Title: Spekkensâ€™ toy model in all dimensions and its relationship with stabiliser quantum mechanics

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Abstract: <p>In this talk I am going to describe Spekkensâ€™ toy model, a non-contextual hidden variable model with an epistemic restriction, a constraint on what an observer can know about reality. The aim of the model, developed for continuous and discrete prime degrees of freedom, is to advocate the epistemic view of quantum theory, where quantum states are states of incomplete knowledge about a deeper underlying reality. In spite of its classical flavour, many aspects that were thought to belong only to quantum mechanics can be reproduced in the model. </p>

<p>I am going to describe our results regarding the formulation of rules for the update of states after measurement. I will do it for systems of discrete prime dimensions and I will then give the idea on how to proceed in the non-prime dimensional case. </p>

<p>I will also depict the relationship between Spekkensâ€™ model, stabiliser quantum mechanics and Gross' theory of discrete Wigner functions (they are equivalent theories in odd dimensions) in terms of measurement update rules.</p>

<p>I will conclude by briefly discussing a project we have been recently working on that consists of characterising sub theories of Spekkensâ€™ model that are operationally equivalent to sub theories of QM (in particular in the case of qubits) and use them to represent the non-contextual classically simulable part of state-injection schemes of computation with contextuality as a resource.</p>

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Spekkens' toy model in all dimensions and its relationship with stabiliser quantum mechanics

arXiv:1701.07801 (2017).

Lorenzo Catani, Dan Browne

University College London

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Contents

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- Motivations
- Introduction to Spekkens' toy model
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  - General case
- Measurement update rules
  - Prime dimensions
  - Non-prime dimensions
- Relationship with stabiliser quantum mechanics
- Spekkens' sub-theories compatible with QM
- Conclusions
- Future directions

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Motivations

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- What does a quantum state describe?

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Motivations

- What does a quantum state describe?
- Classify the inherent non-classical features.

Spekkens toy model

Model to support the epistemic view of QT.

Quantum mechanics $\stackrel{?}{=}$ *Classical theory + epistemic restriction*

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Elementary system: 1 bit

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Epistemic restriction

Knowledge balance principle:
When the observer has the maximum knowledge about reality, his amount of knowledge has to equal his ignorance.

↓

The observer can at maximum answer one of the two questions.

"Evidence for the epistemic view of quantum states: A toy theory". *Phys Rev A* **75** (3): 032110 (2007).

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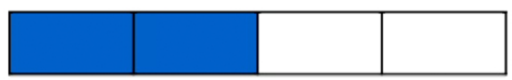
Elementary system: 1 bit

Ontic states = states of reality

Epistemic states = states of (incomplete) knowledge



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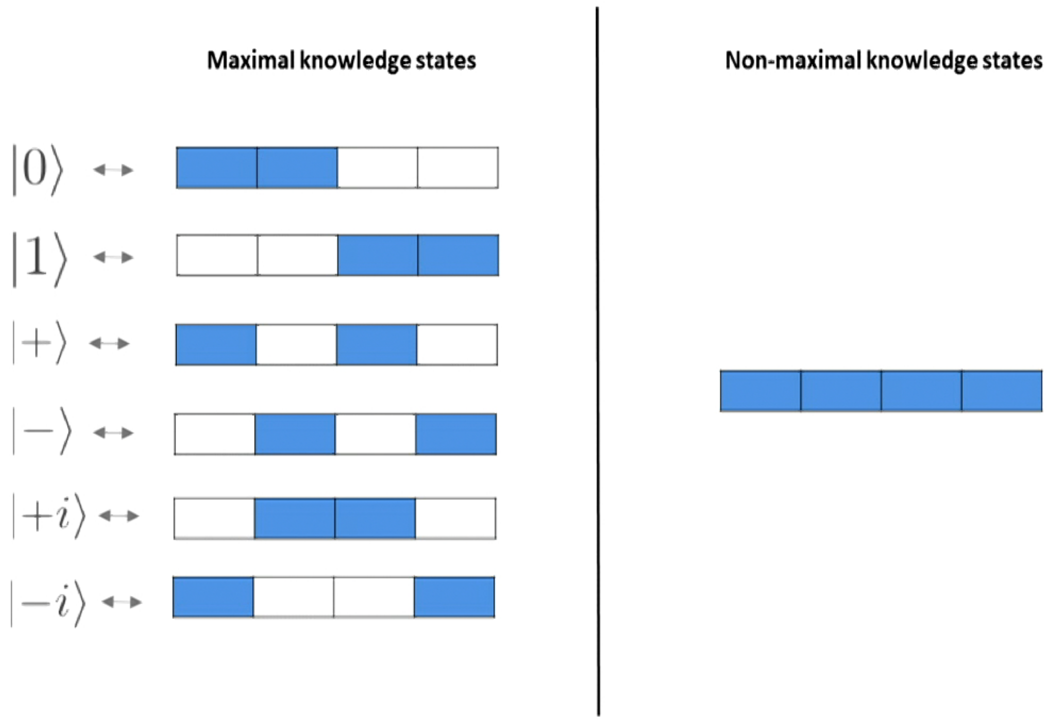
Epistemic states of one bit



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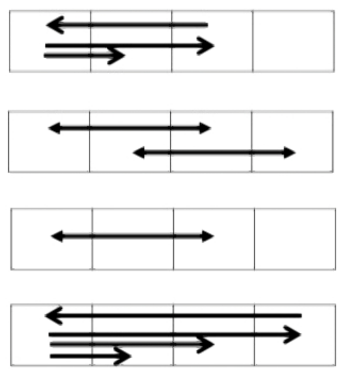
Transformations

Permutations of the ontic states



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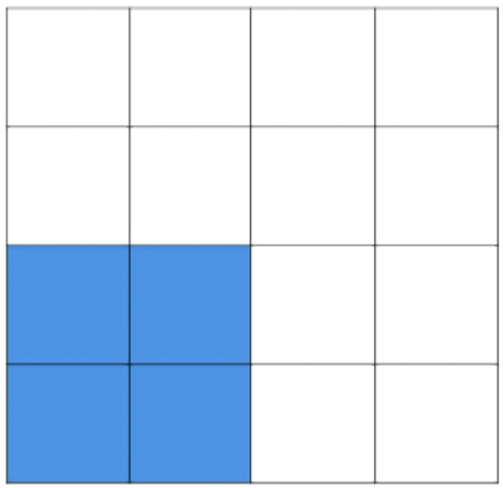
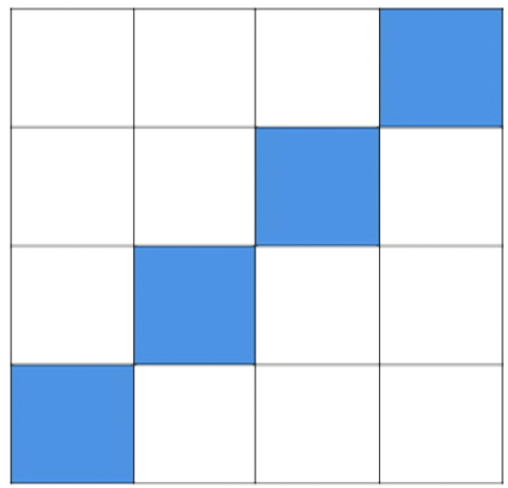
Epistemic states of couple of bits

Maximal knowledge states

Uncorrelated state



Perfectly correlated state



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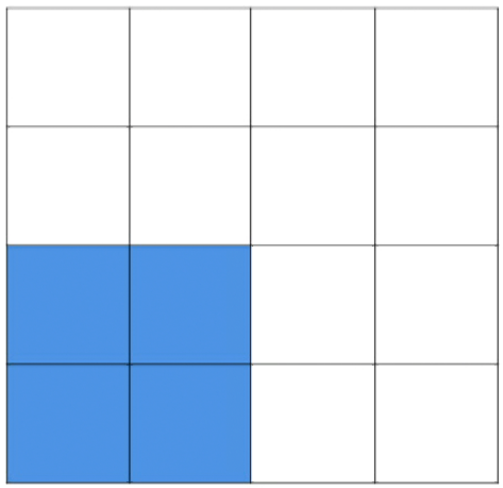
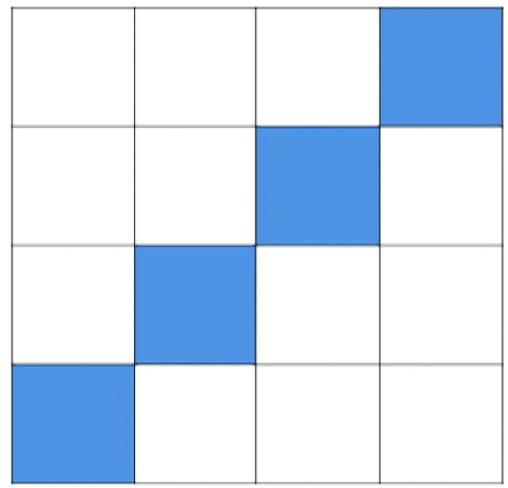
Epistemic states of couple of bits

Maximal knowledge states

Uncorrelated state



Perfectly correlated state



Maximum knowledge of the individual ontic states.

trade-off

Maximum knowledge of their relation.

No classical analogue

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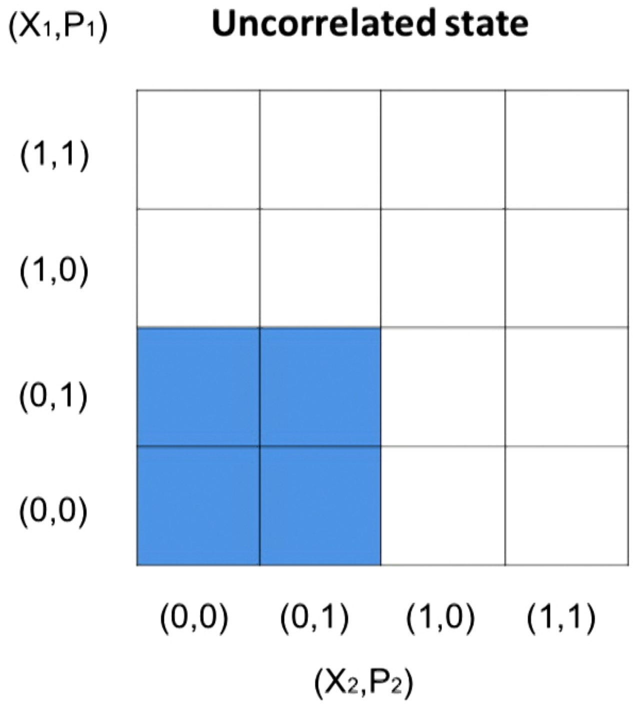
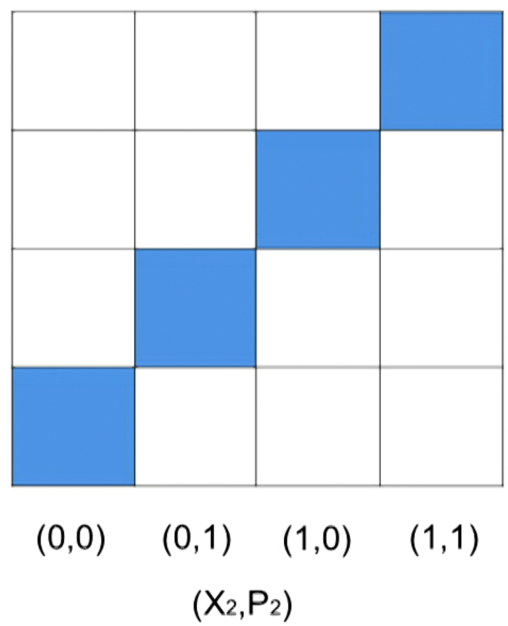
Epistemic states of couple of bits

Maximal knowledge states

Uncorrelated state



Perfectly correlated state



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Phase-space formalism

- Phase space $\Omega \equiv \mathbb{Z}_d^{2n}$; d= dimension, n= number of systems.
- Quadrature variables X_j, P_j .
- Ontic states $\lambda \in \Omega$. ⟶ Vectorial representation: $\lambda = (x_0, p_0, \ldots, x_{n-1}, p_{n-1})$.
- Observables $\Sigma = \sum_m (a_m X_m + b_m P_m)$. ⟶ Vec. representation: $\mathbf{\Sigma} = (a_0, b_0, \ldots, a_{n-1}, b_{n-1})$.
- Outcomes $\sigma = \mathbf{\Sigma}^T \lambda = \sum_j (a_j x_j + b_j p_j)$. Inner product.

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Epistemic restriction

Classical complementarity (C.C.) principle

The observer can at maximum jointly know the values of a set of variables that Poisson commute.

Poisson commute means that the symplectic inner product is zero:

$\langle \mathbf{\Sigma_1}, \mathbf{\Sigma_2} \rangle \equiv \mathbf{\Sigma_1}^T J \mathbf{\Sigma_2} = 0$, where $J = \bigoplus_{j=1}^{n} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}_j$.

Quasi-quantization: classical statistical theories with an epistemic restriction, R. W. Spekkens Fund.TheoryPhys,181(2016)

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Epistemic states

- Defined by $(V, \mathbf{w})$.
- $V = span\{\Sigma_1, \ldots, \Sigma_n\} \subseteq \Omega$. Isotropic subspace of known variables.
- $\mathbf{w}$ $is\ such\ that$ $\mathbf{\Sigma}_j^T \mathbf{w} = \sigma_j$. Evaluation vector.
- The set of ontic states consistent with the epistemic state $(V, \mathbf{w})$ is $V^{\perp} + \mathbf{w}$.

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Spekkens' Toy Model

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Epistemic states

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- $\mathbf{w}$ *is such that* $\mathbf{\Sigma}_j^T \mathbf{w} = \sigma_j$. Evaluation vector.
- The set of ontic states consistent with the epistemic state $(V, \mathbf{w})$ is $V^\perp + \mathbf{w}$.
- The associated uniform probability distribution is

$$P_{(V,\mathbf{w})}(\lambda) = \frac{1}{d^n} \delta_{V^\perp + \mathbf{w}}(\lambda)$$

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Valid Transformations

Symplectic affine transformations: $\lambda \rightarrow S\lambda + \mathbf{a}$

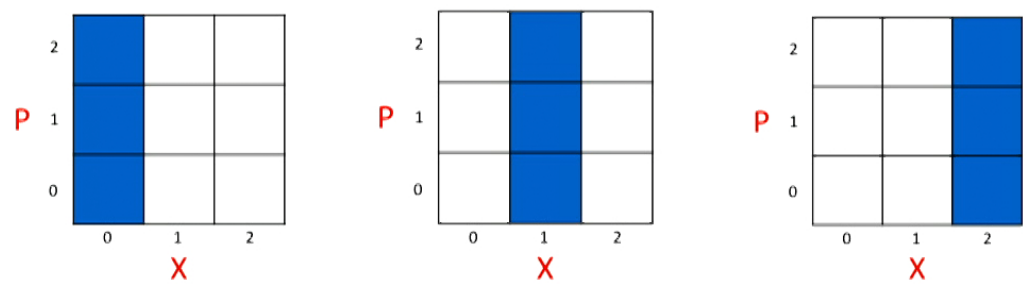
Epistemic restriction preserved: $(S\lambda)^T J(S\lambda') = \lambda^T J\lambda', \quad \forall\lambda, \lambda' \in \Omega\,.$

Associated probability distribution: $\Gamma_{S,\mathbf{a}}(\lambda|\lambda') = \delta_{S\lambda'+\mathbf{a}}(\lambda)\,.$

Valid Measurements

The elements of a measurement are represented as epistemic states.

Example: *One trit – measure of the observable* $\Pi = X$.



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Operational statistics

Probability of obtaining outcome $k \in V_\Pi$:

$$P(k) = \sum_{\lambda,\lambda' \in \Omega} P_{(V_\Pi,\mathbf{r}_k)}(k|\lambda)\Gamma_{(S,\mathbf{a})}(\lambda|\lambda')P_{(V,\mathbf{w})}(\lambda')$$

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Achievements

Phenomena arising in epistricted theories	Phenomena not arising in epistricted theories
Noncommutativity	Bell inequality violations
Coherent superposition	Noncontextuality inequality violations
Collapse	Computational speed-up (if it exists)
Complementarity	Certain aspects of items on the left
No-cloning	
No-broadcasting	
Interference	
Teleportation	
Remote steering	
Key distribution	
Dense coding	
Entanglement	
Monogamy of entanglement	
Choi-Jamiolkowski isomorphism	
Naimark extension	
Stinespring dilation	
Ambiguity of mixtures	
Locally immeasurable product bases	
Unextendible product bases	
Pre and post-selection effects	
Quantum eraser	
And many others...	

TABLE II: Categorization of quantum phenomena.

Quasi-quantization: classical statistical theories with an epistemic restriction, R. W. Spekkens Fund.TheoryPhys,181(2016)

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Result

Spekkens' model is *operationally equivalent* to SQM in odd dimensions.

↓

Proven through Gross' Wigner functions.

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Measurement update rules – prime case

State / Measurement / State after measurement

$$\underbrace{(V, \mathbf{w})}_{\text{State}}, \underbrace{(V_\Pi, \mathbf{r})}_{\text{Measurement}} \to \underbrace{(V', \mathbf{w}')}_{\text{State after measurement}}$$

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Measurement update rules – prime case

State — Measurement — State after measurement

$$\underbrace{(V, \mathbf{w})}_{\text{State}}, \underbrace{(V_\Pi, \mathbf{r})}_{\text{Measurement}} \rightarrow \underbrace{(V', \mathbf{w}')}_{\text{State after measurement}}$$

Classical complementarity implies that we can *learn* only the generators of the state that commute with the measurement.

⇓

State after measurement given by the generators of the measurement and the generators of the original state that commute with them.

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UCL

Adding/Removing generators

- Adding a generator Σ' to $V = span\{\Sigma_1, \Sigma_2, \ldots, \Sigma_n\}$:

$$V' = V \oplus span\{\Sigma'\} \longrightarrow \boxed{V'^{\perp} = V^{\perp} \cap (span\{\Sigma'\})^{\perp}}$$

- Removing a generator, say Σ_n, from $V = span\{\Sigma_1, \Sigma_2, \ldots, \Sigma_n\}$:

$$V' = span\{\Sigma_1, \ldots, \Sigma_{n-1}\}$$

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Measurement update rules – prime case

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$$V' = span\{\Sigma_1, \ldots, \Sigma_{n-1}\} \longrightarrow \boxed{V'^{\perp} = V^{\perp} \oplus V_n}$$

Proof.

$V'^{\perp}$ *given by all* λ *such that* $\boldsymbol{\Sigma}_j^T \lambda = 0$ *for all* $j < n$ *, but* $\boldsymbol{\Sigma}_n^T \lambda \neq 0.$

Need to add $\lambda' = c\gamma$ *to* $V^{\perp}$*, where* $c \in \mathbb{Z}_d \neq 0$ *and* γ *such that* $\Sigma_n^T \gamma = 1.$

$$\Sigma_n^T(\lambda + \lambda') = \Sigma_n^T(\lambda + c\gamma) = 0 + c \neq 0.$$

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Measurement update rules – prime case

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$$\Sigma_n^T(\lambda + \lambda') = \Sigma_n^T(\lambda + c\gamma) = 0 + c \neq 0.$$

$$V'^{\perp} = \bigcup_c (V^{\perp} + c\gamma) \equiv \bigcup_{w_n \in V_n} (V^{\perp} + w_n) = V^{\perp} \oplus V_n.$$

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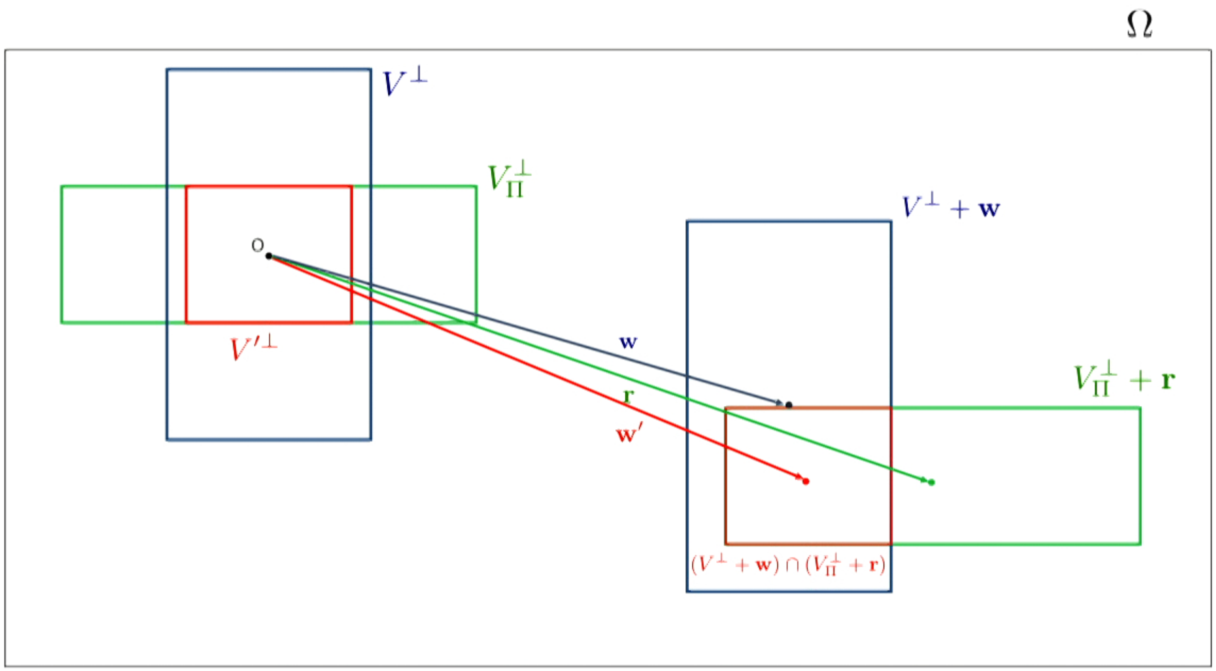
Commuting (non-disturbing) case

- $V'^{\perp} = (V^{\perp} + \mathbf{w} - \mathbf{w}') \cap (V_{\Pi}^{\perp} + \mathbf{r} - \mathbf{w}')$



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Measurement update rules – prime case

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Commuting (non-disturbing) case

- $V'^{\perp} = (V^{\perp} + \mathbf{w} - \mathbf{w}') \cap (V_{\Pi}^{\perp} + \mathbf{r} - \mathbf{w}')$

- $\mathbf{w}' = \mathbf{w} + \sum_i \mathbf{\Sigma}'^{T}_i (\mathbf{r} - \mathbf{w})\gamma_i$, $\quad \mathbf{\Sigma}'^{T}_i \gamma_i = 1$.

Proof.

Let us assume only one generator of the measurement, Σ', *whose associated outcome is* σ'.

Say $\mathbf{w}$ *not compatible with this outcome, then* $\mathbf{\Sigma}'^{T}\mathbf{w} = \sigma' + x$, *where* $x \in \mathbb{Z}_d$.

We want $\mathbf{w}'$ *such that* $\mathbf{\Sigma}'^{T}\mathbf{w}' = \sigma'$.

$\mathbf{w}' = \mathbf{w} - x\gamma,$ *where* $\Sigma'^{T}\gamma = 1$.

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Measurement update rules – prime case

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We want $\mathbf{w}'$ *such that* $\mathbf{\Sigma}'^{T}\mathbf{w}' = \sigma'$.

$\mathbf{w}' = \mathbf{w} - x\gamma$, *where* $\Sigma'^{T}\gamma = 1$.

Therefore $\mathbf{w}' = \mathbf{w} + (\sigma' - \mathbf{\Sigma}'^{T}\mathbf{w})\gamma = \mathbf{w} + \mathbf{\Sigma}'^{T}(\mathbf{r} - \mathbf{w})\gamma$.

In prime dimensions $\gamma = k^{-1}\mathbf{\Sigma}'$, where $k = \mathbf{\Sigma}'^{T}\mathbf{\Sigma}'$.

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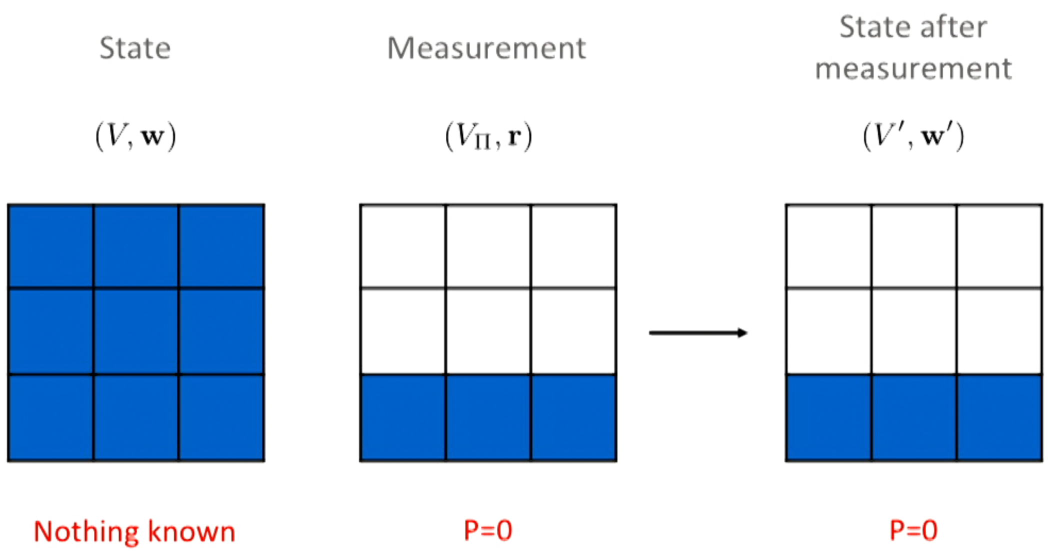
Measurement update rules – prime case

Commuting (non-disturbing) case

Simple example:



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Measurement update rules – prime case

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Non-commuting (disturbing) case

- $V'^{\perp} = (V^{\perp}_{commute} + \mathbf{w} - \mathbf{w}') \cap (V^{\perp}_{\Pi} + \mathbf{r} - \mathbf{w}')$

$$V^{\perp}_{commute} = V^{\perp} \oplus V_{other},$$

$$V = V_{commute} \oplus V_{other}.$$

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Measurement update rules – prime case

Recap and issues

Prime dimensions.	Non disturbing case	$V \to V' = V \oplus V_{\Pi}$ $V^{\perp} \to V'^{\perp} = V^{\perp} \cap V_{\Pi}^{\perp}$

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Recap and issues

Prime dimensions.

Non disturbing case

$$V \to V' = V \oplus V_{\Pi}$$

$$V^{\perp} \to V'^{\perp} = V^{\perp} \cap V_{\Pi}^{\perp}$$

Disturbing case

$$V^{\perp} \to V'^{\perp} = V_{commute}^{\perp} \cap V_{\Pi}^{\perp}$$

$$V_{commute}^{\perp} = \bigcup_{c} (V^{\perp} + c\gamma).$$

Updated shift vector

$$\mathbf{w} \to \mathbf{w}' = \mathbf{w} - x\gamma \qquad \Sigma'^{T}\gamma = 1$$

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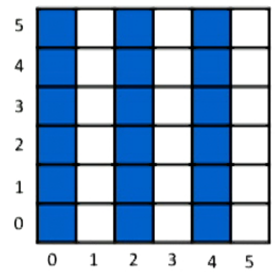
Problematic observables

Example

Epistemic state



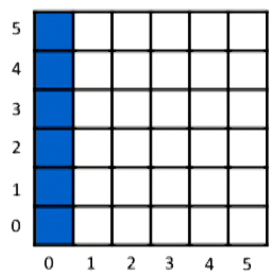
Coarse-graining observable

$$3X = 0$$

$$d = 6$$

$$X = 0$$

Fine-graining observables



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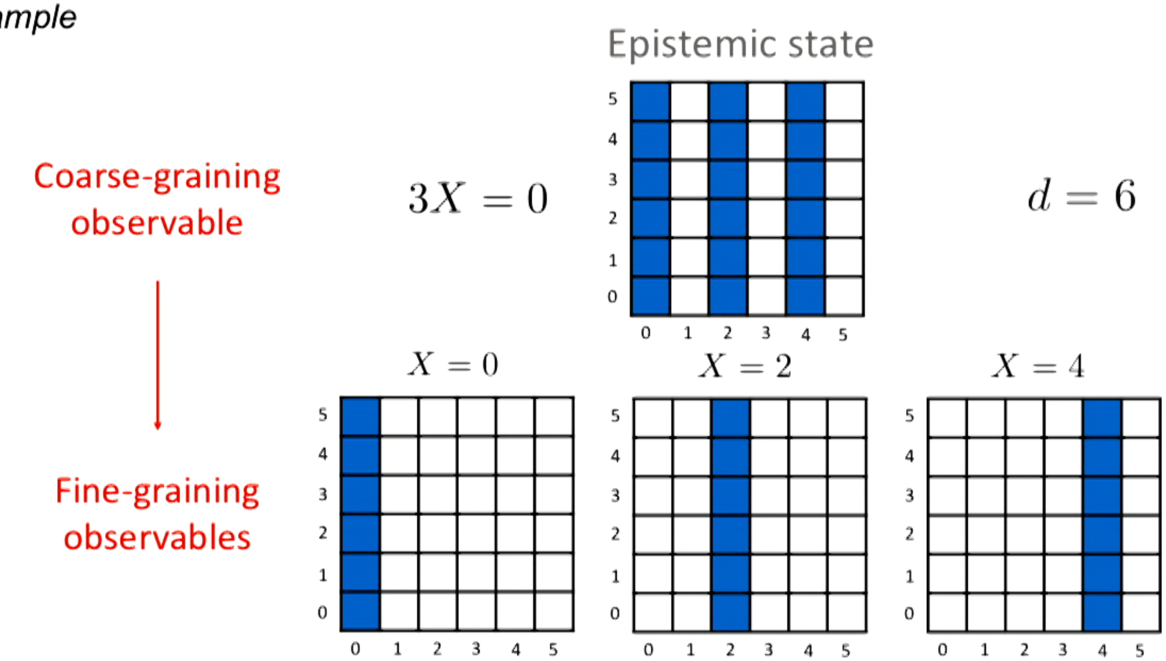
Problematic observables

Coarse graining observable = $O_{cg} = aX + bP = D(a'X + b'P)$, D shared by a, b.

Fine graining observables = $O_{fg} = a'X + b'P$ ⟷ γ exists.

Example



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States and Measurements

Stabilizer state = joint eigenstates of a set of commuting pauli operators.

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Stabilizer quantum mechanics

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States and Measurements

Stabilizer state = joint eigenstates of a set of commuting pauli operators.

↓

$$\rho = \rho_1 \cdot \rho_2 \cdots \rho_n \ , \qquad \rho_j = (\mathbb{I}_d + g_j + g_j^2 + \cdots + g_j^{d-1}).$$

↓

$$\rho \rightarrow \langle g_1, \ldots, g_n \rangle$$

Transformations

Clifford group (unitary representation of the symplectic affine group)

- Map Pauli to Pauli
- Preserve commuting relations

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Gross' theory

- Theorem (d=Odd) : *Pure state + non-negative W.f.* ⟷ *Stabilizer state.*

- Stabilizer Wigner function $W_\rho(\lambda) = Tr(\rho A(\lambda)) = \frac{1}{d^n}\delta_{M^C+\mathbf{w}}(\lambda)$.

Hudson's theorem for finite-dimensional quantum systems, D. Gross, J. Math. Phys. **47**, 122107 (2006).

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Equivalence of the theories

Wigner functions of stabilizer states = Spekkens epistemic states.

$$P_{(V,\mathbf{w})}(\lambda) = \frac{1}{d^n}\delta_{V^\perp+\mathbf{w}}(\lambda) \;=\; W_{(M,\mathbf{w})}(\lambda) = \frac{1}{d^n}\delta_{M^C+\mathbf{w}}(\lambda)$$

$$M = JV$$

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Measurement update rules

- Non-disturbing (commuting) case

$$V^{\perp} \to V'^{\perp} = V^{\perp} \cap V_{\Pi}^{\perp}$$

$$\mathbf{w} \to \mathbf{w}' = \mathbf{w} + k^{-1} \sum_{j} \boldsymbol{\Sigma}'^{T}_{j} (\mathbf{r} - \mathbf{w}) \boldsymbol{\Sigma}'_{j}$$

$\longleftrightarrow$

$$W_{\rho'}(\lambda) = \frac{1}{N} W_{\rho}(\lambda) R_{\Pi}(\lambda)$$

(Product rule)

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Equivalence of the theories – prime case

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	Non-disturbing Measurements (Localization stage) $[\rho, \Pi] = 0$	→	**Disturbing Measurements** (Localization + randomization stage) $[\rho, \Pi] \neq 0$
Stabilizer Quantum Mechanics	$\rho \to \langle g_1, \ldots, g_N \rangle$ $\Pi \to \langle p_1, \ldots, p_M \rangle$ Add generators ↓ $\rho' \to \langle g_1, g_2, \ldots, g_N, p_1, p_2, \ldots, p_M \rangle$		$\rho \to \langle g_1, \ldots, g_N \rangle$ $\Pi \to \langle p_1, \ldots, p_M \rangle$ Add generators ↓ Remove g_N $\rho' \to \langle g_1, g_2, \ldots, g_{N-1}, p_1, p_2, \ldots, p_M \rangle$
Spekkens Theory	$V' = V \oplus V_\Pi$ $V'^{\perp} = V^{\perp} \cap V_\Pi^{\perp}$ $\mathbf{w}' = \mathbf{w} + \sum_i^n \mathbf{\Sigma}'^T_i (\mathbf{r} - \mathbf{w})\gamma_i$		$V' = V_{commute} \oplus V_\Pi$ $V'^{\perp} = (V^{\perp} \oplus V_{other}) \cap V_\Pi^{\perp}$ $\mathbf{w}' = \mathbf{w} + \sum_i^n \mathbf{\Sigma}'^T_i (\mathbf{r} - \mathbf{w})\gamma_i$
Wigner Functions	$W_{\rho'}(\lambda) = \frac{1}{N} W_\rho(\lambda) R_\Pi(\lambda)$		$W_{\rho'}(\lambda) = \frac{1}{N} \sum_{\mathbf{t} \in V_{other}} W_\rho(\lambda - \mathbf{t}) R_\Pi(\lambda)$

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Equivalence of the theories

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	Prime dimensional systems	Non-prime dimensional systems
Spekkens Theory	$V'^{\perp} = (V^{\perp}_{commute} + \mathbf{w} - \mathbf{w}') \cap (V^{\perp}_{\Pi} + \mathbf{r} - \mathbf{w}')$ $\mathbf{w}' = \mathbf{w} + \sum_{i}^{n} \mathbf{\Sigma}'^{T}_{i} (\mathbf{r} - \mathbf{w})\gamma_i$	$V'^{\perp} = \bigcup_{j=0}^{\bar{D}-1} [(V^{\perp}_{commute} + \mathbf{w} - \mathbf{w}') \cap (V^{\perp}_{fg} + \mathbf{r}^{(j)}_{fg} - \mathbf{w}')]$ $\mathbf{w}' = \mathbf{w}'_j = \mathbf{w} + \sum_{i=0}^{n} \mathbf{\Sigma}'^{T}_{i} (\mathbf{r}^{(j)}_{fg} - \mathbf{w})\gamma_i$
Wigner Functions	$W_{\rho'}(\lambda) = \frac{1}{N} \sum_{\mathbf{t} \in V_{other}} W_{\rho}(\lambda - \mathbf{t}) R_{\Pi}(\lambda)$	$W_{\rho'}(\lambda) = \frac{1}{N} \frac{1}{\bar{D}} \sum_{\mathbf{t} \in V_{other}} \sum_{j=0}^{\bar{D}-1} W_{\rho}(\lambda - \mathbf{t}) R^{(j)}_{fg}(\lambda)$

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Basic idea

State-injection schemes (qubits/qudits):

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Spekkens' sub-theories

A set of q. states, transformations and measurements $(\mathcal{S}, \mathcal{T}, \mathcal{O})$ such that

i. Close subtheory. $\forall\, U \in \mathcal{T}, U\rho U^\dagger \in \mathcal{S}\ \forall \rho \in \mathcal{S}.$

ii. Spekkens representability. $\exists\, W_\rho, W_\pi, P_U \geq 0\ \ such\ that$,

$$W_\rho(\lambda) = \frac{1}{N} Tr(\rho A(\lambda)) = \frac{1}{N}\delta_{(V^\perp + \mathbf{w})}$$

$$W_\Pi(\mathbf{k}/\lambda') = \frac{1}{N'} Tr(\Pi_\mathbf{k} A(\lambda)) = \frac{1}{N'}\delta_{(V^\perp_{\Pi_k} + \mathbf{r})}(\lambda)$$

$$P_U(\lambda/\lambda') = \frac{1}{N''}\delta_{\lambda, S\lambda' + \mathbf{a}}$$

This exists if W_ρ is covariant.

$$p(\mathbf{k}|\rho, U, \Pi) = \frac{1}{N} Tr(\Pi_k U \rho U^\dagger) = \frac{1}{N}\sum_{\lambda\in\Omega} W_\Pi(\mathbf{k}/\lambda) \sum_{\lambda'\in\Omega} P_U(\lambda/\lambda') W_\rho(\lambda')$$

Maximal SS if it is not possible to add any other ρ, Π, U without contradicting i),ii).

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Conclusions and future directions

UCL

Conclusions

- Measurement update rules for Spekkens' theory, both prime and non-prime dimensional systems.
- Measurement update rules for Gross Wigner functions.
- Enforced the equivalence between Spekkens' theory and SQM in odd dimensions and depict the equivalence in terms of updating rules.
- What are the sub-theories of Spekkens' model that are compatible with QM (qubit)?
- Use Spekkens' sub-theories to represent the non-contextual cheap part of state-injection schemes of computation.

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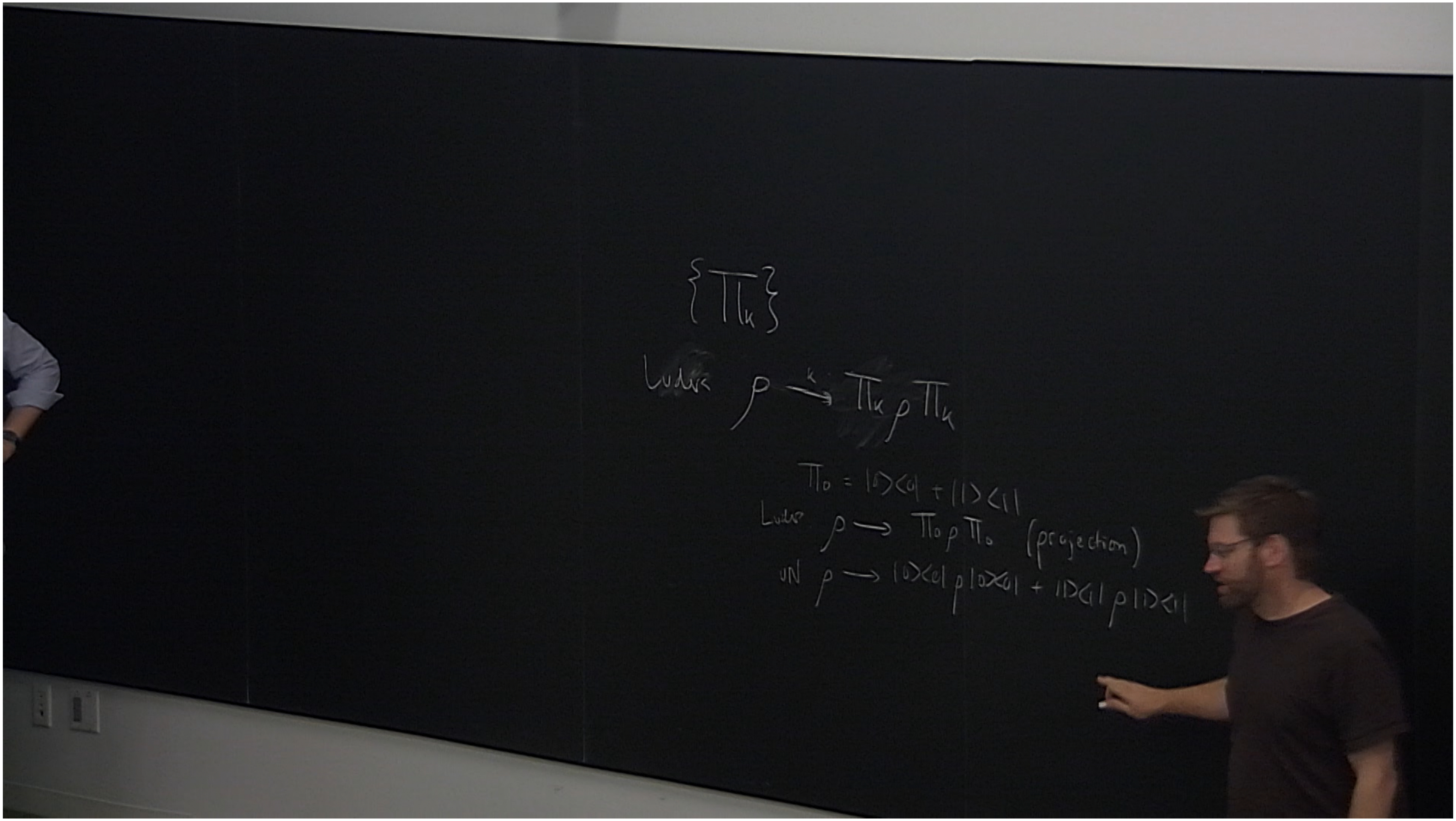
{Πk}
Lüders ρ → Πk ρ Πk
Π0 = |0⟩⟨0| + |1⟩⟨1|
Lüders ρ → Π0 ρ Π0 (projection)
vN ρ → |0⟩⟨0| ρ |0⟩⟨0| + |1⟩⟨1| ρ |1⟩⟨1|