Title: Intrinsic Non-commutativity of Closed Strings

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Abstract: <p>Abstract TBA</p>

Non-commutativity

- traditionally, in open strings, $\underline{\text{non-triv. BG}}$

  $\uparrow$ strength of NC

- here, assoc. w/ closed strings, triv. BG
  - scale of NC $\longleftrightarrow$ λ

"Spacetime" coordinate X

worldsheet: $X^\mu(\tau,\sigma)$ $\Big\{$ target sp: index in fields $\Phi(X)$

✓ non-compact space-times

× compact spacetimes

will consider symplectic structure

(in compact case)

– Scale of NC $\longleftrightarrow$ λ

"Spacetime" coordinate X

worldsheet: $X^\mu(\tau, \sigma)$

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✓ non-compact space-times

× compact spacetimes

will consider symplectic structure (in compact case)

– encodes Poisson brackets $\simeq$ comm. rel^s in qu. thy

classically:

variation of action

$\updownarrow$

pre-symplectic 1-form

quantum:

Causality (on the worldsheet)

\+

locality

$\mathbb{R} \longrightarrow \mathbb{R}/\mathbb{Z}$.

– must include winding modes

$$X(\tau, \sigma + 2\pi) = X(\tau, \sigma) + 2\pi w R$$

classical sol$^{\underline{ns}}$

$$X(\tau, \sigma) = X_R(\tau + \sigma) + X_L(\tau - \sigma)$$

$$\left.\begin{aligned} \hat{X}_L(\tau - \sigma) &= \underline{\hat{x}_L} + \frac{\alpha'}{2}\underline{\hat{p}_L}(\tau - \sigma) + \text{osc}_L \\ \hat{X}_R(\tau + \sigma) &= \underline{\hat{x}_R} + \frac{\alpha'}{2}\underline{\hat{p}_R}(\tau + \sigma) + \text{osc}_R \end{aligned}\right\}$$

$$X(\tau, \sigma) = x + \alpha' p \tau + \alpha' \hat{p} \sigma + \ldots$$

$$x = x_L + x_R$$

$$p = \frac{p_L + p_R}{2}, \quad \tilde{p} = \frac{p_R - p_L}{2}$$

classical sol$^{\underline{ns}}$

$$X(\tau,\sigma) = X_R(\tau+\sigma) + X_L(\tau-\sigma)$$

$$\hat{X}_L(\tau-\sigma) = \underline{\hat{x}_L} + \frac{\alpha'}{2}\underline{\hat{p}_L}(\tau-\sigma) + osc_L$$

$$\hat{X}_R(\tau+\sigma) = \underline{\hat{x}_R} + \frac{\alpha'}{2}\underline{\hat{p}_R}(\tau+\sigma) + osc_R$$

$$X(\tau,\sigma) = x + \alpha' p \tau + \alpha' \tilde{p} \sigma + \cdots$$

$$x = x_L + x_R$$

$$p = \frac{p_L + p_R}{2}, \quad \tilde{p} = \frac{p_R - p_L}{2}$$

T-duality : $X_R \to X_R$, $X_L \to -X_L$

$$\tilde{X}(\tau,\sigma) = X_R(\tau-\sigma) - X_L(\tau+\sigma) = \tilde{x} + \alpha'\tilde{p}\tau + \alpha' p \sigma + \cdots$$

$$\tilde{x} = x_R - x_L$$

$$* \quad \partial_\tau X = \partial_\sigma \tilde{X}$$

$$(\tilde{R} R \equiv 2\lambda^2)$$

Constraint : $m^2 = \left(\frac{n}{R}\right)^2 + \left(\frac{w}{\tilde{R}}\right)^2 + \frac{N_L + N_R}{\lambda^2}$

$\frac{n}{R} = p$, $\quad \frac{w}{\tilde{R}} = \tilde{p}$

$$nw = \frac{N_L - N_R}{2\lambda^2}$$

$$X(\tau, \sigma + 2\pi) = X(\tau, \sigma) + 2\pi w R$$

classical sol^{ns}

$$X(\tau, \sigma) = X_R(\tau+\sigma) + X_L(\tau-\sigma)$$

$$\hat{X}_L(\tau-\sigma) = \hat{x}_L + \frac{\alpha'}{2}\hat{p}_L(\tau-\sigma) + osc_L$$

$$\hat{X}_R(\tau+\sigma) = \hat{x}_R + \frac{\alpha'}{2}\hat{p}_R(\tau+\sigma) + osc_R$$

$$X(\tau,\sigma) = x + \alpha' p \tau + \alpha' \tilde{p} \sigma + \dots$$

$$x = x_L + x_R$$

$$p = \frac{p_L + p_R}{2}, \quad \tilde{p} = \frac{p_R - p_L}{2}$$

$$\tilde{x} = x_R - x_L$$

$$x \quad \partial_\tau X = \partial_\sigma \tilde{X}$$

constraint: $m^2 = \left(\frac{n}{R}\right)^2 + \left(\frac{w}{\tilde{R}}\right)^2 + \frac{N_L + N_R}{\lambda^2}$, with $\frac{n}{R} = p$, $\frac{w}{\tilde{R}} = \tilde{p}$

$$(\tilde{R} R = 2\lambda^2)$$

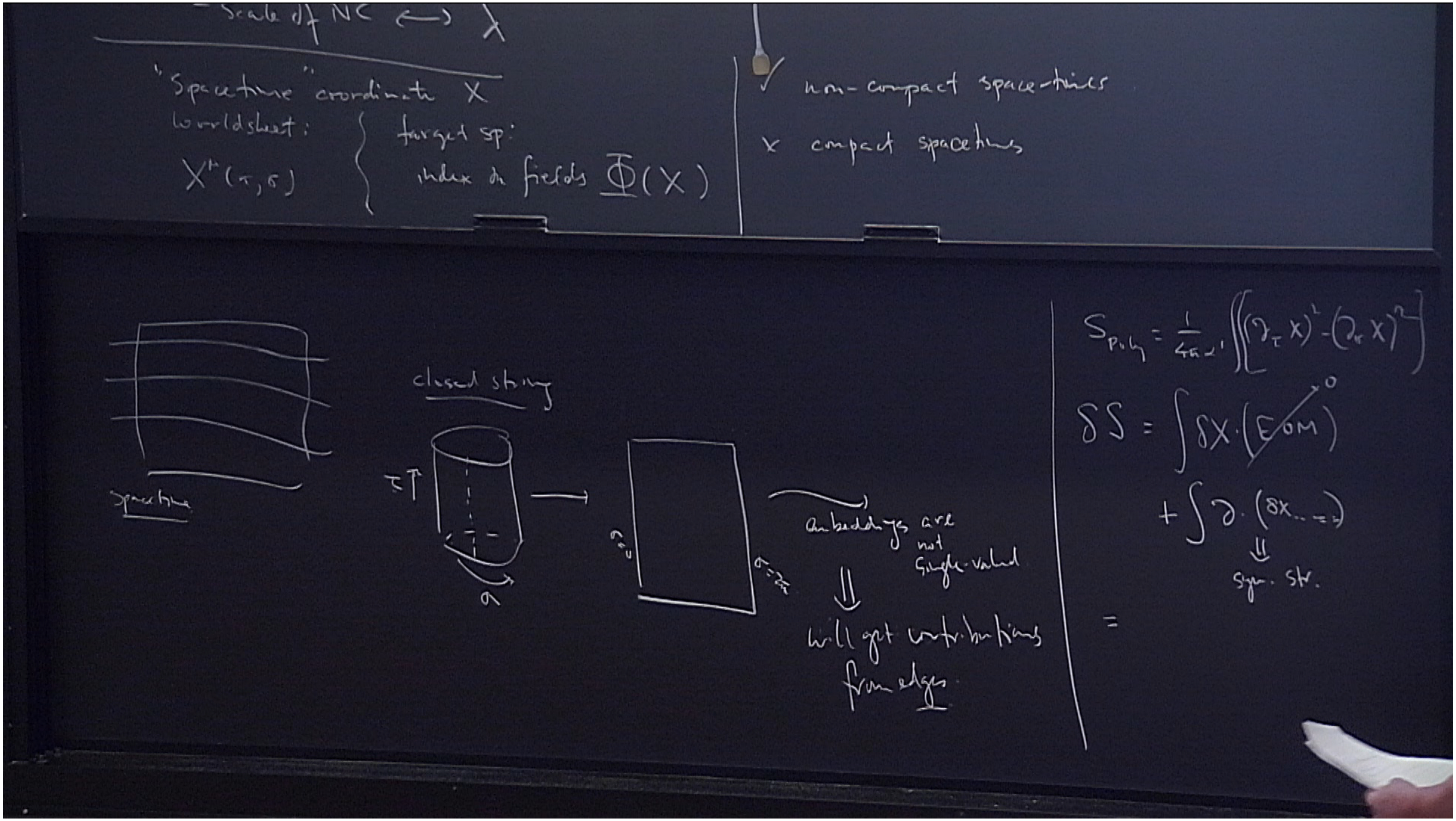
$$\frac{nw}{R\tilde{R}} = \frac{N_L - N_R}{2\lambda^2}$$

T-duality

$$(n, w)_{R,\tilde{R}} \longleftrightarrow (w, n)_{\tilde{R}, R}$$

eg $R \to \infty$: $w \to 0$, $n \to$ cts | non-compact coord ~ x; $\tilde{x}$ decouples

$R \to 0$: $n \to 0$, $w \to$ cts | $\tilde{x}$ looks like coord; x decouples



scale of NC ⟷ λ
"Spacetime" coordinate X
worldsheet:
X^μ(τ, σ)
target sp:
index on fields Φ(X)
✓ non-compact space-times
✗ compact spacetimes
spacetime
closed string
τ
σ
σ = 0
σ = 2π
embeddings are not single-valued
⇓
will get contributions from edges.
S_{Pol} = 1/(4πα') ∫∫[(∂_τ X)² − (∂_σ X)²]
δS = ∫ δX · (EOM)
+ ∫ ∂ · (δX ...)
⇓
sym. str.
=

for Polyakov: $S = \frac{1}{4\pi\alpha'}\int dX \wedge * dX$)

pre-symplectic 1-form.)

$$\delta S = \frac{1}{2\pi\alpha'}\int_0^{2\pi} d\sigma \, (\delta X \cdot \partial_\tau X)\Big|_{\tau_0}^{\tau_1} - \underbrace{\frac{1}{2\pi\alpha'}\int_{\tau_0}^{\tau_1} d\tau \; \delta X(\tau,\sigma)\cdot\partial_\sigma X \Big|_{\sigma=0}^{\sigma=2\pi}}_{-\frac{1}{2\pi\alpha'}\int_{\tau_0}^{\tau_1} d\tau \left(\delta X(\tau,2\pi) - \delta X(\tau,0)\right)\cdot \partial_\sigma X(\tau,0)}$$

$$-\frac{1}{2\pi\alpha'}\int_{\tau_0}^{\tau_1} d\tau\, \underbrace{\left(\delta X(\tau,2\pi) - \delta X(\tau,0)\right)}_{\delta\tilde{p}} \cdot \underbrace{\partial_\sigma X(\tau,0)}_{\partial_\tau \tilde{X}(\tau,0)}$$

$$-\delta\tilde{p}\cdot\tilde{X}(\tau,0)\Big|_{\tau_0}^{\tau_1}$$

"corner term".

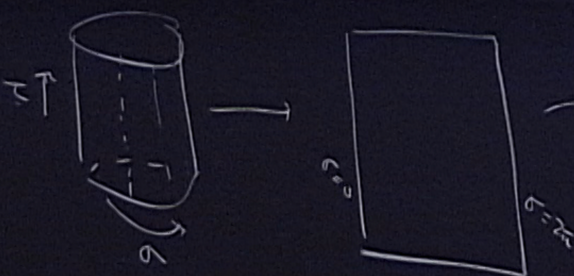
pre-sympl. 1-form

$$\Theta(\tau) = -\delta\tilde{p}\cdot\tilde{X}(\tau,0) + \frac{1}{2\pi\alpha'}\int d\sigma\, \delta X(\tau,\sigma)\,\partial_\tau X(\tau,\sigma)$$

sympl 2-form

$$\Omega(\tau) = \delta\Theta = \delta p\wedge\delta x + \delta\tilde{p}\wedge(\delta\tilde{x} - \pi\alpha'\delta p) + \text{osc}$$

spacetime



embeddings are not single-valued

$\Downarrow$

will get contributions from edges.

$$+\int\partial\cdot(\delta X\ldots)$$

$\Downarrow$

sym. str.

$=$

pre-sympl. 1-form

$$\Theta(\tau) = -\delta\hat{\tilde{p}}\cdot\hat{X}(\tau,0) + \frac{1}{2\pi\alpha'}\int d\sigma\,\delta X(\tau,\sigma)\,\partial_\tau X(\tau,\sigma)$$

$$-\delta\tilde{p}\cdot X(\tau,0)\Big|_{\tau_0}$$

"corner term"

Sympl 2-form

$$\Omega(\tau) = \delta\Theta = \delta p\wedge\delta x + \delta\tilde{p}\wedge(\delta\tilde{x} - \pi\alpha'\delta p) + \text{osc}$$

$$Z^\alpha = (p, x, \tilde{p}, \tilde{x})$$

$$\Omega = \tfrac{1}{2}\Omega_{\alpha\beta}\,\delta Z^\alpha\wedge\delta Z^\beta$$

$$\Omega_{\alpha\beta} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & \pi\alpha' \\ 0 & 0 & 0 & -1 \\ 0 & -\pi\alpha' & 1 & 0 \end{pmatrix}$$

$$[\hat{Z}^\alpha, \hat{Z}^\beta] = i\hbar\,\Omega^{\alpha\beta}\,\hat{\mathbb{1}}$$

$$[p_a, \tilde{p}^b] = 0$$

$$[x^a, p_b] = i\hbar\,\delta^a{}_b$$

$$[\tilde{x}_a, \tilde{p}^b] = i\hbar\,\delta_a{}^b$$

$$[x^a, \tilde{x}_b] = 2\pi i\lambda^2\,\delta^a{}_b$$

Consider equal-τ commutators

$$[\hat{X}(\tau,\sigma_1), \hat{\tilde{X}}(\tau,\sigma_2)]$$

$$= [\hat{x}, \hat{\tilde{x}}] + \alpha'[x,p]\sigma_2 + \alpha'[\tilde{p},\tilde{x}]\sigma_1 + \text{osc}$$

$$= 2i\lambda^2\left[\pi - \Theta(\sigma_{12})\right]$$

$\Theta(\sigma) = \pi$ on $\sigma\in(0,2\pi)$

$\Theta'(\sigma) = \delta(\sigma)$

pre-sympl. 1-form

$$\Theta(\tau) = -\delta\hat{\bar p}\cdot\tilde X(\tau,0) + \frac{1}{2\pi\alpha'}\int d\sigma\,\delta X(\tau,\sigma)\,\partial_\tau X(\tau,\sigma)$$

$$-\delta\bar p\cdot X(\tau,0)\Big|_{\tau_0}$$

"corner term"

sympl 2-form

$$\Omega(\tau) = \delta\Theta = \delta p\wedge\delta x + \delta\bar p\wedge(\delta\tilde x - \pi\alpha'\delta p) + \text{osc}$$

$$[\hat X(\tau,\sigma_1), \partial_\tau\hat X(\tau,\sigma_2)] = \delta(\sigma_{12})$$

$$\partial_\tau\hat X = \partial_\sigma\tilde X$$

integrate → not constant.

consider equal-τ commutators

$$[\hat X(\tau,\sigma_1), \hat{\tilde X}(\tau,\sigma_2)]$$

$$= [\hat x,\hat{\tilde x}] + \alpha'[x,p]\sigma_2 + \alpha'[\bar p,\tilde x]\sigma_1 + \text{osc}$$

$$= 2i\lambda^2[\pi - \Theta(\sigma_{12})]$$

$\Theta(\sigma) = \pi$ on $\sigma\in(0,2\pi)$

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classical sol$^{\text{ns}}$

$$X(\tau,\sigma) = X_R(\tau+\sigma) + X_L(\tau-\sigma)$$

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$$X(\tau,\sigma) = x + \alpha' p\tau + \alpha'\tilde{p}\sigma + \cdots$$

$$x = x_L + x_R$$

$$p = \frac{p_L + p_R}{2}, \quad \tilde{p} = \frac{p_R - p_L}{2}$$

Mutual locality

physical vertex ops : single-valued

eg. tachyon vertex op.

$$W_{k,\tilde{k}} = :e^{ik\cdot\hat{X}(\tau,\sigma)} e^{i\tilde{k}\cdot\hat{\tilde{X}}(\tau,\sigma)}:$$

$$\sim \underbrace{U_{k,\tilde{k}}}_{\text{zero mode}} \underbrace{V_{k_L}(z) V_{k_R}(\bar{z})}_{\text{osc}}$$

$$\mathbb{K} = (k,\tilde{k})$$

$$U_{\mathbb{K}} \cdot U_{\mathbb{K}'} = \epsilon_{\mathbb{K},\mathbb{K}'} U_{\mathbb{K}+\mathbb{K}'}$$

find: $$\frac{\epsilon_{\mathbb{K},\mathbb{K}'}}{\epsilon_{\mathbb{K}',\mathbb{K}}} = e^{2\pi i \lambda^2 (k\cdot\tilde{k}' + k'\cdot\tilde{k})}$$

Polchinski:

$$U_{k,\tilde{k}} = e^{ik_L\cdot\hat{x}_L + ik_R\hat{x}_R}\,\hat{C}_{k,\tilde{k}}(\hat{p}_L,\hat{p}_R)$$

$$\hat{C}_{k,\tilde{k}} = e^{i\pi\alpha'\tilde{k}\cdot\hat{p}}$$

w/ the correct comm. rel^ns.

$$\hat{C}_{k,\tilde{k}} = \hat{1}$$

$U_{\mathbb{K}}$: Weyl op on $(\hat{x},\hat{\tilde{x}})$

satisfies Heisenberg group

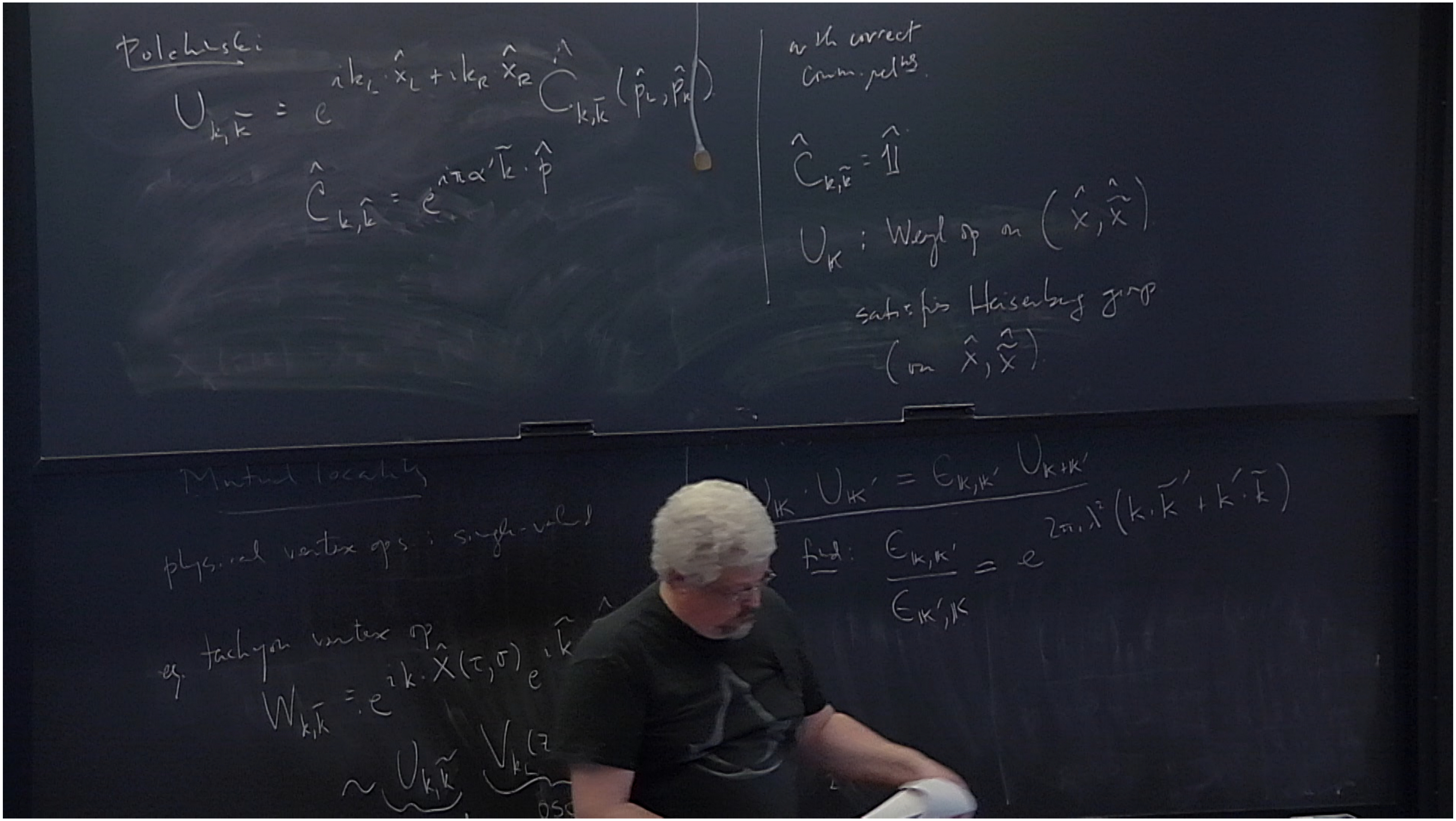
Mutual locality

pl... vertex ops: single-valued

vertex op $\hat{X}(\tau,\sigma)$ $i\hat{k}\cdot\hat{\tilde{X}}(\tau,\sigma)$

$$U_{\mathbb{K}}\cdot U_{\mathbb{K}'} = \epsilon_{\mathbb{K},\mathbb{K}'}\,U_{\mathbb{K}+\mathbb{K}'}$$

find: $$\frac{\epsilon_{\mathbb{K},\mathbb{K}'}}{\epsilon_{\mathbb{K}',\mathbb{K}}} = e^{2\pi i\lambda^2(k\cdot\tilde{k}'+k'\cdot\tilde{k})}$$


Polchinski:
$U_{k,\tilde{k}} = e^{ik_L \cdot \hat{x}_L + ik_R \cdot \hat{x}_R} \hat{C}_{k,\tilde{k}}(\hat{p}_L, \hat{p}_R)$
$\hat{C}_{k,\tilde{k}} = e^{i\pi\alpha' \tilde{k} \cdot \hat{p}}$
w/ the correct comm. rel'ns
$\hat{C}_{k,\tilde{k}} = \hat{1}$
$U_{\mathbb{K}}$: Weyl op on $(\hat{x}, \hat{\tilde{x}})$
satisfies Heisenberg group (on $\hat{x}, \hat{\tilde{x}}$)
Mutual locality
physical vertex ops: single-valued
e.g. tachyon vertex op
$W_{k,\tilde{k}} = e^{ik\cdot\hat{X}(\tau,\sigma)} e^{i\hat{\tilde{k}}}$
$\sim U_{k,\tilde{k}} V_{k_L}$
$U_{\mathbb{K}} \cdot U_{\mathbb{K}'} = \epsilon_{\mathbb{K},\mathbb{K}'} U_{\mathbb{K}+\mathbb{K}'}$
find: $\frac{\epsilon_{\mathbb{K},\mathbb{K}'}}{\epsilon_{\mathbb{K}',\mathbb{K}}} = e^{2\pi i\lambda^2(k\cdot\tilde{k}' + k'\cdot\tilde{k})}$

Polchinski:

$$U_{k,\bar{k}} = e^{ik_L\cdot\hat{x}_L + ik_R\cdot\hat{x}_R}\,\hat{C}_{k,\bar{k}}(\hat{p}_L,\hat{p}_R)$$

$$\hat{C}_{k,\hat{k}} = e^{i\pi\alpha'\tilde{k}\cdot\hat{p}}$$

w/ the correct comm. rel^ns

$$\hat{C}_{k,\tilde{k}} = \hat{\mathbb{1}}$$

U_K : Weyl op on $(\hat{x},\hat{\tilde{x}})$

satisfies Heisenberg grp (on $\hat{x},\hat{\tilde{x}}$)

$$W_{k,\bar{k}} = e^{ik\cdot\ldots}\,e$$

$$\sim \underbrace{U_{k,\tilde{k}}}\;\underbrace{V_{k_L}(z)V_{k_R}(\bar{z})}_{\text{osc}}$$

pre-sympl. 1-form

$$\Theta(\tau) = -\delta\hat{\bar{p}}\cdot\hat{X}(\tau,0) + \frac{1}{2\pi\alpha'}\int d\sigma\, \delta X(\tau,\sigma)\,\partial_\tau X(\tau,\sigma)$$

$$-\delta\bar{p}\cdot\hat{X}(\tau,0)\Big|_{\tau_0}$$

"corner term"

sympl 2-form

$$\Omega(\tau) = \delta\Theta = \delta p\wedge\delta x + \delta\bar{p}\wedge(\delta\tilde{x} - \pi\alpha'\delta p) + \text{osc}$$

- with these comm. relations

→ obtain covariance wrt $O(d,d)$

eg turning on constant B-field

$$[x^a, x^b] = 0\ ,\ [x^a, \tilde{x}_b] = 2\pi i\lambda^2\delta^a{}_b$$

$$\mathbb{X}^A = (x^a, \tilde{x}_a) \qquad [\tilde{x}_a, \tilde{x}_b] = -4\pi i\lambda^2 B_{ab}$$

$$[\hat{\mathbb{X}}^A, \hat{\mathbb{X}}^B] = \omega^{AB}\,\hat{\mathbb{1}}\ ;\quad \text{B-field} \longrightarrow g_B = \begin{pmatrix}\delta^a{}_b & B_{ab}\\ 0 & \delta_a{}^b\end{pmatrix}$$

$$\omega \longrightarrow g_B^T\,\omega\, g_B .$$

Consider equal-τ commutators

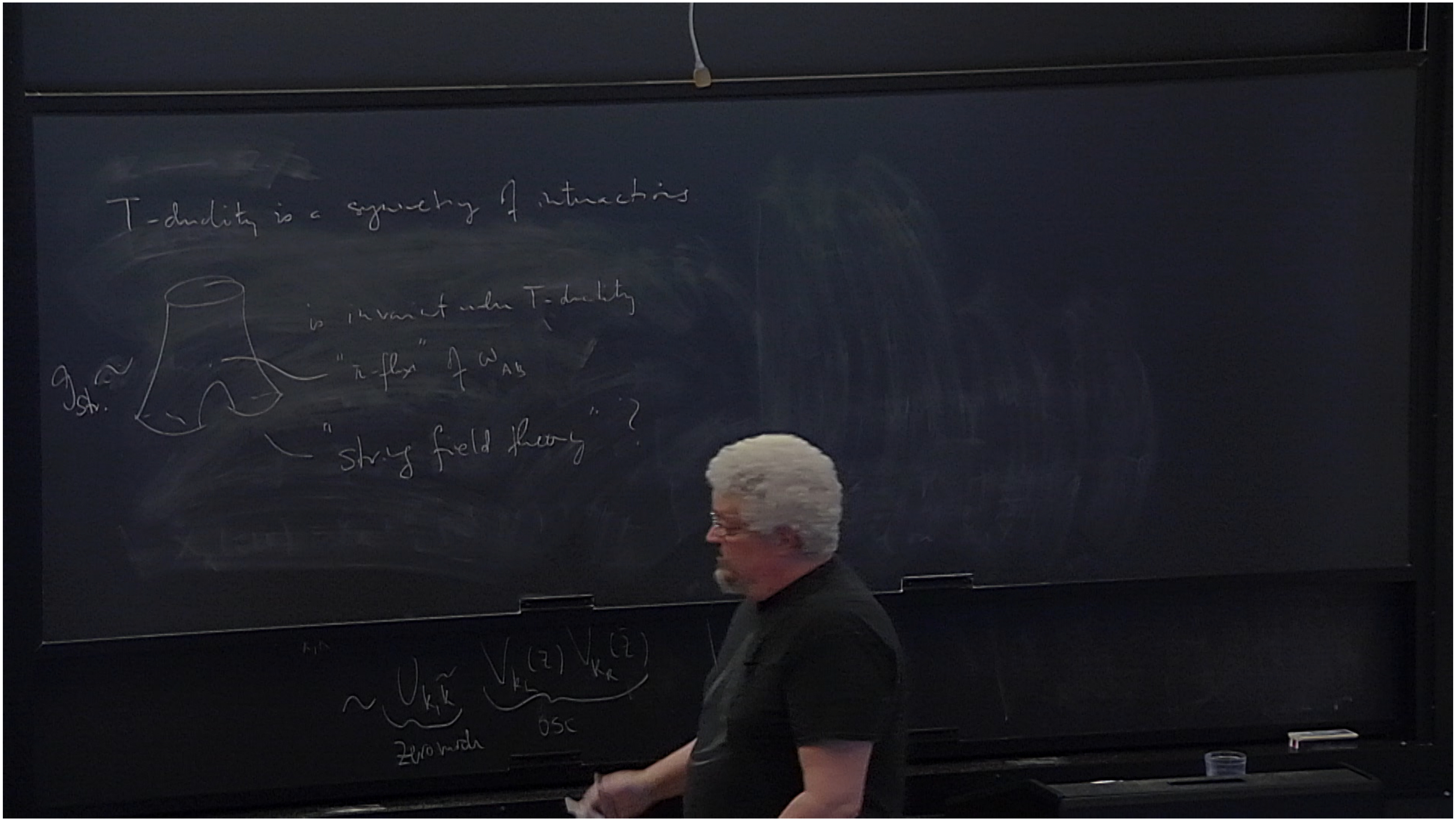
$$[\hat{X}(\tau,\sigma_1), \hat{\tilde{X}}(\tau,\sigma_2)]$$

$$= [\hat{x}, \hat{\tilde{x}}] + \alpha'[x,p]\sigma_2 + \alpha'[\bar{p},\tilde{x}]\sigma_1 + \text{osc}$$

$$i\lambda^2\left[\pi - \Theta(\sigma_{12})\right]$$

$$\Theta(\sigma) = \pi \ \text{ on } \sigma\in(0,2\pi)$$

$$\Theta'(\sigma) = \delta(\sigma)$$


T-duality is a symmetry of interactions
is invariant under T-duality
"π-flux" of ω_AB
"string field theory"?
zero mode
osc