Title: Black Hole Entropy from BMS Symmetry at the Horizon

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Abstract: <p>One of the basic puzzles of black hole thermodynamics is the simplicity and universality of the Bekenstein-Hawking entropy. The idea that this entropy might be governed by a symmetry at the horizon is an old one, but until now efforts have focused on conformal symmetries, either at infinity or on a "stretched horizon." I argue that a better approach uses a BMS-like symmetry of the horizon itself. This avoids the limitations of previous attempts (including my own), and explains the entropy in terms of a generalization of the Cardy formula for the density of states.</p>

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Black hole entropy
from BMS symmetry at the horizon

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The problem of universality: v1

$$S_{BH} = \frac{A_{hor}}{4\hbar G}$$

– for black holes, strings, rings, branes, Saturns, . . .
– in any dimension
– for any charges
– for any spins
– for “dirty” black holes with distorted horizons

Can change entropy by changing action, but the change is universal

The problem of universality: v2

Black hole entropy can be derived from:

- string theory
  - weakly coupled strings and branes
  - AdS/CFT
  - "fuzzballs"
- loop quantum gravity
  - real (tuned) Barbero-Immirzi parameter
  - self-dual formulation
  - spin networks *inside* the horizon
- entanglement entropy across the horizon
- induced gravity a la Sakharov
- instanton calculations
  - single instanton (Gibbons-Hawking)
  - pair production
- topology with no reference to local states
- QFT with no reference to quantum gravity

All have limitations, but all give the same answer . . .

Are we missing some deep structure?

Why universality?

$S \sim A_{hor} \overset{?}{\Rightarrow}$ degrees of freedom on the horizon

But why the universal factor of $1/4$?

Old(ish) idea: entropy is governed by a symmetry at the horizon

- Works for (2+1)-dimensional BTZ black hole
- Some progress for general dimensions

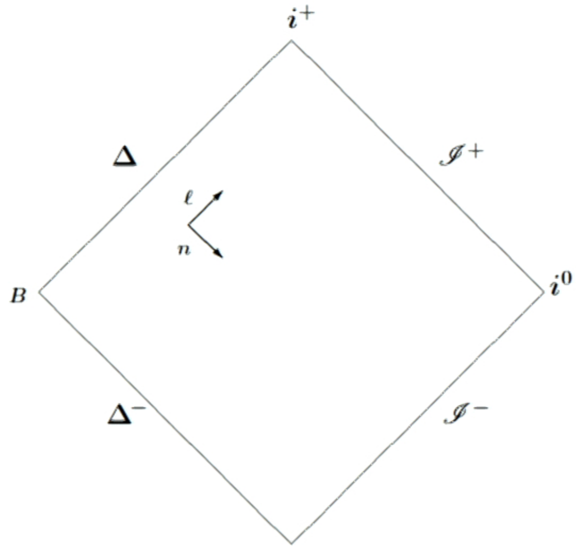
But. . .

- Methods usually require symmetry at infinity or at "stretched horizon"
- Parameters typically blow up at horizon
- Horizon limit is not always unique
- Approach typically fails badly for two-dimensional dilaton black hole
- Symmetry ought to depend only on null generators of horizon, but doesn't

New ingredients:

- Covariant phase space methods $\Rightarrow$ symmetry generators on horizon
- Near-horizon symmetry enhanced: conformal$\rightarrow$BMS

Two-dimensional dilaton gravity



Black hole exterior

Null dyad (ℓ, n), $\ell \cdot n = -1$

Normalization $\nabla_a n^a = 0$, $\nabla_a \ell^a = \kappa$

$g_{ab} = -\ell_a n_b - n_a \ell_b$

Cauchy surface $\Delta \cup \mathscr{I}^+$

$$I_{grav} = \frac{1}{16\pi G}\int_M (\varphi R + V[\varphi])\,\epsilon = \frac{1}{8\pi G}\int_M \left(-\kappa n^a \nabla_a \varphi + \frac{1}{2}V[\varphi]\right)\epsilon$$

$$E_{ab} = \nabla_a \nabla_b \varphi - g_{ab}\Box\varphi + \frac{1}{2} g_{ab} V = 8\pi G T_{ab}$$

Dilaton $\varphi \sim$ "transverse area"

Horizons and symmetries

Define $D = \ell^a \nabla_a$

Isolated horizon:

- $D\varphi \triangleq 0$ (vanishing expansion)
- $DR \triangleq 0$ (stationary geometry)
- conformal class of metric fixed at horizon [can be weakened]

Horizon-preserving diffeomorphisms:

$\delta_\xi \varphi = \xi D\varphi$
$\delta_\xi g_{ab} = -(D+\kappa)\xi\, g_{ab}$
with $n^a \nabla_a \xi \triangleq 0$

But also have an enhanced symmetry near the horizon

Near-horizon shift symmetry:

$$\hat{\delta}_\eta \varphi = \nabla_a(\eta \ell^a) = (D + \kappa)\eta$$
$$\hat{\delta}_\eta g_{ab} = (D\varphi)\delta\omega_\eta\, g_{ab}$$

with $n^a \nabla_a \eta \triangleq 0$

$$\delta_\eta I \sim \int \eta\, [A\, D\varphi + B\, DR]\, \epsilon$$

~ 0 if $\eta \to 0$ fast enough away from horizon

Equations of motion preserved up to terms $\sim \eta D\varphi$, except

$$\ell^a \ell^b \hat{\delta}_\eta T_{ab} \triangleq \frac{1}{8\pi G}(D - \kappa)D(D + \kappa)\eta$$

(standard conformal anomaly)

Covariant canonical formalism

Basic idea:

phase space (space of initial data) $\sim$ space of classical solutions

- can define symplectic structure on space of classical solutions
- generators of transformations are integrals over Cauchy surface

Simple example: point particle

$$L = (p\dot{q} - H)\, dt$$

$$\delta L = \textit{eqns of motion} + d\Theta \qquad \text{with } \Theta = p\delta q$$

$$\omega[\delta_1, \delta_2] = \delta_1 \Theta[\delta_2] - \delta_2 \Theta[\delta_1] = \delta_1 p\, \delta_2 q - \delta_2 p\, \delta_1 q \qquad \text{(symplectic form)}$$

Can generalize to field theory: integrate over Cauchy surface

$$\omega[\delta_1, \delta_2] \to \Omega[\delta_1, \delta_2] = \int_\Sigma \omega[\delta_1, \delta_2]$$

Hamiltonian: for 1-parameter family of transformations δ_τ

$$\delta H[\tau] = \Omega[\delta, \delta_\tau] \quad \Leftrightarrow \quad \delta_\tau \Phi^A = (\omega^{-1})^{AB} \frac{\delta H[\tau]}{\delta \Phi^B}$$

(may not exist for all transformations)

Apply to dilaton gravity

Choose (partial) Cauchy surface $\Delta \cup \mathscr{I}^+$

$$\Omega_\Delta[(\varphi, g); \delta_1(\varphi, g), \delta_2(\varphi, g)] = \frac{1}{8\pi G}\int_\Delta [\delta_1\varphi\, \delta_2\kappa - \delta_1\varphi\, \delta_2\kappa]\, n_a$$

Symmetries δ_ξ and $\hat{\delta}_\eta$ are integrable: generators

$$L[\xi] = \frac{1}{8\pi G}\int_\Delta [\xi D^2\varphi - \kappa\xi D\varphi]\, n_a$$

$$M[\eta] = \frac{1}{8\pi G}\int_\Delta \eta\left(D\kappa - \frac{1}{2}\kappa^2\right) n_a$$

Algebra and state-counting

Use covariant canonical formalism to find symmetry algebra:

General formalism:

$$\{H[\tau_1], H[\tau_2]\} = \Omega[\delta_{\tau_1}, \delta_{\tau_2}]$$

For dilaton gravity:

$$i\{L_m, L_n\} = (m-n)L_{m+n}$$
$$i\{M_m, M_n\} = 0$$
$$i\{L_m, M_n\} = (m-n)M_{m+n} + c_{LM}m(m^2-1)\delta_{m+n,0}$$

with $c_{LM} = 1/4G$

This is a centrally extended BMS_3 (or Galilean Conformal) algebra

Cardy (1986):

For a unitary two-dimensional conformal field theory, density of states is determined by the central charge

Bagchi, Detournay, Fareghbal, and Simón:

This generalizes to a theory with a BMS_3 symmetry

For state with eigenvalues h_L and h_M of L_0 and M_0, entropy is

$$S \sim 2\pi h_L \sqrt{\frac{c_{LM}}{2h_M}}$$

For black hole, this gives

$$S_{BH} = \frac{\varphi_{hor}}{4\hbar G}$$

Correct Bekenstein-Hawking entropy!

What next?

- Generalize boundary conditions (allow conformal class of metric to vary)
- Lift explicitly to higher dimensions
- Look for hidden BMS symmetry in other derivations of black hole entropy
- Couple to matter?