Title: Monte Carlo methods in Dynamical Triangulations - 3

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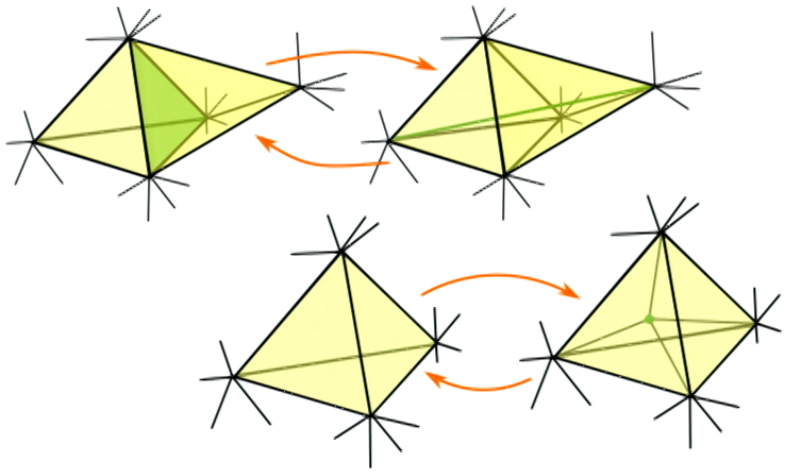
Abstract:

Making Quantum Gravity Computable, 22-06-2017

Monte Carlo methods in Dynamical Triangulations

Part II: Higher dimensions

Timothy Budd



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Outline

- Day 1: 2D random geometry
  - Combinatorial representation
  - Markov Chain Monte Carlo (MCMC) methods
  - Matter coupling
  - Observables
- Day 2: Dynamical Triangulations in higher dimensions
  - Quantum gravity
  - Combinatorial representation
  - MCMC methods
  - Phase diagram
  - Causal Dynamical Triangulations
- Tutorials: numerical analysis of various 2D random geometries
  - Measure observables for random geometries (produced by black box)
  - Extract critical exponents.
  - Experiment with (new?) observables.
  - Conclusions will be collected at the end and be discussed.

A space-time path integral?

$$\int_{\substack{\text{Lorentzian}\\ \text{metrics}}} \frac{\mathrm{d}\mu(g_{\alpha\beta})}{\mathrm{Diff}}\, e^{iS[g_{\alpha\beta}]}$$

Difficulties:

- QFT in perturbative regime: non-renormalizable
- Infinite-dimensional integral
- What is a good diffeo-invariant measure?
- Destructive interference is delicate
- How to interpret integrand?
- Numerical evaluation is hard

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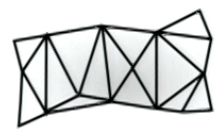
- Numerical evaluation is ~~hard~~ doable in principle
- Classical solutions? Action is unbounded below.
- Does the integral converge?
- Does it possess a continuum limit?

Cut-off



$$\sum_{\{\text{PL geometries } g_{ab}\}} \iint \frac{\mathrm{d}\mu(\ell_i)}{\mathrm{Diff}} e^{-S[g_{ab}]}$$

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Quantum → Statistical
Lorentzian → Euclidean

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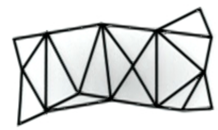

$$\sum_{\{\text{PL geometries } g_{ab}\}} \iint \frac{\mathrm{d}\mu(\ell_i)}{\mathrm{Diff}} e^{-S[g_{ab}]}$$

Discretize

$$\sum_{\{\substack{\text{Combinatorial}\\ \text{geometries}}\}} e^{-S[g_{ab}]}$$

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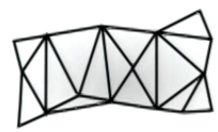
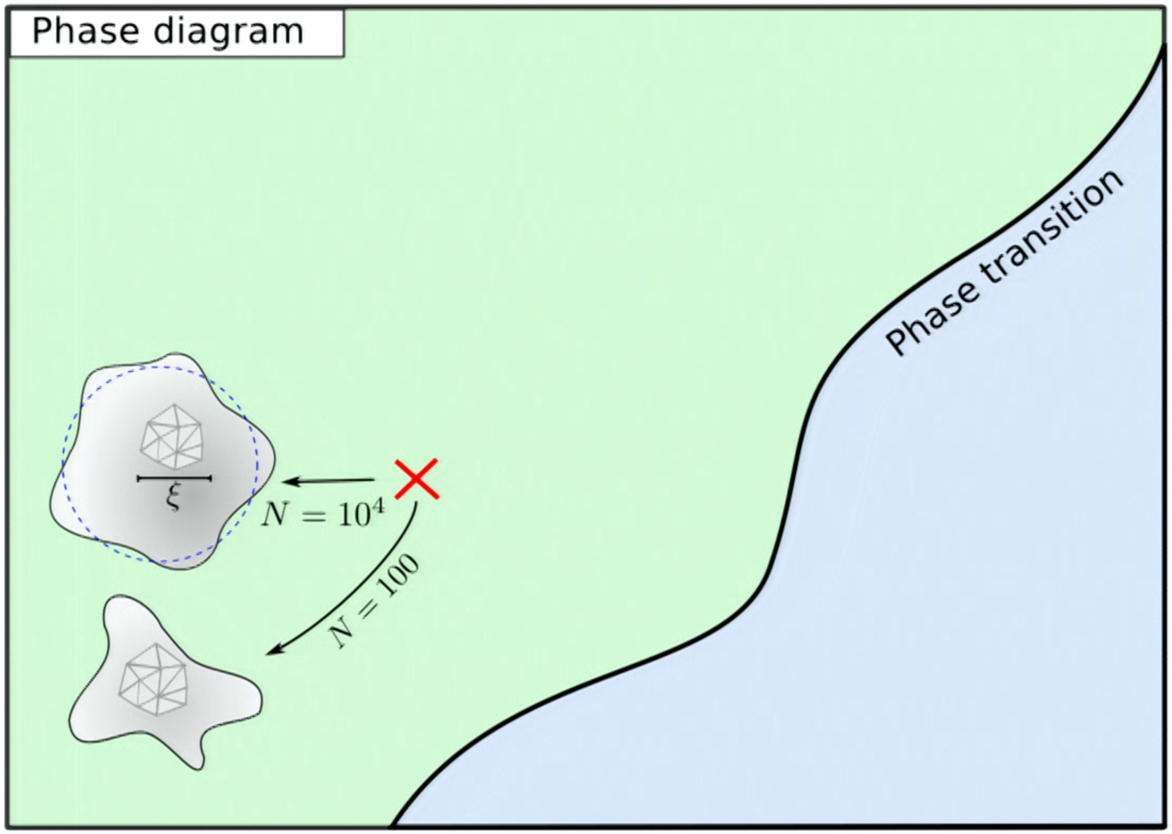


$$\sum_{\{\text{Combinatorial geometries}\}} e^{-S[g_{ab}]} \xleftarrow{\text{Discretize}} \sum_{\{\text{PL geometries } g_{ab}\}} \iint \frac{\mathrm{d}\mu(\ell_i)}{\mathrm{Diff}} e^{-S[g_{ab}]}$$

What would be the best scenario?



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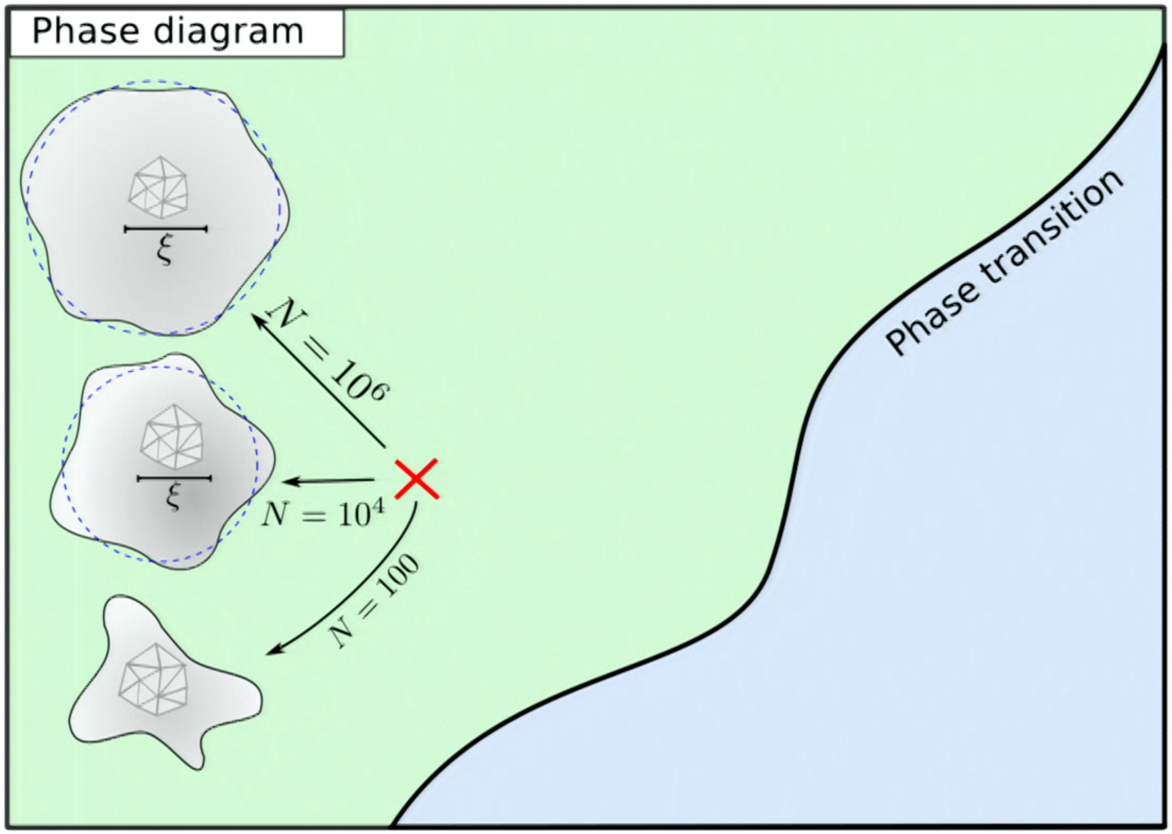

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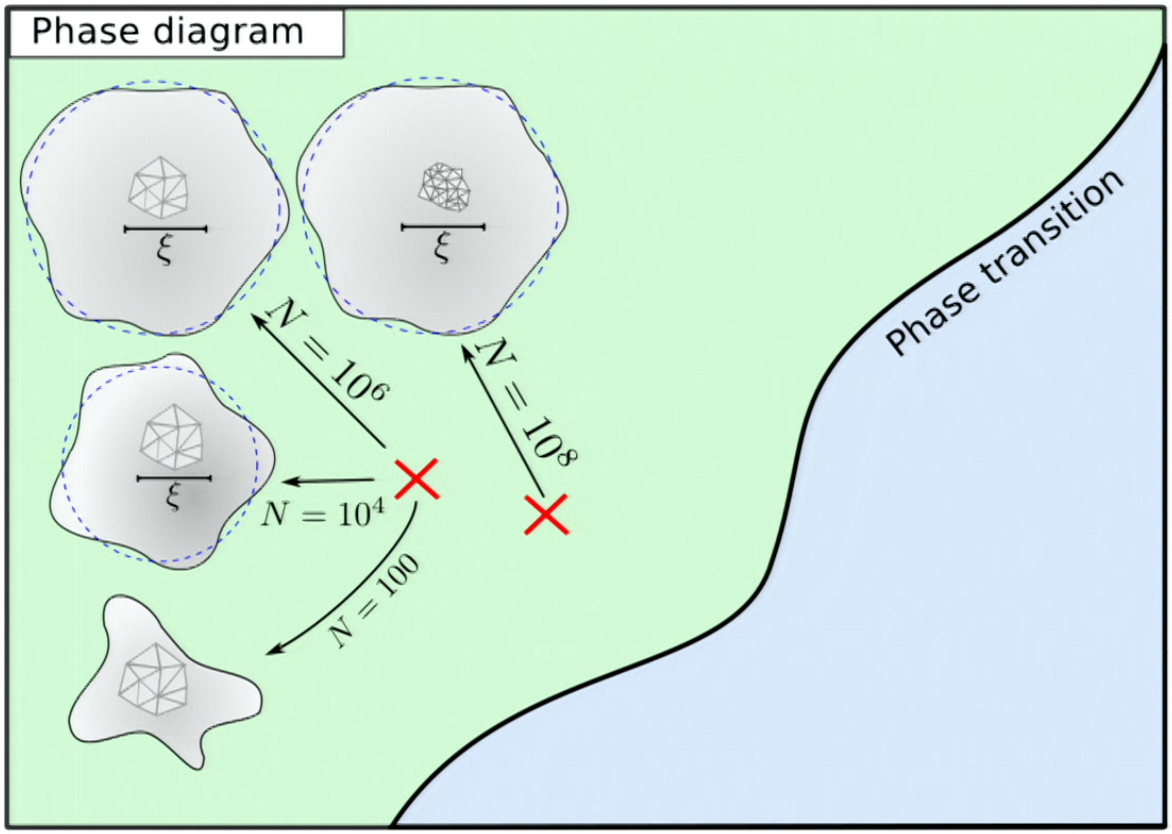

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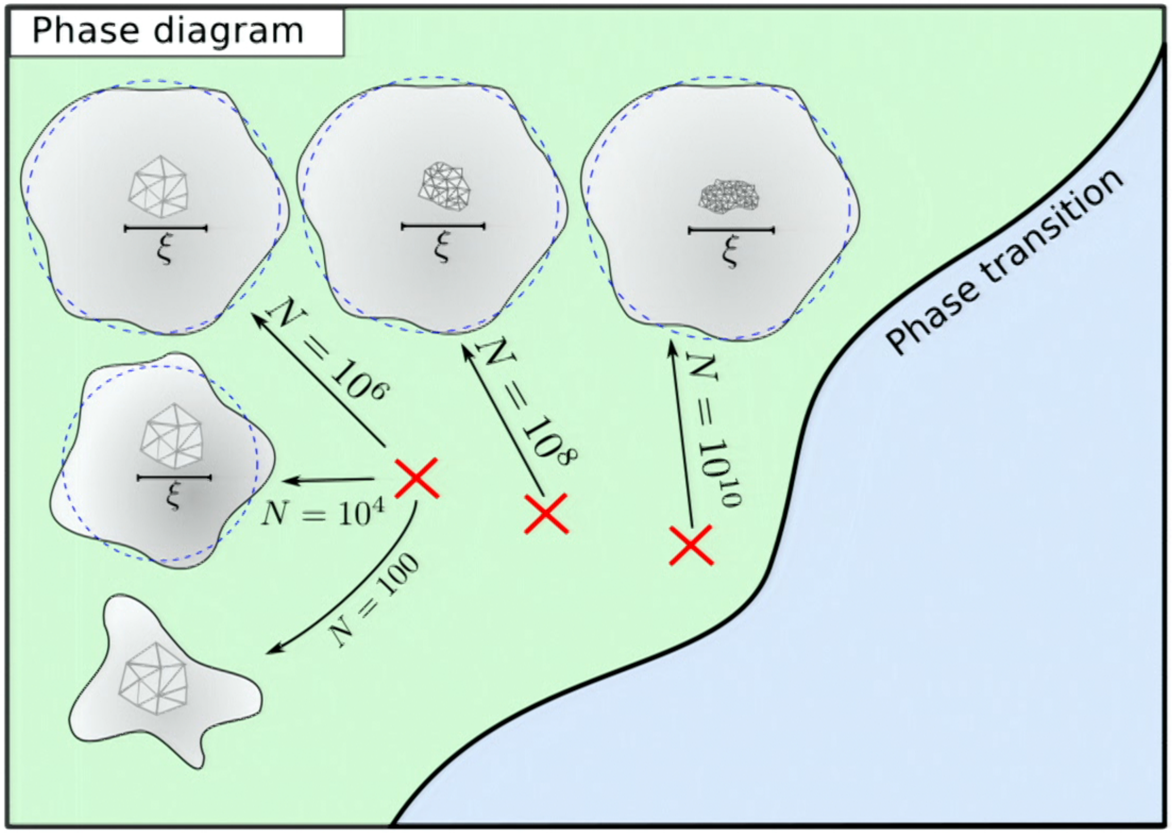

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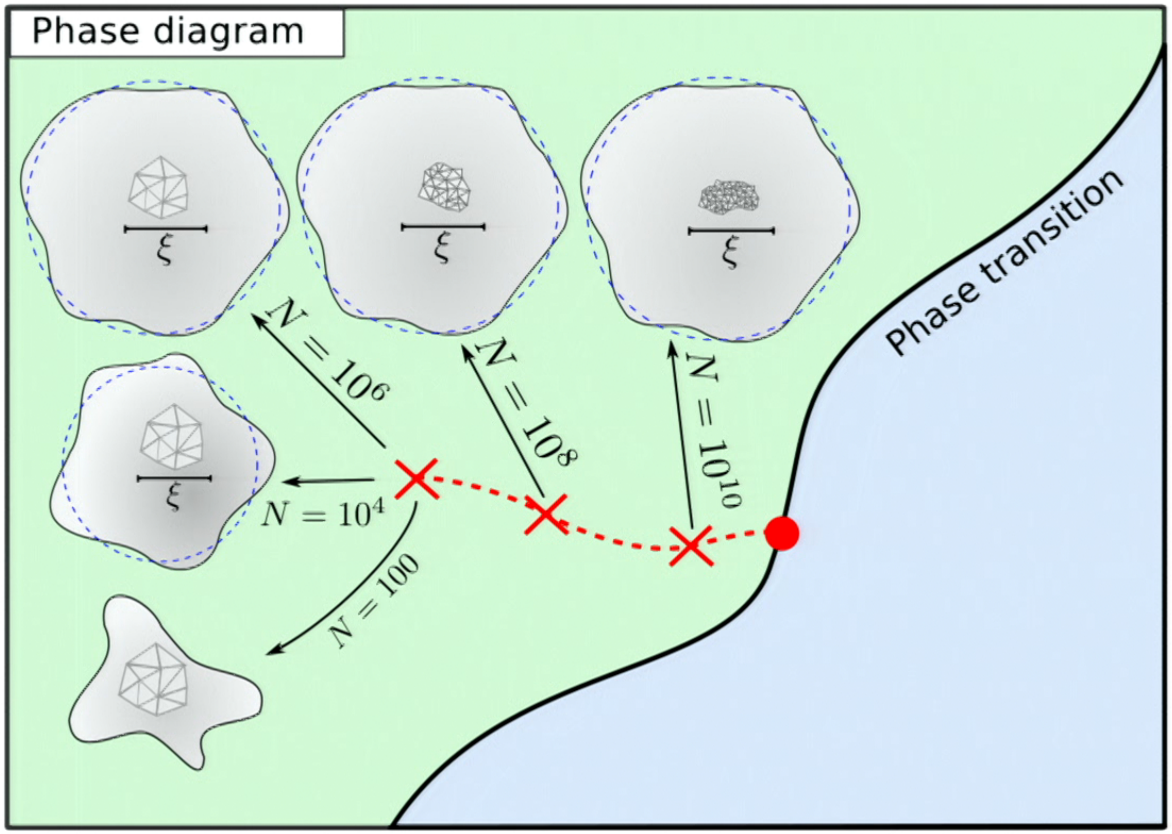

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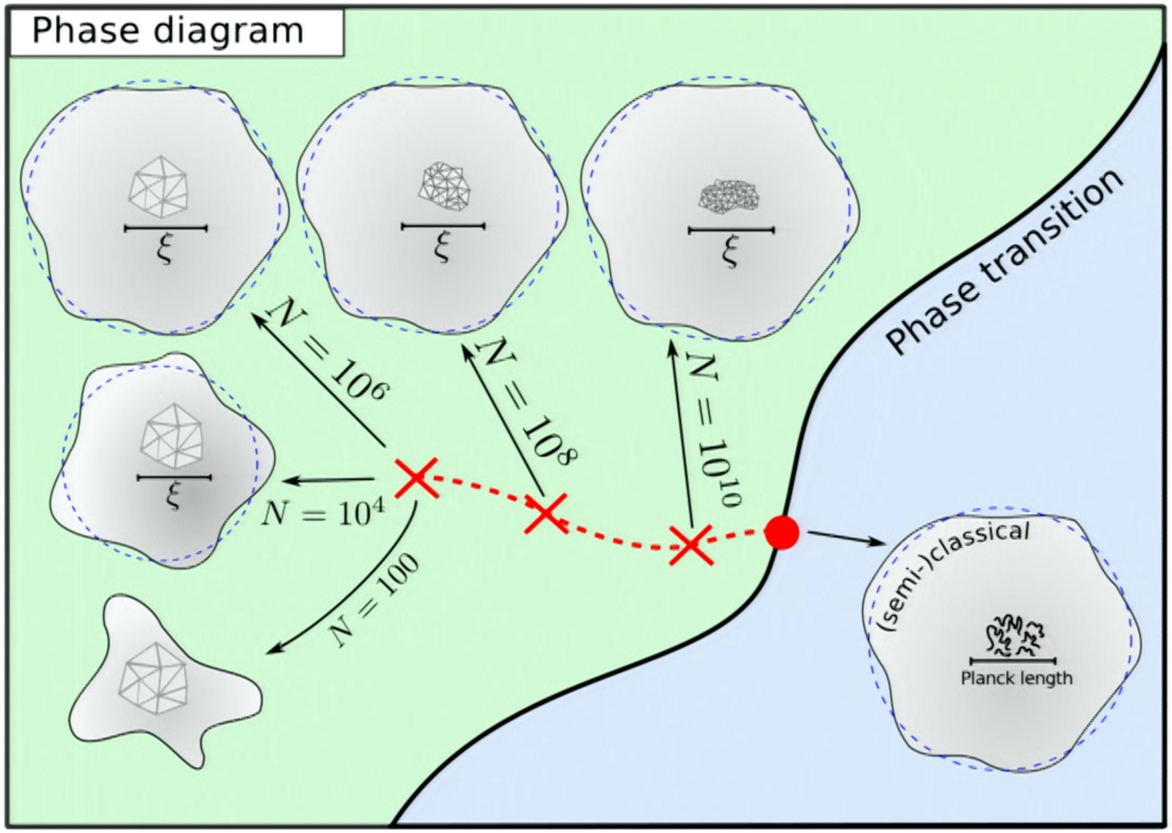
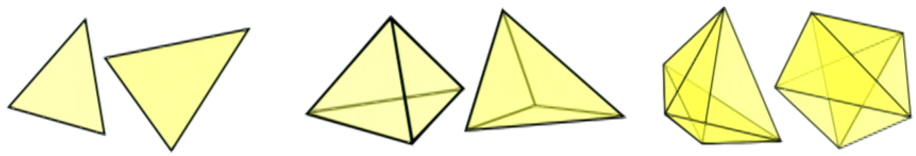


Piecewise linear geometry

- D-simplex: $\{\sum_{i=0}^{D} \lambda_i \mathbf{x}_i : \lambda_i \in [0,1], \sum \lambda_i = 1\} \subset \mathbb{R}^D$ with Euclidean geometry.



$D = 2$ $\quad$ $D = 3$ $\quad$ $D = 4$

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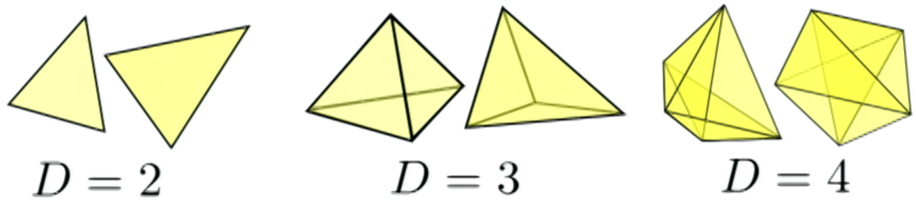
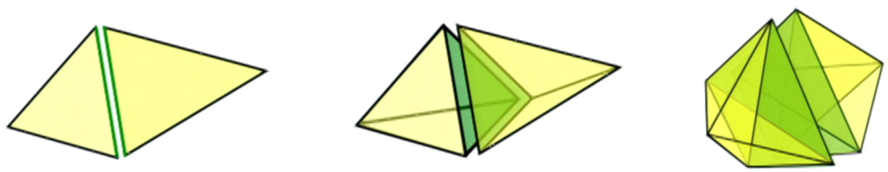


- D-simplices can be glued into larger metric spaces along matching $(D-1)$-simplices.



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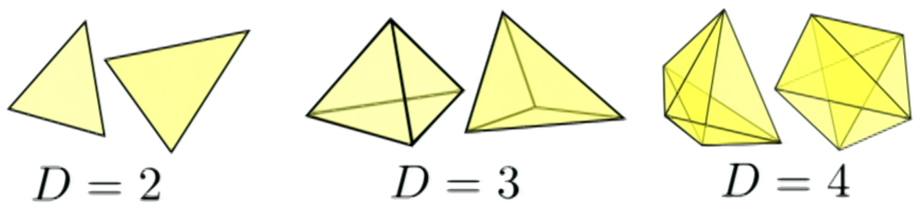

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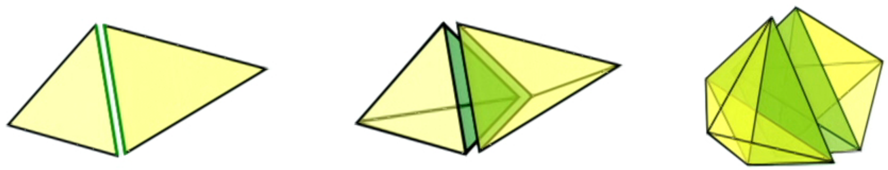
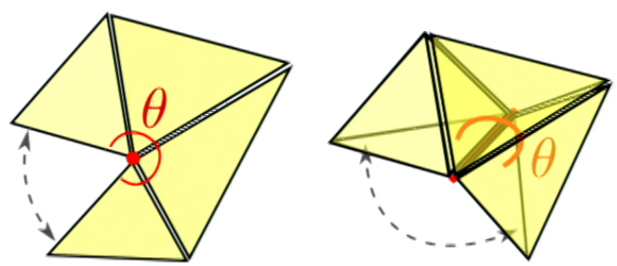


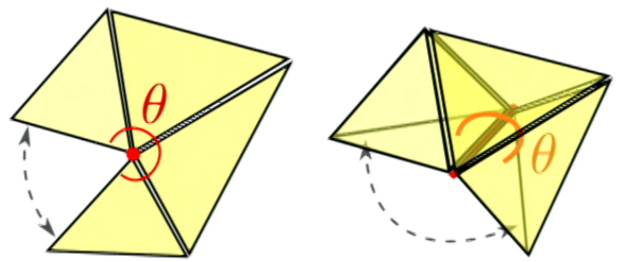
- Resulting geometry has curvature supported on $(D-2)$-simplices.



Einstein-Hilbert action

- Integrated curvature is naturally expressed in terms of deficit angles [Regge, '61]

$$\int \mathrm{d}^D x \sqrt{g} R \longrightarrow \sum_{(D-2)-\text{simplices } \sigma} |\sigma|(2\pi - \theta_\sigma)$$



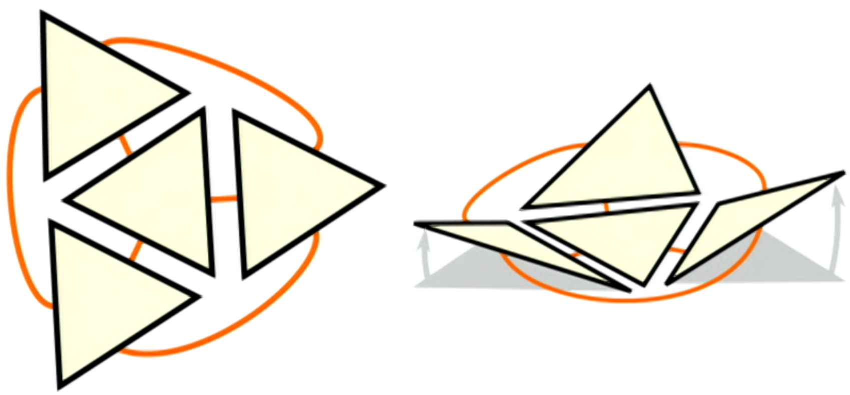
- If all simplices are taken of equal shape (say, equilateral) then linearity of Regge action implies that EH ($+ \int \mathrm{d}^D x \sqrt{g} \Lambda$) is a simple linear combination

$$\kappa_D N_D - \kappa_{D-2} N_{D-2}.$$

- Makes sense to include in MCMC at least such two terms in Boltzmann weight.

Combinatorial representation

- Recall: need next and adjacent to navigate a map, or a polyhedron.



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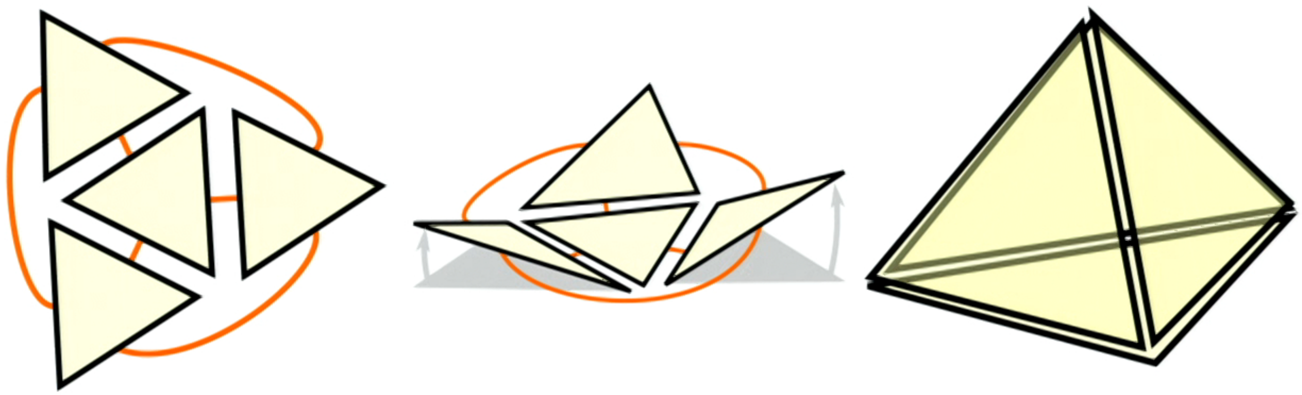

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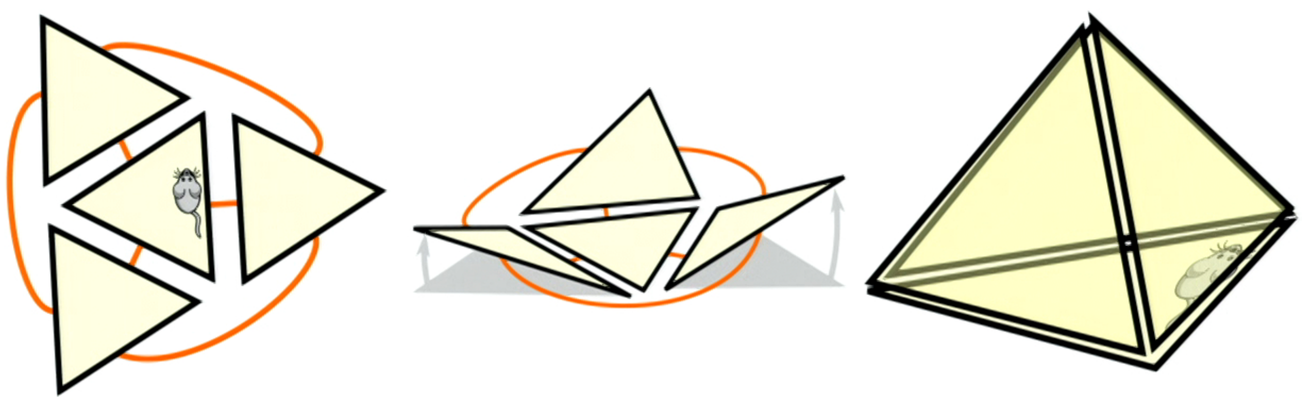

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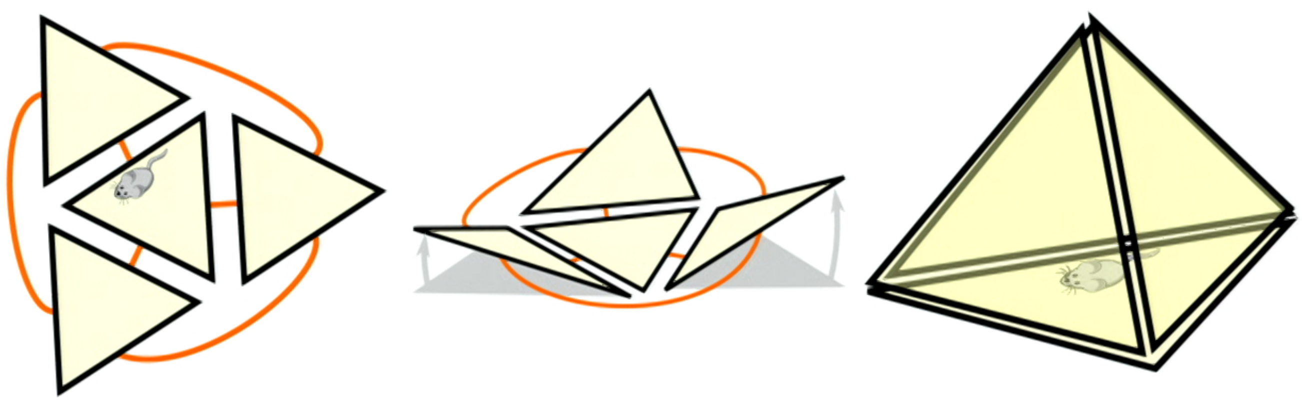

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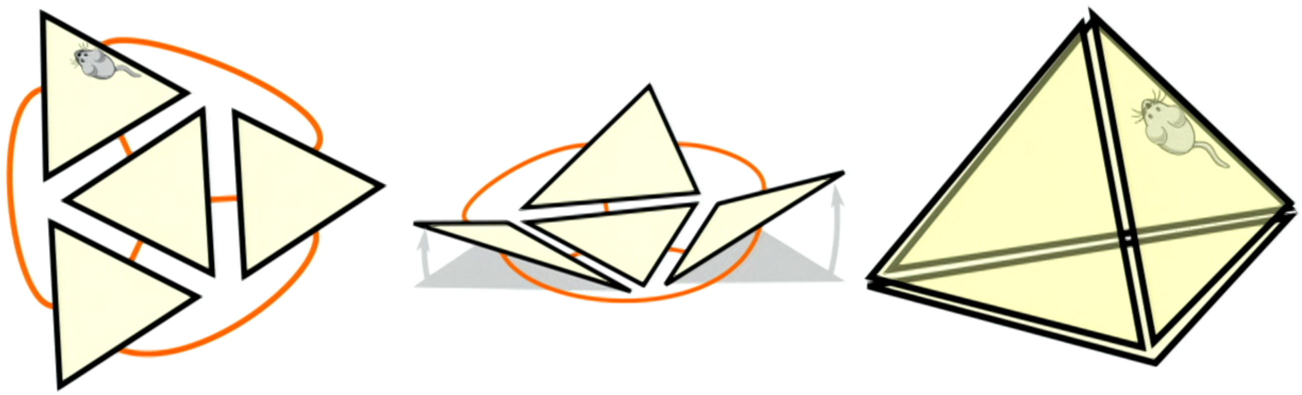

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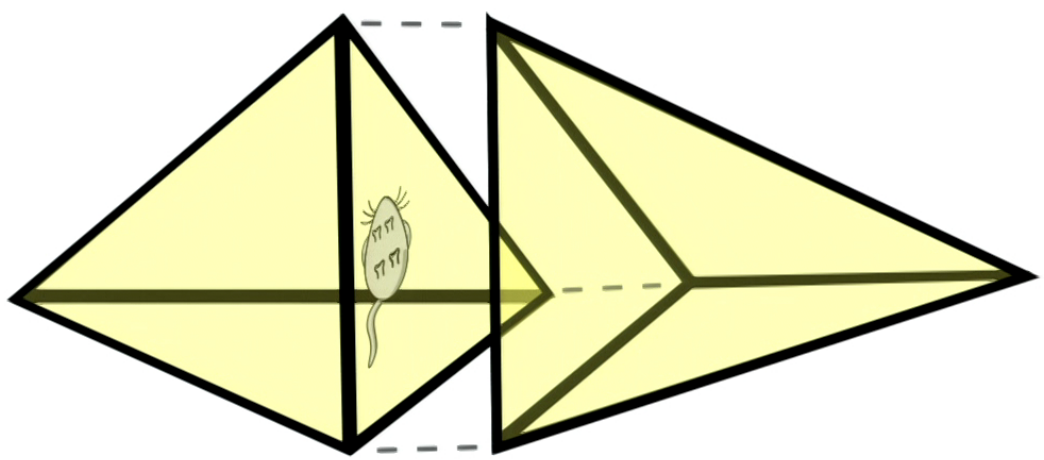
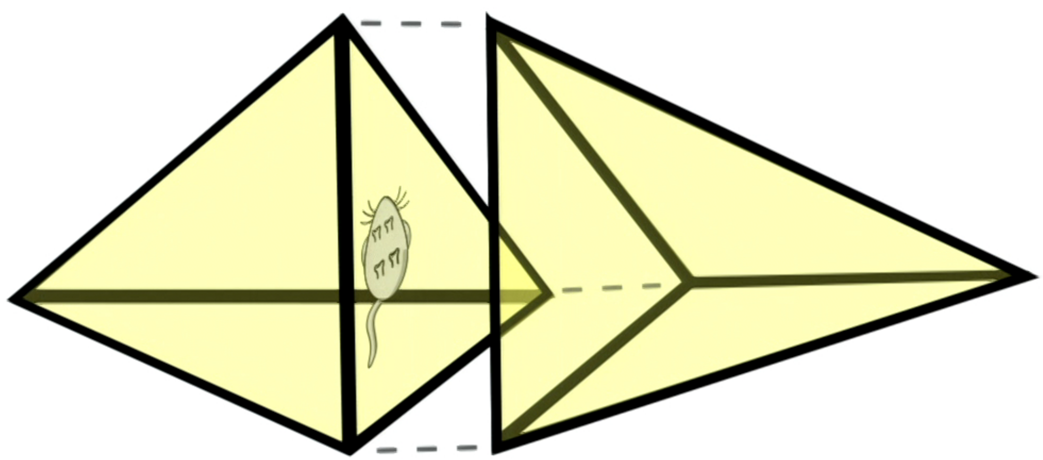


Combinatorial representation

- Recall: need next and adjacent to navigate a map, or a polyhedron.
- Generalize: For D-dimensional geometry, $(n, a) \rightarrow (n, a_2, \cdots, a_D)$.
  a_d maps half-edge i to its d-dimensional neighbor $a_d(i)$.



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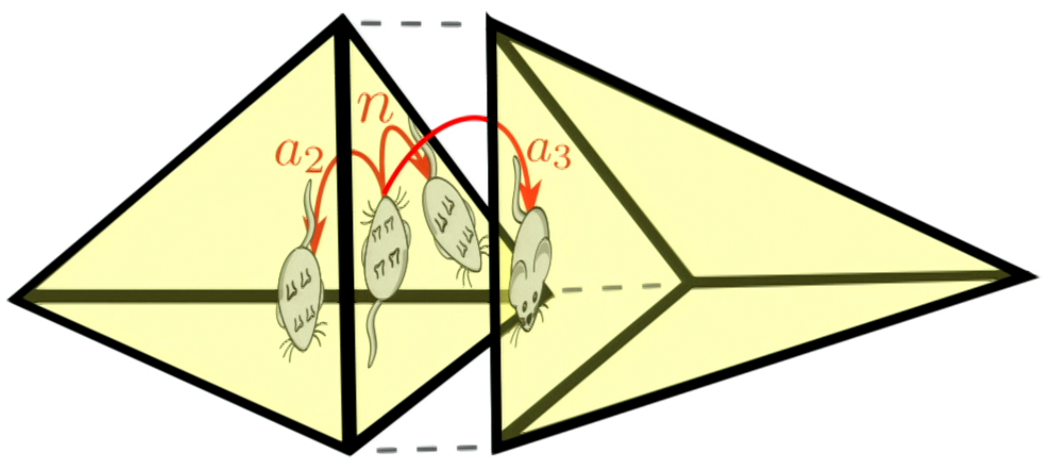

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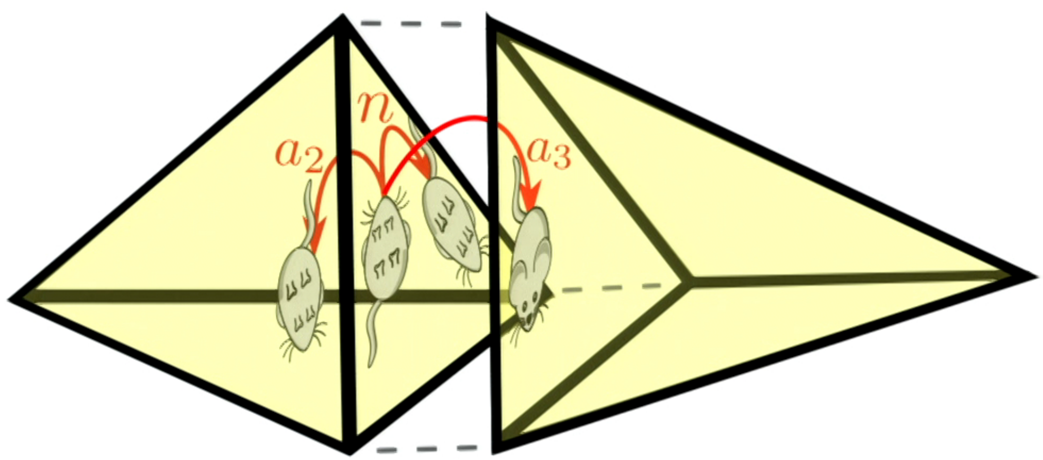
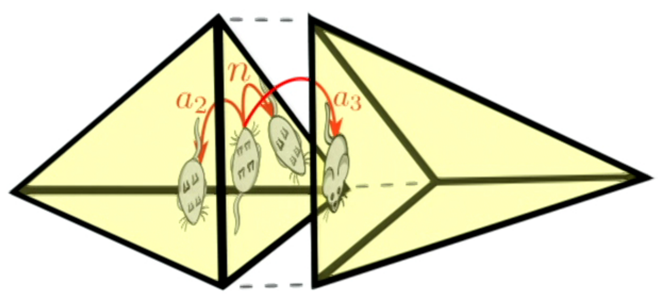
- Cells of various dimensions are identified as orbits. In 3D: $(n, a_2) \to$ polyhedra, $(n, a_3) \to$ faces, $(a_2, a_3) \to$ edges, $(n \circ a_2, n \circ a_3) \to$ vertices.



Remarks on combinatorics in 3D

- When does a triple of permutations (n, a_2, a_3) determine a topological 3-manifold?



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  - Proper gluing: $n \circ a_3 \circ n = a_3$.
  - Polyhedra (orbits under n, a_2) should have 3-ball topology (i.e. boundary S^2): Euler formula!
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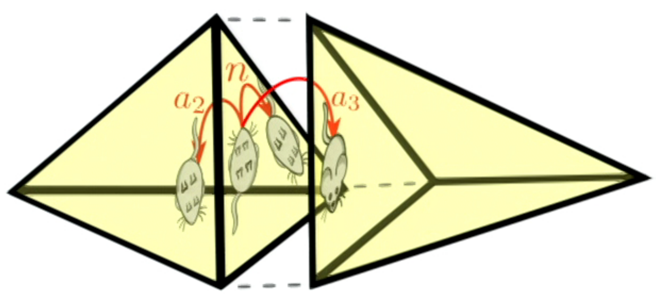

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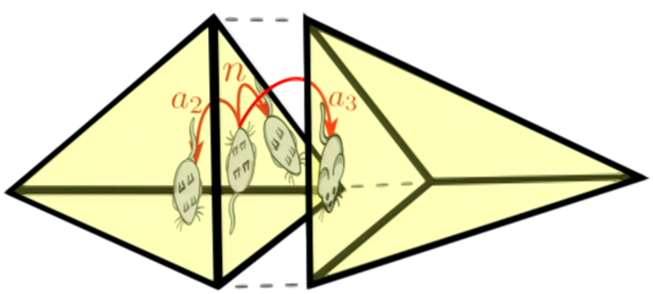

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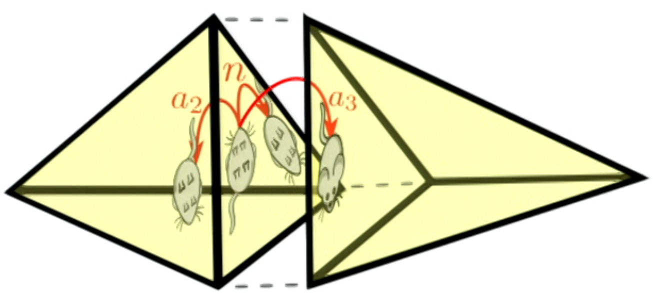
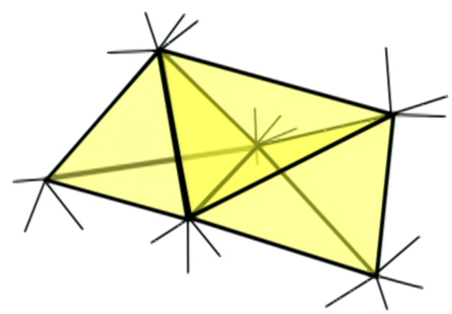
- What is the topology of the resulting 3-manifold?
  - Unfortunately, no simple combinatorial/algorithmic way to decide!
  - Luckily, any two geometries with equal topology are connected by a finite sequence of local moves!
- Situation very similar in 4D (and higher).



Simplicial manifolds

- *D-triangulation*: all D-cells are taken to be D-simplices (i.e. triangles in $2D$, tetrahedra in $3D$, 4-simplices in $4D$).



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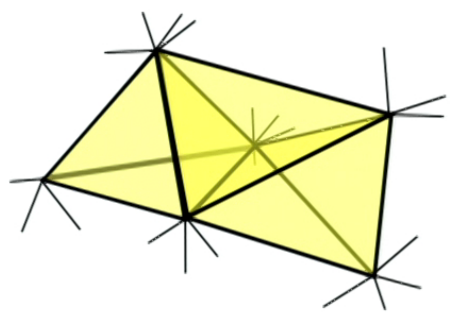
- Amounts to $(D+1)!/2$ numbers to store/update per D-simplex.



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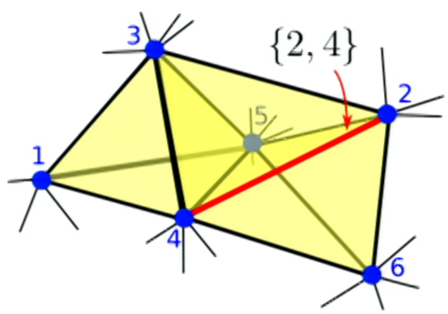
- Label the vertices of a D-triangulation.
- *Simplicial D-triangulation*: each edge, face, ..., D-simplex must be uniquely characterized by its set of incident vertices.



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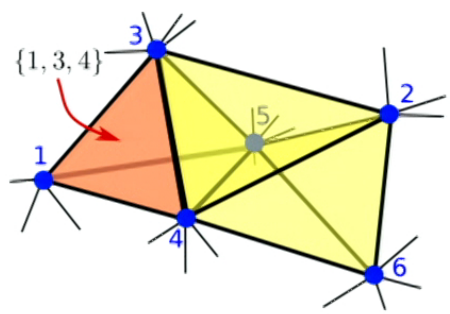

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- Knowing the set $\{\{1,3,4,5\},\{2,3,4,5\},\{2,4,5,6\},\ldots\}$ of D-simplices, can reproduce the triple (n, a_2, a_3) up to relabeling (and orientation).