Title: Coarse-graining of Spin Foams - 3

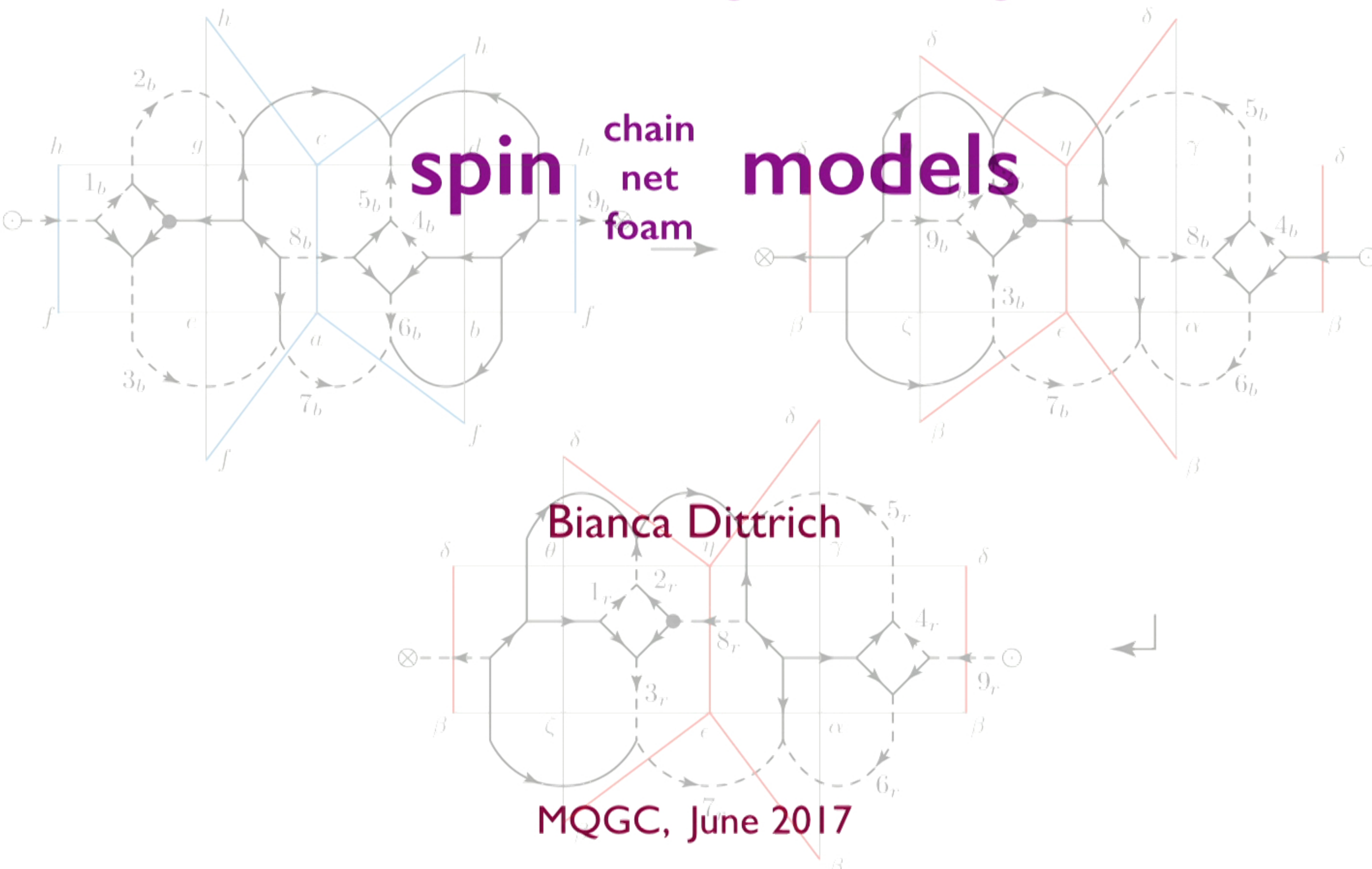
Date: Jun 22, 2017 11:00 AM

URL: http://pirsa.org/17060084

Abstract:

Coarse graining

spin chain net foam models



Bianca Dittrich

MQGC, June 2017

Plan

1. Challenges for coarse graining spin foams (and others)

2. 2D models with global symmetry:
   Symmetry protection and making use of observables.

3. Higher dimensions and gauge symmetries:
   Decorated tensor networks allow direct access to observables.

1. Conceptual background: [BD 2014 in book: 30 years of LQG]

2. [BD, E. Schnetter, C. Seth, S. Steinhaus PRD 2016]

3. [BD, S. Mizera, S. Steinhaus NJP 2016] [C. Delcamp, BD 2016]

Challenges

Spin foam models have complex amplitudes.

Good! Avoids deep problems of Euclidean Quantum Gravity.

But does not allow for Monte Carlo simulations.

Tensor networks do not rely on a sampling, so can deal with complex amplitudes.

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Tensor networks do not rely on a sampling, so can deal with complex amplitudes.

Renormalization flow is in an infinite dimensional coupling space.

Indeed tensor network flow is in a huge (but of course finite) coupling space (the entire tensor can change).

The method implements a cut-off "informed by the dynamics of the theory".

Decorated tensor networks: give more direct access to observables and better understanding of induced cut-offs and extraction of couplings.

Challenges

Spin foams involve a priori infinitely many variables.

Is a range of these variables dynamically suppressed?
Without cosmological constant rather unlikely due to diffeomorphism symmetry.

Implementing a cosmological constant (with Euclidean signature metrics):
Only finitely many variables.

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Implementing a cosmological constant (with Euclidean signature metrics):
Only finitely many variables.

Still many many variables … and very complicated models.

Main point of this talk.

Challenges

Spin foams are very poorly understood.

How to describe phases (what are good order parameters: again diffeomorphism symmetry).
Good or best choice of variables for coarse graining (we already have many different choices!).
What is the space of generalized geometries we are flowing in?
Are lattice gauge theory variables are a good choice for describing geometries?

Phase diagram for spin foams?

Diffeomorphism symmetry.

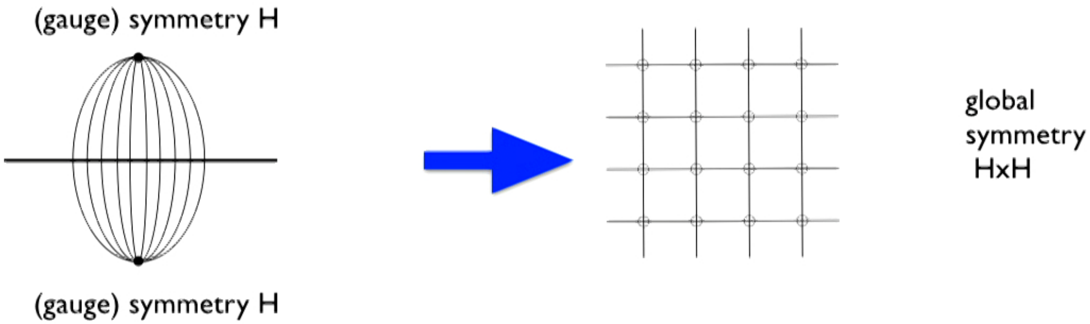
Is broken but needs to be restored.

Lack of (easier but not too easy) examples:
Do not have any diffeomorphism invariant QFT with propagating degrees of freedom.

Models with global symmetry:
Symmetry protection and making use of observables.

Spin net and intertwiner models

(dimensionally reduced spin foams)



We would be actually interested in H=GxG with G=SU(2).

Symmetry: GxGxGxG.

Interesting point: How do all these copies interact with each other (Simplicity constraints).

Spin net and intertwiner models

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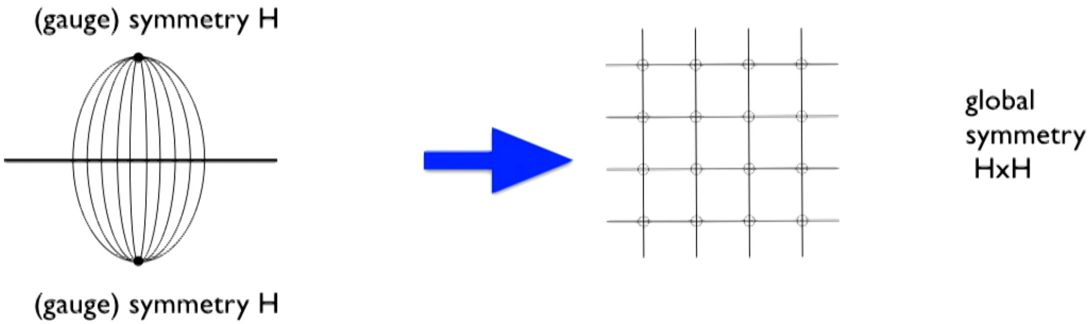

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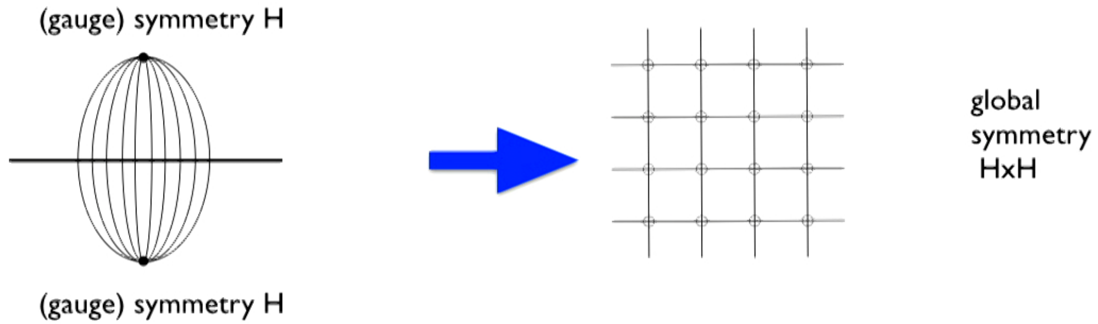

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Interesting point: How do all these copies interact with each other (Simplicity constraints).

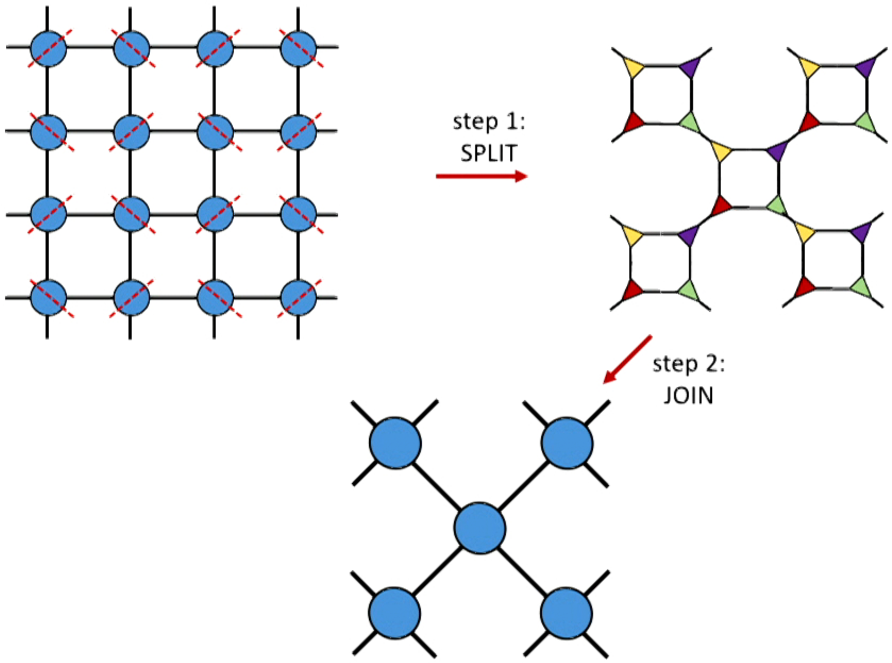
Consider here a symmetry just given by G= quantum group (SU(2)_k) or finite group.

Intertwiner models:

- admit interesting geometrical interpretation and also Hamiltonian and diffeomorphism constraints
- describe intertwiner degrees of freedom in spin foams (Reisenberger's condition)
- related to (quantum group) spin chains: rich phase diagrams, integrable models etc.

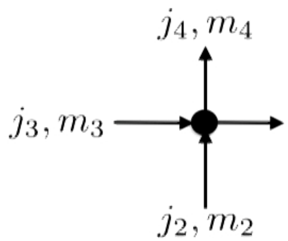
So could apply

tensor network renormalization



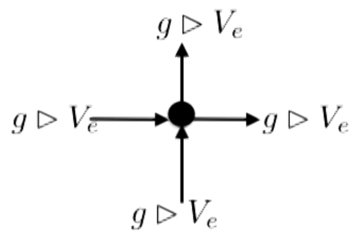
[from Guifre]

Models with a global symmetry



$V_e \simeq \oplus_{j_1} V_{j_1} \; : \; j_1, m_1$

$$T\,[\underbrace{(j_1, m_1)}_{a}, \cdots, \underbrace{(j_4, m_4)}_{d}] \quad \cdots$$



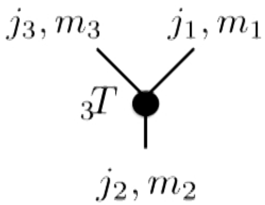
Invariance condition: $$(\otimes^4 g) \triangleright T = T$$

Two consequences:

1. Tensors have special form.
2. Invariance is preserved under 'gluing'.

   Z(boundary data) has a special form.

Tensors have a special form



$$ {}_3T(\{(j_i, m_i)\}) = \underbrace{{}_3\hat{T}(\{j_i\})}_{\text{(almost) free data}} \; C^{\{j_i\}}\underbrace{(\{m_i\})}_{\text{symmetry-determined structure}} $$

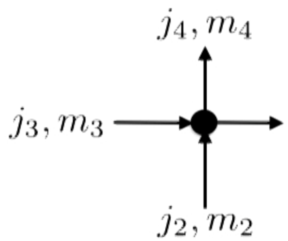
Coupling rules, e.g.: $j = 0, \frac{1}{2}, 1, \ldots, \frac{\mathrm{k}}{2}$

$$ j_1 \leq j_2 + j_3, \ldots, \; j_1 + j_2 + j_3 \in \mathbb{N}, \; j_1 + j_2 + j_3 \leq \mathrm{k} $$

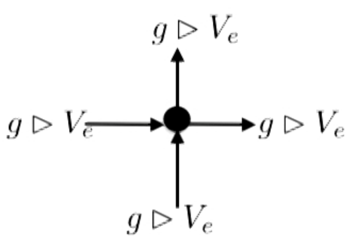
If violated: ${}_3\hat{T} = 0$

Can lead to a lot of (memory and time) savings!

Models with a global symmetry



$$V_e \simeq \oplus_{j_1} V_{j_1} \; : \; j_1, m_1 \quad \longrightarrow \quad T\,[\underbrace{(j_1, m_1)}_{a}, \cdots, \underbrace{(j_4, m_4)}_{d}]$$



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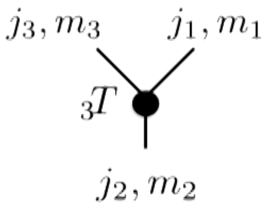

$$ {}_3T(\{(j_i, m_i)\}) = \underbrace{{}_3\hat{T}(\{j_i\})} \; \underbrace{C^{\{j_i\}}(\{m_i\})} $$

(almost) free data

symmetry-determined structure

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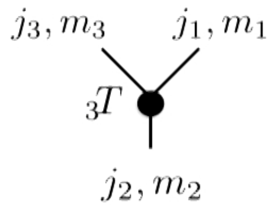

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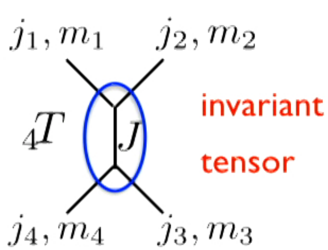


$$ {}_4T(\{(j_i,m_i)\}) = \sum_J \underbrace{{}_4\hat{T}^J(\{j_i\})}_{\text{(almost) free data}} \; \underbrace{{}_4C^{j_i;J}(\{m_i\})}_{\text{symmetry-determined structure}} $$

$$ {}_4C^{j_i;J}(\{m_i\}) = \sum_M C^{j_1,j_2,J}(m_1,m_2,M)\, C^{J,j_3,j_4}(M,m_3,m_4) $$

Tensors have a special form

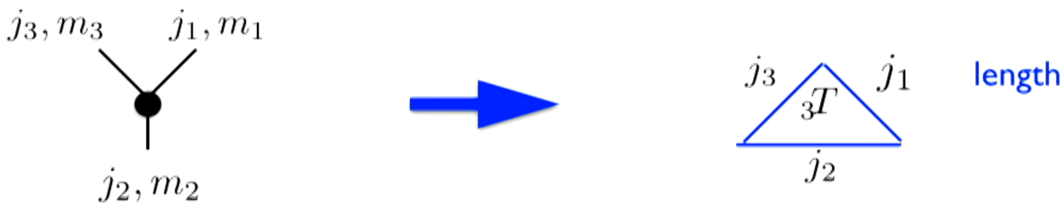
We only want to deal with the (almost) free part: $_N\hat{T}^{J_1,J_2,\cdots}(\{j_i\})$

Taken care of coupling rules can lead to a lot of (memory and time) savings!

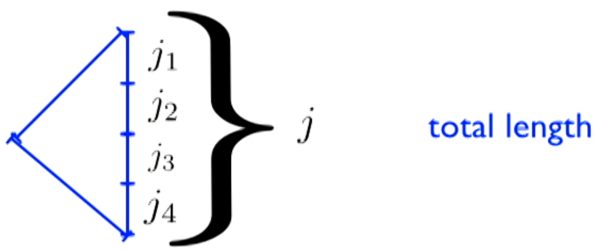
The representation labels might actually be interesting observables:

Labels carry more information than without symmetry!

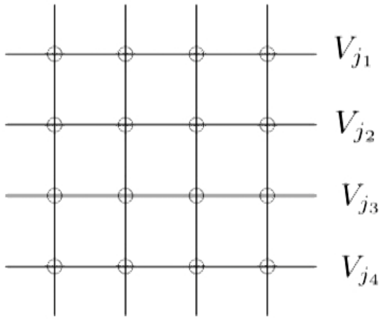
Geometric interpretation



Glue triangles (or quadrangles) to larger 2D geometries.



Tensors keep their special form



$$\Big\} \oplus_j \alpha(j)\ V_j$$

j might be an interesting (coarse grained) boundary variable.

change of basis

$$(\{j_i\},\{m_i\}) \longrightarrow (j, D, m)$$

j: coarse grained variable

D: degeneracy index

m: symmetry determined behaviour

change of basis

$$_{16}T(\{(j_i, m_i)\}) \longrightarrow {}_4T'(\{(j_I, D_I, m_I)\})$$

This tensor has same special form as before.

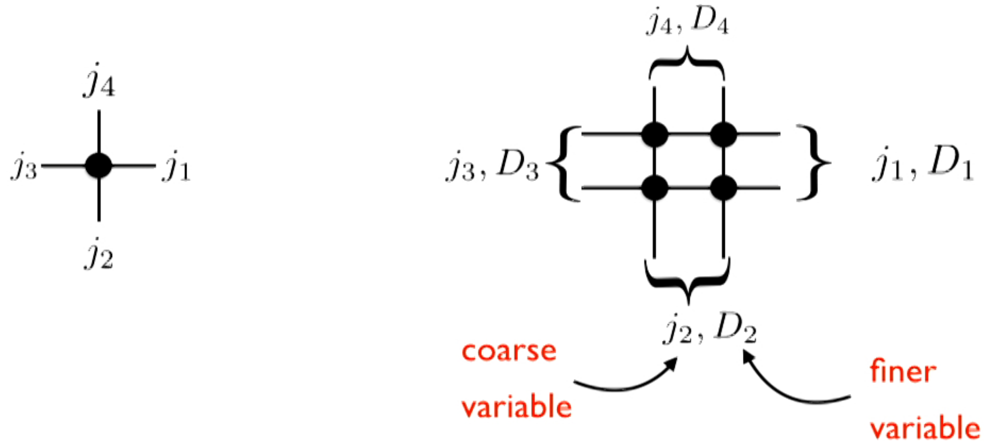
(For each value D-indices.)

$$\longrightarrow {}_4\hat{T}'^J(\{(j_I, D_I)\})$$

Requirement for (interesting) models

Get partition function for a square with different refinements.

How to compare these?



Want to compare only coarse observables.

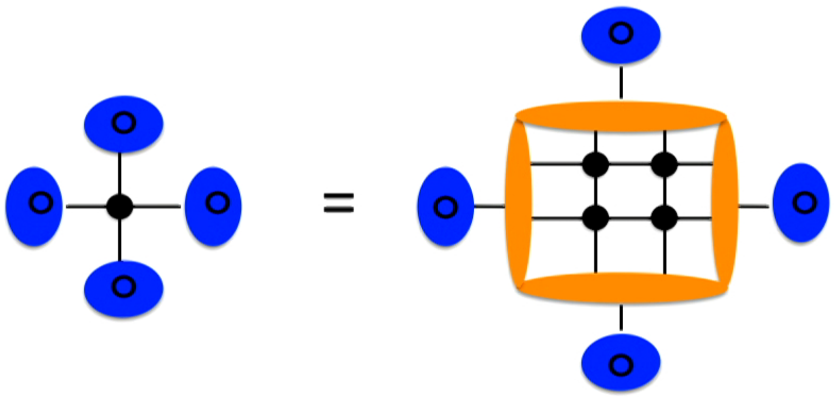
Consistency conditions

finer degrees of freedom

$$T(\{j_i, D_i, m_i\}) = \sum_{D'_i} T'(\{j_i, D'_i, m_i\})\ \psi(\{j_i, D_i; D'_i\})$$

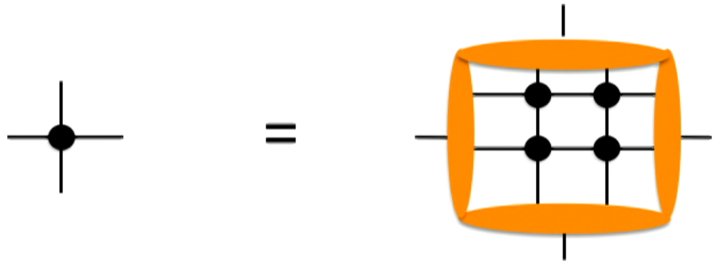
coarse degrees of freedom

"put finer degrees of freedom into vacuum state"



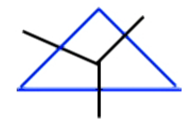
Such tensors and embedding maps are produced as fixed points of the tensor network renormalization flow.

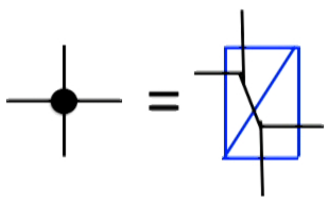
Consistency conditions: a class of examples

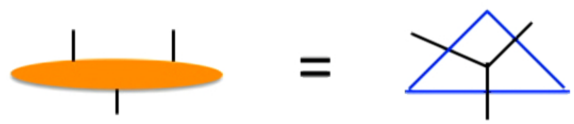


Can these be satisfied at all?

Yes, if you have a lattice TQFT.
(aka gapped phase)



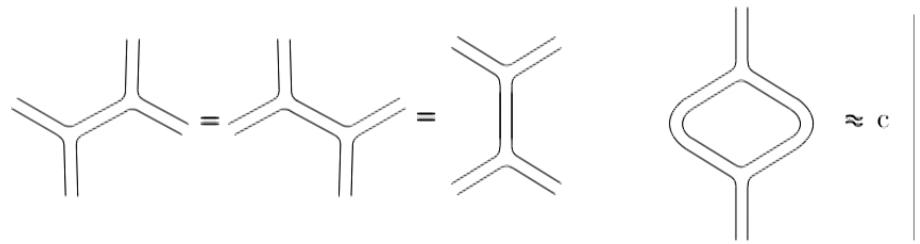


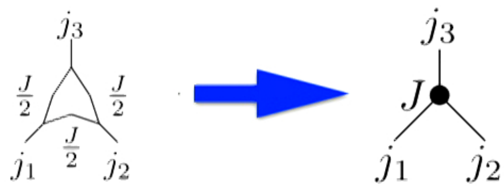


Exercise:

Show that this is true, using invariance under Pachner moves of the TQFT partition function.

Simplest examples of TQFTs: "CDL"





$$ {}_3T = a^J_{CDL}(j_1, j_2, j_3)\ {}_q\mathcal{C}^{j_1\, j_2\, j_3}_{m_1\, m_2\, m_3} $$

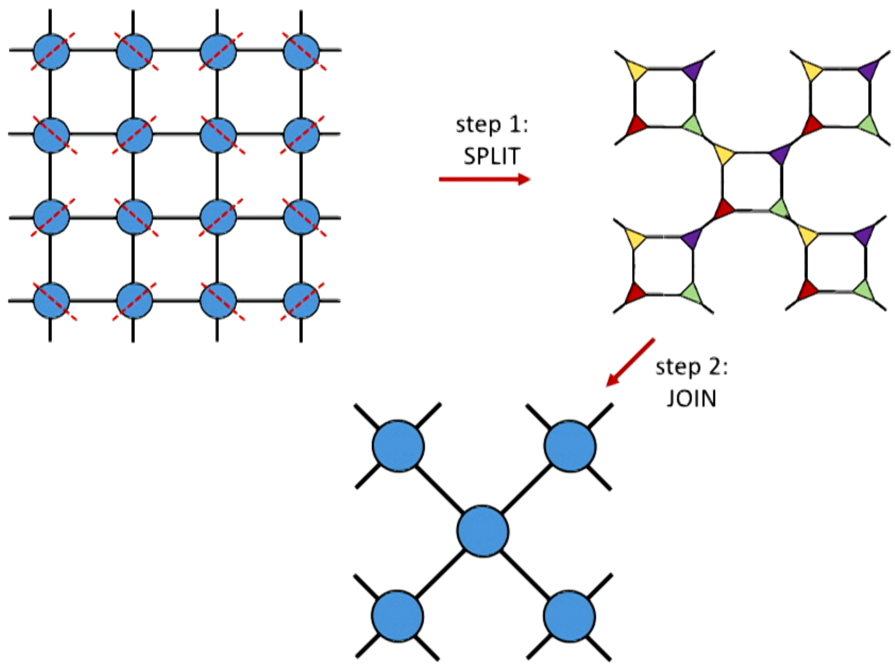
maximal spin value: j=J

Fixed points, describing phases in spin chain phases
(or symmetry protected models).

Further (triangulation invariant) more complicated fixed point models: [BD, W. Kaminski 2013]
(Classified by module categories over SU(2)_k.)

Very rich structure of (gapped and non-gapped) fixed points.
In particular if one considers 2 or 4 copies. (Classification?)

Find fixed points of tensor network renormalization flow



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