Title: Space(-time) structure in models of loop quantum gravity

Date: Jun 27, 2017 02:00 PM

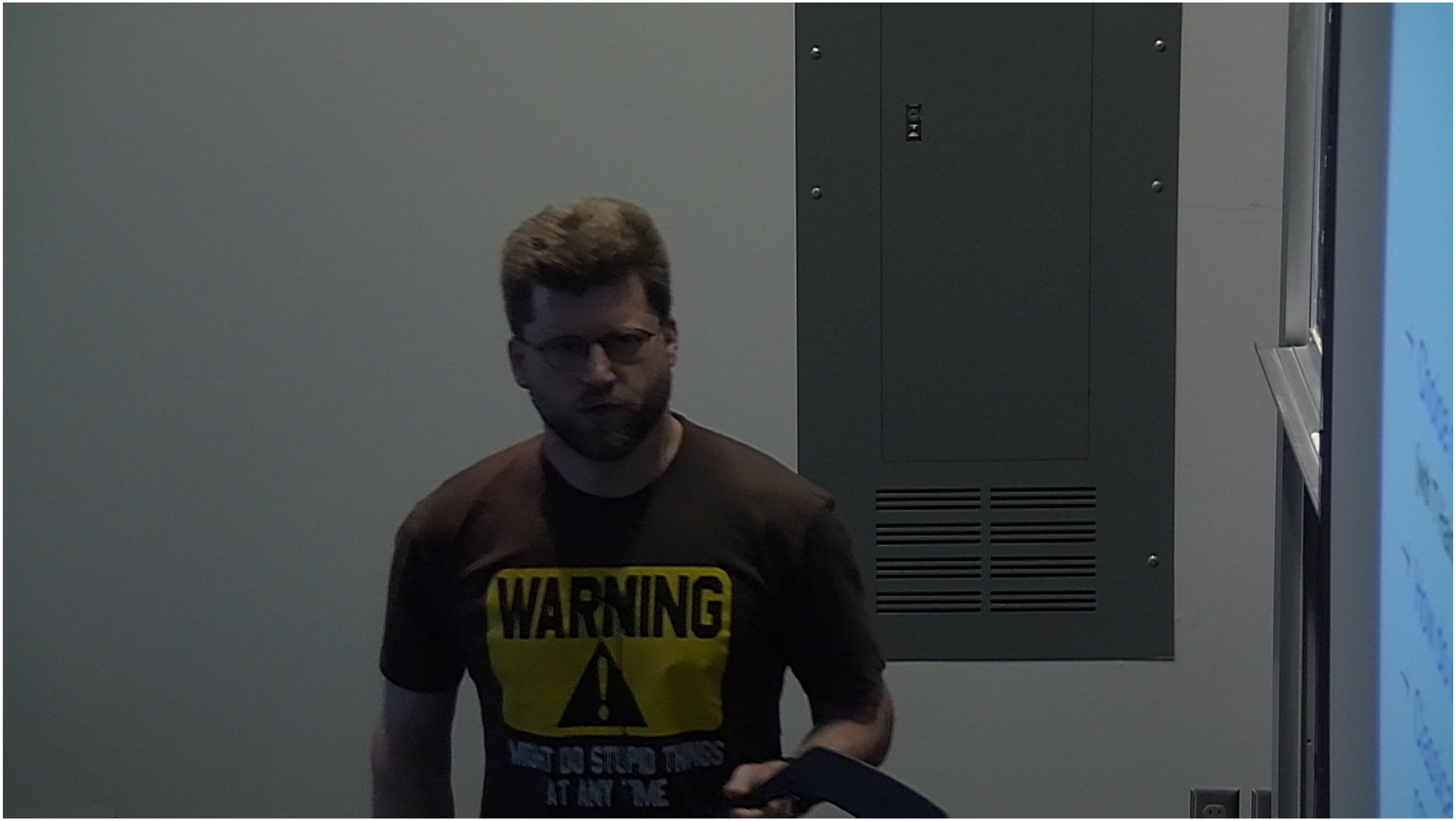
URL: http://pirsa.org/17060102

Abstract: Loop quantum gravity has suggested modifications of the dynamics of cosmological models that could lead to a bounce at large curvature. However, the same modifications may alter the gauge structure of the theory, which is related to the structure of space-time. In a large class of examples the space-time structure has been derived and shown to imply signature change just in the bounce region. The picture of a cyclic universe with a deterministic bounce then has to be replaced by the scenario of a non-singular beginning some finite time ago.

Space(-time) structure in models of loop quantum gravity

Martin Bojowald

The Pennsylvania State University
Institute for Gravitation and the Cosmos
University Park, PA


WARNING

PENNSTATE

Facts vs. Alternative Facts

"There is a huge body of work in Loop Quantum Cosmology comprising of several thousand journal articles."

[from an abstract at this workshop]

"loop quantum cos ×

https://inspirehep.net/search?ln=en&ln=en&p="loop+quantum+cosmology"+OR+"LQC"&of=hb&action_search=Search&sf=earliestdate&so=d&rm=&rg=25&sc

iNSPIRE HEP

Welcome to INSPIRE, the High Energy Physics information system. Please direct questions, comments or concerns to feedback@inspirehep.net.

HEP :: HEPNAMES :: INSTITUTIONS :: CONFERENCES :: JOBS :: EXPERIMENTS :: JOURNALS :: HELP

"loop quantum cosmology" OR "LQC" | Brief format | Search | Easy Search | Advanced Search

find j "Phys.Rev.Lett.,105*" :: more

Sort by: earliest date | desc. | - or rank by -

Display results: 25 results | single list

HEP **765** records found 1 - 25 jump to record: 1 Search took 0.11 seconds.

1. **Diffeomorphism invariant cosmological sector in loop quantum gravity**
Christopher Beetle, Jonathan Steven Engle, Matthew Ernest Hogan, Phillip Mendonca. Jun 7, 2017. 25 pp.
e-Print: **arXiv:1706.02424 [gr-qc]** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record

2. **Viscous Modified Chaplygin Gas in Classical and Loop Quantum Cosmology**
D. Aberkane, N. Mebarki, S. Benchikh (Constantine U.). 2017. 5 pp.
Published in **Chin.Phys.Lett. 34 (2017) no.6, 069801**
DOI: 10.1088/0256-307X/34/6/069801
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Detailed record

3. **Pre-inflationary universe in loop quantum cosmology**
Tao Zhu (Zhejiang U. Tech. & Baylor U.), Anzhong Wang (Baylor U. & Zhejiang U. Tech.), Gerald Cleaver (Baylor U.), Klaus Kirsten, Qin Sheng (Baylor U., Math. Dept.). May 21, 2017. 32 pp.
e-Print: **arXiv:1705.07544 [gr-qc]** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record

4. **Measuring the effects of Loop Quantum Cosmology in the CMB data**
Spyros Basilakos (RCAAM, Academy of Athens), Vahid Kamali, Ahmad Mehrabi (Bou Ali Sina U.). May 16, 2017. 7 pp.
e-Print: **arXiv:1705.05585 [gr-qc]** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record

5. **Testing different approaches to quantum gravity with cosmology: An overview**

Space(-time) structure – p. 4

PENNSTATE

Space-time structure

Easy to avoid divergences by introducing discreteness/bounded functions, in particular in minisuperspace models.

- → Consistent with covariance? If not, how can one avoid low-energy problems? [J Polchinski: arXiv:1106.6346]
- → How exactly are singularity theorems evaded? Example: “Bounce” in some models of loop quantum cosmology without violating energy conditions.

Many open questions at different levels in loop quantum cosmology. Related to covariance:

- → Choice of (internal) time.
- → Averaging volume in minisuperspace truncation.
- → Space(-time) structure.

Minisuperspace deparameterization

Quantum constraint of the form $\hat{C} = \hat{p}_\phi^2 - \hat{H}^2$.

If H does not depend on ϕ, can write constraint equation $\hat{C}\psi = 0$ for states as "evolution" equation

$$-\hat{p}_\phi \psi = i\hbar \frac{\partial \psi}{\partial \phi} = \pm |\hat{H}| \psi$$

→ Choice of ϕ as time affects quantum corrections.
[with T Halnon: arXiv:1612.00353]

→ How do we choose initial states for ϕ-"evolution"?

→ Classical constraint strongly restricted by reduction from covariant theory.

Restrictions on quantum corrections in minisuperspace model?

PENNSTATE

Minisuperspace deparameterization

Quantum constraint of the form $\hat{C} = \hat{p}_\phi^2 - \hat{H}^2$.

If H does not depend on ϕ, can write constraint equation $\hat{C}\psi = 0$ for states as "evolution" equation

$$-\hat{p}_\phi \psi = i\hbar \frac{\partial \psi}{\partial \phi} = \pm |\hat{H}| \psi$$

→ Choice of ϕ as time affects quantum corrections.
[with T Halnon: arXiv:1612.00353]

→ How do we choose initial states for ϕ-"evolution"?

→ Classical constraint strongly restricted by reduction from covariant theory.

Restrictions on quantum corrections in minisuperspace model?

PENNSTATE

Model minisuperspace model

[with S Brahma: arXiv:1509.00640]

Lagrangian $L = \int \mathrm{d}^3x \left(\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}|\nabla\phi|^2 - W(\phi)\right)$ reduced to

$$L_{\rm mini} = V_0 \left(\frac{1}{2}\dot{\phi}^2 - W(\phi)\right)$$

with $V_0 = \int \mathrm{d}^3x$, ϕ spatially constant.

Momentum $p = \partial L_{\rm mini}/\partial\dot{\phi} = V_0\dot{\phi}$.

Hamiltonian

$$H_{\rm mini} = \frac{1}{2}\frac{p^2}{V_0} + V_0 W(\phi)$$

quantized to

$$\hat{H}_{\rm mini} = \frac{1}{2}\frac{\hat{p}^2}{V_0} + V_0 W(\hat{\phi})$$

PENNSTATE

Minisuperspace deparameterization

Quantum constraint of the form $\hat{C} = \hat{p}_\phi^2 - \hat{H}^2$.

If H does not depend on ϕ, can write constraint equation $\hat{C}\psi = 0$ for states as "evolution" equation

$$-\hat{p}_\phi \psi = i\hbar \frac{\partial \psi}{\partial \phi} = \pm |\hat{H}| \psi$$

→ Choice of ϕ as time affects quantum corrections. [with T Halnon: arXiv:1612.00353]

→ How do we choose initial states for ϕ-"evolution"?

→ Classical constraint strongly restricted by reduction from covariant theory.

Restrictions on quantum corrections in minisuperspace model?

PENNSTATE

Model minisuperspace model

[with S Brahma: arXiv:1509.00640]

Lagrangian $L = \int \mathrm{d}^3x \left(\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}|\nabla\phi|^2 - W(\phi)\right)$ reduced to

$$L_{\mathrm{mini}} = V_0 \left(\frac{1}{2}\dot{\phi}^2 - W(\phi)\right)$$

with $V_0 = \int \mathrm{d}^3x$, ϕ spatially constant.

Momentum $p = \partial L_{\mathrm{mini}}/\partial\dot{\phi} = V_0\dot{\phi}$.

Hamiltonian

$$H_{\mathrm{mini}} = \frac{1}{2}\frac{p^2}{V_0} + V_0 W(\phi)$$

quantized to

$$\hat{H}_{\mathrm{mini}} = \frac{1}{2}\frac{\hat{p}^2}{V_0} + V_0 W(\hat{\phi})$$

PENNSTATE

Effective potentials

Minisuperspace:

$$W_{\rm eff}^{\rm mini}(\phi) = W(\phi) + \frac{1}{2V_0}\hbar\sqrt{W''(\phi)}$$

Coleman–Weinberg potential: [S Coleman, E Weinberg: PRD 7 (1973) 1888]

$$W_{\rm eff}(\phi) = W(\phi) + \frac{1}{2}\hbar \int \frac{{\rm d}^4 k}{(2\pi)^4} \log\left(1 + \frac{W''(\phi)}{||\mathbf{k}||^2}\right)$$

After k^0-integration (or canonical derivation):

$$W_{\rm eff}(\phi) = W(\phi) + \frac{1}{2}\hbar \int \frac{{\rm d}^3 k}{(2\pi)^3} \left(\sqrt{|\vec{k}|^2 + W''(\phi)} - |\vec{k}|\right)$$

[with S Brahma: arXiv:1411.3636]

PENNSTATE

Comparison

Minisuperspace potential $W(\phi) + \frac{1}{2V_0}\hbar\sqrt{W''(\phi)}$ as infrared-contribution from quantum field theory: Integrate

$$\frac{1}{2}\hbar \int \frac{\mathrm{d}^3k}{(2\pi)^3} \left(\sqrt{|\vec{k}|^2 + W''(\phi)} - |\vec{k}| \right)$$

over $|\vec{k}| \leq k_{\max} = 2\pi/V_0^{1/3}$

$$W_{\text{eff}}(\phi) \approx W(\phi) + \frac{\hbar}{12\pi^2} k_{\max}^3 \sqrt{W''(\phi)} = W(\phi) + \frac{2\pi}{3V_0}\hbar\sqrt{W''(\phi)}$$

$\longrightarrow$ Minisuperspace truncation can capture some quantum effects of full theory, as long as $V_0 \not\to \infty$.

$\longrightarrow$ Covariance in minisuperspace models?
Fixed infrared scale breaks local Poincaré transformations.

PENNSTATE

Models of loop quantum gravity

Isotropic connection $A_a^i = c\delta_a^i$, $E_j^b = p\delta_j^b$ with $c = \gamma\dot{a}$, $|p| = a^2$

Friedmann equation (flat space)

$$-\frac{c^2}{\gamma^2|p|} + \frac{8\pi G}{3}\rho = 0$$

modified by using "holonomies"

$$\frac{c^2}{|p|} \mapsto \frac{\sin(\ell c/\sqrt{|p|})^2}{\ell^2}$$

Taken *in isolation,* holonomy modifications imply a "bounce" of isotropic models:

$$\frac{\sin(\ell c/\sqrt{|p|})^2}{\gamma^2\ell^2} = \frac{8\pi G}{3}\rho$$

PENNSTATE

Models of loop quantum gravity

Isotropic connection $A_a^i = c\delta_a^i$, $E_j^b = p\delta_j^b$ with $c = \gamma\dot{a}$, $|p| = a^2$

Friedmann equation (flat space)

$$-\frac{c^2}{\gamma^2|p|} + \frac{8\pi G}{3}\rho = 0$$

modified by using "holonomies"

$$\frac{c^2}{|p|} \mapsto \frac{\sin(\ell c/\sqrt{|p|})^2}{\ell^2}$$

Taken *in isolation,* holonomy modifications imply a "bounce" of isotropic models:

$$\frac{\sin(\ell c/\sqrt{|p|})^2}{\gamma^2\ell^2} = \frac{8\pi G}{3}\rho$$

PENNSTATE

Models of loop quantum gravity

Isotropic connection $A_a^i = c\delta_a^i$, $E_j^b = p\delta_j^b$ with $c = \gamma\dot{a}$, $|p| = a^2$

Friedmann equation (flat space)

$$-\frac{c^2}{\gamma^2|p|} + \frac{8\pi G}{3}\rho = 0$$

modified by using "holonomies"

$$\frac{c^2}{|p|} \mapsto \frac{\sin(\ell c/\sqrt{|p|})^2}{\ell^2}$$

Taken *in isolation,* holonomy modifications imply a "bounce" of isotropic models:

$$\frac{\sin(\ell c/\sqrt{|p|})^2}{\gamma^2\ell^2} = \frac{8\pi G}{3}\rho$$

Higher-curvature contributions?

If unknown, should not trust full function $\sin^2(\ell c/\sqrt{|p|})/\ell^2$ but expand:

$$\frac{\sin(\ell c/\sqrt{|p|})^2}{\ell^2} \sim \frac{c^2}{|p|}\left(1-\frac{1}{3}\ell^2\frac{c^2}{|p|}+\cdots\right)$$

If $\ell \sim \ell_{\rm P}$, leading corrections $\ell_{\rm P}^2 c^2/|p| \sim \rho/\rho_{\rm P}$.

Same order as expected for higher-curvature terms.

Higher-derivative terms general feature in effective equations of quantum mechanics, state dependent.

$\longrightarrow$ Not clear whether "bounce" is generic if higher-derivative terms uncontrolled.

$\longrightarrow$ More controlled than specific dynamics:
Space-time structure.

PENNSTATE

Space(-time) structure

⟶ Lagrangian density and *measure* in

$$S[g] = \frac{1}{16\pi G} \int \mathrm{d}^4x \, \sqrt{|\det g|} \, (R[g] + \cdots)$$

may be subject to quantum corrections.

⟶ Quantum-field theory on curved space-time different from quantum gravity.

⟶ Covariance in canonical quantum gravity:
Quantum version of Dirac's hypersurface deformations.

PENNSTATE

QFT vs. QG

Perturbative inhomogeneity $A(x) = \bar{A} + \delta A(x)$, $h[A] := A(x)^2$.

Modified background dynamics: $\bar{A} \longrightarrow \ell^{-1}\sin(\ell\bar{A})$.

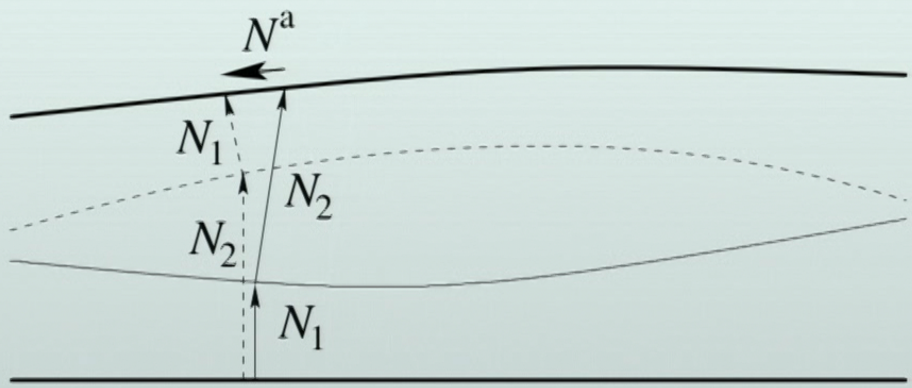
→ *Classical:* $h[\bar{A}, \delta A] = \bar{A}^2 + 2\bar{A}\delta A(x) + \delta A(x)^2$

→ *QFT on modified space-time:*
$h_\ell^{\rm QFT}[\bar{A}, \delta A] = \ell^{-2}\sin(\ell\bar{A})^2 + 2\bar{A}\delta A(x) + \delta A(x)^2$
[Martín-Benito, Garay, Mena Marugán; Ashtekar, Lewandowski, Kaminski; Dapor, Lewandowski, Puchta; Agulló, Ashtekar, Nelson]

→ *Effective quantum gravity:*
$h_\ell^{\rm QG}[\bar{A}, \delta A] = \ell^{-2}\sin(\ell\bar{A})^2 + F_\ell(\bar{A})\delta A(x) + G_\ell(\bar{A})\delta A(x)^2$ with $\lim_{\ell\to 0} F_\ell(\bar{A}) = 2\bar{A}$ and $\lim_{\ell\to 0} G_\ell(\bar{A}) = 1$.
Subject to *covariance conditions.*
$F_\ell/\bar{A}$ and G_ℓ comparable to $(\ell\bar{A})^{-2}\sin(\ell\bar{A})^2$.
[MB, Hossain, Kagan, Shankaranarayanan; Barrau, Cailleteau, Grain, Mielczarek; Wilson–Ewing]

PENNSTATE

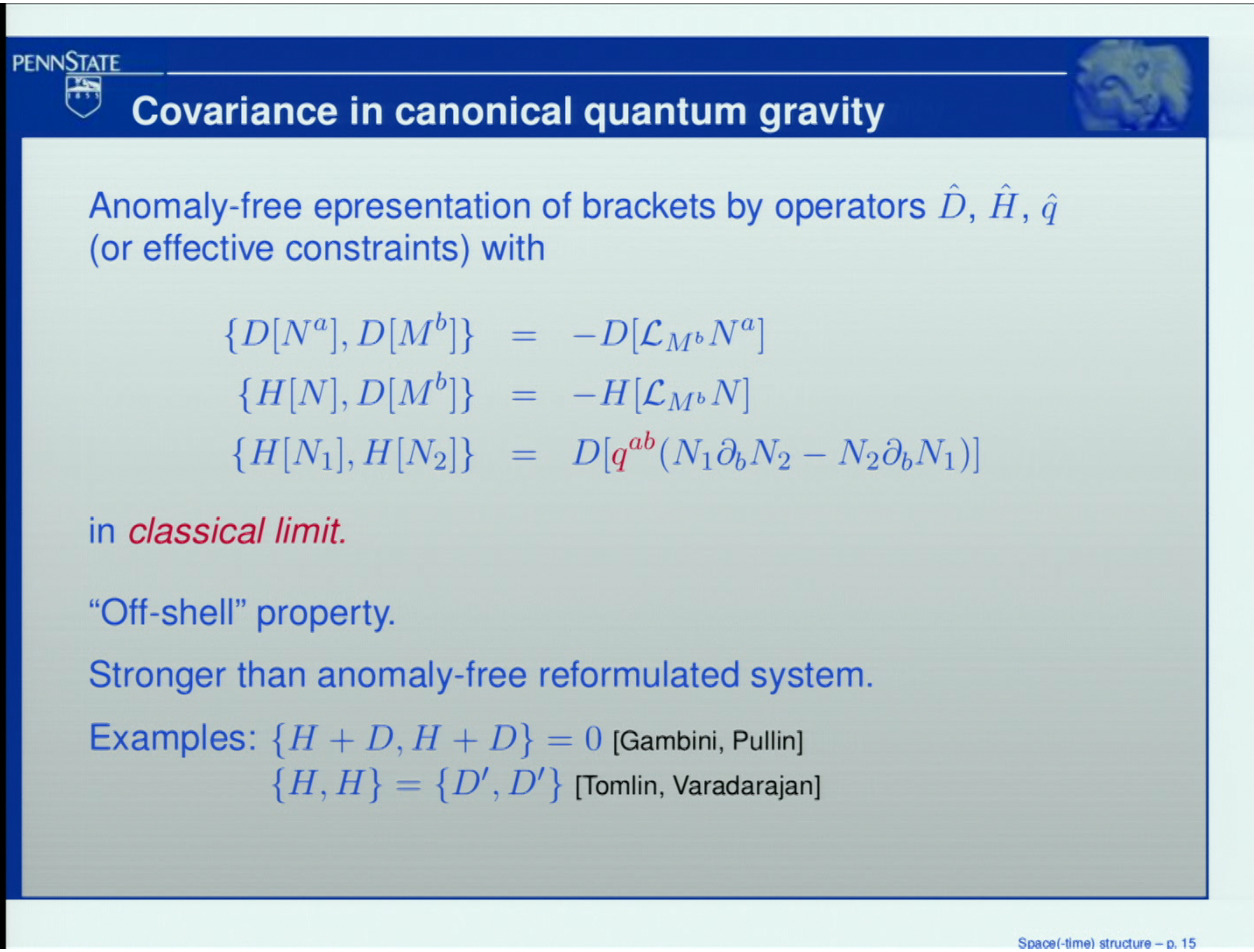
Covariance: Hypersurface deformations



Generators $D[N^a]$ (tangential deformations along $N^a(x)$) and $H[N]$ (normal deformations by $N(x)$) obey

$$
\begin{aligned}
[D[N^a], D[M^b]] &= -D[\mathcal{L}_{M^b} N^a] \\
[H[N], D[M^b]] &= -H[\mathcal{L}_{M^b} N] \\
[H[N_1], H[N_2]] &= D[q^{ab}(N_1 \partial_b N_2 - N_2 \partial_b N_1)]
\end{aligned}
$$

with induced metric q_{ab} on spatial slice. (Lie algebroid.)


PENNSTATE
Covariance in canonical quantum gravity
Anomaly-free epresentation of brackets by operators $\hat{D}$, $\hat{H}$, $\hat{q}$ (or effective constraints) with
$$\begin{aligned}\{D[N^a], D[M^b]\} &= -D[\mathcal{L}_{M^b} N^a]\\ \{H[N], D[M^b]\} &= -H[\mathcal{L}_{M^b} N]\\ \{H[N_1], H[N_2]\} &= D[q^{ab}(N_1\partial_b N_2 - N_2\partial_b N_1)]\end{aligned}$$
in *classical limit.*
"Off-shell" property.
Stronger than anomaly-free reformulated system.
Examples: $\{H+D, H+D\} = 0$ [Gambini, Pullin]
$\{H, H\} = \{D', D'\}$ [Tomlin, Varadarajan]
Space(-time) structure – p. 15

Model

Scalar field $\phi(x)$, momentum $p(x)$, one spatial dimension.

$$H[N] = \int \mathrm{d}x N \left(f(p) - \frac{1}{4}(\phi')^2 - \frac{1}{2}\phi\phi'' \right) \quad , \quad D[w] = \int \mathrm{d}x w \phi p'$$

Spatial diffeomorphisms:

$$\delta_w \phi = \{\phi, D[w]\} = -(w\phi)' \quad , \quad \delta_w p = \{p, D[w]\} = -wp'$$

H-bracket:

$$\{H[N], H[M]\} = D[\beta(p)(N'M - NM')]$$

with $\beta(p) = \frac{1}{2}\mathrm{d}^2 f/\mathrm{d}p^2$.

Lorentzian-type hypersurface deformations for $f(p) = p^2$.

PENNSTATE

Signature change

"Holonomy" modifications, $f(p) = p_0^2 \sin^2(p/p_0)$:

$$\beta(p) = \frac{1}{2}\mathrm{d}^2 f/\mathrm{d}p^2 = \cos(2p/p_0)$$

can be negative. At maximum of $f(p)$:

$$\{H[N], H[M]\} = D[-(N'M - NM')]$$

Euclidean signature:

- → Linear N and M give boosts for $\beta(p) = 1$: $\Delta x = v\Delta t$. Rotation if $\beta(p) = -1$: $\Delta x = -\theta\Delta y$ if y transversal to hypersurfaces.
- → Opposite sign if hypersurface-deformation brackets derived for Euclidean gravity.
- → Elliptic field equations if $\beta(p) < 0$.

PENNSTATE

Covariance

$\{H[N], D[w]\}$ does not close in the scalar model, but does so in several gravity versions: spherical symmetry and cosmological perturbations. [Reyes; Barrau, Cailleteau, Grain, Mielczarek]

Replacing $K^2 \longrightarrow f(K)$ modifies bracket

$$\{H[N_1], H[N_2]\} = D[\beta q^{ab}(N_1\partial_b N_2 - N_2\partial_b N_1)]$$

with

$$\beta(K) = \frac{1}{2}\mathrm{d}^2 f(K)/\mathrm{d}K^2 = \cos(2\ell K)$$

for $f(K) = \ell^{-2}\sin^2(\ell K)$.

Signature change: $\beta(K) < 0$ around maximum of $f(K)$.

"Bounce" indeterministic.

PENNSTATE

Properties

- → Not undone by quantum back-reaction or higher time derivatives. Distinct from higher-curvature corrections.
- → No effective line element on standard space-time: $\mathrm{d}x^a$ in

$$\mathrm{d}s^2_{\mathrm{eff}} = \tilde{q}_{ab}\mathrm{d}x^a\mathrm{d}x^b$$

do not transform by changes dual to deformed gauge transformations $\{\tilde{q}_{ab}, H[N] + D[w]\}$.

Field redefinition to standard q_{ab} possible as long as β does not change sign.

With signature change: New model of non-classical space-time. [with Brahma, Buyukcam, D'Ambrosio: arXiv:1610.08355]

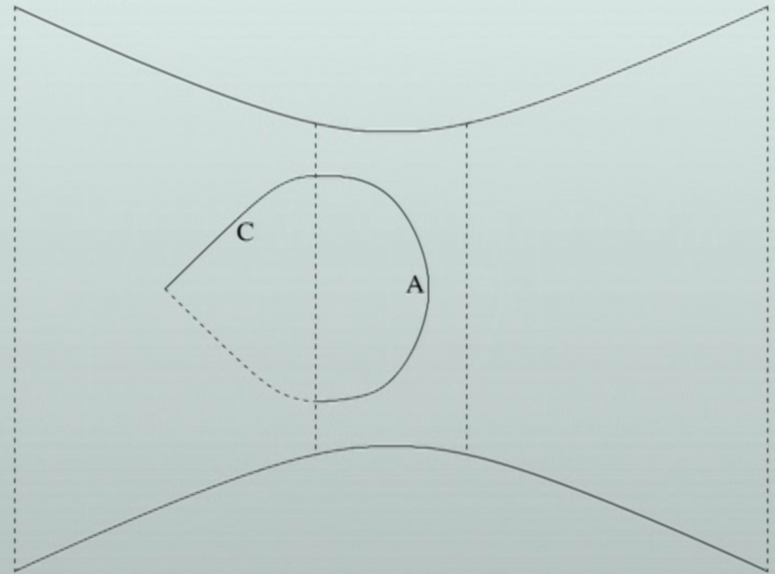
- → Evaluate theory using canonical observables of deformed gauge theory.

PENNSTATE

Tricomi problem

[with Mielczarek: arXiv:1503.09154]

Well-posed $-\frac{\partial^2 u}{\partial t^2} + \beta(\mathcal{H})\Delta u = 0$: Data on characteristic C, arc A.



→ Need future data: No deterministic evolution.
→ Poles generic: Cosmic boom.
→ Non-singular beginning.

PENNSTATE

Information loss

Non-singular black-hole model:

Evolve through classical singularity by quantum evolution of homogeneous interior.

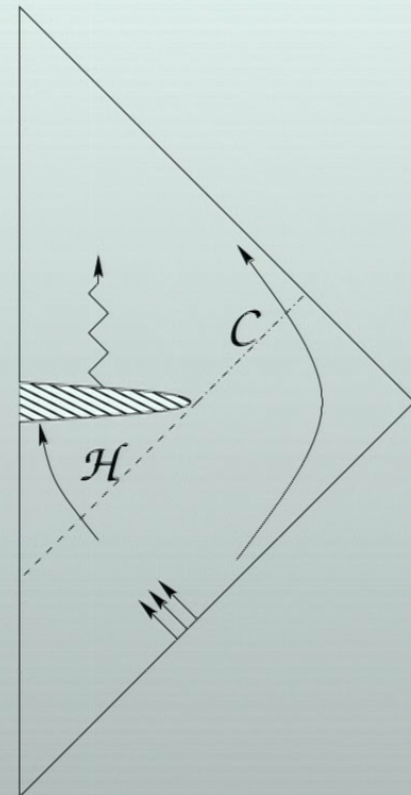
No event horizon. [Ashtekar, MB 2006]

Quantum space-time structure:

High-curvature region Euclidean.

Arbitrary boundary values affect future space-time.

Event horizon $\mathcal{H}$ and Cauchy horizon $\mathcal{C}$.



[arXiv:1409.3157]

PENNSTATE

Structure functions

Systems with several constraints $\hat{C}_I$: $[\hat{C}_I, \hat{C}_J] = \hat{f}^K_{IJ}\hat{C}_K$.

→ Effective constraints: $C_I = \langle \hat{C}_I \rangle$ and "fluctuation constraints" ΔC_I expanded in terms of $\langle \cdot \rangle$ and $\Delta(\cdot)$ for basic operators $\cdot$.

→ Poisson bracket for $\langle \cdot \rangle$ and $\Delta(\cdot)$.

→ No quantum corrections in structure functions: [arXiv:1407.4444]

$$\{C_I, C_J\} = f^K_{IJ}(\langle \cdot \rangle) C_K + \cdots$$

Consistent with higher-curvature effective actions in gravity.

Holonomy modifications in $\hat{C}_I$ change $\hat{f}^K_{IJ}$.

PENNSTATE

Related results (real connections)

- → Closely related behavior in spherically symmetric models and cosmological perturbations.
  [with Barrau, Calcagni, Grain, Kagan: arXiv:1404.1018]
- → Operator version in spherical symmetry. [Brahma: arXiv:1411.3661]
- → Different operator versions in $2+1$ dimensional models, based on reformulations of constraint algebra.
  [Perez, Pranzetti; Henderson, Laddha, Tomlin, Varadarajan]
- → Partially Abelianized constraints: [Gambini, Pullin]
  After holonomy modifications, can reconstruct hypersurface-deformation brackets only if deformed.
  [with Brahma, Reyes: arXiv:1507.00329]
- → Obstructions to anomaly freedom in models with local physical degrees of freedom. [with Brahma: arXiv:1507.00679]

Not much is known about dynamics of loop quantum gravity.
Modified space-time structures generic.