Title: Spinor driven cosmic bounces and their (in)stability

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Abstract: Resolving the big bang singularity with a non-singular classical bounce usually requires the introduction of some sort exotic matter which violates the null-energy condition (NEC), such as a scalar field that undergoes ghost condensation, or models involving Galileon fields. In such models an NEC violating phase is not difficult to achieve on its own, but the situation becomes much more restrictive once observational and stability requirements are taken into consideration. In this talk I discuss whether a more desirable outcome might be achieved by making use of fermionic rather than scalar matter. In particular, I describe bouncing scenarios which arise naturally within the context of Einstein-Cartan-Holst gravity coupled to classical Dirac spinors. As I will show, it is relatively easy to construct backgrounds which not only undergo a bounce, but which also accommodate other interesting dynamics outside the bouncing phase, such as inflation or ekpyrosis. Unfortunately, things work less well when considering perturbations in such bouncing backgrounds as I explain within the context of the simplest models: the comoving curvature perturbation diverges as the moment of NEC violation is approached, and hence the models of greatest interest break down before reaching the bounce.

Spinor driven cosmic bounces and their (in)stability.

Shane Farnsworth
Collaboration with Jean-Luc Lehners and Taotao Qiu

Max Planck Institute for Gravitational Physics (Albert Einstein Institute),
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Perimeter Institute, June 2017

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Overview

1. Introduction
   - Motivation: Why a classical bounce and Why Spinors?
   - Preliminaries: Torsion and Spinors
2. Dynamics
   - The Action
   - Deriving the Equations of Motion
3. Background cosmology
   - Assumptions: Classical Spinors and Flat FRW
   - Bouncing Solutions, Ekpyrosis and Inflation
4. Stability
   - Linearised Equations of motion and the Co-moving Curvature Perturbation
   - Dynamical (In)stabilities
5. Conclusion and Open Questions

Introduction: Why a bounce and why spinors?

- **We need a theory of initial conditions or we need a bounce.** Bouncing cosmologies offer tantalizing alternatives to the standard inflationary scenario, offering solutions to the usual cosmological problems: flatness, horizon problem, etc...
- **Null energy condition violation requires exotic matter.** Gravity with torsion coupled to spinors is perhaps the most natural and minimal solution (see e.g. hep-th/0301129, 1105.6127, 1212.0585, 1111.4595, 1402.5880, 1406.1456...)

Preliminaries: Torsion, spinors, and all that.

In DG the covariant derivative is not unique:

$$\Gamma^{\gamma}{}_{\mu\nu} + \Theta^{\gamma}{}_{\mu\nu}$$

In GR we add two assumptions

$$\triangle_{\rho} g_{\mu\nu} = \partial_{\rho} g_{\mu\nu} - \Gamma^{\gamma}{}_{\rho\mu} g_{\gamma\nu} - \Gamma^{\gamma}{}_{\rho\nu} g_{\gamma\mu} = 0 \qquad \text{(Metric Compatibility)}$$

$$\Gamma^{\tau}_{\mu\nu} - \Gamma^{\tau}_{\nu\mu} = 0 \qquad \text{(Torsion Free)}$$

This gives us the Christoffel Symbols

$$\Gamma^{\gamma}{}_{\mu\nu} = \frac{1}{2} g^{\gamma m} (g_{m\mu,\nu} + g_{m\nu,\mu} - g_{\nu\mu,m})$$

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~~$\Gamma^{\tau}_{\mu\nu} - \Gamma^{\tau}_{\nu\mu} = 0$ (Torsion Free)~~

This gives us the Christoffel Symbols

$$\Gamma^{\gamma}{}_{\mu\nu} = \frac{1}{2} g^{\gamma m}(g_{m\mu,\nu} + g_{m\nu,\mu} - g_{\nu\mu,m}) + C^{\gamma}{}_{\mu\nu},$$

where $C^{\gamma}{}_{\mu\nu} = \frac{1}{2}(T_{\nu}{}^{\tau}{}_{\mu} + T_{\mu}{}^{\tau}{}_{\nu} + T^{\tau}_{\mu\nu})$, and $T^{\tau}{}_{\mu\nu} = \Gamma^{\tau}_{\mu\nu} - \Gamma^{\tau}_{\nu\mu}$.

Preliminaries: Torsion, spinors, and all that.

Some notation:

$$D\Psi = dx^{\mu} e^{I}_{\mu} (\partial_I - \tfrac{1}{8}\omega_{JKI}[\gamma^J, \gamma^K])\Psi$$

where

$$\omega_{JKI} = \widetilde{\omega}_{JKI} + C_{JKI}$$

So:

$$T^I = De^I = \widetilde{D}e^I + C^I{}_J e^J = C^I{}_J e^J = C^I{}_{JK} e^K e^J$$

Dynamics: The action

$$S = S_G + S_\Psi$$

Dynamics: The gravitational Action.

$$S_G = \kappa \int (\epsilon_{IJKL} + \frac{2}{\gamma}\eta_{I[K}\eta_{J]L})e^I e^J R^{KL}$$

- Einstein-Hilbert Term

Dynamics: The gravitational Action.

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- Einstein-Hilbert Term
- Holst Term

Dynamics: The matter action Action.

- Fermion Kinetic term

$$S_\psi = \frac{i}{2.3!}\int \epsilon_{IJKL}e^I e^J e^K(\overline{\Psi}\gamma^L D\Psi - \overline{D\Psi}\gamma^L\Psi)$$
$$+\frac{1}{4}\int (De^I)e^J e^K[\epsilon_{IJKL}(\alpha V^L + \beta A^L) + \eta_{I[K}\eta_{J]L}(\tau V^L + \lambda A^L)]$$
$$-\frac{1}{4!}\int \epsilon_{IJKL}e^I e^J e^K e^L U(\overline{\Psi}\Psi)$$

where:

$$V^L = \overline{\Psi}\gamma^L\Psi, \qquad A^L = \overline{\Psi}\gamma_5\gamma^L\Psi.$$

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- Potential
- Axial and Vector source term

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Deriving the equations of motion.

Variation with respect to the spin connection ω

$$2\kappa(\epsilon_{IJMN} + \tfrac{2}{\gamma}\eta_{I[M}\eta_{N]J})(De^I)e^J = -\tfrac{1}{4!}\epsilon_{IJKL}e^I e^J e^K \varepsilon^{DL}_{\bullet\bullet MN}A_D - \tfrac{1}{4}\epsilon_{[M|JKL}e^J e^K e_{|N]}\Theta^L - \tfrac{1}{4}e^J e^K e_{[N}\eta_{M]K}\eta_{JL}\Omega^L.$$

where:

$$\Theta^L = (\alpha V^L + \beta A^L), \qquad \Omega^L = (\tau V^L + \lambda A^L).$$

Deriving the equations of motion.

After a long and tedious calculation...

$$C^{TXS} = \frac{\gamma^2}{8\kappa(1+\gamma^2)}\Big[\tfrac{1}{2}\varepsilon^{QXST}\big(\tfrac{1}{\gamma}\Theta_Q - (A_Q + \Omega_Q)\big) + \eta^{S[T}\delta_A^{X]}\big(\Theta^A + \tfrac{1}{\gamma}(A^D + \Omega^D)\big)\Big].$$

Deriving the equations of motion.

Variation of the action with respect to the spinor $\overline{\Psi}$:

$$\tfrac{i}{3!}\epsilon_{IJKL}\varepsilon^{IJKM}\gamma^L\widetilde{D}_M\Psi = -\tfrac{1}{4}\epsilon_{IJKL}\varepsilon^{PQJK}C^I_{\bullet QP}\frac{\delta\Theta^L}{\delta\overline{\Psi}} - \tfrac{1}{4}\eta_{I[K}\eta_{J]l}\varepsilon^{PQJK}C^I_{\bullet QP}\frac{\delta\Omega^L}{\delta\overline{\Psi}}$$
$$+\tfrac{i}{8.3!}\epsilon_{IJKL}\varepsilon^{IJKM}C_{ABM}\gamma^L[\gamma^A,\gamma^B]\Psi + \frac{\delta U}{\delta\overline{\Psi}},$$

Deriving the equations of motion.

Once again, after a long calculation...

$$i\gamma^L \widetilde{D}_L \Psi = \frac{\delta W}{\delta \overline{\Psi}},$$

where:

$$W = U(E) - \frac{3\pi G\gamma^2}{2(1+\gamma^2)}\Big[A_I A^I\big(\tfrac{2}{\gamma}\beta(1+\lambda) + \beta^2 - (1+\lambda)^2\big) + V_I V^I\big(\tfrac{2}{\gamma}\alpha\tau + \alpha^2 - \tau^2\big) + 2V^I A_I\big(\alpha(\beta + \tfrac{1}{\gamma}(1+\lambda)) + \tau(\tfrac{1}{\gamma}\beta - (1+\lambda))\big)\Big].$$

Deriving the equations of motion.

Variation with respect to the vierbein e:

$$
\begin{aligned}
0 = {} & 2\kappa(\epsilon_{SJKL} + \tfrac{2}{\gamma}\eta_{SK}\eta_{JL})e^J(\widetilde{R}^{KL} + \widetilde{D}C^{KL} + C^K_{\ P}C^{PL}) \\
& + \tfrac{i}{4}\epsilon_{SJKL}e^J e^K X^L - \tfrac{1}{3!}\epsilon_{SJKL}e^J e^k e^L U + \tfrac{1}{4}\epsilon_{SJKL}e^J e^K D\Theta^L \\
& + \tfrac{1}{2}(\eta_{S[K}\eta_{J]L} + \eta_{J[K}\eta_{S]L})(De^J)\Omega^L + \tfrac{1}{4}\eta_{SK}\eta_{JL}e^J e^K D\Omega^L,
\end{aligned}
$$

Deriving the equations of motion.

After some calculation...:

$$4\kappa\widetilde{G}_{\mu\nu} = -\tfrac{i}{2}e_{L(\mu}\widetilde{X}^{L}_{\nu)} + \tfrac{i}{2}g_{\mu\nu}e^{\tau}_{L}\widetilde{X}^{L}_{\tau} - g_{\mu\nu}W$$
$$- \tfrac{i}{8}e_{L\mu}e_{N\nu}\left[\overline{\Psi}[\gamma^N,\gamma^L]\gamma^P\widetilde{D}_P\Psi + \overline{\widetilde{D}_P\Psi}[\gamma^N,\gamma^L]\gamma^P\Psi\right]$$

- Anti-symmetric terms zero on-shell

where $\widetilde{X}^{L}_{\tau} = (\overline{\Psi}\gamma^L\widetilde{D}_\tau\Psi - \overline{\widetilde{D}_\tau\Psi}\gamma^L\Psi)$.

Deriving the equations of motion.

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Deriving the equations of motion.

Putting it all together:

$$4\kappa\widetilde{G}_{\mu\nu} = -\tfrac{i}{2}e_{L(\mu}\widetilde{X}^{L}_{\nu)} + \tfrac{i}{2}g_{\mu\nu}e^{\tau}_{L}\widetilde{X}^{L}_{\tau} - g_{\mu\nu}W$$
$$i\gamma^{L}\widetilde{D}_{L}\Psi = \frac{\delta W}{\delta\overline{\Psi}},$$

where:

$$\widetilde{X}^{L}_{\tau} = (\overline{\Psi}\gamma^{L}\widetilde{D}_{\tau}\Psi - \overline{\widetilde{D}_{\tau}\Psi}\gamma^{L}\Psi),$$
$$W = U(E) - \frac{3\pi G\gamma^2}{2(1+\gamma^2)}\big[A_I A^I(\tfrac{2}{\gamma}\beta(1+\lambda) + \beta^2 - (1+\lambda)^2) + V_I V^I(\tfrac{2}{\gamma}\alpha\tau + \alpha^2 - \tau^2) + 2V^I A_I(\alpha(\beta + \tfrac{1}{\gamma}(1+\lambda)) + \tau(\tfrac{1}{\gamma}\beta - (1+\lambda)))\big].$$

Questions?

Brief Intermission for Questions...

Background Cosmology: Assumptions

Assumption 1. Flat FRW background:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = a^2(\tau)(-d\tau^2 + \delta^{ij} dx^i dx^j).$$

Background Cosmology: Assumptions

Assumption 2. Classical spinor:
The observed classical gravitational field is sourced by expectation values of spinor invariants such as $\langle \Psi\Psi \rangle$ and $\langle A^I A_I \rangle$. We define:

$$\Psi_{cl} = \langle \Psi \rangle$$

such that

$$\frac{\langle \overline{\Psi}\Psi \rangle - \langle \overline{\Psi} \rangle \langle \Psi \rangle}{\langle \overline{\Psi} \rangle \langle \Psi \rangle} \simeq 0.$$

Background Cosmology: Assumptions

Classical spinor: $\Psi = \{a, b, c, d\}$

$$\langle A_a A^a \rangle = -\langle V_a V^a \rangle = E^2 + B^2,$$
$$\langle A_a V^a \rangle = 0.$$

where $E = \langle \overline{\Psi} \Psi \rangle$, and $iB = \langle \overline{\Psi} \gamma_5 \Psi \rangle$.
This means:

$$W = U(E) + \xi(E^2 + B^2),$$

where

$$\xi = -\frac{3\pi G \gamma^2}{2(1+\gamma^2)} \left[\left(\tfrac{2}{\gamma}\beta(1+\lambda) + \beta^2 - (1+\lambda)^2 \right) - \left(\tfrac{2}{\gamma}\alpha\tau + \alpha^2 - \tau^2 \right) \right].$$

NOTE: A^a **is space-like!**

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Dynamics: The matter action Action.

- Fermion Kinetic term

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- Potential
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where:

$$V^L = \overline{\Psi}\gamma^L\Psi, \qquad A^L = \overline{\Psi}\gamma_5\gamma^L\Psi.$$

Bouncing solutions: Classical Spinors and Flat FRW backgrounds

Background Einstein:

$$12\kappa\mathcal{H}^2 = a^2\left[U + \xi(E^2 + B^2)\right]$$
$$-4\kappa(2\mathcal{H}' + \mathcal{H}^2) = a^2[\xi(E^2 + B^2) + U'E - U],$$

Background Dirac

$$\gamma^0\partial_0\Psi + \tfrac{3}{2}\gamma^0\mathcal{H}\Psi = -ia[(U' + 2\xi E)\Psi - 2i\xi B\gamma_5\Psi].$$

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Background Dirac

$$\dot{E} = -3\mathcal{H}E + 4a\xi BA^0,$$
$$\dot{B} = -3\mathcal{H}B - 2a(U' + 2\xi E)A^0,$$
$$\dot{A}^0 = -3\mathcal{H}A^0 + 2a[(U' + 2\xi E)B - 2\xi BE].$$

Bouncing solutions: Parity invariance $\gamma^0\Psi = \Psi$

Dirac

$$\dot{E} + 3HE = 0 \longrightarrow E = \frac{M}{a^3},$$

Einstein

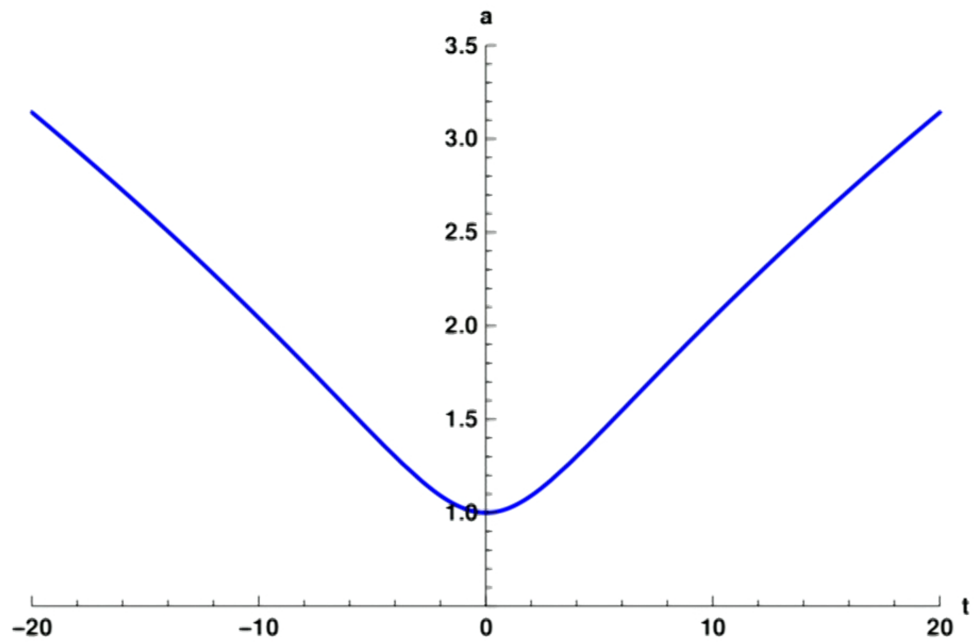
$$12\kappa H^2 = U[E] + \xi E^2,$$

Bouncing solutions: Parity invariance $\gamma^0\Psi = \Psi$

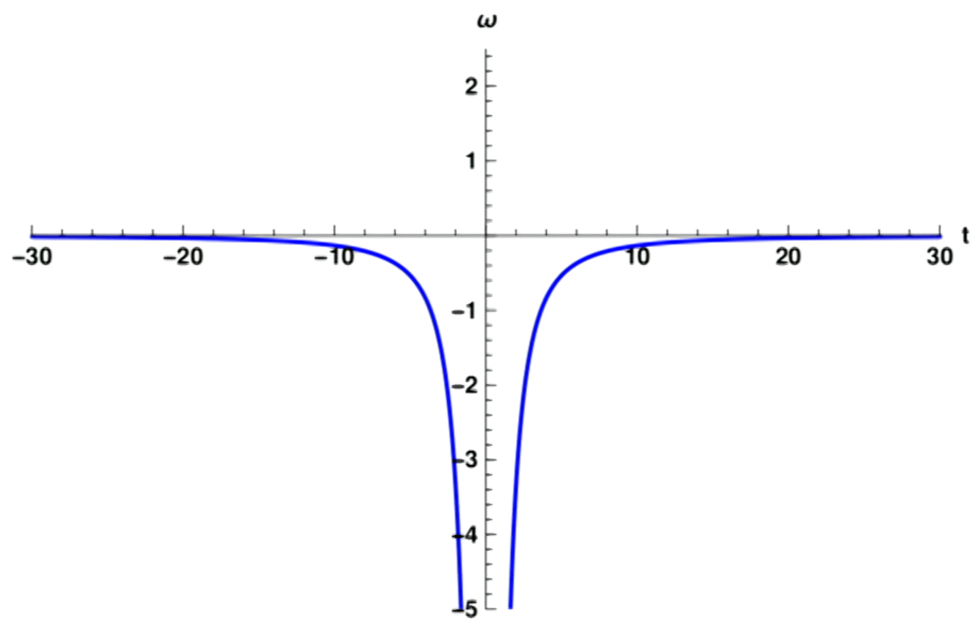
Example 1. Simple mass term:

$$U = mE, \qquad a(t) = \left[M(-\frac{\xi}{m} + \frac{3mt^2}{16\kappa})\right]^{1/3}$$

The Background: Bouncing solutions



The Background: Bouncing solutions



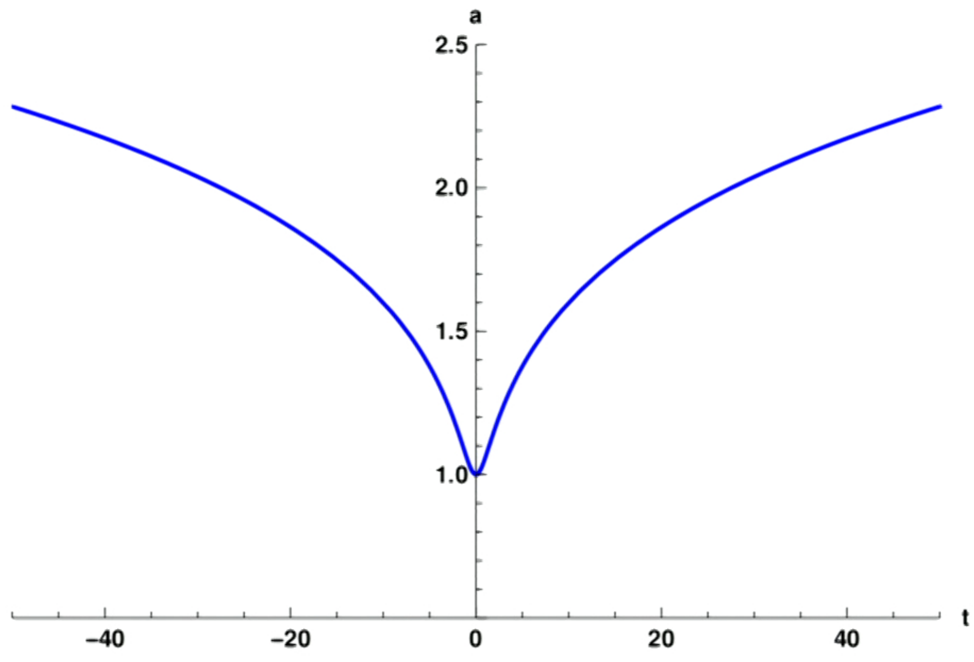
Bouncing solutions: Parity invariance $\gamma^0\Psi = \Psi$

Example 2. Ekpyrotic model:

$$U(E) = -\xi E^2 + b_1 E^n + b_2 E^{2n},$$

$$\omega = \frac{P}{\rho} = \frac{U'E - U + \xi E^2}{U + \xi E^2} = (n-1) + \frac{n b_2 E^{2n}}{b_1 E^n + b_2 E^{2n}}$$

The Background: Bouncing solutions



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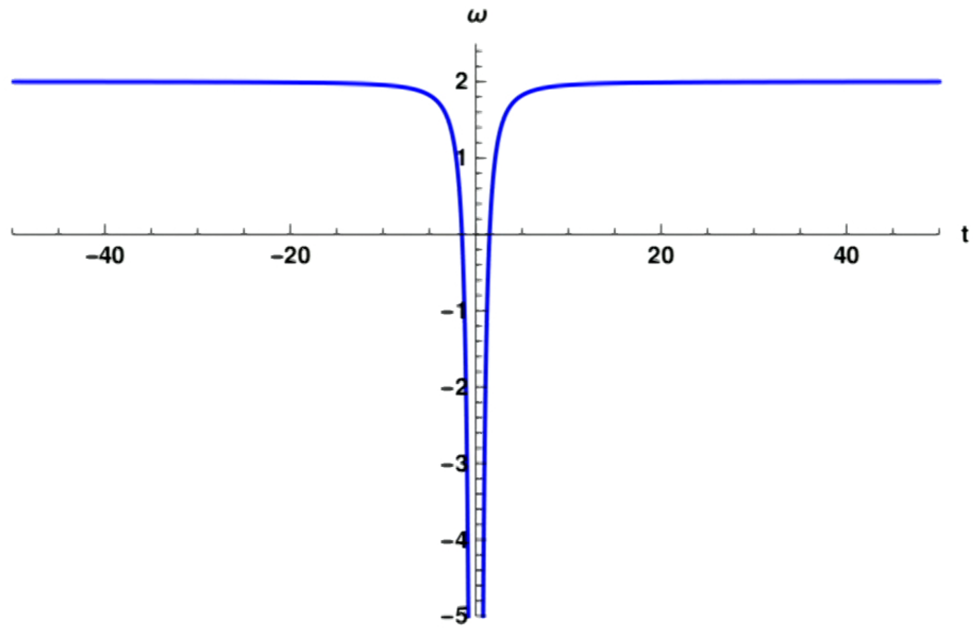

Questions?

Brief Questions Intermission...

Stability: Peturbed FRW

Peturbed line element:

$$ds^2 = a(\tau)^2[-(1+2\psi)d\tau^2 + 2B_i d\tau dx^i + ((1-2\phi)\delta_{ij} + h_{ij})dx^i dx^j],$$

Peturbed spinor

$$\Psi = \Psi_{(0)} + \Psi_{(1)}$$

Stability: Linearised equations

Einstein Equations:

$$4\kappa G_{00} = -8\kappa[\delta^{ij}k_i k_j \phi + 6\mathcal{H}\phi']$$
$$= a^2(1+2\psi)(U+\xi E^2) + \tfrac{i}{4}a[\,B_{[i}k_{k]}\varepsilon^{0ikj}A_j\, + 2k_i C^i]$$
$$\ldots$$

- SVT mixing term!

where $iC^i_{(1)} = (\overline{\Psi}_{(0)}\gamma^0\gamma^i\Psi_{(1)} + \overline{\Psi}_{(1)}\gamma^0\gamma^i\Psi_{(0)})$.

Dirac Equations:

$$\dot{E} = -3\mathcal{H}E + 3\phi' E - \partial_i V^i$$
$$\ldots$$

Stability: Real Fluid Form

$$T_{\mu\nu} = u_\mu u_\nu (P + \rho) + g_{\mu\nu} P + \Sigma_{\mu\nu},$$

where:

$$\Sigma_{\mu\nu} = \Sigma_{\nu\mu}, \qquad \Sigma_{\mu\nu} u^\nu = 0, \qquad \Sigma^\mu_\mu = 0,$$

with $u_\mu = \{-a(1+\psi), av_i\}$.

Stability: Real Fluid Form

$$\rho = (U + \xi E^2) + \tfrac{i}{4a}[B_{[i}k_{k]}\varepsilon^{0ikj}A_j + 2k_kC^k],$$
$$P = [\xi E^2 + U'E - U] + \tfrac{i}{12a}[B_k k_l \varepsilon^{kl0m}A_m + 2k_kC^k],$$
$$\Sigma_{ij} = \tfrac{i}{4}a\Big[2C_{(i}k_{j)} - \tfrac{2}{3}\eta_{ij}k_kC^k + \tfrac{1}{2}[(i\dot{h}_{jl} + B_jk_l)\eta_{ik} + (i\dot{h}_{il} + B_ik_l)\eta_{jk} - \tfrac{2}{3}\eta_{ij}B_kk_l]\varepsilon^{kl0m}A_m\Big],$$

$$q_i \equiv (\rho + P)v_i = \tfrac{i}{4a}[2k_i\widetilde{E} - ((1-3\phi)\eta_{in} + \tfrac{1}{2}h_{in})k_m\varepsilon^{0mnk}A_k].$$

Scalar components of q_i and Σ_{ij}:

$$\Sigma = \tfrac{i}{8k^kk_k}a(k_lB_k\varepsilon^{kl0m}A_m - 4k_iC^i),$$
$$q = \tfrac{1}{2}\widetilde{E}.$$

Stability: Co-moving Curvature perturbation

Simplification: Restrict to Parity Invariant Perturbations and potential $U = mE$!

$$\mathcal{R} = \phi - \frac{H}{P_{(0)} + \rho_{(0)}} q.$$

Differentiate twice:

$$\ddot{\mathcal{R}} = \theta(t)\dot{\mathcal{R}}$$

where

$$\theta(t) = \frac{\dot{H}}{H} + 3H\frac{-\rho + 3P}{\rho + P} = \frac{1}{t} - \frac{12m^2 t}{3m^2t^2 + 16\kappa\xi}$$

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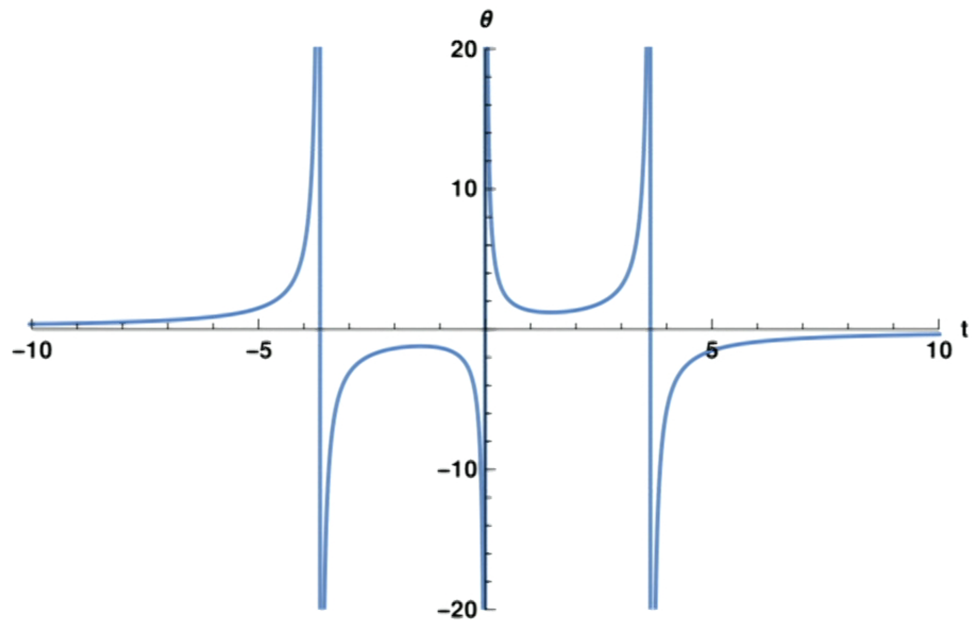

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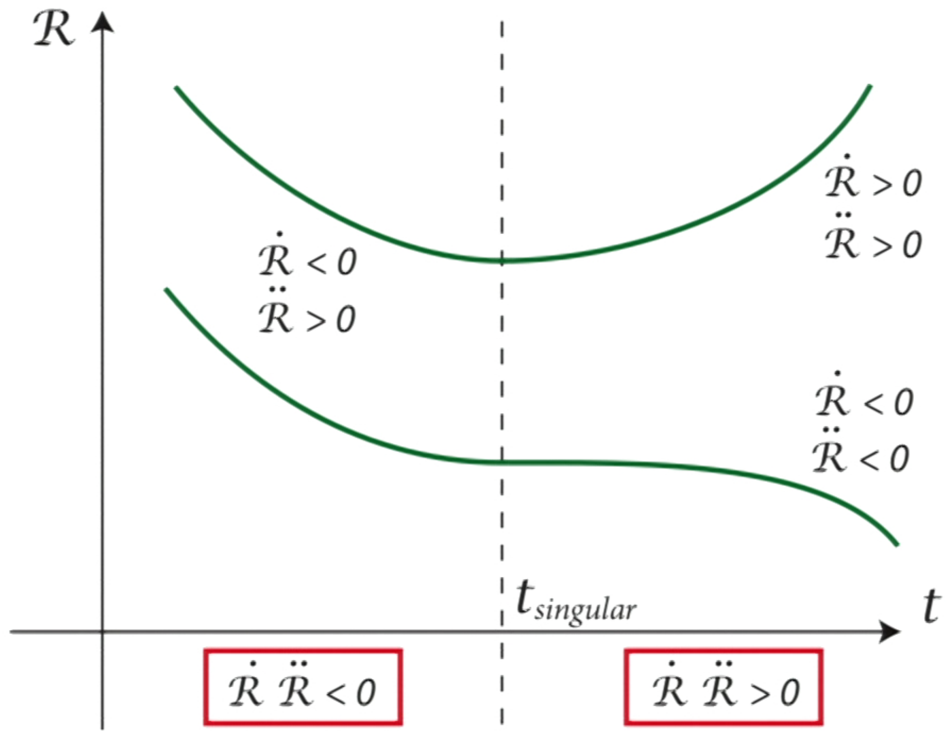
Stability: First order Solutions



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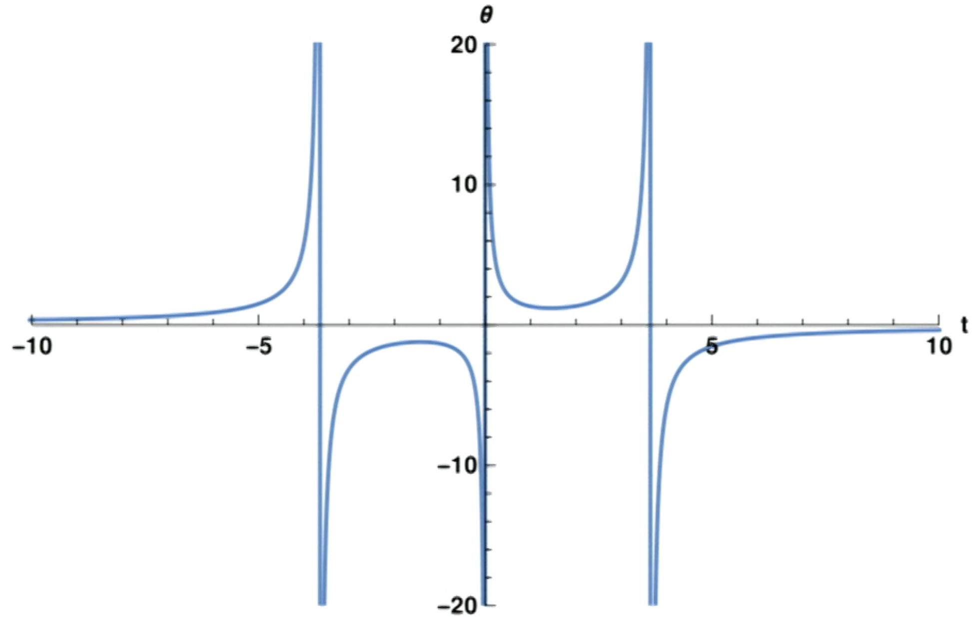
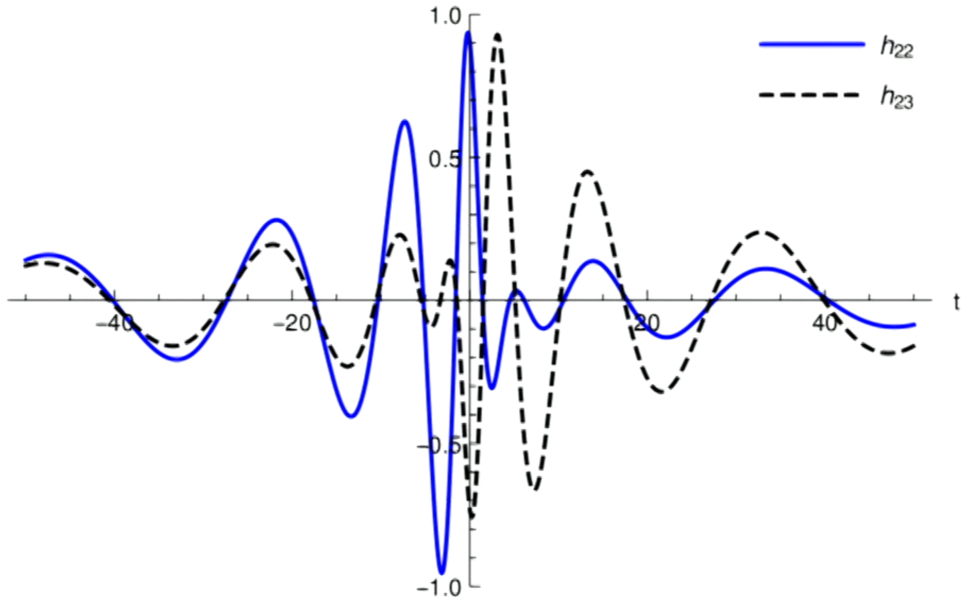


Tensor Perturbations



Discussion

1. **Q.** What is the new Physics responsible for the bounce?
   - **A.** Torsion coupled to classical spinors.
2. **Q.** Might this new physics resolve the singularity for the perturbations as well as the background?
   - **A.** It looks pretty bad...
3. **Q.** Does this new physics have any observational signature?
   - **A.** Possibly the preferred direction picked by A^I?
4. **Q.** What physical principles underlie the theory beyond working to resolve the singularity?
   - **A.** Making most Minimal extension to known physics possible.
5. **Q.** Does a consistent picture for cosmology require that both the background and the perturbations are quantized?
   - **A.** Not for mathematical consistency. Perhaps the 'classical' spinor assumption is however a little too strong.
6. **Q.** Does the bounce or pre-bounce phase help in setting initial conditions?
   - **A.** Yes in the sense that it is very easy to accommodate a phase of ekpyrosis or inflation either side of the bounce.