Title: Evolution of Black Holes in Inflation

Date: Jul 25, 2017 11:00 AM

URL: http://pirsa.org/17070002

Abstract: <p>We present an analytic, perturbative solution that describes dynamical black holes in slow-roll inflation with a general potential. It is shown that the spacetime evolves quasi-statically through a sequence of quasi-Schwarzchild-deSitter metrics with time dependent cosmological constant and mass parameters, such that the cosmological constant is instanteously equal to the value of the scalar potential. The areas of the black hole and cosmological horizons each increase in time as the effective cosmological constant decreases, and the the factional area increase is proportional to the fractional change of the cosmological constant, times a geometrical factor. For black holes ranging in size from much smaller than to comparable to the cosmological horizon, the pre-factor varies from very small to order one. The change in the horizon areas happens in such a way that the first law of thermodynamics is obeyed between successive time steps.</p>

main Refs

0. Chadburn & Gregory
"Time Dep BHs & Scalar Hair"

1. Gregory, Kastor, J.T. "BH Thermo w/ dynamical Lambda" (ArXiv)

2. —— " —— "Evolution of BHs in Inflat." (WIP)

3. Dolan, Kastor, Kubiznak, JT, "Thermo. Vols & Isoperimetric Inequalities" (ArXiv)

4. McInerny, Satishchandran, J.T. "Cosmography of KNdS BHs & Isentropic Phase Transitions" (ArXiv)

"Evolution of BHs in Inflation"

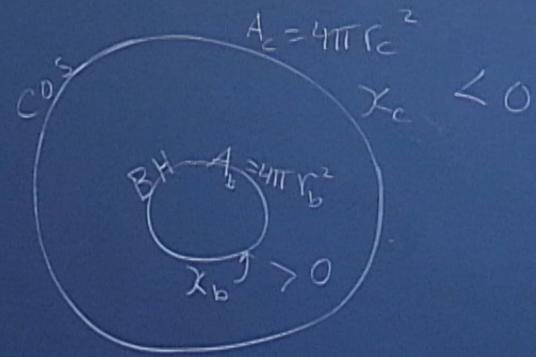
I. Static SdS

$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2$$

$$= \frac{\Lambda}{3}(r_c - r)(r - r_b)(r + r_b + r_c)$$

$$(M, \Lambda) \leftrightarrow (\Lambda, r_b)$$



$$\chi_h = \frac{1}{2} f'(r_h)$$

"Evolution of BHs in Inflation"

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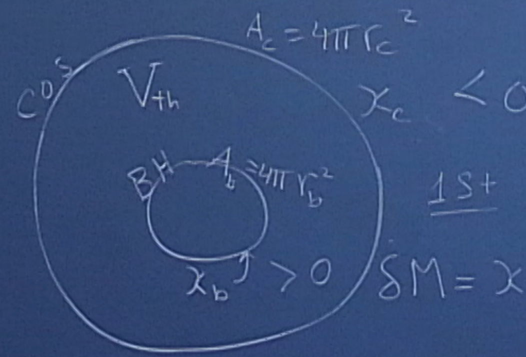

1st Law

$$\delta M = \chi_b \frac{\delta A_b}{8\pi} - V_{BH}\,\delta\Lambda$$

$$\chi_h = \frac{1}{2} f'(r_h)$$

"Evolution of BHs in Inflation"

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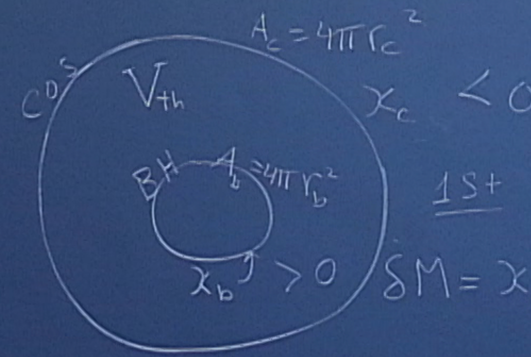

$$\chi_h = \frac{1}{2} f'(r_h)$$

1st Law

$$\delta M = \chi_b \frac{\delta A_b}{8\pi} - V_{BH}\,\delta\Lambda$$

$$+ V(-\delta P)$$

"Evolution of BHs in Inflation"

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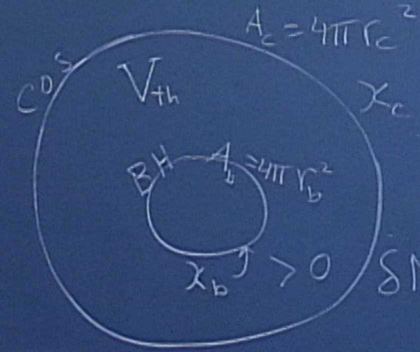
$$(M, \Lambda) \leftrightarrow (\Lambda, r_b)$$



1st Law

$$\delta M = \varkappa_b \frac{\delta A_b}{8\pi} - V_{BH}\,\delta\Lambda$$

$$\varkappa_b\,\delta A_b + |\varkappa_c|\,\delta A_c = -V_{th}\,\delta\Lambda$$

"Evolution of BHs in Inflation"

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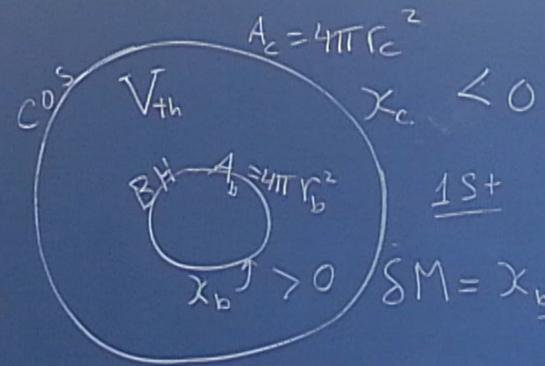
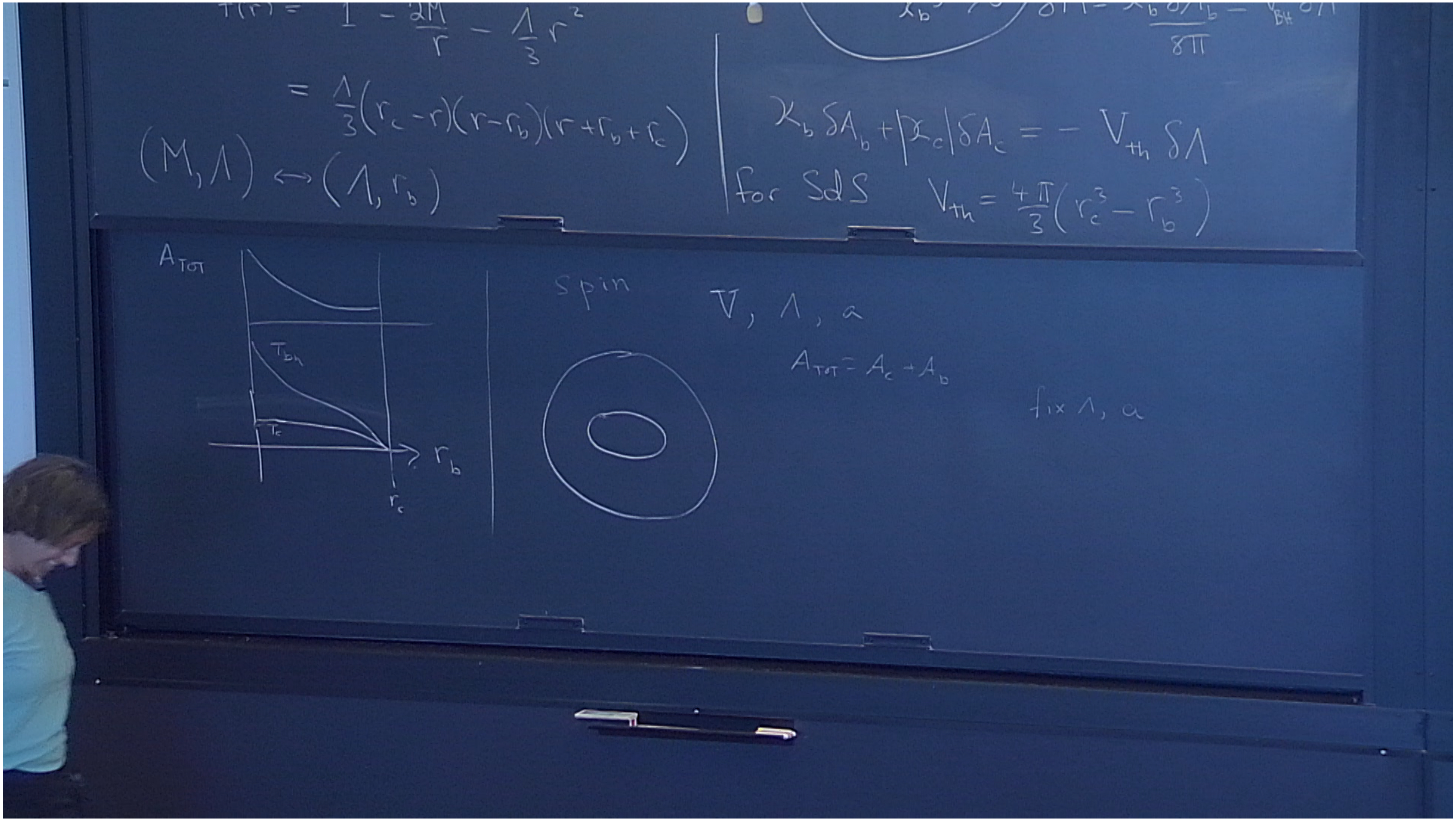


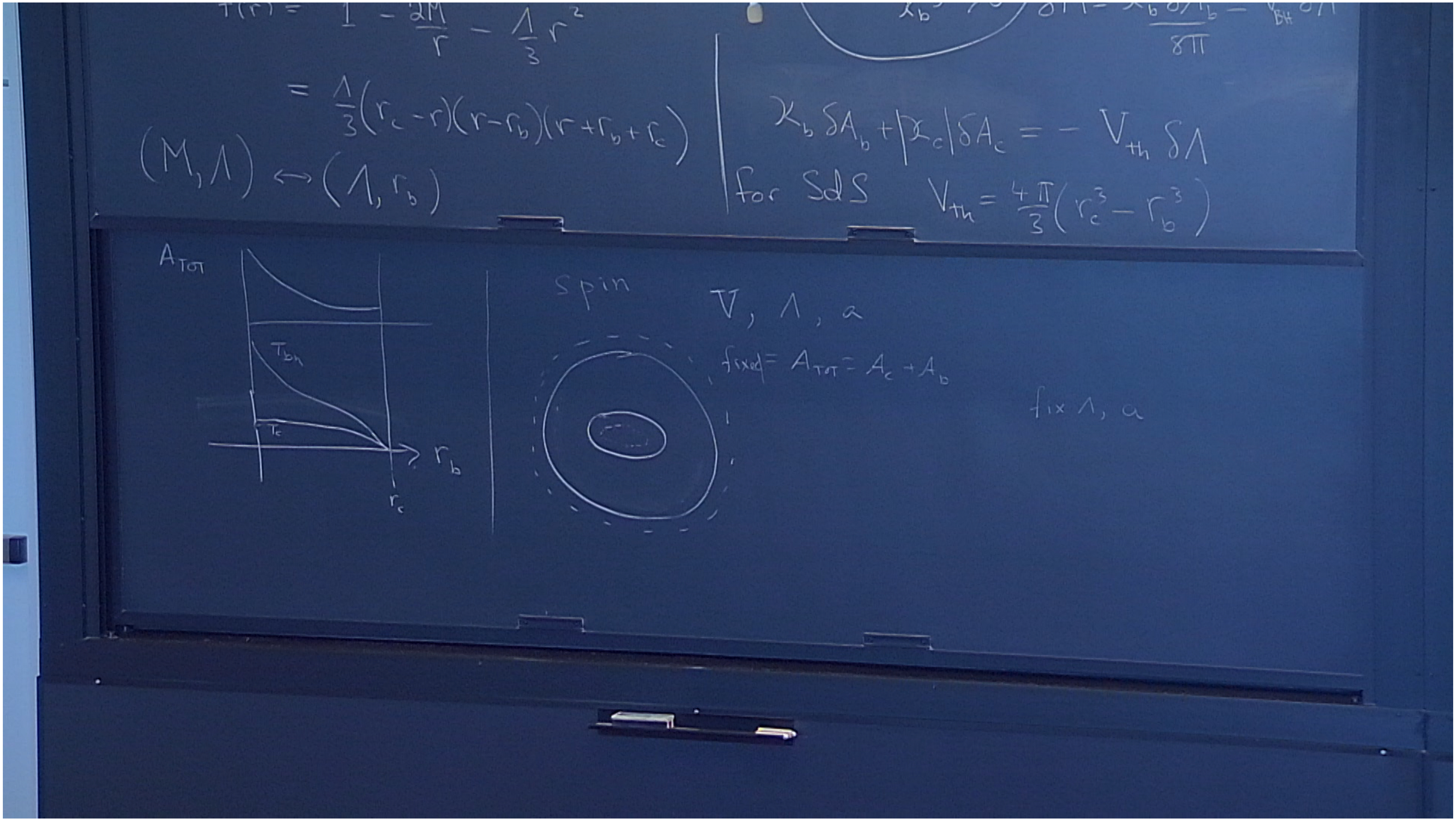
1st Law

$$\delta M = \kappa_b \frac{\delta A_b}{8\pi} - V_{BH}\,\delta\Lambda$$

$$\kappa_b\,\delta A_b + |\kappa_c|\,\delta A_c = -V_{th}\,\delta\Lambda$$

for SdS $\quad V_{th} = \frac{4\pi}{3}(r_c^3 - r_b^3)$


$= \frac{\Lambda}{3}(r_c - r)(r - r_b)(r + r_b + r_c)$
$(M, \Lambda) \leftrightarrow (\Lambda, r_b)$
$\kappa_b \delta A_b + |\kappa_c| \delta A_c = - V_{th} \delta \Lambda$
for SdS $V_{th} = \frac{4\pi}{3}(r_c^3 - r_b^3)$
A_{TOT}
T_{bh}
T_c
r_b
r_c
spin
V, Λ, a
$A_{TOT} = A_c + A_b$
fix Λ, a



= Λ/3 (r_c - r)(r - r_b)(r + r_b + r_c)
(M, Λ) ↔ (Λ, r_b)
8π
κ_b δA_b + |κ_c| δA_c = - V_th δΛ
for SdS V_th = 4π/3 (r_c^3 - r_b^3)
A_TOT
T_bh
T_c
r_b
r_c
spin
V, Λ, a
fixed = A_TOT = A_c + A_b
fix Λ, a

$$f(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2$$

$$= \frac{\Lambda}{3}(r_c - r)(r - r_b)(r + r_b + r_c)$$

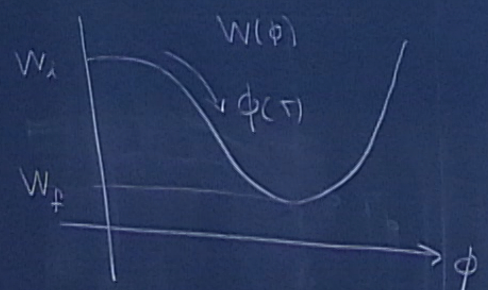
$$(M, \Lambda) \leftrightarrow (\Lambda, r_b)$$

$$\kappa_b \ldots \quad \delta M = \frac{\kappa_b \, \delta A_b}{8\pi} - V_{BH}\, \delta\Lambda$$

$$\kappa_b \, \delta A_b + |\kappa_c| \, \delta A_c = - V_{th} \, \delta \Lambda$$

for SdS $\quad V_{th} = \frac{4\pi}{3}(r_c^3 - r_b^3)$

II $\quad$ dS $\to$ dS w/ BH



a) Using techniques of (O.)

$f(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2$

$= \frac{\Lambda}{3}(r_c - r)(r - r_b)(r + r_b + r_c)$

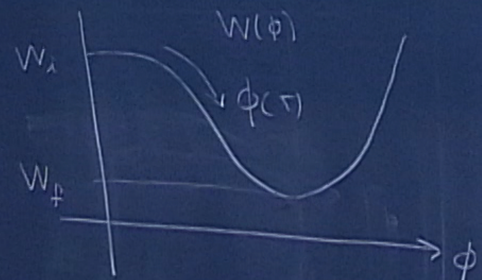
$(M, \Lambda) \longleftrightarrow (\Lambda, r_b)$

$\kappa_b \ldots \quad \delta M = \frac{\kappa_b \, \delta A_b}{8\pi} - V_{BH} \, \delta\Lambda$

$\kappa_b \, \delta A_b + |\kappa_c| \, \delta A_c = - V_{th} \, \delta\Lambda$

for SdS $\quad V_{th} = \frac{4\pi}{3}(r_c^3 - r_b^3)$

II $\quad dS \rightarrow dS \quad$ w/ BH



a) Using techniques of (O.)

$\Delta A_b = A_b^f - A_c^i \, , \quad \Delta A_c \propto |\Delta\Lambda| = W_i - W_f$

b) found that 1st Law Sat $i \longrightarrow f$

$\kappa_b \, \Delta A_b + |\kappa_c| \, \Delta A_c =$

$$f(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2$$

$$= \frac{\Lambda}{3}(r_c - r)(r - r_b)(r + r_b + r_c)$$

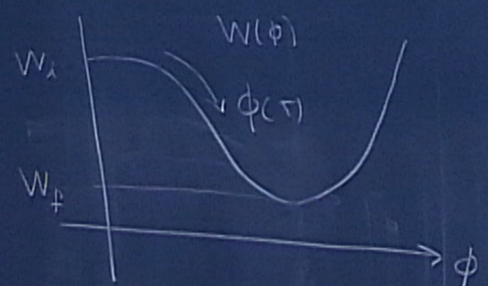
$$(M,\Lambda) \leftrightarrow (\Lambda, r_b)$$

$$\delta M = \frac{\kappa_b \delta A_b}{8\pi} - V_{BH}\,\delta\Lambda$$

$$\kappa_b \delta A_b + |\kappa_c| \delta A_c = - V_{th}\, \delta \Lambda$$

for SdS $V_{th} = \frac{4\pi}{3}(r_c^3 - r_b^3)$

II dS → dS w/ BH

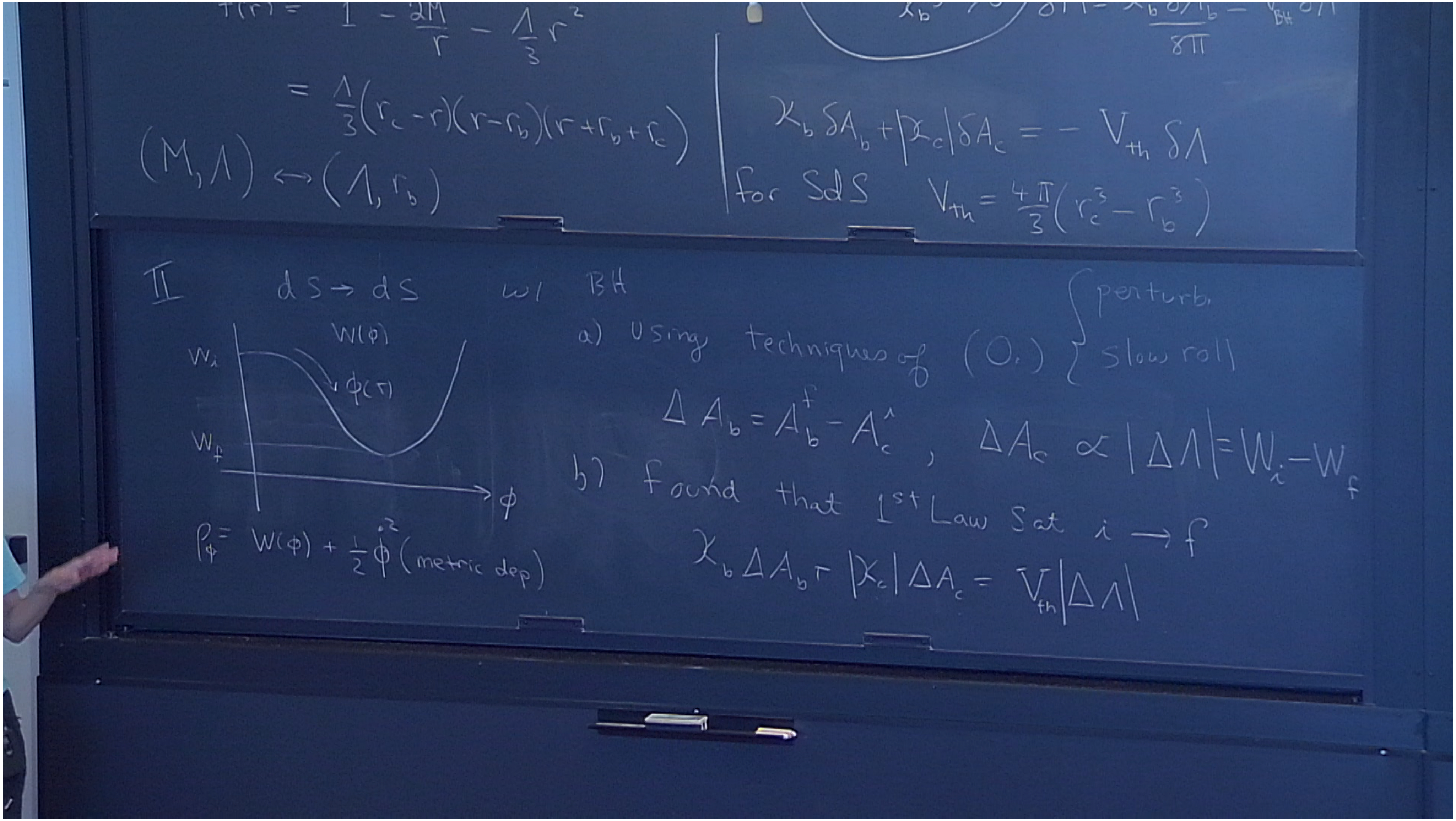


a) Using techniques of (O.) $\left\{ \begin{array}{l} \text{perturb.} \\ \text{slow roll} \end{array} \right.$

$$\Delta A_b = A_b^f - A_c^i, \quad \Delta A_c \propto |\Delta\Lambda| = W_i - W_f$$

b) found that 1st Law Sat $i \to f$

$$\kappa_b \Delta A_b + |\kappa_c| \Delta A_c = V_{th} |\Delta \Lambda|$$


$f(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2$
$= \frac{\Lambda}{3}(r_c - r)(r - r_b)(r + r_b + r_c)$
$(M, \Lambda) \leftrightarrow (\Lambda, r_b)$
$\kappa_b \delta A_b + |\kappa_c| \delta A_c = - V_{th} \delta\Lambda$
for SdS $V_{th} = \frac{4\pi}{3}(r_c^3 - r_b^3)$
II dS → dS w/ BH
$W(\phi)$
W_i
$\phi(\tau)$
W_f
ϕ
$\rho_\phi = W(\phi) + \frac{1}{2}\dot{\phi}^2$ (metric dep)
a) Using techniques of (O.) perturb. / slow roll
$\Delta A_b = A_b^f - A_c^i$, $\Delta A_c \propto |\Delta\Lambda| = W_i - W_f$
b) found that 1st Law Sat $i \to f$
$\kappa_b \Delta A_b + |\kappa_c| \Delta A_c = V_{th}|\Delta\Lambda|$

$$f(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2$$

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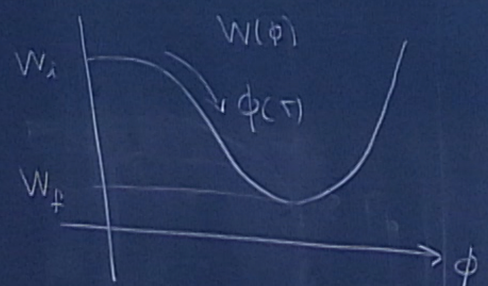
$$(M, \Lambda) \longleftrightarrow (\Lambda, r_b)$$

$$\delta M = \frac{\kappa_b \, \delta A_b}{8\pi} - V_{BH} \, \delta\Lambda$$

$$\kappa_b \, \delta A_b + |\kappa_c| \, \delta A_c = - V_{th} \, \delta\Lambda$$

for SdS $\quad V_{th} = \frac{4\pi}{3}\left(r_c^3 - r_b^3\right)$

II $\quad dS \to dS \quad$ choose $W(\varphi)$, r_b^i, slow roll $\varphi(T)$, perturb Einst



$$\rho_\phi = W(\phi) + \frac{1}{2}\dot{\phi}^2 \ (\text{metric dep})$$

a) Infl, No BH, $\varphi(T)$, $W(\varphi)$

$$\boxed{\Lambda(T) \simeq W(\varphi(T))} \ ?$$

b) Infl, with BH is true

And $M(T)$

BHs & Isentropic Phase Transitions" (ArXiv)

2. ——— " ——— "Evolution of BHs in Inflat.
(WIP)

3. Dolan, Kastor, Kubiznak, JT, "Thermo. Vols & Isoperimetric Inequalities" (ArXiv)

III. Metric

a) whose time?

choose $T = t + h(r)$

s.t. (1) slow-roll ($\ddot{\phi}$ = h.o.t.)

$\exists$ sols $\varphi(T)$ in SdS

(2) φ in-going at BH, out ... cos

"Evolution of BHs in Inflation"

$T \to \begin{cases} \text{in} & \text{BH} \\ \text{out} & \cos \end{cases}$

I. Static SdS

$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2 = -dT^2 f + 2\eta\, dT\, dr + \frac{(1-\eta^2)}{f} dr^2 + r^2 d\Omega^2$$

$$f_{SdS}(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2$$

$$\eta_{SdS} = -\gamma r + \beta / r^2 \to \begin{cases} 1 & \text{BH} \\ -1 & \text{cosm.} \end{cases}$$

"Evolution of BHs in Inflation"

$T \to \begin{cases} \text{in} & \text{BH} \\ \text{out} & \text{cos}; \; \varphi = \varphi(T) \end{cases}$

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BHs & Isentropic Phase Transitions" (ArXiv)

2. ——— " ——— " Evolution of BHs in Inflat. (WIP)

3. Dolan, Kastor, Kubiznak, JT, "Thermo. Vols & Isoperimetric Inequalities" (ArXiv)

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s.t. (1) slow-roll ($\ddot{\phi}$ = h.o.t.)

$\exists$ sols $\varphi(T)$ in SdS

(2) φ in-going at BH, out ... cos

$$-3\gamma\dot{\varphi} = \frac{\partial W}{\partial \varphi}$$

"Evolution of BHs in Inflation"

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$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2 = -dT^2 f + 2\eta\, dT\, dr + \frac{(1-\eta^2)}{f} dr^2 + r^2 d\Omega^2$$

with $\Lambda(T) \simeq W(T)$, $\varphi = \varphi(T)$

$$f_{SdS}(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2$$

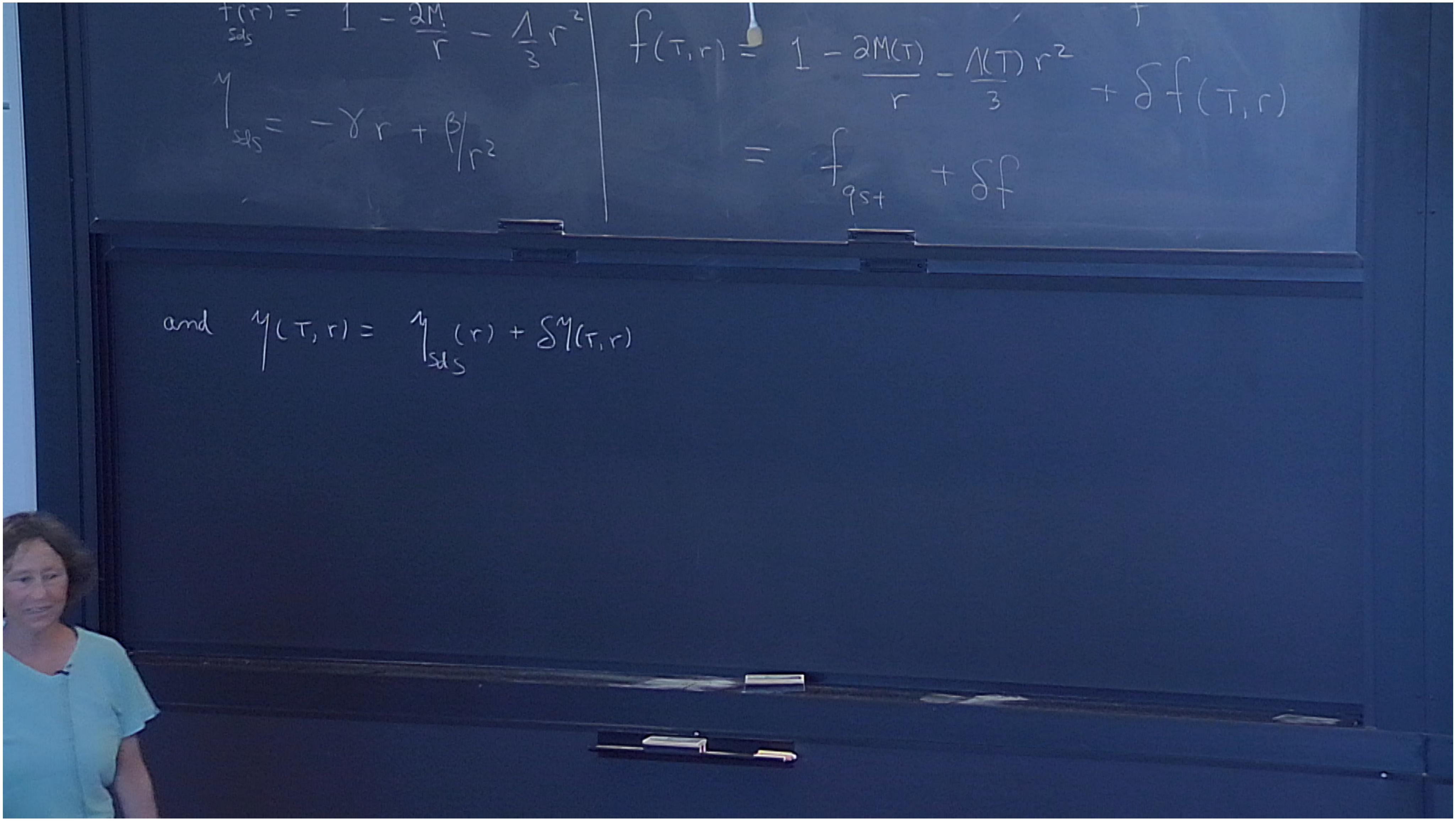
$$\eta_{SdS} = -\gamma r + \beta/r^2$$

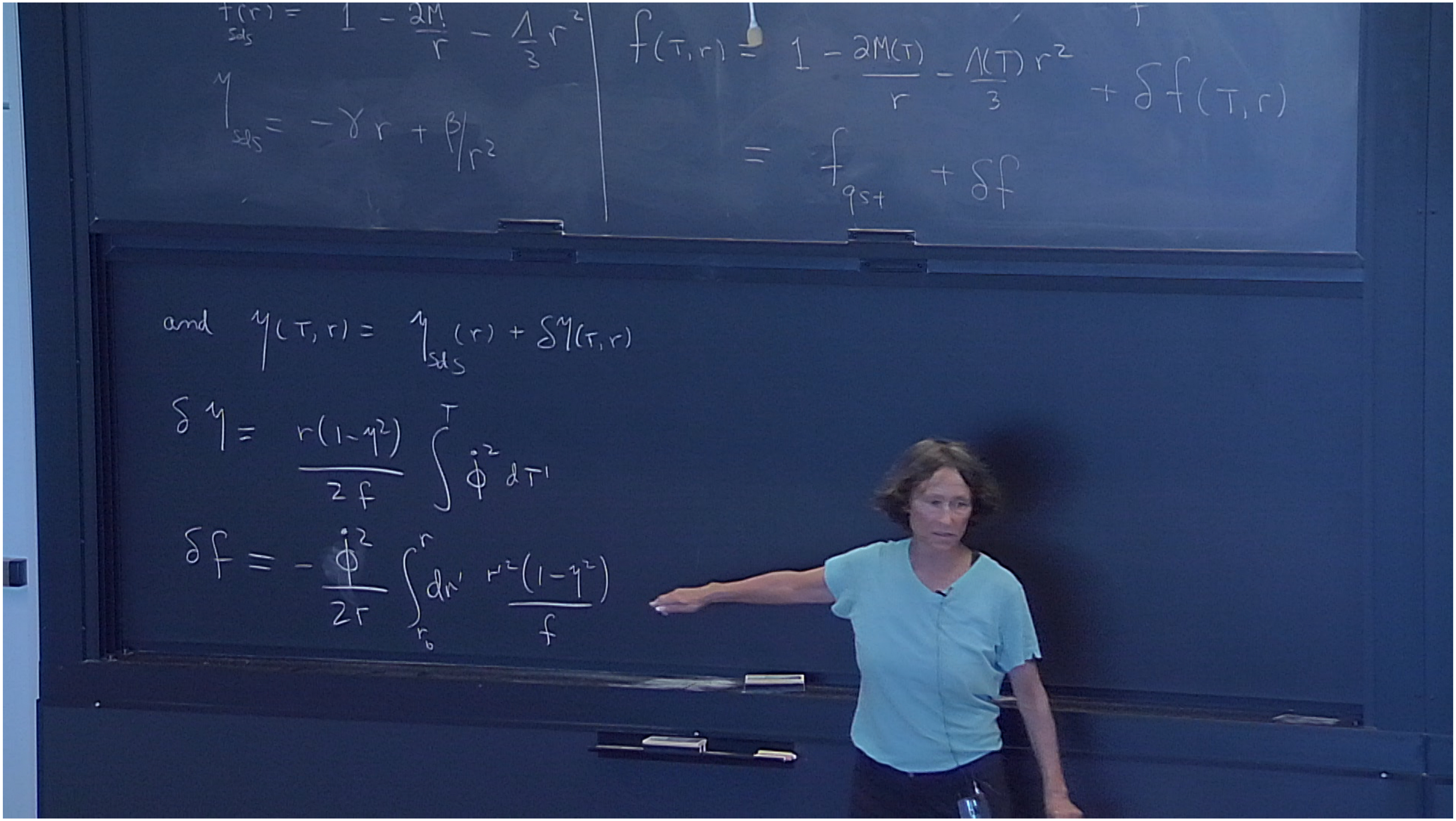
$$f(T,r) = 1 - \frac{2M(T)}{r} - \frac{\Lambda(T)}{3} r^2 + \delta f(T,r)$$

$$= f_{qst} + \delta f$$

$= W(\varphi)$, r_b^i, slow roll $\varphi(T)$, perturb Einst

a) Infl, No BH, $\varphi(T)$, $W(\varphi)$


$f_{SdS}(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2$
$\eta_{SdS} = -\gamma r + \beta / r^2$
$f(T,r) = 1 - \frac{2M(T)}{r} - \frac{\Lambda(T)}{3} r^2 + \delta f(T,r)$
$= f_{qst} + \delta f$
and $\eta(T,r) = \eta_{SdS}(r) + \delta\eta(T,r)$


$f_{SdS}(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2$
$\eta_{SdS} = -\gamma r + \beta / r^2$
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and $\eta(T,r) = \eta_{SdS}(r) + \delta\eta(T,r)$
$\delta\eta = \frac{r(1-\eta^2)}{2f} \int^{T} \dot{\phi}^2 \, dT'$
$\delta f = -\frac{\dot{\phi}^2}{2r} \int_{r_0}^{r} dr' \, \frac{r'^2(1-\eta^2)}{f}$

$$f_{sds}(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2$$

$$\eta_{sds} = -\gamma r + \beta / r^2$$

$$f(T,r) = 1 - \frac{2M(T)}{r} - \frac{\Lambda(T)}{3} r^2 + \delta f(T,r)$$

$$= f_{qst} + \delta f$$

and $\eta(T,r) = \eta_{sds}(r) + \delta\eta(T,r)$

$$\delta\eta = \frac{r(1-\eta^2)}{2f} \int^T \dot{\phi}^2 \, dT'$$

$$\delta f = -\frac{\dot{\phi}^2}{2r} \int_{r_b}^{r} dr' \, \frac{r'^2(1-\eta^2)}{f}$$

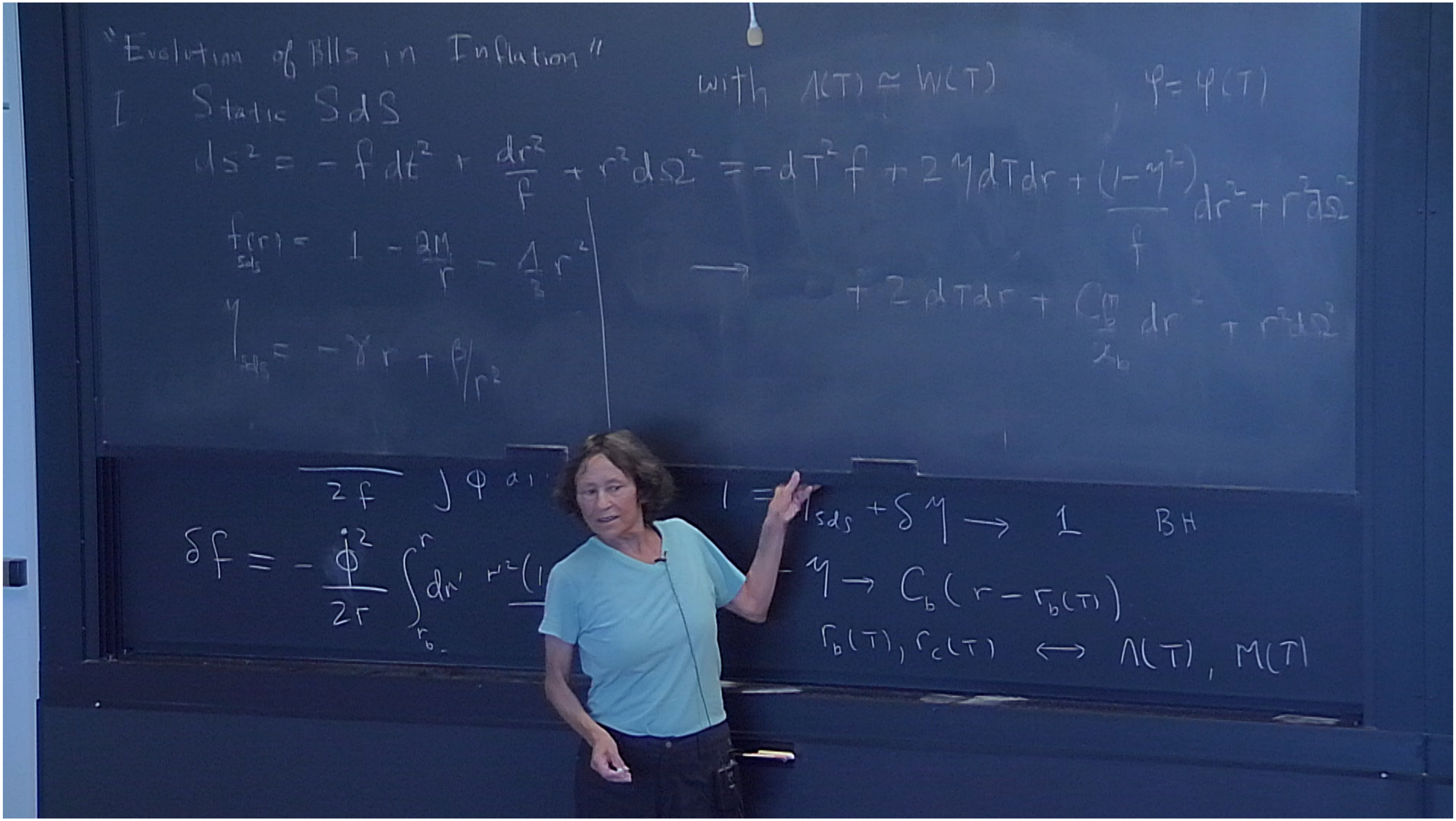
$$2\dot{M} = \beta \dot{\phi}^2, \quad \dot{\Lambda} = -3\gamma \dot{\phi}$$

or $\Lambda = W(\varphi)$

$\eta = \eta_{sds} + \delta\eta \to 1$ BH

$1 - \eta \to C_b(r - r_b(T))$

$r_b(T), r_c(T) \longleftrightarrow \Lambda(T), M(T)$


"Evolution of BHs in Inflation"
I. Static SdS
with Λ(T) ≃ W(T)
φ = φ(T)
ds² = − f dt² + dr²/f + r²dΩ² = − dT² f + 2η dT dr + (1−η²)/f dr² + r²dΩ²
f_SdS(r) = 1 − 2M/r − Λ/3 r²
η_SdS = − γr + β/r²
→ + 2 dT dr + C_b dr² + r²dΩ²
δf = − φ̇²/2r ∫ dr' r²(
η = η_SdS + δη → 1 BH
η → C_b (r − r_b(T))
r_b(T), r_c(T) ⟷ Λ(T), M(T)

$$f_{SdS}(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2$$

$$\eta_{SdS} = -\gamma r + \beta/r^2$$

$$\longrightarrow \quad + 2\, dT\, dr + \frac{C_b(T)}{\chi_b} dr^2 + r^2 d\Omega^2$$

and $\eta(T,r) = \eta_{SdS}(r) + \delta\eta(T,r)$

$$\delta\eta = \frac{r(1-\eta^2)}{2f} \int^T \dot{\phi}^2\, dT'$$

$$\delta f = -\frac{\dot{\phi}^2}{2r} \int_{r_b}^{r} dr' \, \frac{r'^2(1-\eta^2)}{f}$$

$$2\dot{M} = \beta\dot{\phi}^2, \quad \dot{\Lambda} = -3\gamma\dot{\phi} \longrightarrow$$

$$\Lambda = W(\varphi)$$

$$\eta = \eta_{SdS} + \delta\eta \to 1 \quad \text{BH}$$

$$1 - \eta \to C_b(r - r_b(T))$$

$$r_b(T), r_c(T) \longleftrightarrow \Lambda(T), M(T)$$

BHs & Isentropic Phase Transitions" (ArXiv)

2. ——— " ——— " Evolution of BHs in Inflat. (WIP)

3. Dolan, Kastor, Kubiznak, JT, "Thermo. Vols & Isoperimetric Inequalities" (ArXiv)

$$\Rightarrow \quad \dot{r}_h = \frac{r_h}{2|x_h|}\dot{\phi}^2 \; ; \quad \delta A_h(T) = \frac{A_h}{|x_h|}\frac{V_{Th}}{A_{TOT}}\Big[W_i - W(\varphi(T)) \Big]$$

$$\Rightarrow \quad x_b \dot{A}_b + |x_c|\dot{A}_c = -V_{th}\dot{\Lambda}$$

BHs & Isentropic Phase Transitions" (ArXiv)

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$$\Rightarrow \dot{r}_h = \frac{r_h}{2|x_h|}\dot{\phi}^2 \; ; \quad \delta A_h(T) = \frac{A_h}{|x_h|}\frac{V_{Th}}{A_{TOT}}\left[W_i - W(\varphi(T)) \right]$$

$$\Rightarrow x_b \dot{A}_b + |x_c|\dot{A}_c = -V_{th}\dot{\Lambda} \quad \Bigg| \quad \frac{\delta A_b}{A_b} \sim \frac{|\Delta\Lambda|}{\Lambda} \cdot \begin{cases} (r_b\sqrt{\Lambda}) & \text{small} \\ 1 & \text{large} \end{cases}$$