Title: What do we learn about quantum theory from Kochen-Specker quantum contextuality?

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Abstract:

What do we learn about quantum theory from Kochen-Specker contextuality?

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"Contextuality: Conceptual Issues, Operational Signatures, and Applications",
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July 24, 2017

Motivation: Where does quantum theory come from?

Quantum theory does not trouble me at all. (...) What eats me (...) is to understand (...) [w]here does it come from?

J. A. Wheeler, quoted in J. Bernstein, "John Wheeler: Retarded learner", in *Quantum Profiles* (Princeton University Press, Princeton, New Jersey, 1989).

Plan

- I. Quantum contextuality

Is QT about non-existent properties?

[W]e have so far no reason to believe that there are some inner properties of the atom which condition a definite outcome (. . .). Ought we hope later to discover such properties (. . .)? Or ought we to believe that the agreement of theory and experiment (. . .) is a pre-established harmony founded on the nonexistence of such conditions?

M. Born, Zur Quantenmechanik der Stossvorgänge Zeitschrift für Physik **37**, 863 (1926) [On the quantum mechanics of collisions, in *Quantum Theory and Measurement*, edited by J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton, NJ, 1983), p. 52].

The Kochen-Specker theorem

The KS theorem addresses the following:

- Question: Is it possible that, at any instant of time, the "observed quantities" (i.e., those represented by projection valued measures) each possess a definite value, regardless of whether they have been measured?

- Answer: No, if the system is represented by a Hilbert space of dimension larger than two.
  Yes, if the system is represented by a Hilbert space of dimension two.

S. Kochen and E. P. Specker, The problem of hidden variables in quantum mechanics, J. Math. Mech. **17**, 59 (1967).

Why is it impossible?

Two options:

- *Outcome indeterminism*. It is because the observables do not have predetermined values; the values are created when the measurements are performed
- *Outcome contextuality*. It is because the observables have predetermined values, but they are contextual; i.e., they depend on which other compatible observables are measured

The basic assumption

- *Outcome noncontextuality for sharp measurements.*

Sharp measurements in general probabilistic theories

- Problem: sharp measurements were only defined *within quantum theory* (sharp measurements = observables represented by projection valued measures).

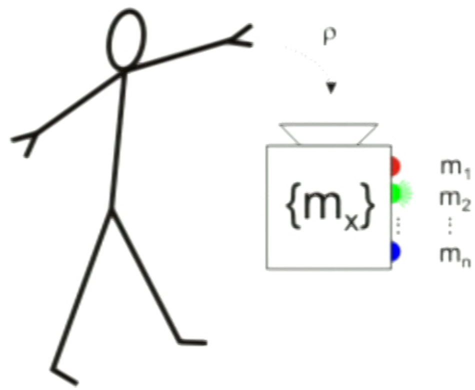
Sharp measurements in general probabilistic theories

- Problem: sharp measurements were only defined *within quantum theory* (sharp measurements = observables represented by projection valued measures).
- Solution: an operational definition of sharp measurements for general probabilistic theories.

M. Kleinmann, Sequences of projective measurements in generalized probabilistic models, J. Phys. A **47**, 455304 (2014).
G. Chiribella and X. Yuan, Measurement sharpness cuts nonlocality and contextuality in every physical theory, arXiv:1404.3348.

Measurement

A *measurement* is an interaction between a system and a device, which produces an outcome. It is described by a collection of *events*, each of them labeled by an outcome $x \in X$. In the case of a *demolition measurement*, the events are called *effects* and the measurement is described by the collection of effects $\{m_x\}_{x \in X}$. A general probabilistic theory assigns, for every state ρ, a probability $P(m_x|\rho)$ of the outcome x.

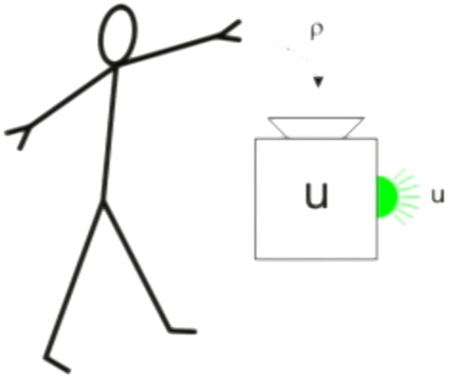


Unit effect

For every system there is an effect u, called *unit*, such that, for all states ρ and all measurements $\{m_x\}_{x\in X}$,

$$\sum_{x\in X} P(m_x|\rho) = P(u|\rho) = 1. \tag{1}$$

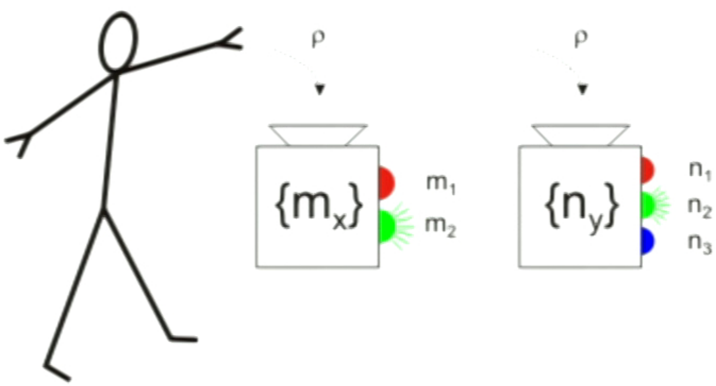
In quantum theory u is represented by the identity operator on the Hilbert space of the system and Eq. (1) expresses the fact that quantum measurements are resolutions of the identity.



Refinement

$\{n_y\}_{y\in Y}$ is a *refinement* (or *fine-graining*) of $\{m_x\}_{x\in X}$ if for all y there is an x such that, for all ρ,

$$P(n_y|\rho) \leq P(m_x|\rho). \qquad (2)$$

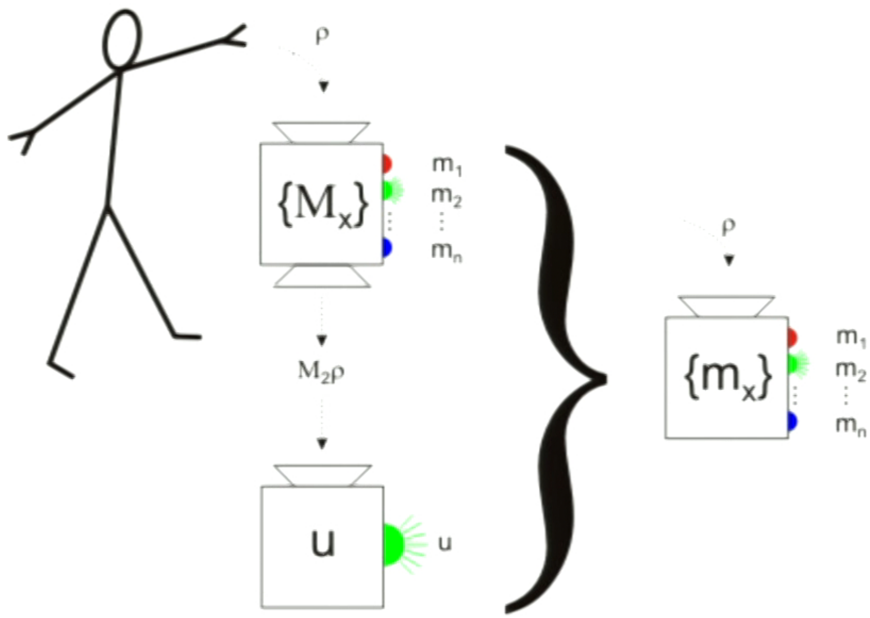


Non-demolition measurement

A *non-demolition measurement* (or *instrument*) is a measurement which transforms the state of the system into a new state. It is therefore a collection $\{\mathcal{M}_x\}_{x\in X}$ of *transformations* of the state for each outcome x. If the initial state is ρ, the state after outcome x is denoted by $\mathcal{M}_x\rho$. Every $\{\mathcal{M}_x\}_{x\in X}$ is associated to a $\{m_x\}_{x\in X}$ via the relation

$$P(u|\mathcal{M}_x\rho) = P(m_x|\rho)$$

for all $x \in X$.

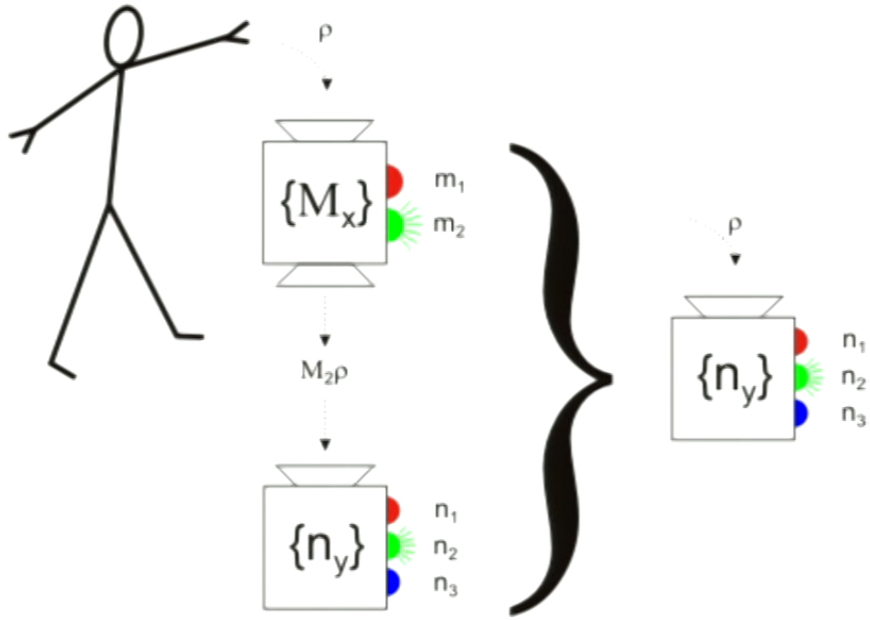


Sharp measurement

A *sharp measurement* is a non-demolition measurement which cannot be detected when performed before any of its refinements. That is,

$$P(n_y|\mathcal{M}_x\rho) = P(n_y|\rho) \qquad (4)$$

for all ρ, all $\{n_y\}_{y\in Y}$ refinement of $\{m_x\}_{x\in X}$ associated to $\{\mathcal{M}_x\}_{x\in X}$, and all (x, y) in Eq. (2).

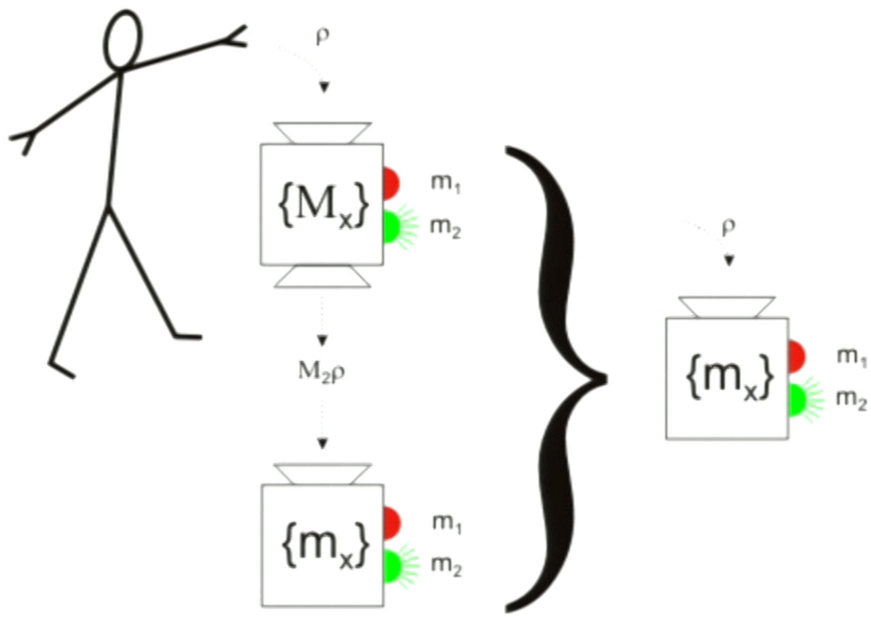


Properties of a sharp measurement

(I) *Repeatability*: it gives the same outcome when performed consecutive times. This implies,

$$P(m_x|\mathcal{M}_x\rho) = P(m_x|\rho) \qquad (5)$$

for all ρ and all $x \in X$.

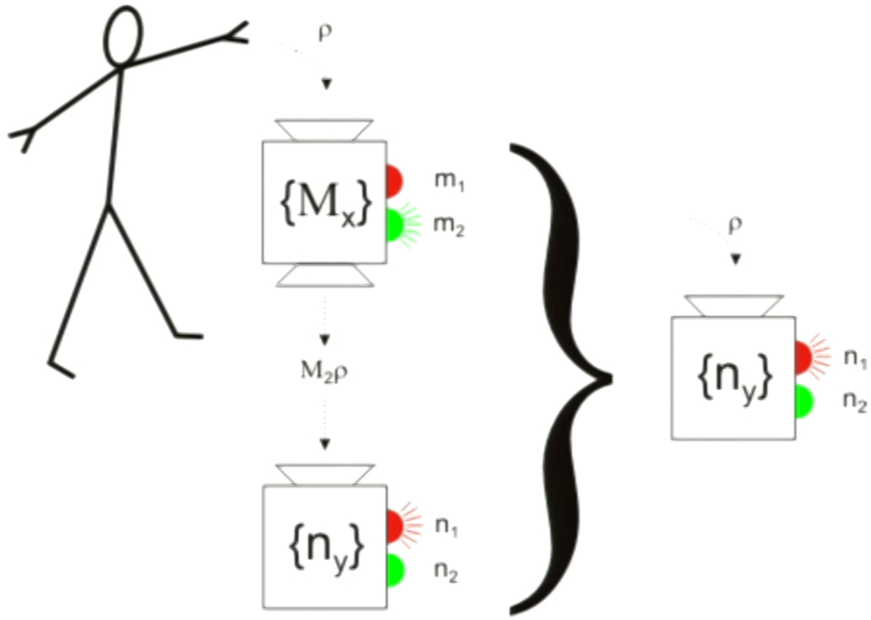


Properties of a sharp measurement

(II) *Minimal disturbance*: it affects only the statistics of incompatible measurements. That is, if $\{n_y\}_{y\in Y}$ is compatible with $\{m_x\}_{x\in X}$, then

$$P(n_y|\mathcal{M}_x\rho) = P(n_y|\rho) \qquad (6)$$

for all ρ, all $x \in X$, and all $y \in Y$.

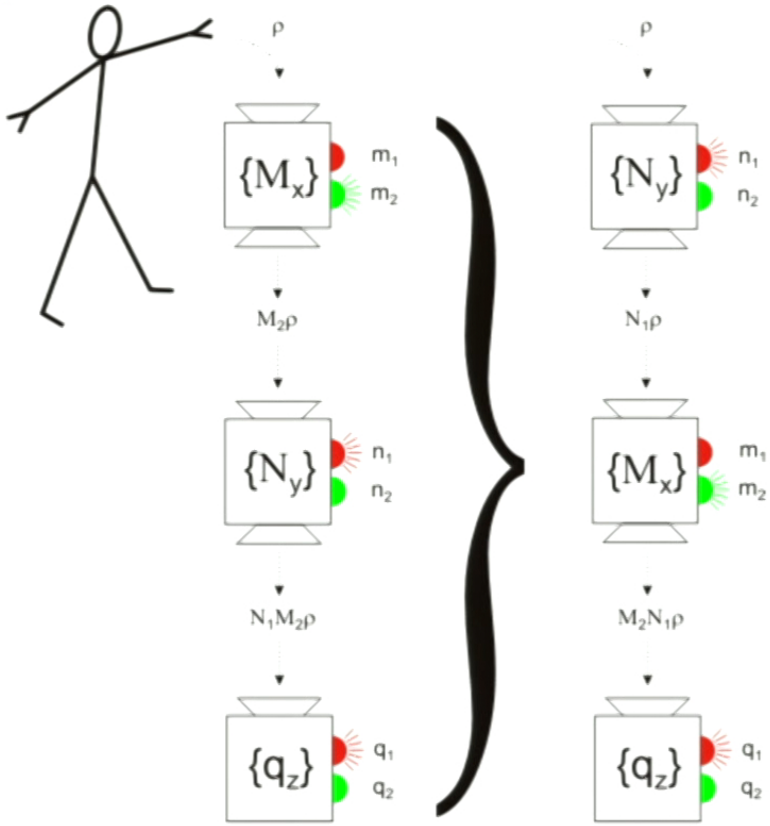


Compatibility

Two sharp measurements $\{\mathcal{M}_x\}_{x\in X}$ and $\{\mathcal{N}_y\}_{y\in Y}$ are *compatible* if and only if, for any measurement $\{q_z\}_{z\in Z}$,

$$P(q_z|\mathcal{N}_y\mathcal{M}_x\rho) = P(q_z|\mathcal{M}_x\mathcal{N}_y\rho)$$

for all ρ, all $x \in X$, all $y \in Y$, and all $z \in Z$.



Compatibility, context

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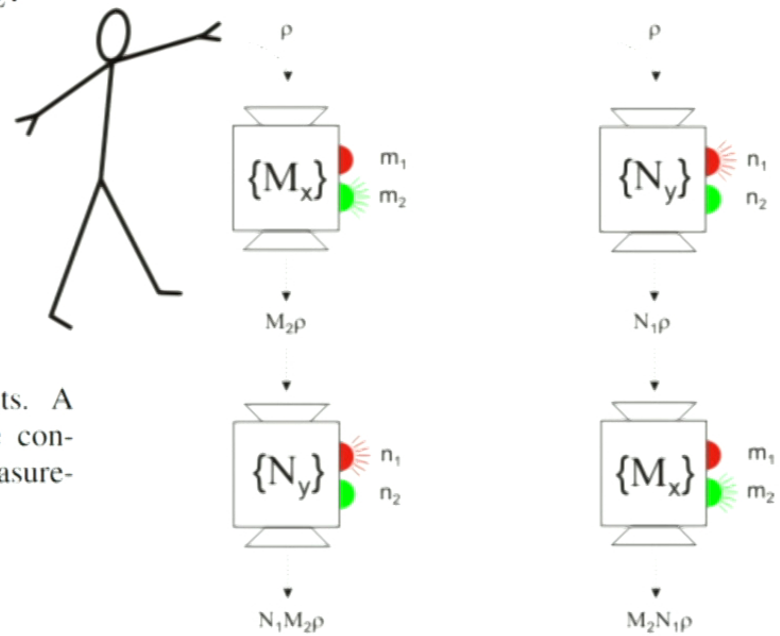


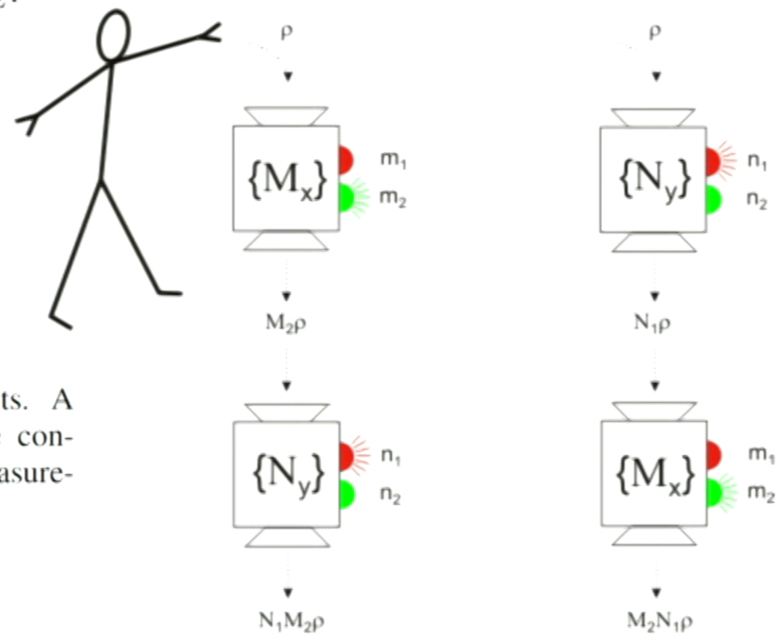
A *context* is a set of compatible sharp measurements. A mother measurement associated to a context can be constructed by sequentially measuring each of the sharp measurements in any order.

Compatibility, context, event

Two sharp measurements $\{\mathcal{M}_x\}_{x\in X}$ and $\{\mathcal{N}_y\}_{y\in Y}$ are *compatible* if and only if, for any measurement $\{q_z\}_{z\in Z}$,

$$P(q_z|\mathcal{N}_y\mathcal{M}_x\rho) = P(q_z|\mathcal{M}_x\mathcal{N}_y\rho)$$

for all ρ, all $x \in X$, all $y \in Y$, and all $z \in Z$.

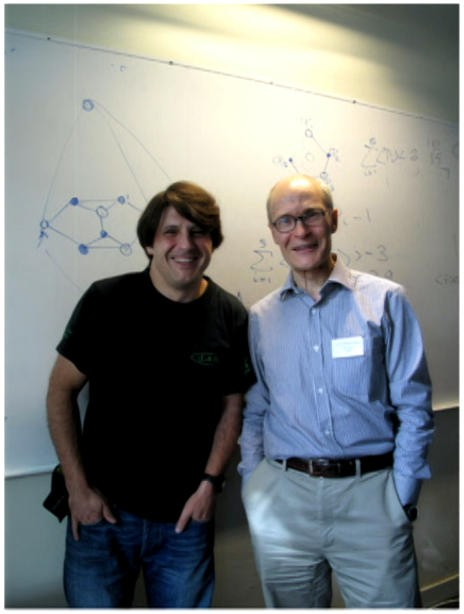


A *context* is a set of compatible sharp measurements. A mother measurement associated to a context can be constructed by sequentially measuring each of the sharp measurements in any order.

We will focus on experiments involving sequential compatible sharp measurements. In these experiments, an *event* is a transformation between the initial state and the state after the last measurement.

Contextuality meets inequalities

- Sharp measurements allows us to define noncontextuality (NC) inequalities for *contextuality scenarios* (sets of sharp measurements and their compatibility relations) and detect
  - Single qutrit contextuality
  - State-independent contextuality



A. A. Klyachko, M. A. Can, S. Binicioğlu, and A. S. Shumovsky, Simple Test for Hidden Variables in Spin-1 Systems, Phys. Rev. Lett. **101**, 020403 (2008).
A. Cabello, Experimentally Testable State-Independent Quantum Contextuality, Phys. Rev. Lett. **101**, 210401 (2008).

Tight NC inequalities and the noncontextual polytope

P. Suppes and M. Zanotti, When are probabilistic explanations possible?, Synthese **48**, 191 (1981).
A. Fine, Hidden Variables, Joint Probability, and the Bell Inequalities, Phys. Rev. Lett. **48**, 291 (1982).
I. Pitowsky, *Quantum Probability, Quantum Logic*, Lecture Notes in Physics **321** (Springer, Heidelberg, 1989).

For a given *contextuality scenario* (defined as a set of sharp measurements and their compatibility relations), there are inequalities involving linear combinations of correlations between the outcomes of compatible sharp measurements, which provide *necessary and sufficient conditions for the existence of a joint probability distribution*. We refer to these inequalities as *tight noncontextuality inequalities* and to the set they define as the *noncontextual polytope*.

M. Araújo, M. T. Quintino, C. Budroni, M. Terra Cunha, and A. Cabello, All noncontextuality inequalities for the n-cycle scenario, Phys. Rev. A **88**, 022118 (2013).

Tight NC inequalities and the noncontextual polytope

P. Suppes and M. Zanotti, When are probabilistic explanations possible?, Synthese **48**, 191 (1981).
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M. Araújo, M. T. Quintino, C. Budroni, M. Terra Cunha, and A. Cabello, All noncontextuality inequalities for the n-cycle scenario, Phys. Rev. A **88**, 022118 (2013).

Alternative (without sharp measurements):

A. Acín, T. Fritz, A. Leverrier, and A. B. Sainz, A Combinatorial Approach to Nonlocality and Contextuality, Comm. Math. Phys. **334**, 533 (2015).

KS quantum contextuality

We define *quantum contextuality* as those quantum correlations for compatible sharp measurements which are outside the *noncontextual polytope*.

Why *this* notion?

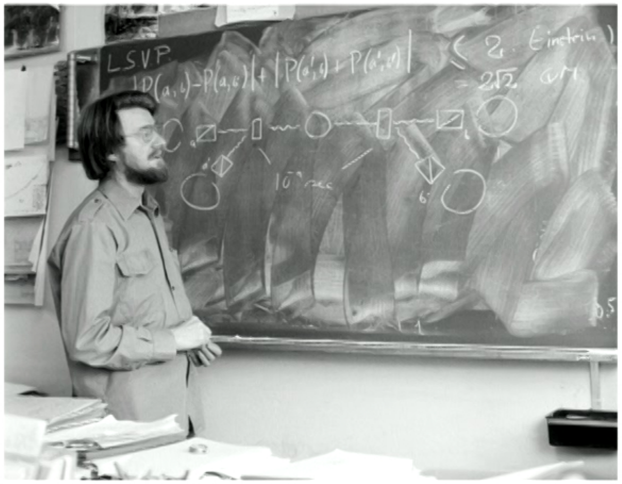
Hereafter, I will argue why *this* notion of quantum contextuality teaches us more about what *is* quantum theory and what we can infer from the world from the effectiveness of quantum theory than any other notion of "non-classicality".

Plan

- I. Quantum contextuality
- II. The "contextuality wars" and their lessons

First contextuality war, 1966. Contextuality vs nonlocality

Different possibilities require different experimental arrangements; there is no *a priori* reason to believe that the results for [a measurement in different contexts] should be the same.



It would be interesting (...) replacing the arbitrary axioms objected to above by some condition of locality, or of separability of distant systems.

J. B. Bell, On the Problem of Hidden Variables in Quantum Mechanics, Rev. Mod. Phys. **38**, 447 (1966).

Reasons 1-3

Reason 1: Contextuality generalizes nonlocality and provides a unifying paradigm for the resources of quantum information.

Reason 2: Contextuality requires less and has a broader scope. It does not presuppose the existence of parts or that causal influences propagate at finite speed. It neither privileges composite systems, nor entangled states, nor space-like separated measurements.

Reason 3: Contextuality put the emphasis on measurements.

Reason 4

Reason 4: A "device-independent approach" does not allows us to recover quantum correlations (not even for Bell inequality scenarios).

Proof: Real Hilbert space quantum theory is enough to simulate any quantum correlation in Bell inequality scenarios.

M. McKague, M. Mosca, and N. Gisin, Simulating quantum systems using real Hilbert spaces, Phys. Rev. Lett. **102**, 020505 (2009).

Second contextuality war, 1999. (i) C vs finite precision

Only finite precision measurements are experimentally reasonable, and they cannot distinguish a dense subset from its closure. We show that the rational vectors, which are dense in S^2, can be [KS] colored so that the contradiction with hidden variable theories provided by Kochen-Specker constructions does not obtain.

D. A. Meyer, Finite Precision Measurement Nullifies the Kochen-Specker Theorem, Phys. Rev. Lett. **83**, 3751 (1999).

Second contextuality war, 1999. (i) C vs finite precision

D. A. Meyer, Finite Precision Measurement Nullifies the Kochen-Specker Theorem, Phys. Rev. Lett. **83**, 3751 (1999).

A. Kent, Noncontextual Hidden Variables and Physical Measurements, Phys. Rev. Lett. **83**, 3755 (1999).

R. Clifton and A. Kent, Simulating quantum mechanics by non-contextual hidden variables, Proc. R. Soc. Lond. A **456**, 2101 (2000).

J. Barrett and A. Kent, Non-contextuality, finite precision measurement and the KochenSpecker theorem, Stud. Hist. Philos. Mod. Phys. Part B: Stud. Hist. Philos. Mod. Phys. **33**, 151 (2004).

A. Cabello, Comment on "Non-Contextual Hidden Variables and Physical Measurements", quant-ph/9911024.

H. Havlicek, G. Krenn, J. Summhammer, and K. Svozil, Colouring the rational quantum sphere and the Kochen-Specker theorem, J. Phys. A: Math. Gen. **34**, 3071 (2001).

N. D. Mermin, A Kochen-Specker theorem for imprecisely specified measurement, quant-ph/9912081.

D. M. Appleby, Contextuality of approximate measurements, quant-ph/0005010.

D. M. Appleby, Existential contextuality and the models of Meyer, Kent, and Clifton, Phys. Rev. A **65**, 022105 (2002).

C. Simon, Č. Brukner, and A. Zeilinger, Hidden-variable theorems for real experiments, Phys. Rev. Lett. **86**, 4427 (2001).

J.-Å. Larsson, A Kochen-Specker inequality, Europhys. Lett. **58**, 799 (2002).

C. Simon, *The Foundations of Quantum Information and Feasible Experiments*, Ph.D. thesis, University of Vienna, 2000; quant-ph/0103057.

A. Cabello, Finite precision measurement does not nullify the Kochen-Specker theorem, Phys. Rev. A **65**, 5, 052101 (2002).

C. F. Boyle and R. L. Schafir, Remarks on noncontextual hidden variables and physical measurements, quant-ph/0106040.

D. M. Appleby, Nullification of the nullification, quant-ph/0109034.

D. M. Appleby, The BellKochenSpecker theorem, Stud. Hist. Philos. Mod. Phys. Part B: Stud. Hist. Philos. Mod. Phys. **36**, 1 (2005).

A. Cabello and J.-Å. Larsson, Quantum contextuality for rational vectors, Phys. Lett. A **375**, 99 (2010).

Contextuality is more than the KS proof

The contradiction with noncontextual hidden variable theories can be proven using the dense set proposed by Meyer: Contextuality is much more than the KS theorem.

A. Cabello and J.-Å. Larsson, Quantum contextuality for rational vectors, Phys. Lett. A **375**, 99 (2010).

Second contextuality war, 1999. (ii) C vs advantage

[I]n contrast to violation of the Bell inequalities, no quantum-over-classical advantage for information processing can be derived from the Kochen-Specker theorem alone.

D. A. Meyer, Finite Precision Measurement Nullifies the Kochen-Specker Theorem, Phys. Rev. Lett. **83**, 3751 (1999).

Reason 5

Reason 5: Contextuality allows us to identify new quantum-over-classical advantages.

Simulating quantum contextuality requires classical systems with higher memory.

Simulating contextuality with classical systems with a finite number of states produces heat due to Landauer's principle.

M. Kleinmann, O. Gühne, J. R Portillo, J.-Å. Larsson, and A. Cabello, Memory cost of quantum contextuality, New J. Phys. **13**, 113011 (2011).

A. Cabello, M. Gu, O. Gühne, J.-Å. Larsson, and K. Wiesner, Thermodynamical cost of some interpretations of quantum theory, Phys. Rev. A **94**, 052127 (2016).

Reason 5

The GHZ violation of Mermin inequality computes.

If a $l2$-measurement-based quantum computer deterministically computes a non-linear Boolean function $f : 2^m \rightarrow 2^l$, then the resource must be fully contextual.

There is an equivalence between contextuality and the possibility of universal quantum computation via magic state distillation.

J. Anders and D. E. Browne, Computational Power of Correlations, Phys. Rev. Lett. **102**, 050502 (2009).
R. Raussendorf, Contextuality in measurement-based quantum computation, Phys. Rev. A **88**, 022322 (2013).
M. Howard, J. Wallman, V. Veitch, and J. Emerson, Contextuality supplies the 'magic' for quantum computation, Nature (London) **510**, 351 (2014).

Second contextuality war, 1999. (iii) Bell inequalities again

Since violations of Bell inequalities can be verified without requiring that the observables whose correlations figure in the inequalities be measured with arbitrarily high precision, Bell's theorem yields a method of falsifying local hidden variable theories.

R. Clifton and A. Kent, Simulating quantum mechanics by non-contextual hidden variables, Proc. R. Soc. Lond. A **456**, 2101 (2000).

Third contextuality war, 2005. (i) C vs unsharpness

The outcome of a measurement depends deterministically on the ontic state of the system being measured if and only if the measurement is sharp.

[A]ny realistic measurement necessarily has some nonvanishing amount of noise and therefore never achieves the ideal of sharpness.

R. W. Spekkens, Contextuality for preparations, transformations, and unsharp measurements, Phys. Rev. A **71**, 052108 (2005).

R. W. Spekkens, The status of determinism in proofs of the impossibility of a noncontextual model of quantum theory, Found. Phys. **44**, 1125 (2014).

Third contextuality war, 2005. (i) C vs unsharpness

Any realistic measurement of the speed of falling bodies necessarily has some air resistance.

Third contextuality war, 2005. (i) C vs unsharpness

Any realistic measurement of the speed of falling bodies necessarily has some air resistance.

Ignoring air resistance was wise.

Reason 6

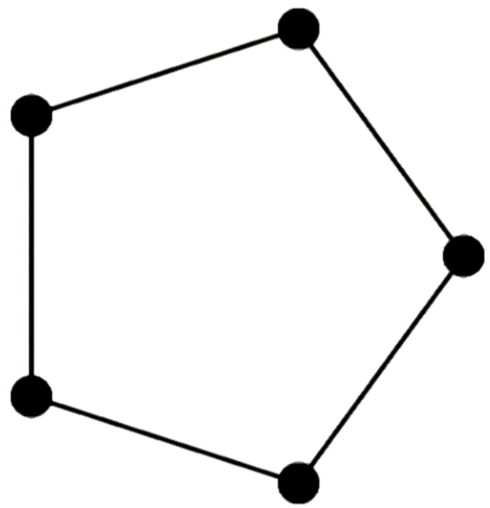
Reason 6: The fundamental feature of quantum theory is that *all measurements are sharp* in the sense that all can be *conceived* as sharp measurements on a larger system.

M. A. Neumark, Izv. Akad. Nauk SSSR Ser. Mat. **4**, 53 (1940); Izv. Akad. Nauk SSSR Ser. Mat. **4** 277 (1940); C.R. (Dokl.) Acad. Sci. URSS (N.S.) **41** 359 (1943).
N. I. Akhiezer and I. M. Glazman, *Theory of Linear Operators in Hilbert Space* (Dover, New York, 1993), Vol. II, p. 121.
A. Peres, Neumark's theorem and quantum inseparability, Found. Phys. **20**, 1441 (1990).
A. Peres, *Quantum Theory: Concepts and Methods* (Kluwer, New York, 1995), p. 285.

Reason 7

Reason 7: Our quantum contextuality leads to a simple definition of "quantum correlations" which suggests a simple reason for their physical bounds and ultimately suggests what is quantum theory and why quantum theory is so effective.

Exclusivity graphs



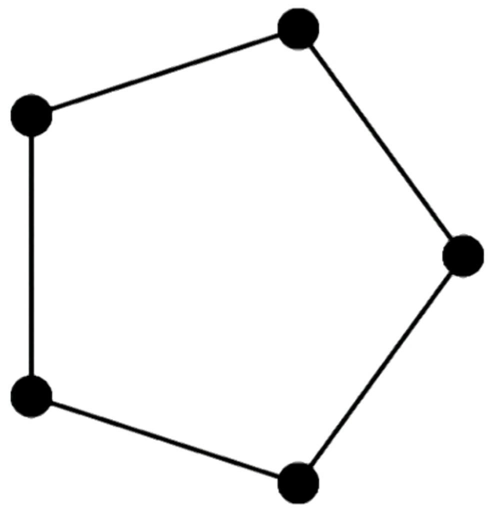
Event: state transformation when compatible sharp measurements are performed on state ρ

Two events are *equivalent* if they correspond to indistinguishable transformations.

Two events are *exclusive* if there exists a sharp measurement that contains both.

An *exclusivity graph* is a graph in which vertices represent events and edges exclusivity relations.

Exclusivity graphs



Event: state transformation when compatible sharp measurements are performed on state ρ

Two events are *equivalent* if they correspond to indistinguishable transformations.

Two events are *exclusive* if there exists a sharp measurement that contains both.

An *exclusivity graph* is a graph in which vertices represent events and edges exclusivity relations.

Question: What are the possible probability assignments in QT to the vertices of a given exclusivity graph (no matter how)?

Contextuality meets exclusivity graphs

Result 2: For any graph G, there is always a NC inequality such that the quantum maximum is *exactly* $\vartheta(\overline{G})$ and the set of quantum probabilities is *exactly* the Grötschel-Lovász-Schrijver theta body $\mathrm{TH}(\overline{G})$.

A. Cabello, S. Severini, and A. Winter, Graph-Theoretic Approach to Quantum Correlations, Phys. Rev. Lett. **112**, 040401 (2014).

Comments

Result 2 identifies $\vartheta(\overline{G})$ as a fundamental physical limit for quantum correlations and $\mathrm{TH}(\overline{G})$ as *the* set of *physical* correlations for a given G.

The strategy of focusing on graphs without referring to any specific experimental scenario substantially simplifies the problem of characterizing the quantum set of correlations.

The E principle

- <u>The exclusivity principle</u>: Every set of *n* pairwise exclusive events is *n*-wise exclusive.
- The sum of the probabilities of *n* *n*-wise exclusive events cannot be higher than 1. Therefore, the sum of the probabilities of *n* pairwise exclusive events cannot be higher than 1.
- The E principle <u>does not</u> follow from Kolmogorov's axioms.

The E principle

- The exclusivity principle: Every set of n pairwise exclusive events is n-wise exclusive.
- The sum of the probabilities of n n-wise exclusive events cannot be higher than 1. Therefore, the sum of the probabilities of n pairwise exclusive events cannot be higher than 1.
- The E principle does not follow from Kolmogorov's axioms.
- It is the only principle known capable to single out convex sets of quantum correlations.

A. Cabello, Simple Explanation of the Quantum Violation of a Fundamental Inequality, Phys. Rev. Lett. **110**, 060402 (2013).
B. Yan, Quantum Correlations are Tightly Bound by the Exclusivity Principle, Phys. Rev. Lett. **110**, 260406 (2013).
B. Amaral, M. Terra Cunha, and A. Cabello, Exclusivity principle forbids sets of correlations larger than the quantum set, Phys. Rev. A **89**, 030101(R) (2014).
A. Cabello, Simple Explanation of the Quantum Limits of Genuine n-Body Nonlocality, Phys. Rev. Lett. **114**, 220402 (2015).

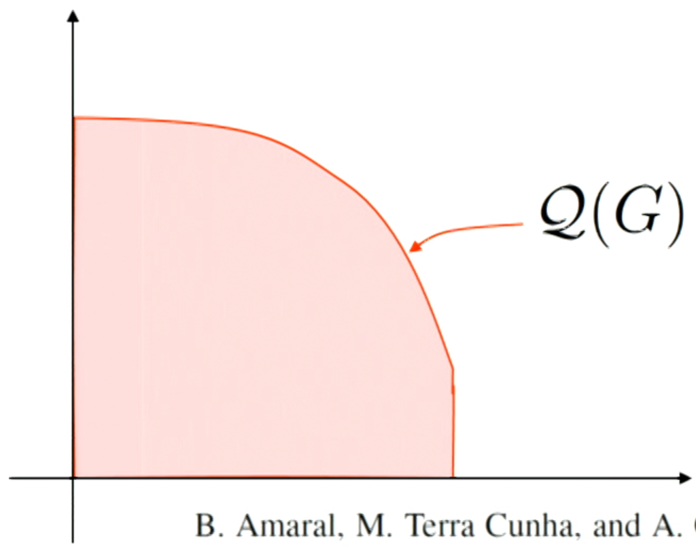
Some results

Result 1: Given the quantum set $\mathcal{Q}(\overline{G})$, the E principle singles out the quantum set $\mathcal{Q}(G)$.

B. Amaral, M. Terra Cunha, and A. Cabello, Exclusivity principle forbids sets of correlations larger than the quantum set, Phys. Rev. A **89**, 030101(R) (2014).

Some results

Result 2: If G is a self-complementary graph, the E principle, without any further assumptions, excludes any set of probability distributions strictly larger than the quantum set.



B. Amaral, M. Terra Cunha, and A. Cabello, Exclusivity principle forbids sets of correlations larger than the quantum set, Phys. Rev. A **89**, 030101(R) (2014).

Fundamental sharpness implies the E principle

- <u>Principle 1</u> (*Fundamental sharpness of measurements*): Every measurement arises from a sharp measurement performed jointly on the system and on the environment.
- <u>Principle 2</u>: The set of sharp measurements is closed under coarse-graining.
- Imply the E principle.

G. Chiribella and X. Yuan, Measurement sharpness cuts nonlocality and contextuality in every physical theory, arXiv:1404.3348.

Fundamental sharpness implies Born's rule

- (At least) for the probability assignments on self-complementary exclusivity graphs.

Attitudes

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Attitudes

- Noise is fundamental.
- Noise is not fundamental and only arises from the fact that the realistic measurements do not measure only the system but also the environment.
- The volume of GPTs with fundamental sharpness (FS) is negligible. Quantum theory is a GPT with FS. Therefore, to understand QT it is not wise to view noise as fundamental.
- The focus of the theory on interactions out of which one can construct sharp measurements (and only on them!) is the best possible trick to deal with slices of nature for which nature has no laws.

Third contextuality war, 2005. (ii) C and experiments

Plan

- I. Quantum contextuality
- II. The “contextuality wars” and their lessons
- III. Testing contextuality in experiments

How to analize a contextuality test. Options:

- (i) Assume that noise is not fundamental and keep the KS notion of contextuality.
  - (ia) Use hypothesis testing to evaluate the probability that the data can be explained by a noncontextual model assuming that the noise is due to well-traceable reasons.
  - (ib) Use Winter's or, better, Kujala et al.'s method to quantify the contextuality of the data.

A. Winter, What does an experimental test of quantum contextuality prove or disprove?, J. Phys. A **47**, 424031 (2014).
J. V. Kujala, E. N. Dzhafarov, and J.-Å. Larsson, Necessary and Sufficient Conditions for Maximal Noncontextuality in a Broad Class of Quantum Mechanical Systems, Phys. Rev. Lett. **115**, 150401 (2015).

How to analize a contextuality test. Options:

- (i) Assume that noise is not fundamental and keep the KS notion of contextuality.
  - (ia) Use hypothesis testing to evaluate the probability that the data can be explained by a noncontextual model assuming that the noise is due to well-traceable reasons.
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- (ii) Assume that noise is fundamental and adopt Spekkens' notion.

R. W. Spekkens, Contextuality for preparations, transformations, and unsharp measurements, Phys. Rev. A **71**, 052108 (2005).

How to analize a contextuality test. Options:

- (i) Assume that noise is not fundamental and keep the KS notion of contextuality.
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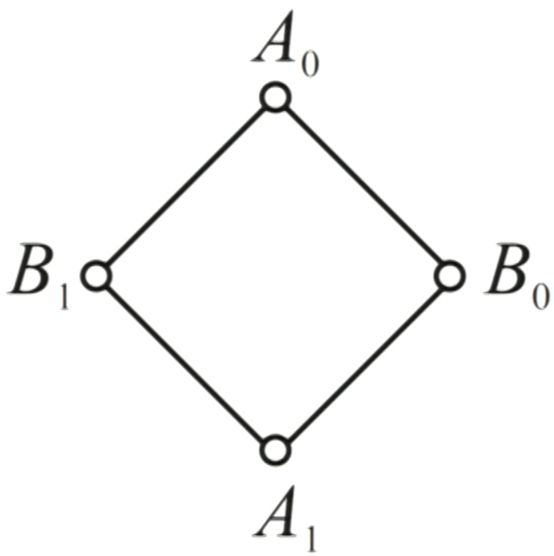
  - (ib) Use Winter's or, better, Kujala et al.'s method to quantify the contextuality of the data.
- (ii) Assume that noise is fundamental and adopt Spekkens' notion.
- (iii) Convert *any* KS contextuality test into a Bell inequality test.

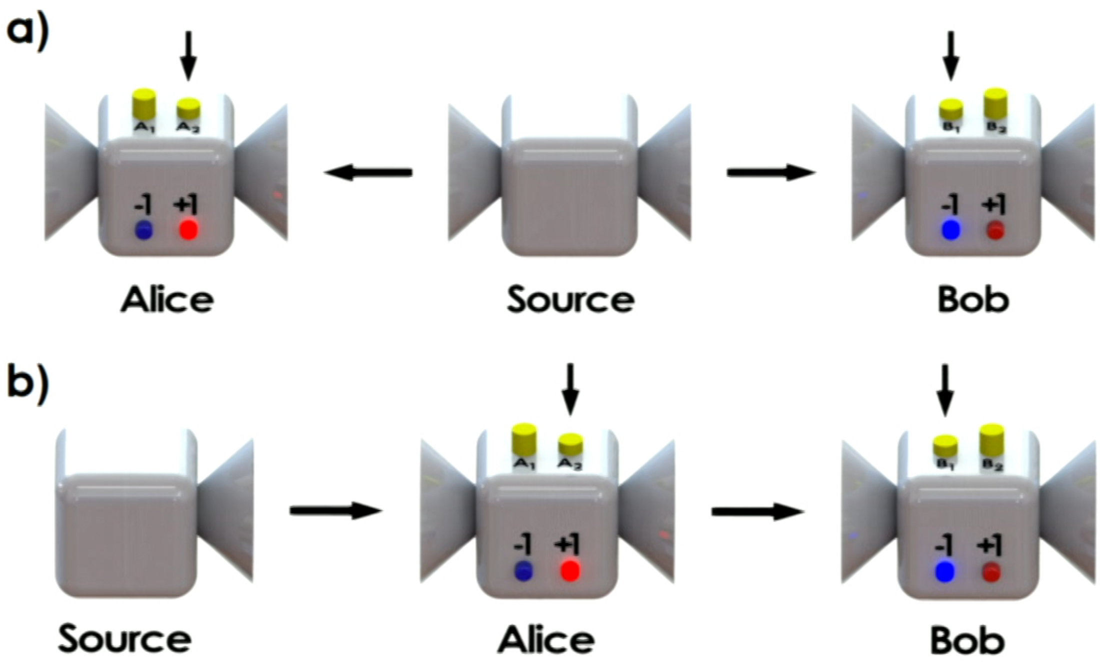
Plan

- I. Quantum contextuality
- II. The “contextuality wars” and their lessons
- III. Testing contextuality in experiments
- IV. Entanglement-amplified contextuality

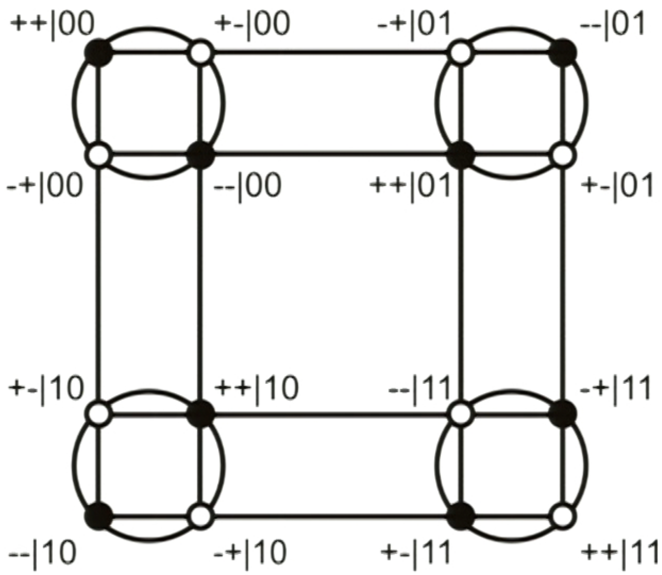
Compatibility graph of the CHSH scenario



The CHSH scenario(s)



Exclusivity graph of the CHSH scenario



Event: state transformation represented by *ab*|*xy*: outcomes *a* and *b* are respectively obtained when compatible sharp measurements *x* and *y* are performed on state ρ

Two events are *equivalent* if they correspond to indistinguishable transformations.

Two events are *exclusive* if there exists a sharp measurement that contains both.

Events (nodes) in the same straight line or circumference are pairwise exclusive

Transforming correlations into a positive linear combination of probabilities of events

$$\beta = \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle \overset{\mathrm{LR}}{\leq} 2$$

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$$\pm \langle A_i B_j \rangle = 2[P(1, \pm 1 \,|\, i, j) + P(-1, \mp 1 \,|\, i, j)] - 1$$

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$$S = P(1,1\,|\,0,0) + P(-1,-1\,|\,0,0) + P(1,1\,|\,0,1) + P(-1,-1\,|\,0,1) + P(1,1\,|\,1,0) + P(-1,-1\,|\,1,0) + P(1,-1\,|\,1,1) + P(-1,1\,|\,1,1)$$

$$S = \frac{\beta}{2} + 2$$

$$S \overset{\text{LR}}{\leq} 3$$

Transforming correlations into a positive linear combination of probabilities of events

$$\beta = \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle \overset{\mathrm{LR}}{\leq} 2$$

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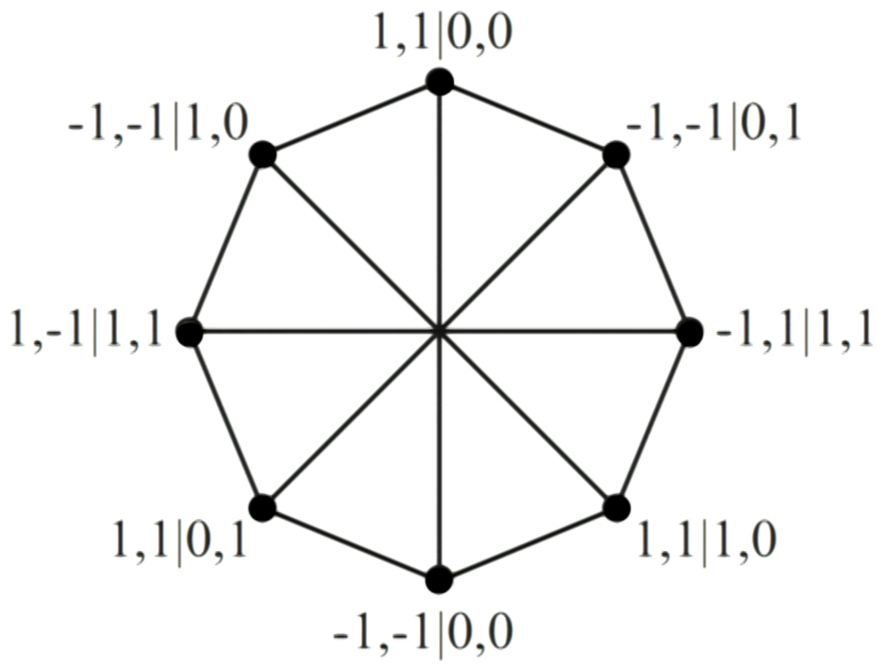
$$S = \frac{\beta}{2} + 2$$

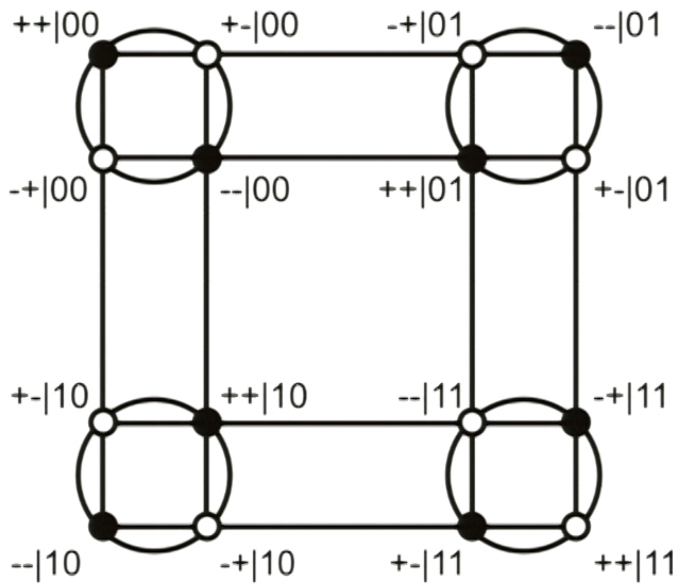
$$S \overset{\mathrm{LR}}{\leq} 3$$

The exclusivity graph for *S*



Vertices linked by an edge represent pairwise exclusive events

Is a subgraph of the E graph of the CHSH scenario



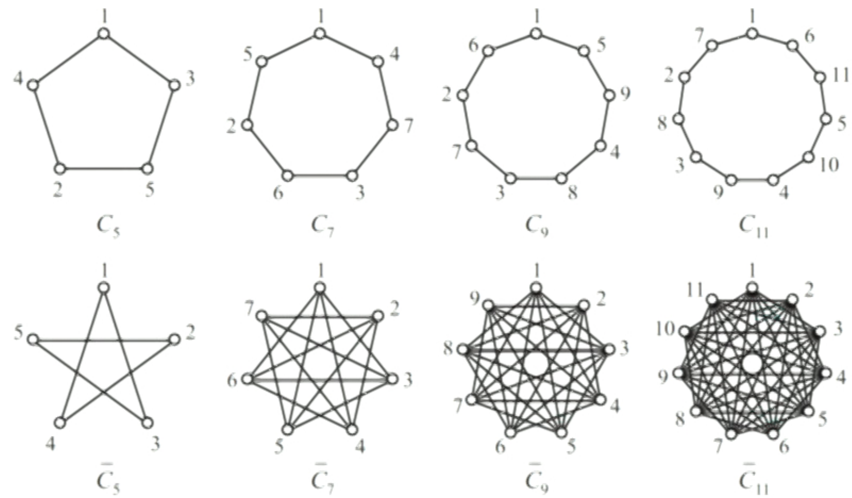
Events (nodes) in the same straight line or circumference are exclusive

Necessary and sufficient condition for no joint prob.

- QT violates a given NC inequality *iff* the exclusivity graph contains at least one induced pentagon, or heptagon, or nonagon, etc. (i.e., a “hole”) or their complements (i.e., an “antihole”)

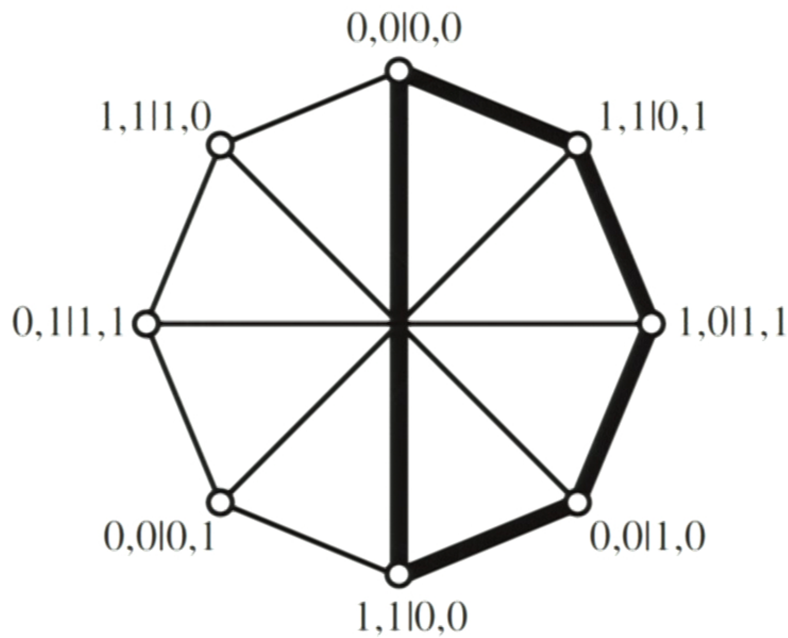
Holes:

Antiholes:



A. Cabello, S. Severini, and A. Winter, Graph-Theoretic Approach to Quantum Correlations, Phys. Rev. Lett. **112**, 040401 (2014).

Example of an induced basic graph



Basic E graphs of Bell and NC inequalities and KS sets

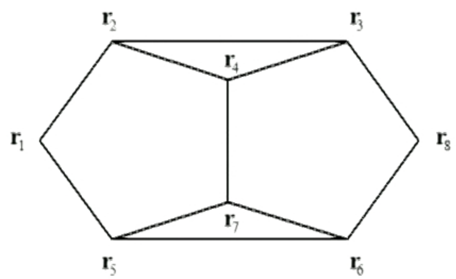
ADÁN CABELLO *et al.* PHYSICAL REVIEW A **88**, 032104 (2013)

TABLE I. Number of induced basic exclusivity graphs in some NC inequalities and KS proofs. The column "Graph" gives the standard name in graph theory, "Vertices" indicates its number of vertices, "Dimension" indicates the minimum dimension of the quantum system needed to define events with the corresponding exclusivity relationships.

NC inequality/KS proof	Graph	Vertices	Dimension	C_5	C_7	$\bar{C}_7$	C_9	$\bar{C}_9$
KCBS [11]	C_5	5	3	1	0	0	0	0
CHSH [20]	$Ci_8(1,4)$	8	4	8	0	0	0	0
S_3 [8,43]		10	4	10	0	0	0	0
KCBS-twin [44]	$J(5,2)$	10	6	12	0	0	0	0
Mermin [45]	Complement of Shrikhande	16	8	96	0	0	0	0
KS-18 [46,47]		18	4	144	108	0	12	0
YO [48] and its tight version [49]		22	3	288	384	0	0	0
KS-24 [50]		24	4	576	576	0	192	0
KS-31 [51]		31	3	70	184	0	248	0
KS-33 [50]		33	3	72	84	0	128	0

Any basic E graph can be converted into a DPS

Definite prediction sets for C_5



S. Kochen and E. P. Specker, The problem of hidden variables in quantum mechanics, J. Math. Mech. **17**, 59 (1967).

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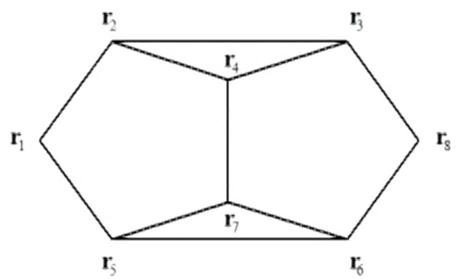
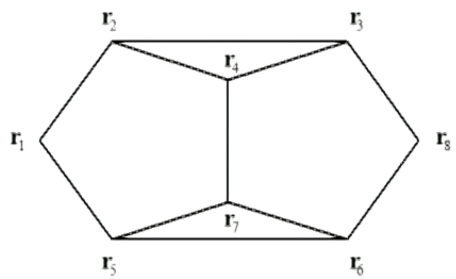
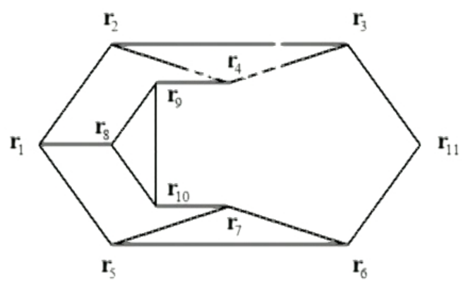


S. Kochen and E. P. Specker, The problem of hidden variables in quantum mechanics, J. Math. Mech. **17**, 59 (1967).

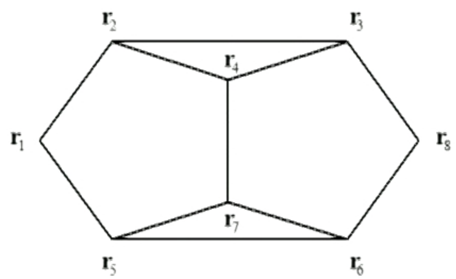
Definite prediction sets for C_5, C_7

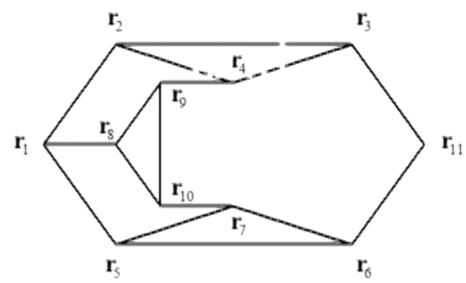


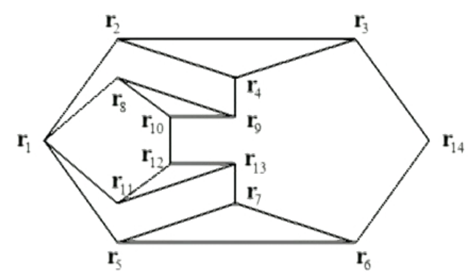


J. B. Bell, On the Problem of Hidden Variables in Quantum Mechanics, Rev. Mod. Phys. **38**, 447 (1966).

Definite prediction sets for C_5, C_7 and C_9 for $d=3$

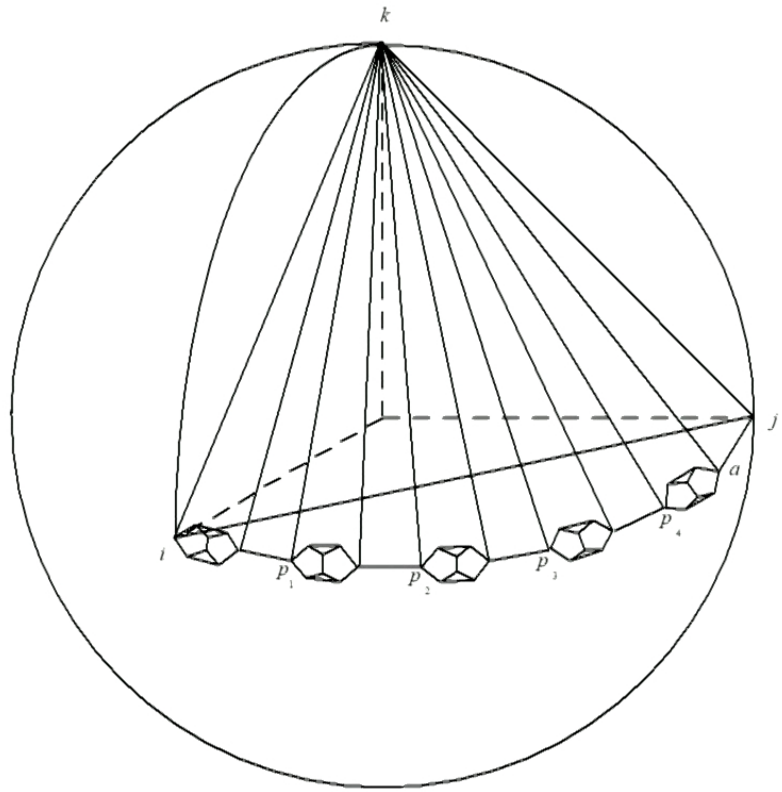




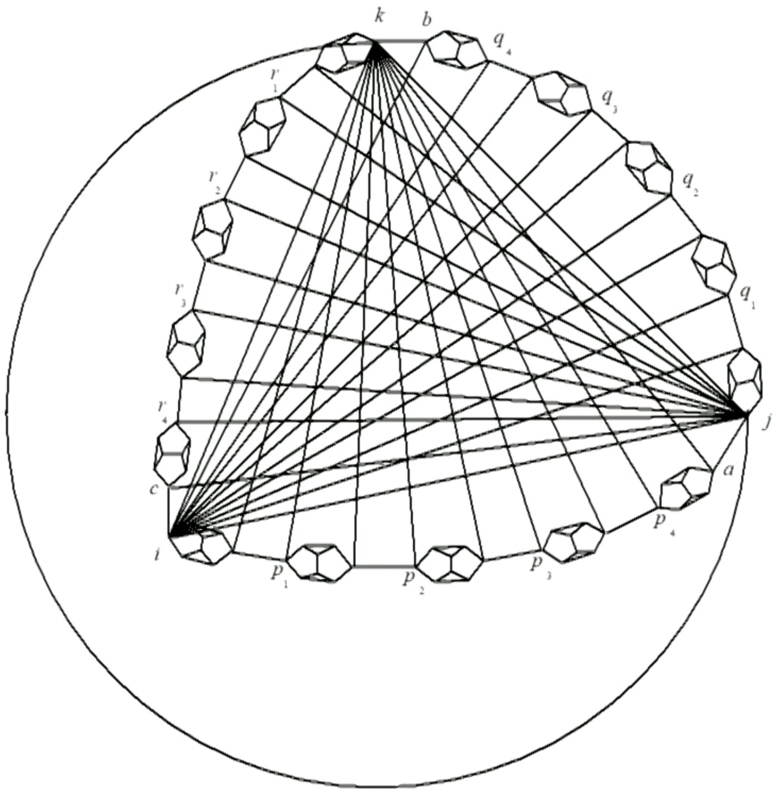


A. Cabello and G. García-Alcaine, A hidden-variables versus quantum mechanics experiment, J. Phys. A: Math. Gen. **28**, 3719 (1995).

Connect the initial state with an orthogonal one



Connect a complete basis

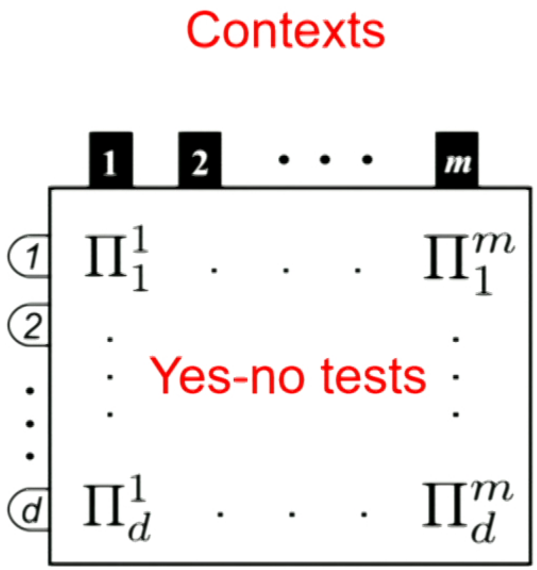


Similarly in any *d*

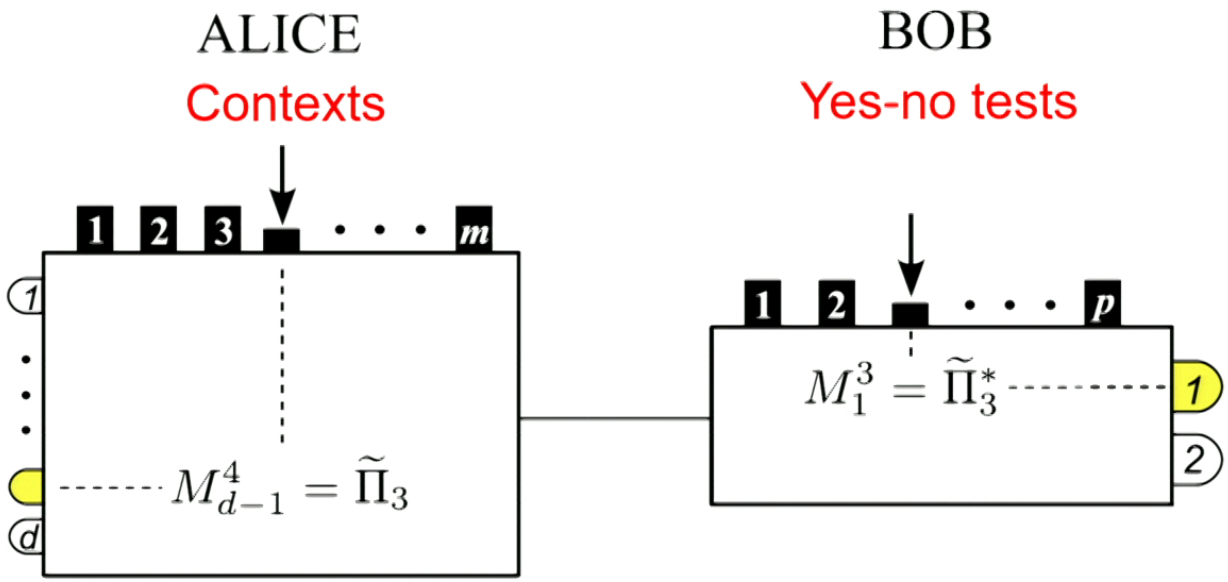
A. Cabello and G. García-Alcaine, Bell-Kochen-Specker theorem for any finite dimension $n \geq 3$, J. Phys. A: Math. Gen. **29**, 1025 (1996).

Repeat the process for any basic induced subgraph

Result: a big KS set



Converting a KS set into a Bell inequality



L. Aolita, R. Gallego, A. Acín, A. Chiuri, G. Vallone, P. Mataloni, and A. Cabello, Fully nonlocal quantum correlations, Phys. Rev. A **85**, 032107 (2012).

Features

- *Free of* the (conceptual) problems of finite precision and unsharpness.
- *Every* quantum point out of the noncontextual polytope can be tested.
- One-to-one correspondence with the original states and measurements.
- One-to-one correspondence with the *basic* structures that make the original states and measurements to violate the original NC inequality.
- The method for producing a KS set from *any* quantum violation of a NC inequality is of interest by itself.

Motivation: Where does quantum theory come from?

Quantum theory does not trouble me at all. (...) What eats me (...) is to understand (...) [w]here does it come from?

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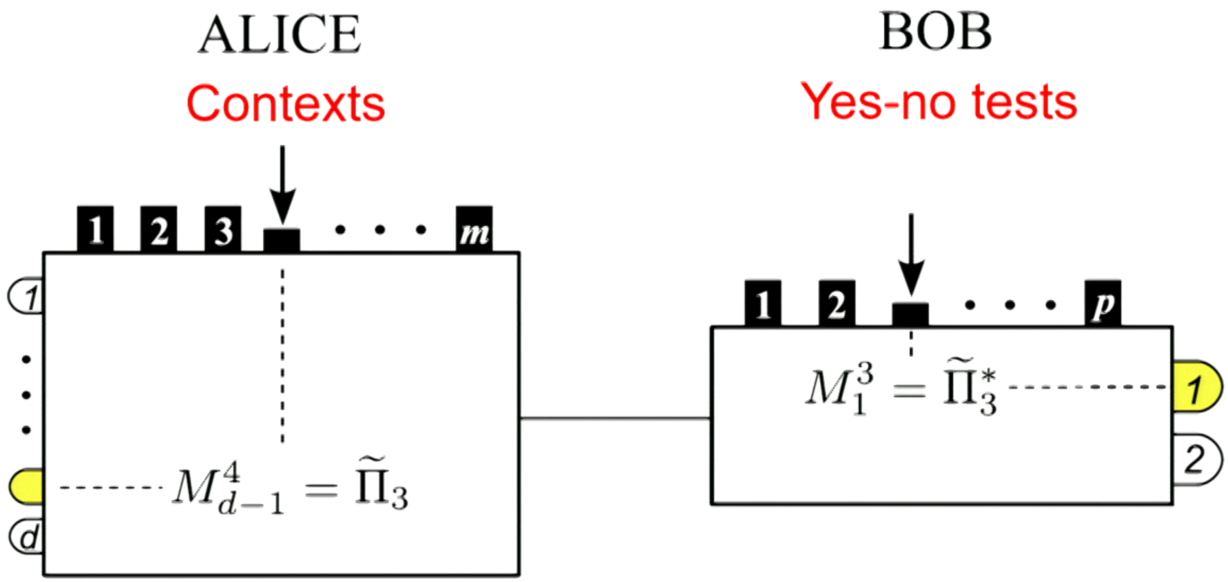

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