Title: Unitary Cosmological Bounces

Date: Jun 27, 2017 10:00 AM

URL: http://pirsa.org/17060099

Abstract: A non-singular cosmological bounce in the Einstein frame can only take place if the Null Energy Condition (NEC) is violated. I will explore the constraints imposed by demanding tree level unitarity on a cosmological background in single scalar field theories before focusing on the explicit constraints that arise in P(X) theories. In that context, perturbative unitarity makes it impossible for the NEC violation to occur within the region of validity of the effective field theory but I will show explicitly how unitarity may be restored by involving irrelevant operators that arise at a higher scale.

Bounce Scenarios in Cosmology Perimeter Institute for Theoretical Physics June 2017

Unitary Cosmological Bounces

Imperial College



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Setting different EFTs apart

• There has recently been an explosion of models that can play important roles for cosmology

(eg. DBI, K-inflation, G-inflation, gauge inflation, ghost inflation, Axion Monodromy, Chromo-Natural Inflation, f(R), Chameleon, Symmetron, ghost condensate, Galileon, generalized galileon, Horndeski, beyond Horndeski, beyond beyond Horndeski, Fab4, beyond Fab4, EST, DHOST, K-essence, DGP, cascading gravity, massive gravity, minimal massive gravity, bi-gravity, multi-gravity, mass-varying massive gravity, f(R) massive gravity, mass-varying massive gravity, quasi-dilaton, extended quasi-dilaton, superfuid dark matter, Proca dark energy, generalized Proca, beyond generalized Proca, gauge field dark energy, Galileon genesis, extended Galileon genesis, SLED, mimetic gravity, unimodular gravity, dipolar dark matter, ..., ..., ...) CDCJ

Setting different EFTs apart

- We could simply wait for observations to tell them apart (but they evolve with observations...)
- As low EFTs they often have very peculiar features that can make us question their whole validity

Do these models: 1. make sense in the "traditional" strong coupling regime?

- 2. preserve perturbative unitarity ?
- 3. have any chance of ever admitting
 - a standard Wilsonian UV completion ?

4. ... 5. ...

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Decoupling limit

- Most of these theories have been developed with a cosmological motivation in mind and are therefore fundamentally gravitational theories
- However a huge insight on the consistency of the theory can be gained by focusing on the scalar (or other) degree of freedom on a particular cosmological background
- This is justified by taking an appropriate decoupling limit where the gravitational degrees of freedom decouple (eg. $M_{\rm Pl} \rightarrow \infty$ While keeping the scale at which the other fields interact fixed).
- This approximation can then be checked a posteriori

 $\mathcal{L}[g_{\mu\nu}, \Phi] \xrightarrow{a = \bar{g}(t) \Phi = \bar{g}(t) + c} \mathcal{L}[\varphi, t]$

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Applications for inflation, dark energy, dark matter....

Silverstein, Tong, PRD70, 2004 Alishahiha Silverstein Tong PRD70, 2004 В

Kiding on irrelevant Operators

- Eg. of DBI $\mathcal{L}_{\text{DBI}} \sim -\sqrt{1-X} \sim -\frac{1}{2}(\partial \phi)^2 + \frac{1}{\Lambda^4}(\partial \phi)^4 + \cdots$
 - + higher derivative operators
- The interesting phenomenology requires the irrelevant operators to be important $\left|rac{(\partial\phi)^2}{\Lambda^4}
 ight|\sim 1$
- In this type of theories, the breakdown of the EFT is not measured by " $\partial \phi$ " but rather by " ∂ " itself.
- So we can trust a regime where $~\partial \phi \sim \Lambda^2$
- So long as $\,\partial\partial\phi\ll\Lambda^3$

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Riding on irrelevant Operators

- Eg. of DBI $\mathcal{L}_{\text{DBI}} \sim -\sqrt{1-X} \sim -\frac{1}{2}(\partial \phi)^2 + \frac{1}{\Lambda^4}(\partial \phi)^4 + \cdots$
 - + higher derivative operators
- $\frac{\delta^{2}\mathcal{L}}{\delta\phi^{2}} = Z^{\mu\nu}[\phi]\partial_{\nu}\partial_{\nu} \qquad \qquad \mathcal{L}_{\text{DBI}}[\bar{\phi} + \delta\varphi] \sim Z^{\mu\nu}(\bar{\phi})\partial_{\mu}\delta\varphi\partial_{\nu}\delta\varphi$
- We expect loop corrections of the form

 $\Gamma_{1-\text{loop}} \sim \left(\frac{\partial Z}{Z}\right)^2 + \frac{\partial^2 Z}{Z}$

Which can be under control even if $\partial \phi \sim \Lambda^2$ so long as the gradients are under control...

Caveats: sound speed can typically be very small (hierarchy of eigenvalues in $Z^{\mu\nu}$)

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Do these models:

- 1. make sense in the "traditional" strong coupling regime?
- 2. preserve perturbative unitarity $?^{\circ}$
- 3. have any chance of ever admitting a standard Wilsonian UV completion ?

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2. Perturbative Unitarity (with small sound speed)

 Many new models of inflation (with non-trivial kinetic terms) with very low sound speed have emerged

eg. could lead to enhanced non-gaussianities $\,f_{
m NL}pprox c_s^{-2}$

• In itself this may be the first hint of perturbative unitarity breaking...

 $\mathcal{L} = \dot{\phi}^2 - c_s^2 \left(\partial_i \phi\right)^2 + \text{interactions}$

From Optical theorem:

$$|\mathcal{A}_{2
ightarrow 2}|\lesssim c_s^3$$

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2. Perturbative Unitarity (with small sound speed)

• Sound speed effect on perturbative unitarity breaking...

$$S = \int dt d^3x \left(\dot{\phi}^2 - c_{\downarrow s}^2 (\partial_i \phi)^2 + \frac{(\partial_i \phi)^{2\ell}}{\Lambda^{4(\ell-1)}} + \cdots \right)$$
$$x^i = c_s \hat{x}^i$$
$$\phi = c_s^{-3/2} \hat{\phi}$$
$$S = \int dt d^3 \hat{x} \left(\dot{\hat{\phi}}^2 - (\hat{\partial}_i \hat{\phi})^2 \left(+ \frac{(\hat{\partial}_i \hat{\phi})^{2\ell}}{c_s^{5\ell-3} \Lambda^{4(\ell-1)}} + \cdots \right) \right)$$

Strong coupling energy scale $\longrightarrow 0$ as $c_s \rightarrow 0$

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Eg. of $P(\phi, X)$ bounces

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 $c_{s}^{2} = 0$

• Cosmological bounces are known to be "possible" in $P(\phi,X)$, $X=-(\partial\phi)^2=\dot{\phi}^2$

 $M_{\rm Pl}^2 \dot{H} = -\frac{1}{2}(\rho + p) = -XP'(X)$

, WEC for (
ho+p)<0 i.e. P'(X)<0

 $c_s^2 = \frac{P'(X)}{2XP''(X) + P'(X)}$

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Eq. of $P(\phi, X)$ bounces

Cosmological bounces are known to be "possible" in $P(\phi, X)$, $X = -(\partial \phi)^2 = \dot{\phi}^2$



 $\land H$

 $M_{\rm Pl}^2 \dot{H} = -\frac{1}{2}(\rho + p) = -XP'(X)$

NEC for
$$(
ho+p)<0$$
 i.e. $P'(X)<0$

$$c_s^2 = \frac{P'(X)}{2XP''(X) + P'(X)}$$

Unitarity requires $|\mathcal{A}_{2
ightarrow 2}| \lesssim c_s^3$

unitarity necessarily breaks down before the onset of NEC

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 In practice any EFT will have to involve some irrelevant operators that enter at high energy



No Ostrogadski ghost,

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 In practice any EFT will have to involve some irrelevant operators that enter at high energy

Λ : Traditional strong coupling scale

 $\mathcal{L} = \Lambda^4 P(X/\Lambda^4) + \frac{(\Box \phi)^2}{\mathcal{M}^2} + \cdots$



Scale at which *background is strongly coupled* EFT for perturbations on top of that background can see a different strong coupling scale, (need to remain weakly coupled to trust that background)

\mathcal{M} : Cutoff of the low energy EFT

background has to remain below cutoff to trust low energy EFT perturbations have to remain below redressed cutoff

 $X = -(\partial \phi)^2$

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 In practice any EFT will have to involve some irrelevant operators that enter at high energy

$$\mathcal{L} = \Lambda^4 P(X/\Lambda^4) + rac{(\Box \phi)^2}{\mathcal{M}^2} + \cdots_{X = -(\partial \phi)^2}$$

Effective Speed of sound:

$$c_{\text{eff}}^{2} = \frac{P'}{\underbrace{P' + XP''}_{c_{s}^{2}}} + \frac{k^{2}}{\mathcal{M}^{2}} + \mathcal{O}\left(\frac{k^{4}}{\mathcal{M}^{4}}\right)$$
$$k \ll \mathcal{M}$$

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 In practice any EFT will have to involve some irrelevant operators that enter at high energy

$$\mathcal{L} = \Lambda^4 P(X/\Lambda^4) + rac{(\Box \phi)^2}{\mathcal{M}^2} + \cdots_{X = -(\partial \phi)^2}$$

Effective Speed of sound: $c_{
m eff}^2$

$$f_{\rm ff} = \frac{P'}{\underbrace{P' + XP''}_{c_s^2}} + \frac{k^2}{\mathcal{M}^2} + \mathcal{O}\left(\frac{k^4}{\mathcal{M}^4}\right)$$

Subtle trade-off between having the high energy operators "save" unitarity without spoiling the low-energy FET В

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The low-energy EFT

$$\mathcal{L} = \frac{A}{2}\dot{\varphi}^2 - \frac{B}{2a^2}(\partial_i\varphi)^2 - \frac{m^2}{2}\varphi^2 + \frac{1}{\mathcal{M}^2}\left(\ddot{\varphi} - \frac{1}{a^2}(\partial_i^2\varphi)\right)^2 \quad \textbf{+} \dots$$



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The low-energy EFT

 μ_c : scale at which transition between relativistic and non-relativistic form

$$\mathcal{L} = \frac{A}{2}\dot{\varphi}^2 - \frac{B}{2a^2}(\partial_i\varphi)^2 - \frac{m^2}{2}\varphi^2 + \frac{1}{\mathcal{M}^2}\left(\ddot{\varphi} - \frac{1}{a^2}(\partial_i^2\varphi)\right)^2 \quad \textbf{+} \dots$$

Modified Dispersion relation: (mass can be ignored)

$$\begin{array}{ll} \text{Modified} \\ \text{Dispersion relation:} \\ \text{(mass can be ignored)} \end{array} \qquad F = A\omega^2 - B\tilde{k}^2 - \frac{1}{\mathcal{M}^2}(\omega^2 - \tilde{k}^2)^2 = 0 \\ \\ \omega^2(k) = c_{\text{eff}}^2(k)\tilde{k}^2 = c_s^2\tilde{k}^2 + \frac{\tilde{k}^4}{A\mathcal{M}^2} + \cdots \qquad \omega_{\text{new mode}}^2(k) = 0 \\ \end{array}$$

$$\omega_{\rm new\,mode}^2(k) = A\mathcal{M}^2$$

Energy Cutoff (upper bound):

$$\Lambda_{\rm cutoff} \leq \sqrt{A}\mathcal{M}$$

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Low vs high energy modes



 μ_c : scale at which transition between relativistic and non-relativistic form

The effective strong coupling scale derived in that regime should be larger than μ_c otherwise we are in the same situation as earlier

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The low-energy EFT

$$\mathcal{L} = \frac{A}{2}\dot{\varphi}^2 - \frac{B}{2a^2}(\partial_i\varphi)^2 - \frac{m^2}{2}\varphi^2 + \frac{1}{\mathcal{M}^2}\left(\ddot{\varphi} - \frac{1}{a^2}(\partial_i^2\varphi)\right)^2 + \dots$$



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Low vs high energy modes



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For a
$$P(\phi, X)$$
 theory, $\mathcal{L} = \Lambda^4 \sum_{\ell, n} \frac{c_{n,\ell}}{\Lambda^{n+4\ell}} \Phi X^{\ell}$

The EFT for the fluctuations then looks like

$$\mathcal{L}[\varphi] = A\dot{\varphi}^2 - \frac{B}{a^2}(\partial_i\varphi)^2 - \frac{1}{2}m^2\varphi^2 + \sum \frac{\tilde{c}}{\Lambda^{n+4\ell+2m-4}}\varphi^n\dot{\varphi}^m(\partial_i\varphi)^{2\ell}$$

$$c_{n,\ell} \sim \mathcal{O}(1) \quad \Rightarrow \quad A, \tilde{c}_{n,m,\ell} \sim \mathcal{O}(1)$$

 ${\cal L}_{
m int} \supset {arphi^5\over \Lambda}$

For modes with $\omega \lesssim \mu_c$, the effective strong coupling scale associated with that operator is $\Lambda_* \sim A^{1/4} B^{9/4} \Lambda \gtrsim \mu_c \sim \frac{B}{A^{1/2}} \mathcal{M}$

 $A \ge B^{-5/3} (\mathcal{M}/\Lambda)^{4/3} \ggg 1$

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Explict (tuned) $P(\phi, X)$ model

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}R + P(\Phi, X) + \frac{1}{2\Lambda_c^2}(\Box\Phi)^2$$
$$P(\Phi, X) = -\Lambda^4 V(\Phi) + p(\Phi)X + \frac{q(\Phi)}{\Lambda^4}X^2$$

$$V(\Phi) = -\frac{1}{4}q(\Phi)\frac{\Phi^4}{\Lambda^4} + \left(3\frac{\Lambda^2}{M_{\rm Pl}^2}h^2(\Phi) + \Phi h'(\Phi)\right) - \frac{\Phi^2}{2\mathcal{M}^2}\left(1 + 3\frac{\Lambda^2}{M_{\rm Pl}^2}h(\Phi)\right)$$
$$p(\Phi) = -q(\Phi)\frac{\Phi^2}{\Lambda^2} - 2\frac{\Lambda^2}{\Phi}h'(\Phi) + \frac{2\Lambda^2}{\mathcal{M}^2}\left[1 + 3\frac{\Lambda^2}{M_{\rm Pl}^2}h(\Phi) + 3\frac{\Lambda^2}{M_{\rm Pl}^2}\Phi h'(\Phi)\right]$$

So that the background profile be simply

with

$$\dot{\phi} = \Lambda \phi, \quad H = \frac{\Lambda^3}{M_{\rm Pl}^2} h(\phi)$$

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$2 \rightarrow 2$ Scattering Amplitude

For a low energy EFT described by a massive Lorentz invariant scalar field



Mandelstam variables: s: center of mass energy² t: momentum transfer $u = 4m^2 - s - t$

 $|\text{initial state}\rangle \longrightarrow |\text{final state}\rangle = \hat{S} |\text{initial state}\rangle$

 $\hat{S} = 1 + i\hat{T}$

Scattering amplitude $\mathcal{A}=\langle \mathrm{final}\,|\,\hat{T}\,|\,\mathrm{initial}
angle$

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Optical theorem:

$\mathcal{A} = \langle \text{final} \, | \, \hat{T} \, | \, \text{initial} \rangle$

$$\sigma(s) = \frac{\operatorname{Im}\mathcal{A}(s,0)}{\sqrt{s(s-4m^2)}} \gtrsim 0 \qquad \begin{array}{c} \text{Physical scattering for } s \geq 4 m^2 \\ \text{In the forward scattering limit, i} \end{array}$$

$$2 \operatorname{Im} = \sum_{X} \left| \sum_{X} x \right|^2 \geq \left| \sum_{X} \right|^2$$

Physical scattering for $s \ge 4 m^2$ In the forward scattering limit, ie. t = 0

Analyticity (implied by causality) & locality imply:

$$B''(s) \sim \int_{4m^2}^{\infty} \mathrm{d}\mu \frac{\mathrm{Im}A(\mu)}{(\mu - s)^3}$$

 $|B''(s)|_{s=0}$

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Positivity bounds for P(X)

eg. P(X) model $\mathcal{L} = -\frac{1}{2}(\partial \phi)^2 + \frac{c}{\Lambda^4}(\partial \phi)^4 + \cdots$

$$\mathcal{A}_{2\to 2}^{\text{tree}} = \frac{c}{\Lambda^4} \left(s^2 + t^2 + u^2 - 4m^2 \right)$$

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Positivity bounds requires: c>0

No P(X) model with $c \leq 0$ can ever have an analytic Wilsonian UV completion

Pirsa: 17060099

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Positivity bounds for P(X)

eg.
$$P(X)$$
 model $\mathcal{L} = -\frac{1}{2}(\partial \phi)^2 + \frac{c}{\Lambda^4}(\partial \phi)^4 + \cdots$

$$\mathcal{A}_{2\to 2}^{\text{tree}} = \frac{c}{\Lambda^4} \left(s^2 + t^2 + u^2 - 4m^2 \right)$$

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• There has recently been an explosion of models that can play important roles for cosmology

(eg. DBI, K-inflation, G-inflation, gauge inflation, ghost inflation, Axion Monodromy, Chromo-Natural Inflation, f(R), Chameleon, Symmetron, ghost condensate, Galileon, generalized galileon, Horndeski, beyond Horndeski, beyond beyond Horndeski, Fab4, beyond Fab4, EST, DHOST, K-essence, DGP, cascading gravity, massive gravity, minimal massive gravity, bi-gravity, multi-gravity, mass-varying massive gravity, f(R) massive gravity, mass-varying massive gravity, quasi-dilaton, extended quasi-dilaton, superfuid dark matter, Proca dark energy, generalized Proca, beyond generalized Proca, gauge field dark energy, Galileon genesis, extended Galileon genesis, SLED, mimotia gravity, unimodular gravity, dipolar dark matter, CDCJ

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Summary

- (perturbative) unitarity can place strong constraints on different classes of cosmological models
- In the context of $P(\phi, X)$ cosmological bounces, unitarity is always violated unless some high order operators that enter at or above the cutoff are considered
- Even considering these irrelevant operators, perturbative unitarity is violated in many models (including ghost condensate)
- But a window of opportunities remains open ...

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