Title: Numerical Questions in Causal Set Quantum Gravity

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Abstract:

Numerical Questions in Causal Set Quantum Gravity

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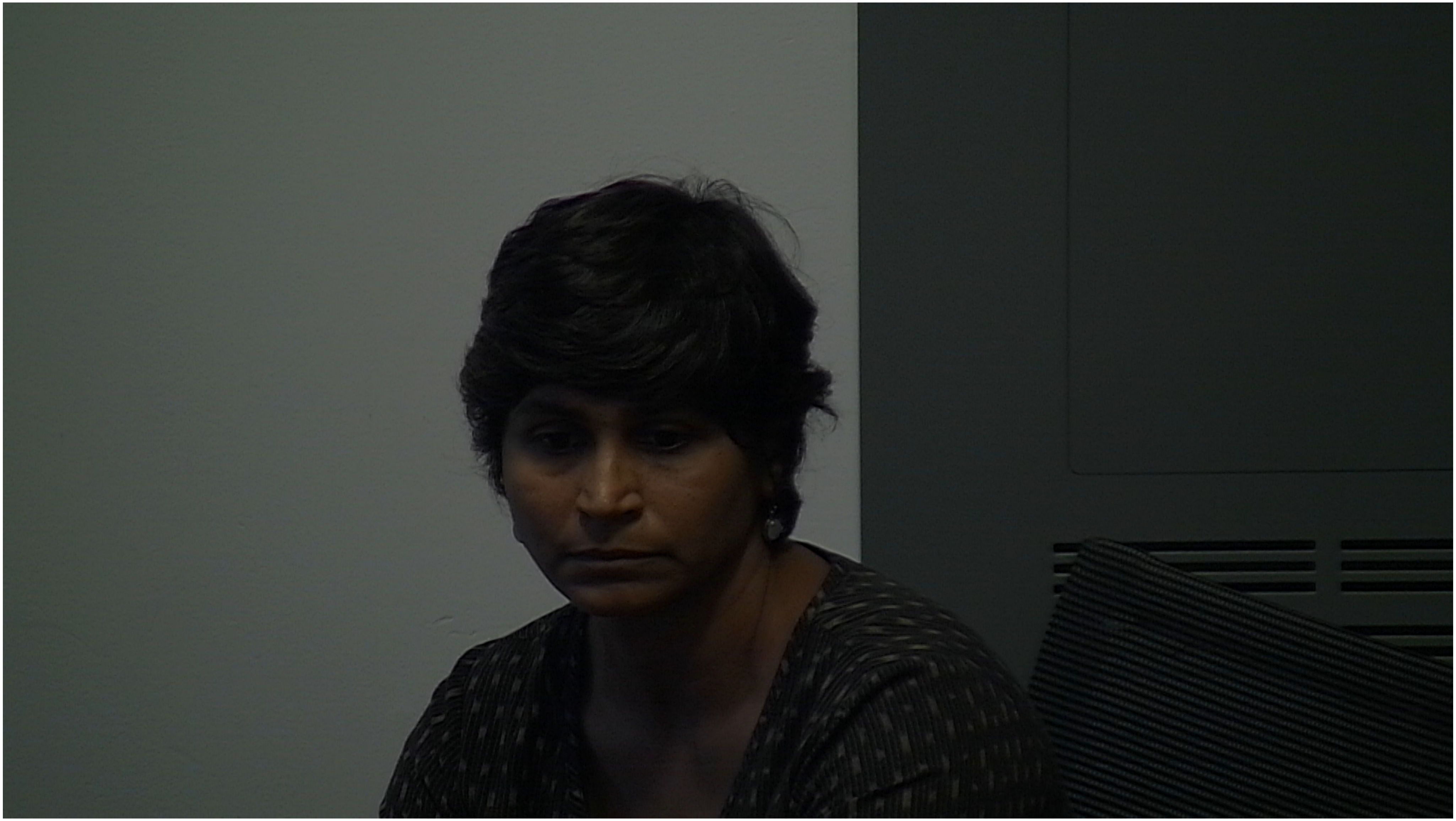


Outline

- A brief review of Causal Sets and what makes them "different"

  Lorentz Inv +Discreteness ⇒ Non-Locality

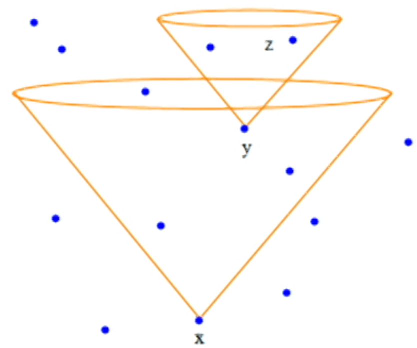
- CST Kinematics and Observables
- CST Dynamics
  - Analytic Continuation
  - Example of 2dCST:
    Phase Transition, Hartle-Hawking Wave Function and Finite Size Scaling
- Summary and Future Directions

S. Surya, Class.Quant.Grav., 2012 [1],
Glaser and Surya, CQG, 2016 [2]
Henson, Rideout, Sorkin and Surya, Experimental Mathematics, 2016 [3]
Glaser, O'Connor and Surya, arXiv:1706.06432[4]

The Causal Set Hypothesis

– L.Bombelli, J.Lee, D. Meyer and R. Sorkin, PRL 1987

- The Causal Structure Poset $(M, \prec) \subset (M, g)$
  - M: the **set** of events.
  - $\prec$:
    - Acyclic: $x \prec y$ and $y \prec x \Rightarrow x = y$
    - Transitive: $x \prec y$ and $y \prec z \Rightarrow x \prec z$



Only signature $(-, +, +, +)$ has a poset/causal structure

The Causal Set Hypothesis

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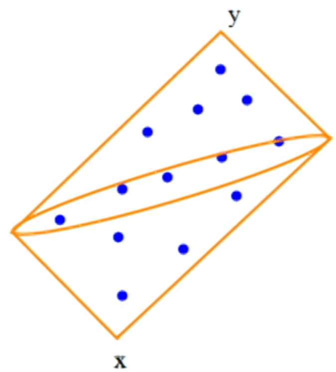


Finite number of "atoms" of spacetime $\sim V/V_p$

The Causal Set Hypothesis – L.Bombelli, J.Lee, D. Meyer and R. Sorkin, PRL 1987

- The Causal Structure Poset $(M, \prec) \subset (M, g)$
- Spacetime Discreteness

The underlying structure of spacetime is a *causal set* or locally finite poset $(C, \prec)$

Motivation

Causal Structure + Volume Element = Spacetime

–Hawking-King-McCarthy-Malament Theorem

S. W. Hawking, A.R. King, P.J. McCarthy, J. Math. Phys. (1976);
D. Malament, J. Math. Phys. (1977)
O. Parrikar, S. Surya, CQG (2011).

Causal Structure $\rightarrow$ Partially Ordered Set

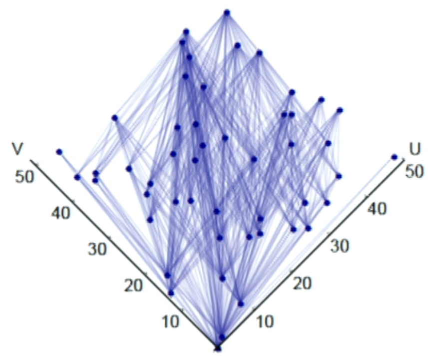
Spacetime Volume $\rightarrow$ Number

Order + Number $\sim$ Spacetime geometry

The Continuum Approximation

- Spacetime emerges as a "random lattice" generated via a Poisson process:

$$P_V(n) \equiv \frac{1}{n!} \exp^{-\rho V} (\rho V)^n$$



- Regular Lattice (eg. a diamond lattice): $n \not\sim V/V_p$ under boosts.
- In a Poisson sprinkling: $< n >= V/V_p$

The Continuum Approximation

- Spacetime emerges as a "random lattice" generated via a Poisson process:

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- Local Lorentz invariance: there are no preferred directions

– L.Bombelli, J.Henson, R. Sorkin, Mod.Phys.Lett. 2009

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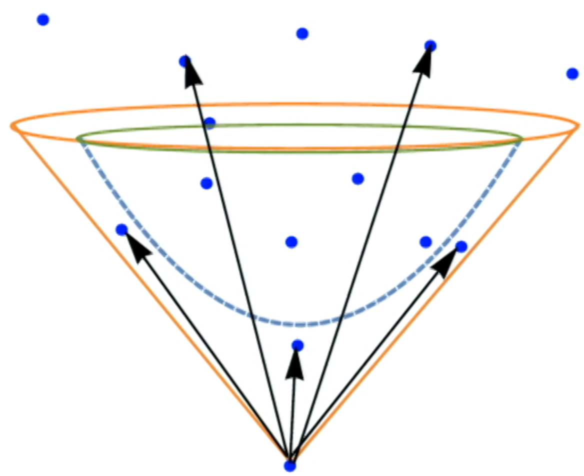
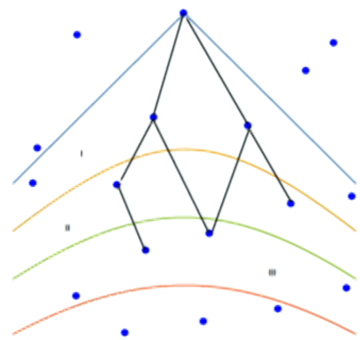
- Non-locality: A causal set need not be a fixed valency graph.



Non-Locality

- Lorentz invariant discretisation *does not* produce a triangulation.
- Not a fixed or finite valency graph
- Non-compactness of Lorentz group gives rise to a layered structure
  - Nearest neighbours/first layer: "links"
  - Next nearest neighbours/second layer: 3-chain intervals
  - Next to next neighbours/third layer: 4-chain intervals or diamonds.



Causal Sets on the Computer

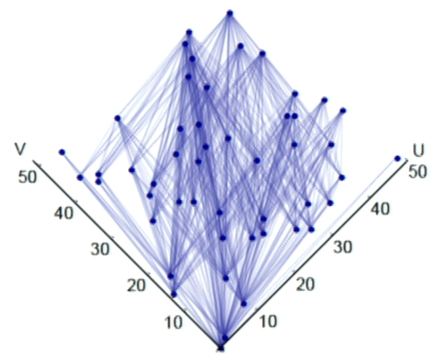
- Local graph of size N and valency k contains $k \times N$ bits of information.
- Nonlocal graph: N is *not* a measure of complexity.
  - Number of possible relations is $\binom{N}{2} \sim N^2/2$ bits $= \sim N^2/16$ bytes of RAM.
  - Not all relations are born equal
    - Links, N_0, 1-element intervals N_1, 2-element intervals N_2, etc.
    - Flat spacetime:

$$N_n \sim N^{2-\frac{2}{d}}, d > 2, \qquad N_n \sim N \ln N, d = 2 \tag{1}$$

    - Simultaneous imposition of these can lead to a lot of information!

CST Kinematics or How to Define Interesting Covariant Observables

When does a causal set look like a spacetime?



Quiz: Is this causal set approximated by a region of spacetime?

Hint: answer hidden in the talk

CST Kinematics or How to Define Interesting Covariant Observables

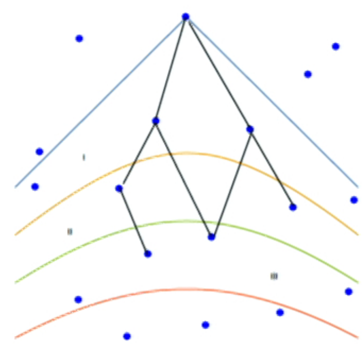
- Geometry from Order
  - Dimension Estimator: Myrheim-Myer Dimension – Myrheim, Myer
  - Timelike Distance –Brightwell and Gregory
  - Spatial Homology –Major, Rideout and Surya
  - Spatial and Spacelike Distance –Rideout and Walden, Eichhorn, Mizera and Surya
  - Benincasa-Dowker Action –Benincasa and Dowker, Dowker and Glaser
  - Locality tests: –Glaser and Surya
- Analytic Results use large N limit
- Numerical Simulations suggest N is not that large.. –Cactus Causal Set Toolkit, David Rideout[5]

A Continuum Inspired Dynamics for Causal sets

Path Integral/Sum over Histories

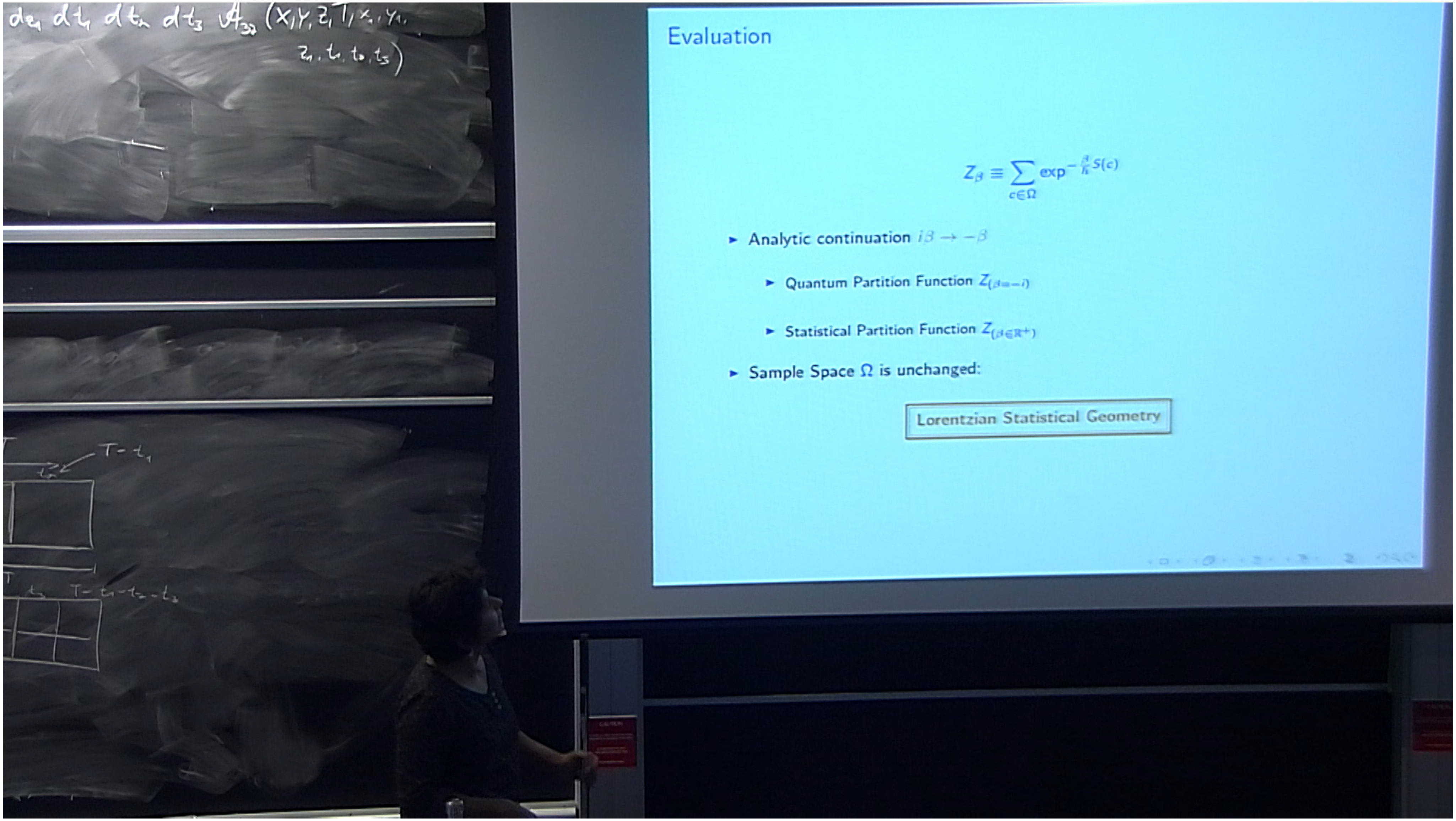
$$Z = \sum_{c \in \Omega} \exp^{\frac{i}{\hbar} S(c)}$$

- $S(c)$ is the discrete Einstein-Hilbert or *Benincasa-Dowker action*.
  - Weighted sum over number of neighbour pairs, next to neighbour pairs, etc.



–D. Benincasa and F. Dowker, Phys.Rev.Lett., 2010

- Ω is a sample space of causal sets


Evaluation
$Z_\beta \equiv \sum_{c\in\Omega} \exp^{-\frac{\beta}{\hbar}S(c)}$
Analytic continuation $i\beta \to -\beta$
Quantum Partition Function $Z_{(\beta=-i)}$
Statistical Partition Function $Z_{(\beta\in\mathbb{R}^+)}$
Sample Space Ω is unchanged:
Lorentzian Statistical Geometry

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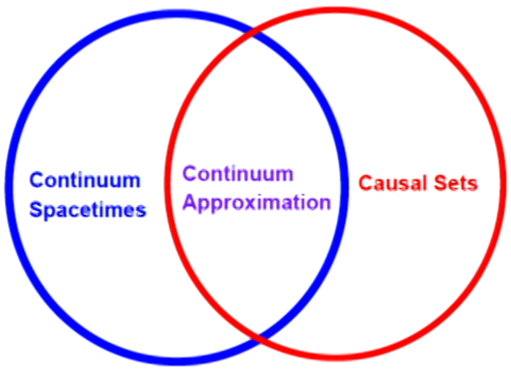
Lorentzian Statistical Geometry

The Sample Space Ω



- Continuum Approximation not Continuum Limit
- Examples:
  - Ω_N : Finite/Fixed element causal sets
  - Ω_{pf}: Countable, past finite causal sets
  - Ω_d: "Dimensionally" restricted causal sets

2d CST: A Dimensional Restriction

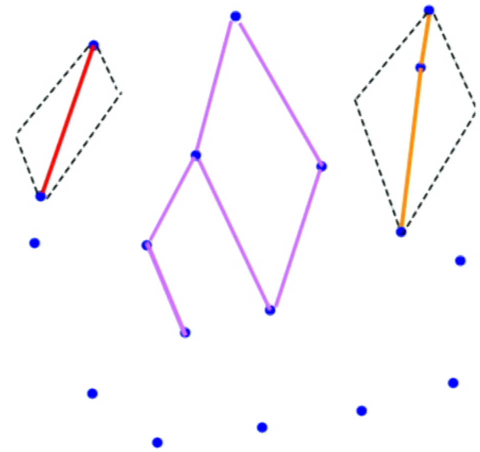
$$Z_\beta \equiv \sum_{\Omega_{2d}} \exp^{-\beta S_{2d}(C)}$$

- CST Einstein-Hilbert or Benincasa-Dowker Action in 2d: $S_{2\mathrm{d}}(c)$
- $\Omega_{2\mathrm{d}}(N) \subset \Omega(N)$: Sample space of N-element 2d-orders.

The CST Einstein-Hilbert or Benincasa-Dowker Action

$$\frac{1}{\hbar}S(c) = 4\epsilon\left(N - 2\epsilon\sum_{n=0}^{N-2} N_n f(n,\epsilon)\right)$$

- N_n: # of n-element order intervals



- Mesoscale $l_k >> l_p$, $\epsilon = \left(\frac{l_p}{l_k}\right)^2 \in (0,1]$ (Smears the layers)
- $f(n,\epsilon) = (1-\epsilon)^n - 2\epsilon n(1-\epsilon)^{n-1} + \frac{1}{2}\epsilon^2 n(n-1)(1-\epsilon)^{n-2}$

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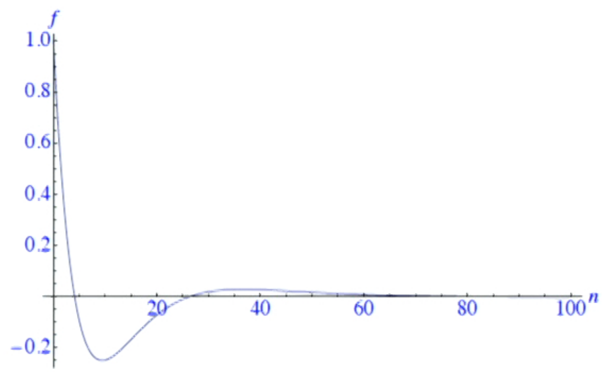
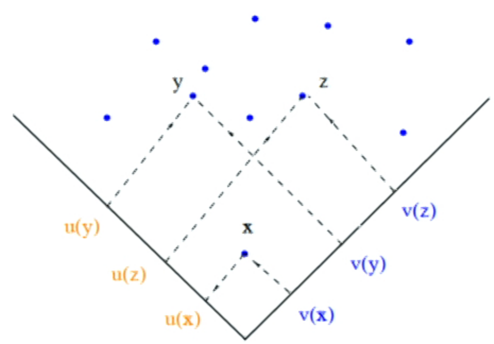


Ω_{2d}: the sample space of 2d Orders

- A 2d order is an *intersection of two linear orders*
  - Base Set: $S = (1, \dots, N)$.
  - $u_i, v_i \in S = (1, \dots, N)$, $i = 1, \dots, N$.
  - $U = (u_1, u_2, \dots u_i, \dots u_j, \dots u_N)$, $V = (v_1, v_2, \dots v_i, \dots v_j, \dots v_N)$ are *total orders or chains*.
  - 2d order $C = U \cap V$: $e_i = (u_i, v_i) \prec e_j = (u_j, v_j)$ iff $u_i < u_j$ and $v_i < v_j$.



$$x \prec y \quad \Leftrightarrow \quad u(x) < u(y) \quad \& \quad v(x) < v(y)$$

- Order theoretic dimension coincides with the spacetime dimension

Ω_{2d}: the sample space of 2d Orders

- A 2d order is an *intersection of two linear orders*
- Examples:
  - Discretisation of topologically trivial 2d spacetimes.
  - Sprinklings into finite regions of $^2\mathbb{M}$ are 2d *random orders*
    - U and V chosen randomly and independently from S.

–Peter Winkler, Order, 1985
El-Zahar and N.W. Sauer, Order, 1988
–G. Brightwell, J. Henson, S.Surya, Class.Quant.Grav. 25, 2008

  - 2d orders which are not manifold-like
- Order theoretic dimension coincides with the spacetime dimension

Markov Chain Monte Carlo

The Move:

- $U = (u_1, \ldots u_i, \ldots u_j, \ldots u_N)$, $V = (v_1, \ldots v_i, \ldots v_j, \ldots v_N)$
- Pick a pair (u_i, v_i) and (u_j, v_j) at random and exchange: $u_i \leftrightarrow u_j$
- $U' = (u_1, u_2, \ldots u_j, \ldots u_i, \ldots u_N)$, $V' = (v_1, v_2, \ldots v_i, \ldots v_j, \ldots v_N)$
- Example:

  $u_2 \leftrightarrow u_3$: $U = (1, 2, 3, 4)$, $V = (1, 2, 3, 4) \longrightarrow U' = (1, 3, 2, 4)$, $V' = (1, 2, 3, 4)$

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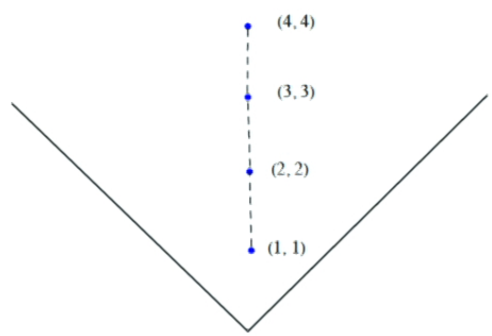


Homework

- Monte Carlo + Metropolis Hastings Algorithm on a 20 element 2d order.
  - Start with a chain and an antichain

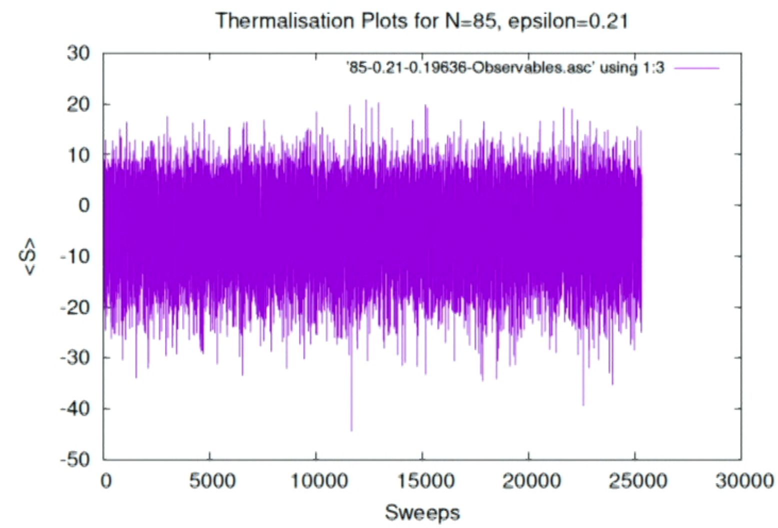
    Chain: $U = (1, 2, \ldots 20)$, $V = (1, 2, \ldots 20)$

    Antichain: $U = (1, 2, \ldots 20)$, $V = (20, 19, 18 \ldots 1)$
  - $\beta = 0$: Reproduce the random 2d order
  - $\beta > 0$ use the Benincasa Dowker action with $\epsilon = 0.21$.

Ergodicity, Thermalisation, Detailed Balance

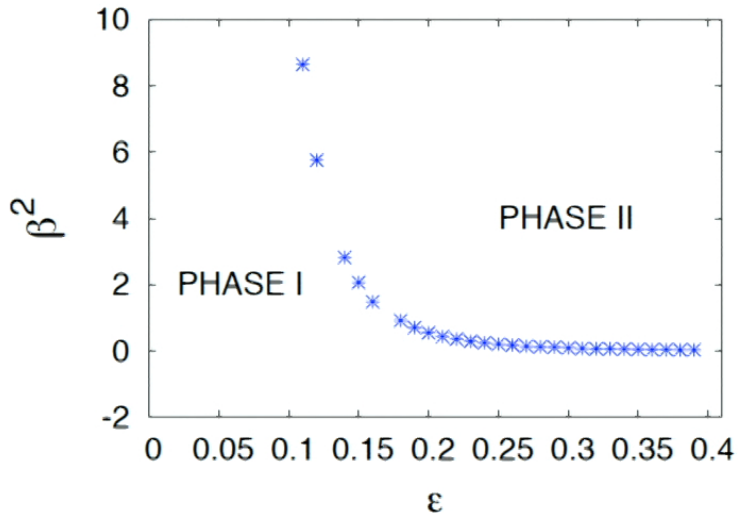
- Detailed Balance
- Metropolis Algorithm
- 7 different initial causal sets.
- Good thermalisation.

Example: $N = 85$, $\epsilon = 0.21$, $\beta = 0.19636$ and $\sim 25,000$ sweeps



Results in Non-perturbative 2d CST

- A Phase Transition from a Continuum Phase to a non-continuum phase



- Hartle-Hawking Wave Function

Results in Non-perturbative 2d CST

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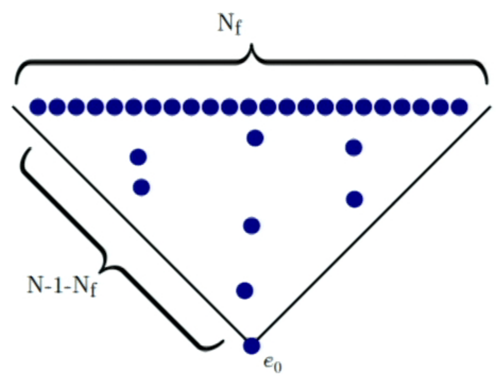
- Hartle-Hawking Wave Function

$$\Psi_0^{(N)}(\mathcal{N}_f, \beta) \equiv A \sum_{c \in \Omega_{2d}} \exp^{-\frac{1}{\hbar}\beta S_{2d}(c)},$$



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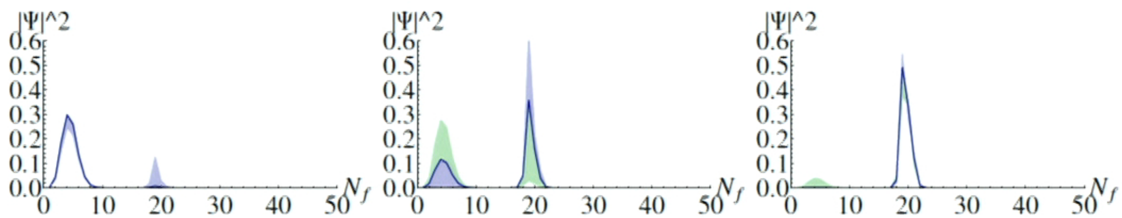

(a) $\beta = 0.10, \epsilon = 0.5$ (b) $\beta = 0.11, \epsilon = 0.5$ (c) $\beta = 0.12, \epsilon = 0.5$

Results in Non-perturbative 2d CST

- A Phase Transition from a Continuum Phase to a non-continuum phase
- Hartle-Hawking Wave Function

Results for "small" $N \sim 50$

Results in Non-perturbative 2d CST

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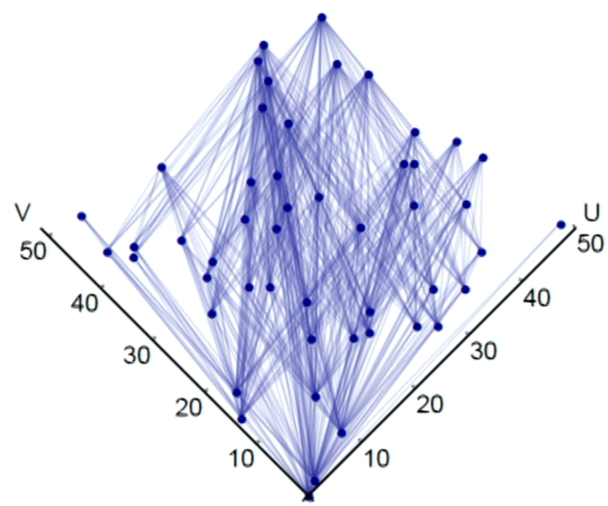

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Big Question

What puts practical limits on the size N of the causal set?

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Non-locality

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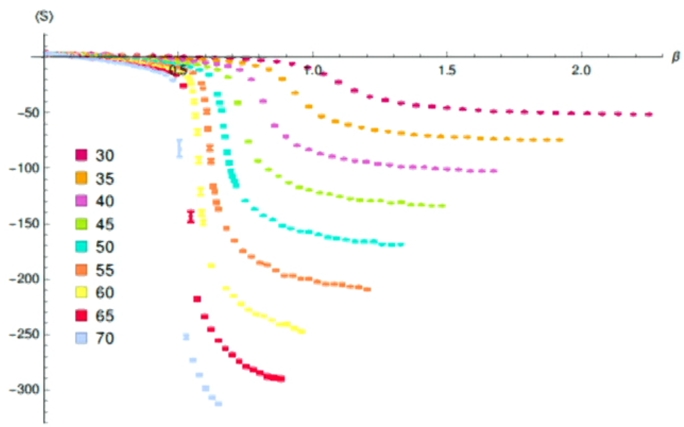
Non-locality

- Each sweep is $\binom{N}{2}$ MCMC steps $\Rightarrow$ longer thermalisation times
- Non-local action is computationally very expensive. –Out of Order Control!
- Metropolis works fine. But no analog of Cluster/Wolff algorithm.

Finite Size Scaling: Or how big should N be?

- Where is the asymptotic regime?
- Continuum approximation v/s Continuum limit
  - Every N is physically relevant!
  - Finite Size Scaling: for thermodynamic not continuum limit
- $S(c)$ is not extensive.
  - $S(C) \not\propto N$
  - Non-locality $\sim$ Long Range Order

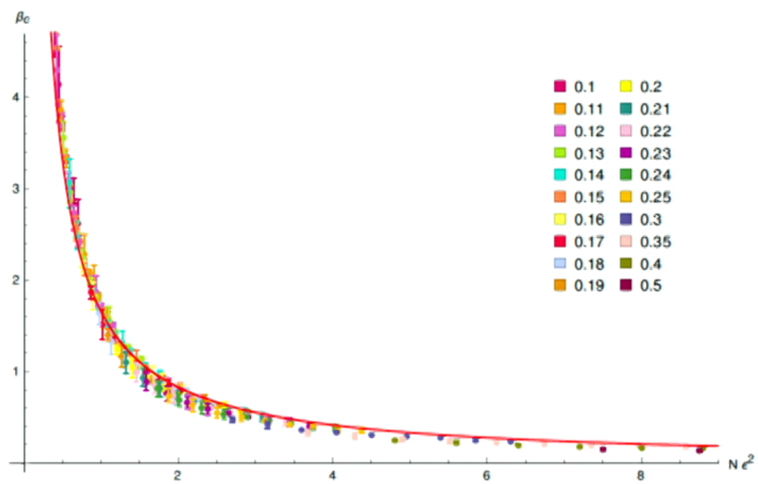
Varying N



Strong Indication of Scaling behaviour

Finite Size Scaling Analysis

- Collapses:
  - $\bar{\beta} = \beta N$



  - Phase I: $\langle S \rangle \propto N$
  - Phase II: $\langle S \rangle \propto N^2$
- First Order Phase Transition –Challa, Binder and Landau,1986
- Asymptotic regime for $N \gtrsim 65$

Finite Size Scaling Analysis

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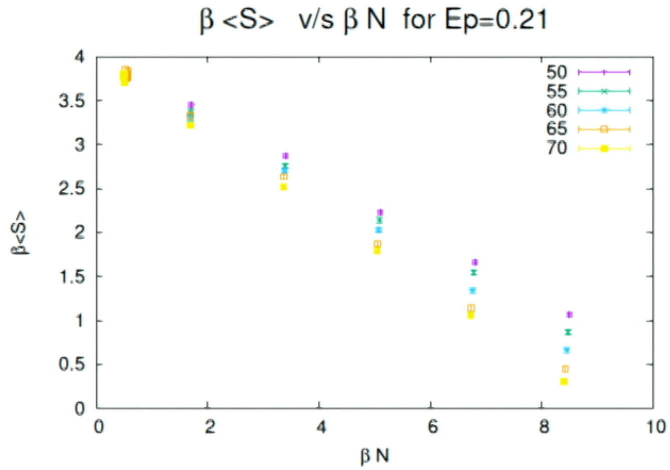
Dynamical Generation of a $\Lambda < 0$



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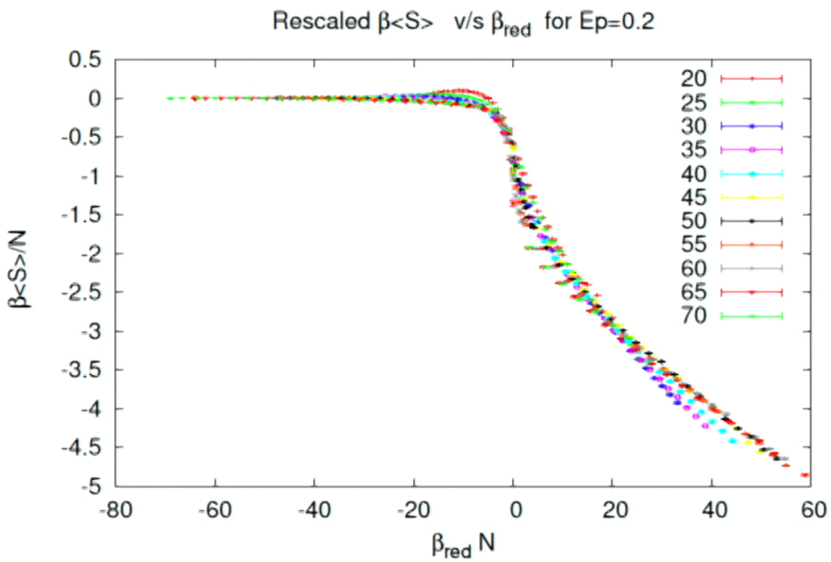

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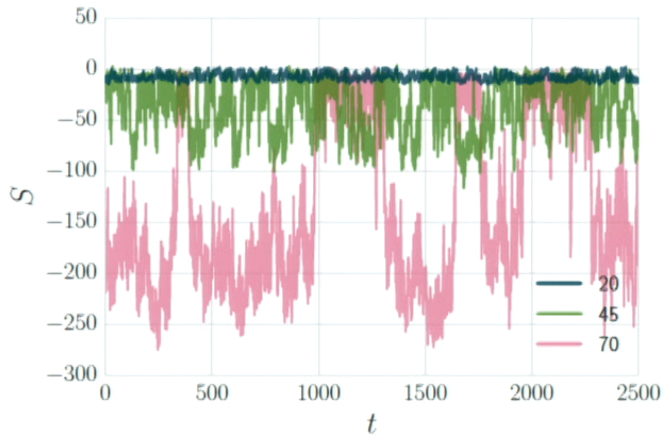
  - Coexistence of Phases



  - Double Gaussian
  - $C_{peak} \sim N^2$
- Asymptotic regime for $N \gtrsim 65$

Finite Size Scaling Analysis

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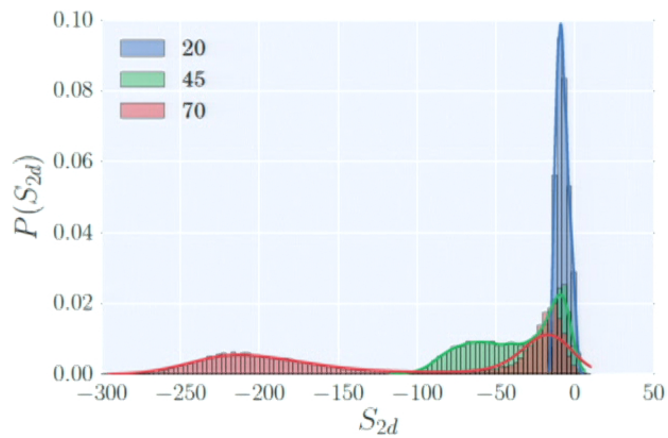

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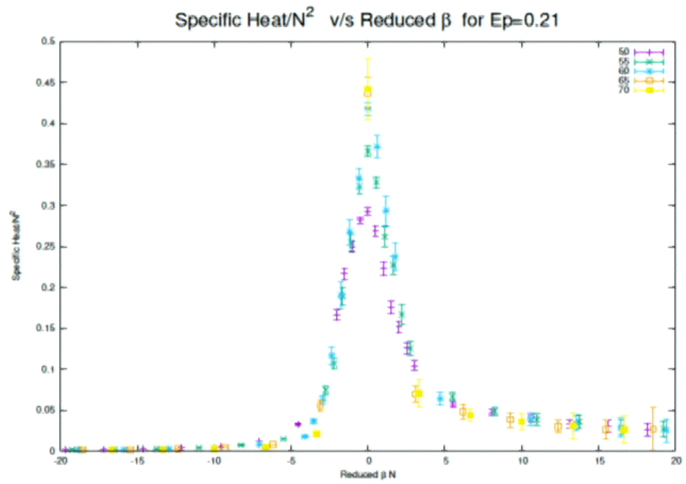

- Asymptotic regime for $N \gtrsim 65$

Summary and Looking Ahead

- Summary
  - A fully non-perturbative 2d quantum gravity ala CST.
  - Finite size scaling suggests that asymptotic regime in $N \gtrsim 65$
  - Observables unambiguously defined.
- Looking Ahead
  - Unrestricted Ω: reproduce entropy for $N \sim 90$ – Henson, Rideout, Sorkin and Surya, 2015[3]
  - Work with Directed Acyclic Graphs M –S.S., unpublished
  - Restrictions of Ω to $d = 3, 4$.
  - New non-local algorithms?
  - Testing different analytic continuations.
  - Explorations of the phase space for smaller ϵ
  - Inclusion of Matter
  - RG analysis possible if N could be made larger –Glaser and Surya, "in progress"
  - BOGEY MAN: Interpretation!

Homework

- Monte Carlo + Metropolis Hastings Algorithm on a 20 element 2d order.
  - Start with a chain and an antichain

    Chain: $U = (1, 2, \ldots 20)$, $V = (1, 2, \ldots 20)$

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References

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