Title: Coarse-graining of Spin Foams - 2

Date: Jun 21, 2017 12:00 PM

URL: http://pirsa.org/17060078

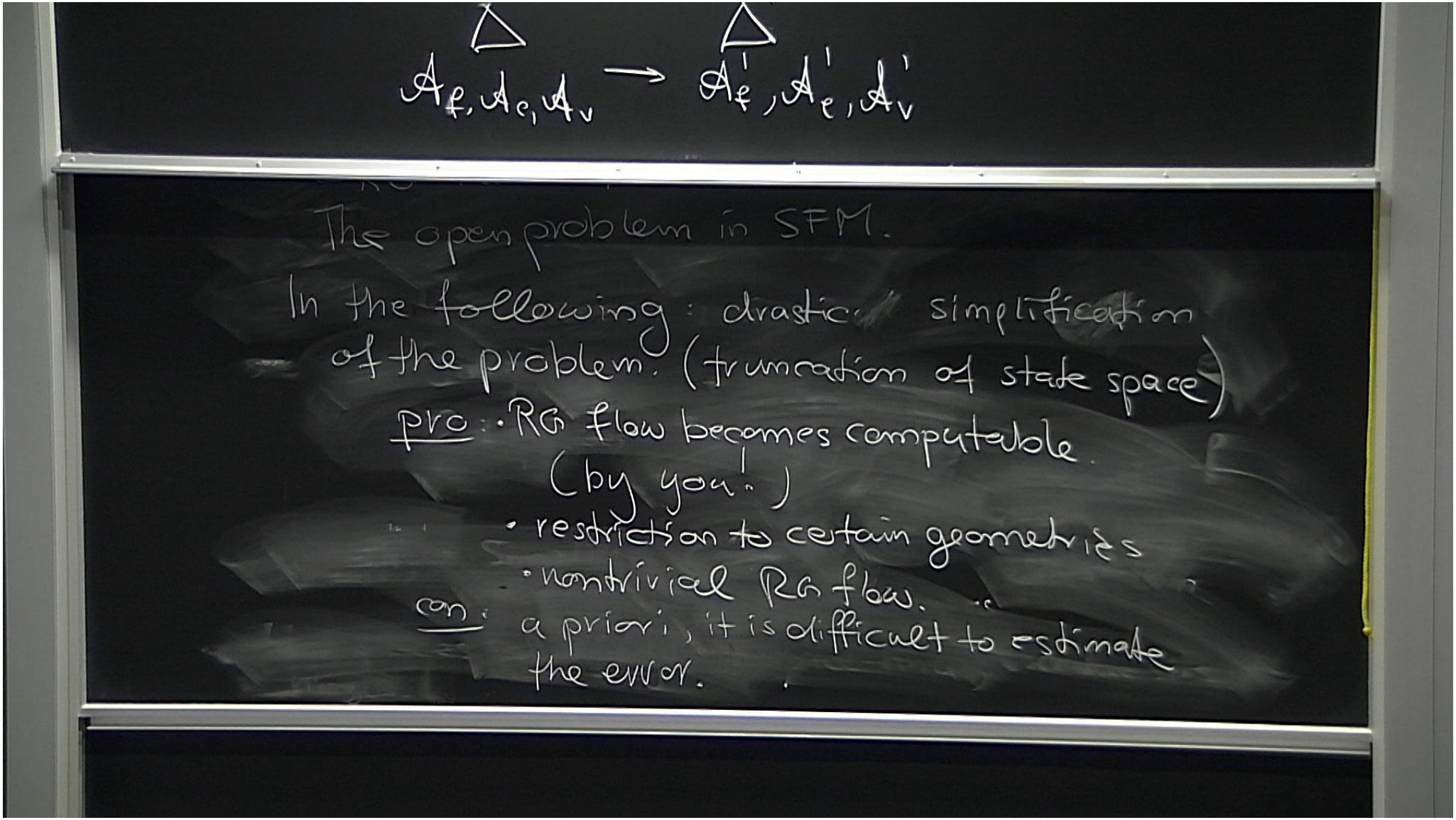
Abstract:

The open problem in SFM.

In the following: drastic simplification of the problem. (truncation of state space)

pro: · RG flow becomes computable. (by you!)
· restriction to certain geometries
· nontrivial RG flow.

con: a priori


$\mathcal{A}_f, \mathcal{A}_e, \mathcal{A}_v \to \mathcal{A}'_f, \mathcal{A}'_e, \mathcal{A}'_v$
The open problem in SFM.
In the following: drastic simplification of the problem! (truncation of state space)
pro: • RG flow becomes computable. (by you!)
• restriction to certain geometries
• nontrivial RG flow.
con: a priori, it is difficult to estimate the error.

fix 3d geometry completely, shape can vary.

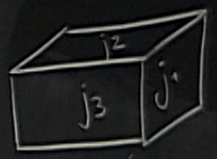
(i,f) -(i,f) (i,f)

Approximations a) Δ will be 4d hypercubic lattices

b) truncation of states: only consider subset of j_f, ι_e

ι_e = "coherent intertwiners"

special form of intertwiners (given by 3 areas) as (quantum) cuboids



c) Work in large-j-asymtatic regime

$j_f = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots = \lambda J_f \leftarrow J_f \in [0,\infty)$ (roughly)

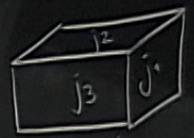
λ: large, fixed

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c) Work in large-j-asymtotic regime

$j_f = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots = \lambda J_f$ on $J_f \in [0, \infty)$ (roughly)

large, fixed

$\Rightarrow \sum_{j_f} \to \int dJ_f$

d) Take the large-j asymptotic expression of Riemannian signature EPRL-Fk model.

depends on coupling constants: α
$\hbar$ (Newton's const.)
γ (Barbero-Immirzi-parameter)
(Λ, cosm. const)

$$\mathcal{A}_f = j_f^{2\alpha}$$

$$\mathcal{A}_e = 1$$

$$\mathcal{A}_v = \frac{1}{D} e^{iS_R} + \frac{1}{D^*} e^{-iS_R} + \frac{1}{|D|}\left(e^{i\gamma S_R} + e^{-i\gamma S_R}\right)$$

D = determinant function of spins j_f

S_R = Regge-like action ($\sim 1/\chi$)

($\triangleq$ discretized Einstein-Hilbert action)

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$e \longmapsto 2e$ SU(2)-invariant tensors ~ 3d shape of 3d piece
("intertwiners")

When changing Δ (coarse graining) amplitudes have to change as well, to be consistent.
(i.e. $Z_\Delta, Z_{\Delta'}$ have to give the same answers to questions that can be asked on Δ')
$\Leftrightarrow$ give the same expectation values of observables on Δ' → see Bianca's talk
see Guifre's talk

$S_R \sim \sum a \cdot e$

D = determinant function of spins j_f

S_R = Regge-like action ($\sim 1/\hbar$)

($\triangleq$ discretized Einstein-Hilbert action)

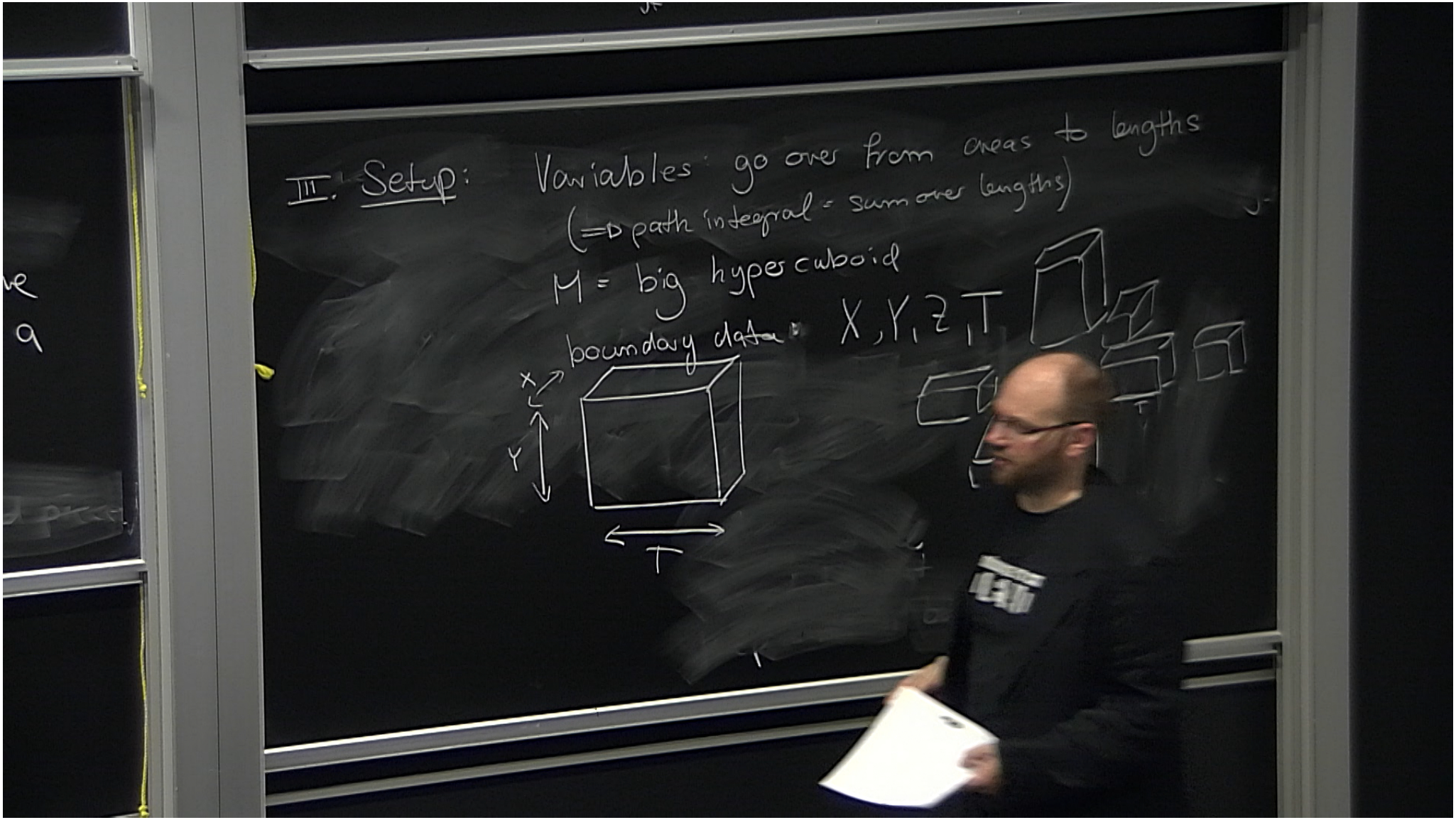
Since we work with quantum cuboids, $S_R = 0$

$\rightarrow$ only α remains as interesting coupling constant.

e) Not allow general flow, but restrict to amplitudes of the type d) (parameterized by α) $\Rightarrow$ Flow in α

$$\Delta \to \Delta' \to \Delta'' \to \dots$$
$$\alpha \to \alpha' \to \alpha'' \to \dots$$

Given Δ, Δ' and α, the renormalized value α' is the one for which $Z_\Delta, Z_{\Delta'}$ agree on a specific observable.

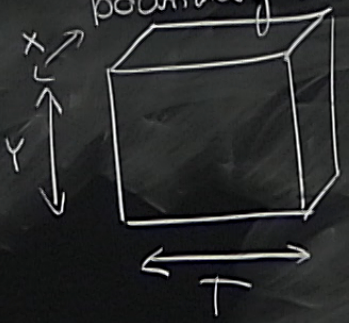

II. Setup: Variables: go over from areas to lengths
(=> path integral = sum over lengths)
M = big hypercuboid
boundary data: X, Y, Z, T
X
Y
T

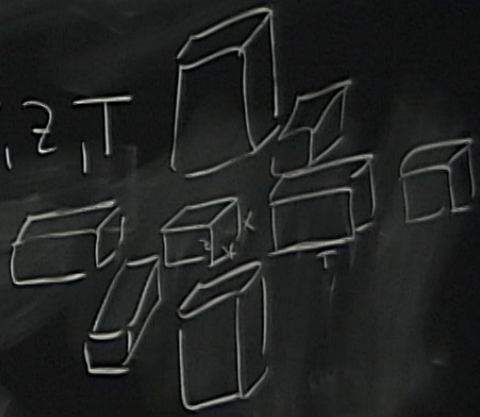
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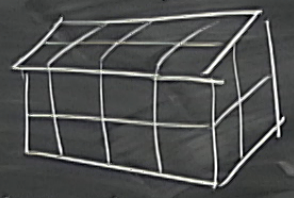
boundary data: X, Y, Z, T





Δ = lattice of $2 \times 2 \times 2 \times 4$ hypercuboids

Δ



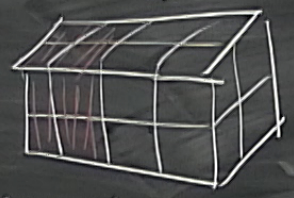
Δ' = lattice of $1 \times 1 \times 1 \times 2$ hypercuboids (coarse grained)



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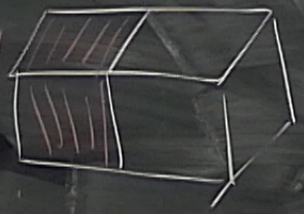
Δ

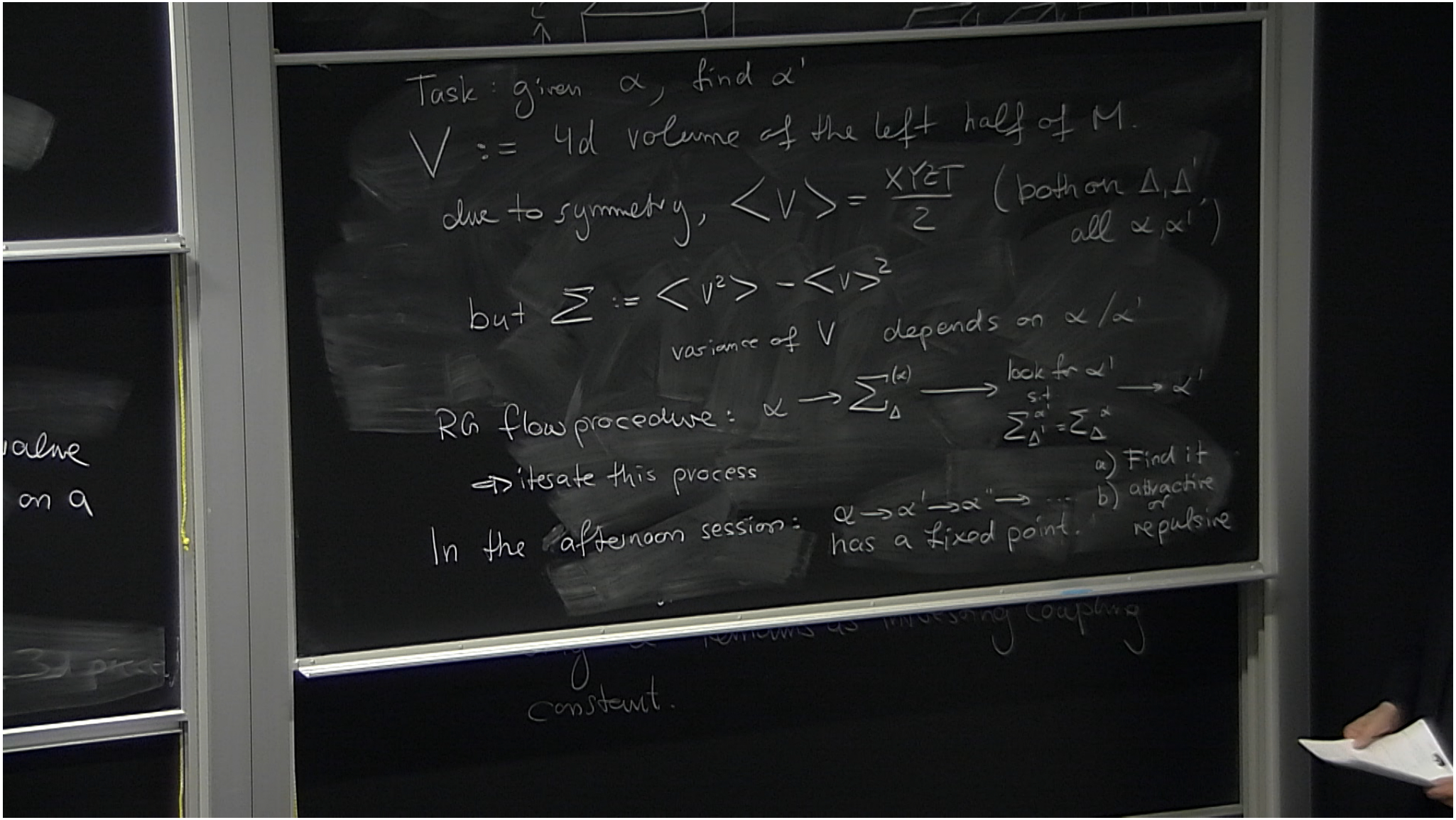
V



Δ' = lattice of $1 \times 1 \times 1 \times 2$ hypercuboids (coarse grained)

V




Task: given α, find α'
$V :=$ 4d volume of the left half of M.
due to symmetry, $\langle V \rangle = \frac{XYZT}{2}$ (path on Δ, Δ', all α, α')
but $\Sigma := \langle V^2 \rangle - \langle V \rangle^2$
variance of V depends on α / α'
RG flow procedure: $\alpha \to \Sigma_\Delta^{(\alpha)} \longrightarrow$ look for α' s.t. $\Sigma_{\Delta'}^{\alpha'} = \Sigma_\Delta^{\alpha} \longrightarrow \alpha'$
$\Rightarrow$ iterate this process
In the afternoon session: $\alpha \to \alpha' \to \alpha'' \to \cdots$ has a fixed point.
a) Find it
b) attractive or repulsive
constant.