Title: Challenges for Bouncing Cosmologies

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Abstract: I will review various approaches to bouncing cosmologies and will discuss challenges which the different approaches face.

Challenges

R. Brandenberger

Introduction

Challenges
- Inflation
- Stability
- Bounce

S-Brane Bounce

String Gas
- Background
- Structure
- Overview
- Analysis

Conclusions

Challenges for Bouncing Cosmologies

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Physics Department, McGill University

Perimeter Institute, June 26, 2017

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Outline

Isotropic CMB Background

Challenges

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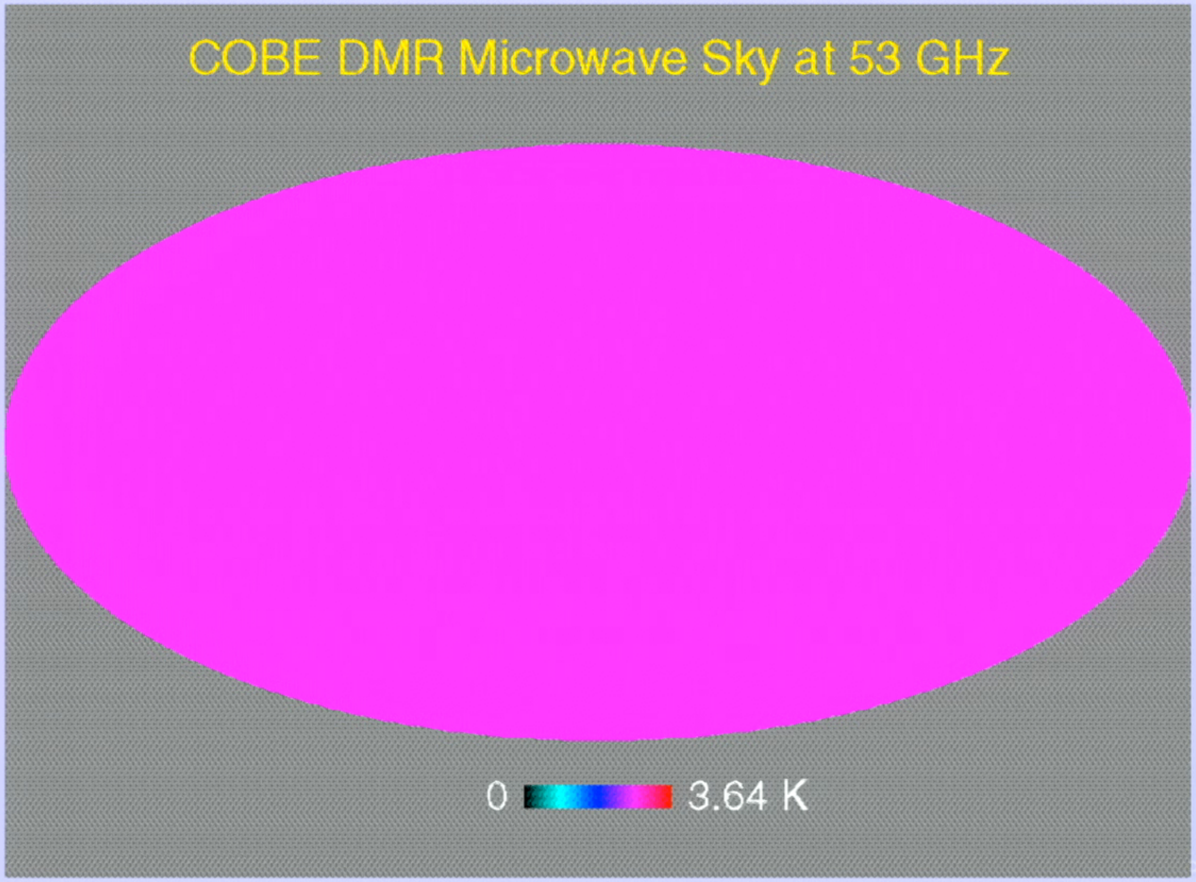
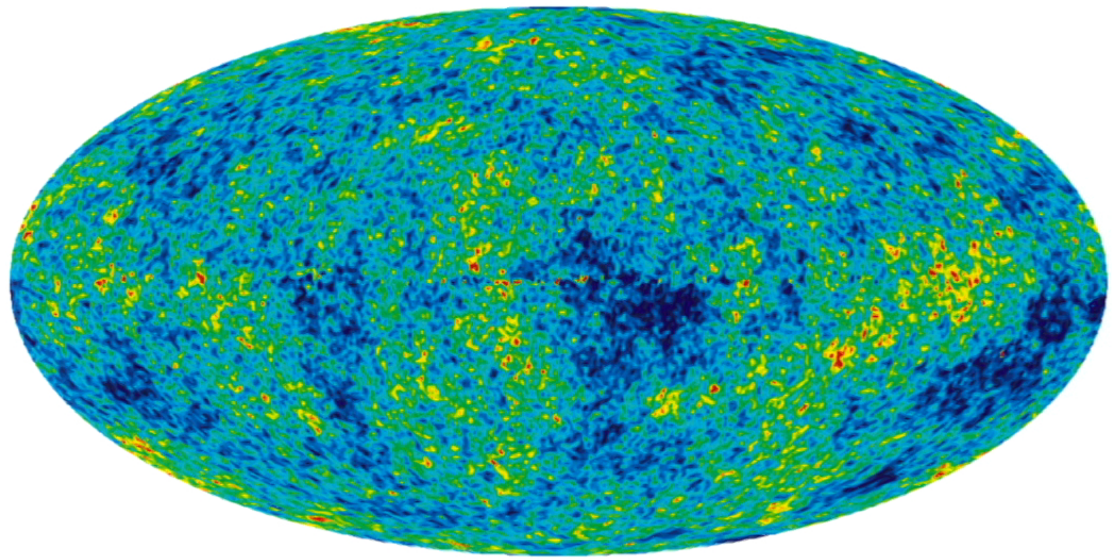


Map of the Cosmic Microwave Background (CMB)



Credit: NASA/WMAP Science Team

Angular Power Spectrum of CMB Anisotropies

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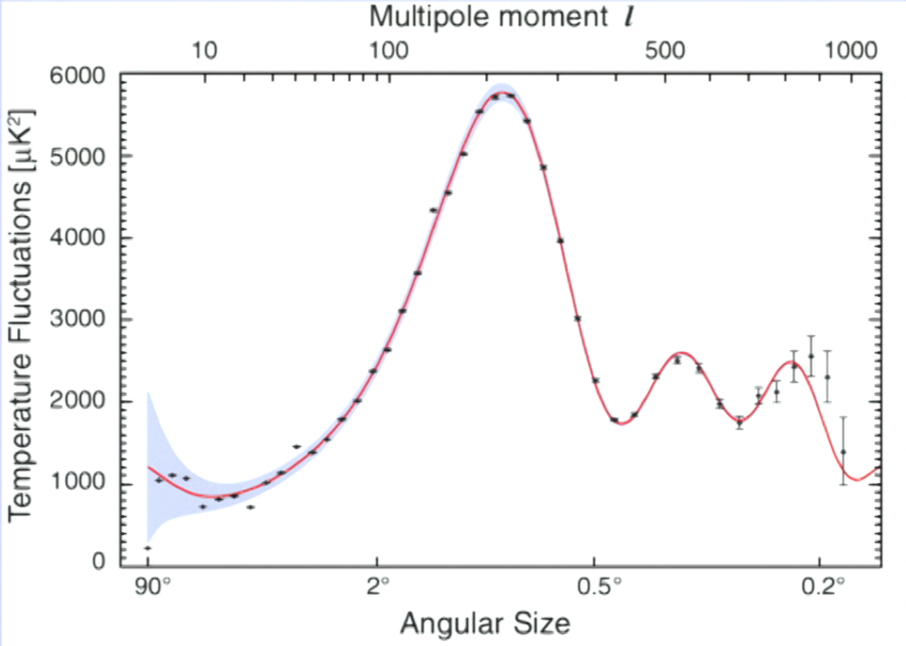

Credit: NASA/WMAP Science Team

Early Work

1970 p

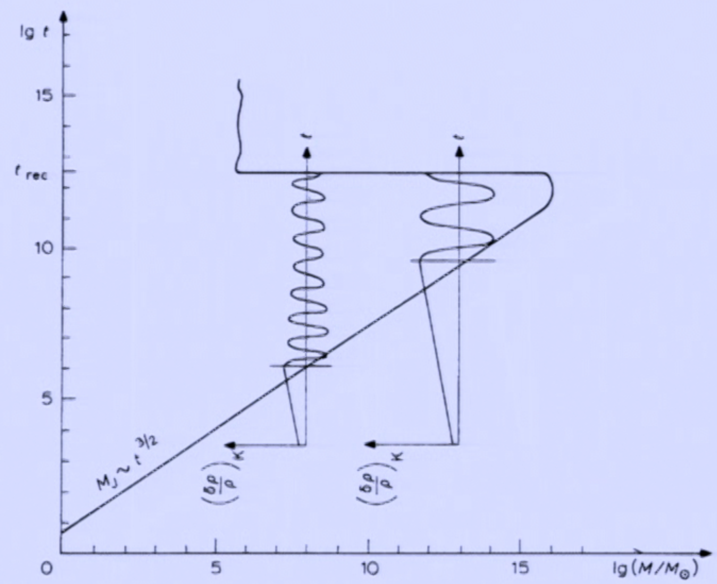

Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

Key Realization

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

- Given a scale-invariant power spectrum of adiabatic fluctuations on "super-horizon" scales before t_{eq}, i.e. standing waves.
- $\rightarrow$ "correct" power spectrum of galaxies.
- $\rightarrow$ acoustic oscillations in CMB angular power spectrum.

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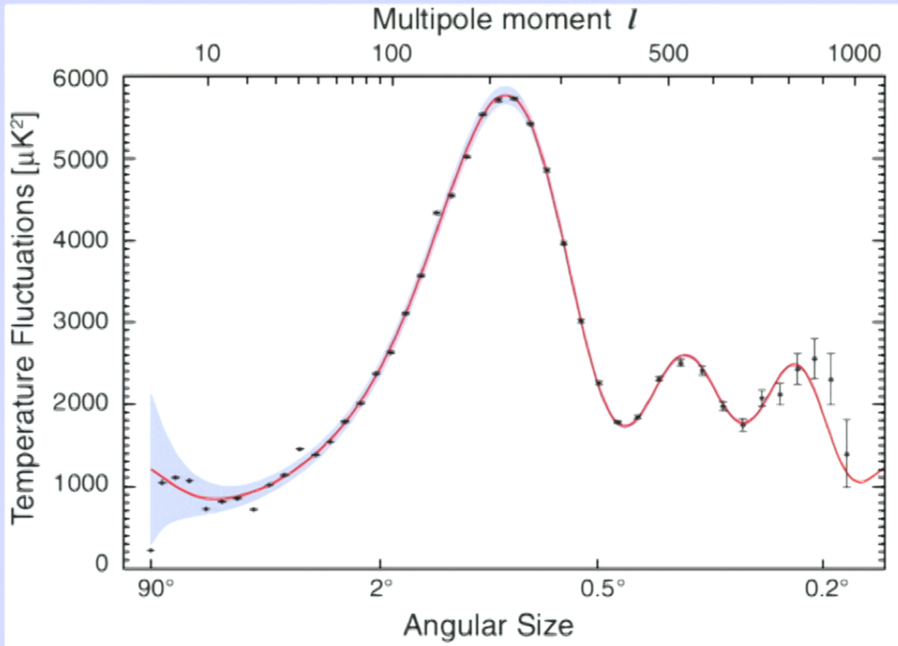

Credit: NASA/WMAP Science Team

Early Work

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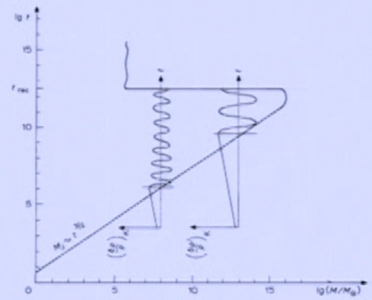
1970 paper



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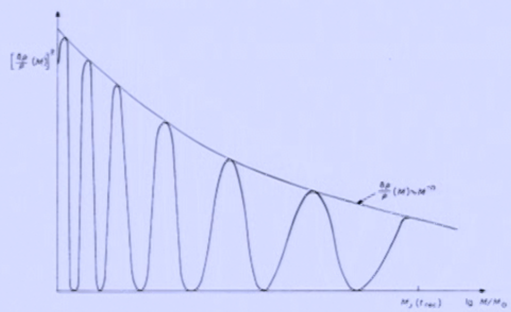

Fig. 1b. The dependence of the square of the amplitude of density perturbations of matter on scale. The fine line designates the usually assumed dependence $(\delta\varrho/\varrho)_M \sim M^{-n}$. It is apparent that fluctuations of relic radiation should depend on scale in a similar manner.

R. Sunyaev & Ya. Zeldovich, Astrophysics and Space Science 7 3-19 (1970

Key Challenge

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How does one obtain such a spectrum?

- Inflationary Cosmology is the first scenario based on causal physics which yields such a spectrum.
- But it is not the only one.

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Hubble Radius vs. Horizon

- Horizon: Forward light cone of a point on the initial Cauchy surface.
- Horizon: region of causal contact.
- Hubble radius: $l_H(t) = H^{-1}(t)$ inverse expansion rate.
- Hubble radius: local concept, relevant for dynamics of cosmological fluctuations.
- In Standard Big Bang Cosmology: Hubble radius = horizon.
- In any theory which can provide a mechanism for the origin of structure: Hubble radius $\neq$ horizon.

Criteria for a Successful Early Universe Scenario

- Horizon $\gg$ Hubble radius in order for the scenario to solve the "horizon problem" of Standard Big Bang Cosmology.
- Scales of cosmological interest today originate inside the Hubble radius at early times in order for a causal generation mechanism of fluctuations to be possible.
- Squeezing of fluctuations on super-Hubble scales in order to obtain the acoustic oscillations in the CMB angular power spectrum.
- Mechanism for producing a scale-invariant spectrum of curvature fluctuations on super-Hubble scales.

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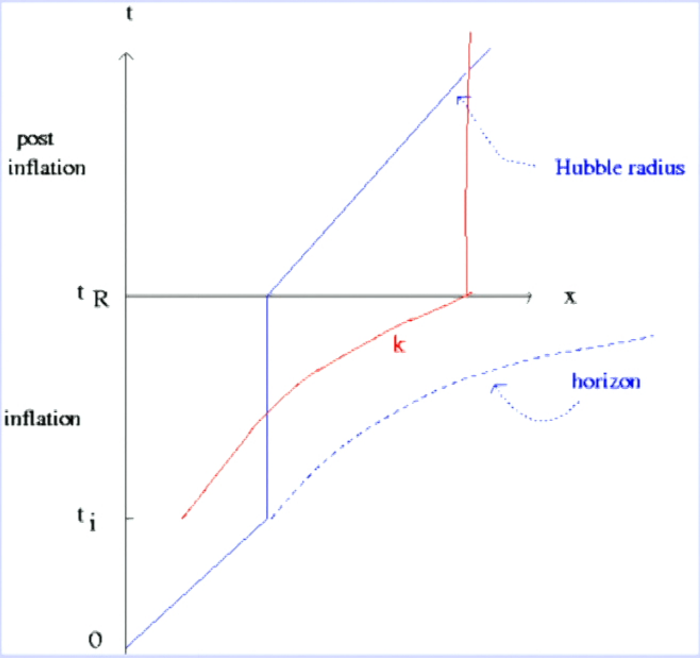
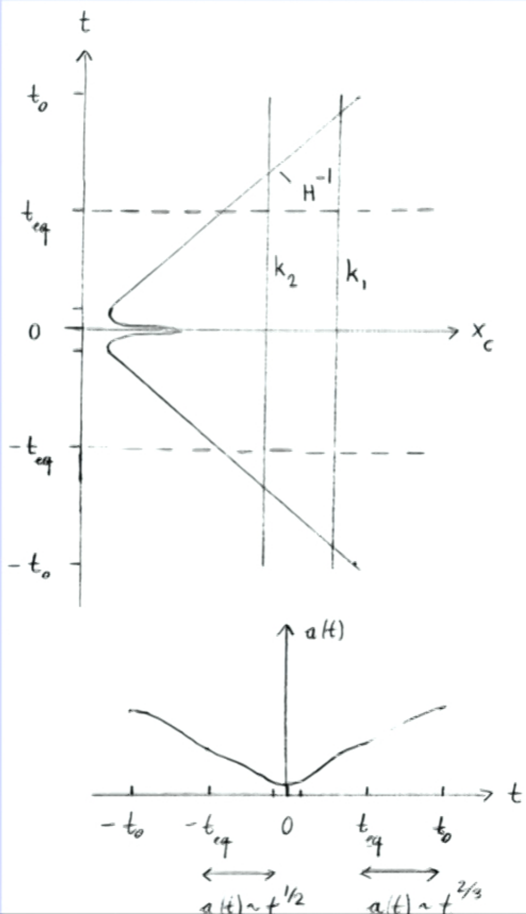
Inflation as a Solution



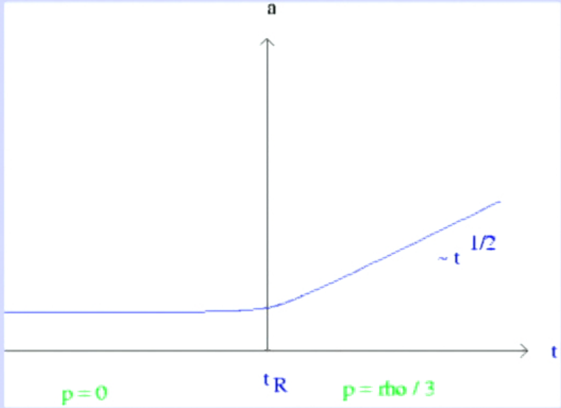
Bouncing Cosmologies as a Solution

M. Gasperini and G. Veneziano (1992); J. Khoury et al (2001); F. Finelli and R.B., (2002), D. Wands(1999)



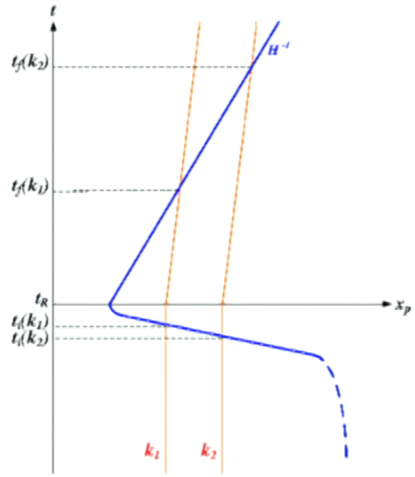
Emergent Universe

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*



Emergent Universe as a Solution

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



Plan

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Conceptual Problems of Inflationary Cosmology

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- Nature of the scalar field φ (the "inflaton")
- Conditions to obtain inflation (initial conditions, slow-roll conditions, graceful exit and reheating)
- Amplitude problem
- Trans-Planckian problem
- Singularity problem
- Cosmological constant problem
- Applicability of General Relativity

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Origin of Inflation?

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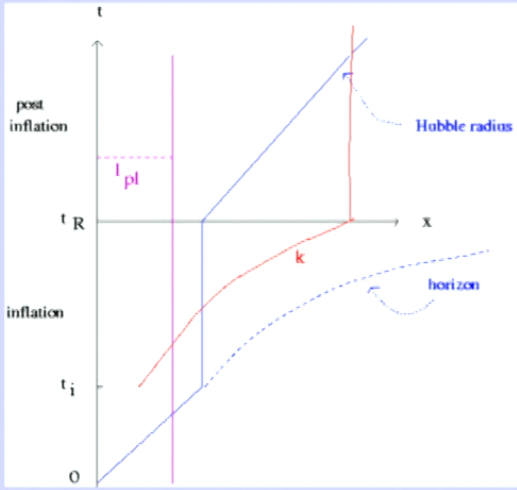
- To obtain inflationary dynamics free of initial condition fine tuning we require super-Planckian field values.
- $\rightarrow$ requires embedding of inflation into a quantum gravitational theory.
- But: No-go theorems on obtaining de Sitter space in string theory.

Trans-Planckian Problem



- **Success of inflation**: At early times scales are inside the Hubble radius → causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation.
- → new physics MUST enter into the calculation of the fluctuations.

Cosmological Constant Problem

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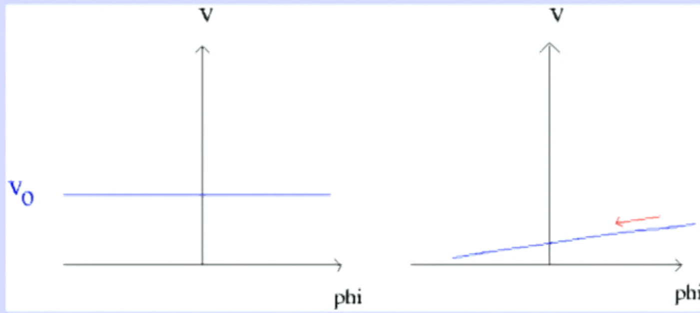

- Quantum vacuum energy does not gravitate.
- Why should the almost constant $V(\varphi)$ gravitate?

$$\frac{V_0}{\Lambda_{obs}} \sim 10^{120}$$

Applicability of GR

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- In all approaches to quantum gravity, the Einstein action is only the leading term in a low curvature expansion.
- Correction terms may become dominant at much lower energies than the Planck scale.
- Correction terms will dominate the dynamics at high curvatures.
- The energy scale of inflation models is typically $\eta \sim 10^{16}$GeV.
- $\rightarrow \eta$ too close to m_{pl} to trust predictions made using GR.

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Zones of Ignorance

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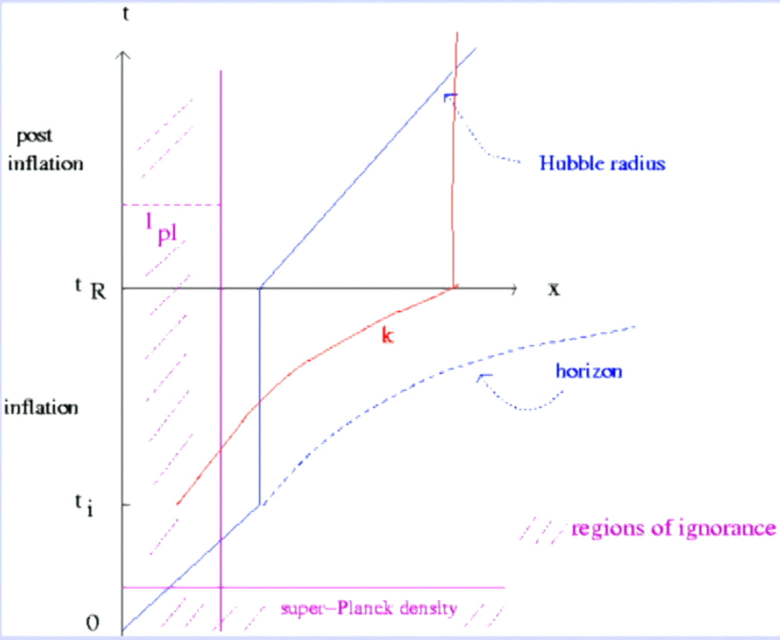

Anisotropy Problem of the Contracting Phase

Y. Cai, R.B. and P. Peter, arXiv:1301.4703

Problem: The energy density in anisotropies increases faster than the energy density in matter and radiation in the contracting phase.

$$ds^2 = dt^2 - a^2(t)\sum_i e^{2\theta_i(t)}\sigma_i^2$$

$$H^2 = \frac{\rho}{3m_{pl}^2} + \frac{1}{6}\sum_i \dot{\theta}_i^2$$

$$\ddot{\theta}_i + 3H\dot{\theta}_i = 0$$

$$\rightarrow \quad \rho_{anis} \sim a^{-6}$$

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Black Hole Formation in the Contracting Phase

J. Quintin and R.B., arXiv:1609.02556

Worry: Cosmological fluctuations become nonlinear on sub-Hubble scales and form black holes.

Starting point: scalar cosmological perturbations in longitudinal gauge:

$$\mathrm{d}s^2 = a(\eta)^2 \left\{ [1 + 2\Phi(\eta, \mathbf{x})]\, \mathrm{d}\eta^2 - [1 - 2\Phi(\eta, \mathbf{x})]\, \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j \right\} .$$

Equation of motion:

$$\Phi_k'' - \frac{6(1 + c_s^2)}{1 + 3w} \frac{1}{(-\eta)} \Phi_k' + \left(c_s^2 k^2 + \frac{12(c_s^2 - w)}{(1 + 3w)^2} \frac{1}{(-\eta)^2} \right) \Phi_k = 0 .$$

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Black Hole Formation (ctd.)

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Resulting fractional density contrast:

$$\delta_k \equiv \frac{\delta\rho_k^{(gi)}}{\rho^{(0)}} = -\frac{2}{3}\left(\frac{k^2}{\mathcal{H}^2}\Phi_k + \frac{3}{\mathcal{H}}\Phi_k' + 3\Phi_k\right) .$$

Criterium for direct black hole formation.

$$\int_{R \leq R_s} \mathrm{d}\delta M \geq M_s .$$

Result: for Bunch-Davies vacuum initial conditions early in the contracting phase the first scale to form black holes is the Hubble scale.

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Black Hole Formation (ctd.)

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The condition that black holes form becomes

$$|H| \sim c_s^{12/5} w^{3/5} \left(\frac{M_{\rm Pl}}{H_{\rm ini}}\right)^{1/5} M_{\rm Pl}$$

- For $c_s \ll 1$ we have $H \ll M_{pl}$.
- For a radiation dominated phase at late stages of contraction no black holes form from the direct channel if $|H_{max}| < M_{pl}$.

Initial Condition Problem of the Contracting Phase

- **Q: Attractor Nature of the Background**
- A: o.k. for Ekpyrotic contraction, not o.k. for matter bounce.
- **Q: What initial conditions for fluctuations?**
- Usual answer: vacuum - but why?
- Note: For inflation the use of vacuum initial conditions for fluctuations can be justified.

Initial Condition Problem of the Contracting Phase

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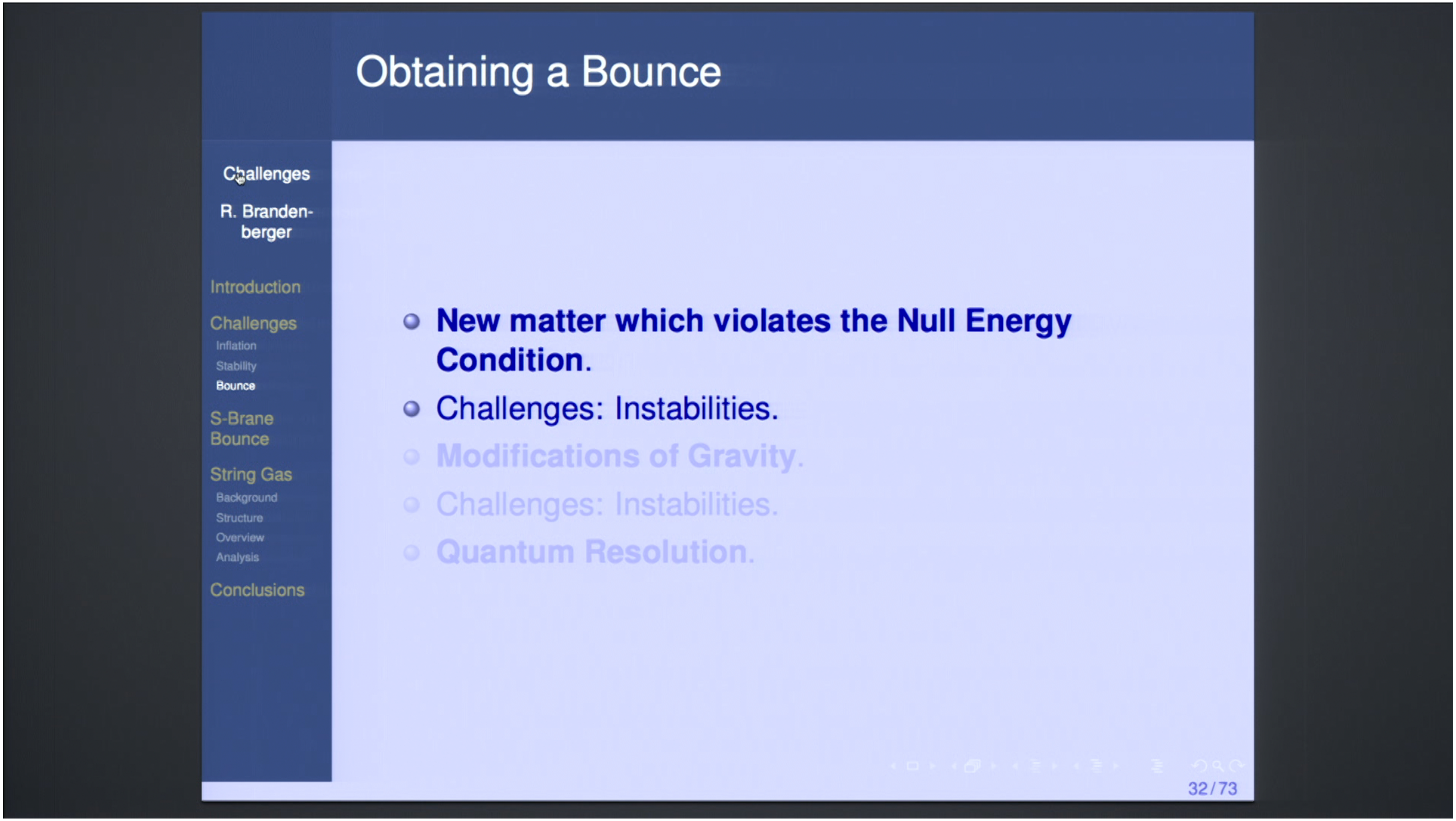
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New matter which violates the Null Energy Condition.
Challenges: Instabilities.
Modifications of Gravity.
Challenges: Instabilities.
Quantum Resolution.
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Obtaining a Bounce

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- **New matter which violates the Null Energy Condition.**
- Challenges: Instabilities.
- **Modifications of Gravity.**
- Challenges: Instabilities.
- **Quantum Resolution.**

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Some Examples

Modified Matter

- Ghost condensate [C. Lin, L. Perreault Levasseur and R.B., arXiv:1007.2654 [hep-th]]
- Galileon matter [A. Ijjas and P. Steinhardt, 2016]

Modified Gravity

- Horava-Lifshitz gravity [R.B., arXiv:0904.2835 [hep-th]]

Quantum Resolution

- Loop quantum cosmology [Lectures by Ashtekar, Bojowald, Barrau, Agullo]
- Perfect bounce [S. Gielen and N. Turok]


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Temporal Duality

R.B., C. Kounnas, H. Partouche, S. Patil and N. Toumbas, arXiv:1312.2524 [hep-th]

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Starting point: Type II superstring theory in the presence of non-trivial gravito-magnetic fluxes (Euclidean background)

Temperature duality:

$$Z(T) = Z(T_c^2/T).$$

T_c: Self-dual temperature (equals the Hagedorn temperature modulo coupling constants)

Physical temperature

$$T_p = T \quad T \ll T_c$$

$$T_p = \frac{T_c^2}{T} \quad T \gg T_c$$

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S-Brane

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- For $T \ll T_c$ and $T \gg T_c$ the dynamics of the low energy modes of string theory is given by **dilaton gravity**
- Begin in a contracting phase with $T \gg T_c$ and T decreasing (i.e. T_p increasing).
- When $T = T_c$ a set of string states becomes massless (enhanced symmetry states)
- These states must be included in the action for the low energy modes.
- S-Brane: term in the action present only at $T = T_c$
- S-brane has $\rho < 0$ and $p = |\rho| > 0 \rightarrow$ S-brane is matter violating the NEC and can mediate a transition from contraction to expansion.
- $\rightarrow$ S-Brane bounce.

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S-Brane

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$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} - \nabla_\mu\phi\nabla^\mu\phi\right] + \int d^4x\sqrt{-g}\, n^*\sigma_r T_E^4$$
$$-\kappa\int d\tau d^3\xi\sqrt{h}e^{\phi}\delta(\tau)\,.$$

Evolution of Fluctuations through the Bounce

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- Consider initially scale-invariant cosmological fluctuations in the contracting phase on super-Hubble scales.
- **Matching conditions** across the S-brane: continuity of the induced metric and extrinsic curvature.
- Note: matching surface uniquely determined!
- **Result**: the spectrum of cosmological perturbations after the bounce on super-Hubble scales is scale-invariant.

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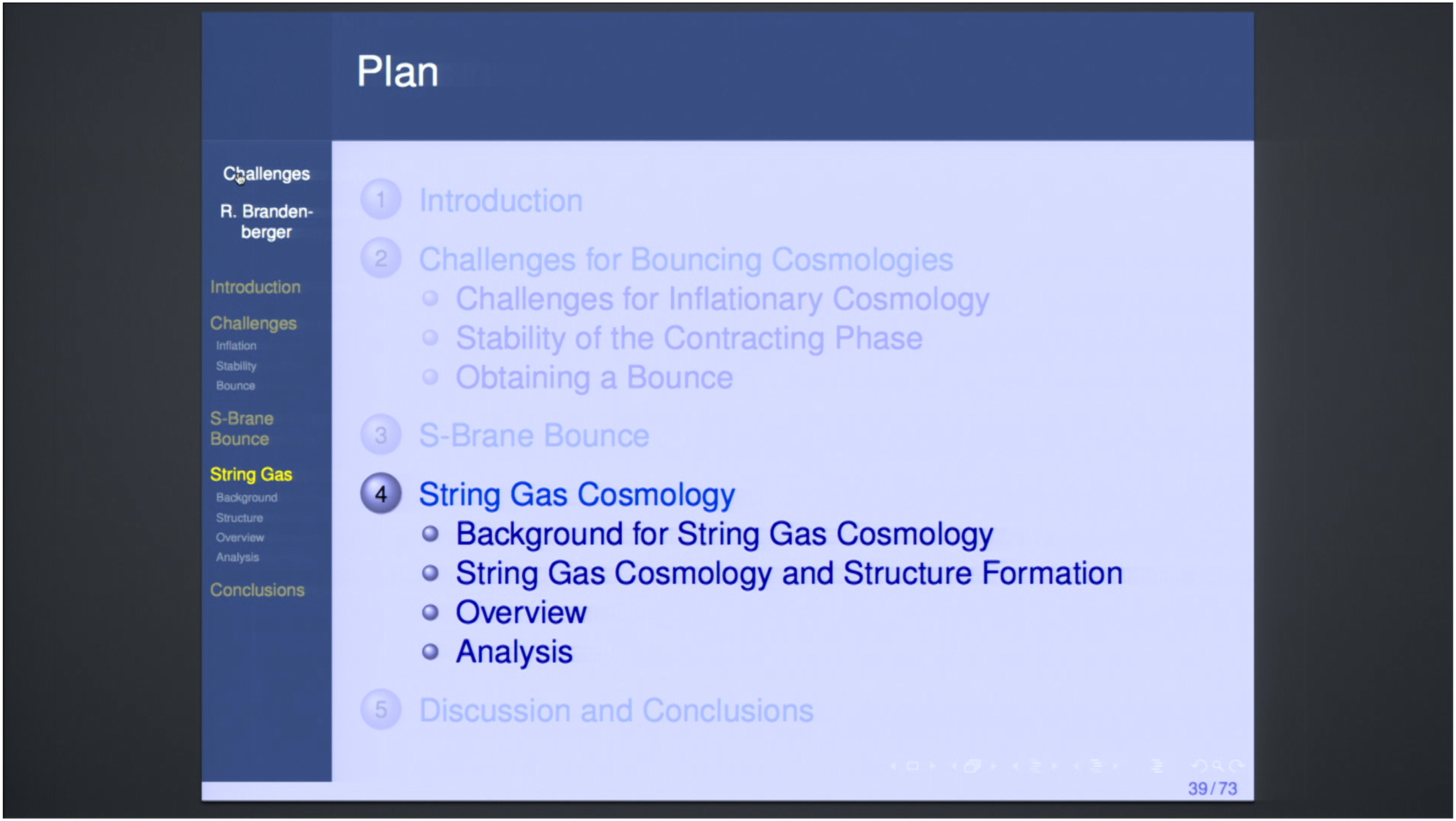
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Principles

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

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Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings

Assumption: Space is compact, e.g. a torus.

Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

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T-Duality

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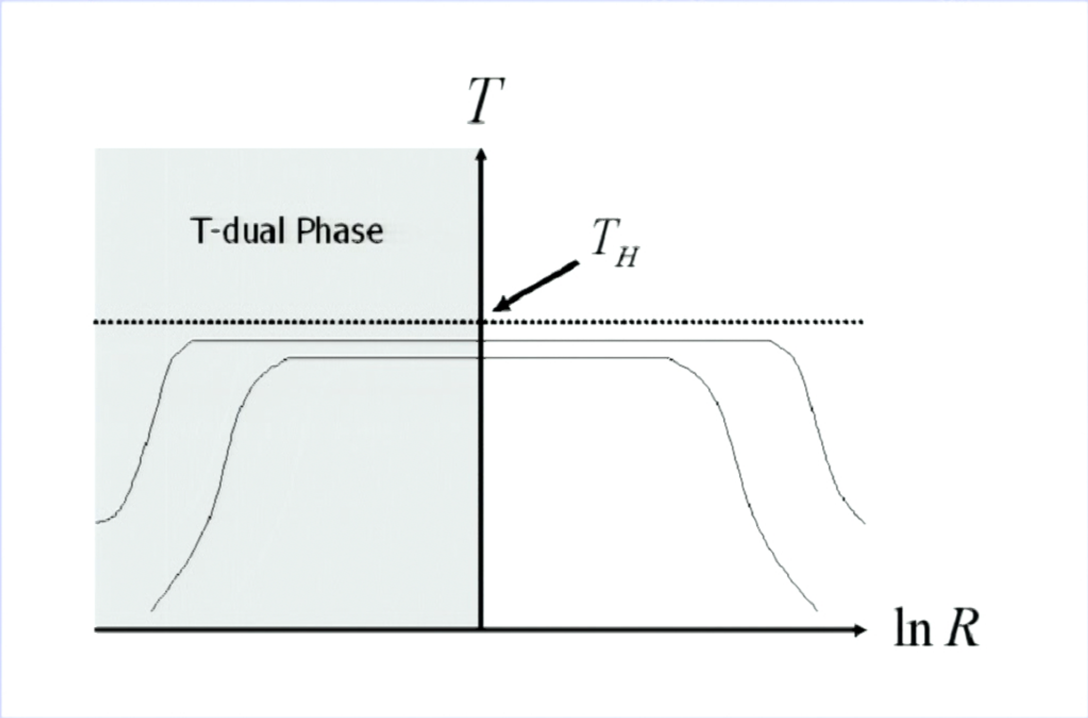
T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \to 1/R \quad (n, m) \to (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\to$ existence of D-branes

Adiabatic Considerations

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Temperature-size relation in string gas cosmology



Singularity Problem in Standard and Inflationary Cosmology

Challenges

R. Brandenberger

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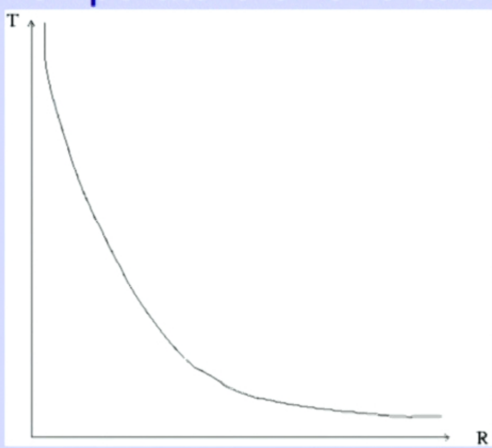
Temperature-size relation in standard cosmology



Dynamics

Challenges

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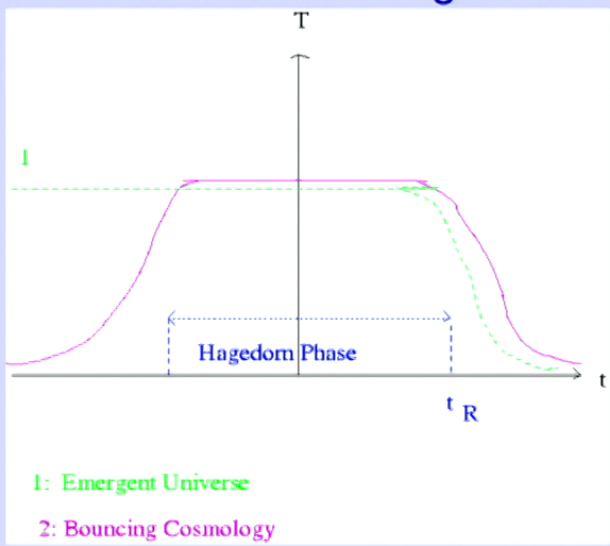
Assume some action gives us $R(t)$



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String Gas Bounce

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Two possibilities:

- Thermal Bounce
- Emergent Scenario

In both cases, a **long Hagedorn phase** will allow thermalization of the string gas on large scales.

→ thermal initial conditions for fluctuations

Doubled Space in SGC

R.B., R. Costa, G. Franzmann, S. Patil and A. Weltman, in prep.

Candidate for dynamics in the Hagedorn phase: Double Field Theory [C. Hull and B. Zwiebach, 2009]
Idea: For each dimension of the underlying topological space there are two position operators [R.B. and C. Vafa]:

- x: dual to the momentum modes
- $\tilde{x}$: dual to the winding modes

We measure **physical length** in terms of the **light** degrees of freedom.

$$l(R) = R \text{ for } R \gg 1 ,$$
$$l(R) = \frac{1}{R} \text{ for } R \ll 1 .$$

Doubled Space in SGC

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$$
\begin{aligned}
l(R) &= R \ \text{ for } \ R \gg 1\,, \\
l(R) &= \frac{1}{R} \ \text{ for } \ R \ll 1\,.
\end{aligned}
$$

Double Field Theory Approach

Idea Describe the low-energy degrees of freedom with an action in doubled space in which the T-duality symmetry is manifest.

$$S = \int dx d\tilde{x} e^{-2d} \mathcal{R},$$

$$\begin{aligned}\mathcal{R} = & \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL} \\ & + 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\partial_M d\partial_N d \\ & + 4\partial_M\mathcal{H}^{MN}\partial_N d + \frac{1}{2}\eta^{MN}\eta^{KL}\partial_M\mathcal{E}^A{}_K\partial_N\mathcal{E}^B{}_L\mathcal{H}_{AB}.\end{aligned}$$

$$\mathcal{H}_{MN} = \begin{bmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij} - b_{ik}g^{kl}b_{lj} \end{bmatrix} . \tag{1}$$

$$X^M = (\tilde{x}_i, x^i), \tag{2}$$

$$\eta^{MN} = \begin{bmatrix} 0 & \delta_i{}^j \\ \delta^i{}_j & 0 \end{bmatrix} . \tag{3}$$

Singularity Resolution in SGC

R.B., R. Costa, G. Franzmann, S. Patil and A. Weltman, in prep.

Challenges

R. Brandenberger

- Consider test particles in a DFT background.
- Derive geodesic equation of motion
- Consider a cosmological background with $b = 0$ and fixed dilaton.
- Find that the geodesics can be extended to infinite proper time in both time directions.
- $\rightarrow$ geodesic completeness.

Singularity Resolution in SGC

R.B., R. Costa, G. Franzmann, S. Patil and A. Weltman, in prep.

Challenges

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Singularity Resolution in SGC

R.B., R. Costa, G. Franzmann, S. Patil and A. Weltman, in prep.

Metric in DFT:

$$dS^2 = -dt^2 + \mathcal{H}_{MN} dX^M dX^N ,$$

Specialization to a cosmological background:

$$ds^2 = -dt^2 + b^2(t)\delta_{ij} dx^i dx^j + b^{-2}(t)\delta^{ij} d\tilde{x}_i d\tilde{x}_j ,$$

Point particle geodesics

$$\frac{d}{dS}\left(\frac{d\tilde{x}_a}{dS}\frac{1}{b^2}\right) = 0$$

$$\frac{d}{dS}\left(\frac{dx^a}{dS} b^2\right) = 0.$$

Singularity Resolution in SGC

R.B., R. Costa, G. Franzmann, S. Patil and A. Weltman, in prep.

Proper distance going forwards in time:

$$\Delta S = \int_{t_0}^{t_2} \gamma(t)^{-1} dt + T_2 \,, \qquad (4)$$

Proper distance going backwards in time:

$$\Delta S = \int_{t_1}^{t_0} \tilde{\gamma}(t) dt + T_1 \,, \qquad (5)$$

geodesic completeness in terms of physical time:

$$\begin{aligned} t_p(t) &= t \quad \text{for} \quad t \gg 1 \,, \\ t_p(t) &= \frac{1}{t} \quad \text{for} \quad t \ll 1 \,. \end{aligned}$$

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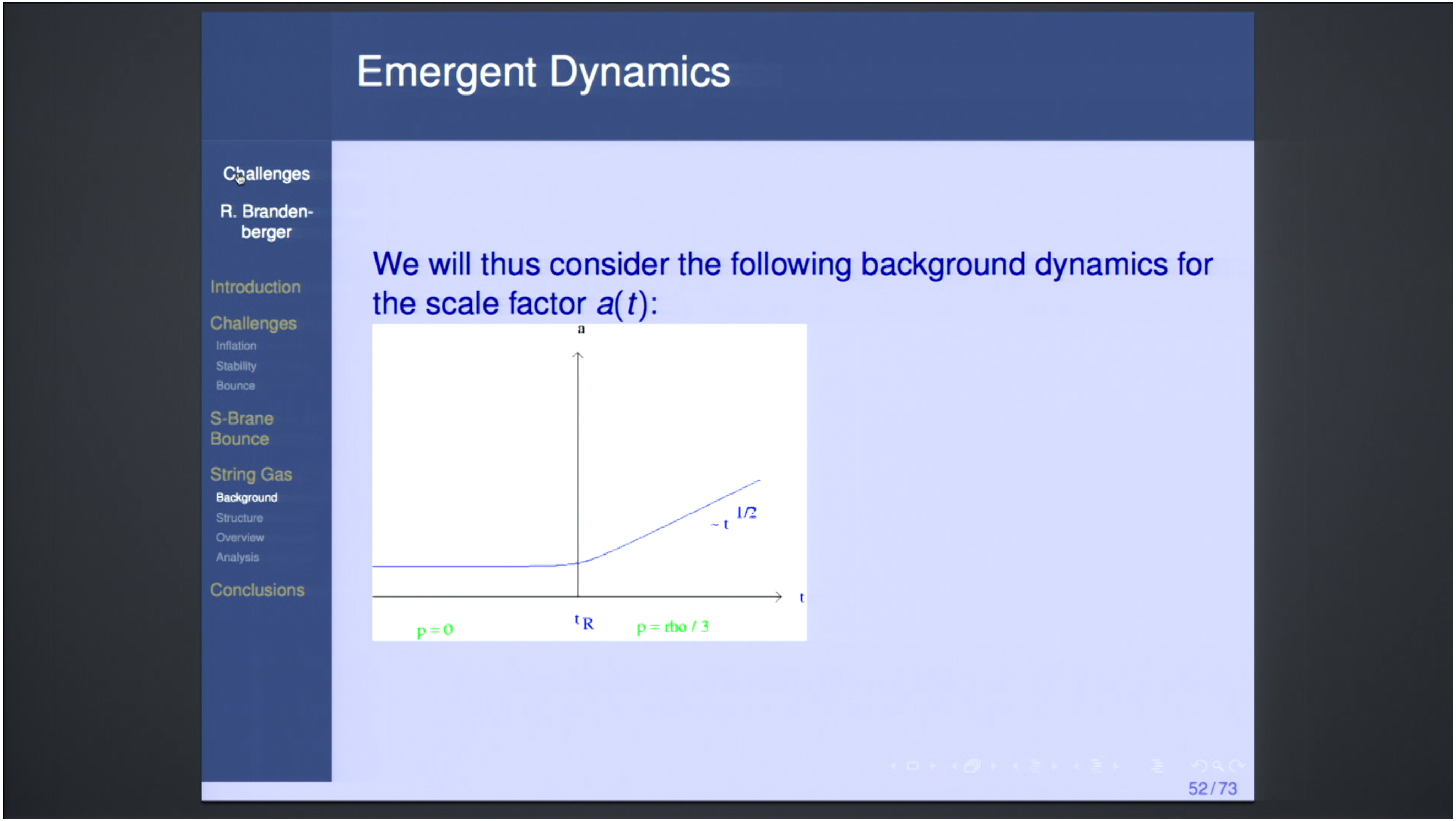
Emergent Dynamics
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We will thus consider the following background dynamics for the scale factor a(t):
a
~ t 1/2
t
p = 0
t R
p = rho / 3
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Dimensionality of Space in SGC

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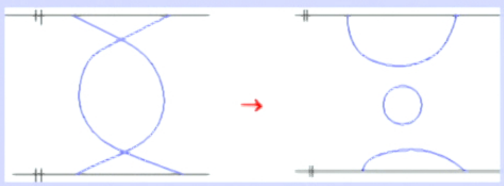
- Begin with all 9 spatial dimensions small, initial temperature close to T_H $\rightarrow$ winding modes about all spatial sections are excited.
- Expansion of any one spatial dimension requires the annihilation of the winding modes in that dimension.



- Decay only possible in three large spatial dimensions.
- $\rightarrow$ dynamical explanation of why there are exactly three large spatial dimensions.

(see also numerical work by M. Sakellariadou)

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Moduli Stabilization in SGC

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Size Moduli [S. Watson, 2004; S. Patil and R.B., 2004, 2005]

- winding modes prevent expansion
- momentum modes prevent contraction
- $\rightarrow$ $V_{eff}(R)$ has a minimum at a finite value of R, $\rightarrow$ R_{min}
- in heterotic string theory there are enhanced symmetry states containing both momentum and winding which are massless at R_{min}
- $\rightarrow V_{eff}(R_{min}) = 0$
- $\rightarrow$ size moduli stabilized in Einstein gravity background

Shape Moduli [E. Cheung, S. Watson and R.B., 2005]

- enhanced symmetry states
- $\rightarrow$ harmonic oscillator potential for θ
- $\rightarrow$ shape moduli stabilized

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Theory of Cosmological Perturbations: Basics

Cosmological fluctuations connect early universe theories with observations

- Fluctuations of matter $\rightarrow$ large-scale structure
- Fluctuations of metric $\rightarrow$ CMB anisotropies
- N.B.: Matter and metric fluctuations are coupled

Key facts:

- 1. Fluctuations are small today on large scales
- $\rightarrow$ fluctuations were very small in the early universe
- $\rightarrow$ can use linear perturbation theory
- 2. Sub-Hubble scales: matter fluctuations dominate
- Super-Hubble scales: metric fluctuations dominate

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Quantum Theory of Linearized Fluctuations

V. Mukhanov, H. Feldman and R.B., *Phys. Rep. 215:203 (1992)*

Step 1: Metric including fluctuations

$$ds^2 = a^2[(1+2\Phi)d\eta^2 - (1-2\Phi)d\mathbf{x}^2]$$

$$\varphi = \varphi_0 + \delta\varphi$$

Note: Φ and $\delta\varphi$ related by Einstein constraint equations

Step 2: Expand the action for matter and gravity to second order about the cosmological background:

$$S^{(2)} = \frac{1}{2}\int d^4x\left((v')^2 - v_{,i}v^{,i} + \frac{z''}{z}v^2\right)$$

$$v = a\left(\delta\varphi + \frac{z}{a}\Phi\right)$$

$$z = a\frac{\varphi_0'}{\mathcal{H}}$$

Step 3: Resulting equation of motion (Fourier space)

$$v_k'' + (k^2 - \frac{z''}{z})v_k = 0$$

Features:

- oscillations on sub-Hubble scales
- squeezing on super-Hubble scales $v_k \sim z$

Quantum vacuum initial conditions:

$$v_k(\eta_i) = (\sqrt{2k})^{-1}$$

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- squeezing on super-Hubble scales $v_k \sim z$

Quantum vacuum initial conditions:

$$v_k(\eta_i) = (\sqrt{2k})^{-1}$$

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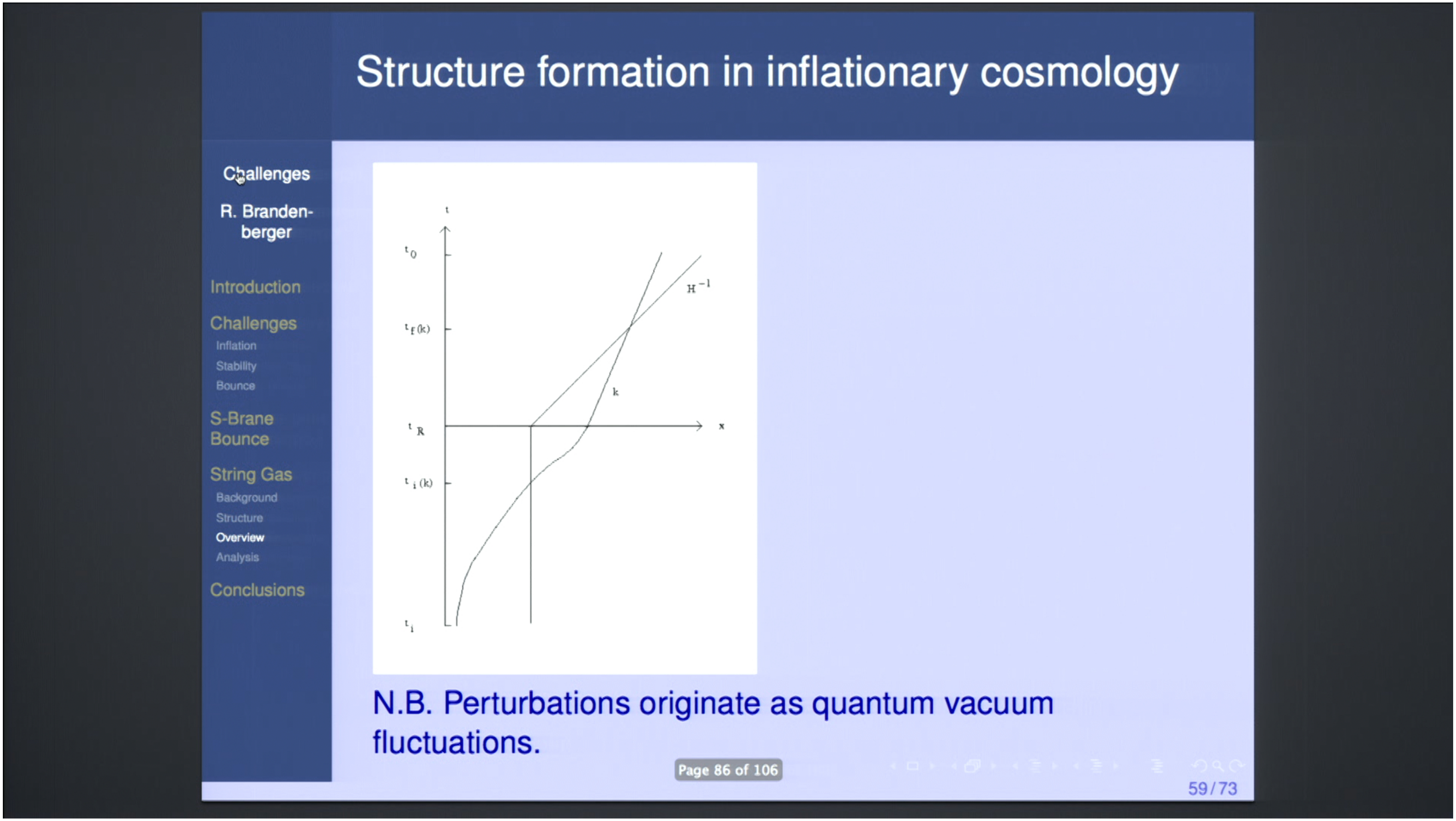
Structure formation in inflationary cosmology
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t
t0
tf (k)
tR
ti (k)
ti
H −1
k
x
N.B. Perturbations originate as quantum vacuum fluctuations.
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Background for string gas cosmology

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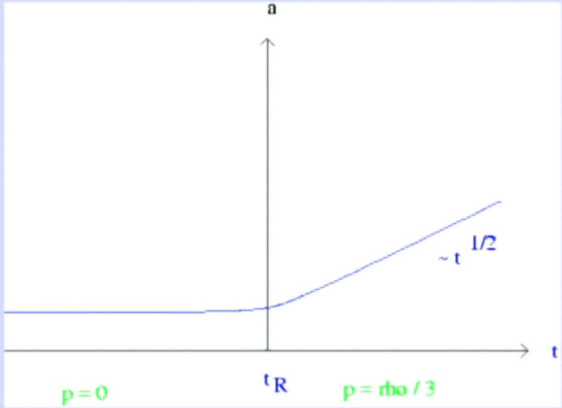
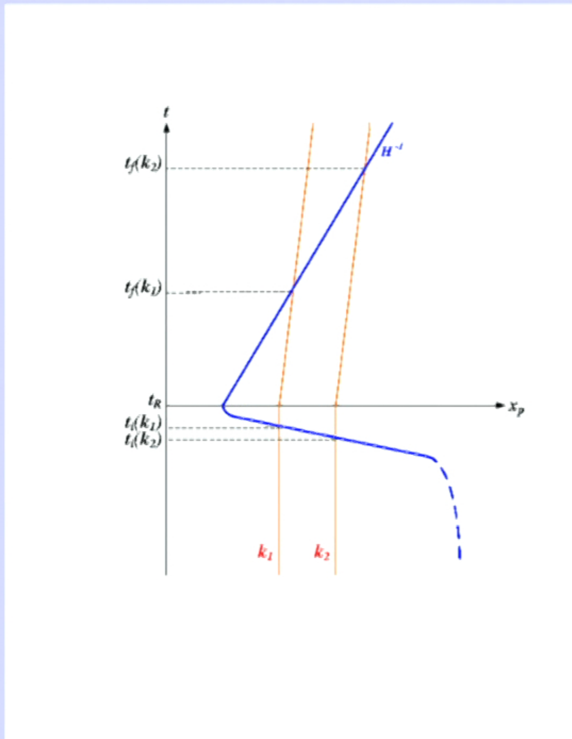


Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal string gas fluctuations.

Method

Challenges

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- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

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Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\left((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\right) .$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0{}_0(k) \delta T^0{}_0(k) \rangle ,$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^i{}_j(k) \delta T^i{}_j(k) \rangle .$$

Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V \, .$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T\,(1 - T/T_H)} \, .$$

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Power spectrum of cosmological fluctuations

$$
\begin{aligned}
P_\Phi(k) &= 8G^2k^{-1} < |\delta\rho(k)|^2 > \\
&= 8G^2k^2 < (\delta M)^2 >_R \\
&= 8G^2k^{-4} < (\delta\rho)^2 >_R \\
&= 8G^2 \frac{T}{\ell_s^3} \frac{1}{1 - T/T_H}
\end{aligned}
$$

Key features:

- scale-invariant like for inflation
- slight red tilt like for inflation

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Power spectrum of cosmological fluctuations

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- scale-invariant like for inflation
- slight red tilt like for inflation

Spectrum of Gravitational Waves

R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett. (2007)*

Challenges

R. Brandenberger

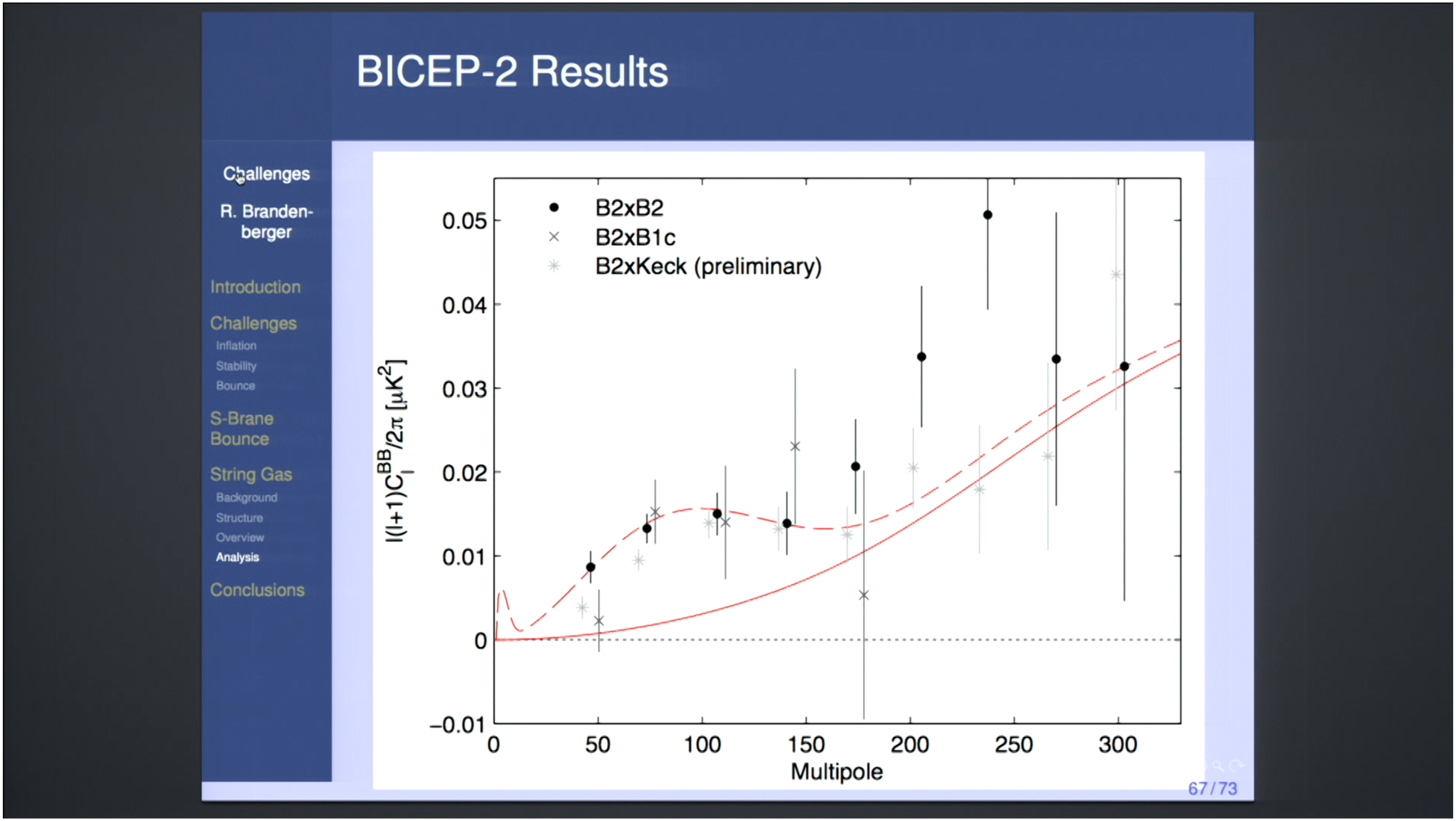
$$
\begin{aligned}
P_h(k) &= 16\pi^2 G^2 k^{-1} < |T_{ij}(k)|^2 > \\
&= 16\pi^2 G^2 k^{-4} < |T_{ij}(R)|^2 > \\
&\sim 16\pi^2 G^2 \frac{T}{\ell_s^3}(1 - T/T_H)
\end{aligned}
$$

Key ingredient for string thermodynamics

$$
< |T_{ij}(R)|^2 > \sim \frac{T}{l_s^3 R^4}(1 - T/T_H)
$$

Key features:

- scale-invariant (like for inflation)
- slight blue tilt (unlike for inflation)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B2xB2
B2xB1c
B2xKeck (preliminary)
l(l+1)C_l^BB/2π [μK²]
Multipole
0.05
0.04
0.03
0.02
0.01
0
−0.01
0
50
100
150
200
250
300
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R. Brandenberger

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Conclusions

- Emergent phase in thermal equilibrium
- $C_V(R) \sim R^2$ obtained from a thermal gas of strings provided there are winding modes which dominate.
- Cosmological fluctuations in the IR are described by Einstein gravity.

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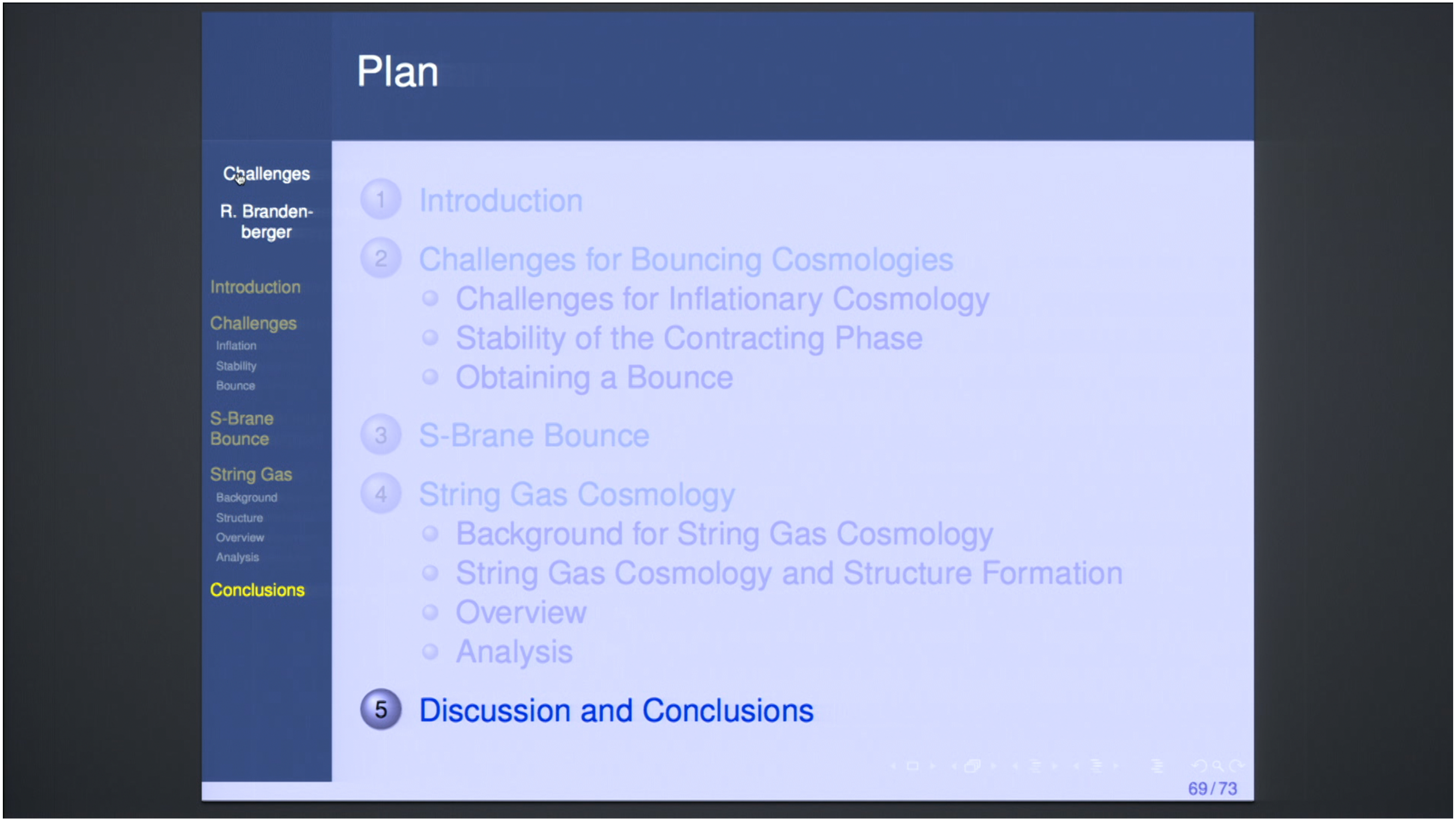
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- **Q: What is the new physics responsible for the bounce?**
- A: Duality Symmetry of Superstring Theory
- **Q: Might this physics resolve the singularity for the perturbations as well as the background?**
- A: yes
- **Does this new physics have any observational signature?**
- A: yes, a slight blue tilt of the spectrum of gravitational waves.
- **A: What general principles underlie the theory, beyond wanting to resolve the singularity?**
- A: Unification of all four forces of nature at a quantum level.

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Questions

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- **Q: Does a consistent picture for cosmology require that both the background and perturbations are quantized?**
- A: No
- **Q: Does the bounce or pre-bounce phase help in setting initial conditions?**
- A: The initial conditions for fluctuations are set in the bounce phase.

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- Current paradigm: cosmological inflation.
- Alternatives to cosmological inflation exist.
- Many of these alternatives are bouncing scenarios.
- Superstring cosmology $\rightarrow$ need to look beyond inflation and beyond point particle effective field theory.
- String Gas Cosmology: Model of cosmology of the very early universe based on new degrees of freedom and new symmetries of superstring theory.
- Thermal string fluctuations lead to a scale-invariant spectrum of cosmological fluctuations with a blue tilt of the tensor modes.
- String Theory testable through cosmological observations.

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Conclusions

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R. Brandenberger

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