Title: Loop Quantum Cosmology, Non-Gaussianity, and CMB anomalies

Date: Jun 28, 2017 10:00 AM

URL: http://pirsa.org/17060105

Abstract: Loop quantum cosmology has become a robust framework to describe the highest curvature regime of the early universe. In this theory, inflation is preceded by a bounce replacing the big bang singularity. I will summarize the theoretical framework, and explore the corrections to the inflationary predictions for the primordial spectrum of cosmological perturbations that this pre-inflationary, quantum gravity phase of the universe introduces. The impact of the bounce on non-Gaussianity and the exciting relation to the observed large scale anomalies in the CMB will be discussed.

Summary of work done in collaboration with several people:

Ashtekar, Bolliet, Gupt, Morris, Nelson, Parker, Shandera, Vijayakumar.

From Abhay's talk:

LQC is a mini-superspace version of LQG:
quantization of spacetimes with cosmological symmetries.

Underlying principles: non-perturbative and background independent quantization of gravity

Rigorous and beautiful mathematics

Hilbert space: $\Psi(c, \phi)$ such that $\hat{H}\Psi(c, \phi) = 0$

From Abhay's talk:

LQC is a mini-superspace version of LQG:
quantization of spacetimes with cosmological symmetries.

Underlying principles: non-perturbative and background independent quantization of gravity

Rigorous and beautiful mathematics

Hilbert space: $\Psi(c,\phi)$ such that $\hat{H}\Psi(c,\phi) = 0$

Analytical results:

All physical observables (e.g. curvature invariants, energy density of ϕ) are bounded from above. No singularity in the entire Hilbert space. For instance:

$$\rho_{\rm sup} = \frac{18\pi}{G^2\hbar\Delta_o} \approx 0.4\,\rho_{Pl} \qquad\qquad R_{\rm sup} = 48\pi G\rho_{\rm sup}$$

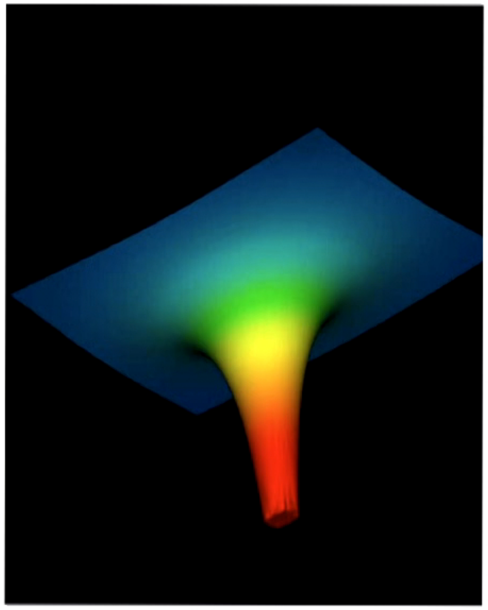
area gap in LQG: minimum area eigenvalue

Additionally:

All states during the evolution go through an instant (in ϕ-time) of minimum volume and maximum curvature: Bounce

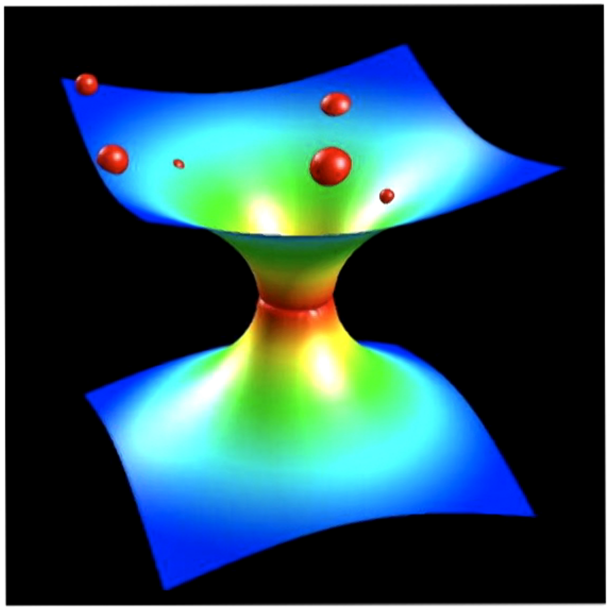
Artistic conceptions of the Big Bang and Big Bounce

Big Bang



Credits: Pablo Laguna

Big Bounce



Credits: Cliff Pikover

LQC effective eqns for "sharply peaked" states $\Psi(c,\phi)$

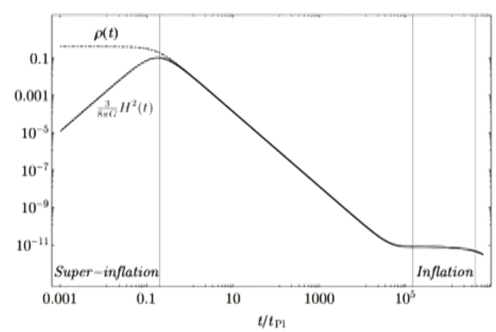
Geometry well approximated by a smooth metric tensor with the FLRW symmetries:

$$H^2 = \frac{8\pi G}{3}\,\rho\left(1-\frac{\rho}{\rho_{\rm sup}}\right) \qquad \textbf{where} \qquad \rho_{\rm sup}\approx 0.4\,\rho_{P\ell}$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}\,\rho\left(1-4\frac{\rho}{\rho_{\rm sup}}\right) - 4\pi G\,P\left(1-2\frac{\rho}{\rho_{\rm sup}}\right)$$

$$\ddot\phi + 3H\dot\phi + \frac{dV(\phi)}{d\phi} = 0$$

and, as usual: $\rho = \frac{1}{2}\dot\phi^2 + V(\phi)$ **and** $P = \frac{1}{2}\dot\phi^2 - V(\phi)$



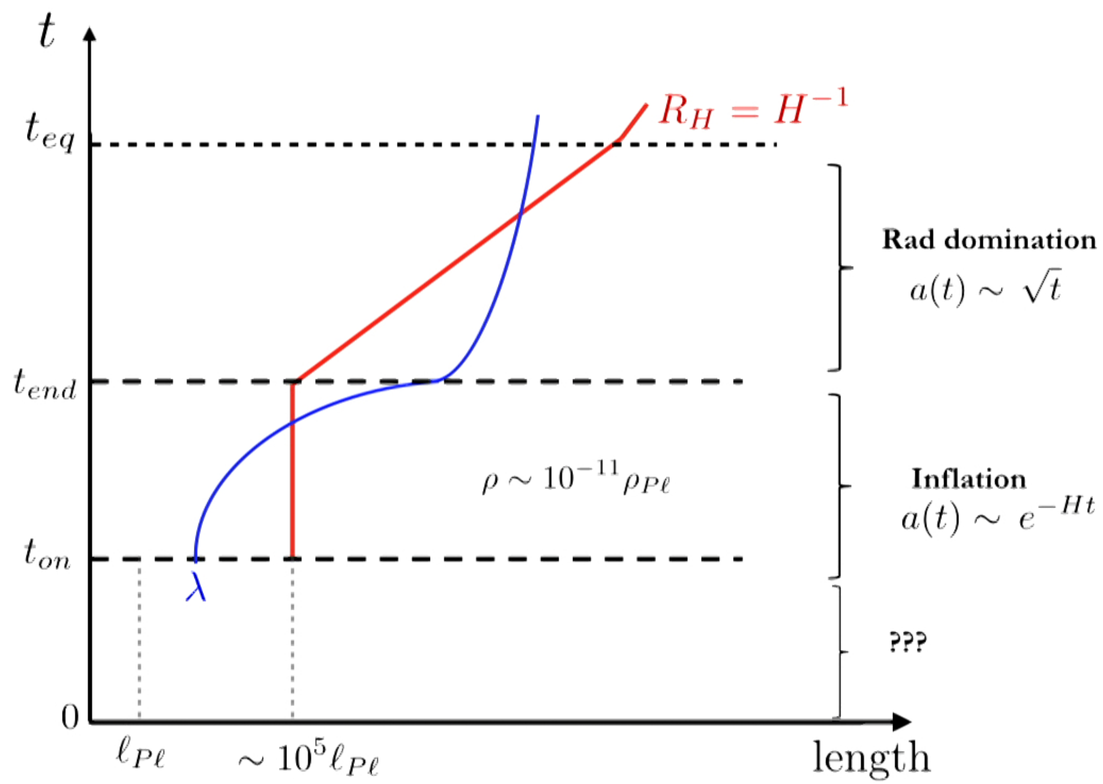
Goal: Explore observational consequences

All my results can be extended to other models with a similar bounce

I will use the bounce to complete inflation, rather than to replace it

Inflation

(Within our observable patch) FLRW metric: $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$



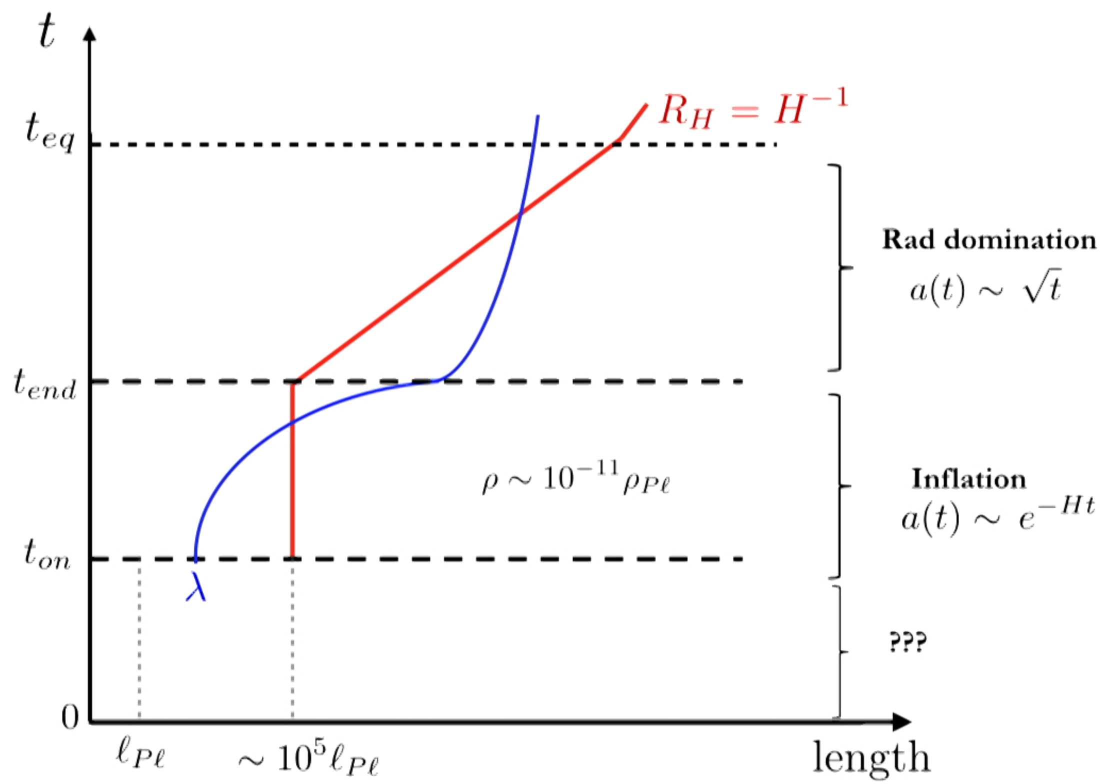
From Robert's talk:

What is ϕ?
What are the initial condition of inflation?
Trans-Planckian problem
Singularity problem
Applicability of GR

"For this audience, replace problem by a window of opportunity"

Inflation

(Within our observable patch) FLRW metric: $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$



From Robert's talk:

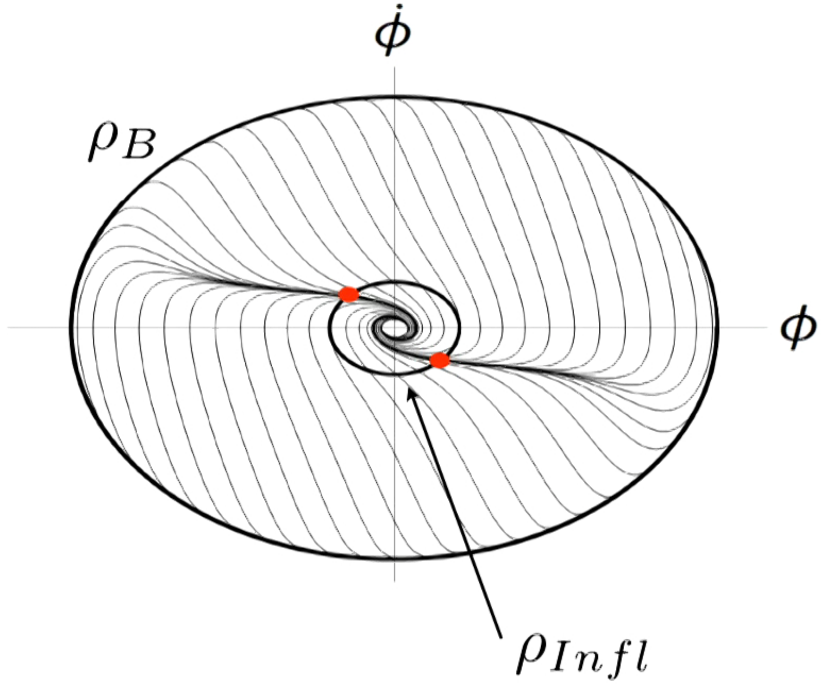
What is ϕ?
What are the initial condition of inflation?
Trans-Planckian problem
Singularity problem
Applicability of GR

"For this audience, replace problem by a window of opportunity"

Is inflation "compatible" with the bounce?

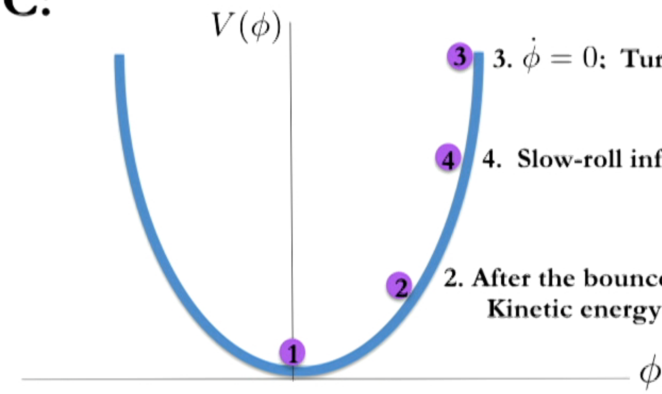
Ashtekar, Sloan

Message: the "attractor" character of inflation remains in presence of a bounce



A fraction greater than 0.99999 of the "volume" of initial data at the bounce leads to an inflationary phase with more than 60 e-folds of slow-roll

Inflation and LQC:



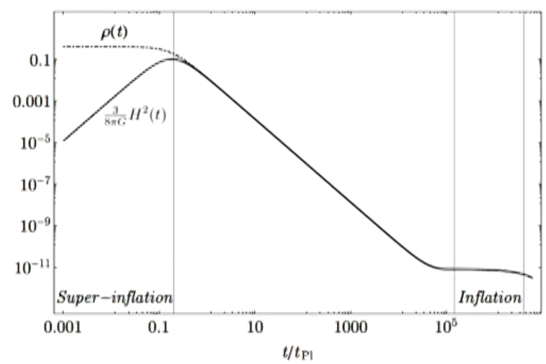
3. $\dot{\phi} = 0$: Turn around point

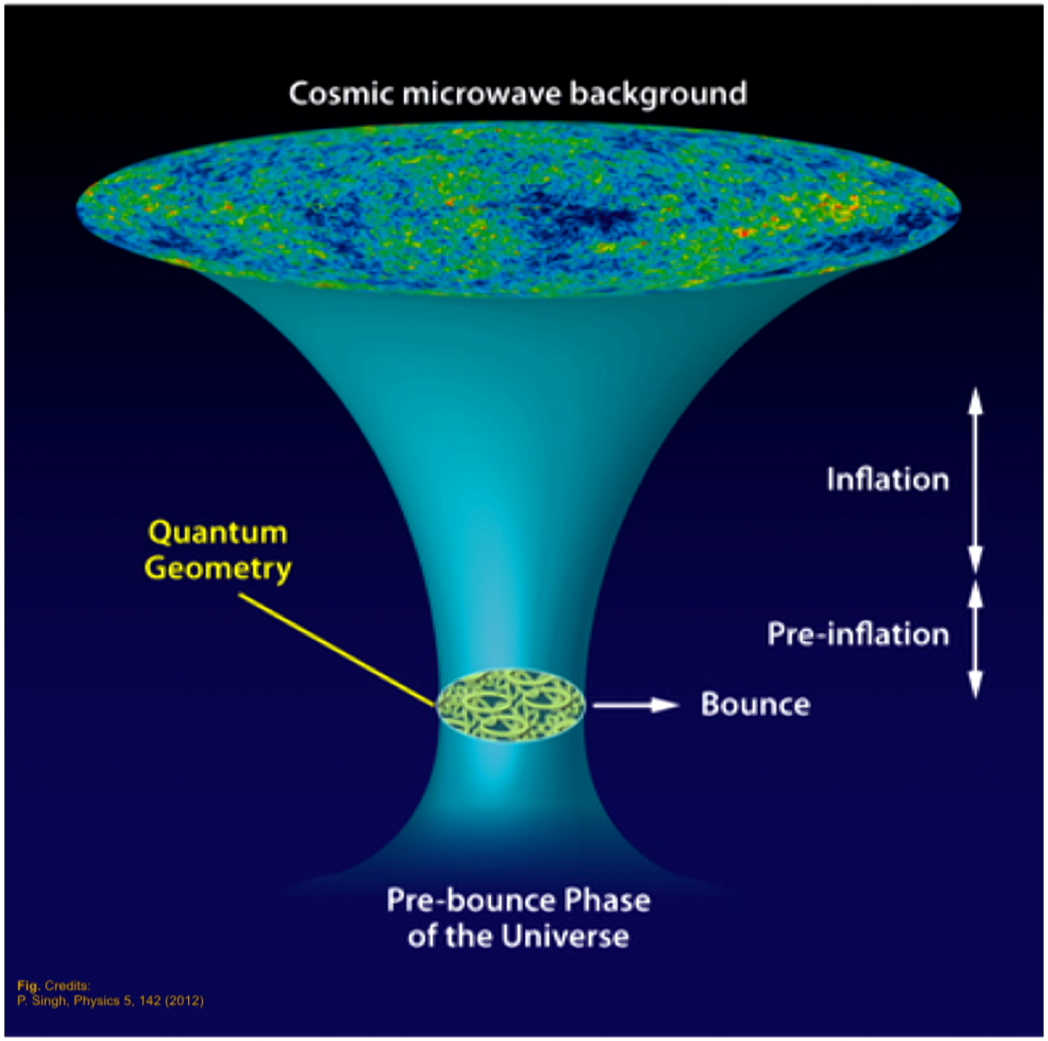
4. Slow-roll inflation begins

2. After the bounce: friction grows, $\dot{\phi}$ slows down
Kinetic energy → Potential energy

1. Before the bounce $\phi \approx 0$ but large anti-friction boosts $\dot{\phi}$.
Kinetic energy dominated

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0$$




Cosmic microwave background
Inflation
Quantum
Geometry
Pre-inflation
Bounce
Pre-bounce Phase
of the Universe
Fig. Credits:
P. Singh, Physics 5, 142 (2012)

3. The Power Spectrum

Quantum field theory on a cosmological, quantum space-time

Abhay Ashtekar[1,*] Wojciech Kaminski[1,2,†] and Jerzy Lewandowski[1,2‡]

[1] *Institute for Gravitation and the Cosmos & Physics Department, Penn State, University Park, PA 16802, U.S.A.*
[2] *Instytut Fizyki Teoretycznej Uniwersytet Warszawski, ul.Hoza 69, PL-00 681 Warsaw, Poland*

An Extension of the Quantum Theory of Cosmological Perturbations to the Planck Era

Ivan Agullo[1,2,*] Abhay Ashtekar[1,†] and William Nelson[1‡]

[1] *Institute for Gravitation and the Cosmos & Physics Department, Penn State, University Park, PA 16802, U.S.A.*
[2] *Center for Theoretical Cosmology, DAMTP, Wilberforce Road, University of Cambridge, Cambridge CB3 OWA, U.K.*

Phenomenology with fluctuating quantum geometries in loop quantum cosmology

Ivan Agullo[1,*] Abhay Ashtekar[2,†] and Brajesh Gupt[2‡]

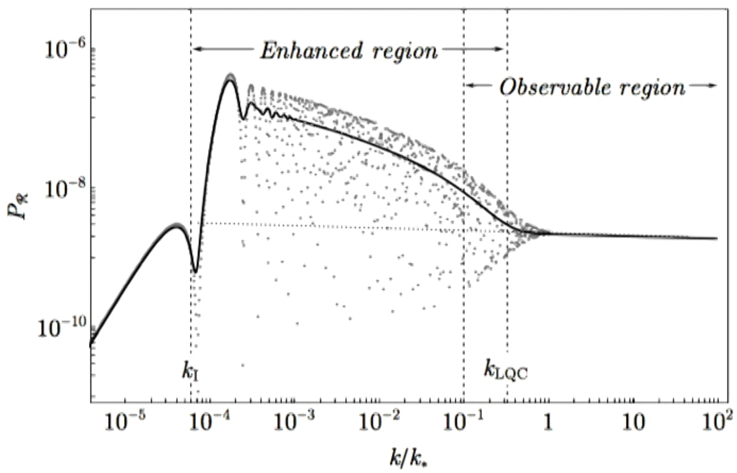
[1] *Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803, U.S.A. and*

[2] *Institute for Gravitation and the Cosmos & Physics Department, The Pennsylvania State University, University Park, PA 16802 U.S.A.*

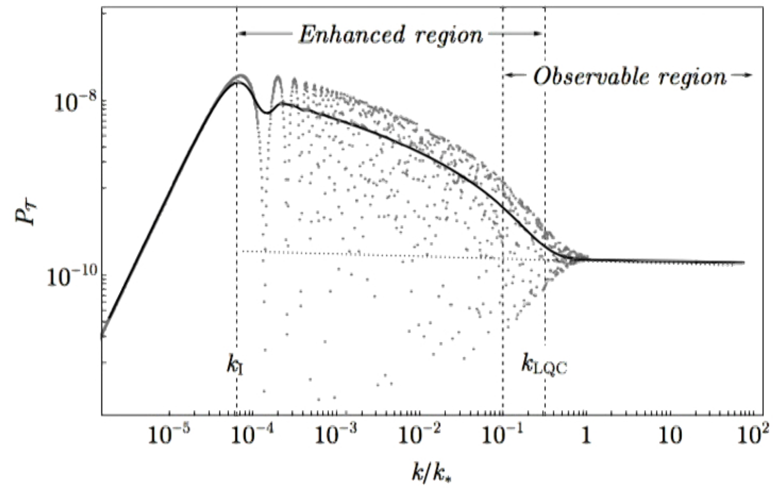
Results of numerical evolution

(I.A.-Ashtekar-Nelson 2012-13, I.A.-Morris 2015)

Scalar Power Spectrum



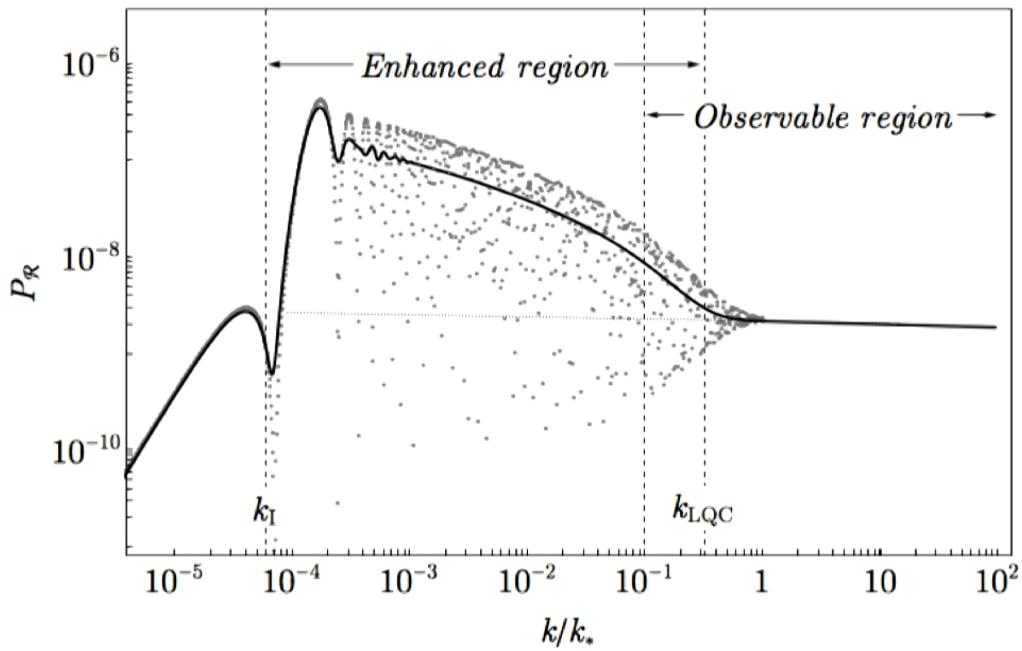
Tensor Power Spectrum



- $\phi_B = 1.22 \quad m = 1.1 \times 10^{-6}$ **and vacuum initial condition in the past**
- **Grey point: numerical result for individual** k **'s**
- **Black line: average of grey points**
- $k_\star/a_0 = 0.002\,\mathrm{Mpc}^{-1}$

Results of numerical evolution

Scalar Power Spectrum



The pre-inflationary evolution modifies the power for low k-values (long wavelengths)

- For large values of ϕ_B predictions are indistinguishable from standard inflation

→ QG extension of the inflationary scenario

- For large values of ϕ_B predictions are indistinguishable from standard inflation

  → QG extension of the inflationary scenario

- For smaller ϕ_B, QG corrections at large angles in CMB. In particular:

- For large values of ϕ_B predictions are indistinguishable from standard inflation

→ QG extension of the inflationary scenario

- For smaller ϕ_B , QG corrections at large angles in CMB. In particular:

Most important:

- enhancement of power for low k
- effects on spectral indices and runnings
- reduction of tensor-to-scalar ratio (slightly alleviates constrains on quadratic potential)
- modification of consistency relation $r < -8\, n_t$

Robustness tests:

- Conclusions robust against change in the potential (Bonga-Gutp 2015-16)

Robustness tests:

- Conclusions robust against change in the potential (Bonga-Gutp 2015-16)
- Conclusions robust against choice of the quantum FLRW geometry (I.A., Ashtekat, Gutp 2016)
- Conclusions are robust against initial conditions for perturbations (I.A.-Ashtekar-Nelson 2013, I.A.-Morris 2015)

Robustness tests:

- Conclusions robust against change in the potential (Bonga-Gutp 2015-16)
- Conclusions robust against choice of the quantum FLRW geometry (I.A., Ashtekat, Gutp 2016)
- Conclusions are robust against initial conditions for perturbations (I.A.-Ashtekar-Nelson 2013, I.A.-Morris 2015)
- Extension to anisotropic bounces (I.A., Olmedo, Vijayakumar)

Robustness tests:

- Conclusions robust against change in the potential (Bonga-Gutp 2015-16)
- Conclusions robust against choice of the quantum FLRW geometry (I.A., Ashtekat, Gutp 2016)
- Conclusions are robust against initial conditions for perturbations (I.A.-Ashtekar-Nelson 2013, I.A.-Morris 2015)
- Extension to anisotropic bounces (I.A., Olmedo, Vijayakumar)
- Other approaches for perturbations (within LQC) produce quite similar results

Robustness tests:

- Conclusions robust against change in the potential (Bonga-Gutp 2015-16)
- Conclusions robust against choice of the quantum FLRW geometry (I.A., Ashtekat, Gutp 2016)
- Conclusions are robust against initial conditions for perturbations (I.A.-Ashtekar-Nelson 2013, I.A.-Morris 2015)
- Extension to anisotropic bounces (I.A., Olmedo, Vijayakumar)
- Other approaches for perturbations (within LQC) produce quite similar results

Important:

Ashtekat, Gutp 2016: there are physically motivated initial conditions for perturbation at the bounce for which the power spectrum is suppressed at low k's

6. Non-Gaussianity from the bounce

I.A., Bolliet, Vijayakumar

Goal: Compute three-point correlation function

$$\langle 0|\hat{\mathcal{R}}_{\vec{k}_1}\hat{\mathcal{R}}_{\vec{k}_2}\hat{\mathcal{R}}_{\vec{k}_3}|0\rangle =: (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3)\, B_{\mathcal{R}}(k_1,k_2,k_3)$$

Motivation: there are observational constraints + we may find new effects

We need:

To go beyond linear perturbation theory: expand Einstein action at third order

Hard calculation. Done for the first time by Maldacena in 2003

Recast Maldacena's result in Hamiltonian language

A summary of the way we proceed:

Phase space variables:

$$\begin{aligned} \Phi(\vec{x}) &= \phi + \delta\phi(\vec{x}) \\ \Pi_\Phi(\vec{x}) &= p_\phi + \delta p_\phi(\vec{x}) \\ q_{ij}(\vec{x}) &= \mathring{q}_{ij} + \delta q_{ij}(\vec{x}) \\ \pi^{ij}(\vec{x}) &= \mathring{\pi}^{ij} + \delta\pi^{ij}(\vec{x}) \end{aligned}$$

where: $\mathring{q}_{ij} = a^2\,\delta_{ij}\,;$ $\qquad \mathring{\pi}^{ij} = \frac{\pi_a}{6\,a}\,\delta^{ij}$

Fourier expansion perturbations:

$$\delta q_{ij}(\vec{x}) = \frac{1}{V_0}\sum_{\vec{k}} \delta\tilde{q}_{ij}(\vec{k})\,e^{i\,\vec{k}\cdot\vec{x}} \qquad \delta\pi_{ij}(\vec{x}) = \frac{1}{V_0}\sum_{\vec{k}} \delta\tilde{\pi}_{ij}(\vec{k})\,e^{i\,\vec{k}\cdot\vec{x}}$$

Expansion in a convenient basis:

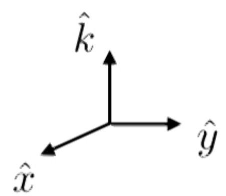
$$\delta\tilde{q}_{ij}(\vec{k}) = \sum_{n=1}^{6} \tilde{\gamma}_n(\vec{k})\, A_{ij}^{(n)} \qquad \delta\tilde{\pi}^{ij}(\vec{k}) = \sum_{n=1}^{6} \tilde{\pi}_n(\vec{k})\, A^{ij}_{(n)}$$

where:

$$A_{ij}^{(1)} = \frac{\mathring{q}_{ij}}{\sqrt{3}} \qquad A_{ij}^{(2)} = \sqrt{\frac{3}{2}}\left(\hat{k}_i\,\hat{k}_j - \frac{\mathring{q}_{ij}}{3}\right)$$

$$A_{ij}^{(3)} = \frac{1}{\sqrt{2}}\left(\hat{k}_i\,\hat{x}_j + \hat{k}_j\,\hat{x}_i\right) \qquad A_{ij}^{(4)} = \frac{1}{\sqrt{2}}\left(\hat{k}_i\,\hat{y}_j + \hat{k}_j\,\hat{y}_i\right)$$

$$A_{ij}^{(5)} = \frac{1}{\sqrt{2}}\left(\hat{x}_i\,\hat{y}_j + \hat{x}_j\,\hat{y}_i\right) \qquad A_{ij}^{(6)} = \frac{1}{\sqrt{2}}\left(\hat{x}_i\,\hat{x}_j - \hat{y}_i\,\hat{y}_j\right)$$



orthonormal triad

Expansion in a convenient basis:

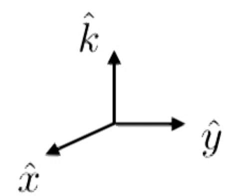
$$\delta\tilde{q}_{ij}(\vec{k}) = \sum_{n=1}^{6} \tilde{\gamma}_n(\vec{k})\, A^{(n)}_{ij} \qquad \delta\tilde{\pi}^{ij}(\vec{k}) = \sum_{n=1}^{6} \tilde{\pi}_n(\vec{k})\, A^{ij}_{(n)}$$

where:

$$A^{(1)}_{ij} = \frac{\mathring{q}_{ij}}{\sqrt{3}} \qquad A^{(2)}_{ij} = \sqrt{\frac{3}{2}}\left(\hat{k}_i\,\hat{k}_j - \frac{\mathring{q}_{ij}}{3}\right)$$

$$A^{(3)}_{ij} = \frac{1}{\sqrt{2}}\left(\hat{k}_i\,\hat{x}_j + \hat{k}_j\,\hat{x}_i\right) \qquad A^{(4)}_{ij} = \frac{1}{\sqrt{2}}\left(\hat{k}_i\,\hat{y}_j + \hat{k}_j\,\hat{y}_i\right)$$

$$A^{(5)}_{ij} = \frac{1}{\sqrt{2}}\left(\hat{x}_i\,\hat{y}_j + \hat{x}_j\,\hat{y}_i\right) \qquad A^{(6)}_{ij} = \frac{1}{\sqrt{2}}\left(\hat{x}_i\,\hat{x}_j - \hat{y}_i\,\hat{y}_j\right)$$



orthonormal triad

$\tilde{\gamma}_1 \;\; \tilde{\gamma}_2$ **scalars**

$\tilde{\gamma}_3 \;\; \tilde{\gamma}_4$ **vectors** **w. r. t. rotation around** $\hat{k}$

$\tilde{\gamma}_5 \;\; \tilde{\gamma}_6$ **tensors**

$\delta\phi$ **scalar**

Focus on scalar perturbations

We can work with gauge invariant variables or fix the gauge. Here, fix the gauge.

Strategy:

1) Expand scalar and vector constraint (we use xTensor in Mathematica)

$$\mathbb{S}(\vec{x}) = \mathbb{S}^{(0)} + \mathbb{S}^{(1)}(\vec{x}) + \mathbb{S}^{(2)}(\vec{x}) + \mathbb{S}^{(3)}(\vec{x}) + \cdots ;$$
$$\mathbb{V}(\vec{x}) = \mathbb{V}^{(0)} + \mathbb{V}^{(1)}(\vec{x}) + \mathbb{V}^{(2)}(\vec{x}) + \mathbb{V}^{(3)}(\vec{x}) + \cdots .$$

Focus on scalar perturbations

We can work with gauge invariant variables or fix the gauge. Here, fix the gauge.

Strategy:

1) Expand scalar and vector constraint (we use xTensor in Mathematica)

$$\begin{aligned}\mathbb{S}(\vec{x}) &= \mathbb{S}^{(0)} + \mathbb{S}^{(1)}(\vec{x}) + \mathbb{S}^{(2)}(\vec{x}) + \mathbb{S}^{(3)}(\vec{x}) + \cdots ;\\ \mathbb{V}(\vec{x}) &= \mathbb{V}^{(0)} + \mathbb{V}^{(1)}(\vec{x}) + \mathbb{V}^{(2)}(\vec{x}) + \mathbb{V}^{(3)}(\vec{x}) + \cdots .\end{aligned}$$

2) Impose gauge conditions: $\gamma_1 = 0\,, \quad \gamma_2 = 0$

4) Use first order constraints $\mathbb{S}^{(1)}(\vec{x}) = 0, \mathbb{V}_i^{(1)}(\vec{x}) = 0$ **to eliminate** $\tilde{\pi}_1, \tilde{\pi}_2$ **in favor of** $\delta\tilde{\phi}$ and $\delta\tilde{p}_\phi$

$$\tilde{\pi}_1 = \tilde{\pi}_1(\delta\tilde{\phi}, \delta\tilde{p}_\phi), \; \tilde{\pi}_2 = \tilde{\pi}_2(\delta\tilde{\phi}, \delta\tilde{p}_\phi)$$

4) Use first order constraints $\mathbb{S}^{(1)}(\vec{x}) = 0, \mathbb{V}_i^{(1)}(\vec{x}) = 0$ **to eliminate** $\tilde{\pi}_1, \tilde{\pi}_2$ **in favor of** $\delta\tilde{\phi}$ and $\delta\tilde{p}_\phi$

$$\tilde{\pi}_1 = \tilde{\pi}_1(\delta\tilde{\phi}, \delta\tilde{p}_\phi),\ \tilde{\pi}_2 = \tilde{\pi}_2(\delta\tilde{\phi}, \delta\tilde{p}_\phi)$$

5) Substitute in the Hamiltonian, and keep terms up to their oder in perturbations

Second order Hamiltonian (free evolution)

$$\mathcal{H}^{(2)} = N\frac{1}{2}\int \mathrm{d}^3x \left[\frac{1}{a^3}\,\delta p_\phi^2 + a^3\,(\vec{\partial}\delta\phi)^2 + a^3\,\mathfrak{A}\,\delta\phi^2\right]$$

$$\mathfrak{A} = -9\frac{p_\phi^4}{a^8\pi_a^2} + \frac{3}{2}\kappa\frac{p_\phi^2}{a^6} - \frac{6\,p_\phi}{a\,\pi_a}V_\phi + V_{\phi\phi} + 6\frac{p_\phi\dot{p}_\phi}{a^4\,\pi_a} - 3\frac{p_\phi^2\,\dot{\pi}_a}{a^4\,\pi_a^2} - 3\,\frac{\dot{a}\,p_\phi^2}{a^5\,\pi_a}$$

$$(\Box - \mathfrak{A}(t))\,\delta\phi(\vec{x},t) = 0$$

Third order Hamiltonian (interaction Hamiltonian)

$$\mathcal{H}^{(3)} = N \int \mathrm{d}^3x \Bigg[\left(\frac{9\,\kappa\, p_\phi^3}{4\, a^4\, \pi_a} - \frac{27\, p_\phi^5}{2\, a^6 \pi_a^3} - \frac{3\, a^2\, p_\phi\, V_{\phi\phi}}{2\, \pi_a} + \frac{a^3\, V_{\phi\phi\phi}}{6} \right) \delta\phi^3$$
$$- \frac{3\, p_\phi}{2\, a^4\, \pi_a}\, \delta p_\phi^2\, \delta\phi - \frac{9\, p_\phi^3}{a^5 \pi_a^2}\, \delta p_\phi \delta\phi^2 - \frac{3\, a^2\, p_\phi}{2\, \pi_a} \delta\phi\, (\vec{\partial}\delta\phi)^2 + \frac{3\, p_\phi^2}{N\, a\, \pi_a}\, \delta\phi^2 \partial^2\chi$$
$$+ \frac{1}{N}\, \delta p_\phi\, \partial_i \delta\phi\, \partial^i \chi + \frac{3\, a^2\, p_\phi}{N^2\, 2\, \kappa\, \pi_a}\, \delta\phi\, \partial^2\chi\, \partial^2\chi - \frac{3\, a^2\, p_\phi}{N^2\, 2\, \kappa\, \pi_a}\, \delta\phi\, \partial_i \partial_j \chi\, \partial^i \partial^j \chi \Bigg].$$

with: $\tilde{\chi} = N \frac{3\,\kappa}{k^2 a^3} \left[\left(\frac{p_\phi}{2} - \frac{a^5\, V_\phi}{\kappa\, \pi_a} \right) \delta\tilde{\phi} - \frac{p_\phi}{\kappa\, a\, \pi_a} \delta\tilde{p}_\phi \right]$

After a Legendre transformation, agrees with Maldacena's third order Lagrangian

To recast the result in therms of coming curvature perturbations at the end of inflation, we need the relation:

$$\mathcal{R}(\vec{x},\eta) = -\frac{a}{z}\,\delta\phi(\vec{x},\eta) + \left[-\frac{3}{2} + 3\frac{V_\phi\, a^2}{\kappa\, p_\phi\, \pi_a} - \frac{\sqrt{\kappa}}{4}\frac{z}{a}\right]\left(\frac{a}{z}\,\delta\phi(\vec{x},\eta)\right)^2 + \cdots$$

subdominant terms at the end of inflation

where $z := -\frac{6}{\kappa}\frac{p_\phi}{\pi_a}$

Calculation of the three-point correlation function

To recast the result in therms of coming curvature perturbations at the end of inflation, we need the relation:

$$\mathcal{R}(\vec{x},\eta) = -\frac{a}{z}\,\delta\phi(\vec{x},\eta) + \left[-\frac{3}{2} + 3\frac{V_\phi\, a^2}{\kappa\, p_\phi\, \pi_a} - \frac{\sqrt{\kappa}}{4}\frac{z}{a}\right]\left(\frac{a}{z}\,\delta\phi(\vec{x},\eta)\right)^2 + \cdots$$

subdominant terms at the end of inflation

where $z := -\frac{6}{\kappa}\frac{p_\phi}{\pi_a}$

Calculation of the three-point correlation function

$$\langle 0|\hat{\mathcal{R}}_{\vec{k}_1}\hat{\mathcal{R}}_{\vec{k}_2}\hat{\mathcal{R}}_{\vec{k}_3}|0\rangle = \left(-\frac{a}{z}\right)^3 \Big[\langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{k}_2}\hat{\delta\phi}_{\vec{k}_3}|0\rangle$$
$$+ \left(-\frac{3}{2}+3\frac{V_\phi\, a^2}{\kappa\, p_\phi\, \pi_a}-\frac{\sqrt{\kappa}}{4}\frac{z}{a}\right)\left(-\frac{a}{z}\right)\int \frac{d^3p}{(2\pi)^3}\,\langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{k}_2}\hat{\delta\phi}_{\vec{p}}\hat{\delta\phi}_{\vec{k}_3-\vec{p}}|0\rangle + (\vec{k}_1\leftrightarrow\vec{k}_3)+(\vec{k}_2\leftrightarrow\vec{k}_3)$$
$$+ \cdots\Big]\,.$$

Calculation of the three-point correlation function

$$\langle 0|\hat{\mathcal{R}}_{\vec{k}_1}\hat{\mathcal{R}}_{\vec{k}_2}\hat{\mathcal{R}}_{\vec{k}_3}|0\rangle = \left(-\frac{a}{z}\right)^3 \Big[\langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{k}_2}\hat{\delta\phi}_{\vec{k}_3}|0\rangle$$
$$+ \left(-\frac{3}{2}+3\frac{V_\phi\, a^2}{\kappa\, p_\phi\, \pi_a}-\frac{\sqrt{\kappa}}{4}\frac{z}{a}\right)\left(-\frac{a}{z}\right)\int\frac{d^3p}{(2\pi)^3}\langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{k}_2}\hat{\delta\phi}_{\vec{p}}\hat{\delta\phi}_{\vec{k}_3-\vec{p}}|0\rangle + (\vec{k}_1\leftrightarrow\vec{k}_3) + (\vec{k}_2\leftrightarrow\vec{k}_3)$$
$$+\cdots\Big].$$

$$\langle 0|\hat{\delta\phi}_{\vec{k}_1}(\eta)\hat{\delta\phi}_{\vec{k}_2}(\eta)\hat{\delta\phi}_{\vec{k}_3}(\eta)|0\rangle = \langle 0|U^\dagger(\eta,\eta_0)\,\hat{\delta\phi}^{\rm I}_{\vec{k}_1}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_2}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_3}(\eta)\,U(\eta,\eta_0)|0\rangle$$

$$U(\eta,\eta_0) = T\exp\left(-i\int_{\eta_0}^{\eta}{\rm d}\eta'\,\hat{H}^{\rm I}_{\rm int}(\eta')\right)$$

Calculation of the three-point correlation function

$$\langle 0|\hat{\mathcal{R}}_{\vec{k}_1}\hat{\mathcal{R}}_{\vec{k}_2}\hat{\mathcal{R}}_{\vec{k}_3}|0\rangle = \left(-\frac{a}{z}\right)^3 \Big[\langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{k}_2}\hat{\delta\phi}_{\vec{k}_3}|0\rangle$$
$$+ \left(-\frac{3}{2} + 3\frac{V_\phi\, a^2}{\kappa\, p_\phi\, \pi_a} - \frac{\sqrt{\kappa}}{4}\frac{z}{a}\right)\left(-\frac{a}{z}\right)\int \frac{d^3p}{(2\pi)^3}\langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{k}_2}\hat{\delta\phi}_{\vec{p}}\hat{\delta\phi}_{\vec{k}_3-\vec{p}}|0\rangle + (\vec{k}_1 \leftrightarrow \vec{k}_3) + (\vec{k}_2 \leftrightarrow \vec{k}_3)$$
$$+ \cdots\Big]\,.$$

$$\langle 0|\hat{\delta\phi}_{\vec{k}_1}(\eta)\hat{\delta\phi}_{\vec{k}_2}(\eta)\hat{\delta\phi}_{\vec{k}_3}(\eta)|0\rangle = \langle 0|U^\dagger(\eta,\eta_0)\,\hat{\delta\phi}^{\rm I}_{\vec{k}_1}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_2}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_3}(\eta)\,U(\eta,\eta_0)|0\rangle$$

$$U(\eta,\eta_0) = T\exp\left(-i\int_{\eta_0}^{\eta} {\rm d}\eta'\,\hat{H}^{\rm I}_{\rm int}(\eta')\right)$$

$$\begin{aligned}\langle 0|\hat{\delta\phi}_{\vec{k}_1}(\eta)\hat{\delta\phi}_{\vec{k}_2}(\eta)\hat{\delta\phi}_{\vec{k}_3}(\eta)|0\rangle &= \langle 0|\hat{\delta\phi}^{\rm I}_{\vec{k}_1}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_2}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_3}(\eta)|0\rangle \\ &- i\int d\eta'\langle 0|\left[\hat{\delta\phi}^{\rm I}_{\vec{k}_1}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_2}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_3}(\eta), \hat{H}^{\rm I}_{\rm int}(\eta')\right]|0\rangle \\ &+ \mathcal{O}(H^2_{\rm int})\,,\end{aligned}$$

Calculation of the three-point correlation function

$$\langle 0|\hat{\mathcal{R}}_{\vec{k}_1}\hat{\mathcal{R}}_{\vec{k}_2}\hat{\mathcal{R}}_{\vec{k}_3}|0\rangle = \left(-\frac{a}{z}\right)^3 \Big[\langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{k}_2}\hat{\delta\phi}_{\vec{k}_3}|0\rangle$$
$$+ \left(-\frac{3}{2}+3\frac{V_\phi\, a^2}{\kappa\, p_\phi\, \pi_a}-\frac{\sqrt{\kappa}}{4}\frac{z}{a}\right)\left(-\frac{a}{z}\right)\int\frac{d^3p}{(2\pi)^3}\langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{k}_2}\hat{\delta\phi}_{\vec{p}}\hat{\delta\phi}_{\vec{k}_3-\vec{p}}|0\rangle + (\vec{k}_1\leftrightarrow\vec{k}_3) + (\vec{k}_2\leftrightarrow\vec{k}_3)$$
$$+\ \cdots\Big]\,.$$

$$\langle 0|\hat{\delta\phi}_{\vec{k}_1}(\eta)\hat{\delta\phi}_{\vec{k}_2}(\eta)\hat{\delta\phi}_{\vec{k}_3}(\eta)|0\rangle = \langle 0|U^\dagger(\eta,\eta_0)\,\hat{\delta\phi}^{\rm I}_{\vec{k}_1}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_2}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_3}(\eta)\,U(\eta,\eta_0)|0\rangle$$

$$U(\eta,\eta_0) = T\exp\left(-i\int_{\eta_0}^{\eta}{\rm d}\eta'\,\hat{H}^{\rm I}_{\rm int}(\eta')\right)$$

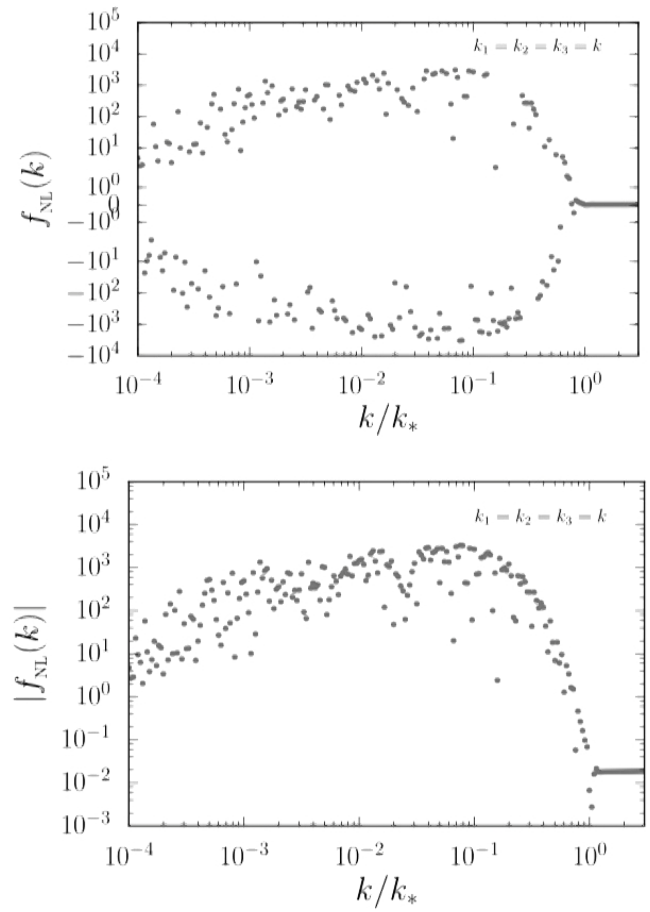
$$\begin{aligned}\langle 0|\hat{\delta\phi}_{\vec{k}_1}(\eta)\hat{\delta\phi}_{\vec{k}_2}(\eta)\hat{\delta\phi}_{\vec{k}_3}(\eta)|0\rangle &= \langle 0|\hat{\delta\phi}^{\rm I}_{\vec{k}_1}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_2}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_3}(\eta)|0\rangle \\ &- i\int d\eta'\langle 0|\left[\hat{\delta\phi}^{\rm I}_{\vec{k}_1}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_2}(\eta)\hat{\delta\phi}^{\rm I}_{\vec{k}_3}(\eta),\hat{H}^{\rm I}_{\rm int}(\eta')\right]|0\rangle \\ &+\ \mathcal{O}(H^2_{\rm int})\,,\end{aligned}$$

Non-Gaussianity usually parameterized in terms of a dimensionless quantity $f_{NL}(k_1, k_2, k_3)$ **, defined as:**

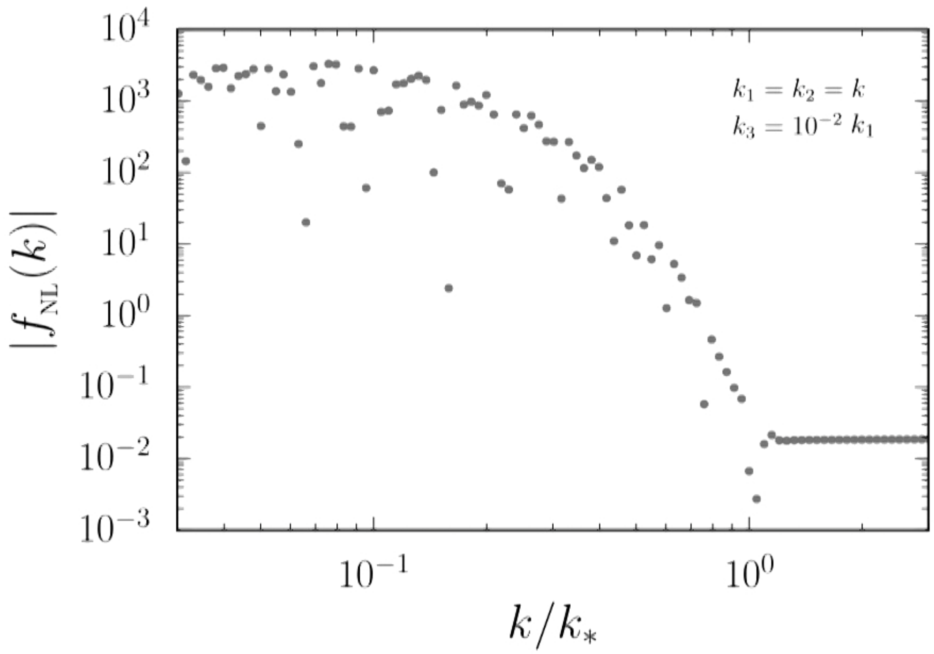
$$B_{\mathcal{R}}(k_1, k_2, k_3) =: -\frac{6}{5} f_{NL}(k_1, k_2, k_3) \times (\Delta_{k_1}\Delta_{k_2} + \Delta_{k_1}\Delta_{k_3} + \Delta_{k_2}\Delta_{k_3})$$

Where $\Delta_k := \frac{2\pi^2}{k^3} P_{\mathcal{R}}(k)$

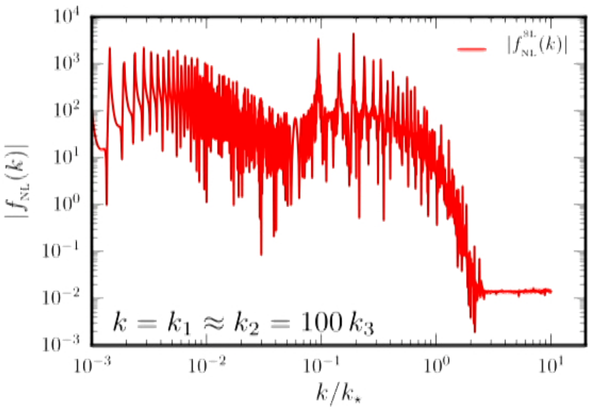
Equilateral configuration:

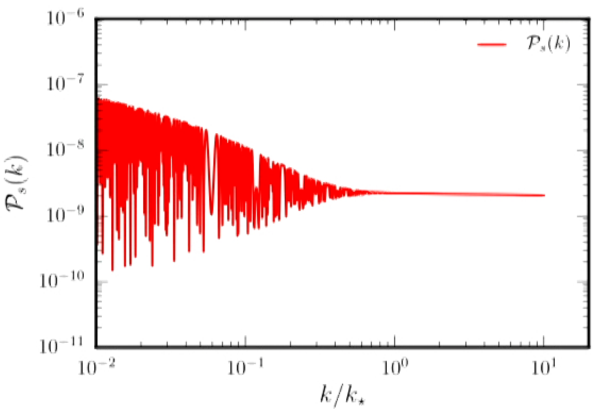


One squeezed configuration:



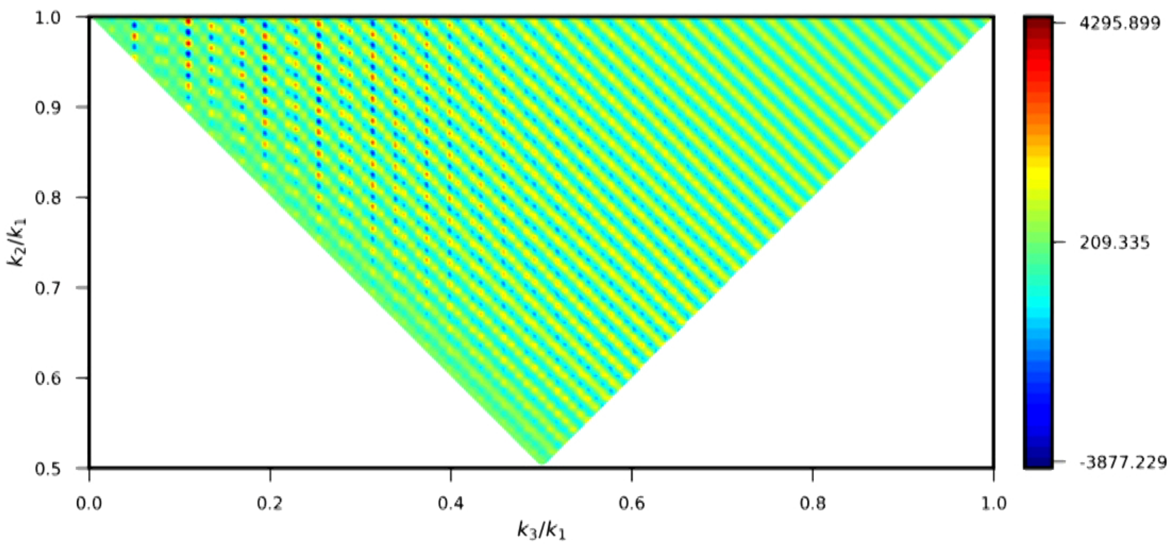
Most important aspects:





- Oscillatory
- Non-Gaussianity significantly more sensitive to the bounce!
- Exponential growth for small k's

The shape: peaked in squeezed (but not too squeezed) configurations



Understanding the origin of the exponential growth:

Remember:

$$\langle 0|\delta\hat{\phi}_{\vec{k}_1}(\eta)\delta\hat{\phi}_{\vec{k}_2}(\eta)\delta\hat{\phi}_{\vec{k}_3}(\eta)|0\rangle = \langle 0|\delta\hat{\phi}^{\rm I}_{\vec{k}_1}(\eta)\delta\hat{\phi}^{\rm I}_{\vec{k}_2}(\eta)\delta\hat{\phi}^{\rm I}_{\vec{k}_3}(\eta)|0\rangle - i\int d\eta'\langle 0|\left[\delta\hat{\phi}^{\rm I}_{\vec{k}_1}(\eta)\delta\hat{\phi}^{\rm I}_{\vec{k}_2}(\eta)\delta\hat{\phi}^{\rm I}_{\vec{k}_3}(\eta), \hat{H}^{\rm I}_{\rm int}(\eta')\right]|0\rangle + \mathcal{O}(H^2_{\rm int})\,,$$

Involves integrals of the type:

$$\int d\eta\, f(\eta)\, \frac{1}{a(\eta)^n}\, e^{i(k_1+k_2+k_3)\,\eta}$$

other background variables $(\phi(\eta), p_\phi(\eta), \pi_a(\eta))$

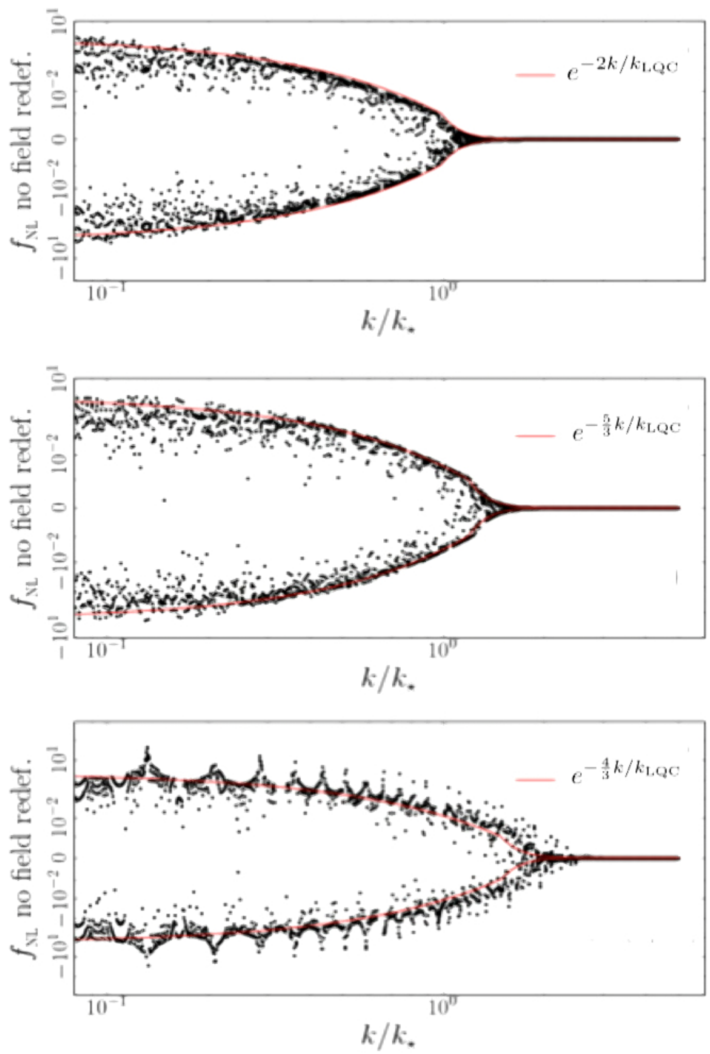
Cauchy's residues theorem + asymptotic techniques:

Integral dominated by the pole with smallest imaginary part

This is the pole in the scale factor: $a(\eta)$

Close to the bounce: $a(\eta) = a(\eta_B) + \frac{1}{2!}\, a''(\eta_B)\, \eta^2 + \cdots$

Poles: $\eta = \pm i\, \frac{1}{\sqrt{\alpha}}$ **with** $\alpha = \frac{1}{2} \frac{a''(\eta_B)}{a(\eta_B)} = \frac{k^2_{\rm LQC}}{2}$


f_{NL} no field redef.
$e^{-2k/k_{LQC}}$
k/k_*
f_{NL} no field redef.
$e^{-\frac{5}{3}k/k_{LQC}}$
k/k_*
f_{NL} no field redef.
$e^{-\frac{4}{3}k/k_{LQC}}$
k/k_*

Observational constraints

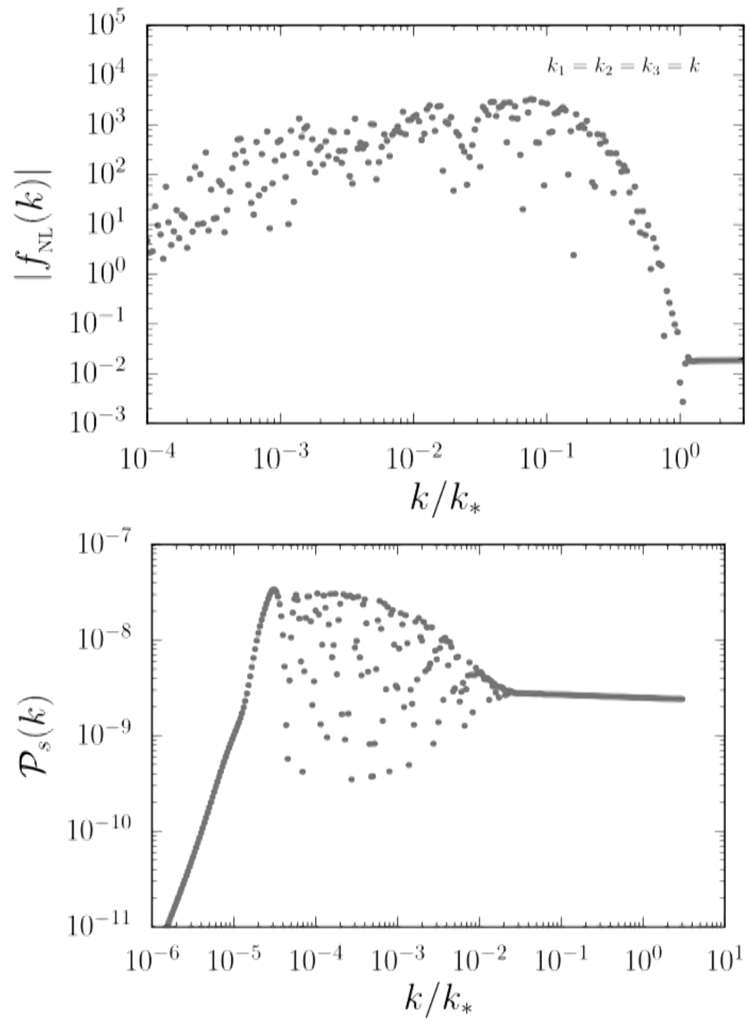
Observational constraints on f_{NL} **(PLANCK 2015) are obtained for scale invariant** $f_{NL}(k_1, k_2, k_3)$

In that case $f_{NL} \lesssim 10$ **for** $\ell \gtrsim 500$

Error bars grow as $1/\ell$ **for small** ℓ

Much weaker constraints for scale dependent $f_{NL}(k_1, k_2, k_3)$ **, as the one we have obtained, particularly for low multipoles**

Hence, our results are OK with observation for large enough ϕ_B


$k_1 = k_2 = k_3 = k$
$|f_{NL}(k)|$
k/k_*
$\mathcal{P}_s(k)$
k/k_*

Is perturbation theory under control?

Higher order corrections to the two-point function:

$$\begin{aligned}\langle 0|\hat{\mathcal{R}}_{\vec{k}_1}\hat{\mathcal{R}}_{\vec{k}_2}|0\rangle &= \left(-\frac{a}{z}\right)^2 \langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{k}_2}|0\rangle \\ &+ 2\left(-\frac{a}{z}\right)^3 \left[-\frac{3}{2}+3\frac{V_\phi\, a^2}{\kappa\, p_\phi\, \pi_a}-\frac{\sqrt{\kappa}}{4}\frac{z}{a}\right]\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{p}}\hat{\delta\phi}_{\vec{k}_2-\vec{p}}|0\rangle \\ &+ \left(-\frac{a}{z}\right)^4 \left[-\frac{3}{2}+3\frac{V_\phi\, a^2}{\kappa\, p_\phi\, \pi_a}-\frac{\sqrt{\kappa}}{4}\frac{z}{a}\right]^2\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{\mathrm{d}^3 q}{(2\pi)^3}\langle 0|\hat{\delta\phi}_{\vec{p}}\hat{\delta\phi}_{\vec{k}_1-\vec{p}}\hat{\delta\phi}_{\vec{q}}\hat{\delta\phi}_{\vec{k}_2-\vec{q}}|0\rangle \\ &+ \cdots\end{aligned}$$

Is perturbation theory under control?

Higher order corrections to the two-point function:

$$
\begin{aligned}
\langle 0|\hat{\mathcal{R}}_{\vec{k}_1}\hat{\mathcal{R}}_{\vec{k}_2}|0\rangle &= \left(-\frac{a}{z}\right)^2 \langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{k}_2}|0\rangle \\
&+ 2\left(-\frac{a}{z}\right)^3 \left[-\frac{3}{2}+3\frac{V_\phi\, a^2}{\kappa\, p_\phi\, \pi_a}-\frac{\sqrt{\kappa}}{4}\frac{z}{a}\right]\int\frac{\mathrm{d}^3p}{(2\pi)^3}\langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{p}}\hat{\delta\phi}_{\vec{k}_2-\vec{p}}|0\rangle \\
&+ \left(-\frac{a}{z}\right)^4 \left[-\frac{3}{2}+3\frac{V_\phi\, a^2}{\kappa\, p_\phi\, \pi_a}-\frac{\sqrt{\kappa}}{4}\frac{z}{a}\right]^2\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{\mathrm{d}^3q}{(2\pi)^3}\langle 0|\hat{\delta\phi}_{\vec{p}}\hat{\delta\phi}_{\vec{k}_1-\vec{p}}\hat{\delta\phi}_{\vec{q}}\hat{\delta\phi}_{\vec{k}_2-\vec{q}}|0\rangle \\
&+ \cdots
\end{aligned}
$$

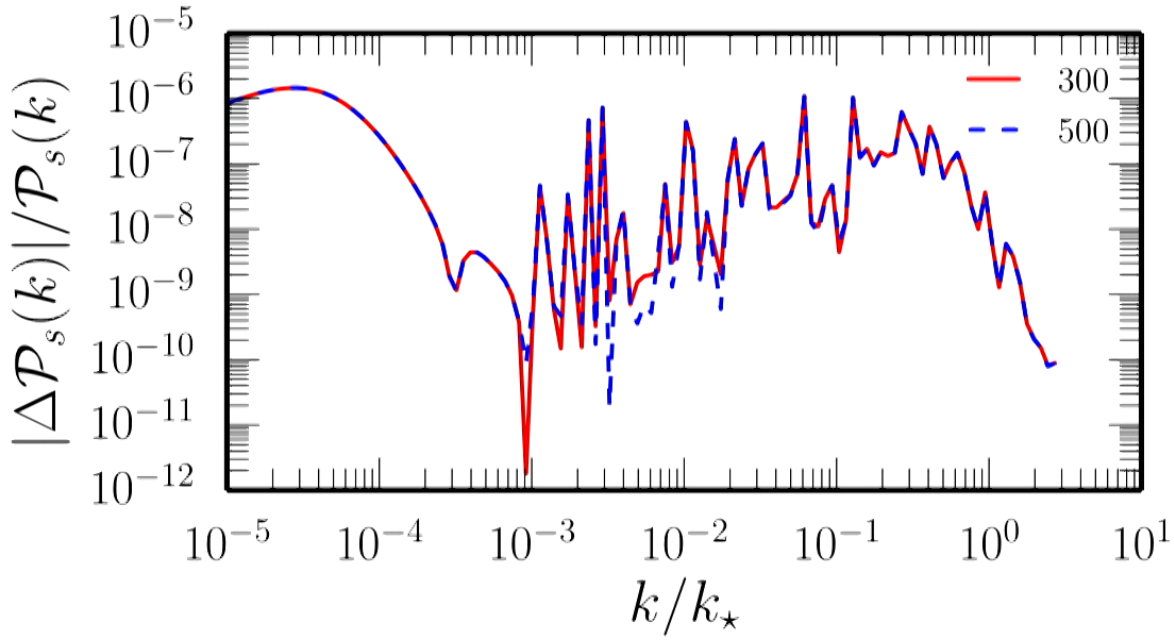
$$
\langle 0|\hat{\mathcal{R}}_{\vec{k}_1}\hat{\mathcal{R}}_{\vec{k}_2}|0\rangle = (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2)\,\frac{2\pi^2}{k_1^3}\,\left[P_{\mathcal{R}}(k_1)+\Delta P_{\mathcal{R}}(k_1)\right]
$$

Is perturbation theory under control?

Higher order corrections to the two-point function:

$$\begin{aligned}\langle 0|\hat{\mathcal{R}}_{\vec{k}_1}\hat{\mathcal{R}}_{\vec{k}_2}|0\rangle &= \left(-\frac{a}{z}\right)^2 \langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{k}_2}|0\rangle \\ &+ 2\left(-\frac{a}{z}\right)^3\left[-\frac{3}{2}+3\frac{V_\phi\, a^2}{\kappa\, p_\phi\, \pi_a}-\frac{\sqrt{\kappa}}{4}\frac{z}{a}\right]\int \frac{\mathrm{d}^3p}{(2\pi)^3}\langle 0|\hat{\delta\phi}_{\vec{k}_1}\hat{\delta\phi}_{\vec{p}}\hat{\delta\phi}_{\vec{k}_2-\vec{p}}|0\rangle \\ &+ \left(-\frac{a}{z}\right)^4\left[-\frac{3}{2}+3\frac{V_\phi\, a^2}{\kappa\, p_\phi\, \pi_a}-\frac{\sqrt{\kappa}}{4}\frac{z}{a}\right]^2\int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{\mathrm{d}^3q}{(2\pi)^3}\langle 0|\hat{\delta\phi}_{\vec{p}}\hat{\delta\phi}_{\vec{k}_1-\vec{p}}\hat{\delta\phi}_{\vec{q}}\hat{\delta\phi}_{\vec{k}_2-\vec{q}}|0\rangle \\ &+ \cdots\end{aligned}$$

$$\langle 0|\hat{\mathcal{R}}_{\vec{k}_1}\hat{\mathcal{R}}_{\vec{k}_2}|0\rangle = (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2)\,\frac{2\pi^2}{k_1^3}\,[P_{\mathcal{R}}(k_1)+\Delta P_{\mathcal{R}}(k_1)]$$


$|\Delta\mathcal{P}_s(k)|/\mathcal{P}_s(k)$
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
300
500
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
$k/k_\star$

Summary:

Bounce close to Planck scale boosts f_{NL}, in a strongly scale dependent way.

Summary:

Bounce close to Planck scale boosts f_{NL}, in a strongly scale dependent way.

Exponential growing dictated by the new scale

Ok with standard inflation for large k

Summary:

Bounce close to Planck scale boosts f_{NL}, in a strongly scale dependent way.

Exponential growing dictated by the new scale

Ok with standard inflation for large k

Perturbation theory under control

Ok with observational constraints for appropriate choice of ϕ_B

Summary:

Bounce close to Planck scale boosts f_{NL}, in a strongly scale dependent way.

Exponential growing dictated by the new scale

Ok with standard inflation for large k

Perturbation theory under control

Ok with observational constraints for appropriate choice of ϕ_B

Room for new effects

5. Large scale anomalies

I.A., Bolliet, Vijayakumar: In Progress…

Planck 2015 results. XVI. Isotropy and statistics of the CMB

Planck Collaboration: P. A. R. Ade[89], N. Aghanim[60], Y. Akrami[65,103], P. K. Aluri[55], M. Arnaud[75], M. Ashdown[72,6], J. Aumont[60], C. Baccigalupi[88], A. J. Banday[100,9,*], R. B. Barreiro[67], N. Bartolo[32,68], S. Basak[88], E. Battaner[101,102], K. Benabed[61,99], A. Benoit[58], A. Benoit-Lévy[26,61,99], J.-P. Bernard[100,9], M. Bersanelli[35,49], P. Bielewicz[85,9,88], J. J. Bock[69,11], A. Bonaldi[70], L. Bonavera[67], J. R. Bond[8], J. Borrill[14,94], F. R. Bouchet[61,92], F. Boulanger[60], M. Bucher[1], C. Burigana[48,33,50], R. C. Butler[48], E. Calabrese[97], J.-F. Cardoso[76,1,61], B. Casaponsa[67], A. Catalano[77,74], A. Challinor[64,72,12],

ABSTRACT

We test the statistical isotropy and Gaussianity of the cosmic microwave background (CMB) anisotropies using observations made by the *Planck* satellite. Our results are based mainly on the full *Planck* mission for temperature, but also include some polarization measurements. In particular, we consider the CMB anisotropy maps derived from the multi-frequency *Planck* data by several component-separation methods. For the temperature anisotropies, we find excellent agreement between results based on these sky maps over both a very large fraction of the sky and a broad range of angular scales, establishing that potential foreground residuals do not affect our studies. Tests of skewness, kurtosis, multi-normality, N-point functions, and Minkowski functionals indicate consistency with Gaussianity, while a power deficit at large angular scales is manifested in several ways, for example low map variance. The results of a peak statistics analysis are consistent with the expectations of a Gaussian random field. The "Cold Spot" is detected with several methods, including map kurtosis, peak statistics, and mean temperature profile. We thoroughly probe the large-scale dipolar power asymmetry, detecting it with several independent tests, and address the subject of a posteriori correction. Tests of directionality suggest the presence of angular clustering from large to small scales, but at a significance that is dependent on the details of the approach. We perform the first examination of polarization data, finding the morphology of stacked peaks to be consistent with the expectations of statistically isotropic simulations. Where they overlap, these results are consistent with the *Planck* 2013 analysis based on the nominal mission data and provide our most thorough view of the statistics of the CMB fluctuations to date.

1. Introduction

foreground-cleaned CMB maps, it was generally considered that the case for anomalous features in the CMB had been strengthened. Hence, such anomalies have attracted considerable attention in the community, since they could be the visible traces of fundamental physical processes occurring in the early Universe.

Observed anomalies at large angular scales (WMAP, PLANCK)

- **Hemispherical anomaly**
- **Quadrupole-octopole alignment**
- **Low power @ large scales (more prominent in angular space)**
- **Parity anomaly**
- **...**

Warning: Significance $\lesssim 3\sigma$ could be just a statistical fluke

But observed in different ways and by different teams (WMAP, PALNCK, etc)

Observed anomalies at large angular scales (WMAP, PLANCK)

- Hemispherical anomaly
- Quadrupole-octopole alignment
- Low power @ large scales (more prominent in angular space)
- Parity anomaly
- ...

Warning: Significance $\lesssim 3\sigma$ could be just a statistical fluke

But observed in different ways and by different teams (WMAP, PALNCK, etc)

Message: all these anomalies have a common feature. They seem to require correlations between different modes:

$$\Delta T(\hat{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n})$$

anisotropies

$$\langle a_{\ell m} a^{\star}_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell + \text{non-diagonal terms}$$

Additionally, effects only appears at large angles (low ℓ's): we may need a scale-dependent non-diagonal (anisotropic) terms!

Recent discussions: we do not need anisotropic physics to modify the statistics in our observable universe. Large correlations between modes can do the job

(Adhikari, Brahma, Bartolo, Bramante, Byrnes, Carrol, Dai, Deutsch, Dimastrogiovanni, Erickcen, **Hui**, Jeong, Kamionkowski, LoVerde, Matarrese, Mota, Nelson, Nurmi, Peloso, Pullen, Ricciardone, Shandera, **Schmidt**, Tasinato, Thorsrud, Urban,...)

A typical realization shows larger anisotropies if the distribution is non-Gaussian

Non-Gaussian modulation of the power spectrum

Non-Gaussian modulation of the power spectrum

In k-space:

Non-Gaussianity

$$\langle \mathcal{R}_{\vec{k}_1} \mathcal{R}^{\star}_{\vec{k}_2} \rangle = P_{\mathcal{R}}(k_1) \Big[(2\pi)^3 \delta(\vec{k}_1 - \vec{k}_2) + f_{NL}(\vec{k}_1, \vec{k}_2, \vec{k}_L)\, \mathcal{R}_{\vec{k}_L} \Big]$$

Where $\vec{k}_L = -(\vec{k}_1 + \vec{k}_2)$

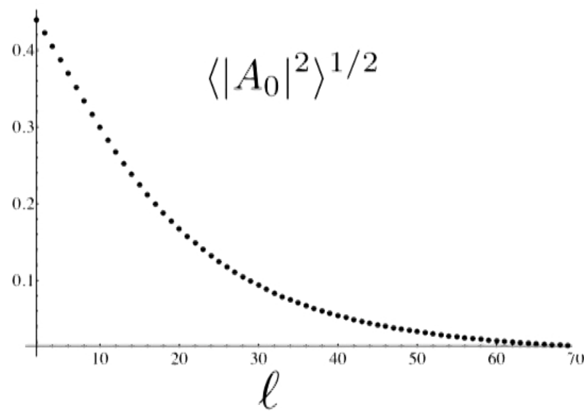
In angular space:

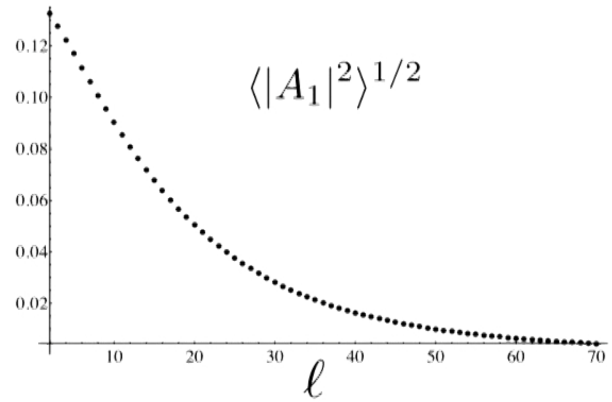
Wigner 3j-symbols

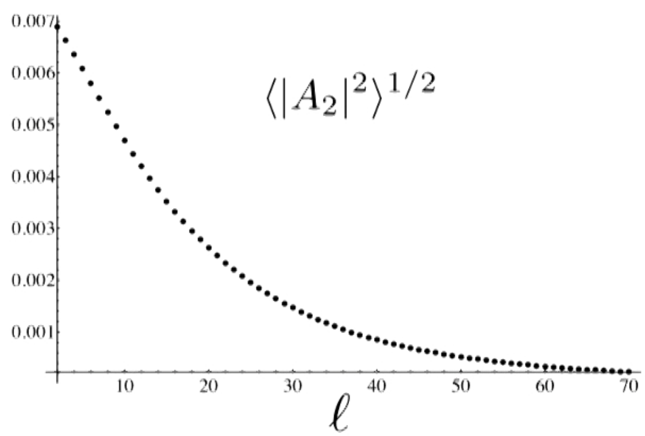
$$\langle a_{\ell m} a^{\star}_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'}\, C_\ell + \sum_{L,M} A_L(\ell, \ell')\, \mathcal{G}^{\ell\ell' L}_{-mm' M}\, (C_\ell + C_{\ell'})$$

off-diagonal = anisotropies

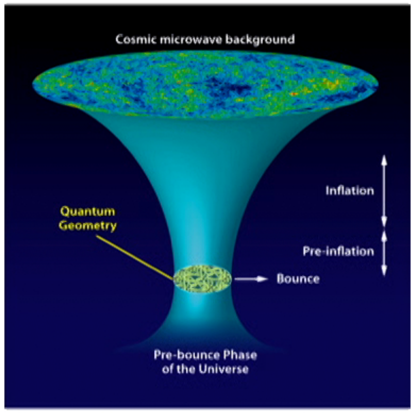
A sample of results for the modulation amplitudes:



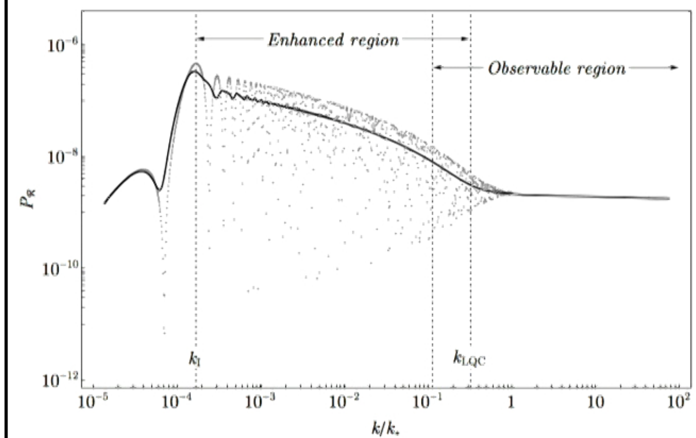




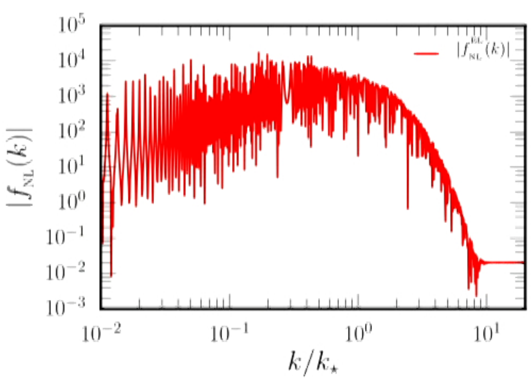
LQC Bounce + inflation



Primordial Power Spectrum



Bounce and Non-Gaussianity



CMB anomalies