Title: Tutorial: Monte Carlo methods in Dynamical Triangulations

Date: Jun 22, 2017 02:00 PM

URL: http://pirsa.org/17060086

Abstract:

example-analysis.nb * - Wolfram Mathematica 11.0 Student Edition - Personal Use Only

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Config

```
(* Provide the location of the randgeom executable *)
(* On windows change randgeom to randgeom.exe *)
programlocation = FileNameJoin[{NotebookDirectory[], "linux", "randgeom"}]

/home/timothy/Documents/web/homepage/randgeom/linux/randgeom

(* Test the program. If it return False, check the programlocation provided. If it still does not work, first test randgeom from the console. *)
FileExistsQ[programlocation] && ListQ[RunThrough["'" <> programlocation <> "'", ""]]

True
```

Useful functions

```
(* the following just runs randgeom with the specified parameters and parses the output as Mathematica code *)
generateMaps[type_, size_, number_] := RunThrough["'" <> programlocation <> "' -t" <> type <> " -s" <> ToString[size] <> " -n" <> ToString[number], ""];
generateMap[type_, size_] := First@generateMaps[type, size, 1];

(* given a permutation p of (1,2,...,n), cycles[p] gives the partition of (1,2,...,n) into cycles *)
cycles[p_] := PermutationCycles[p, Identity];
(* given a list plist of permutations, orbits[plist] gives the partition of (1,2,...,n) into orbits under the permutations *)
orbits[plist_] := GroupOrbits@PermutationGroup[PermutationCycles /@ plist]

(* edges, vertices and faces correspond to cycles of halfedge-permutations *)
edgecycles[map_] := cycles[map[[All, 3]]];
facecycles[map_] := cycles[map[[All, 1]]];
vertexcycles[map_] := cycles[map[[map[[All, 3]], 1]]];
(* We may assign id's to the vertices of map according to their position in vertexcycles[map] *)
halfedgeToVertexId[map_] := Dispatch[Join @@ MapIndexed[#1 → #2[[1]] &, vertexcycles[map], {2}]];
halfedgeToFaceId[map_] := Dispatch[Join @@ MapIndexed[#1 → #2[[1]] &, facecycles[map], {2}]];

(* functions to construct a Mathematica Graph object *)
uniqueEdges[map_] := Union[Sort /@ (edgecycles[map] /. halfedgeToVertexId[map])];
uniqueDualEdges[map_] := Union[Sort /@ (edgecycles[map] /. halfedgeToFaceId[map])];
mapGraph[map_] := With[{edges = uniqueEdges[map]}, Graph[Union @@ edges, #[[1]] ↔ #[[2]] & /@ edges, GraphLayout → None]];
mapDualGraph[map_] := With[{edges = uniqueDualEdges[map]}, Graph[Union @@ edges, #[[1]] ↔ #[[2]] & /@ edges, GraphLayout → None]];
```

Plotting

```
(* the following is a bit of a hack to extract coordinates from GraphPlot3D's embedding of a graph *)
```

100%

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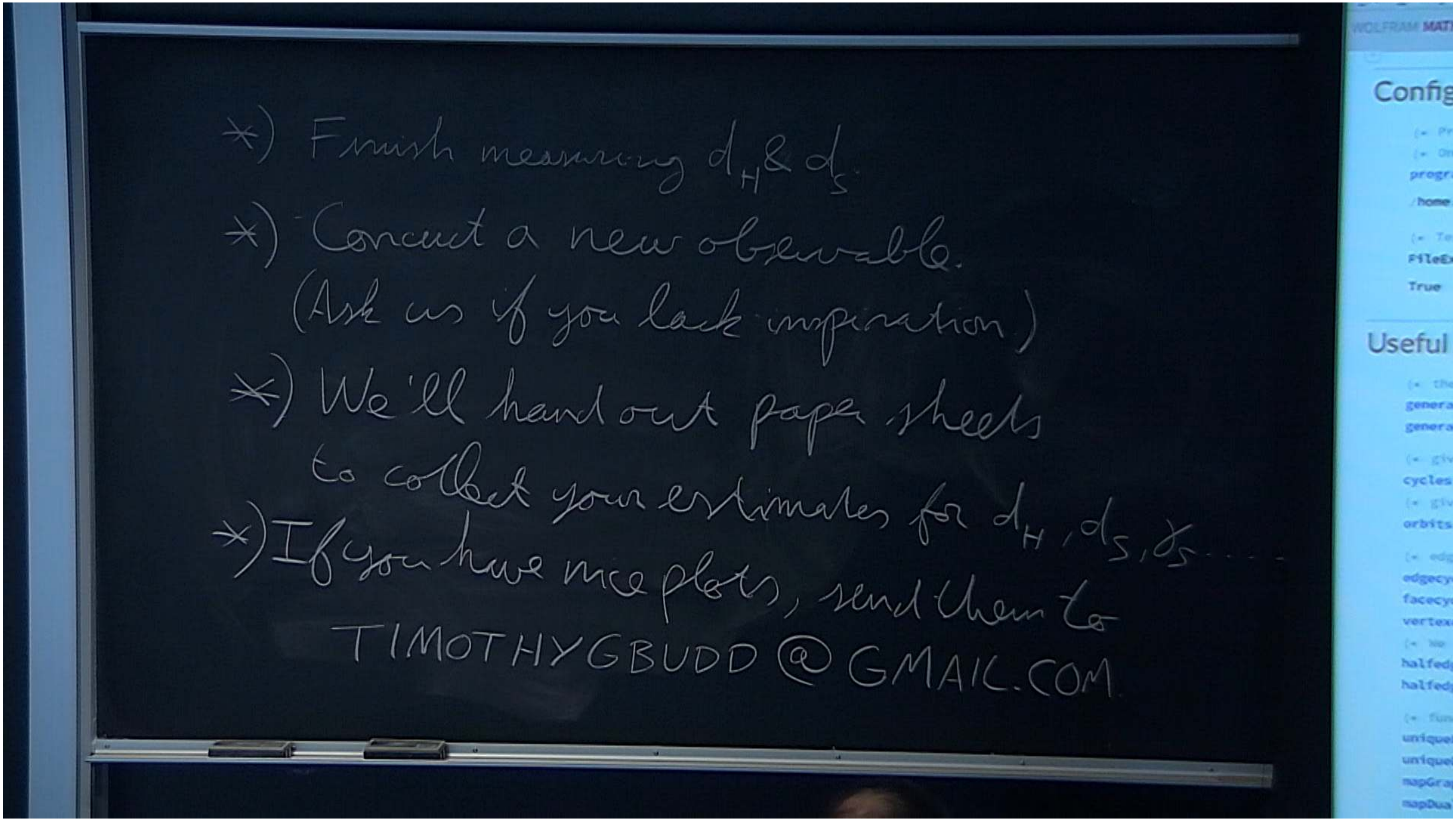
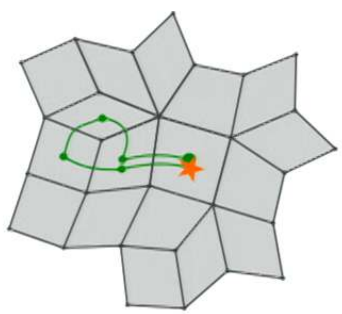

*) Finish measuring d_H & d_S.
*) Construct a new observable.
(Ask us if you lack inspiration)
*) We'll hand out paper sheets
to collect your estimates for d_H, d_S, γ_S ....
*) If you have nice plots, send them to
TIMOTHYGBUDD@GMAIL.COM

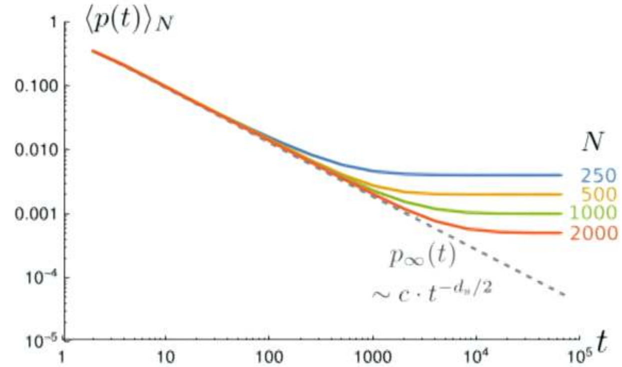
Observables: spectral dimension

- Consider a simple random walk on the faces of $\mathfrak{m}$, started at face x.
- *Return probability* $p_x(t;\mathfrak{m})$ is probability that it is back at x after t steps.
- Averaging over all starting points we get a family of observables

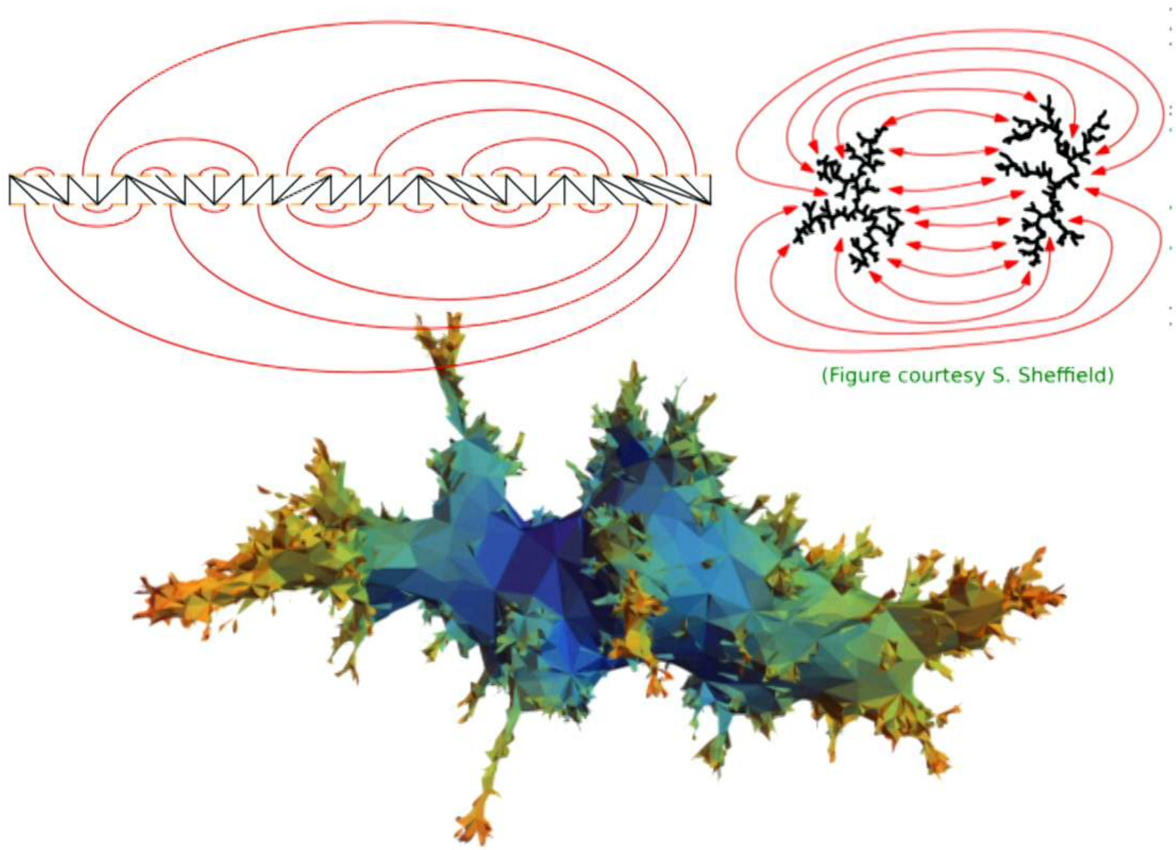
$$p(t;\mathfrak{m}) := \frac{1}{|\mathfrak{m}|}\sum_x p_x(t;\mathfrak{m}).$$

- One expects $\langle p(t;\mathfrak{m})\rangle_N \to p_\infty(t)$ as $N\to\infty$, and $p_\infty(t) \sim t^{-d_s/2}$ as $t\to\infty$.
- d_s is the (annealed) *spectral dimension* (equal to d on $\mathbb{Z}^d$).

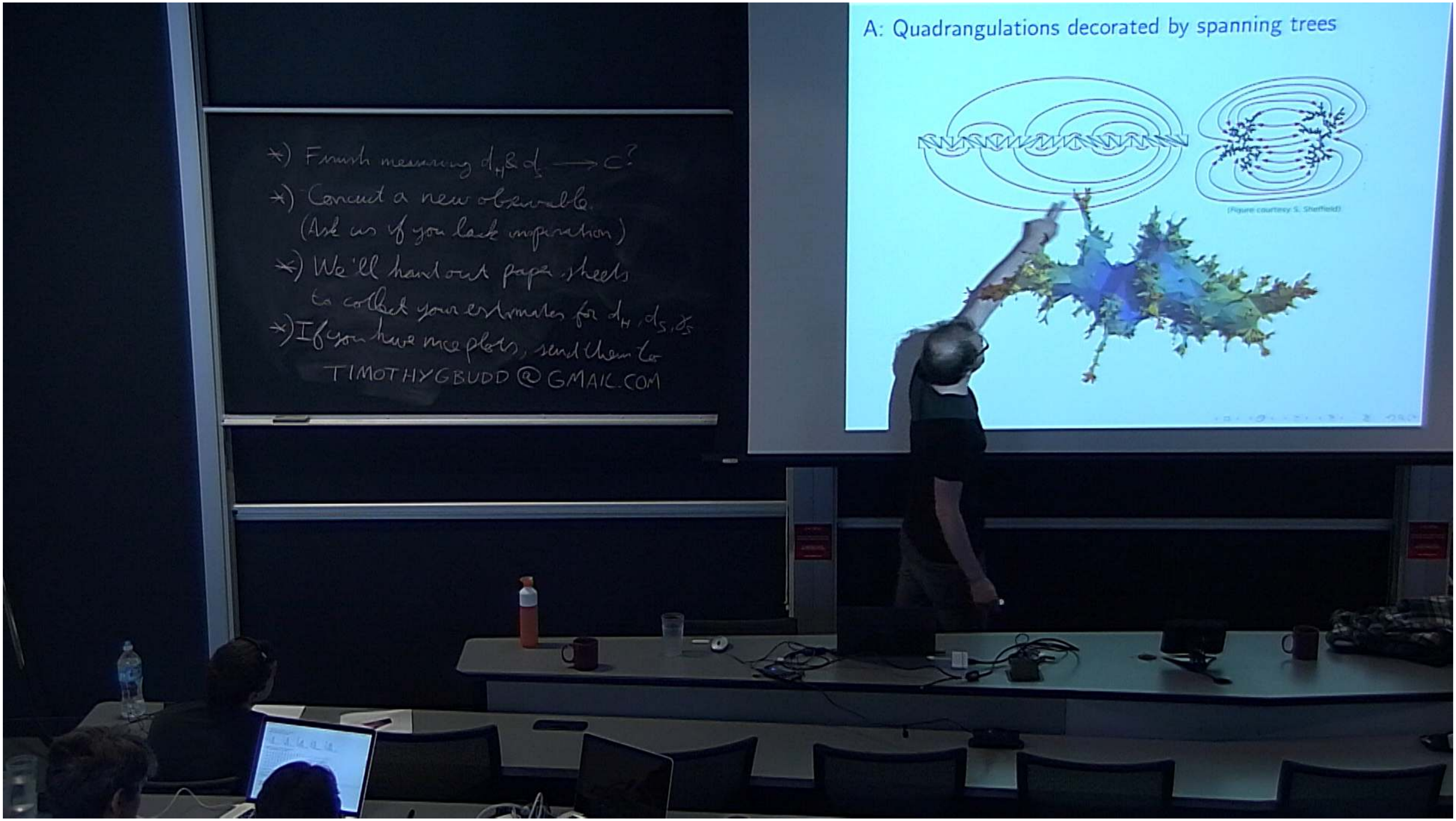




A: Quadrangulations decorated by spanning trees



(Figure courtesy S. Sheffield)



A: Quadrangulations decorated by spanning trees
(Figure courtesy S. Sheffield)
(Ask us if you lack inspiration)
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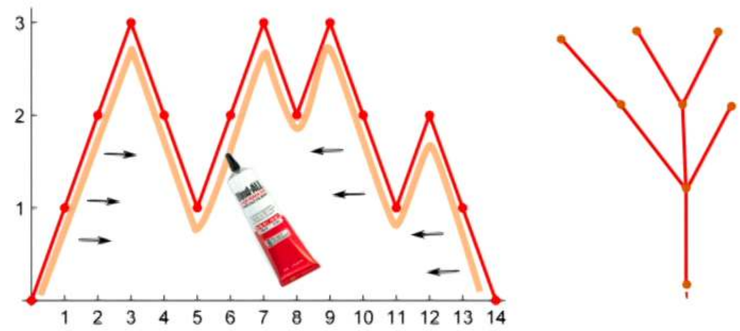
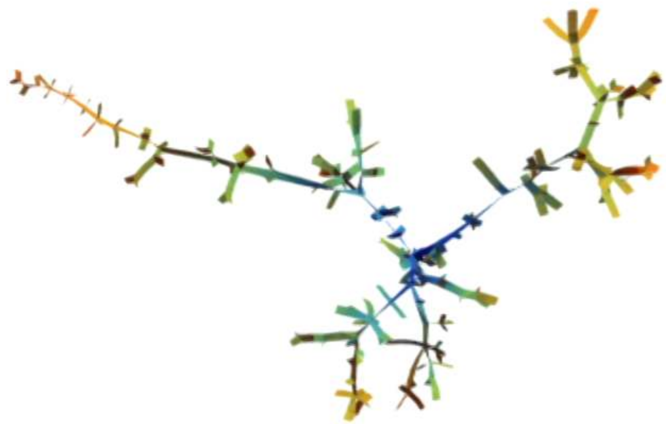
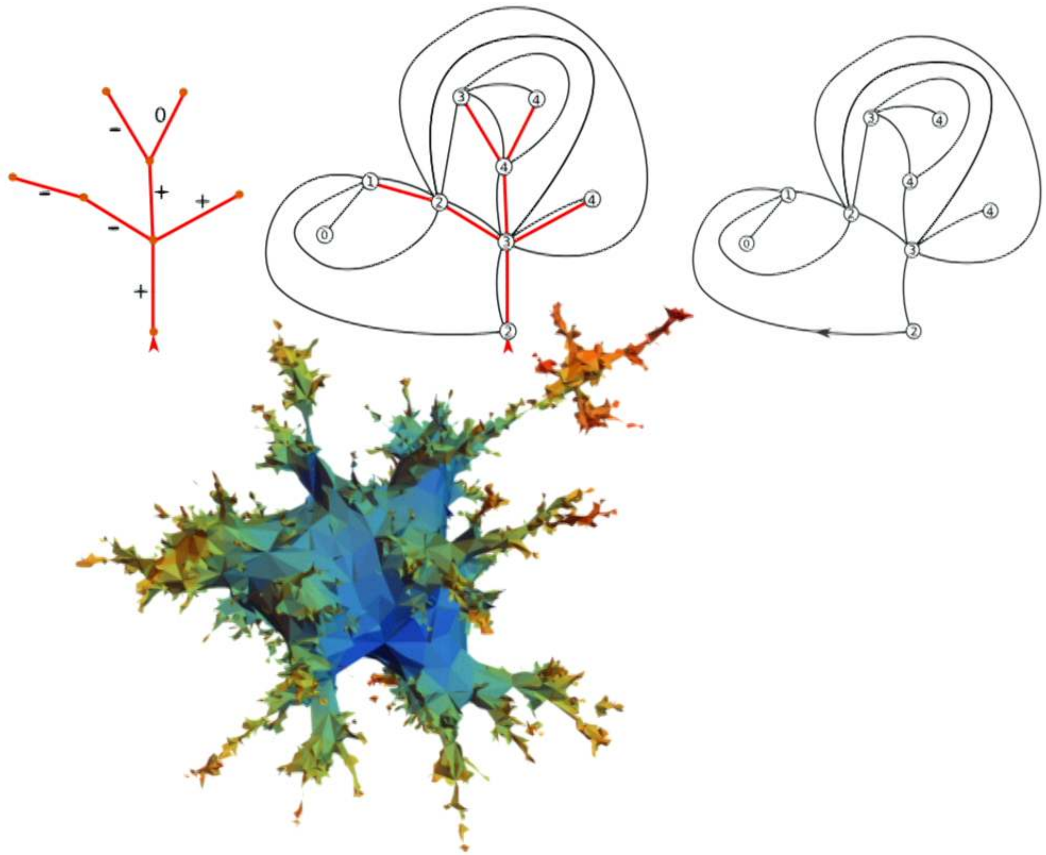
B: Branched polymer / Uniform random tree

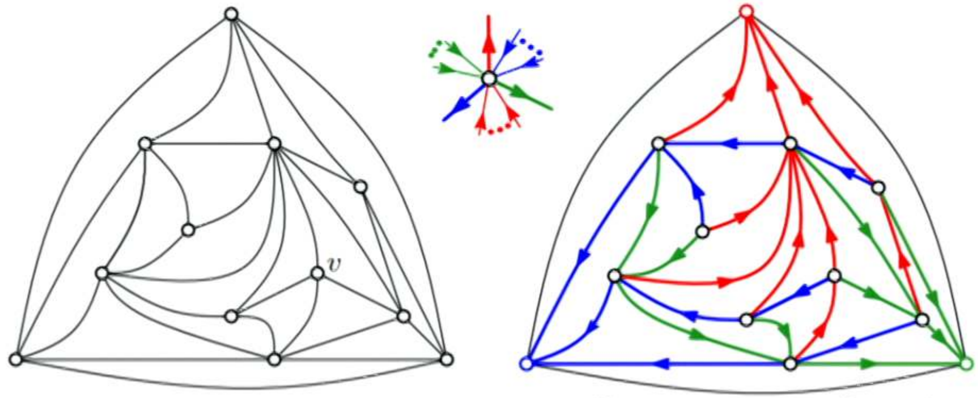




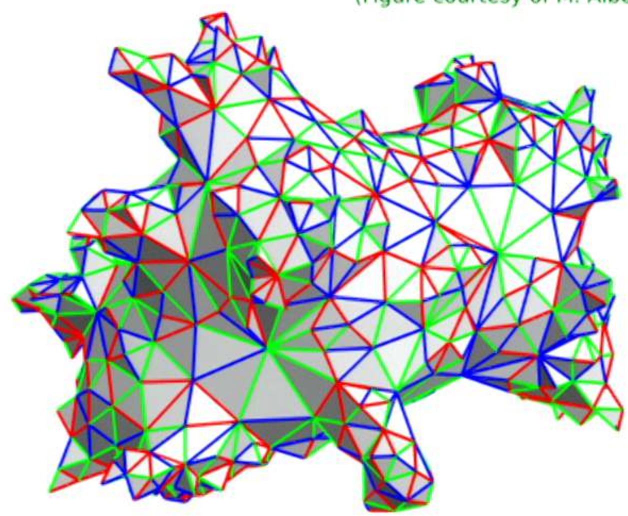
C: Uniform quadrangulation ("pure gravity")



D: Triangulations decorated by Schnyder woods



(Figure courtesy of M. Albenque)



Overview

	A: spanning tree	B: br. polymer	C: uniform quad.	D: Schnyder
c	-2	"∞"	0	$-25/2$
γ_s	-1	$1/2$	$-1/2$	-3
d_s	2	$4/3$	2	2
d_H	$\frac{3+\sqrt{17}}{2} \approx 3.56$	2	4	$\frac{5+\sqrt{41}}{4} \approx 2.85$

(Conjectures in red)