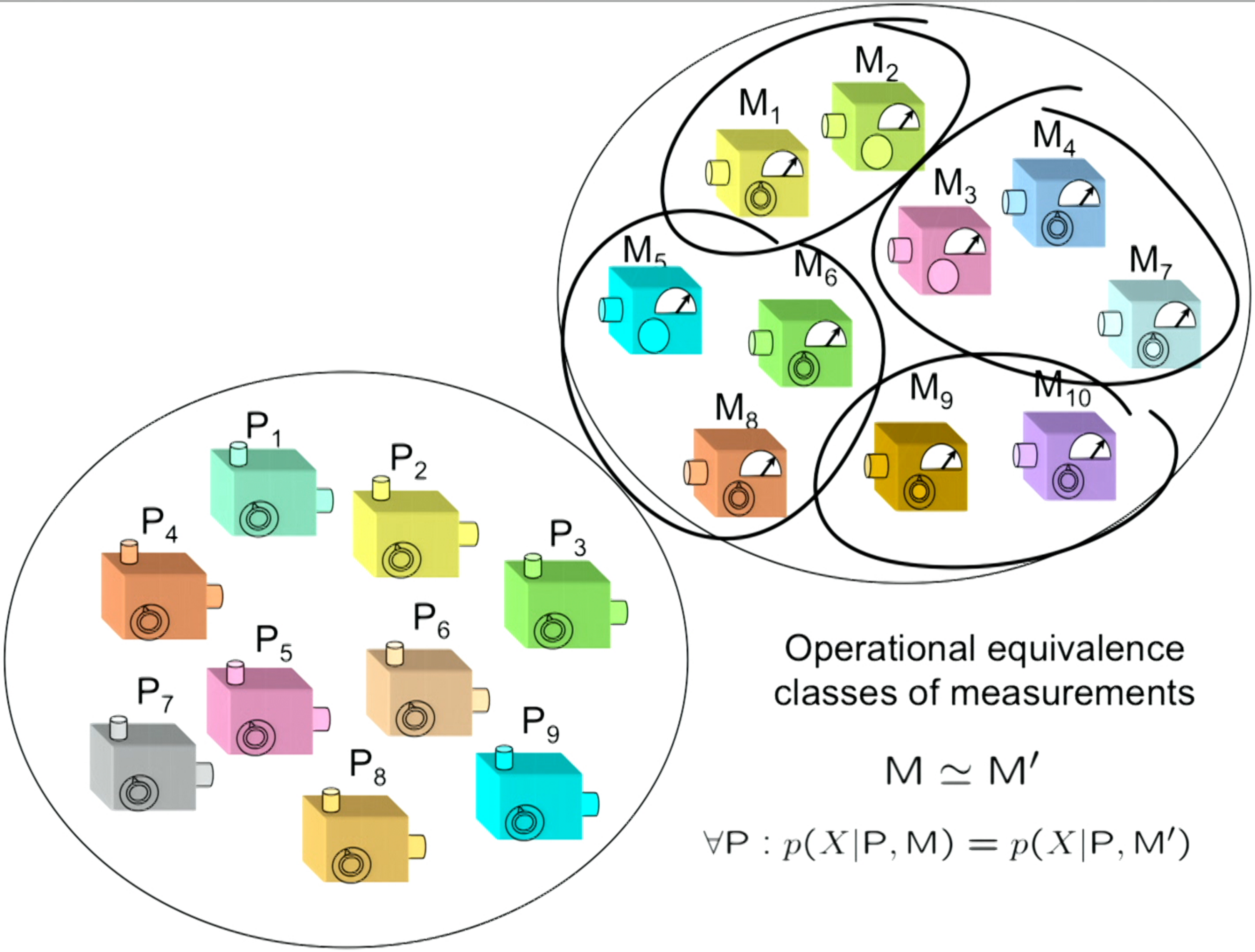
Preparation noncontextual model



Preparation contextual model



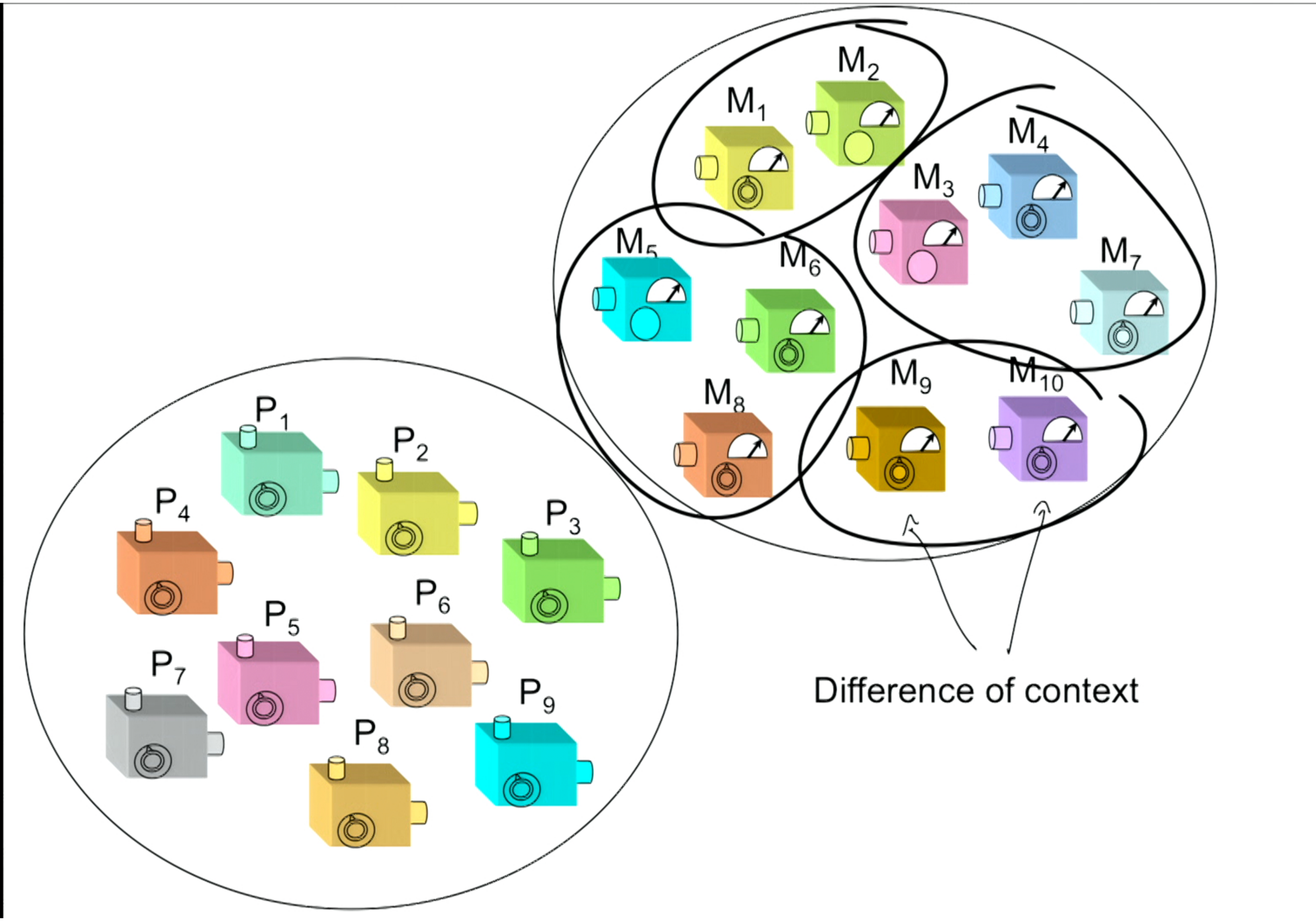
Measurement Noncontextuality



Operational equivalence classes of measurements

$$\mathsf{M} \simeq \mathsf{M}'$$

$$\forall \mathsf{P} : p(X|\mathsf{P}, \mathsf{M}) = p(X|\mathsf{P}, \mathsf{M}')$$



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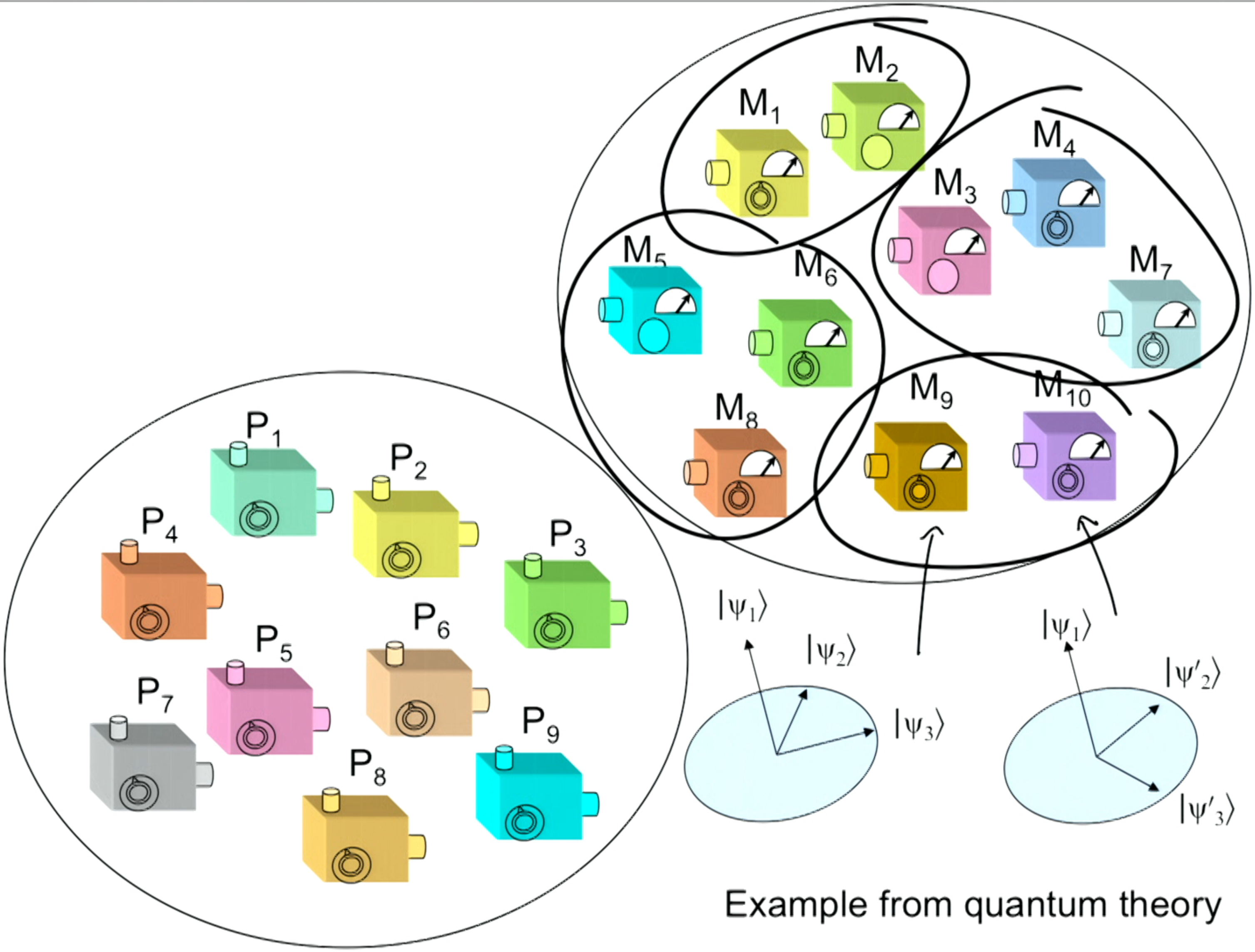
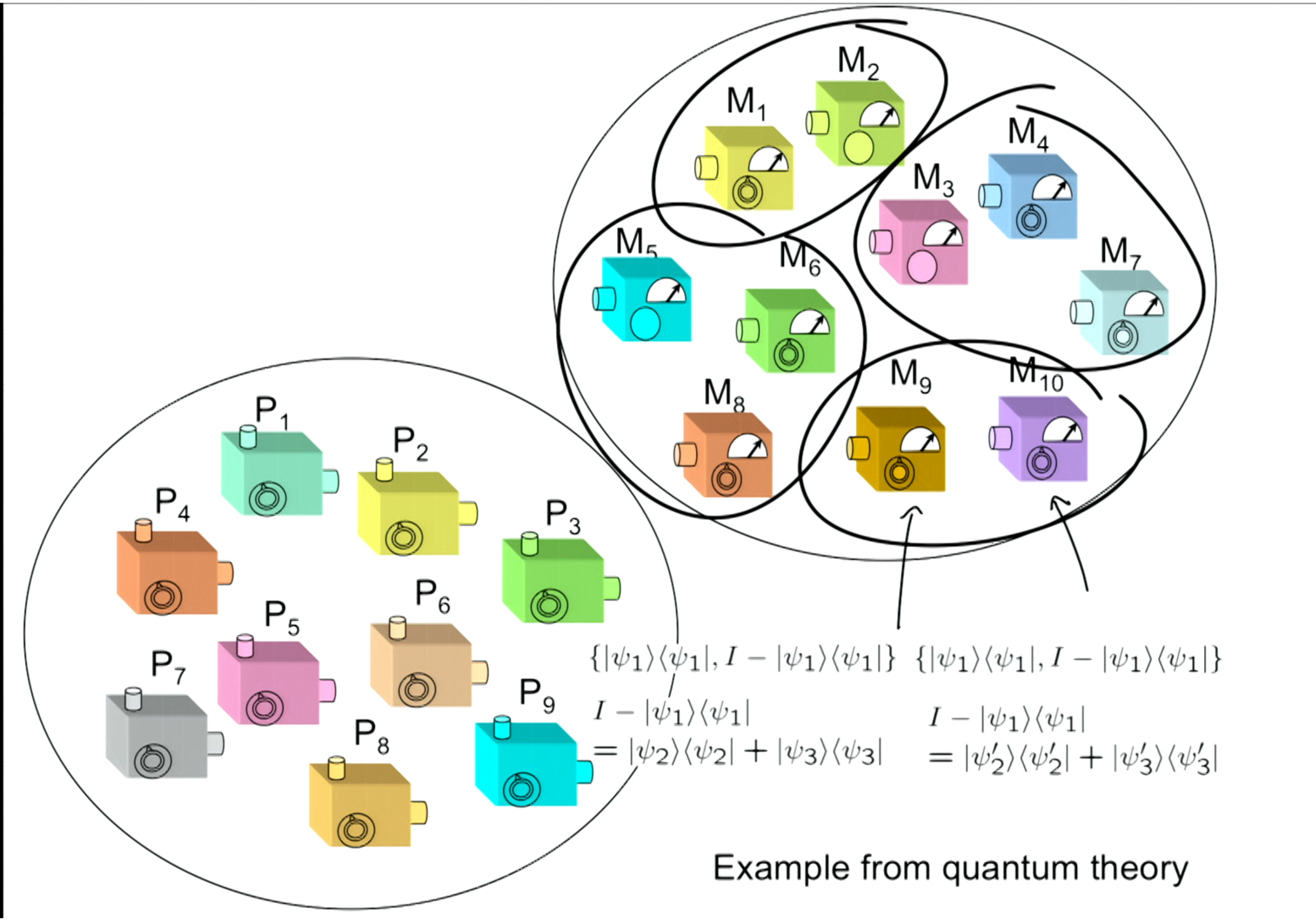
Difference of context



Example from quantum theory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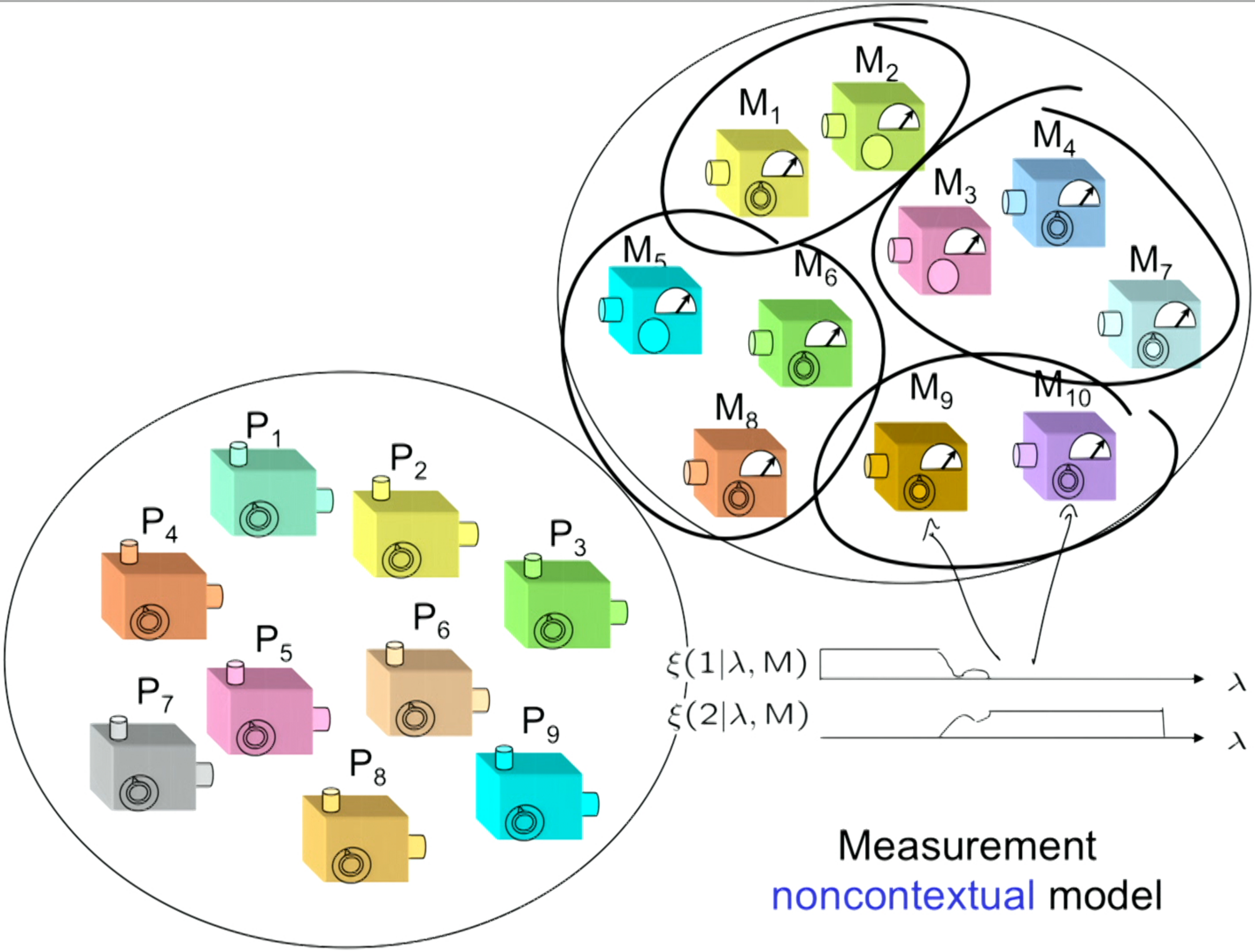
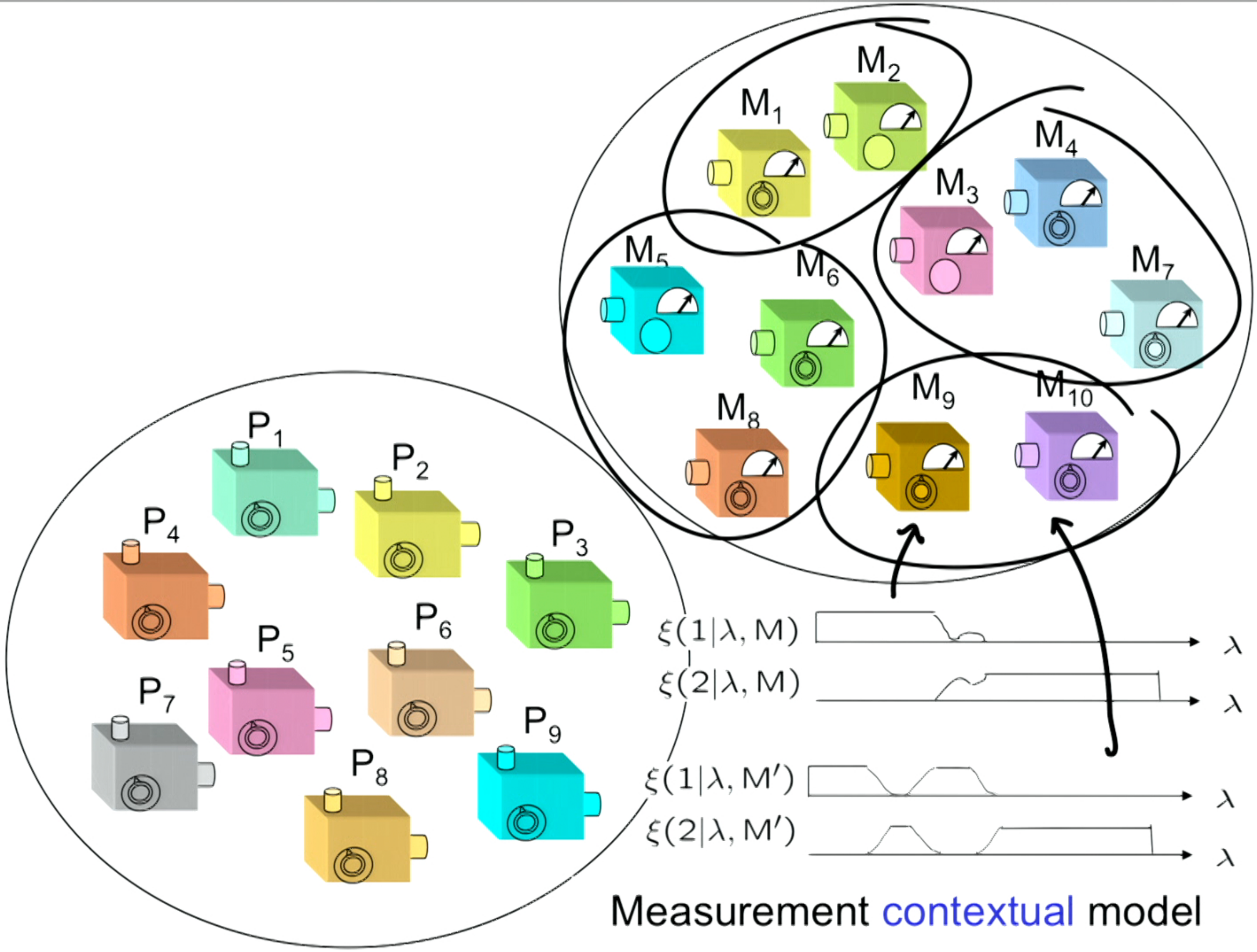
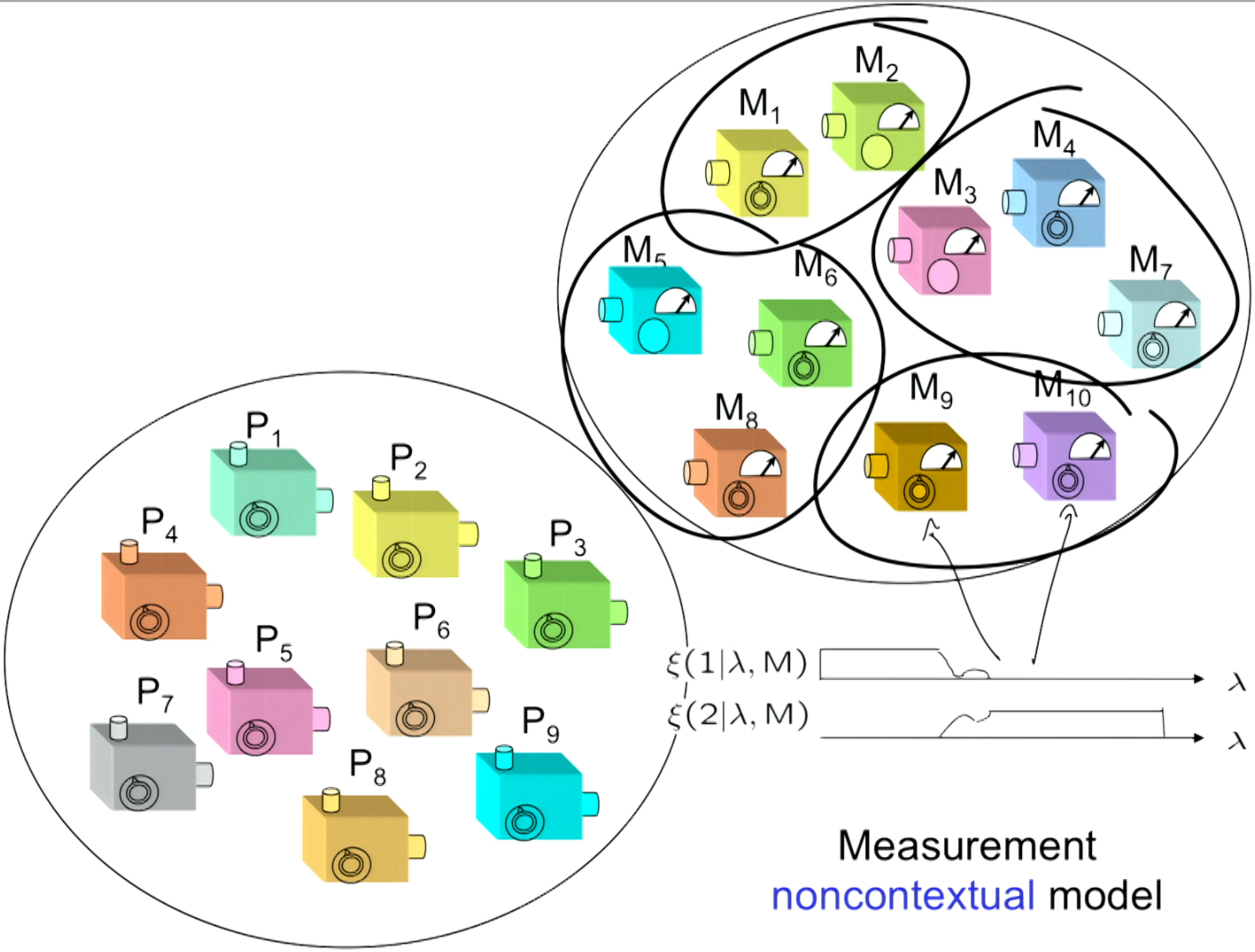
$\{|\psi_1\rangle\langle\psi_1|, I - |\psi_1\rangle\langle\psi_1|\}$
$\{|\psi_1\rangle\langle\psi_1|, I - |\psi_1\rangle\langle\psi_1|\}$
$I - |\psi_1\rangle\langle\psi_1| = |\psi_2\rangle\langle\psi_2| + |\psi_3\rangle\langle\psi_3|$
$I - |\psi_1\rangle\langle\psi_1| = |\psi_2'\rangle\langle\psi_2'| + |\psi_3'\rangle\langle\psi_3'|$
Example from quantum theory



Measurement noncontextual model



Measurement contextual model



Measurement
noncontextual model

$$\mathsf{P} \simeq \mathsf{P}'$$

$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M})$$

Preparation noncontextuality $\longrightarrow$

$$\mu(\lambda|\mathsf{P}) = \mu(\lambda|\mathsf{P}')$$

$$\mathsf{M} \simeq \mathsf{M}'$$

$$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}')$$

Measurement noncontextuality $\longrightarrow$

$$\xi(X|\lambda,\mathsf{M}) = \xi(X|\lambda,\mathsf{M}')$$

The best explanation of context-independence at the operational level is context-independence at the ontological level

Transformation noncontextuality is defined similarly

The only natural assumption is universal noncontextuality

Comparison with other approaches

Other approaches agree that to assume noncontextuality is to assume an inference of the form:

Some equivalence relation holding between two experimental procedures	→	Equivalent representations in the ontological model

But

- focus on measurements
- disagree on the nature of the equivalence relation
- disagree on the nature of the ontological representation

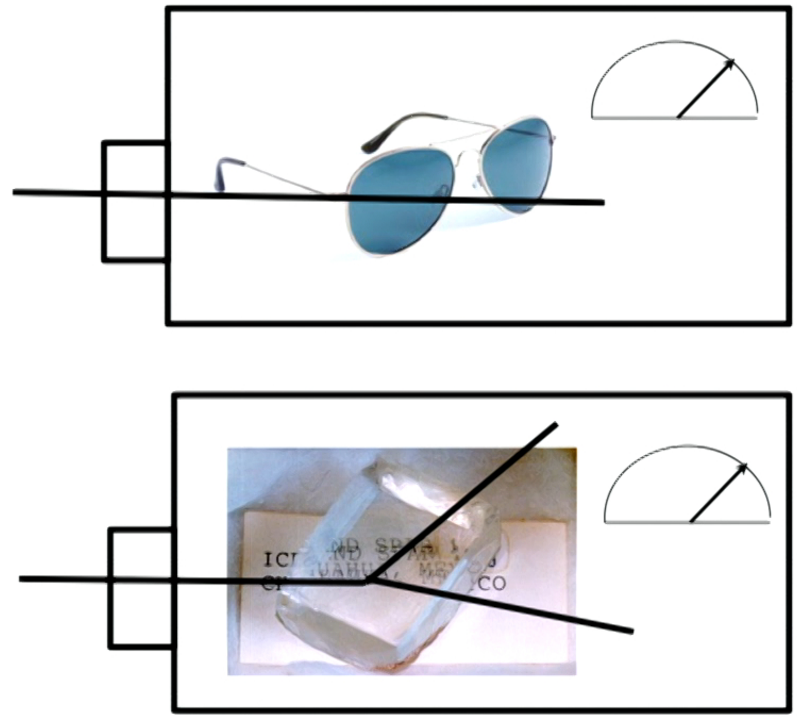
Disagreement #1:
The nature of the equivalence relation between measurement procedures

The competing proposal:

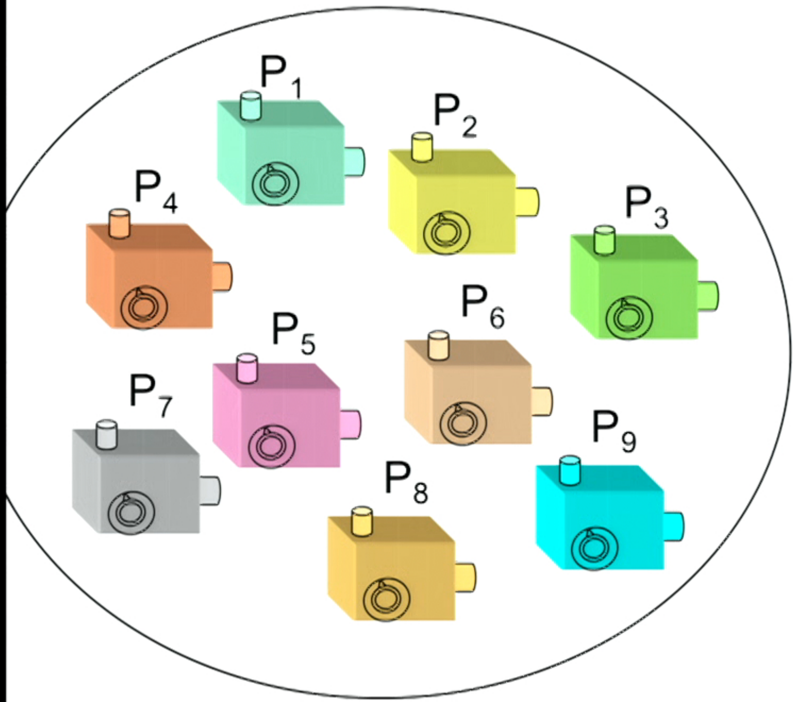
Suppose B and C are two incompatible measurements that are each compatible with A

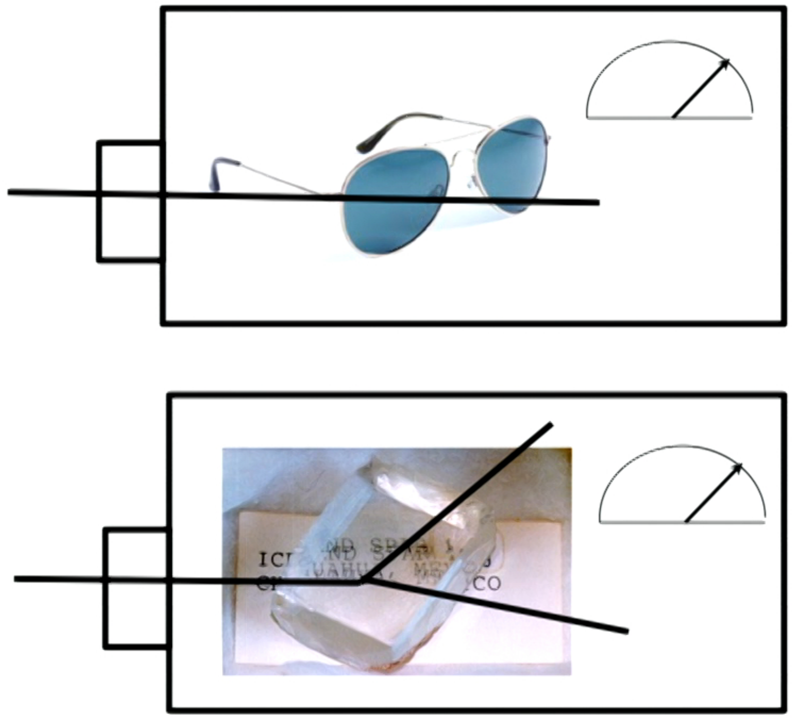
“The experimental apparatus used for measuring, e.g., [A] must be the same when [it] is measured together with [B], and when it is measured together with [C].”
--- A. Cabello, PRL 101, 210401 (2008).

Sameness of apparatus not a necessary condition:

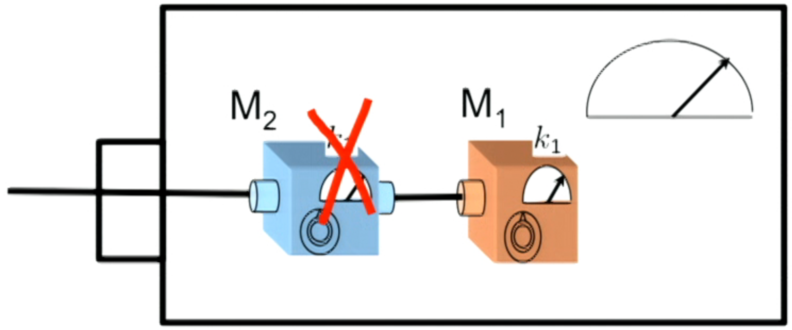


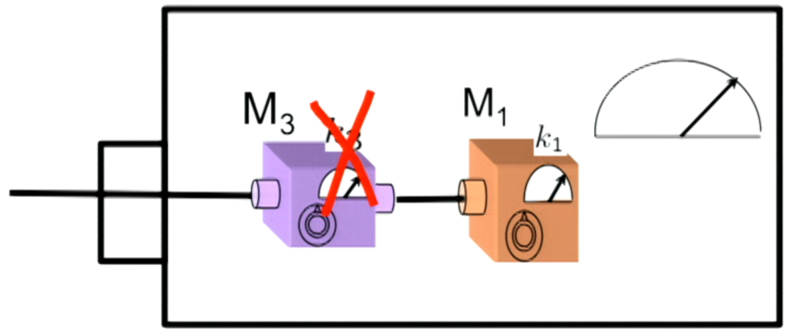
Sameness of apparatus
not a necessary condition:

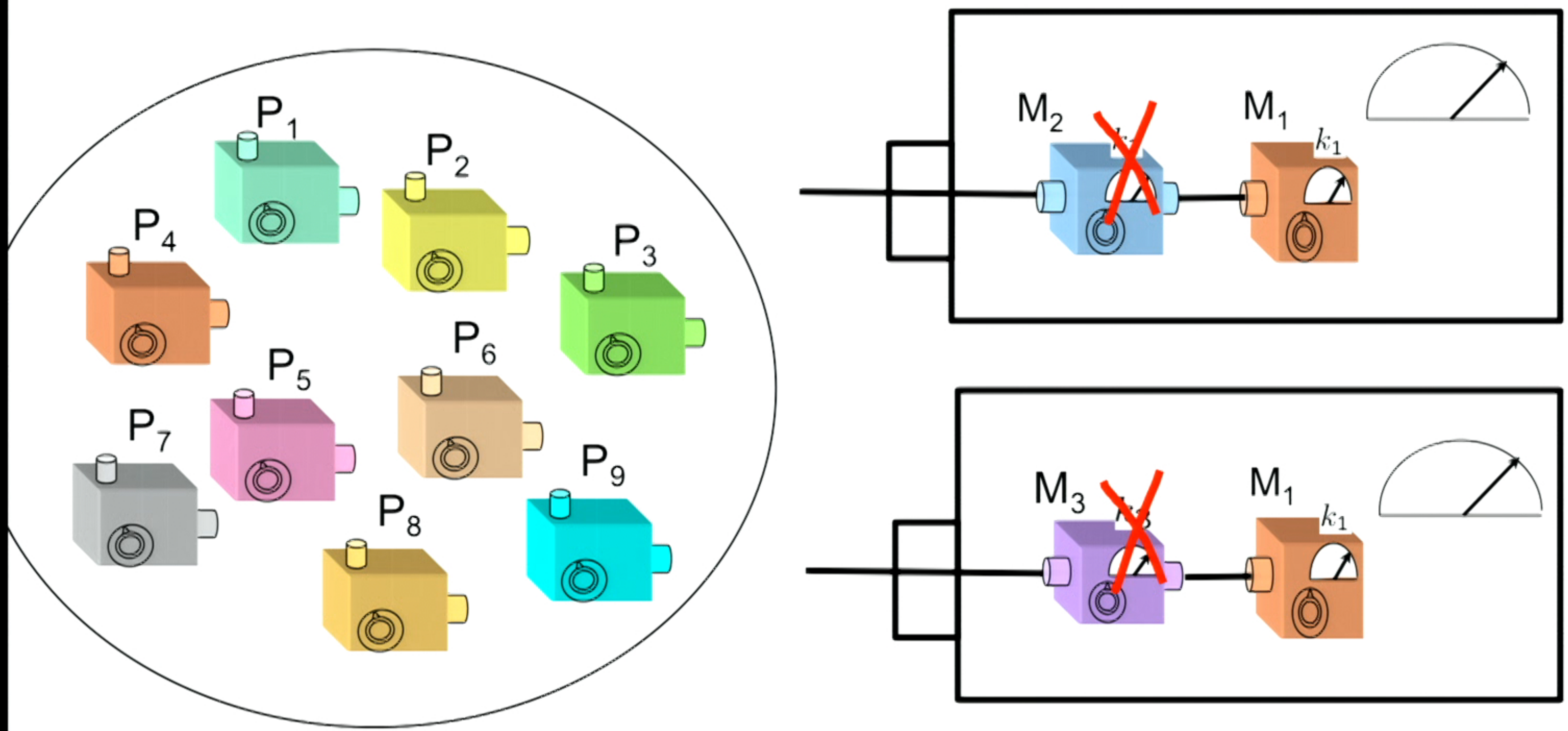




Sameness of apparatus not a sufficient condition:







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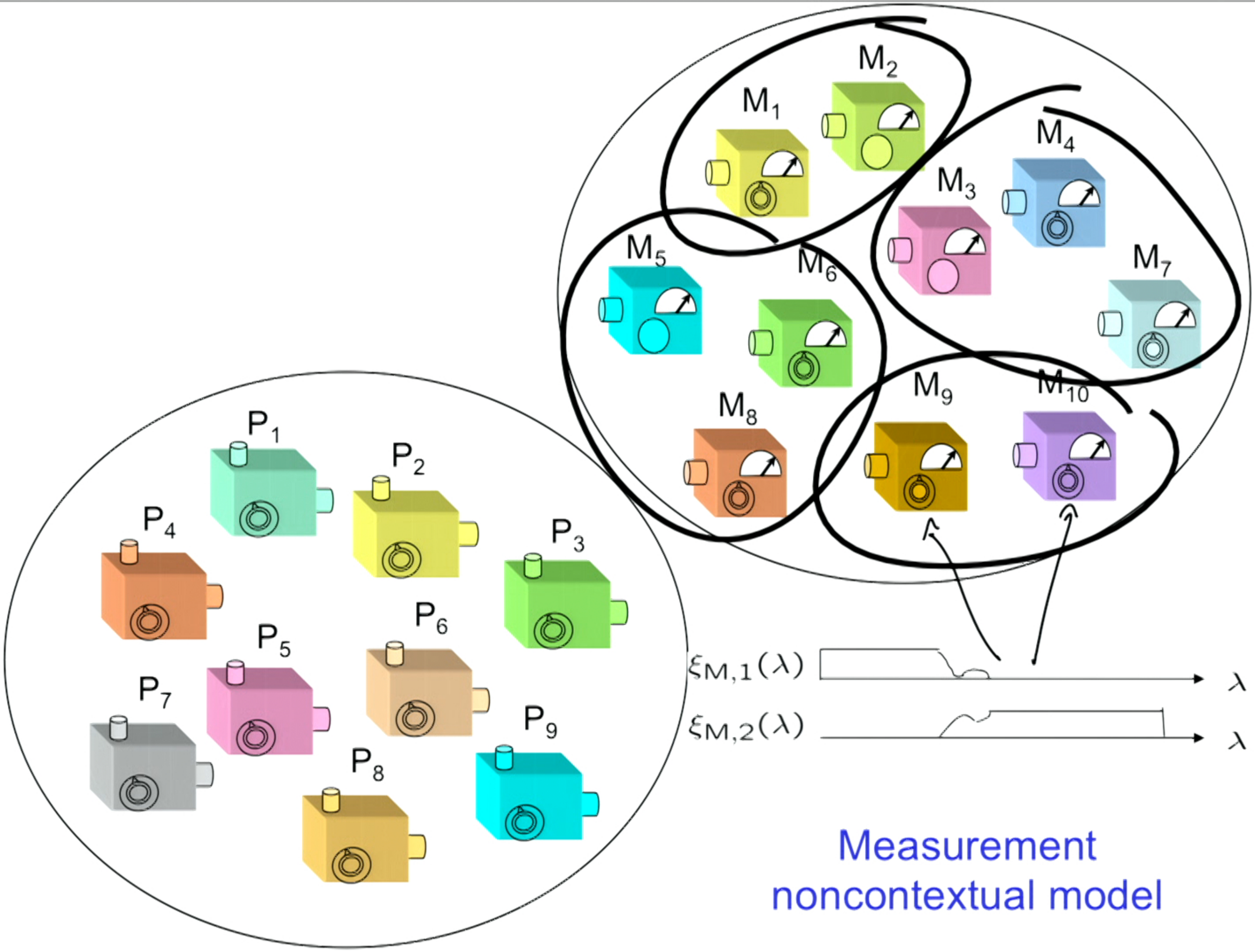
Operational equivalence says: the relevant notion of sameness for two measurement procedures on a system is sameness of statistics for all preparation procedures on that system.

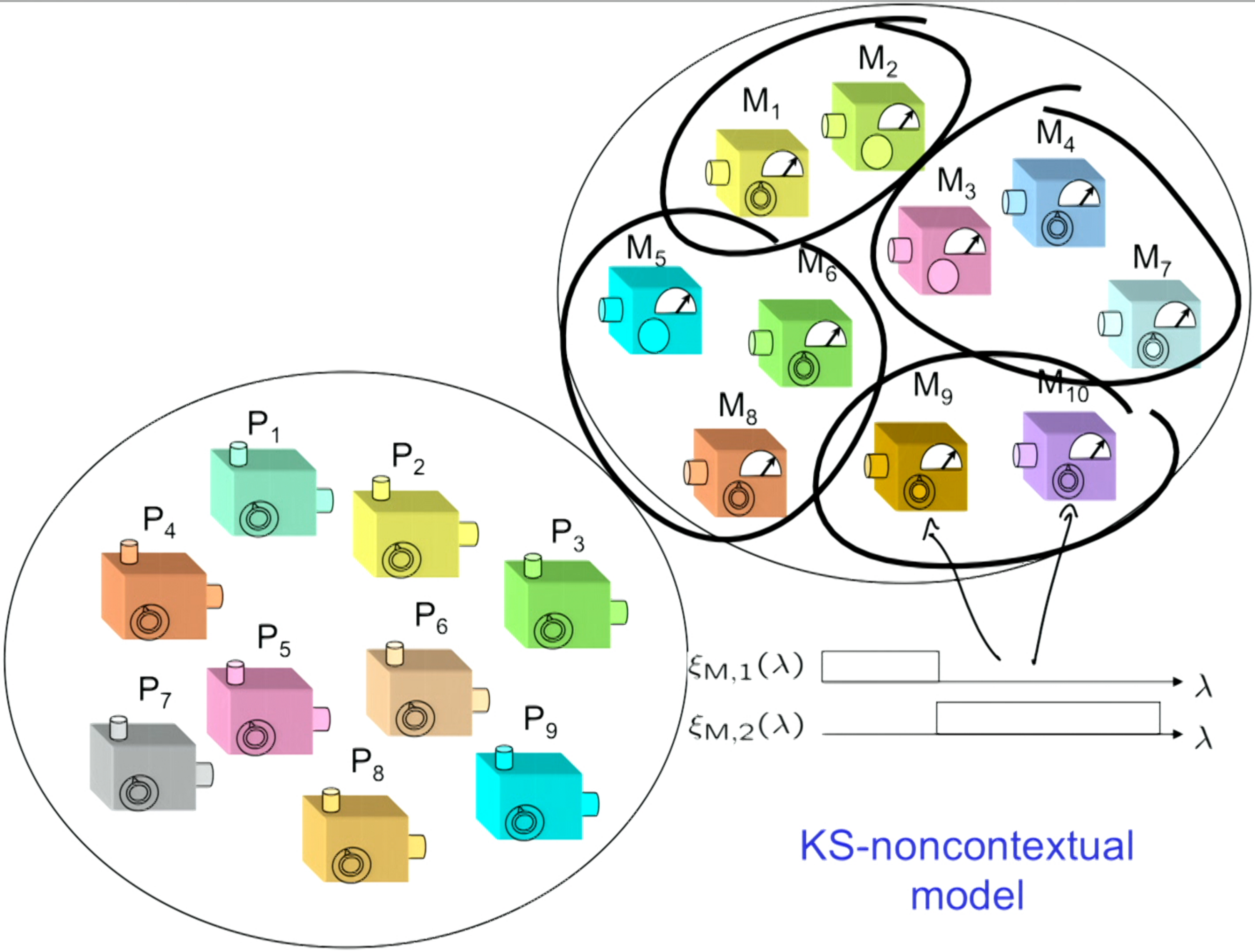
Operational equivalence says: the relevant notion of sameness for two measurement procedures on a system is sameness of statistics for all preparation procedures on that system.

It is the grounds by which we justify modeling two measurements equivalently in QT

Disagreement #2:
The nature of the ontological representation of measurements



Measurement noncontextual model



KS-noncontextual model

measurement noncontextuality

and = KS-noncontextuality

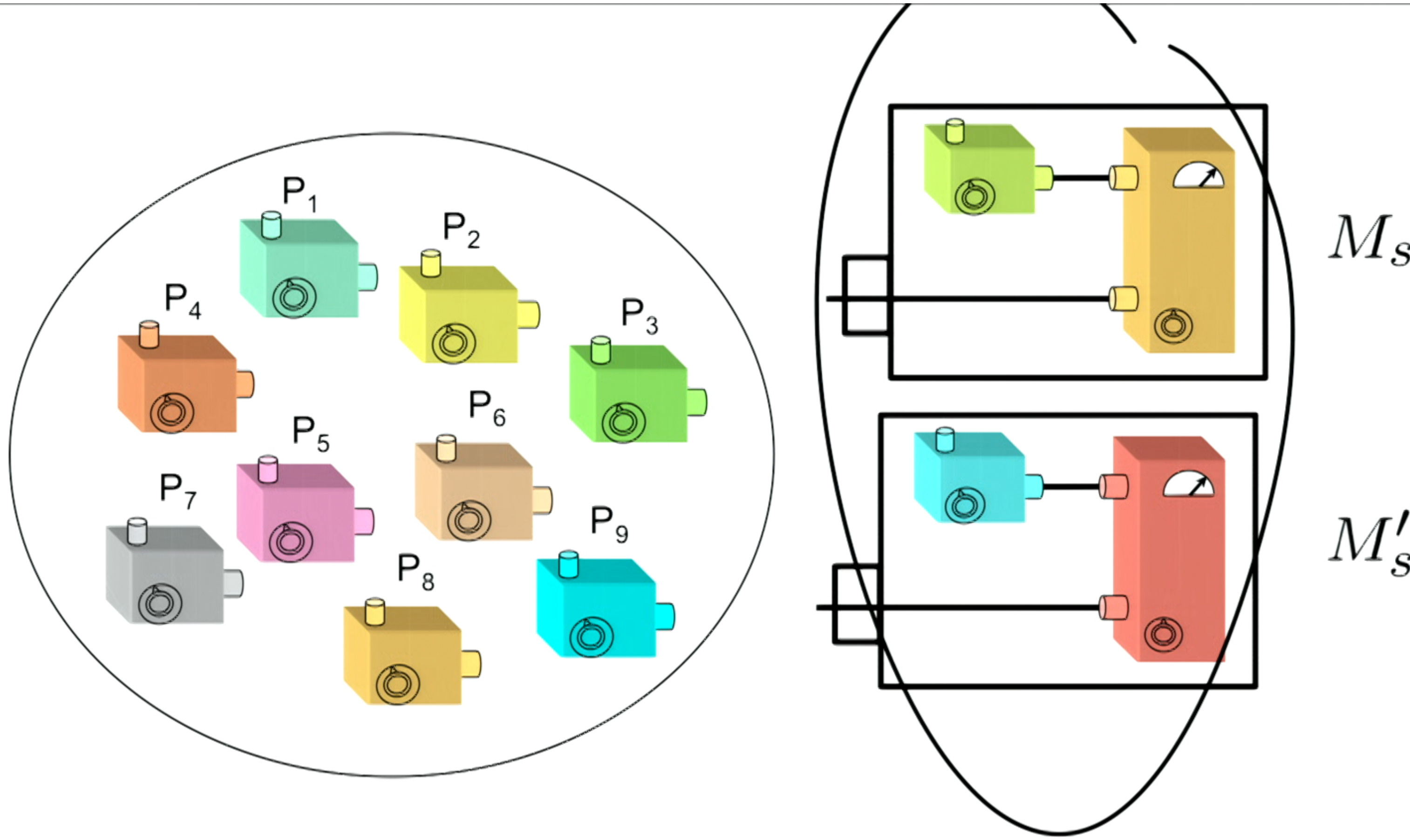
outcome determinism

But, in face of a violation of KS-noncontextuality,
we could give up outcome determinism

Theorem: A universally noncontextual ontological model assigns outcomes to a quantum measurement deterministically if and only if the measurement is sharp

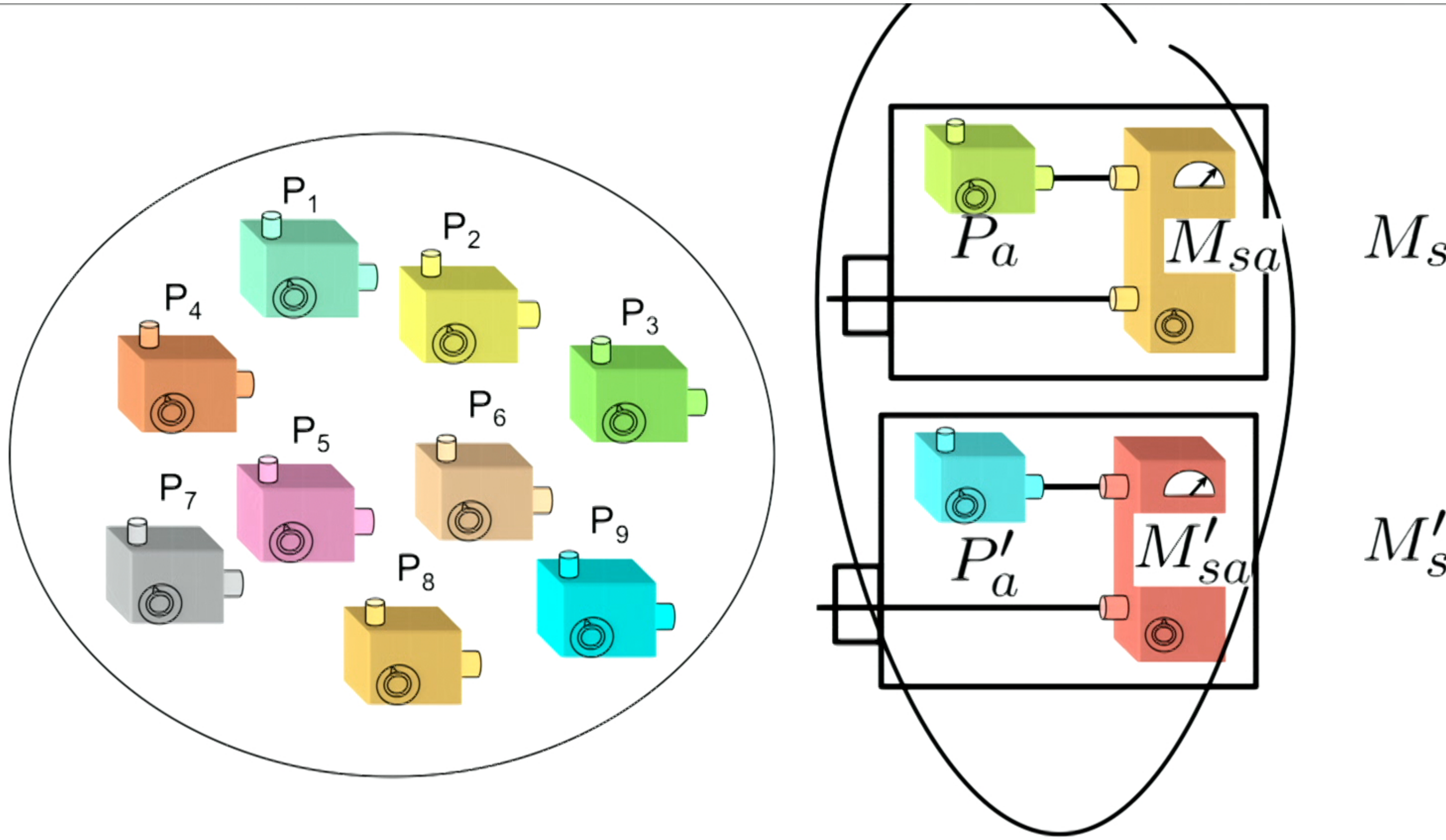
See: RWS, Found. Phys. 44, 1125 (2014)

All real-world measurements have some degree of unsharpness



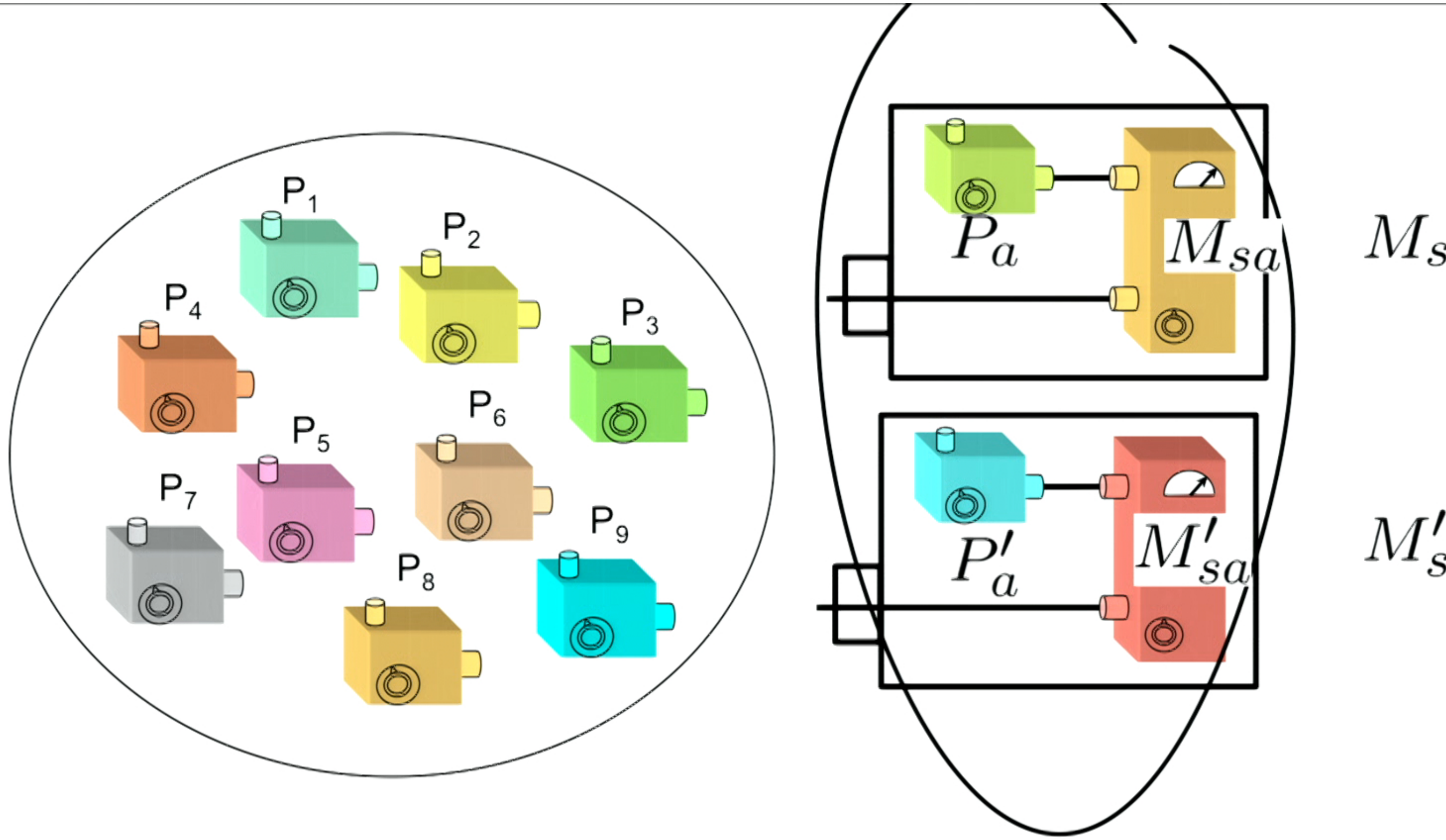
$$M_s \simeq M_s'$$

Thus $\xi(X|\lambda_s, M_s) = \xi(X|\lambda_s, M_s')$



No evidence for $M_{sa} \simeq M'_{sa}$

No justification for $\xi(X|\lambda_s, \lambda_a, M_{sa}) = \xi(X|\lambda_s, \lambda_a, M'_{sa})$



From $M_s \simeq M'_s$ all we can infer is

$$\int \mathrm{d}\lambda_a \xi(X|\lambda_s, \lambda_a, M_{sa})\mu(\lambda_a|P_a) = \int \mathrm{d}\lambda_a \xi(X|\lambda_s, \lambda_a, M'_{sa})\mu(\lambda_a|P'_a)$$

Challenges to direct experimental testability

How do we derive constraints on experimental statistics from noncontextuality when the preparations and measurements are noisy/unsharp?

The problem of contending with unsharpness

How do we experimentally establish operational equivalence?

Problem of inexact operational equivalences

Preparation noncontextuality

$$\mathsf{P} \simeq \mathsf{P}'$$
$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M}) \longrightarrow \mu(\lambda|\mathsf{P}) = \mu(\lambda|\mathsf{P}')$$

Measurement noncontextuality

$$\mathsf{M} \simeq \mathsf{M}'$$
$$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}') \longrightarrow \xi(X|\lambda,\mathsf{M}) = \xi(X|\lambda,\mathsf{M}')$$

How do we experimentally establish operational equivalence?
The problem of the universal quantifier

$$\mathsf{P} \simeq \mathsf{P}'$$
$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M}) \xrightarrow{\text{Preparation noncontextuality}} \mu(\lambda|\mathsf{P}) = \mu(\lambda|\mathsf{P}')$$

$$\mathsf{M} \simeq \mathsf{M}'$$
$$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}') \xrightarrow{\text{Measurement noncontextuality}} \xi(X|\lambda,\mathsf{M}) = \xi(X|\lambda,\mathsf{M}')$$

The problem of contending with unsharpness

Kunjwal and RWS, “From the Kochen-Specker theorem to noncontextuality inequalities without assuming determinism”, PRL 115, 110403 (2015)

Mazurek, Pusey, Kunjwal, Resch, and RWS, “An experimental test of noncontextuality without unphysical idealizations”, Nat. Commun. 7, 11780 (2016)

Kunjwal and RWS, “Translating proofs of the Kochen-Specker theorem without KS-uncolourability into noise-robust noncontextuality inequalities”, forthcoming

(See talk on Friday by Ravi Kunjwal)

Krishna, RWS and Wolfe, “Deriving robust noncontextuality inequalities from algebraic proofs of the Kochen-Specker theorem: the Peres-Mermin square”, arXiv:1704.01153

The Peres-Mermin square

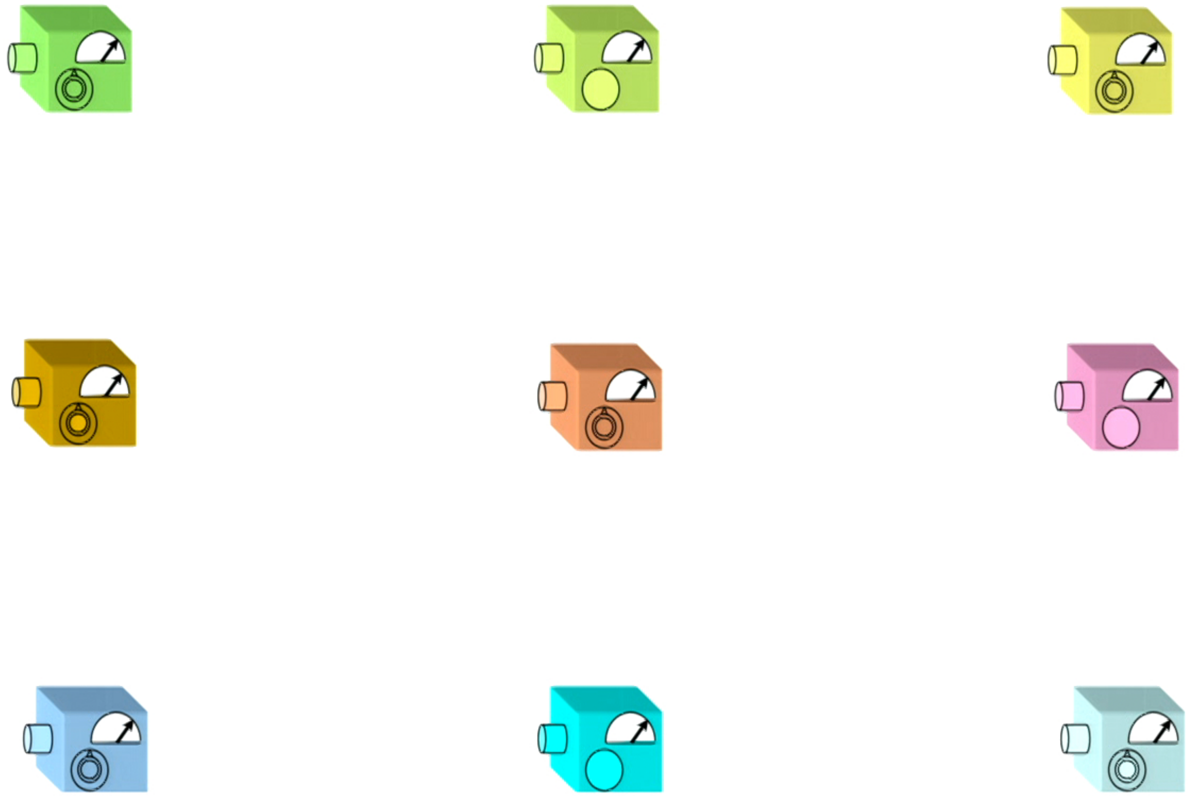
$$\begin{array}{ccc} X \otimes I & I \otimes X & X \otimes X \\ I \otimes Z & Z \otimes I & Z \otimes Z \\ X \otimes Z & Z \otimes X & Y \otimes Y \end{array}$$

$$\begin{aligned}
(X \otimes I)(I \otimes X)(X \otimes X) &= I \otimes I, \\
(I \otimes Z)(Z \otimes I)(Z \otimes Z) &= I \otimes I, \\
(X \otimes Z)(Z \otimes X)(Y \otimes Y) &= I \otimes I, \\
(X \otimes I)(I \otimes Z)(X \otimes Z) &= I \otimes I, \\
(I \otimes X)(Z \otimes I)(Z \otimes X) &= I \otimes I, \\
(X \otimes X)(Z \otimes Z)(Y \otimes Y) &= -I \otimes I.
\end{aligned}$$

The Peres-Mermin square

$X \otimes I$	$I \otimes X$	$X \otimes X$
$I \otimes Z$	$Z \otimes I$	$Z \otimes Z$
$X \otimes Z$	$Z \otimes X$	$Y \otimes Y$

$$
\begin{aligned}
\lfloor X \otimes I \rfloor_\lambda \lfloor I \otimes X \rfloor_\lambda \lfloor X \otimes X \rfloor_\lambda &= +1, \\
\lfloor I \otimes Z \rfloor_\lambda \lfloor Z \otimes I \rfloor_\lambda \lfloor Z \otimes Z \rfloor_\lambda &= +1, \\
\lfloor X \otimes Z \rfloor_\lambda \lfloor Z \otimes X \rfloor_\lambda \lfloor Y \otimes Y \rfloor_\lambda &= +1, \\
\lfloor X \otimes I \rfloor_\lambda \lfloor I \otimes Z \rfloor_\lambda \lfloor X \otimes Z \rfloor_\lambda &= +1, \\
\lfloor I \otimes X \rfloor_\lambda \lfloor Z \otimes I \rfloor_\lambda \lfloor Z \otimes X \rfloor_\lambda &= +1, \\
\lfloor X \otimes X \rfloor_\lambda \lfloor Z \otimes Z \rfloor_\lambda \lfloor Y \otimes Y \rfloor_\lambda &= -1.
\end{aligned}
$$



The Peres-Mermin square

$X \otimes I$	$I \otimes X$	$X \otimes X$
$I \otimes Z$	$Z \otimes I$	$Z \otimes Z$
$X \otimes Z$	$Z \otimes X$	$Y \otimes Y$

$$
\begin{aligned}
\lfloor X \otimes I \rfloor_\lambda \lfloor I \otimes X \rfloor_\lambda \lfloor X \otimes X \rfloor_\lambda &= +1, \\
\lfloor I \otimes Z \rfloor_\lambda \lfloor Z \otimes I \rfloor_\lambda \lfloor Z \otimes Z \rfloor_\lambda &= +1, \\
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\lfloor I \otimes X \rfloor_\lambda \lfloor Z \otimes I \rfloor_\lambda \lfloor Z \otimes X \rfloor_\lambda &= +1, \\
\lfloor X \otimes X \rfloor_\lambda \lfloor Z \otimes Z \rfloor_\lambda \lfloor Y \otimes Y \rfloor_\lambda &= -1.
\end{aligned}
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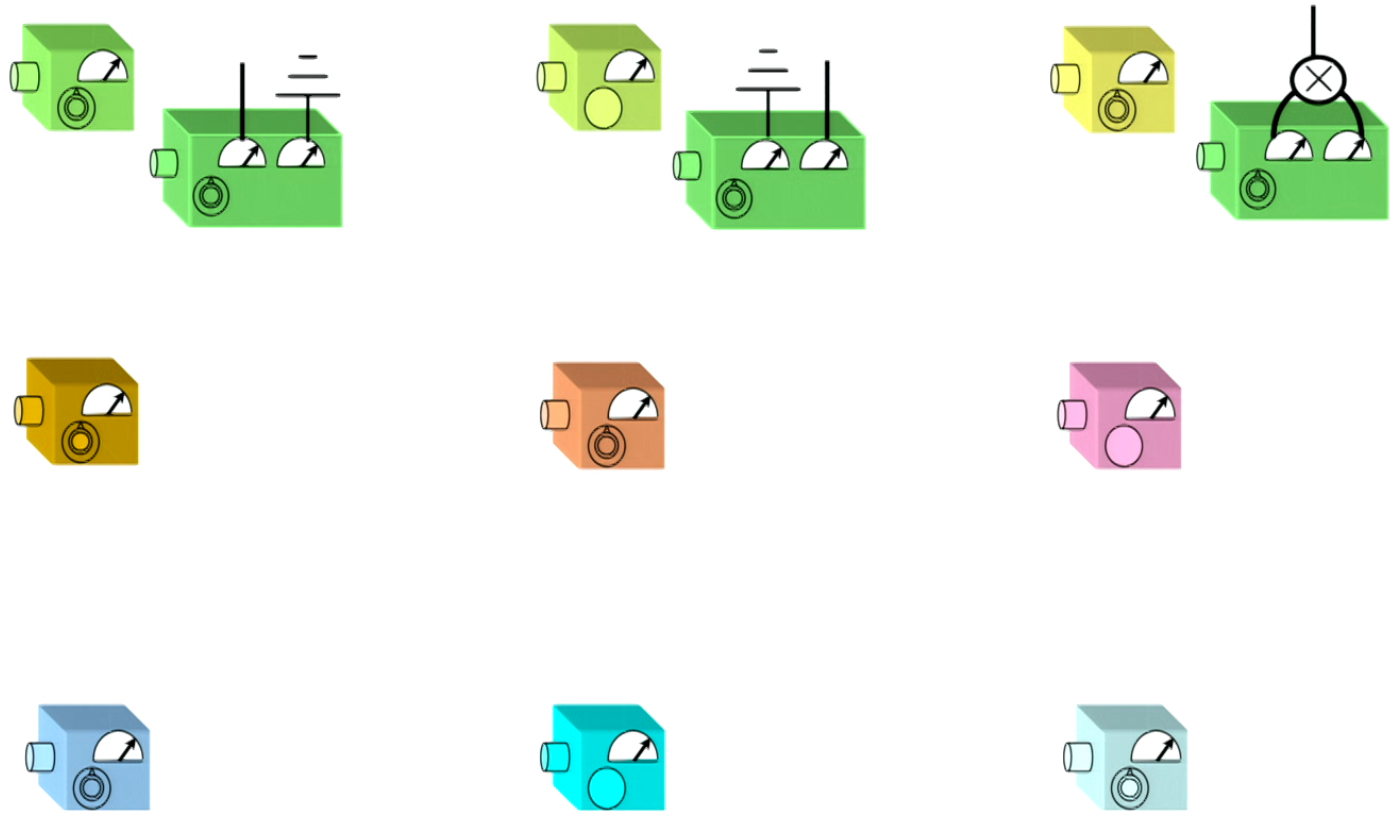

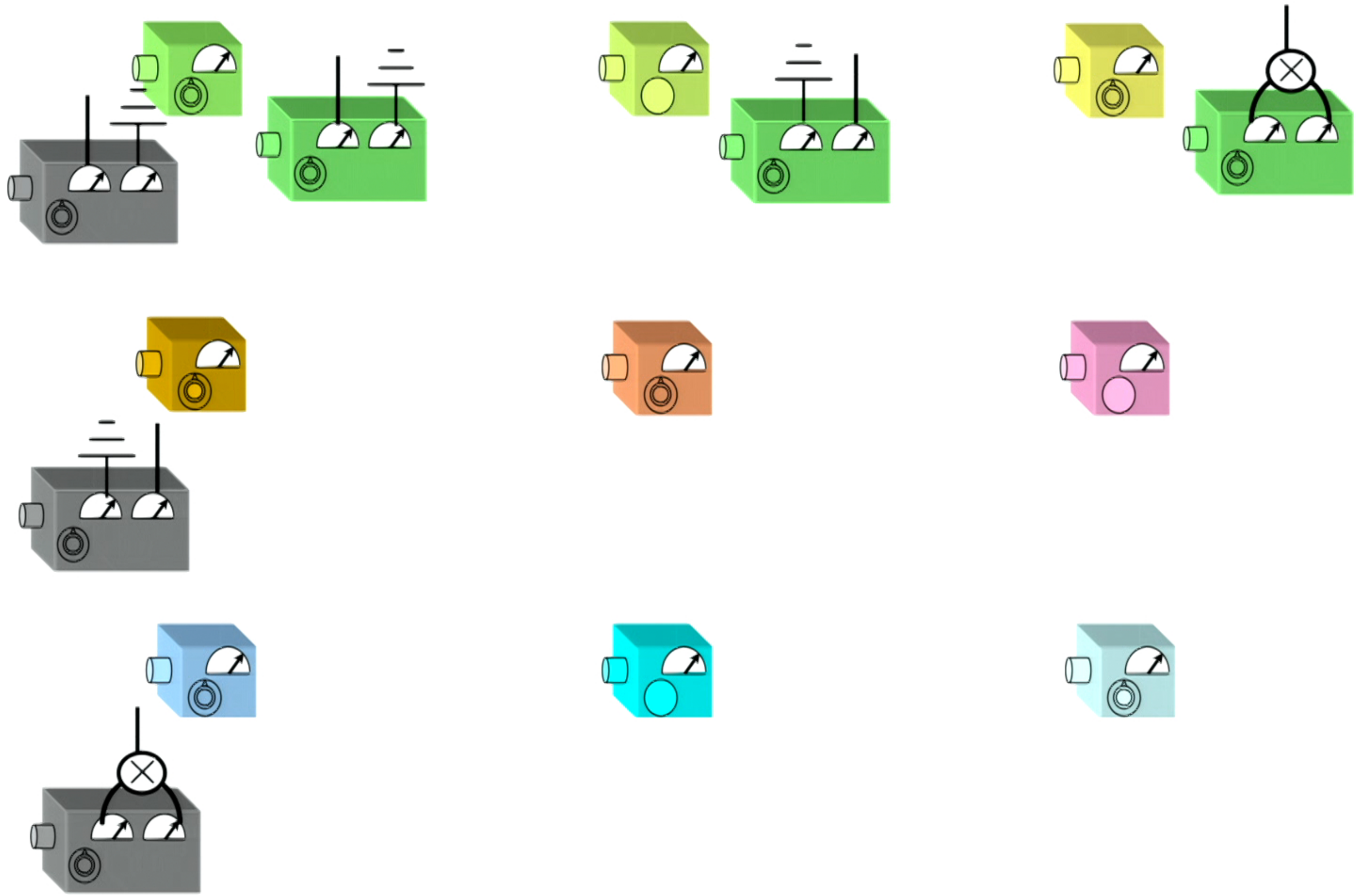

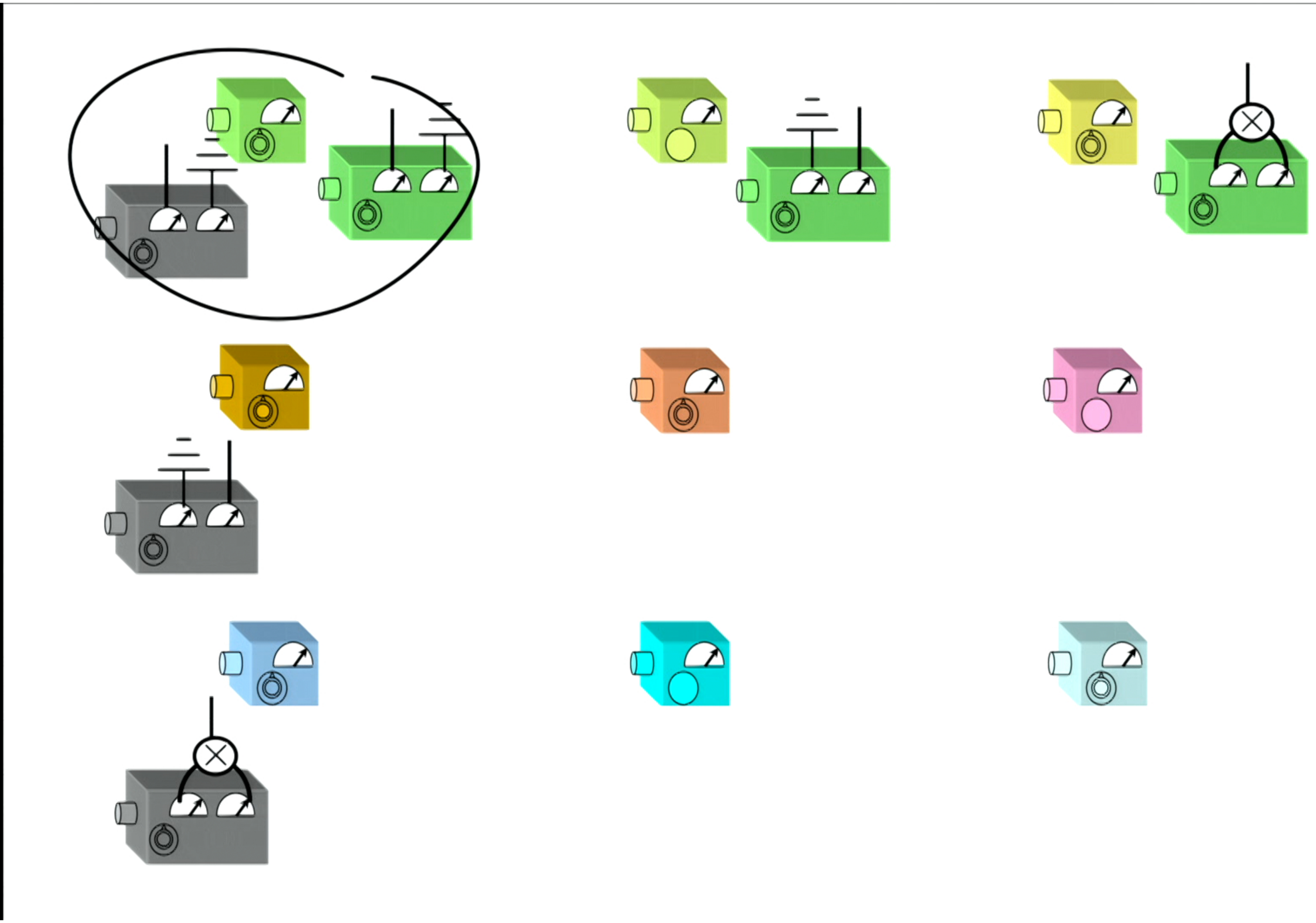

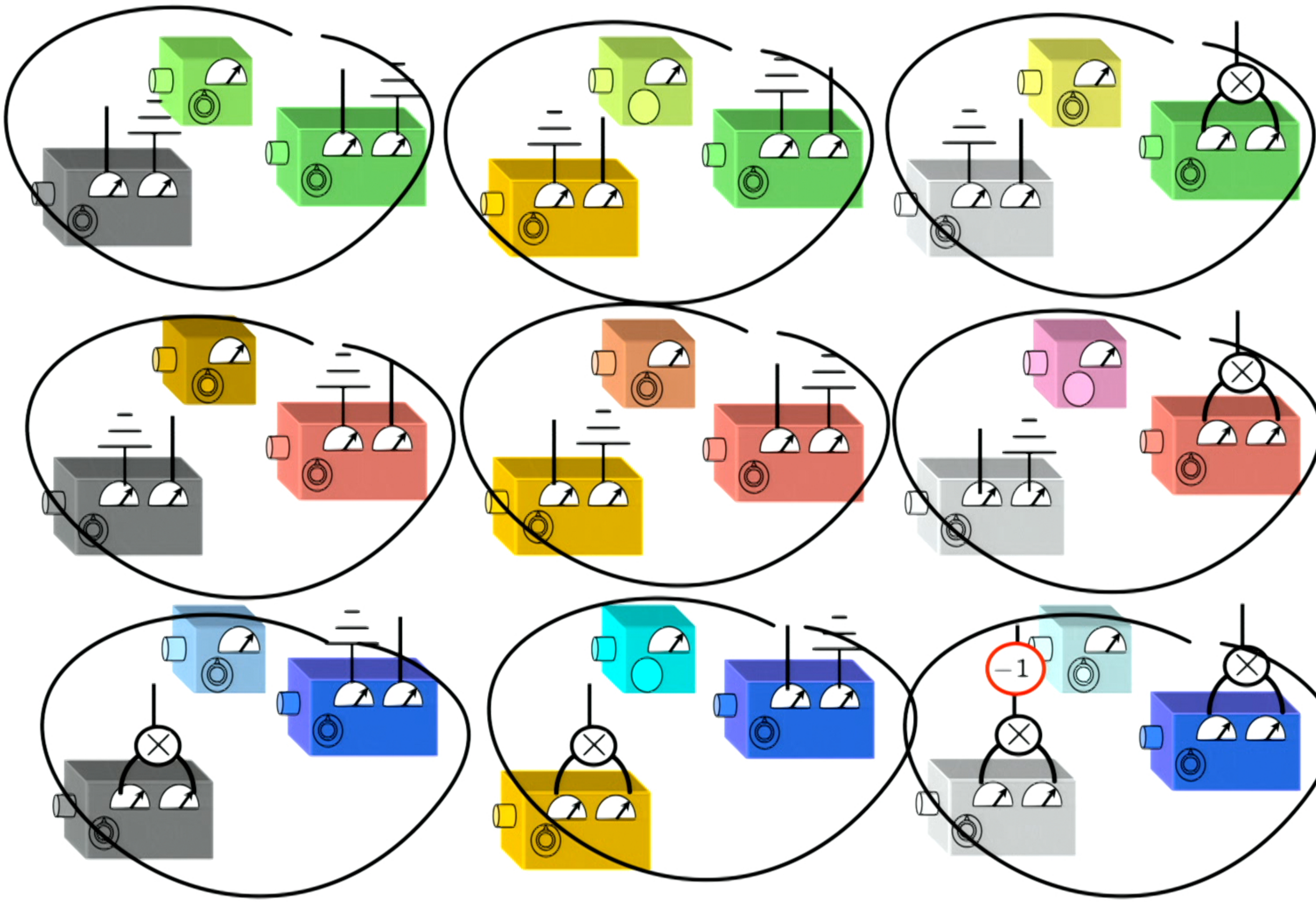
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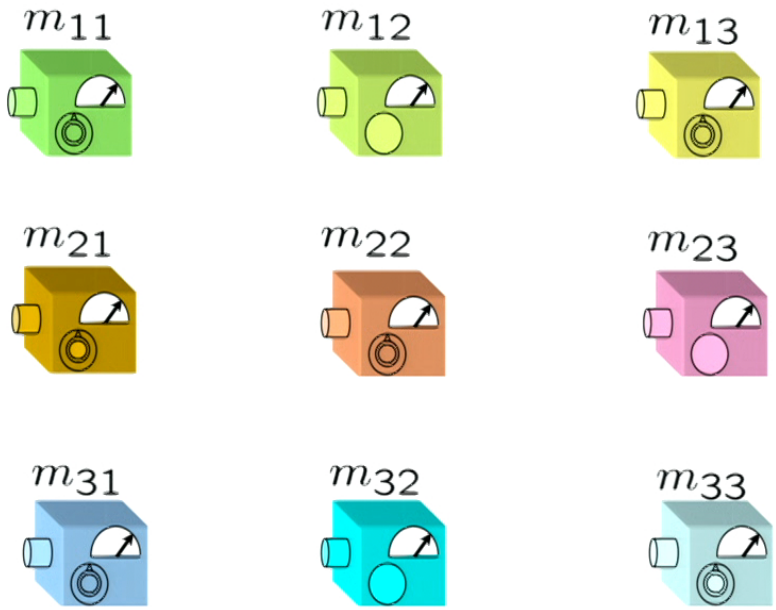


Distn's have support only on values such that

$$m_{11}m_{12}m_{13} = 1$$

$$m_{21}m_{22}m_{23} = 1$$

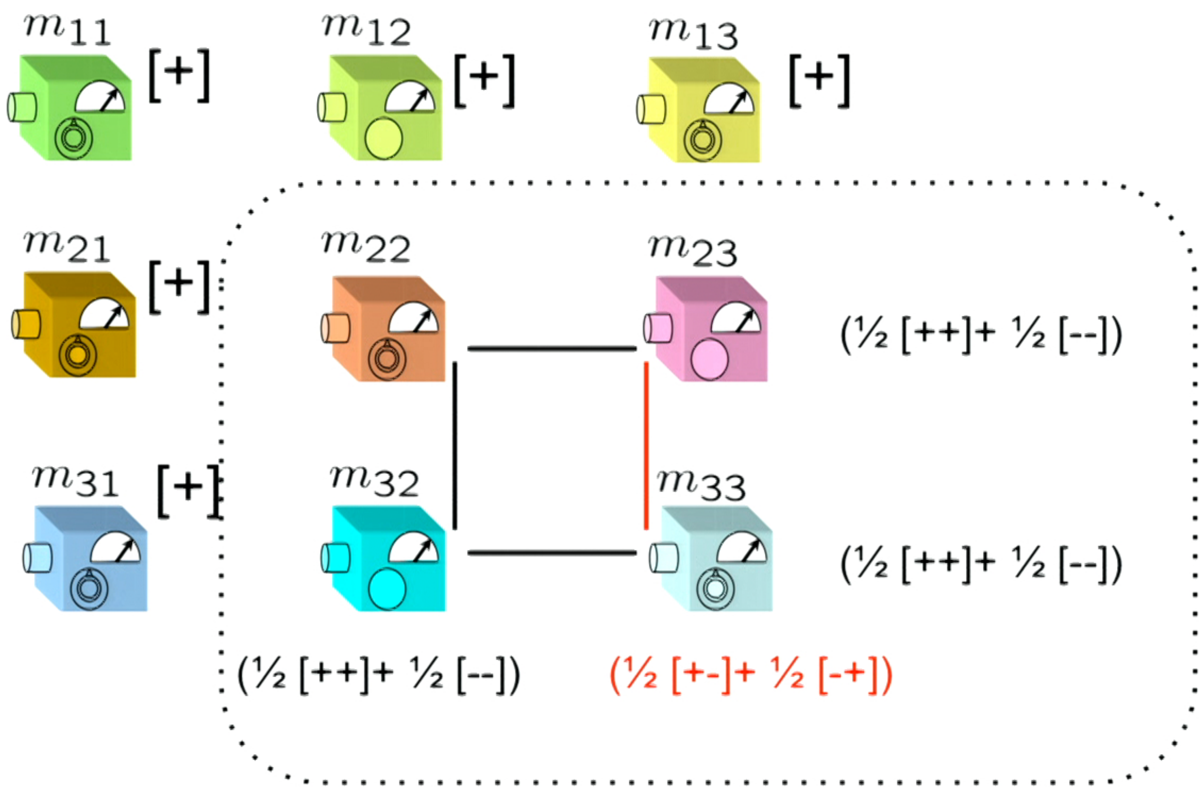
$$m_{31}m_{32}m_{33} = 1$$

$$m_{11}m_{21}m_{31} = 1$$

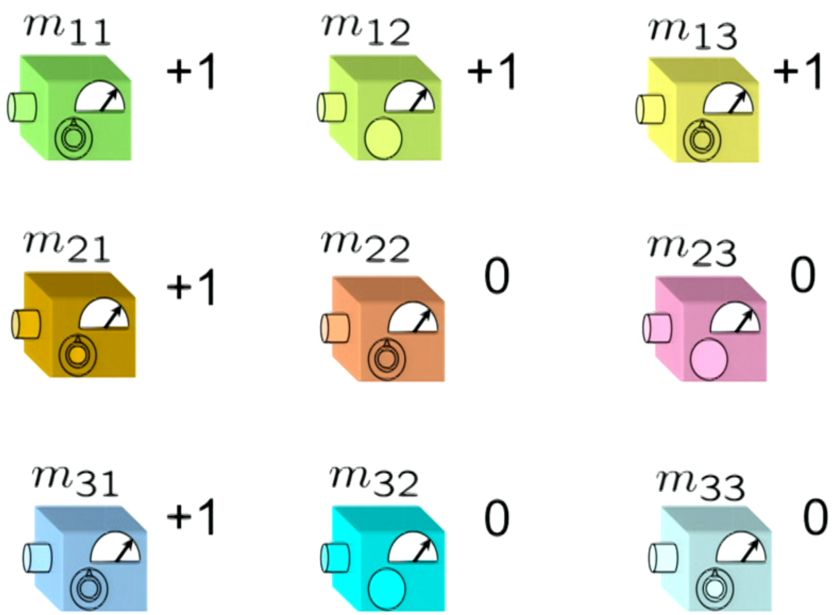
$$m_{12}m_{22}m_{32} = 1$$

$$m_{13}m_{23}m_{33} = -1$$

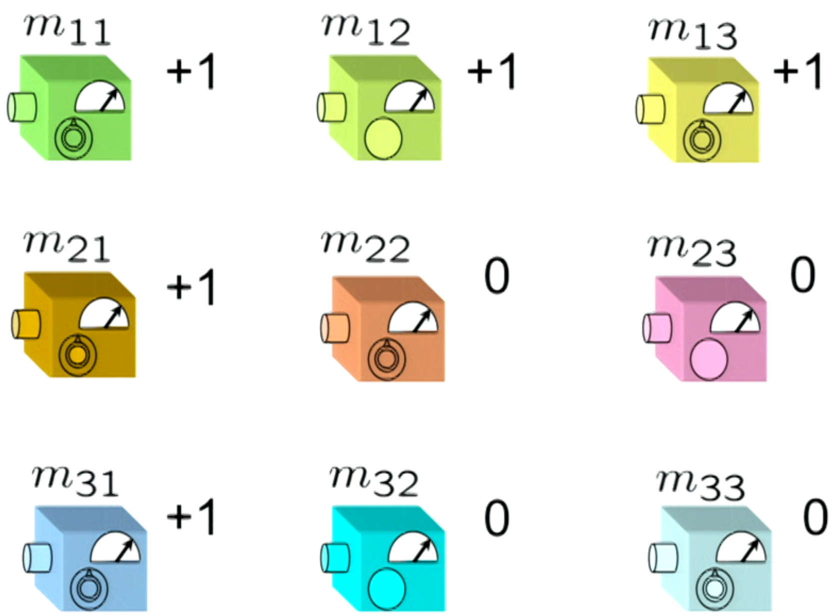
An indeterministic noncontextual assignment



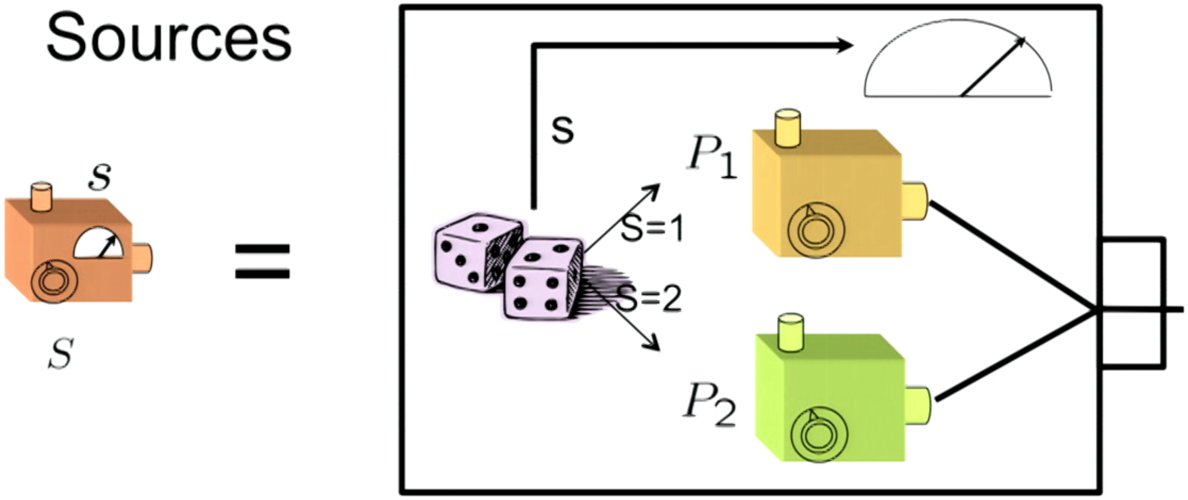
An indeterministic noncontextual assignment



An indeterministic noncontextual assignment



Sources



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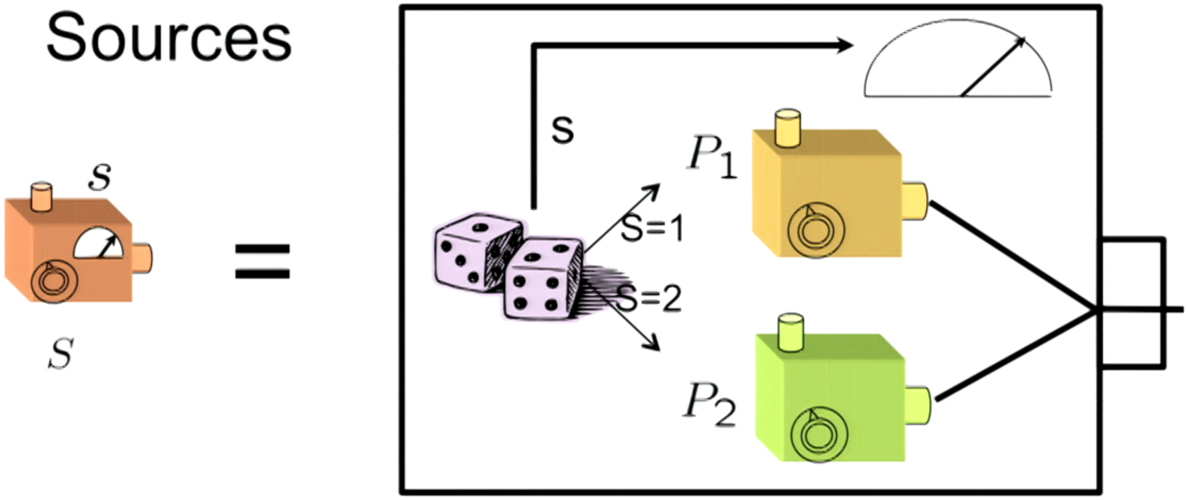
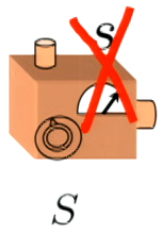
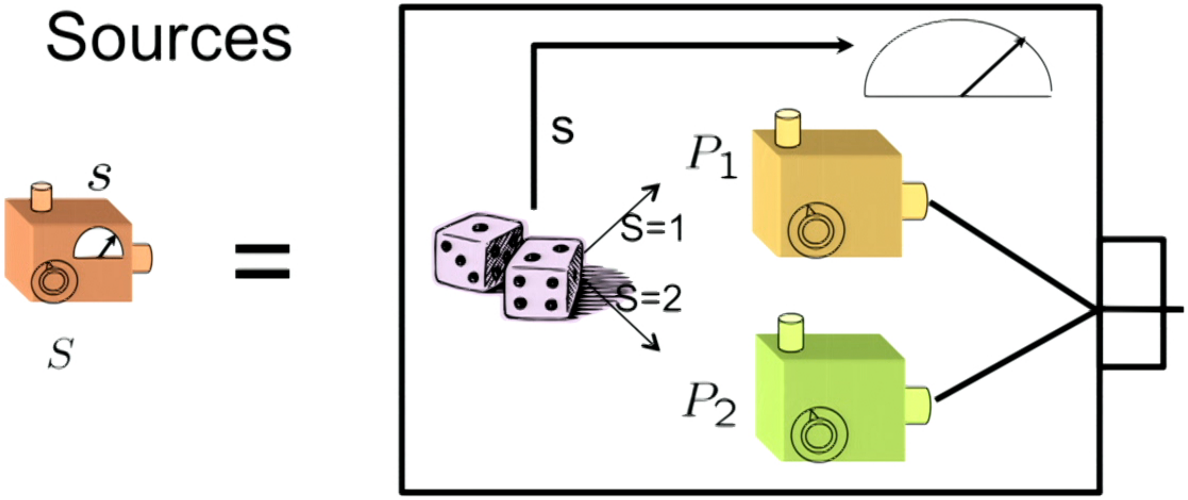


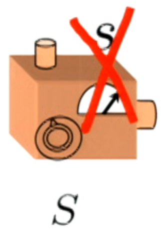
P_S^{ave}



Sources

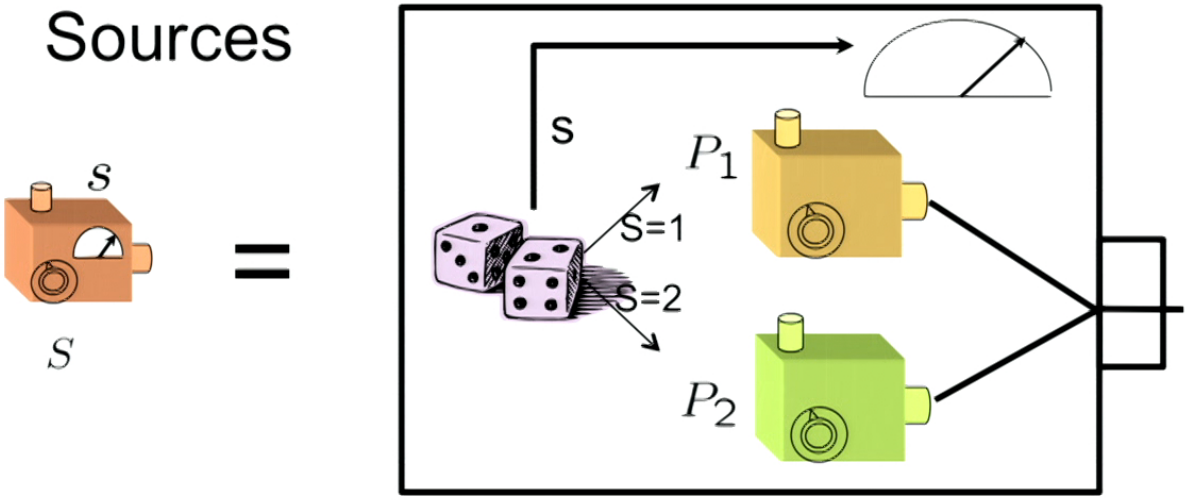


P_S^{ave}

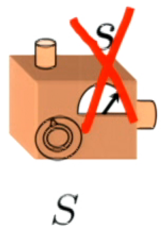


$$\mu(\lambda|P_S^{\text{ave}}) = \sum_s \mu(\lambda, s|S)$$
$$= \mu(\lambda|S)$$

Sources



P_S^{ave}



$$\mu(\lambda|P_S^{\text{ave}}) = \sum_s \mu(\lambda, s|S)$$
$$= \mu(\lambda|S)$$

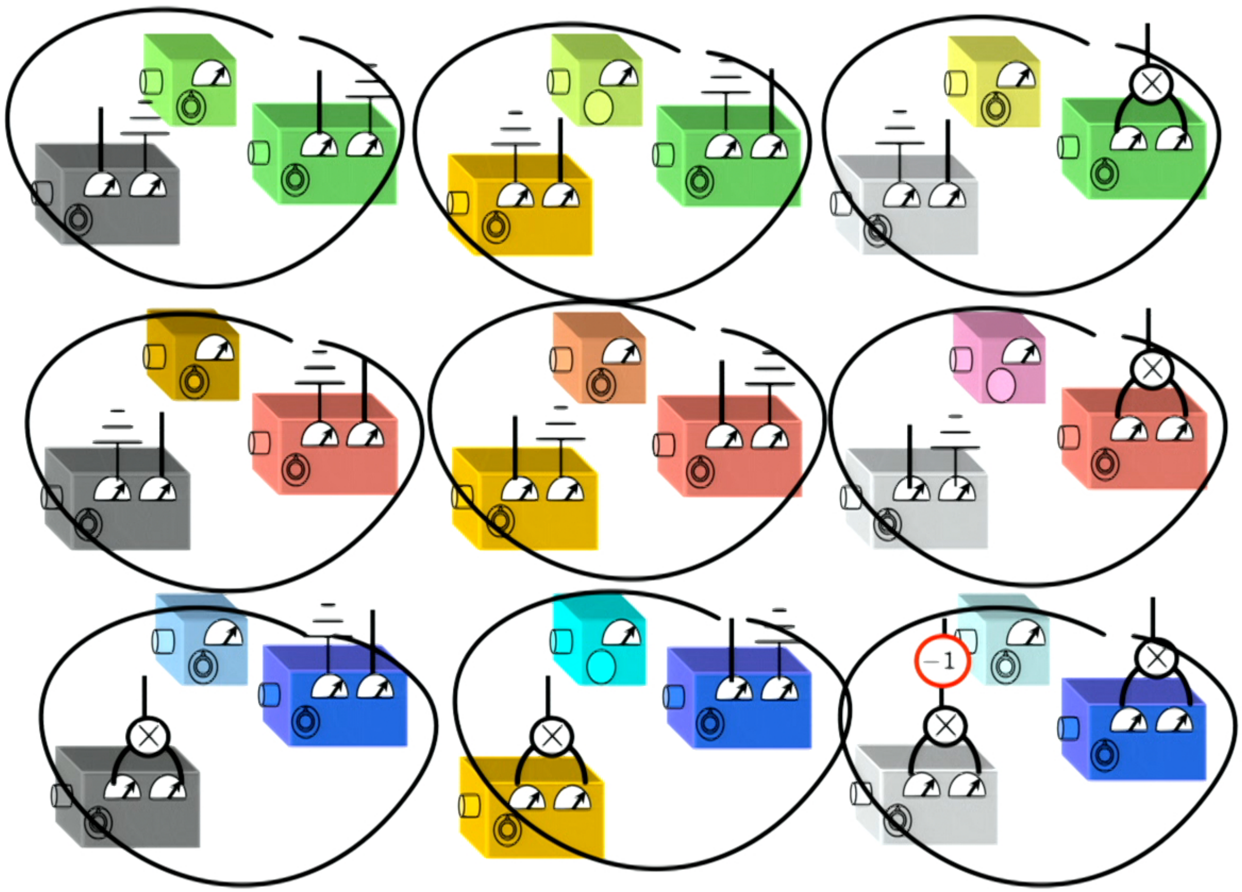
$$P_S^{\text{ave}} \simeq P_{S'}^{\text{ave}} \xrightarrow{\text{Preparation noncontextuality}} \mu(\lambda|S) = \mu(\lambda|S')$$

Source version of the Peres-Mermin square

$X \otimes I$	$I \otimes X$	$X \otimes X$
$I \otimes Z$	$Z \otimes I$	$Z \otimes Z$
$X \otimes Z$	$Z \otimes X$	$Y \otimes Y$

9 binary-outcome sources defining the same average preparation

$$
\begin{aligned}
(X \otimes I)(I \otimes X)(X \otimes X) &= I \otimes I, \\
(I \otimes Z)(Z \otimes I)(Z \otimes Z) &= I \otimes I, \\
(X \otimes Z)(Z \otimes X)(Y \otimes Y) &= I \otimes I, \\
(X \otimes I)(I \otimes Z)(X \otimes Z) &= I \otimes I, \\
(I \otimes X)(Z \otimes I)(Z \otimes X) &= I \otimes I, \\
(X \otimes X)(Z \otimes Z)(Y \otimes Y) &= - I \otimes I.
\end{aligned}
$$



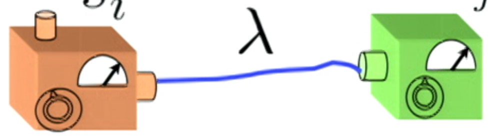
$$\forall i, i' : P^{\mathsf{ave}}_{S_i} \simeq P^{\mathsf{ave}}_{S_{i'}}$$

By preparation noncontextuality

$$\forall i, i' : P^{\text{ave}}_{S_i} \simeq P^{\text{ave}}_{S_{i'}} \quad \Longrightarrow \quad \forall i, i' : \mu(\lambda|\mathcal{S}_i) = \mu(\lambda|\mathcal{S}_{i'}) \equiv \nu(\lambda)$$

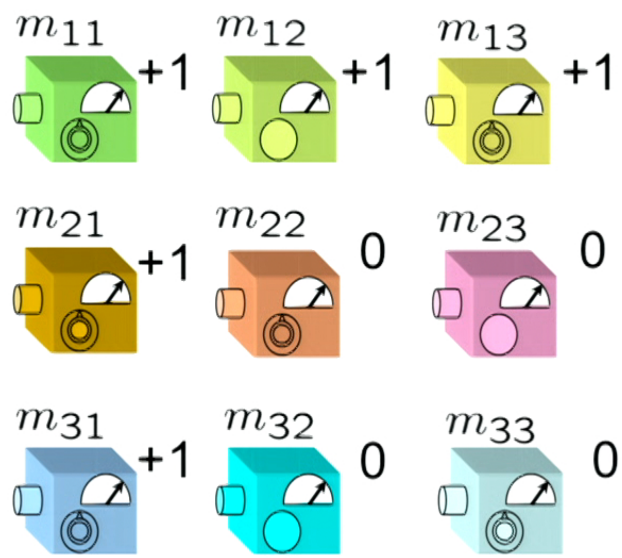
By preparation noncontextuality

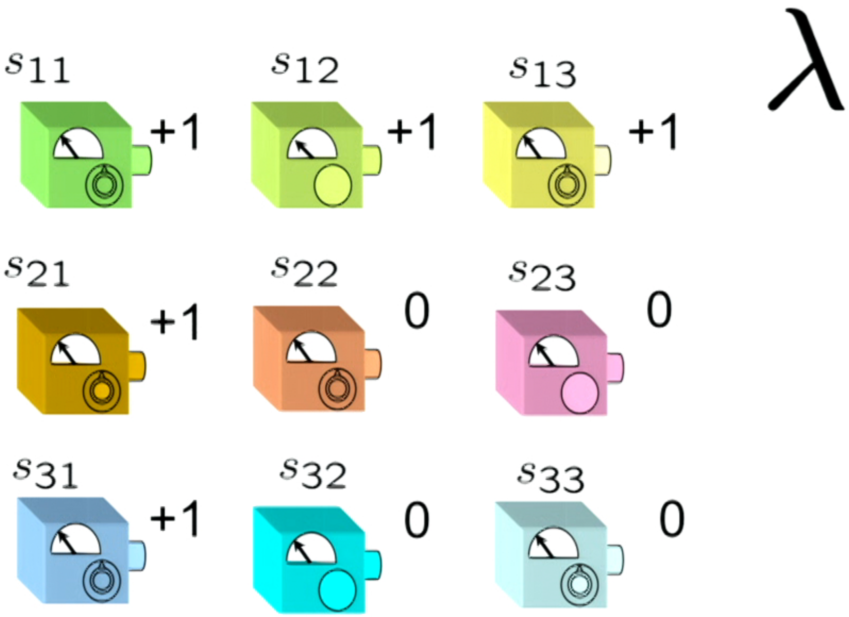
$$\forall i, i' : P^{\text{ave}}_{S_i} \simeq P^{\text{ave}}_{S_{i'}} \implies \forall i, i' : \mu(\lambda|\mathcal{S}_i) = \mu(\lambda|\mathcal{S}_{i'}) \equiv \nu(\lambda)$$



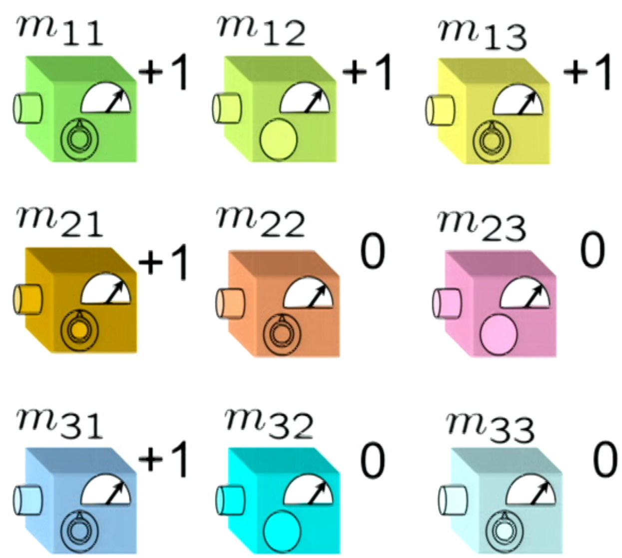
$$\begin{aligned}
\text{pr}(m_j, s_i|M_j, S_i) &= \sum_{\lambda\in\Lambda} \xi(m_j|M_j, \lambda)\mu(s_i, \lambda|S_i) \\
&= \sum_{\lambda\in\Lambda} \xi(m_j|M_j, \lambda)\mu(s_i|\lambda, S_i)\mu(\lambda|S_i) \\
&= \sum_{\lambda\in\Lambda} \xi(m_j|M_j, \lambda)\mu(s_i|\lambda, S_i)\nu(\lambda)
\end{aligned}$$

$$\text{pr}(m_j, s_i | M_j, S_i) = \sum_{\lambda \in \Lambda} \xi(m_j | M_j, \lambda)\mu(s_i | \lambda, S_i)\nu(\lambda)$$

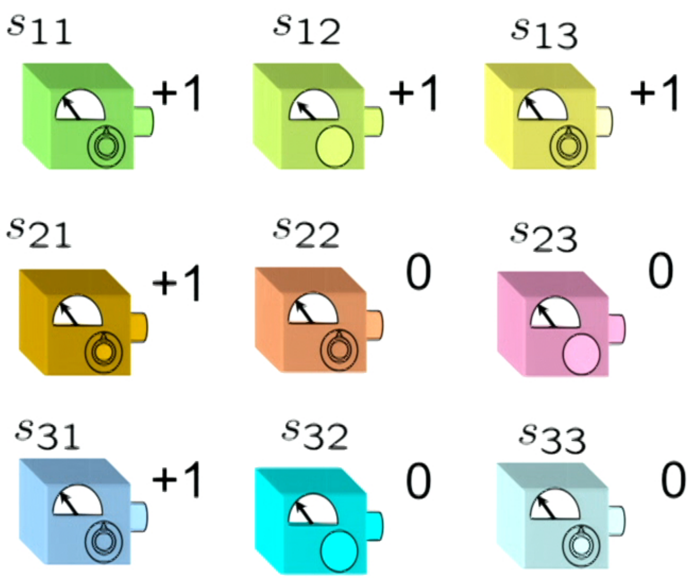




$$\mathrm{pr}(m_j, s_i | M_j, S_i)$$
$$= \sum_{\lambda \in \Lambda} \xi(m_j | M_j, \lambda) \mu(s_i | \lambda, S_i) \nu(\lambda)$$

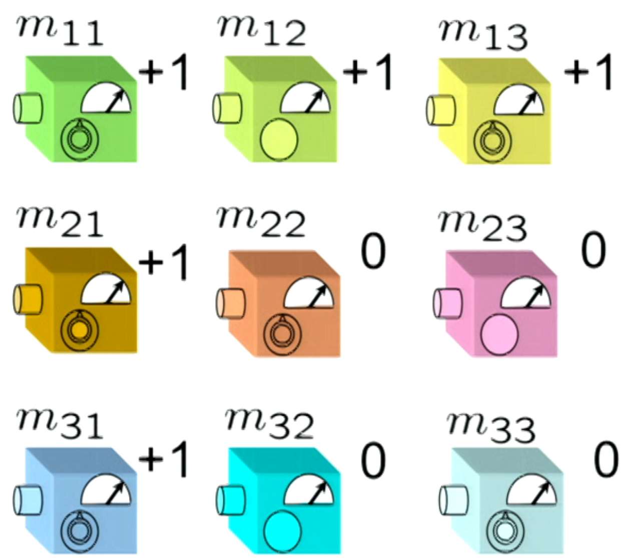


λ

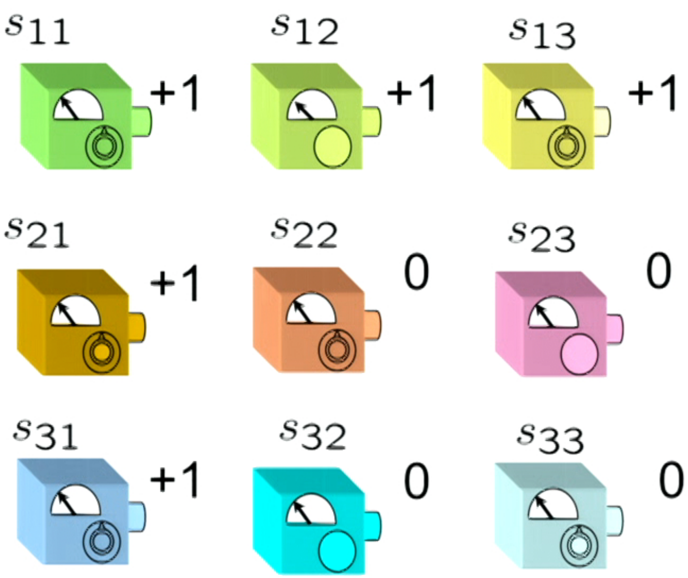


$$\mathrm{Corr} = \frac{1}{9} \sum_{i=1}^{9} \langle s_i m_i \rangle_{S_i, M_i}$$

$$\mathrm{pr}(m_j, s_i | M_j, S_i) = \sum_{\lambda \in \Lambda} \xi(m_j | M_j, \lambda) \mu(s_i | \lambda, S_i) \nu(\lambda)$$



λ

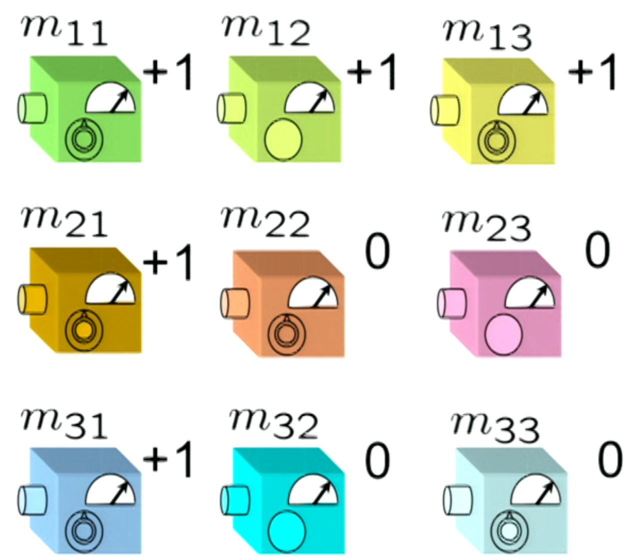


$$\mathrm{Corr} = \frac{1}{9} \sum_{i=1}^{9} \langle s_i m_i \rangle_{S_i, M_i}$$

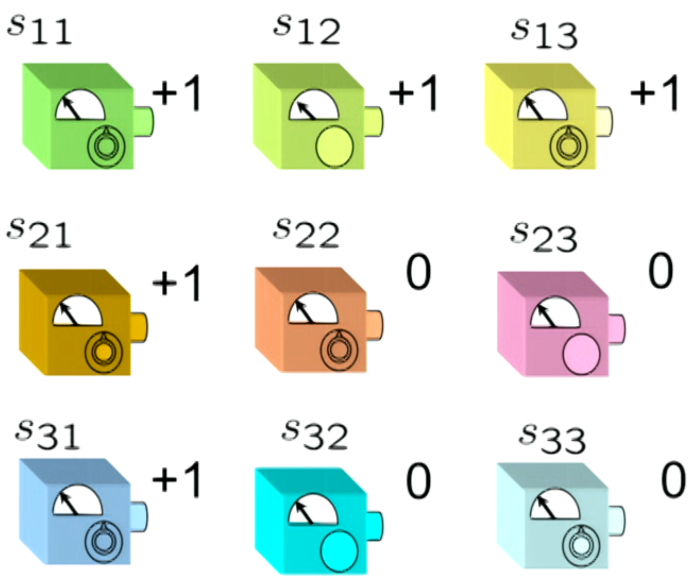
$$= \sum_{\lambda} \mathrm{Corr}(\lambda) \nu(\lambda)$$

$$\mathrm{Corr}(\lambda) = \frac{1}{9} \sum_{i=1}^{9} \langle s_i \rangle_{\lambda} \langle m_i \rangle_{\lambda}$$

$$\text{pr}(m_j, s_i | M_j, S_i) = \sum_{\lambda \in \Lambda} \xi(m_j | M_j, \lambda) \mu(s_i | \lambda, S_i) \nu(\lambda)$$



λ



$$\text{Corr} = \frac{1}{9} \sum_{i=1}^{9} \langle s_i m_i \rangle_{S_i, M_i}$$

$$= \sum_{\lambda} \text{Corr}(\lambda) \nu(\lambda)$$

$$\text{Corr}(\lambda) = \frac{1}{9} \sum_{i=1}^{9} \langle s_i \rangle_{\lambda} \langle m_i \rangle_{\lambda}$$

$$\text{Corr} \leq \frac{5}{9}$$

Noncontextuality inequality

The problem of
inexact operational
equivalences

How do we experimentally establish operational equivalence?

Problem of inexact operational equivalences

$\mathsf{P} \simeq \mathsf{P}'$

$\forall \mathsf{M} : p(X|\mathsf{P}, \mathsf{M}) = p(X|\mathsf{P}', \mathsf{M})$ —Preparation noncontextuality→ $\mu(\lambda|\mathsf{P}) = \mu(\lambda|\mathsf{P}')$

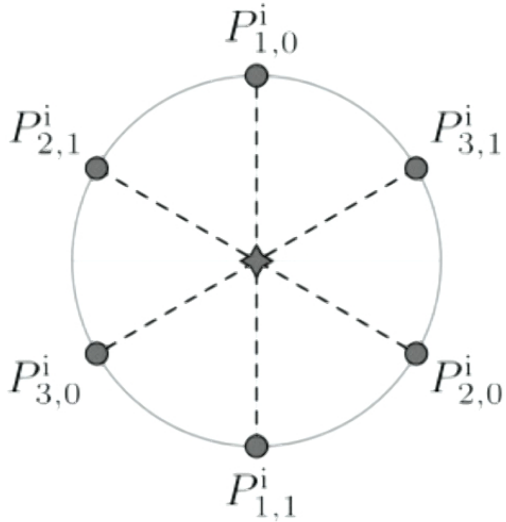
$\mathsf{M} \simeq \mathsf{M}'$

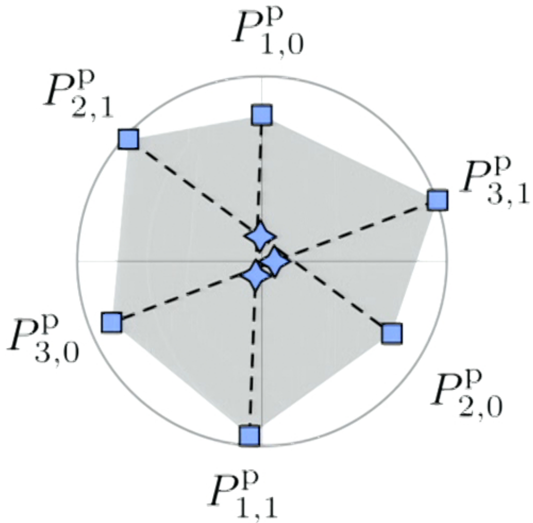
$\forall \mathsf{P} : p(X|\mathsf{P}, \mathsf{M}) = p(X|\mathsf{P}, \mathsf{M}')$ —Measurement noncontextuality→ $\xi(X|\lambda, \mathsf{M}) = \xi(X|\lambda, \mathsf{M}')$

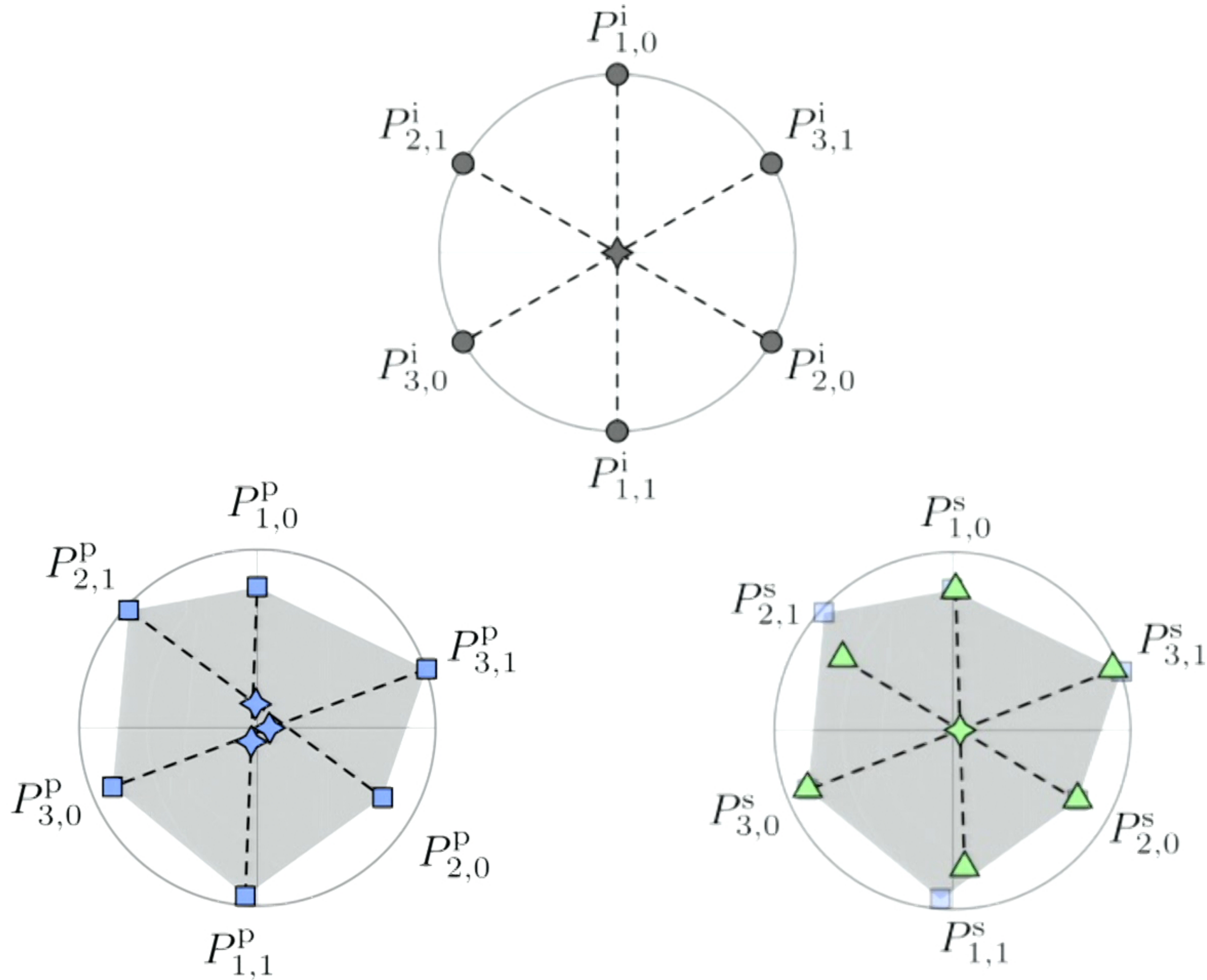
A version of the finite precision loophole

See: Mazurek, Pusey, Kunjwal, Resch, and RWS, “An experimental test of noncontextuality without unphysical idealizations”, Nat. Commun. 7, 11780 (2016)

M. Pusey, “The robust noncontextuality inequalities in the simplest scenario”, arXiv:1506.04178


$P^{\mathrm{i}}_{1,0}$
$P^{\mathrm{i}}_{2,1}$
$P^{\mathrm{i}}_{3,1}$
$P^{\mathrm{i}}_{3,0}$
$P^{\mathrm{i}}_{2,0}$
$P^{\mathrm{i}}_{1,1}$


$P^{\mathrm{p}}_{1,0}$
$P^{\mathrm{p}}_{2,1}$
$P^{\mathrm{p}}_{3,1}$
$P^{\mathrm{p}}_{3,0}$
$P^{\mathrm{p}}_{2,0}$
$P^{\mathrm{p}}_{1,1}$


$P^{\mathrm{i}}_{1,0}$
$P^{\mathrm{i}}_{2,1}$
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$P^{\mathrm{i}}_{1,1}$
$P^{\mathrm{p}}_{1,0}$
$P^{\mathrm{p}}_{2,1}$
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$P^{\mathrm{p}}_{3,0}$
$P^{\mathrm{p}}_{2,0}$
$P^{\mathrm{p}}_{1,1}$
$P^{\mathrm{s}}_{1,0}$
$P^{\mathrm{s}}_{2,1}$
$P^{\mathrm{s}}_{3,1}$
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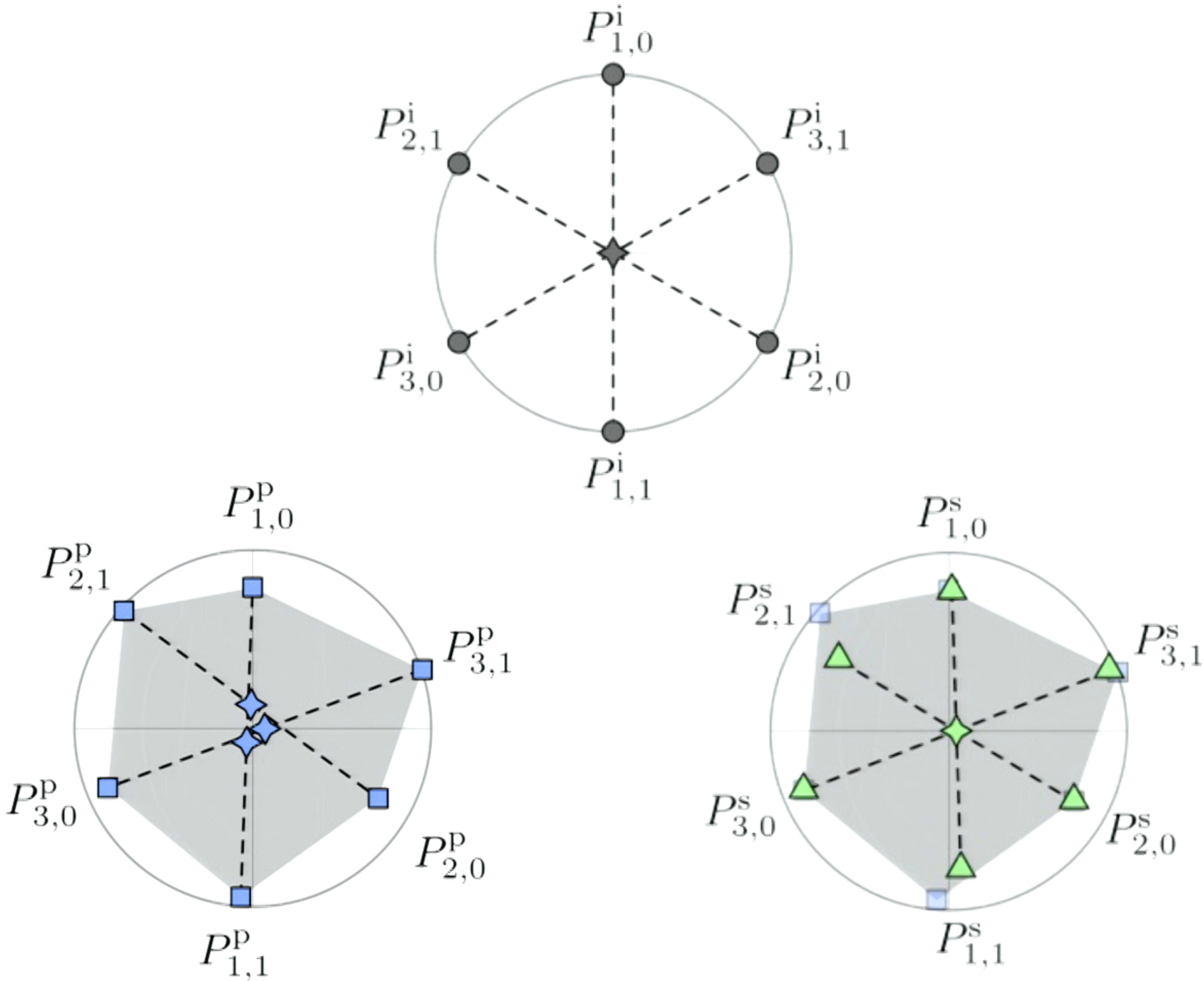
$P^{\mathrm{s}}_{2,0}$
$P^{\mathrm{s}}_{1,1}$

The problem of the universal quantifier



$P^{i}_{1,0}$
$P^{i}_{2,1}$
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$P^{s}_{1,0}$
$P^{s}_{2,1}$
$P^{s}_{3,1}$
$P^{s}_{3,0}$
$P^{s}_{2,0}$
$P^{s}_{1,1}$

The problem of the universal quantifier

How do we experimentally establish operational equivalence?

The problem of the universal quantifier

$$\mathsf{P} \simeq \mathsf{P}'$$
$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M})$$

Preparation noncontextuality $\longrightarrow$

$$\mu(\lambda|\mathsf{P}) = \mu(\lambda|\mathsf{P}')$$

$$\mathsf{M} \simeq \mathsf{M}'$$
$$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}')$$

Measurement noncontextuality $\longrightarrow$

$$\xi(X|\lambda,\mathsf{M}) = \xi(X|\lambda,\mathsf{M}')$$

How do we experimentally establish operational equivalence?

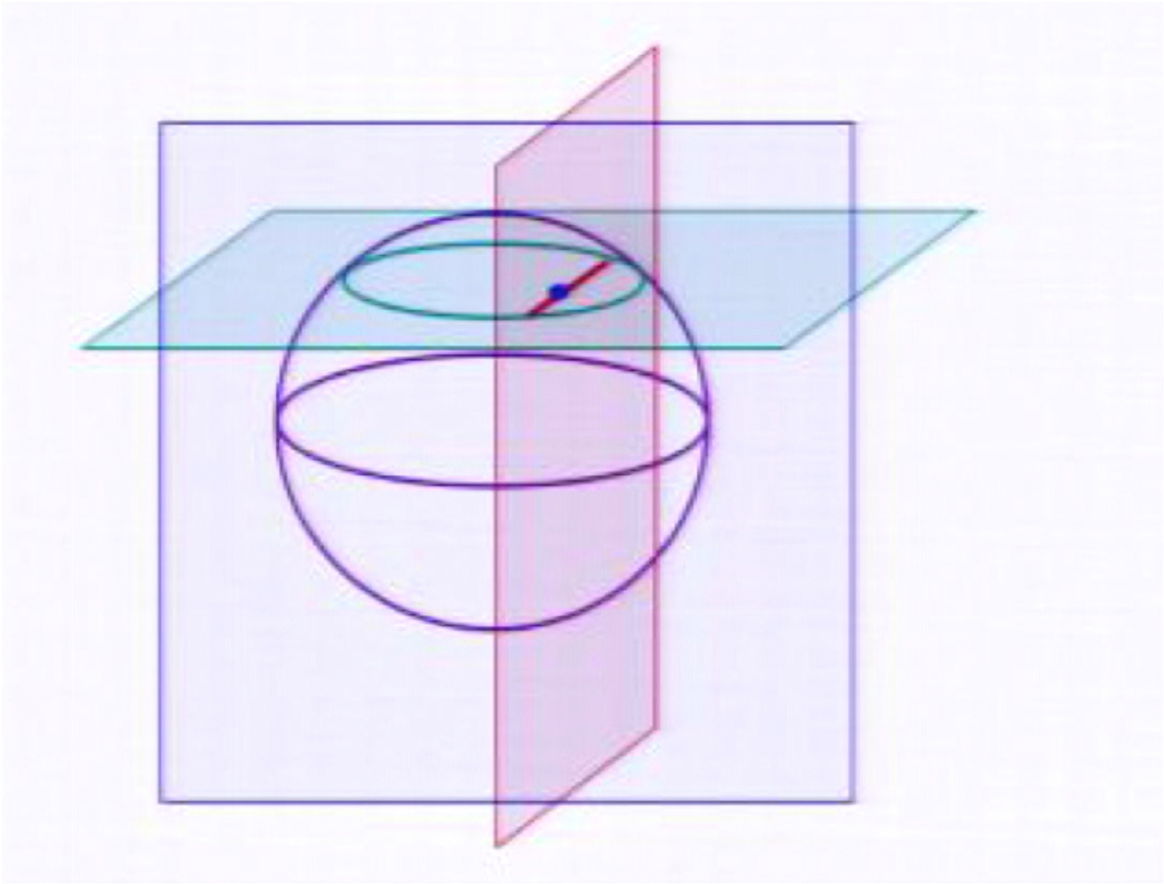
The problem of the universal quantifier

$$\mathsf{P} \simeq \mathsf{P}'$$
$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M}) \xrightarrow{\text{Preparation noncontextuality}} \mu(\lambda|\mathsf{P}) = \mu(\lambda|\mathsf{P}')$$

$$\mathsf{M} \simeq \mathsf{M}'$$
$$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}') \xrightarrow{\text{Measurement noncontextuality}} \xi(X|\lambda,\mathsf{M}) = \xi(X|\lambda,\mathsf{M}')$$

Sufficient to consider a tomographically complete set

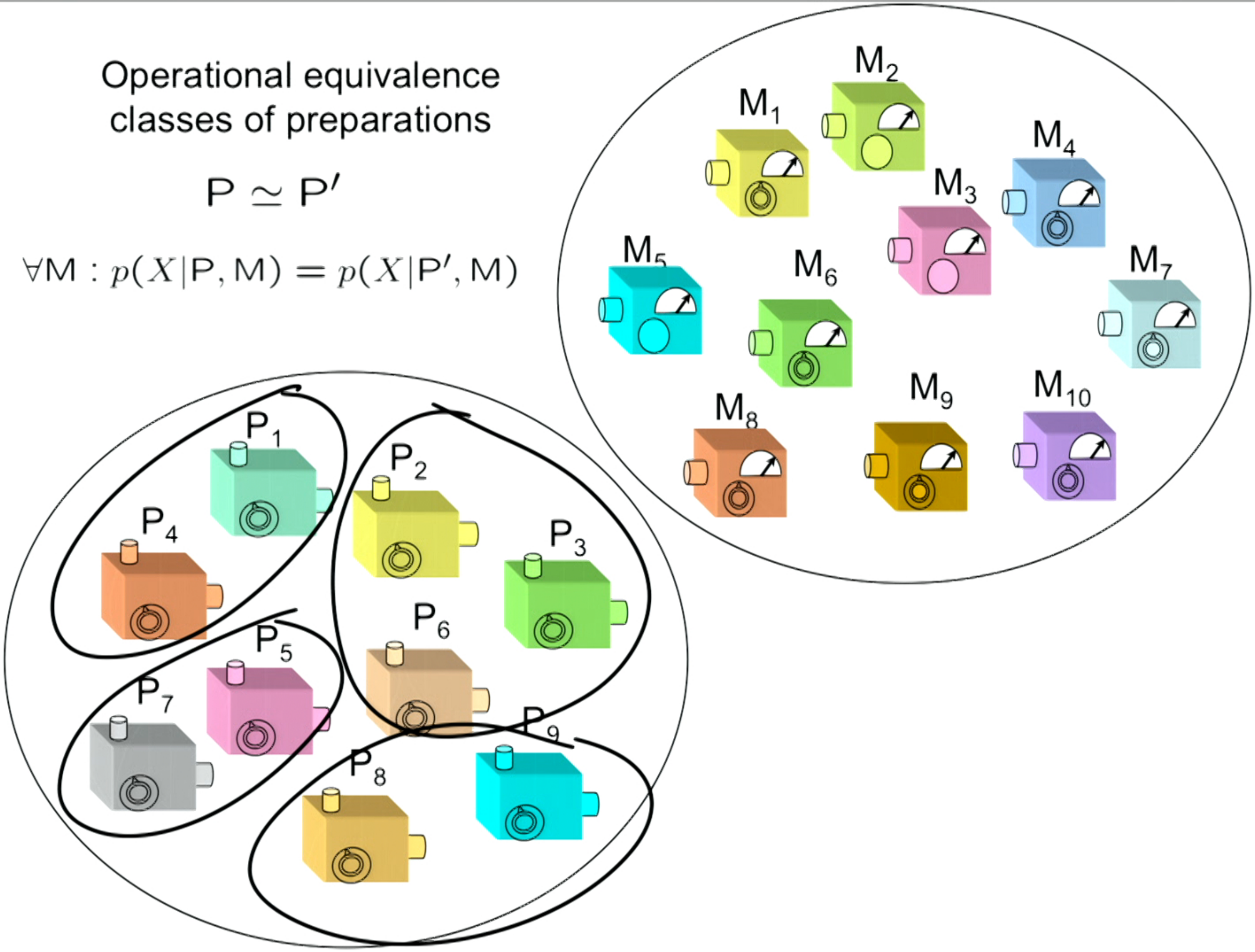
State tomography for a single qubit



Operational equivalence classes of preparations

$$\mathsf{P} \simeq \mathsf{P}'$$

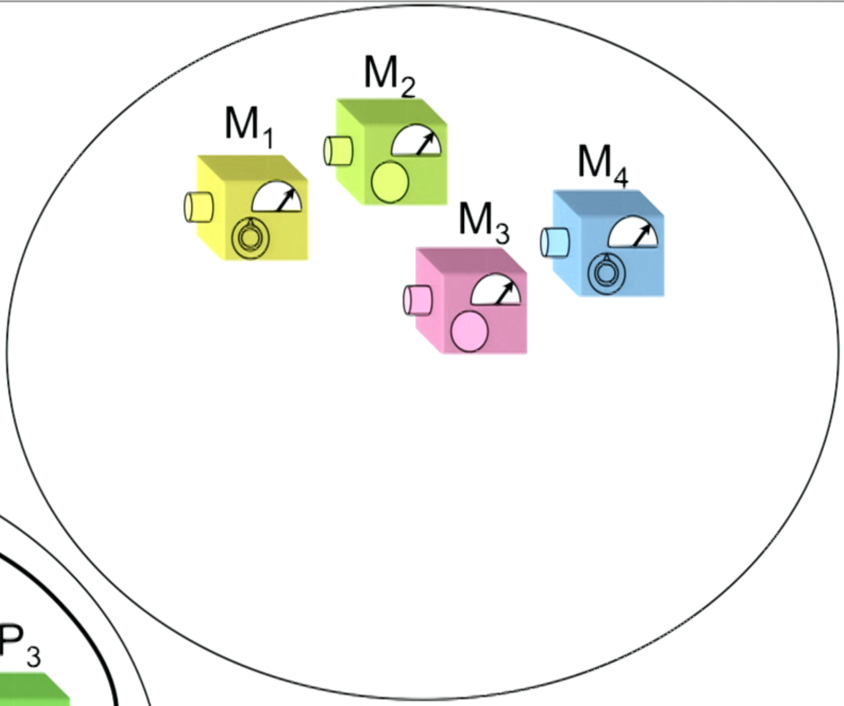
$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M})$$

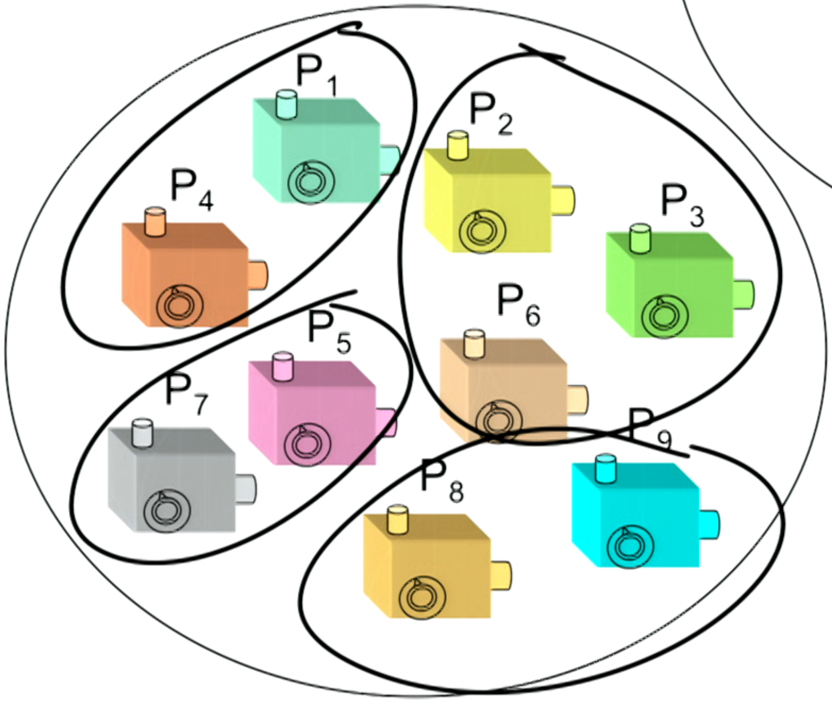


Operational equivalence classes of preparations

$$\mathsf{P} \simeq \mathsf{P}'$$

$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M})$$

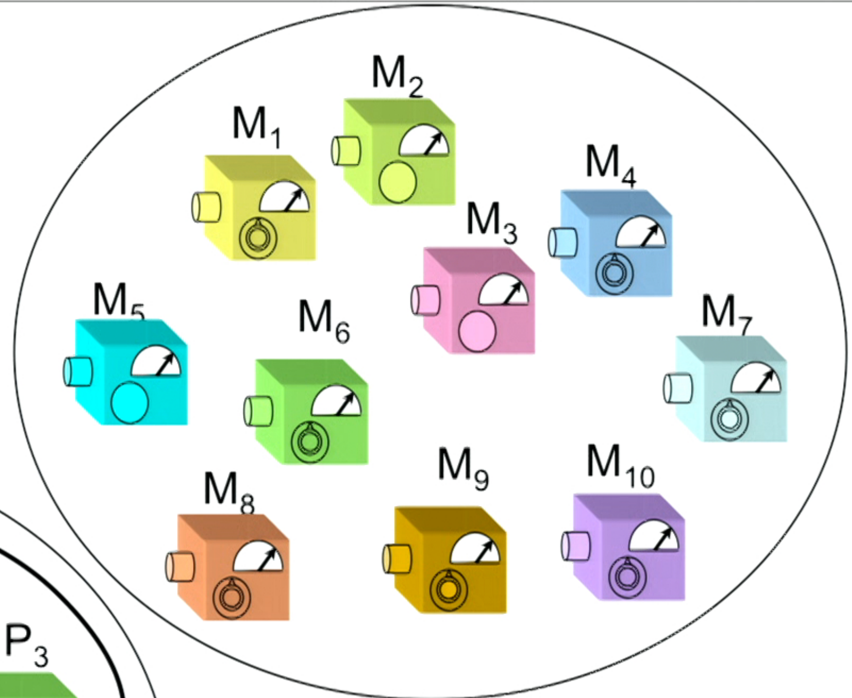


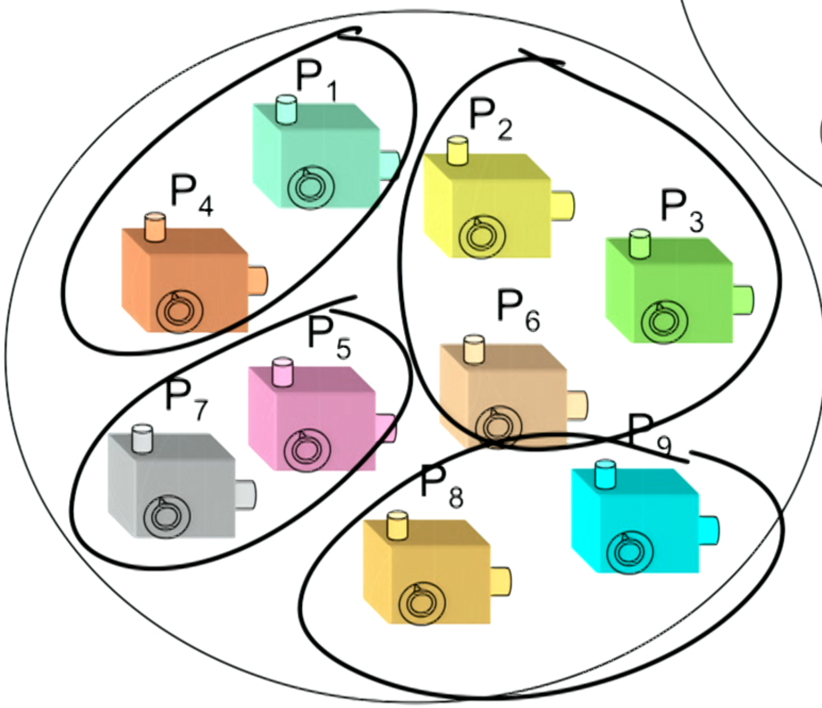


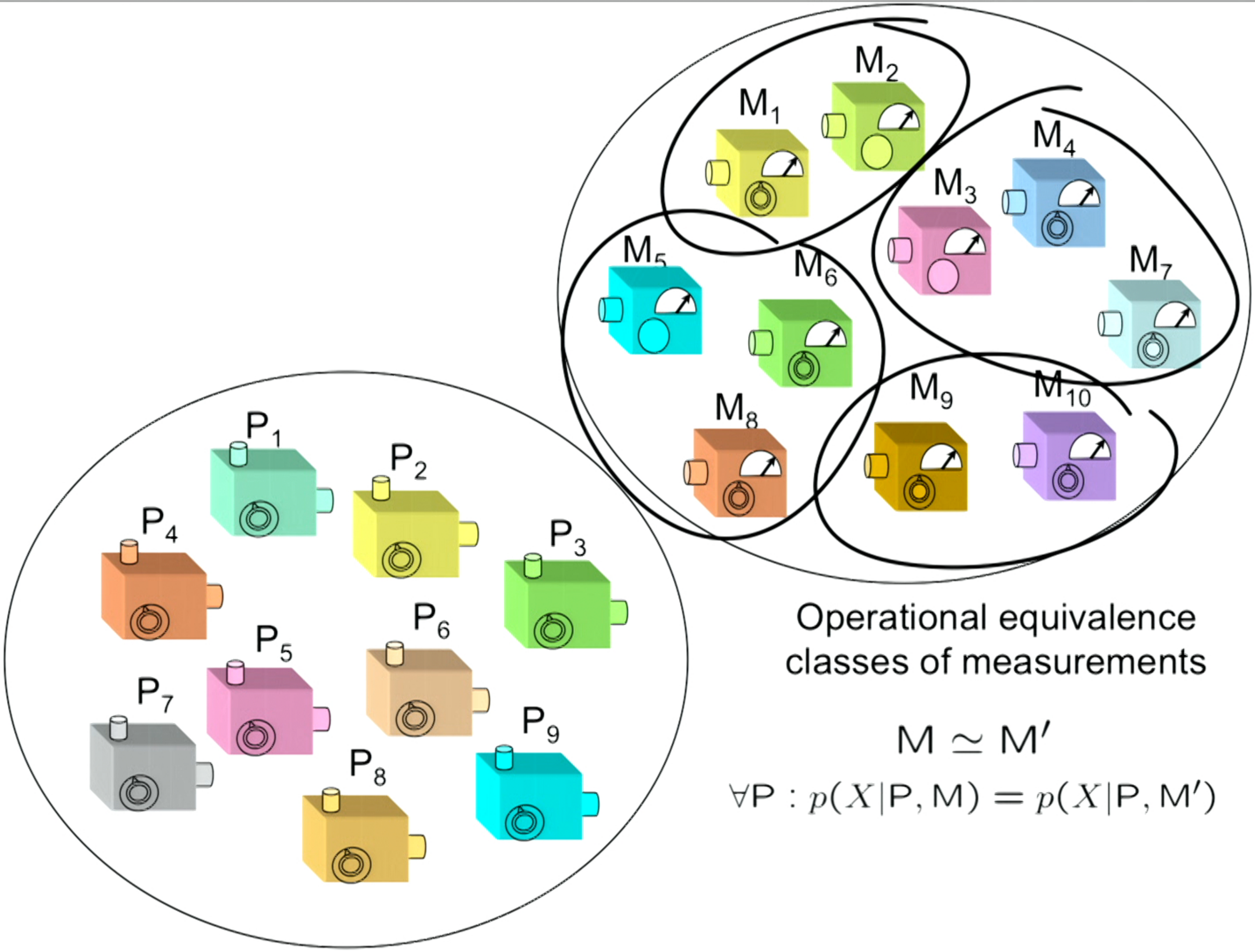
Operational equivalence classes of preparations

$$\mathsf{P} \simeq \mathsf{P}'$$

$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M})$$







Operational equivalence classes of measurements

$$\mathsf{M} \simeq \mathsf{M}'$$

$$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}')$$

Mazurek, Pusey, Resch and RWS, “Deviations from quantum theory in the landscape of generalized probabilistic theories: direct constraints from experimental data”, forthcoming

See talk on friday by Mike Mazurek

Mazurek, Pusey, Resch and RWS, "Deviations from quantum theory in the landscape of generalized probabilistic theories: direct constraints from experimental data", forthcoming

See talk on friday by Mike Mazurek

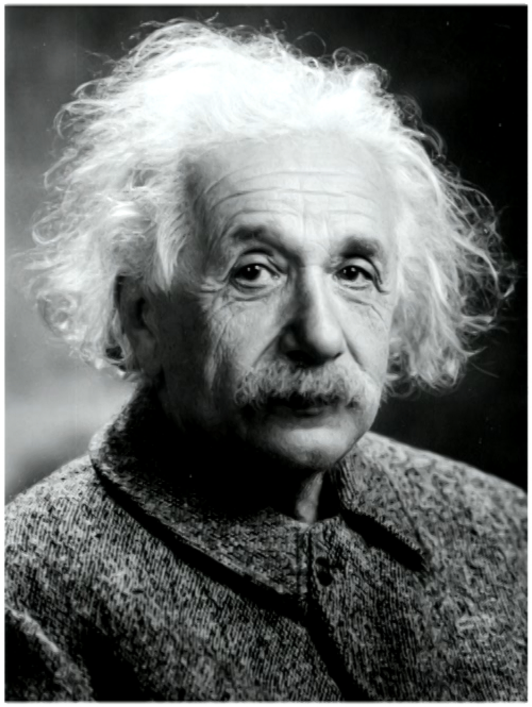
Note: In principle, one can never verify the claim that a given set of procedures is tomographically complete, but one can accumulate more and more evidence for it by trying one's best to falsify the claim and failing to do so.

Why this notion of noncontextuality is natural

Leibniz’s principle of the identity of indiscernibles

If a physical model posits two scenarios that are empirically indistinguishable *in principle* but nonetheless are represented as ontologically distinct, this model should be rejected and replaced with one that makes these two scenarios ontologically the same.

The credentials of Leibniz's principle

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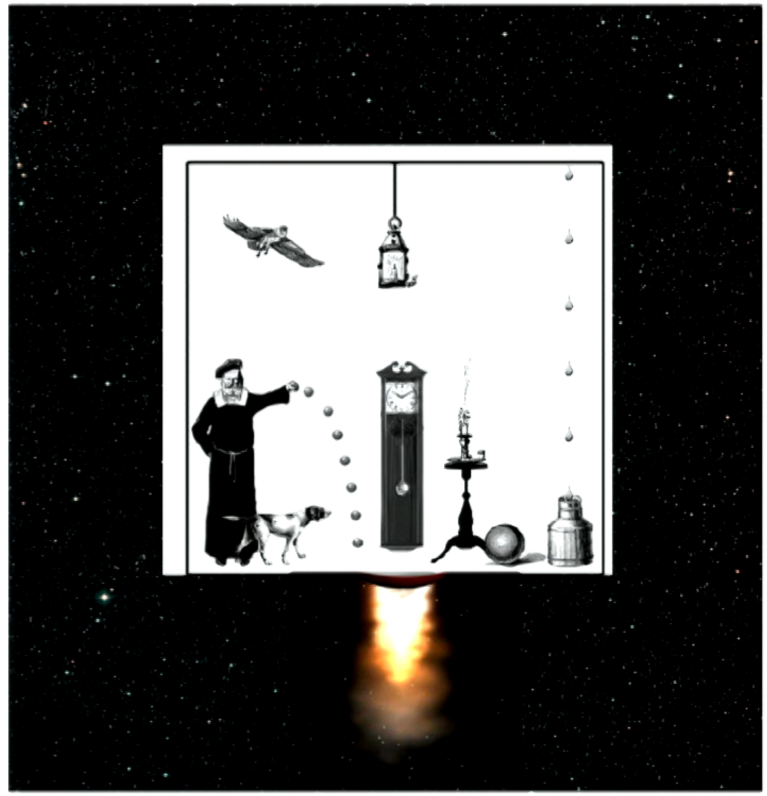
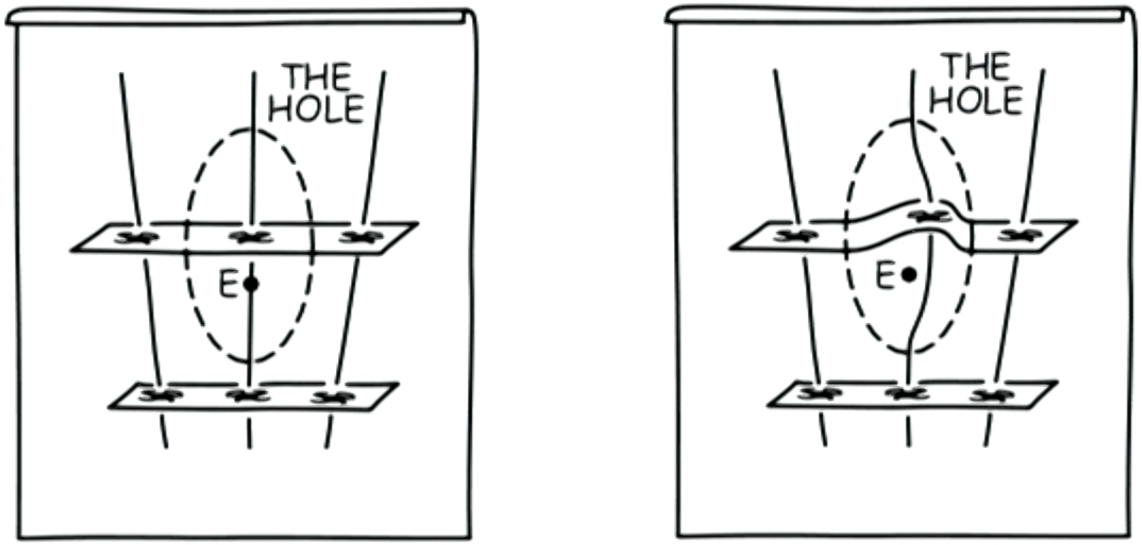
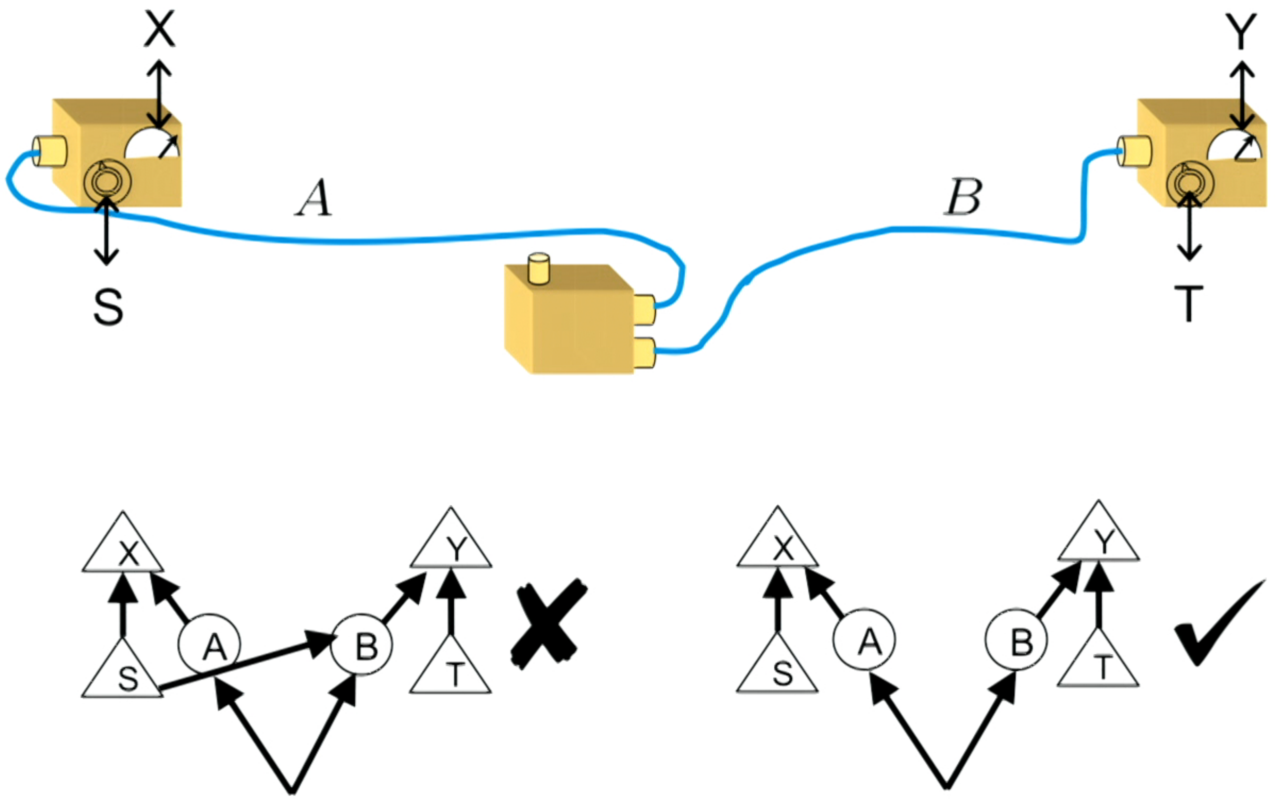




Fields before hole transformation

Fields after hole transformation

John Norton, SEP


X
Y
A
B
S
T
X
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X
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S
B
Y
T

The credentials of Leibniz's principle

No fine-tuning
No overfitting

Nonclassicality of operational theory = Failure to admit of a noncontextual ontological model

Nonclassicality of operational theory	=	Failure to admit of a Leibnizian ontological model

What to do about
the fact that quantum theory and nature
fail to admit of a
Leibnizian ontological model?

Abandon Leibniz's principle? No.

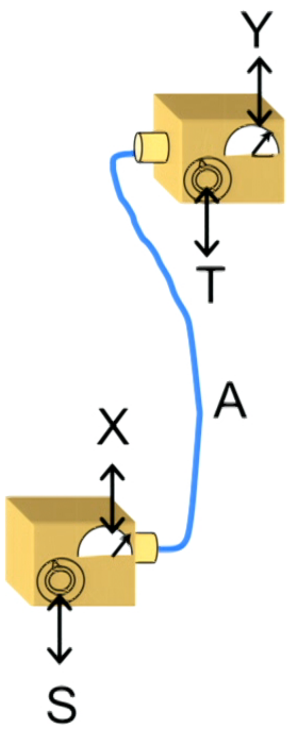
Abandon Leibniz’s principle? No.

Give up on realism? No.

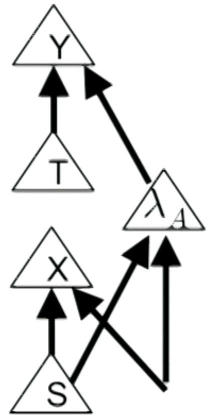
Abandon Leibniz's principle? No.

Give up on realism? No.

Devise a notion of realism that goes beyond the ontological models framework and salvages Leibniz's principle (and therefore the spirit of locality and noncontextuality)



No measurement of A reveals any info about S



$$P(Y|T\lambda_A)$$

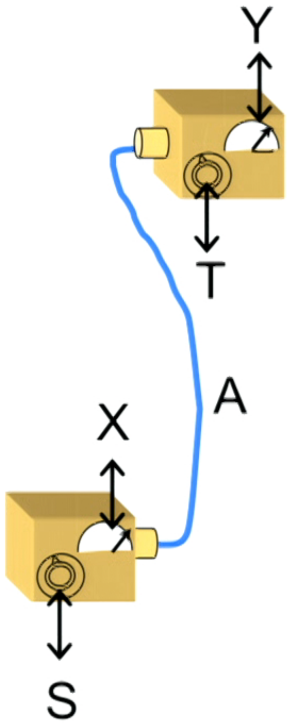
$$P(X\lambda_A|S)$$

where

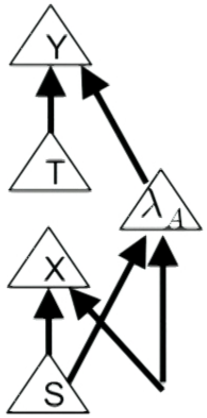
$$P(\lambda_A|S) = P(\lambda_A)$$

$$P(XY|ST) = \sum_{\lambda_A} P(Y|T\lambda_A)P(X\lambda_A|S)$$

Satisfy noncontextuality inequalities



No measurement of A reveals any info about S



$P(Y|T\lambda_A)$

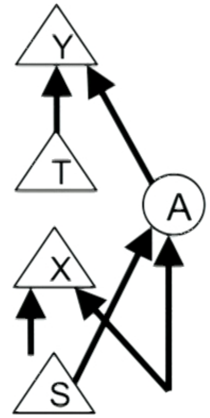
$P(X\lambda_A|S)$

where

$P(\lambda_A|S) = P(\lambda_A)$

$P(XY|ST)$
$= \sum_{\lambda_A} P(Y|T\lambda_A)P(X\lambda_A|S)$

Satisfy noncontextuality inequalities



$\rho_{Y|TA}$

$\rho_{XA|S}$

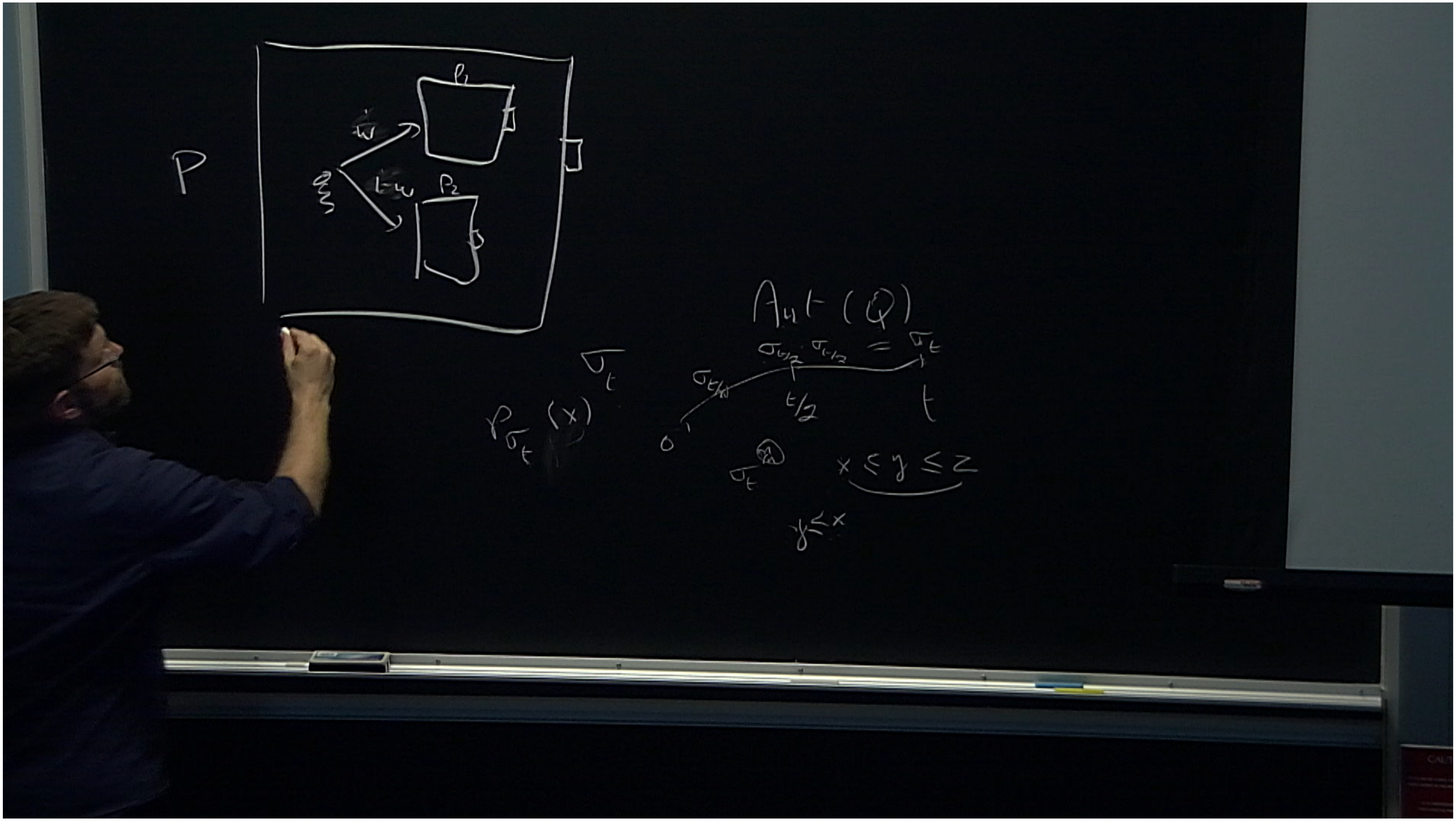
where

$\rho_{A|S} = \rho_A$

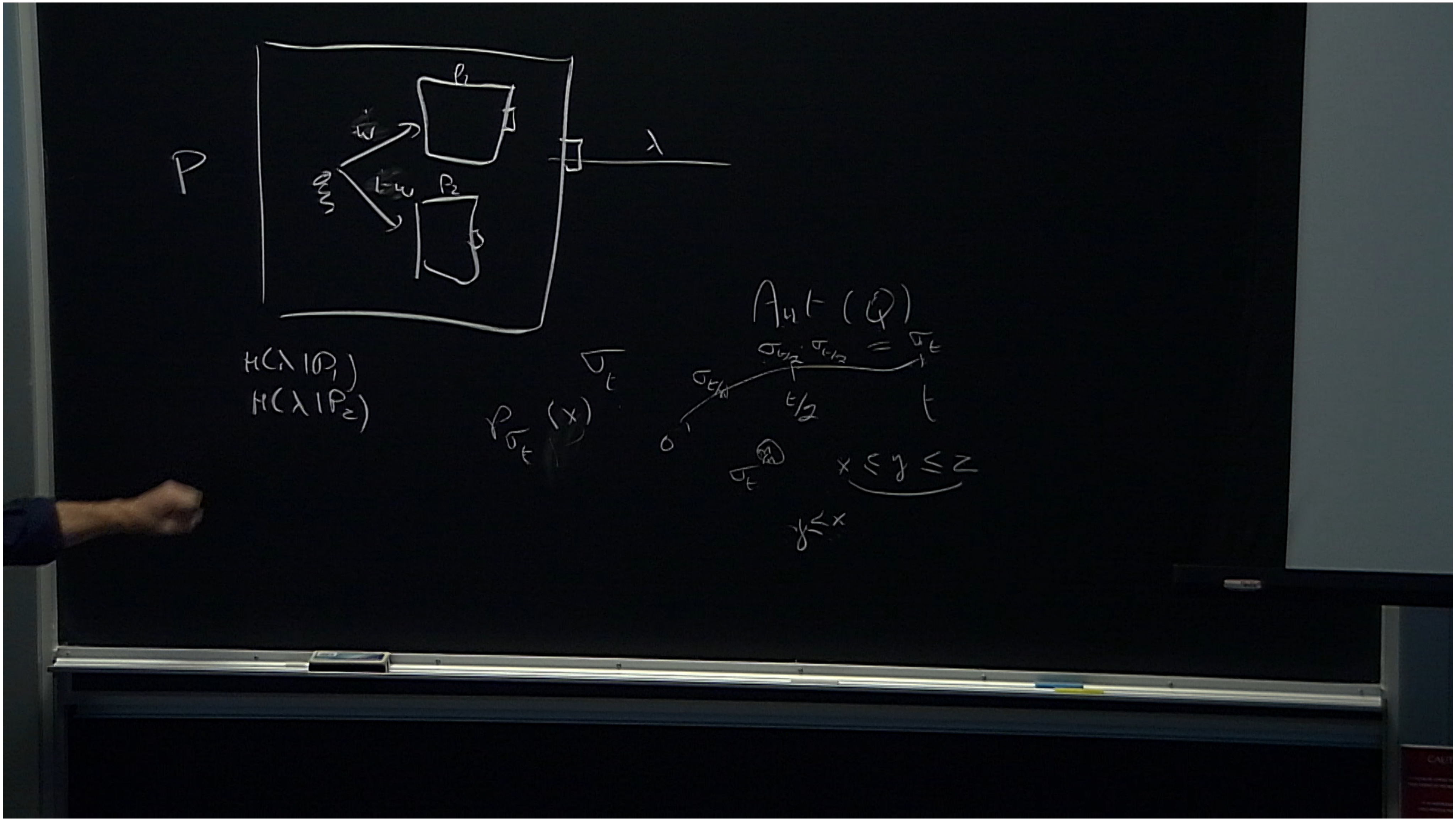
$P(XY|ST)$
$= \mathrm{Tr}_A(\rho_{Y|TA}\rho_{XA|S})$

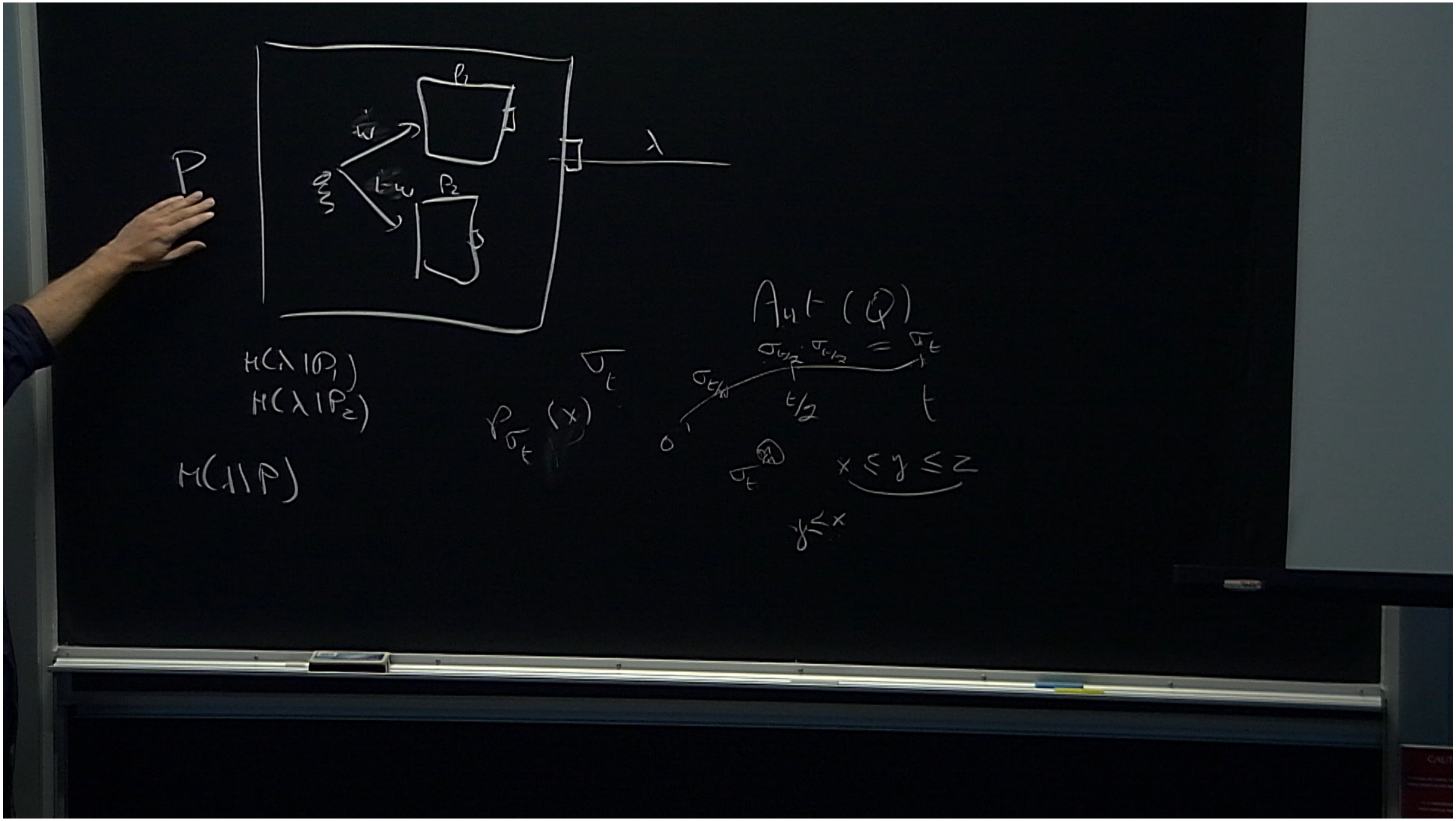
Violate noncontextuality inequalities

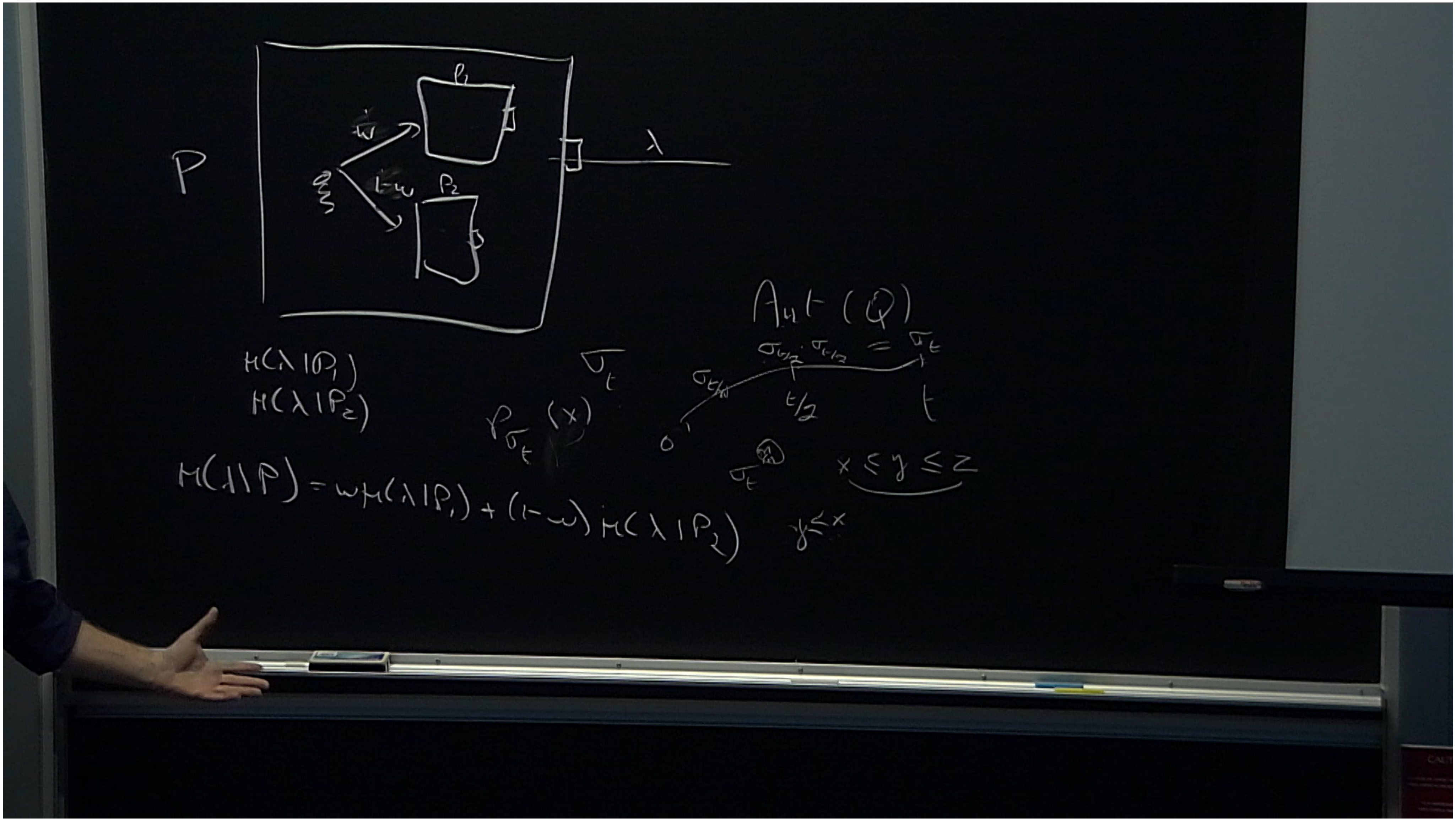
See: Leifer and RWS, PRA 88, 052130 (2013)
Lee, Allen, Horsman, Barrett, RWS, arXiv:1609.09487

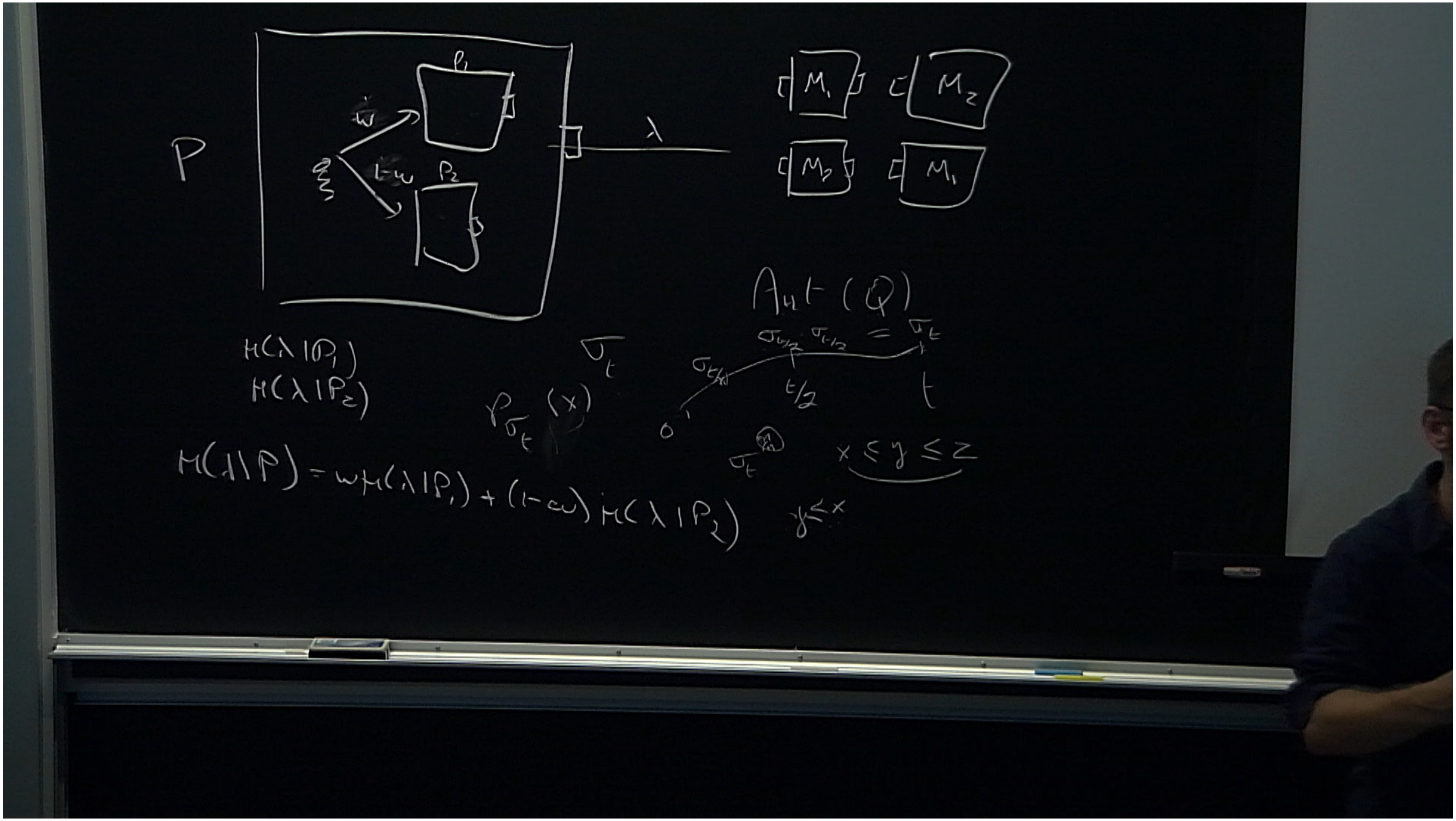


P
Aut (Q)
x ≤ y ≤ z
y ≤ x




P
ρ_1
ρ_2
λ
$H(\lambda|\rho_1)$
$H(\lambda|\rho_2)$
$H(\lambda|\rho)$
σ_t
$\rho_{\sigma_t}(x)$
Aut (Q)
$\sigma_{t/2}\cdot\sigma_{t/2}=\sigma_t$
0
t/2
t
$x\leq y\leq z$
$y\leq x$


P
λ
Aut(Q)
$\mu(\lambda|P) = \omega\mu(\lambda|P_1) + (1-\omega)\mu(\lambda|P_2)$
x ≤ y ≤ z


P
λ
M1
M2
M2
M1
1-ω
Aut (Q)
t/2
t
x ≤ y ≤ z
μ(λ|P1)
μ(λ|P2)
μ(λ|P) = ωμ(λ|P1) + (1-ω)μ(λ|P2)