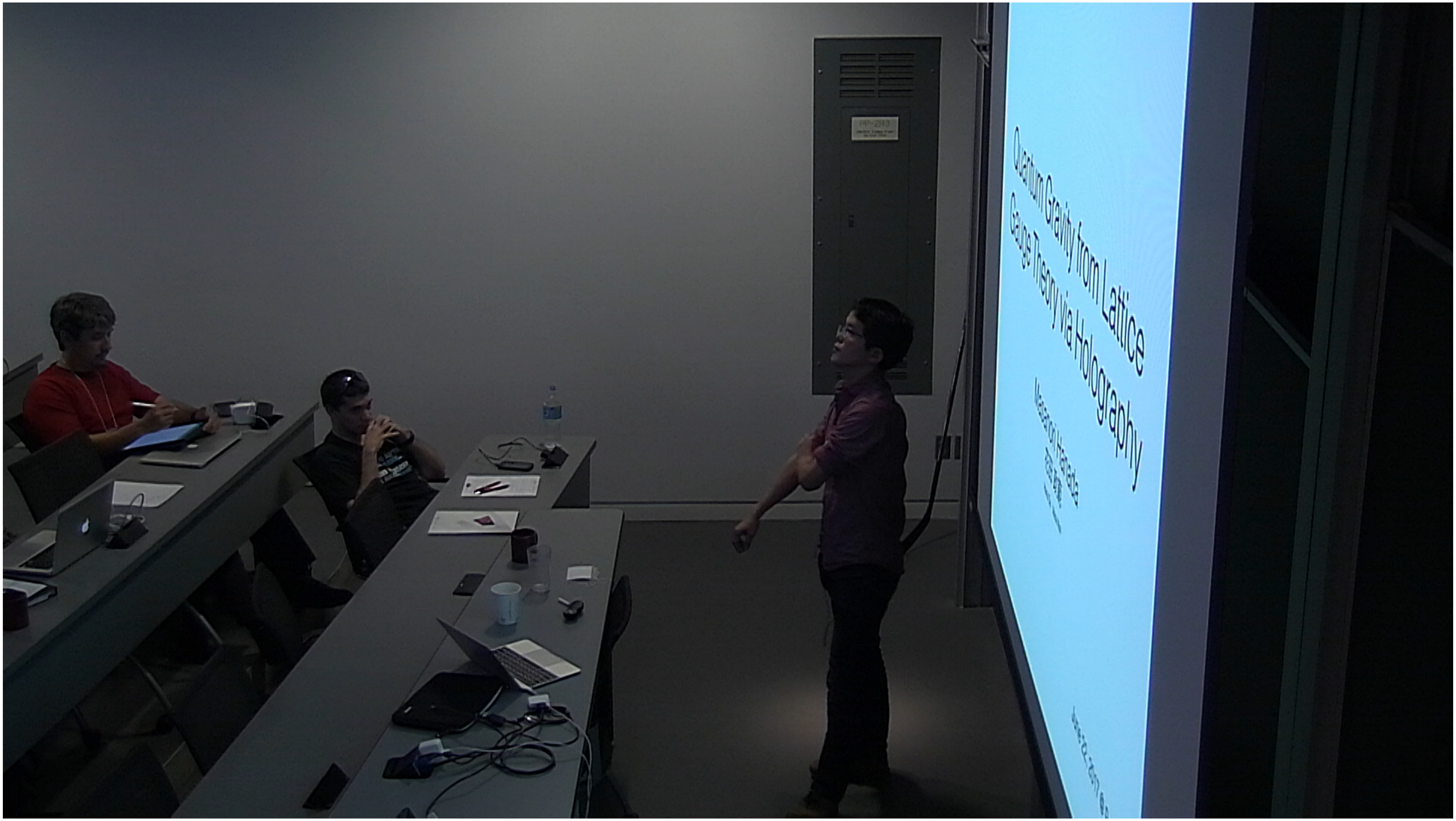
Title: Quantum Gravity from Lattice Gauge Theory via Holography

Date: Jun 22, 2017 05:15 PM

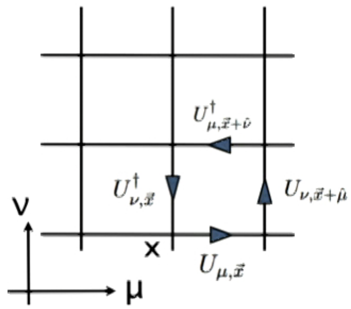
URL: http://pirsa.org/17060088

Abstract: The gauge/gravity duality relates supersymmetric gauge theories to superstring/M-theory.
Powerful Monte Carlo methods developed in lattice QCD can be applied to study the former,and from the simulation data we can extract the quantum gravitational effects. In this talk I will give a brief introduction to lattice gauge theory and supersymmetry on a lattice, and show some applications to quantum gravity via the gauge/gravity duality.


Quantum Gravity from Lattice
Gauge Theory via Holography
Masanori Hanada

Lattice Gauge Theory

- Regularization of gauge theory (and more generic QFT).
- Nonperturbative formulation of QFT.
- Practically useful numerical tool.

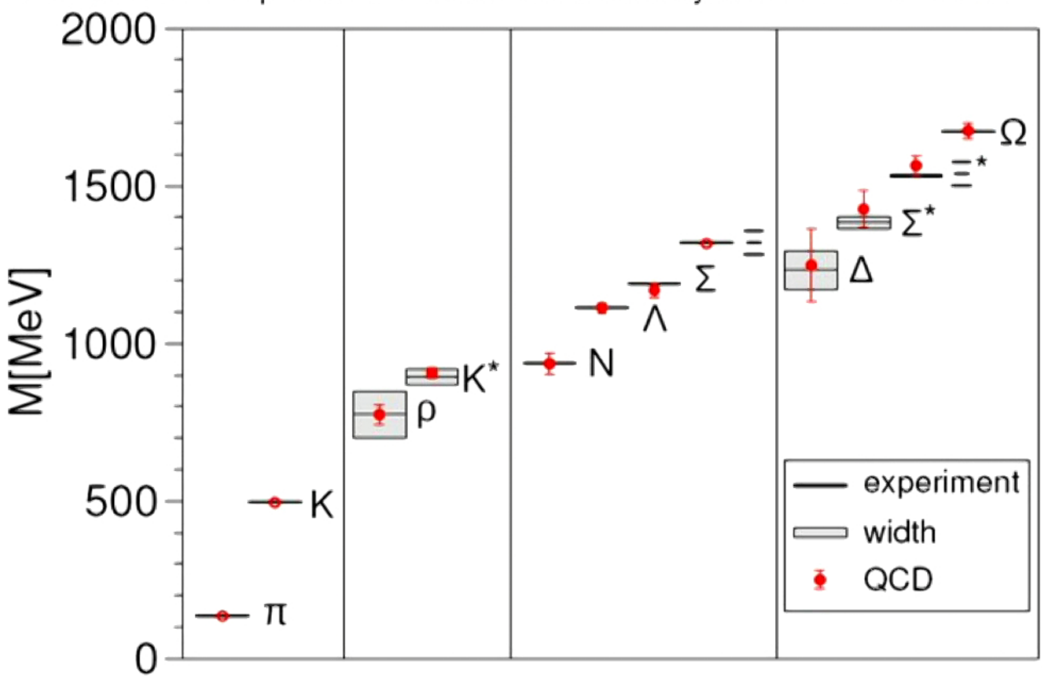


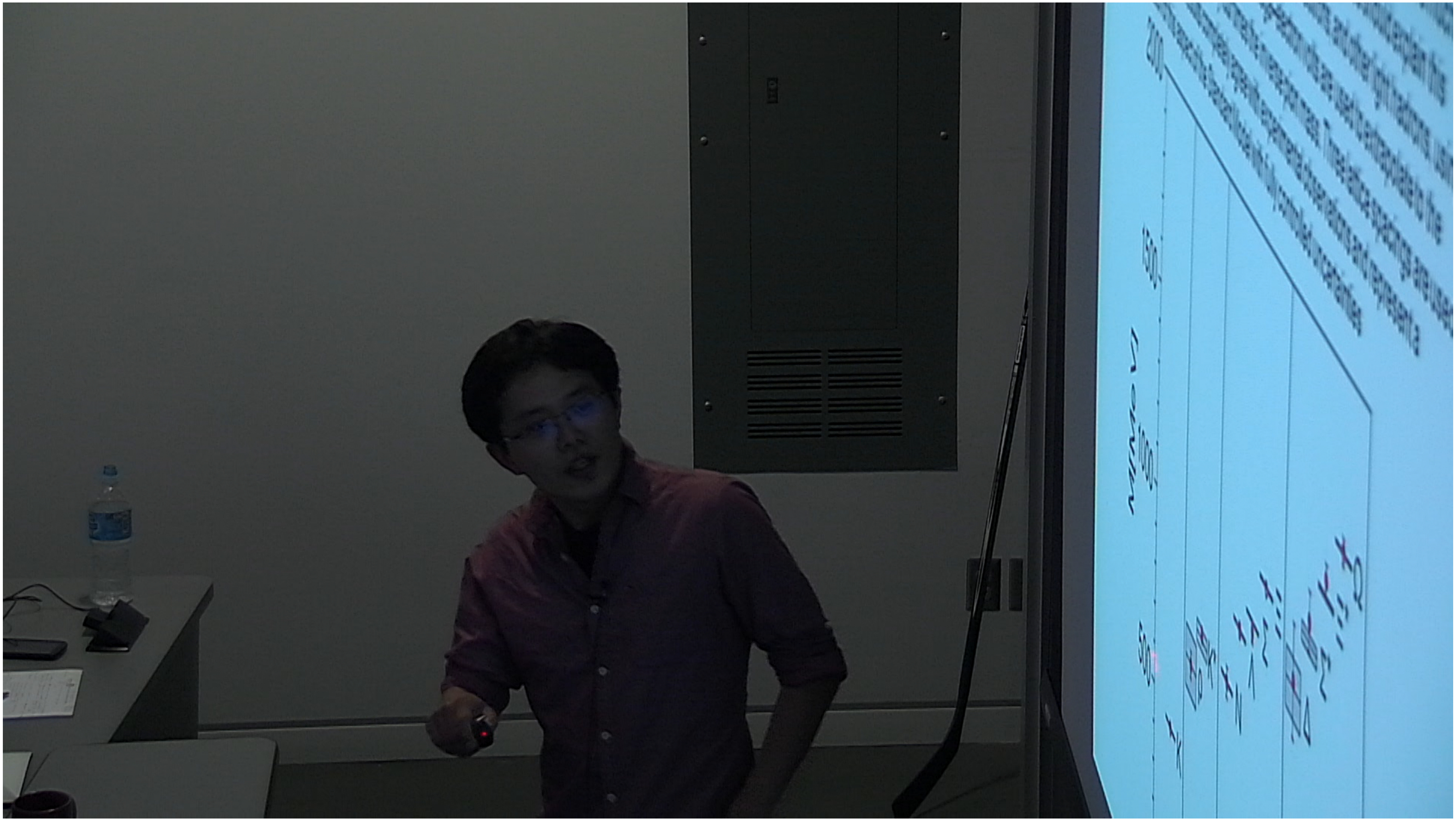
Low-lying Hadron Spectrum

Dürr, Fodor, Lippert et al., BMW Collaboration

Science 322, 1224 November 2008

More than 99% of the mass of the visible universe is made up of protons and neutrons. Both particles are much heavier than their quark and gluon constituents, and the Standard Model of particle physics should explain this difference. We present a full ab initio calculation of the masses of protons, neutrons, and other light hadrons, using lattice quantum chromodynamics. Pion masses down to 190 mega–electron volts are used to extrapolate to the physical point, with lattice sizes of approximately four times the inverse pion mass. Three lattice spacings are used for a continuum extrapolation. Our results completely agree with experimental observations and represent a quantitative confirmation of this aspect of the Standard Model with fully controlled uncertainties





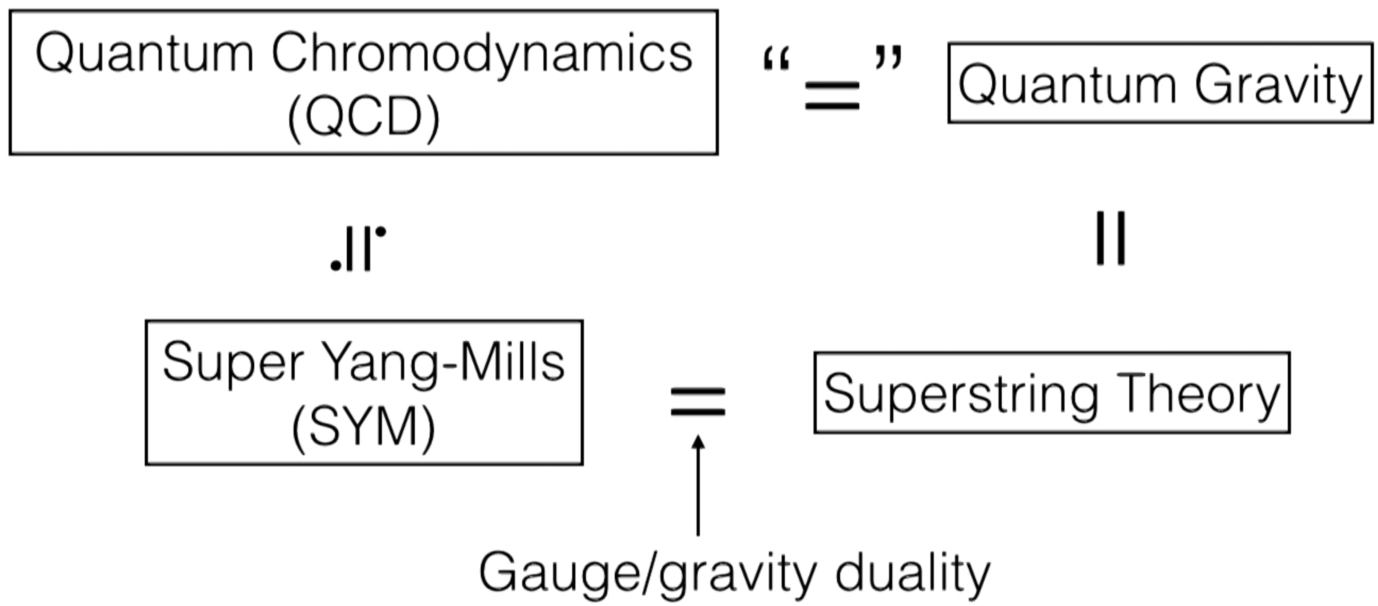
Quantum Chromodynamics (QCD) "=" Quantum Gravity

Quantum Chromodynamics
(QCD)

“=” Quantum Gravity

“=”

Super Yang-Mills
(SYM)


Quantum Chromodynamics (QCD)
"="
Quantum Gravity
Super Yang-Mills (SYM)
=
Superstring Theory
Gauge/gravity duality

AdS/CFT Duality

(Maldacena1997)

IIB string
on $AdS_5 \times S^5$

equivalent

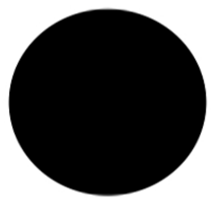
(3+1)-d U(N)
maximal SYM

.ll•

QCD

Gauge/Gravity Duality

(Maldacena1997, Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)



IIA/IIB string around black p-brane

equivalent

(p+1)-d U(N)SYM (Dp-branes+strings)

$p=3 \rightarrow AdS_5 \times S^5$

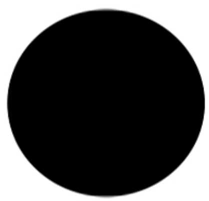
$p=0,1,2,3$

.ll'

QCD

Black p-brane = bunch of Dp-branes

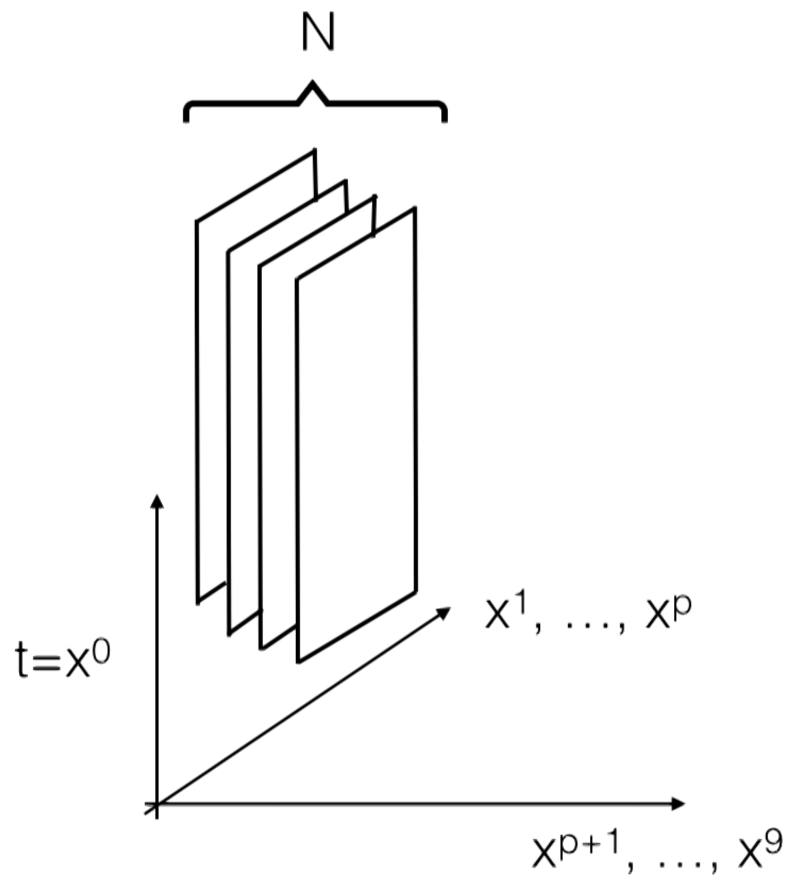
(+ strings between them)



- Dp-brane : (p+1)-d object
- Open string connects Dp-branes

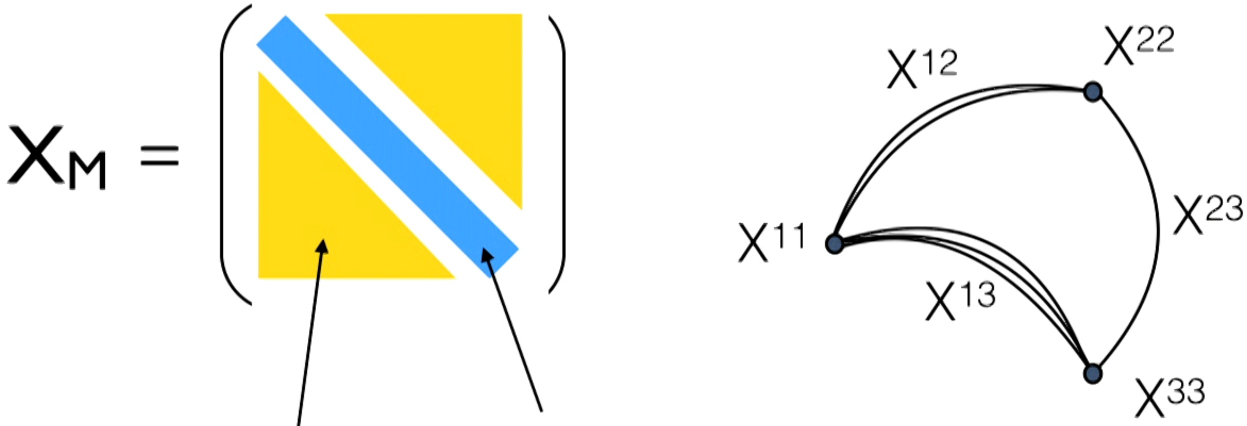
low-energy effective theory of Dp-branes
= (p+1)-d SYM

U(N)   N = number of D-branes



(9−p) transverse directions

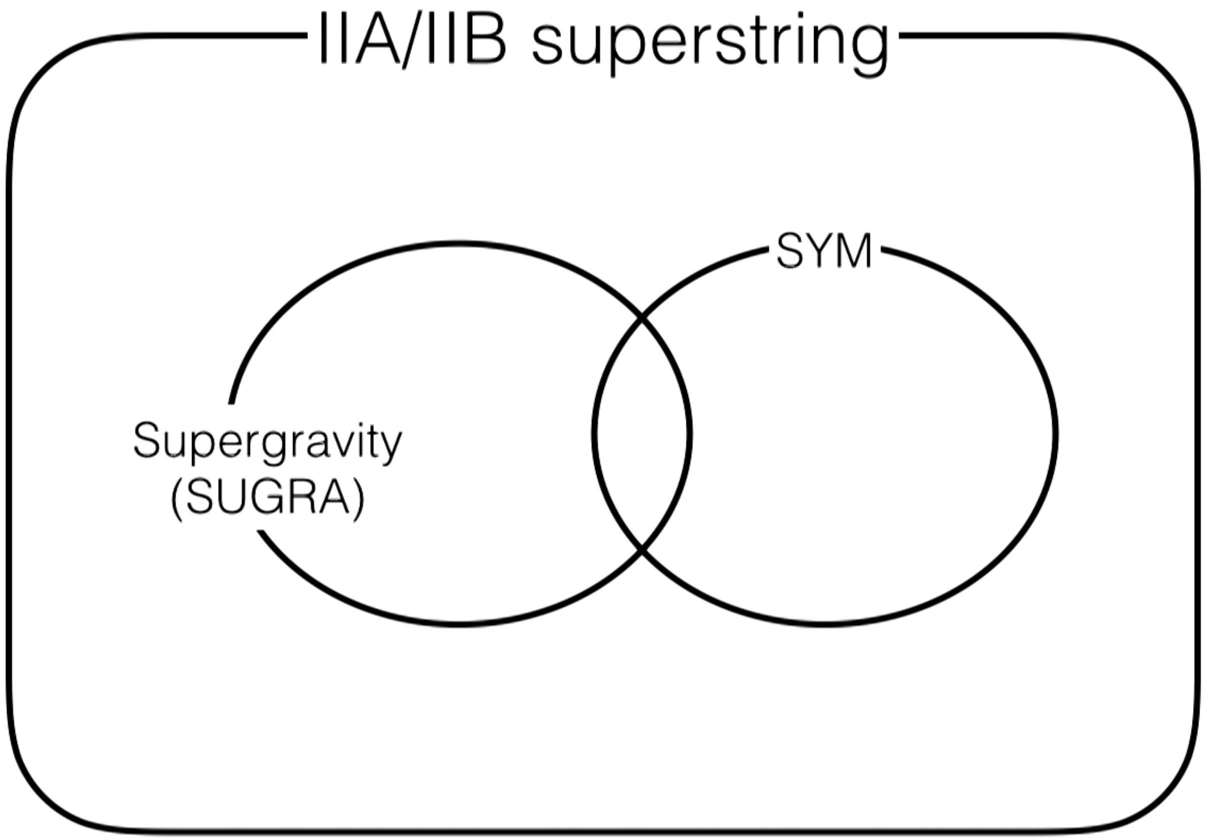
Dp-brane bound state and Gauge Theory

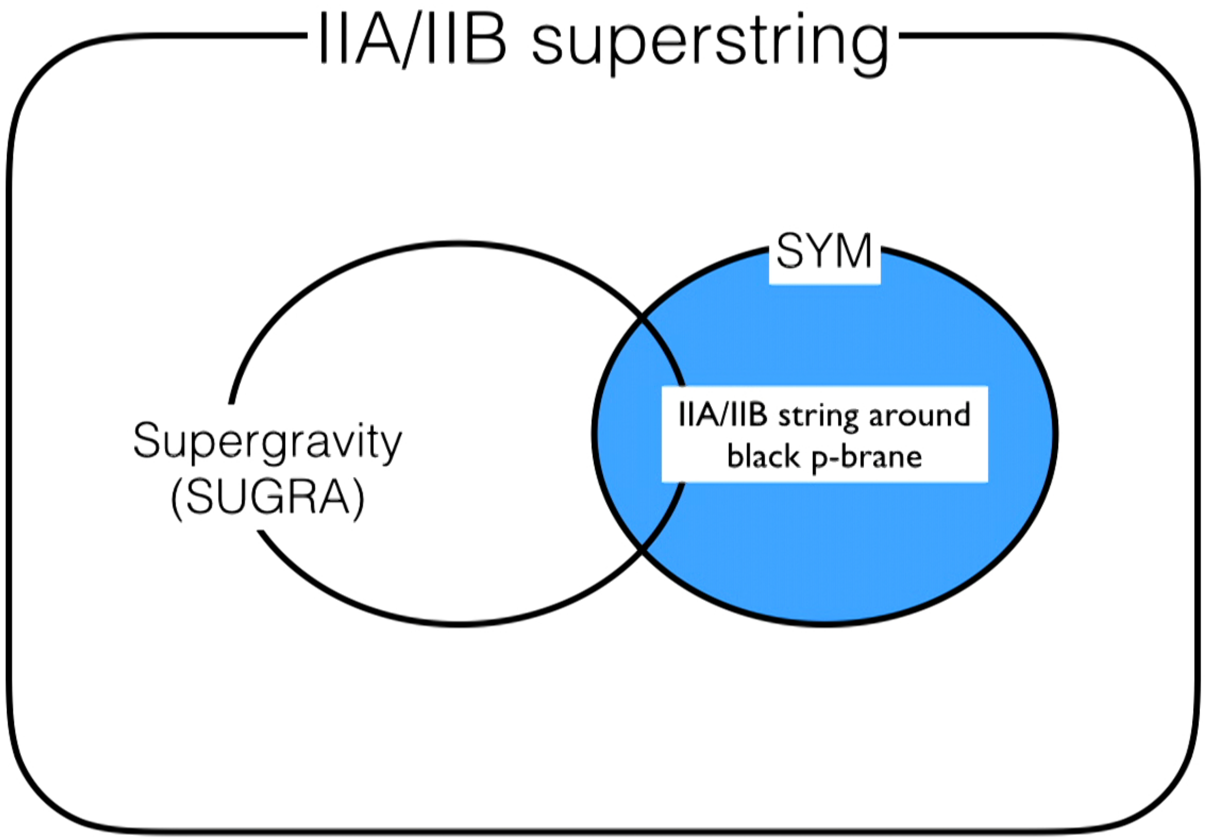


$(X_1^{ii}, X_2^{ii}, \ldots, X_{9-p}^{ii})$
position of i-th Dp-brane in R^{9-p}

X_M^{ij} : open strings connecting i-th and j-th D0-branes.
large value → a lot of strings are excited

(Witten, 1994)


IIA/IIB superstring
SYM
Supergravity
(SUGRA)


IIA/IIB superstring
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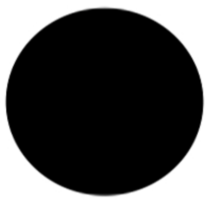
Gauge/Gravity Duality

(Maldacena1997, Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)



IIA/IIB string around black p-brane

equivalent

(p+1)-d U(N)SYM (Dp-branes+strings)

$p=3 \rightarrow AdS_5 \times S^5$

$p=0,1,2,3$

Gauge/Gravity Duality

(Maldacena1997, Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)

IIA/IIB string around black p-brane

definition

(p+1)-d U(N)SYM (Dp-branes+strings)

$p=3 \rightarrow AdS_5 \times S^5$

p=0,1,2,3

By deriving various field theories from string theory and considering their large N limit we have shown that they contain in their Hilbert space excitations describing supergravity on various spacetimes. We further conjectured that the field theories are dual to the full quantum M/string theory on various spacetimes. In principle, we can use this duality to give a definition of M/string theory on flat spacetime as (a region of) the large N limit of the field theories. Notice that this is a non-perturbative proposal for defining such theories, since the corresponding field theories can, *in principle*, be defined non-perturbatively. We

Maldacena,
"The Large N Limit of Superconformal Field Theories and Supergravity"
(1997)

in principle, if not in practice, possible. The situation is much like that in QCD where the only known definition of the theory is in terms of a conjectured limit of lattice gauge theory. Although the practical utility of the lattice theory may be questioned, it is almost certain that an extrapolation to the continuum limit exists. The existence of the lattice gauge Hamiltonian formulation insures that the the theory is unitary and gauge invariant.

Banks-Fischler-Shenker-Susskind,
"M Theory As A Matrix Model: A Conjecture"
(1996)

Lattice Gauge Theory → Gauge Theory

Super Yang-Mills → String/M-theory

<u>Definitions of the theories</u>

doable in practice!

If the conjecture is correct, it would provide us with the first well defined nonperturbative formulation of a quantum theory which includes gravitation. In principle, with a sufficiently big and fast computer any scattering amplitude could be computed in the finite N matrix model with arbitrary precision. Numerical extrapolation to infinite N is in principle, if not in practice, possible. The situation is much like that in QCD where the only known definition of the theory is in terms of a conjectured limit of lattice gauge theory. Although the practical utility of the lattice theory may be questioned, it is almost certain that an extrapolation to the continuum limit exists. The existence of the lattice gauge Hamiltonian formulation insures that the the theory is unitary and gauge invariant.

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(1996)

Can SYM be put on computer?

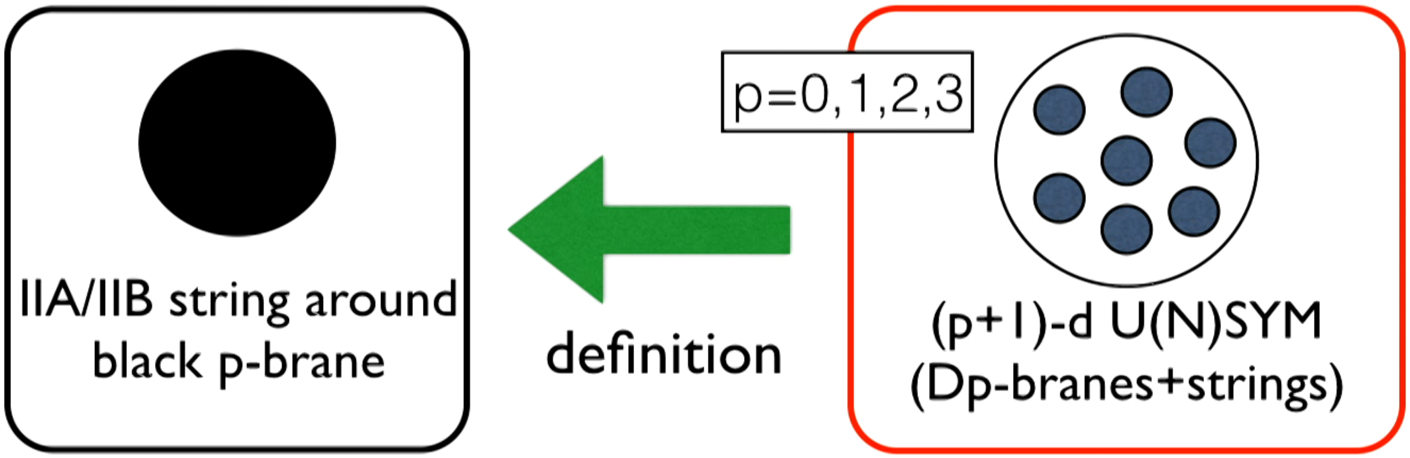
- ‘Fine tuning problem’
- ‘Sign problem’

(will be explained later)

Both problems have been **solved** for interesting classes of theories.

How large N can we go?

(with currently available resources; very rough estimate)



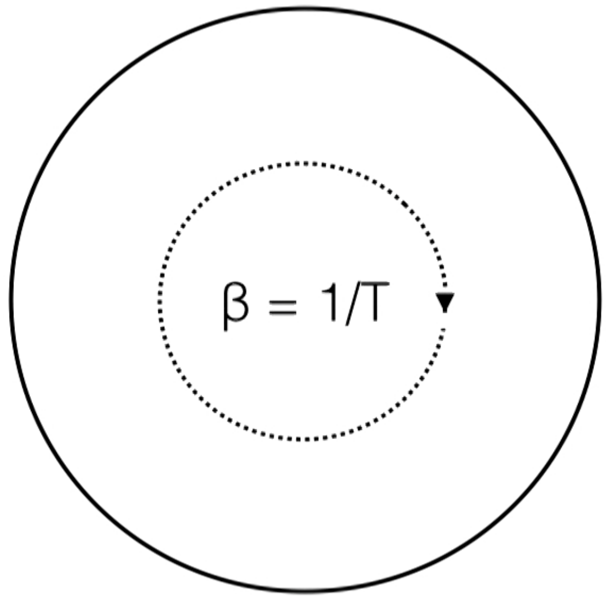
p=0 → N=128 is OK; probably 256

(if you/I write a good code, with not very large volume)

p=1 → N=32?

p=2 → N=8 ∼12?

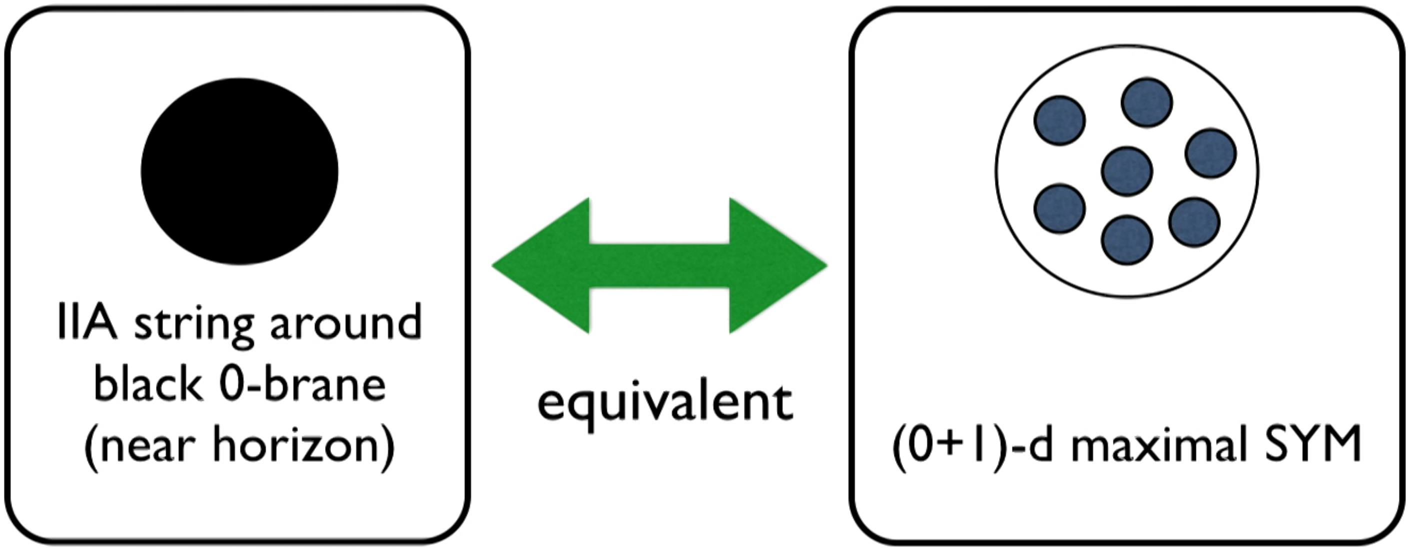
p=3 → N=3∼8?



Thermodynamics → Euclidean time circle with circumference $\beta = 1/T$

Black hole = matrix model

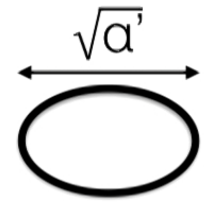
p=0



simulation cost $\sim N^6 T^{-3}$

high temperature is cheap, low temperature is expensive.

SYM		STRING
large-N, strong coupling	←	SUGRA
large-N, finite coupling	→	α' correction
finite-N, finite coupling	→	g_s correction



D0-brane quantum mechanics

$$S = \frac{N}{\lambda}\int dt\, Tr\Big\{\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2 + \frac{1}{2}\bar{\psi} D_t \psi - \frac{1}{2}\bar{\psi}\gamma^i[X_i, \psi]\Big\}$$

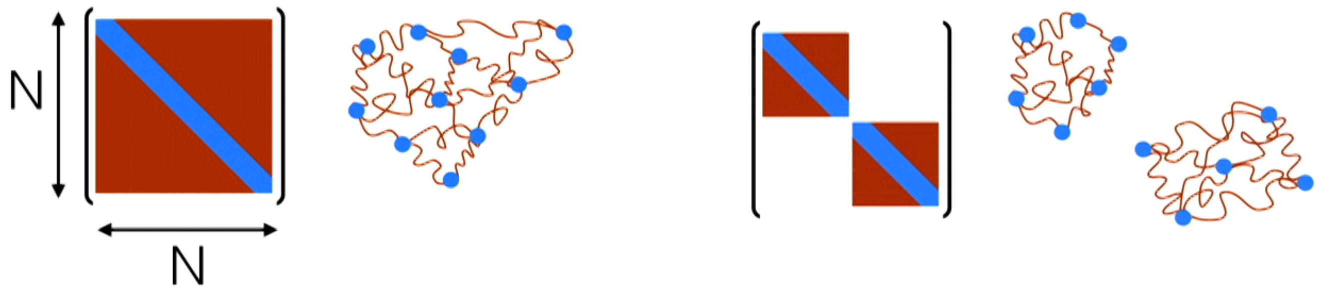
$X_1, \ldots, X_9$
N×N hermitian

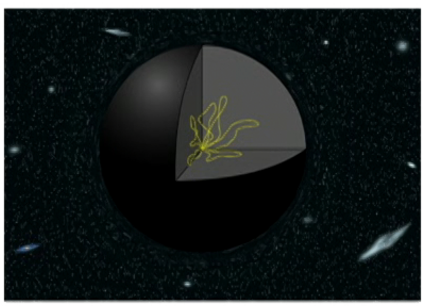
It should reproduce thermodynamics of black 0-brane.

effective dimensionless temperature $T_{eff} = \lambda^{-1/3} T$
strong coupling = low temperature → more simulation cost

Gauge theory description of a black hole (D0-brane quantum mechanics)

(Banks, Fischler, Shenker, Susskind 1996; Itzhaki, Maldacena, Sonnenschein, Yankielowicz1998)





diagonal elements = particles (D0-branes)
off-diagonal elements = open strings

(Witten, 1994)

black hole = bound state of D-branes and strings

(※ The same picture holds for p>0 theories as well)

- ‘Fine tuning problem’
- ‘Sign problem’

Wilson's lattice gauge theory

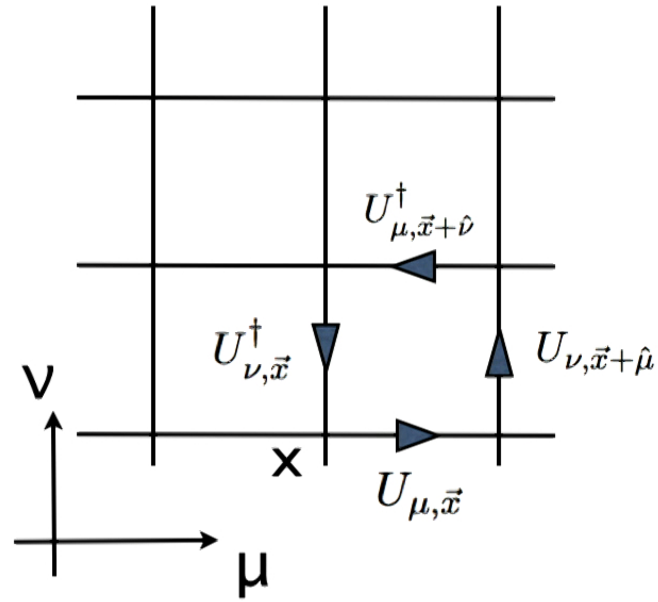
$$S = -\beta N \sum_{\vec{x}} \sum_{\mu \neq \nu} Tr \left(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}} \right)$$

Unitary link variable

$$U_{\mu,\vec{x}} = e^{iaA_\mu(x)}$$

a : lattice spacing

$$\beta = 1/(g^2_{YM}(a) \cdot N)$$



$$S = \frac{1}{4g^2_{YM}} \int d^4x Tr F^2_{\mu\nu} + O(a^4)$$

'Exact' symmetries

- Gauge symmetry

$$U_{\mu,\vec{x}} \to \Omega(x) U_{\mu,\vec{x}} \Omega(x+\hat{\mu})^{\dagger}$$

- 90 degree rotation
- discrete translation
- Charge conjugation, parity

These symmetries exist *at discretized level*.

→ Correct continuum limit is realized without fine tuning.

Quantum Gravity from Lattice Gauge Theory via Holography

Masanori Hanada
花田 政範
Hana Da Masa Nori

June 22, 2017 @ Perimeter

fine tuning problem

Continuum limit on lattice $a \to 0$

respects exact symmetries at discretized level.

Exact symmetries on lattice

↓

Gauge invariance, translational invariance, rotationally invariance, ... in the continuum limit.

If the symmetry is explicitly broken, radiative corrections break the symmetries. The <u>counter-terms</u> are needed, and their coefficients must be <u>fine-tuned</u>.

- SUSY algebra contains infinitesimal translation.

$$\{Q, \bar{Q}\} \sim \partial$$

- ~~Infinitesimal translation~~ on lattice by definition.
- Still a part of supercharges can be kept. (subalgebra which does not contain ∂)
- No general solution is known, but there are many case-by-case solutions.

Which SYM can be simulated?

Possible/Impossible

(without fine tuning; not necessarily lattice)

smaller simulation cost

(0+1)-d

(1+1)-d

any number of SUSY, various matter contents

(2+1)-d maximal SUSY

less SUSY matter fields

larger simulation cost

(3+1)-d

without matter (pure $\mathcal{N}$=1)

SUSY QCD (matter fields)

maximal SUSY

Sign problem (1)

$$S = S_B + S_F, \qquad S_F = \int d^4x \bar{\psi} D \psi$$

Fermions appear in a bilinear form.
(if not, make them bilinear by introducing auxiliary fields.)

→ can be integrated out *by hand*.

$$\int [dA][d\psi] e^{-S_B[A] - S_F[A,\psi]} = \int [dA] \underline{\det D[A] \cdot e^{-S_B[A]}}$$

Monte Carlo cannot be used if it is not real positive

(In maximal SYM, Pfaffian appears instead of determinant)

Sign problem (2)

- In Monte Carlo simulations, configurations are generated with <u>probability</u> $\det D[A] \cdot e^{-S_B[A]}$

$$A_\mu^{(1)}, A_\mu^{(2)}, \cdots$$

- Then by collecting many configuration one can approximate the expectation value as

$$\langle \mathcal{O} \rangle = \frac{\int [dA] \mathcal{O}[A] \cdot \det D[A] \cdot e^{-S_B[A]}}{\int [dA] \det D[A] \cdot e^{-S_B[A]}} \simeq \frac{1}{n} \sum_{i=1}^{n} \mathcal{O}[A^{(i)}]$$

- <u>Crucial assumption</u>:

$$\det D[A] \cdot e^{-S_B[A]} > 0$$

Sign problem (3)

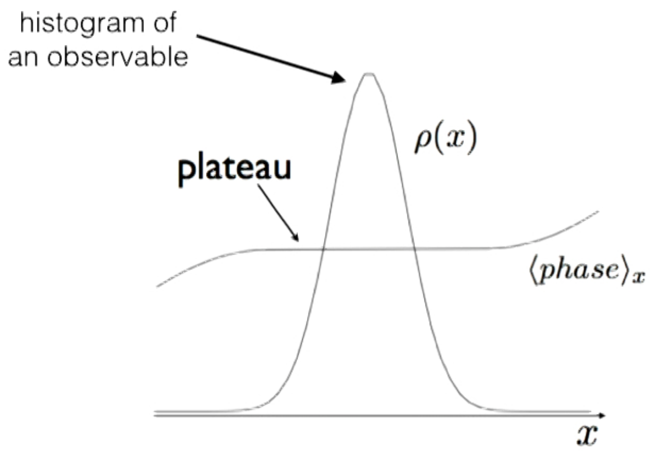
- In maximal SYM, detD is complex.
- The 'phase-quenched' weight can still be studied.

$$\Big|\det D[A]\Big| \cdot e^{-S_B[A]}$$

- Phase can be taken into account by the 'phase reweighting' in principle, but usually it's hopelessly hard.

$$\begin{aligned}\langle \mathcal{O} \rangle &= \frac{\int [dA] \det D \cdot e^{-S_B} \cdot \mathcal{O}}{\int [dA] \det D \cdot e^{-S_B}} \\ &= \frac{\int [dA](phase) \cdot |\det D| \cdot e^{-S_B} \cdot \mathcal{O} / \int [dA] |\det D| \cdot e^{-S_B}}{\int [dA](phase) \cdot |\det D| \cdot e^{-S_B} / \int [dA] |\det D| \cdot e^{-S_B}} \\ &= \frac{\langle (phase) \cdot \mathcal{O} \rangle_{phase\ quench}}{\langle (phase) \rangle_{phase\ quench}} \quad \sim 0/0\end{aligned}$$

No Sign problem (4)

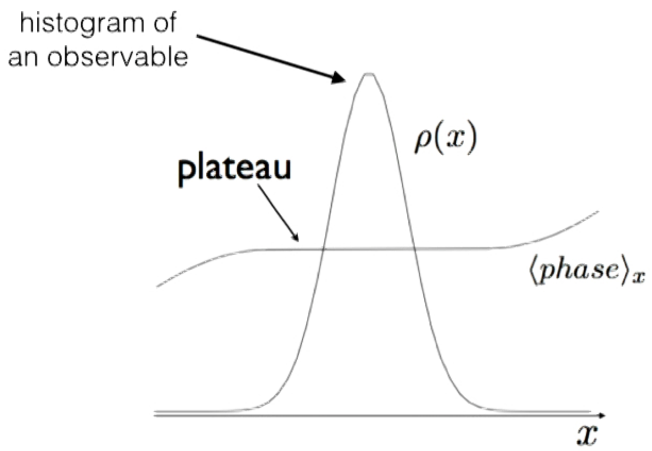


$$\langle \mathcal{O} \rangle_{phase\ quench} = \int dx\ x\rho(x)$$

$$\frac{\langle (phase) \cdot \mathcal{O} \rangle_{phase\ quench}}{\langle (phase) \rangle_{phase\ quench}} = \frac{\int dx\ x\rho(x) \langle phase \rangle_x}{\int dx\ \rho(x) \langle phase \rangle_x} \simeq \frac{c \int dx\ x\rho(x)}{c} = \langle \mathcal{O} \rangle_{phase\ quench}$$

- Numerically observed in (0+1)-d and (1+1)-d.
- Large statistics demonstration is in progress.

No Sign problem (4)



$$\langle \mathcal{O} \rangle_{phase\ quench} = \int dx\ x\rho(x)$$

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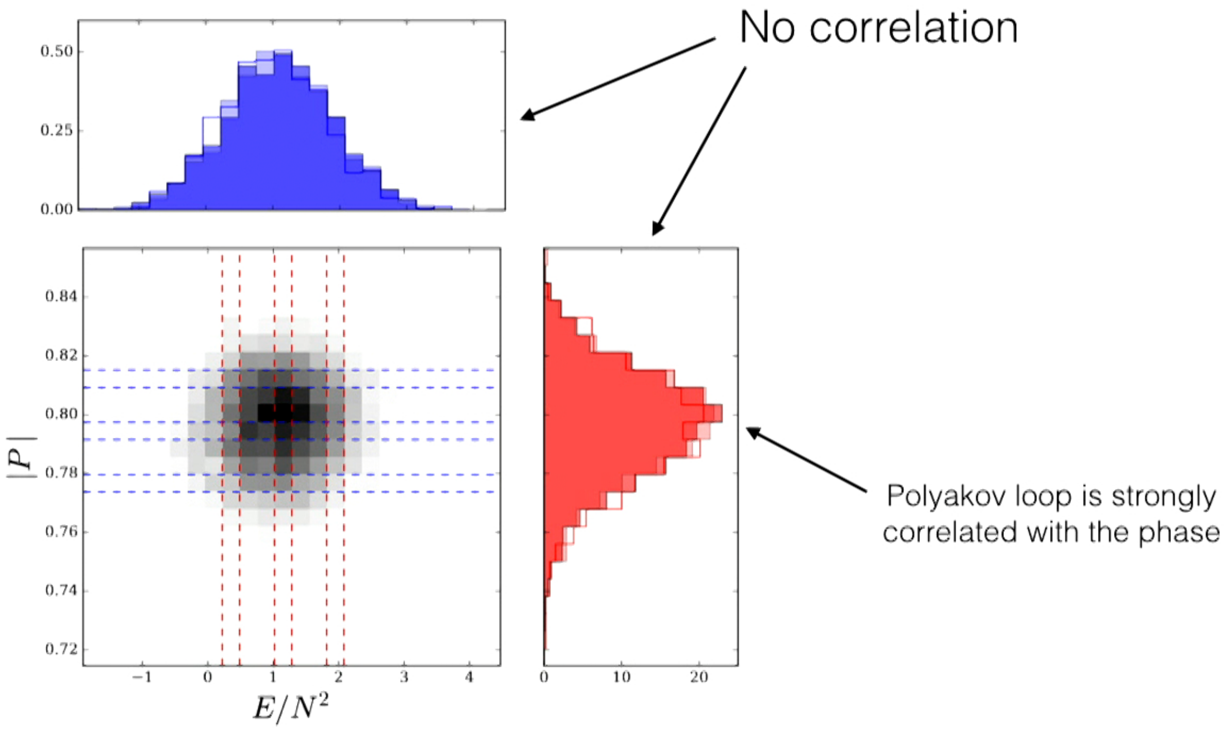
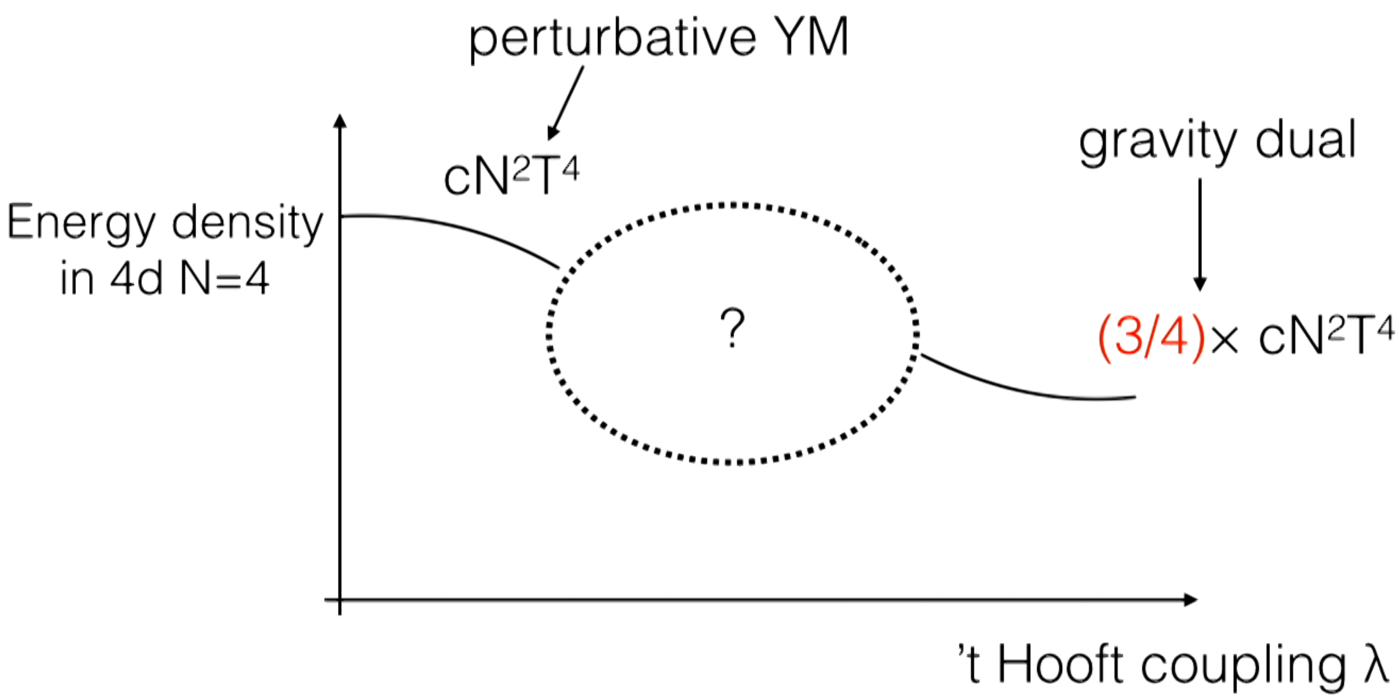
No Sign problem (5)

Even at large N/volume, where phase cannot be calculated:



Numerical solution to 3/4 Problem

3/4 Problem



Let's solve the D0-brane version of this problem.

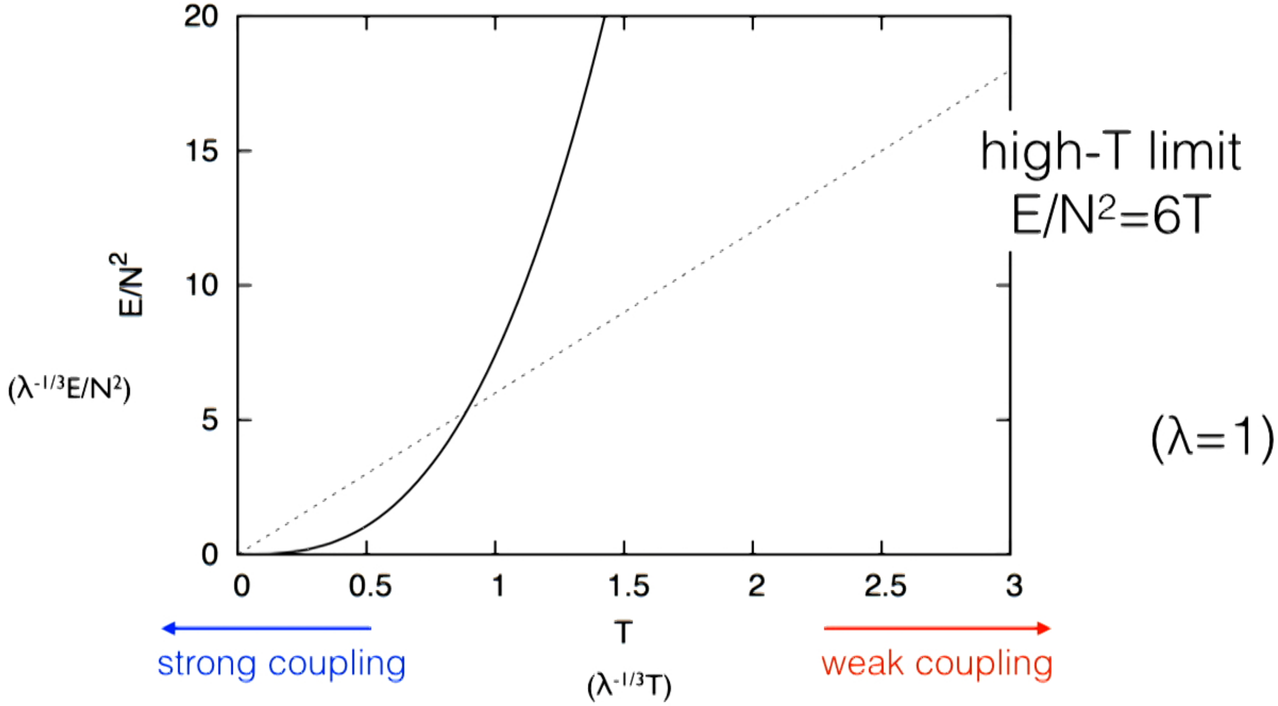
$\lambda^{-1/3}E$
$\lambda^{-1/3}T$
$\lambda=1$

String theory prediction

$$\text{BH mass} = E = -\frac{\partial}{\partial\beta}\log Z$$

$$E/N^2 = 7.41T^{14/5} + b\,T^{23/5} + c\,T^{29/5} + \ldots + O(1/N^2)$$

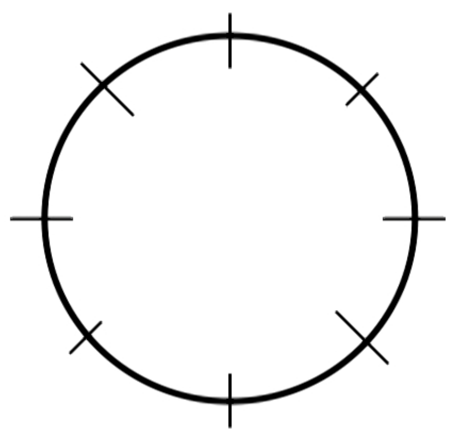
SUGRA (low-T limit)
$E/N^2=7.4T^{14/5}$

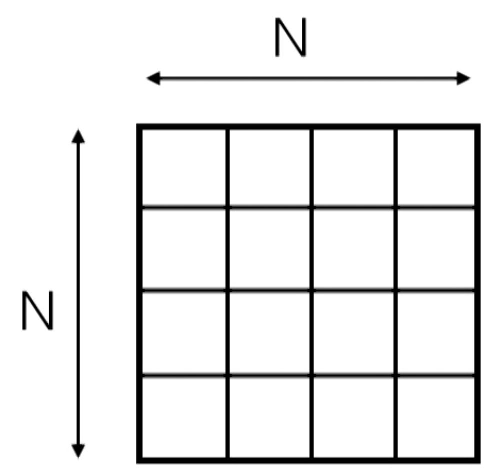


Let's see how they are interpolated.

Basic Strategy:

- Use lattice regularization.
- MPI parallelization.
  (https://sites.google.com/site/hanadamasanori/home/mmmm)
- Rational Hybrid Monte Carlo algorithm.
- Solve various technical problems.
- Burn electricity.





$\lambda^{-1/3}E$
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$$E/N^2 = 7.41T^{14/5} + b\,T^{23/5} + c\,T^{29/5} + \ldots + O(1/N^2)$$

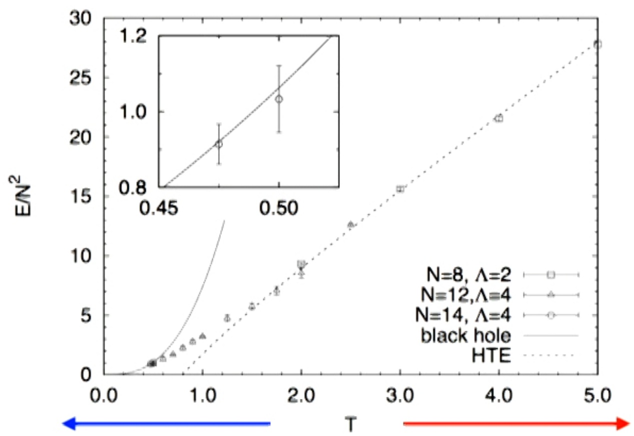
Took $N\rightarrow\infty$, and did 3-parameter fit by

$$E/N^2 = aT^{14/5} + b\,T^{23/5} + c\,T^{29/5}$$

Some history

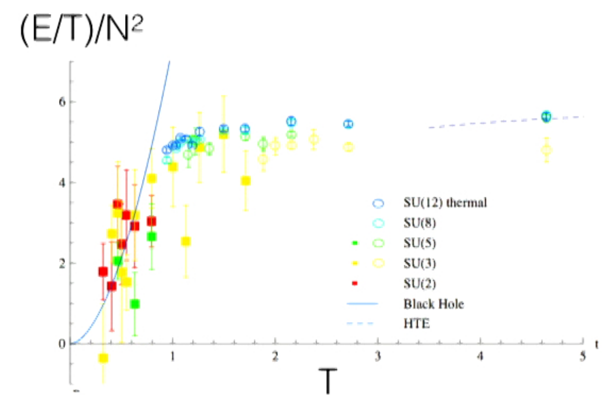
An earlier attempt
with a mean-field method:
Kabat-Lifschytz, 2001

Anagnostopoulos-M.H.-Nishimura-Takeuchi, 0707.4454 [hep-th]



strong coupling

weak coupling

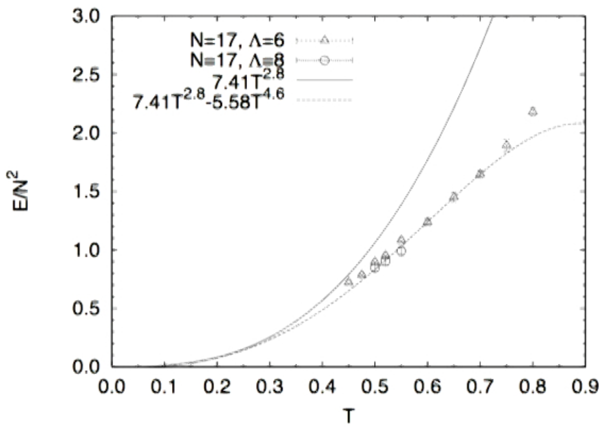


Catterall-Wiseman, 0803.4273 [hep-th]

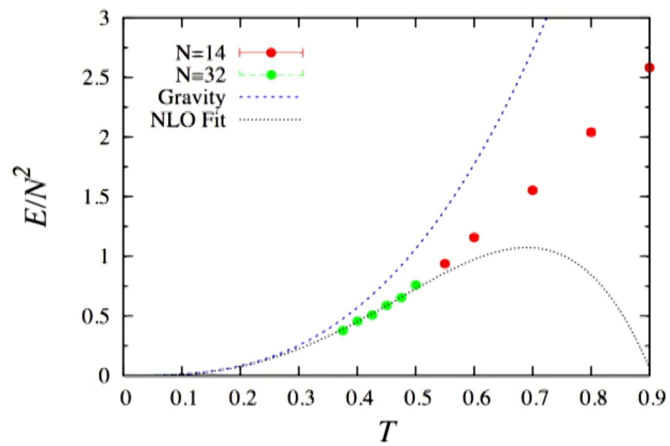
Qualitatively Good, but…

- Not good for any fit…
- Not large-N, not continuum
- ‘consistent with SUGRA’ due to cutoff effect & large error bar.

Some history (cont'd)



M.H.-Hyakutake-Nishimura-Takeuchi, 0811.3102 [hep-th]



Kadoh-Kamata, 1503.08499 [hep-lat]

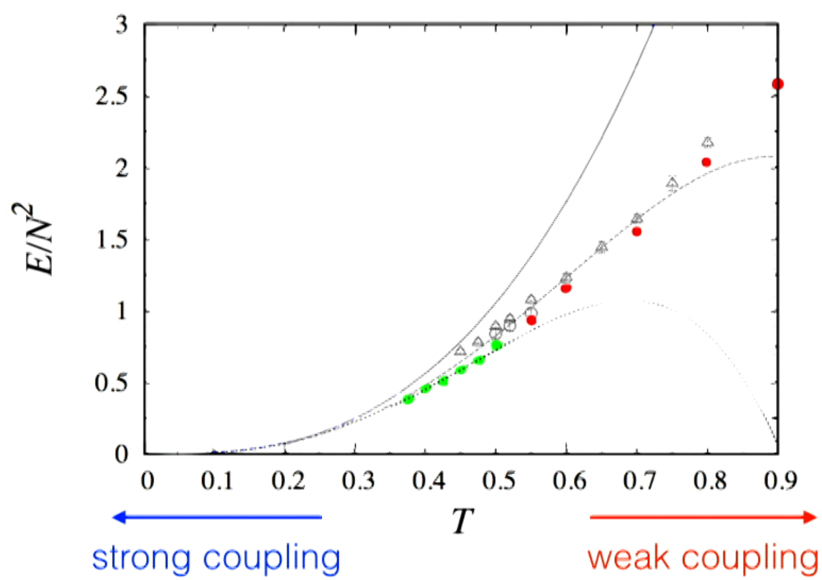
- LO SUGRA part is fixed by hand.
- NLO fit results disagreed…
- Not large-N, not continuum

$$E/N^2 = 7.41T^{14/5} + b\, T^p$$

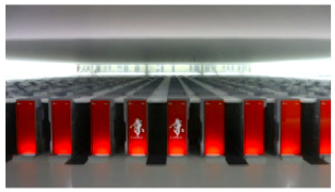
(See also Filev-O'connor 2015)

M.H.-Hyakutake-Nishimura-Takeuchi, 0811.3102 [hep-th]

Kadoh-Kamata, 1503.08499 [hep-lat]



We need large-N and continuum result.



K-supercomputer
(RIKEN, Kobe, Japan)

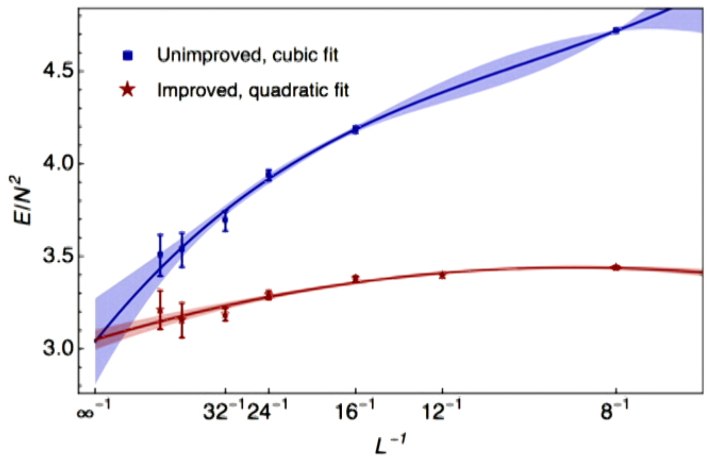
Vulcan
(LLNL, Livermore, USA)

Enrico Rinaldi

Evan Berkowitz

64 — 2047 core parallel
O(100) parameters
(N=16, 24, 32; T=1.0, 0.9, …; L=8, 12, …, 64)

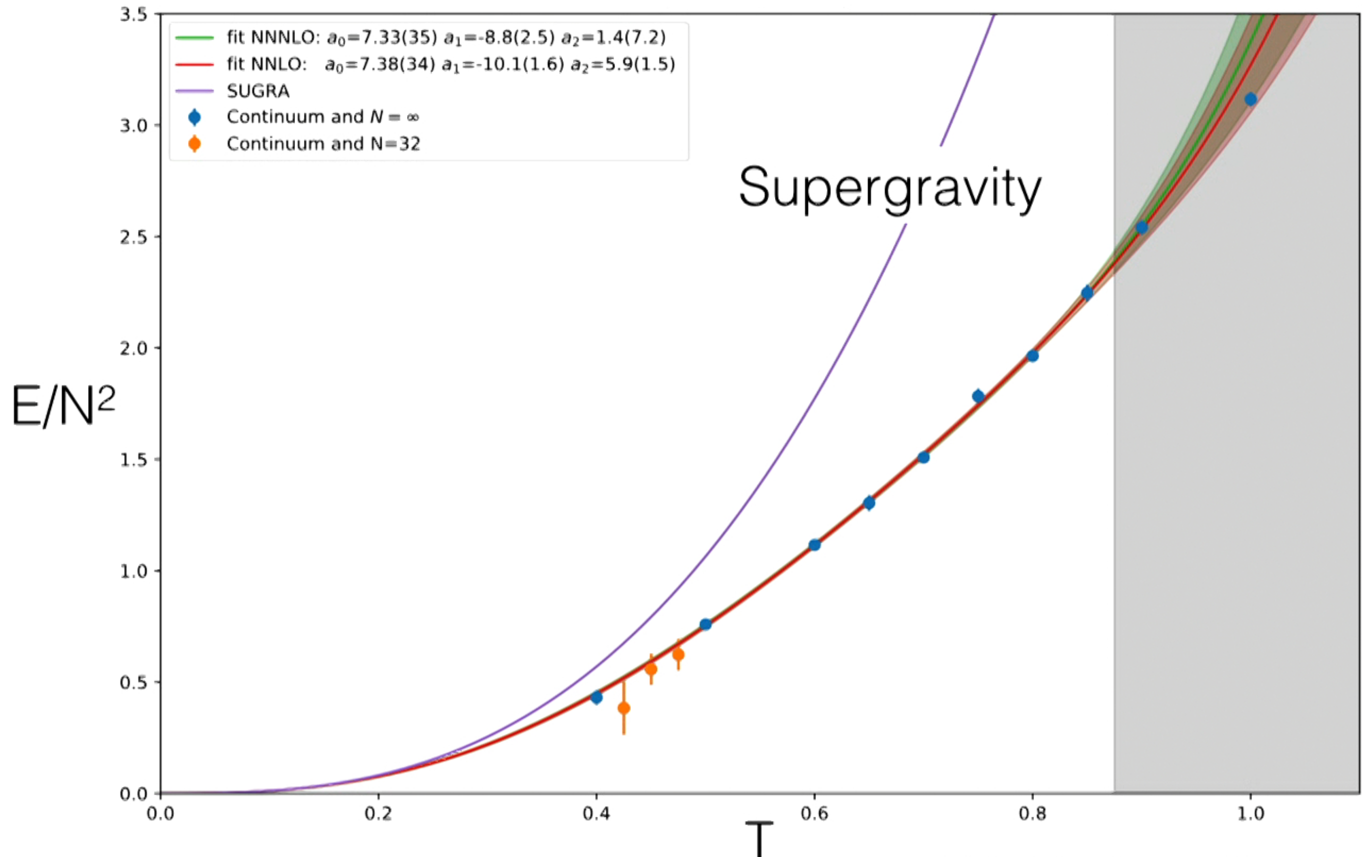
- Simulation at various N & T, with various lattice sizes L.
- Rational Hybrid Monte Carlo algorithm, MPI parallelization.
- Take continuum limit & large-N limit.
- Not more difficult than Ising.



continuum limit, T=1.0, N=16

N=∞ obtained from N=16, 24, 32

Continuum limit from 8,12,16, …, 64 lattice points



1606.04951 [hep-lat] + recent data

SUGRA = SYM @ finite-T

$E/N^2 = aT^{14/5} + b\, T^{23/5} + c\, T^{29/5}$

3-parameter fit
(4-parameter is too much)

$a = 7.33 \pm 0.35$

1606.04951 [hep-lat] + recent data

$E/N^2 = 7.41\, T^{14/5} + b\, T^{23/5} + c\, T^{29/5} + \ldots + O(1/N^2)$

STRING = SYM @ finite-T

$$E/N^2 = 7.41T^{14/5} + b\,T^{p} + c\,T^{p+6/5}$$

3-parameter fit
(4-parameter is too much)

$p = 4.6 +/- 0.3$

1606.04951 [hep-lat]

$$E/N^2 = 7.41T^{14/5} + b\,T^{23/5} + c\,T^{29/5} + \ldots + O(1/N^2)$$

※ We are adding more data points to make the fit even more reliable; especially studying the parameter region where higher order terms become smaller.

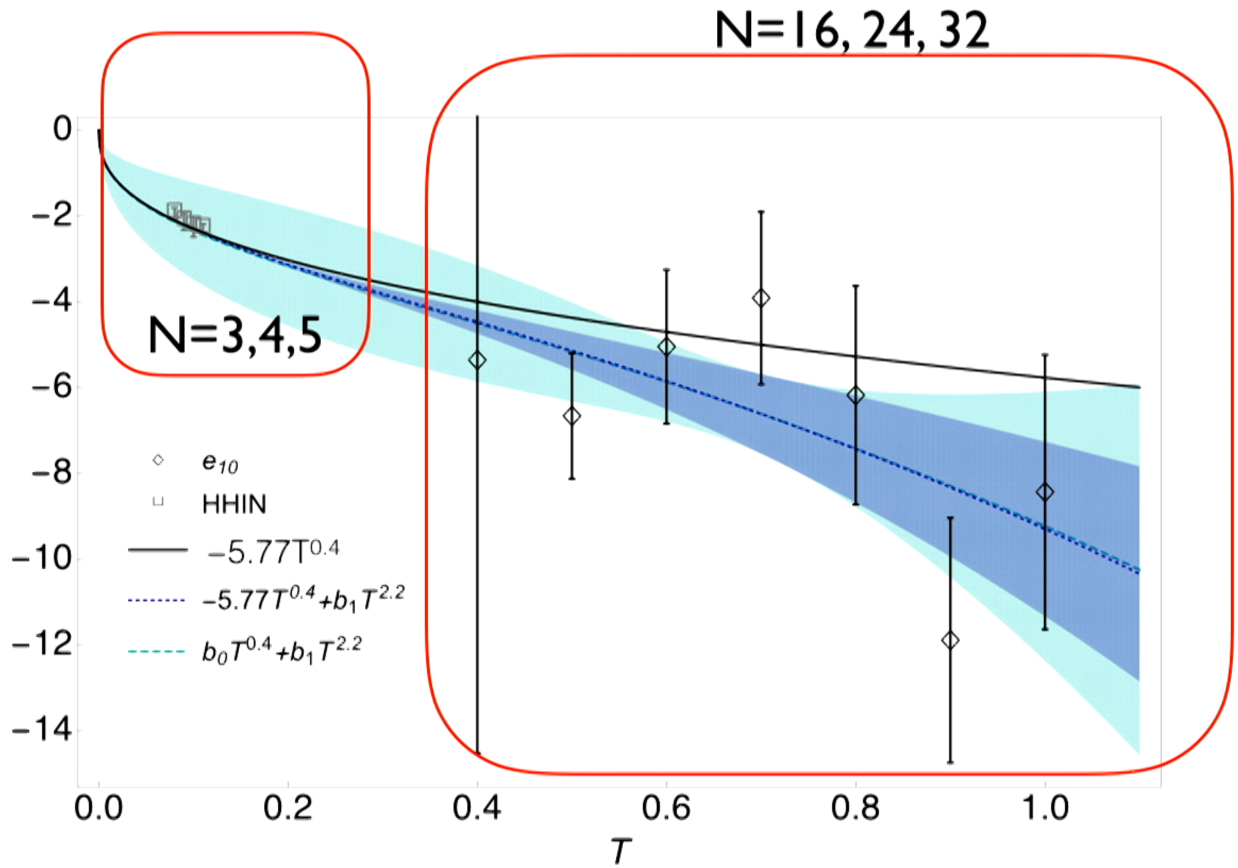
- “3/4 problem” has been solved, in the case of D0-branes.
- SYM correctly describes String α’-corrections.
- D3-brane version can also be solved by lattice simulation. (Within a 5-year span? If more people join the activity.)

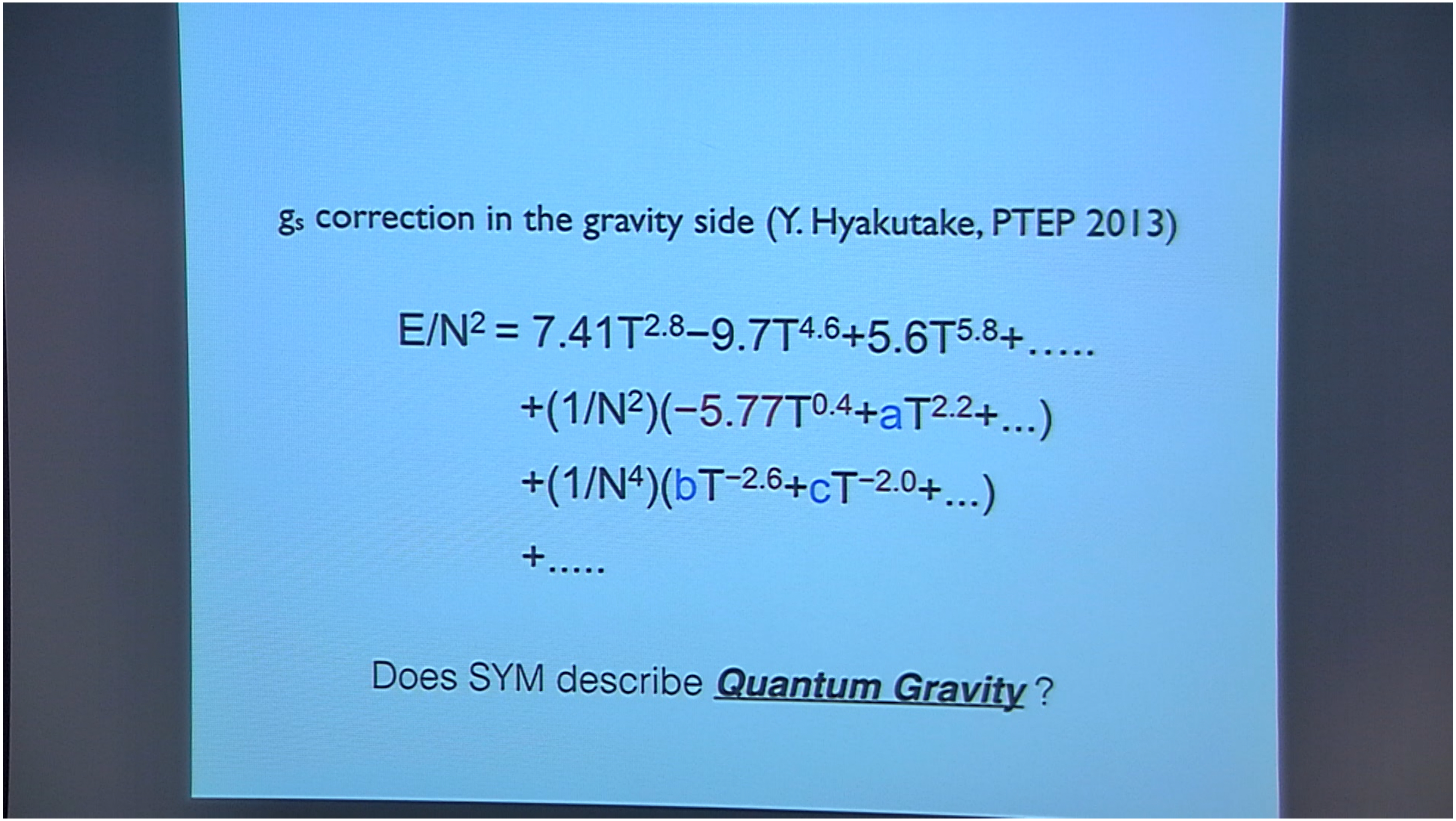
1/N vs Quantum String

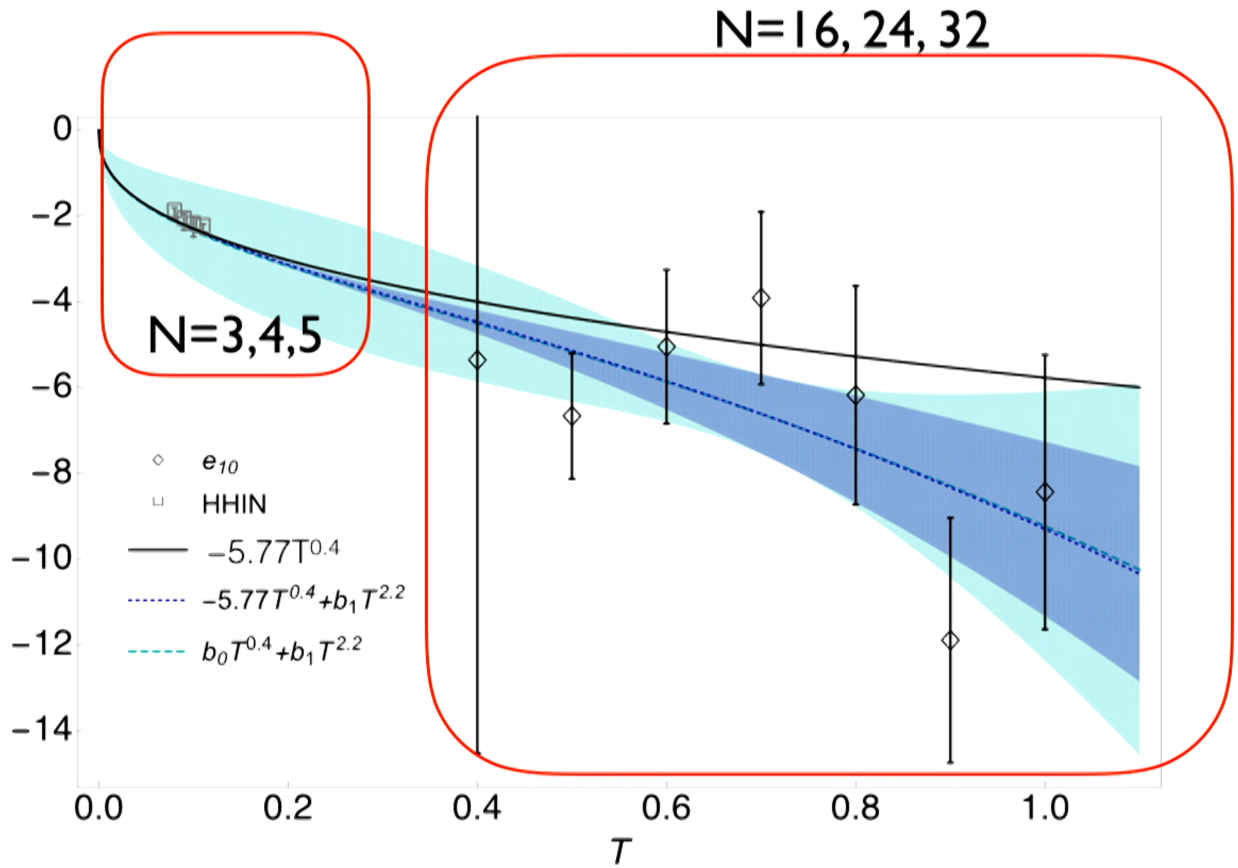
g_s correction in the gravity side (Y. Hyakutake, PTEP 2013)

$$E/N^2 = 7.41T^{2.8} - 9.7T^{4.6} + 5.6T^{5.8} + \ldots..$$
$$+(1/N^2)(-5.77T^{0.4} + aT^{2.2} + \ldots)$$
$$+(1/N^4)(bT^{-2.6} + cT^{-2.0} + \ldots)$$
$$+\ldots..$$

Does SYM describe ***Quantum Gravity*** ?

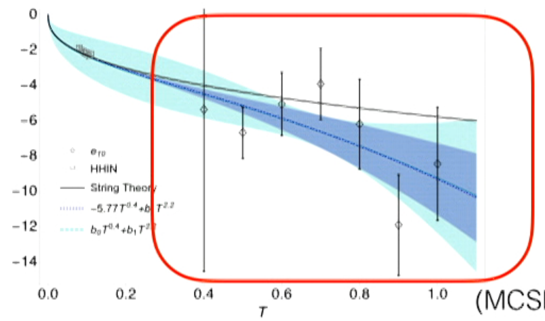

N=16, 24, 32
N=3,4,5
0
−2
−4
−6
−8
−10
−12
−14
0.0
0.2
0.4
0.6
0.8
1.0
T
e10
HHIN
−5.77T0.4
−5.77T0.4+b1T2.2
b0T0.4+b1T2.2


g_s correction in the gravity side (Y. Hyakutake, PTEP 2013)
$E/N^2 = 7.41T^{2.8} - 9.7T^{4.6} + 5.6T^{5.8} + \ldots..$
$+(1/N^2)(-5.77T^{0.4} + aT^{2.2} + \ldots)$
$+(1/N^4)(bT^{-2.6} + cT^{-2.0} + \ldots)$
$+\ldots..$
Does SYM describe Quantum Gravity ?



N=16, 24, 32
N=3,4,5
0
−2
−4
−6
−8
−10
−12
−14
0.0
0.2
0.4
0.6
0.8
1.0
T
e10
HHIN
−5.77T0.4
−5.77T0.4+b1T2.2
b0T0.4+b1T2.2

N=16, 24, 32



(MCSMC, PRD2016)

e_{10}

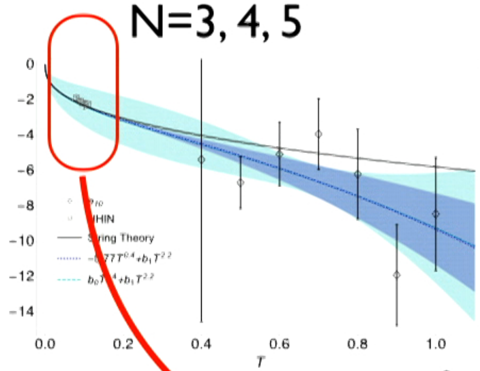
$$\frac{E}{N^2} \approx e_{00} + \frac{e_{01}}{L} + \frac{e_{02}}{L^2} + \frac{e_{10}}{N^2}$$

L : number of lattice points

L = 8, 12, 16, 24, 32, 48, 64

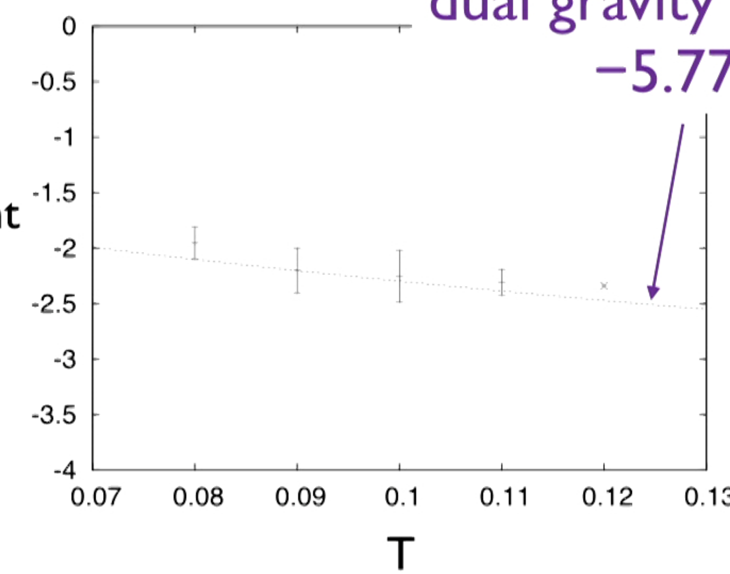
N = 16, 24, 32

N=3, 4, 5



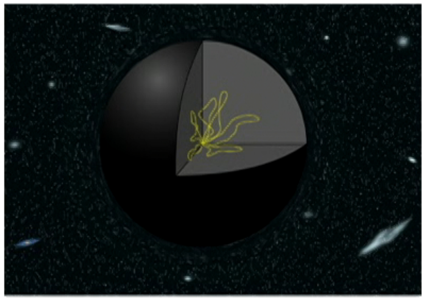
coefficient of $1/N^2$

dual gravity prediction $-5.77T^{0.4}$



M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

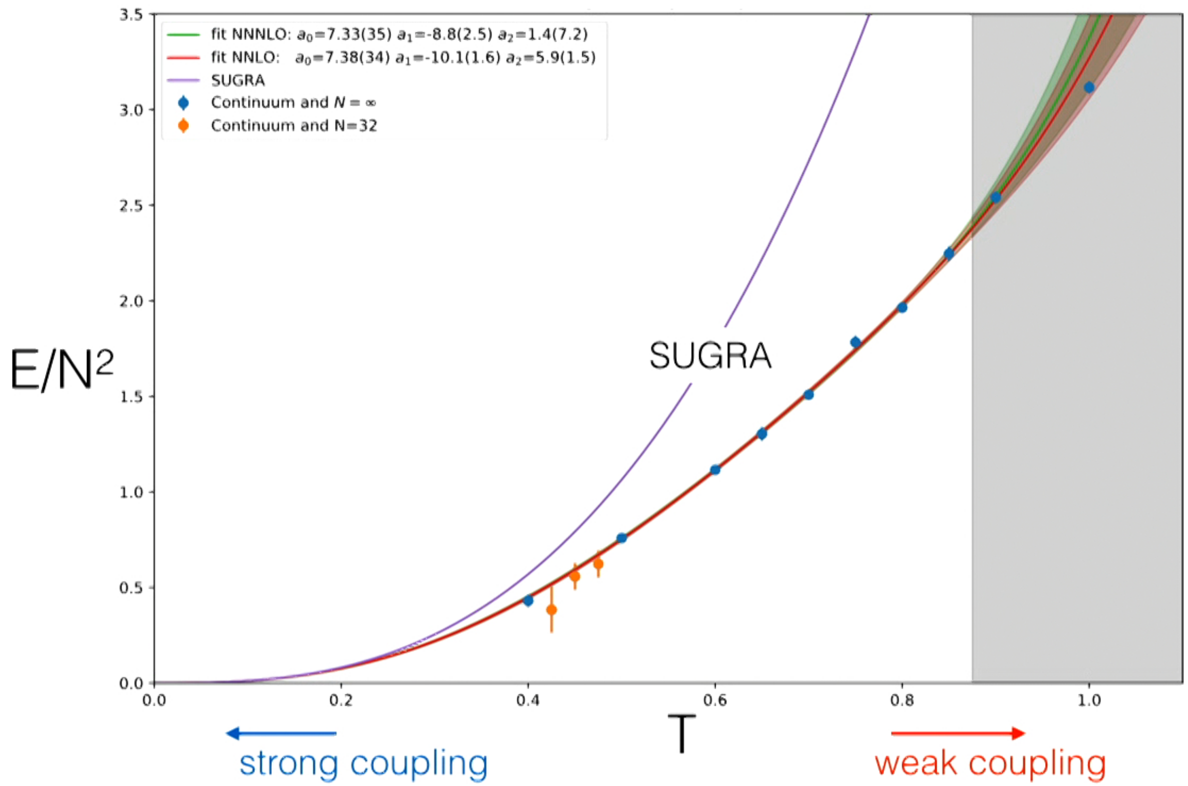
- SYM does describe string theory, including α' and g_s corrections.
- A unitary theory which describes black hole! A counter-example of information loss.
- Numerical methods are practical tools; even give 'predictions' for gravity side.



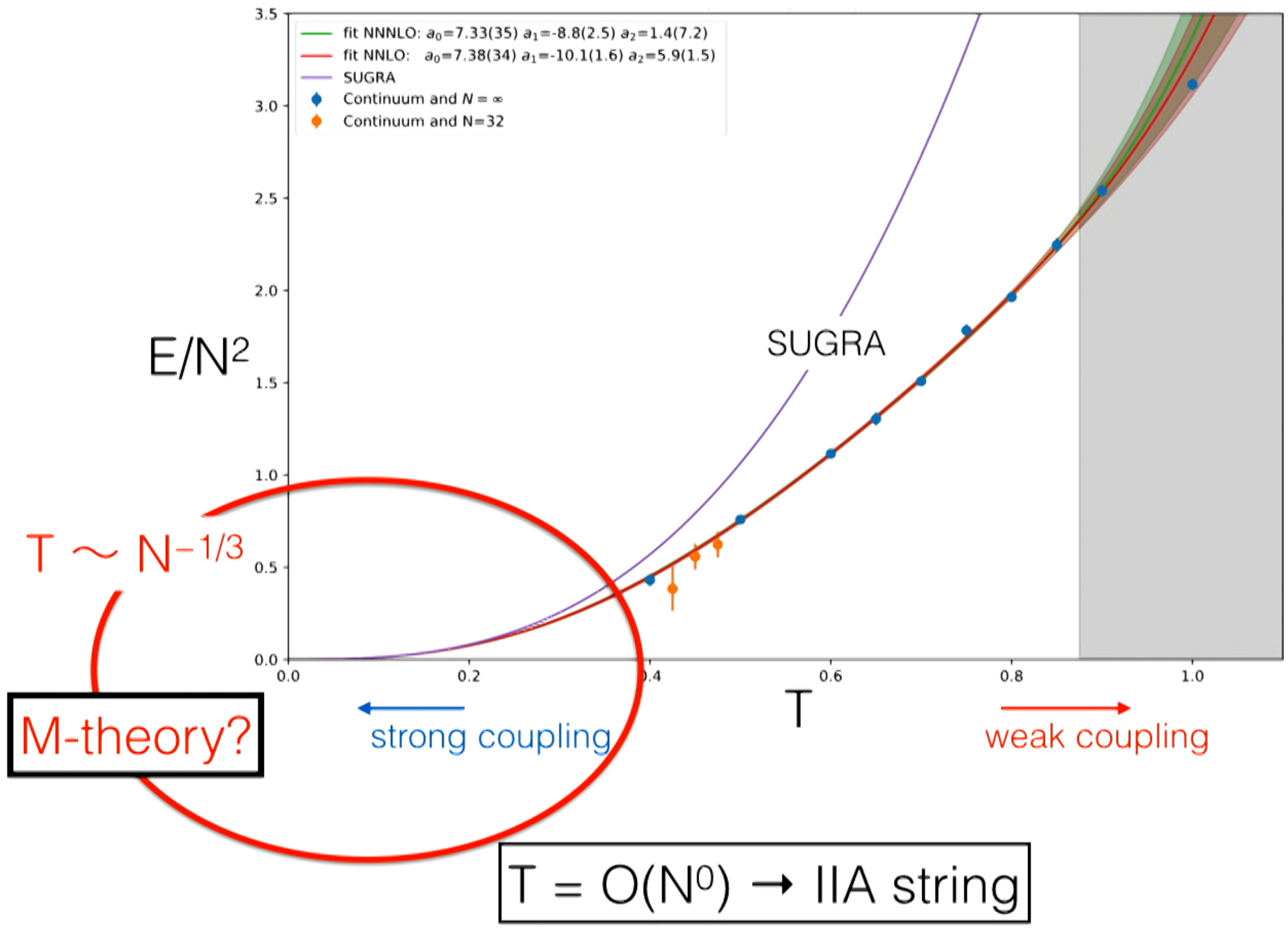
black hole = bound state of D-branes and strings

Next targets

(1) M-theory



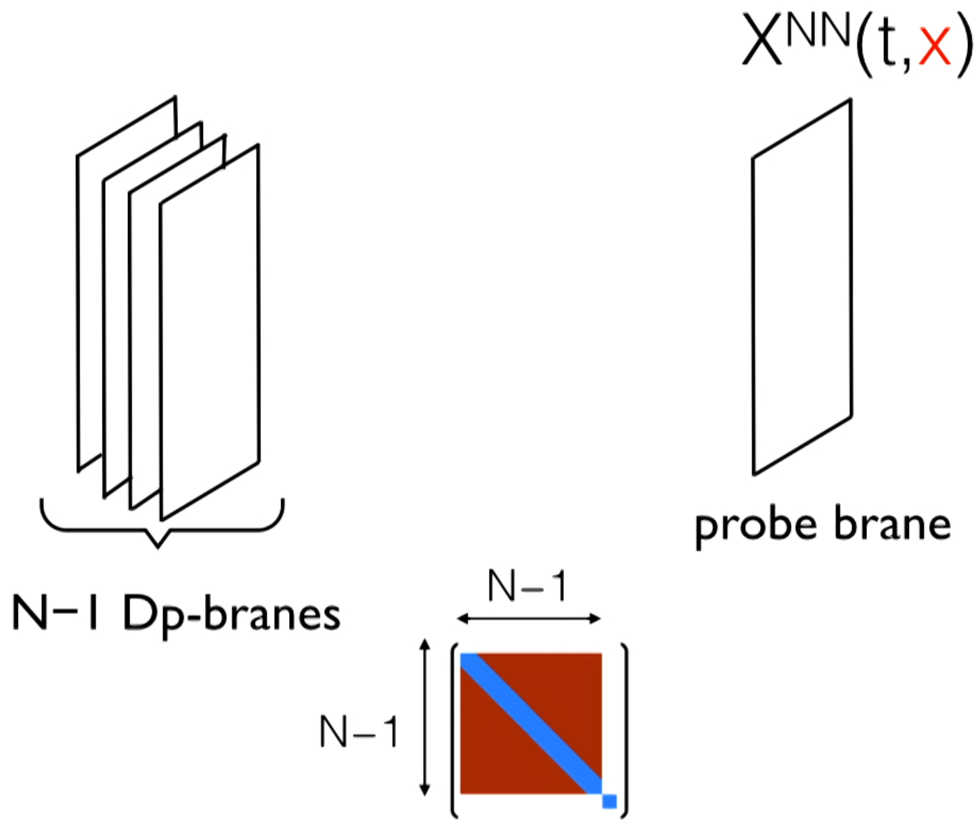
$T = O(N^0) \rightarrow$ IIA string



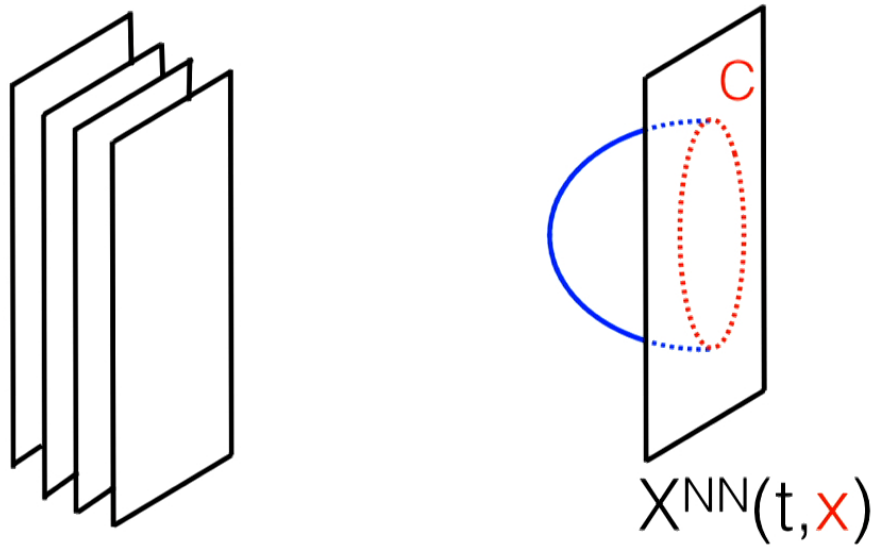
fit NNNLO: a_0=7.33(35) a_1=-8.8(2.5) a_2=1.4(7.2)
fit NNLO: a_0=7.38(34) a_1=-10.1(1.6) a_2=5.9(1.5)
SUGRA
Continuum and $N = \infty$
Continuum and N=32
E/N^2
SUGRA
$T \sim N^{-1/3}$
M-theory?
strong coupling
T
weak coupling
$T = O(N^0) \rightarrow$ IIA string

Next targets

(2) reconstruction of black hole geometry



XNN(t,x)
probe brane
N−1 Dp-branes
N−1
N−1



Let $X^{NN}(t,x)$ to vary for $x \in C$

Fix $X^{NN}(t,x)$ for $x \notin C$

Nontrivial shape → curved geometry in string theory

Other simulations (past & future)

Wilson loop, 2-pt functions

(M.H.-Miwa-Nishimura-Takeuchi, M.H.-Nishimura-Sekino-Yoneya)

(1+1)-d lattice SYM (Suzuki, M.H.-kanamori, Catterall-Joseph-Wiseman, Giguere-Kadoh)

(3+1)-d lattice SYM with fine tuning
(Catterall-Damgaard-Degrand-Schaich)

BMN matrix model → D2, M2, NS5, M5
(formulation: Maldacena-Sheikh Jabbarri-van Raamsdonk)

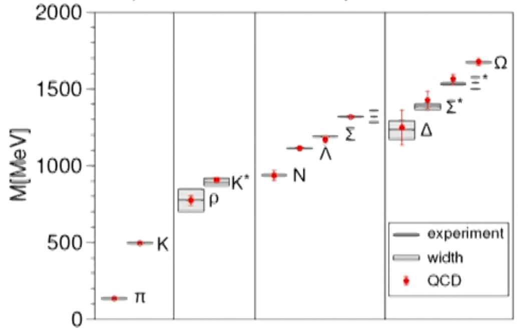
2d SYM + flux deformation→ 4d SYM
(formulation: M.H.-Matsuura-Sugino)

Low-lying Hadron Spectrum

Dürr, Fodor, Lippert et al., BMW Collaboration

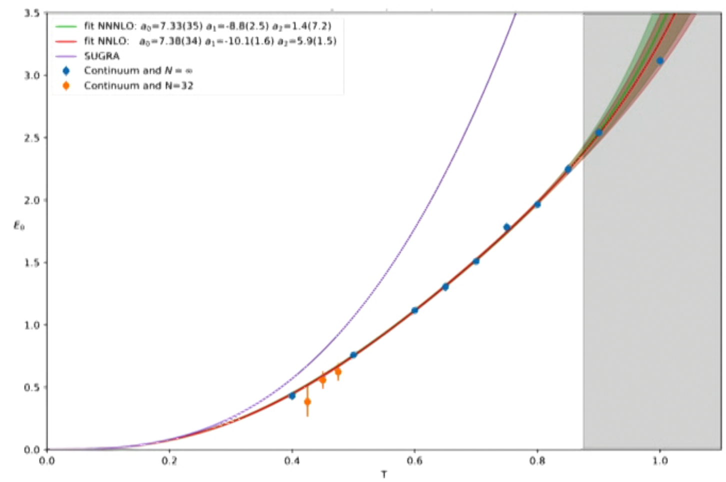
Science 322, 1224 November 2008

More than 99% of the mass of the visible universe is made up of protons and neutrons. Both particles are much heavier than their quark and gluon constituents, and the Standard Model of particle physics should explain this difference. We present a full ab initio calculation of the masses of protons, neutrons, and other light hadrons, using lattice quantum chromodynamics. Pion masses down to 190 mega–electron volts are used to extrapolate to the physical point, with lattice sizes of approximately four times the inverse pion mass. Three lattice spacings are used for a continuum extrapolation. Our results completely agree with experimental observations and represent a quantitative confirmation of this aspect of the Standard Model with fully controlled uncertainties



Lattice QCD

Superstring from Lattice



Lattice(-like) methods are practical tools for superstring theory.

and not more difficult than simulating the Ising model.