Title: Introduction to Tensor Network methods - 4

Date: Jun 20, 2017 12:00 PM

URL: http://pirsa.org/17060071

Abstract:

LECTURES 3 and 4

Tensor Renormalization Group (TRG)

TRG as a computational scheme

- TRG Algorithm
- Non-critical systems
- critical systems
- D=3 dimensions
- Transfer matrix

TRG and the Renormalization Group flow

- The Renormalization Group
- TRG flow
- Tensor Network Renormalization (TNR)

The Renormalization Group

Hamiltonian H

coupling constants
$\vec{k} = (k_1, k_2, k_3, \cdots)$

Leo Kadanoff

Kenneth Wilson

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$$H \to H' \to H'' \to H''' \cdots$$

RG flow

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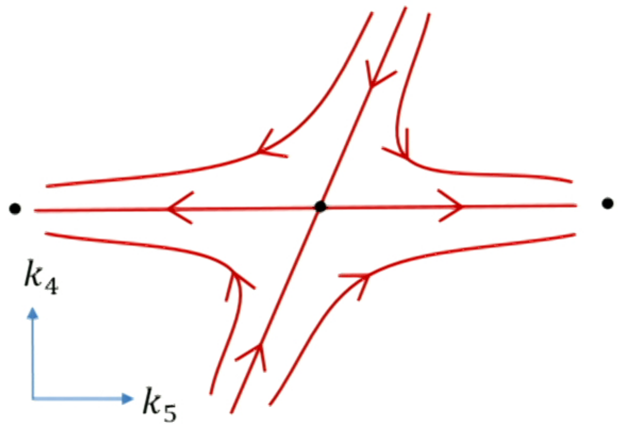
RG flow

RG flow in the space of Hamiltonians



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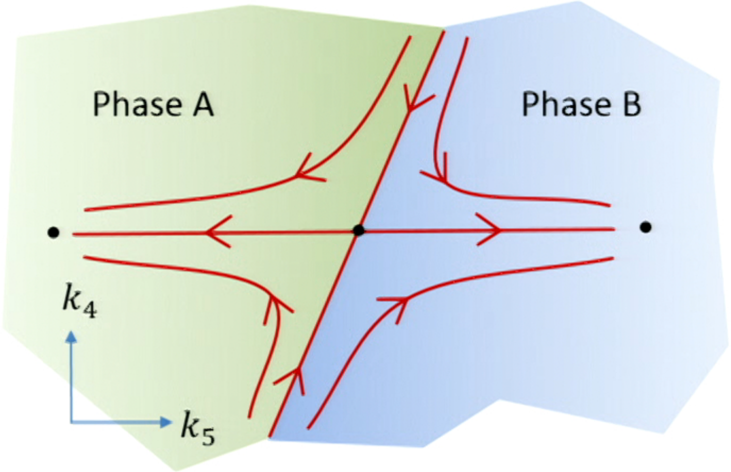

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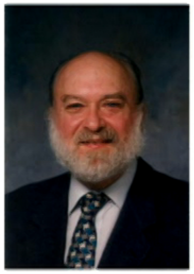

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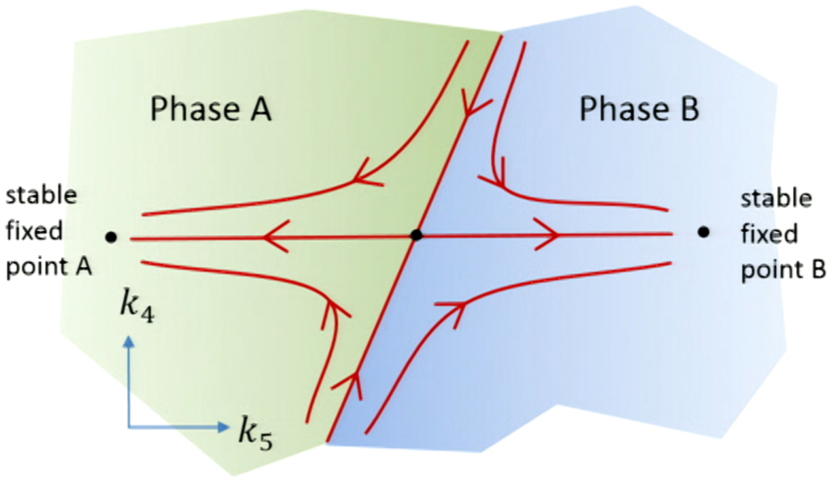

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(yes, but not with TRG...)

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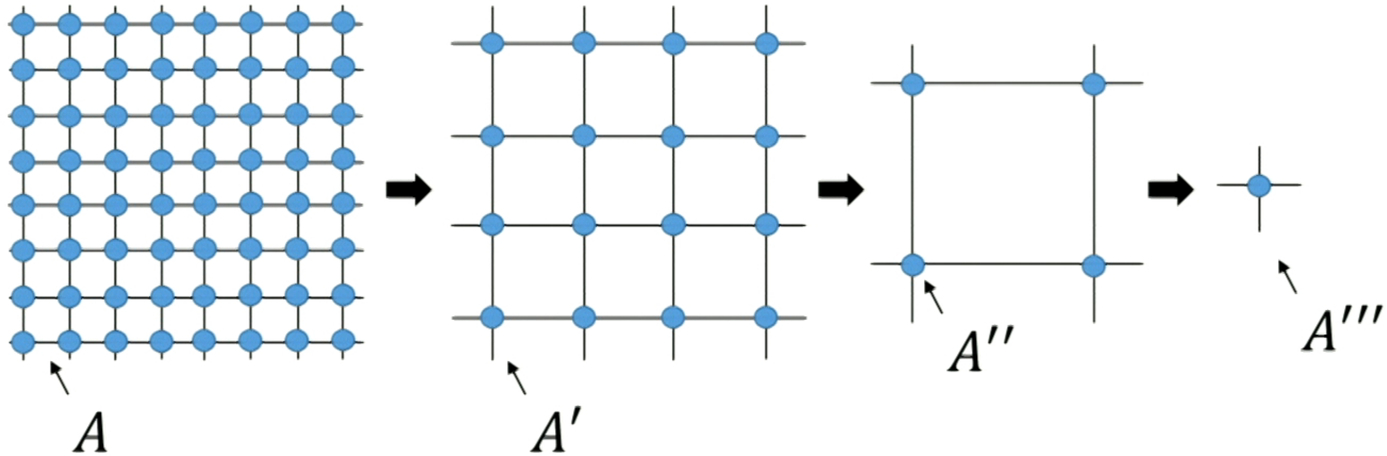

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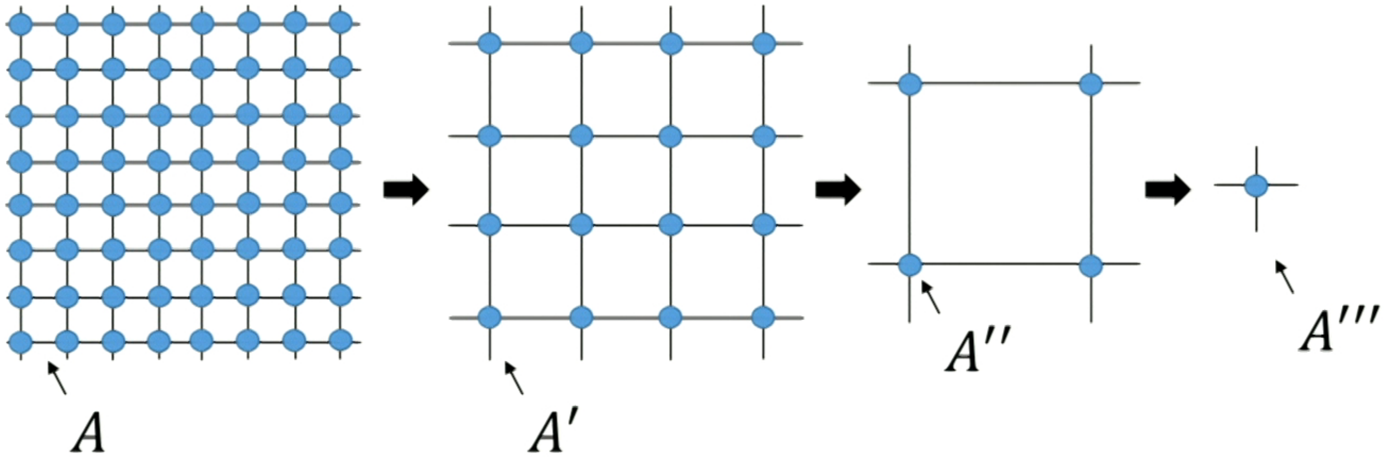

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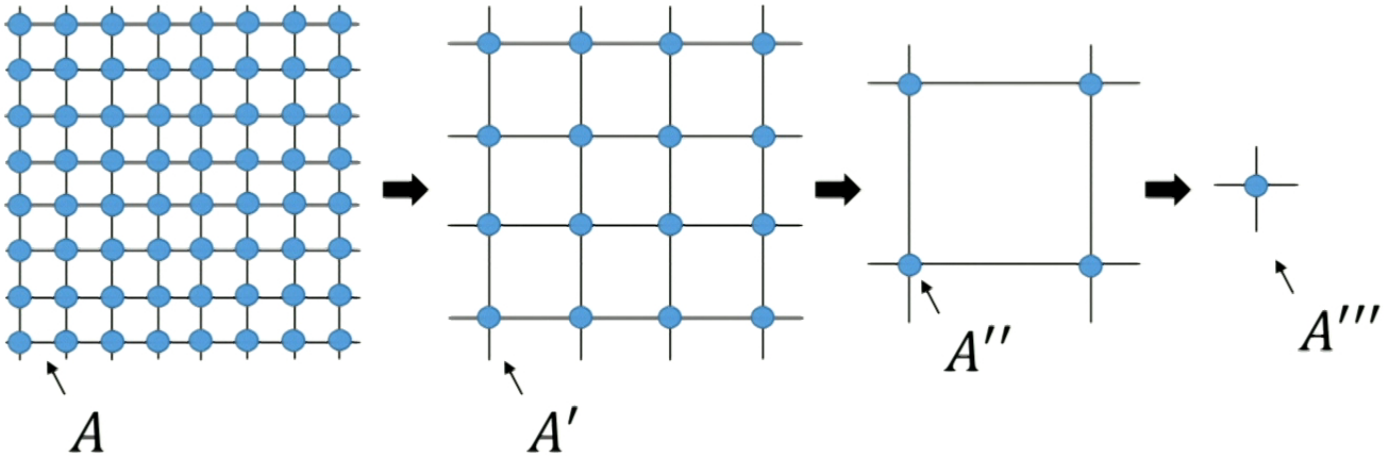

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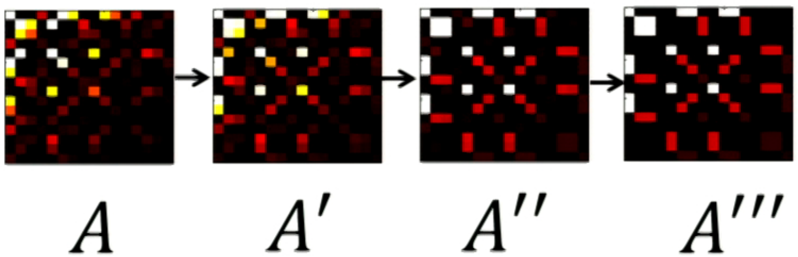
example: Ising model $\beta = 0.9\beta_c$



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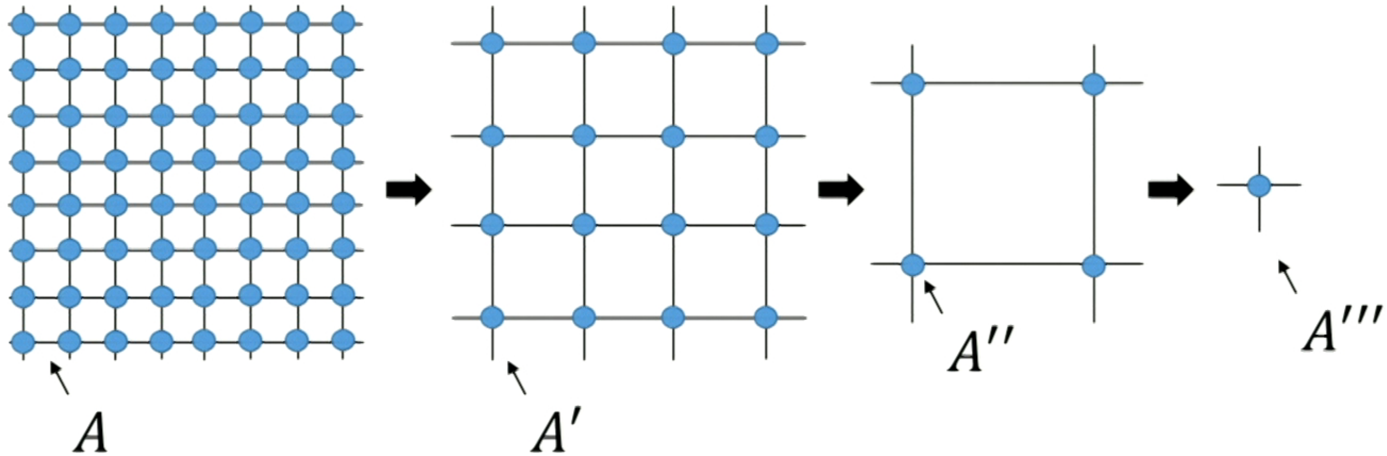

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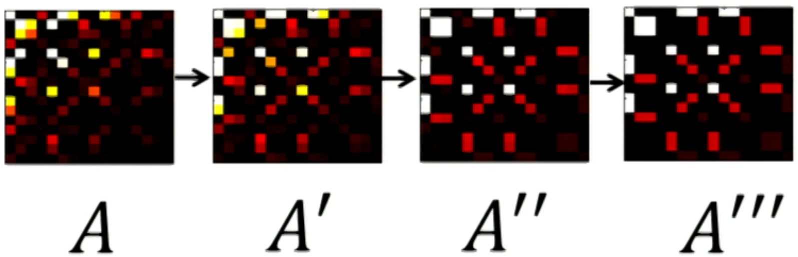
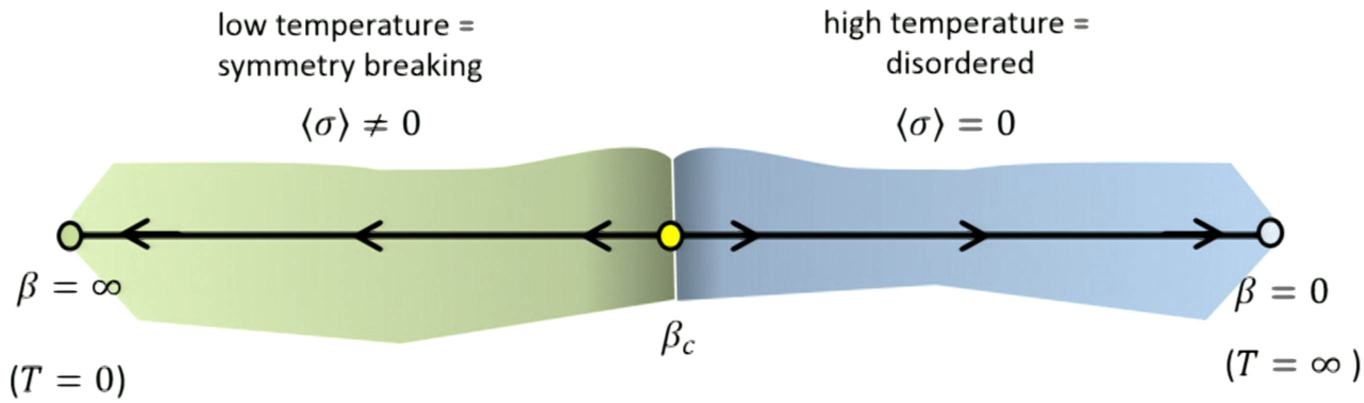


Does this flow have the expected structure of fixed points?

Expected RG flow for 2d classical Ising model



$\beta_c \approx 0.4406$

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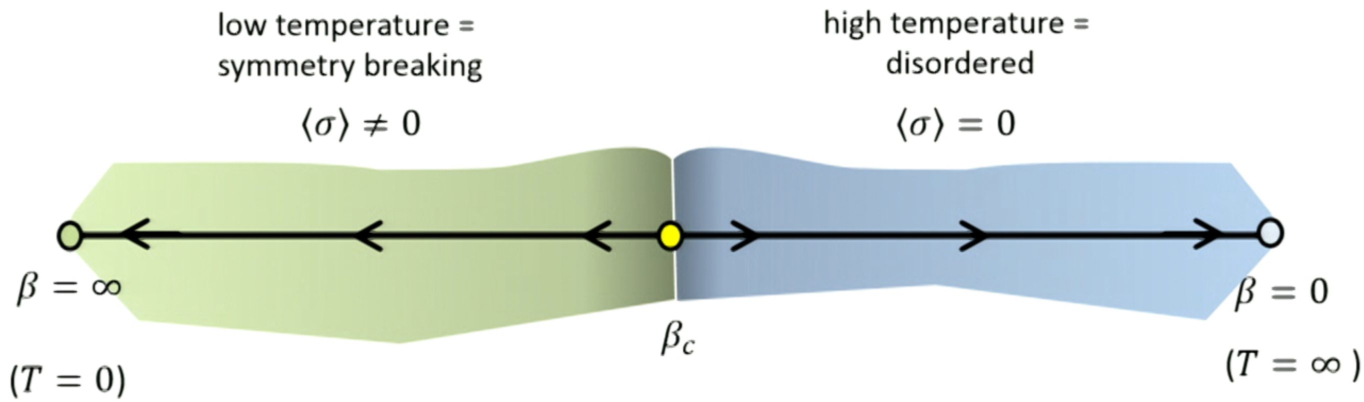

$\beta_c \approx 0.4406$

we expect:

- two stable RG fixed points
  - symmetry breaking $(\beta > \beta_c)$
  - disordered $(\beta < \beta_c)$
- one unstable (critical) RG fixed-point $(\beta = \beta_c)$

However, this is not what TRG produces

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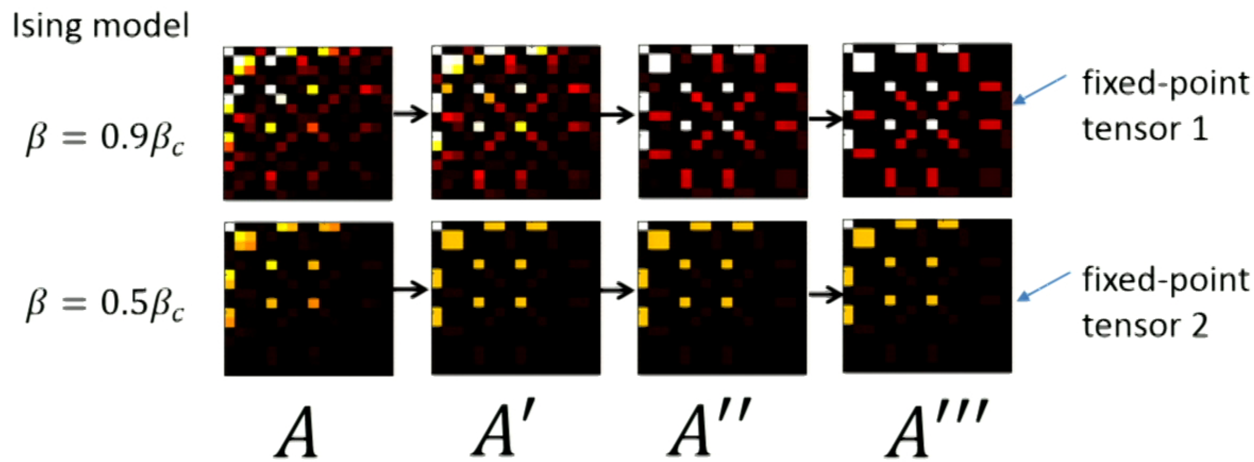

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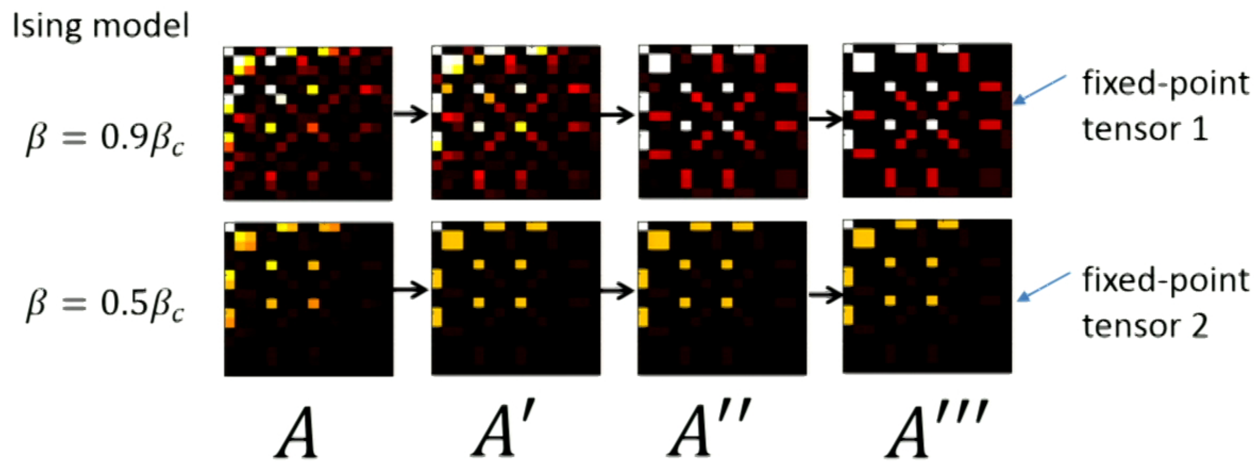

for each $\beta < \beta_c$ we flow to a different fixed-point tensor

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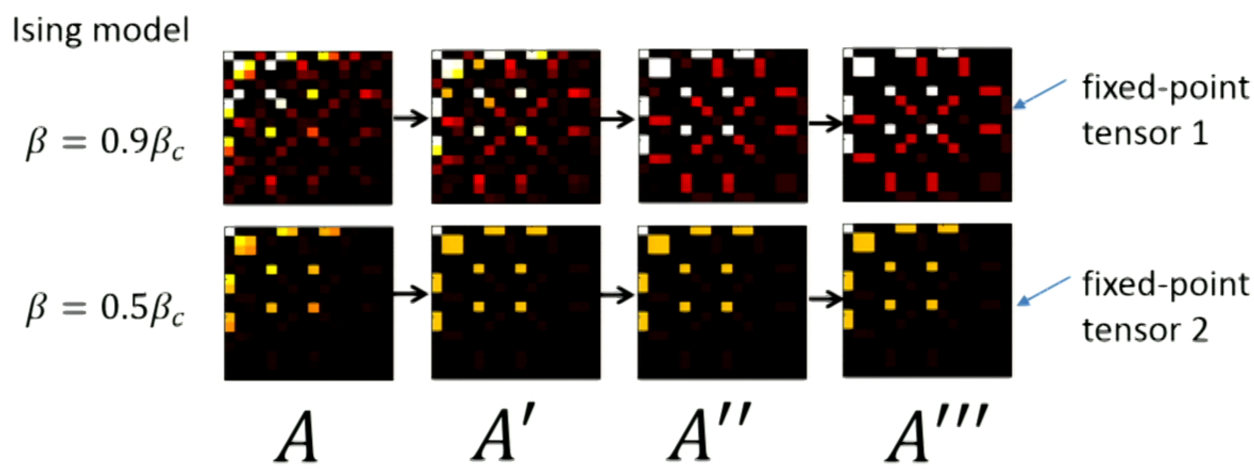

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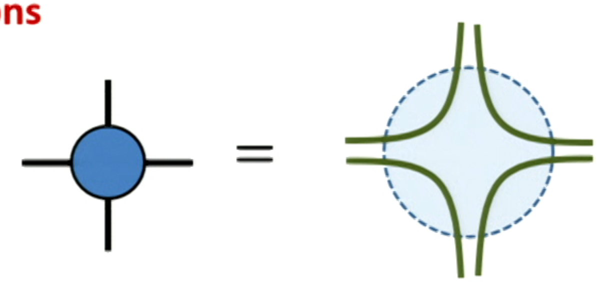
This is ***not*** a proper RG flow

1) What did go wrong?
2) Can we fix it?

TRG cannot remove some short-ranged correlations

Let us apply TRG to CDL
(corner double line) tensors:



CDL

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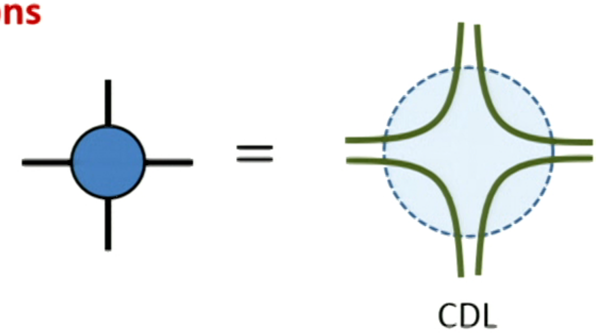

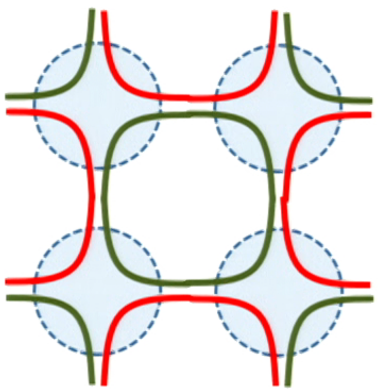

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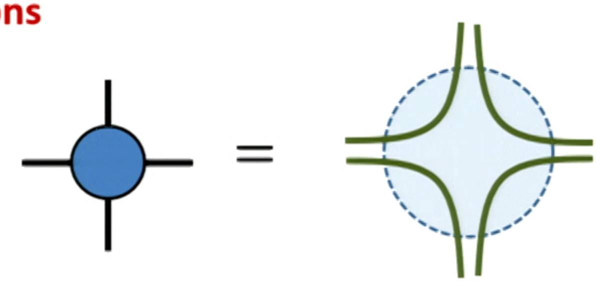

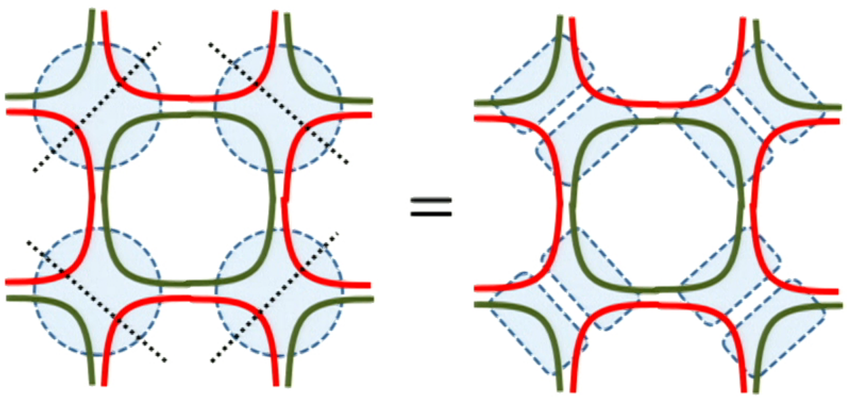

step 1:
SPLIT tensors
(SVD)

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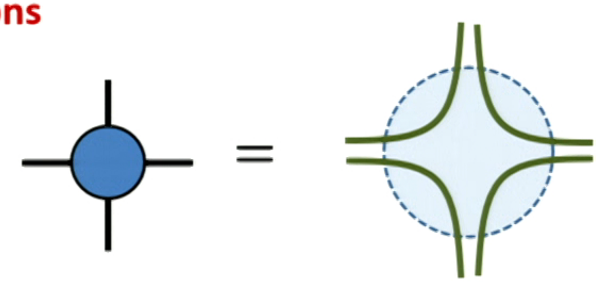

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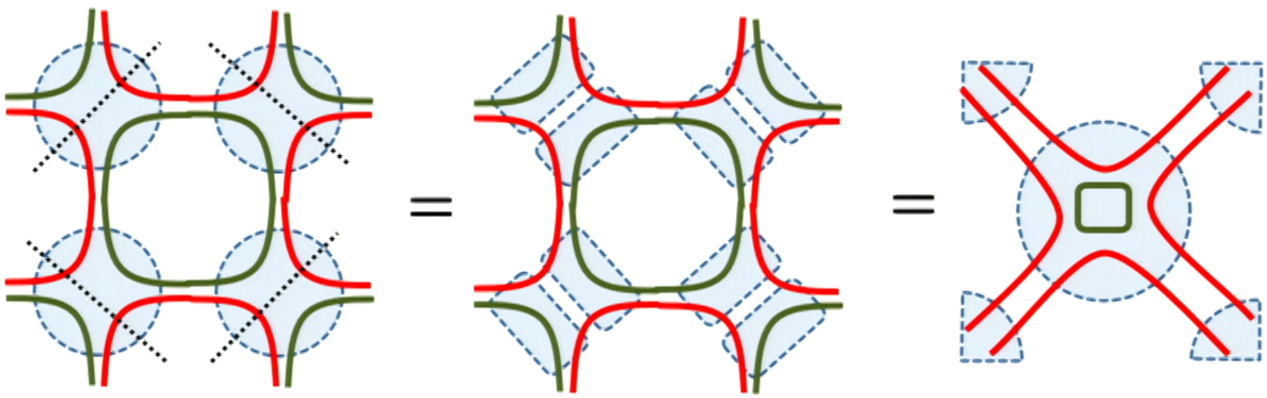

step 1: SPLIT tensors (SVD)

step 2: JOIN tensors (contraction)

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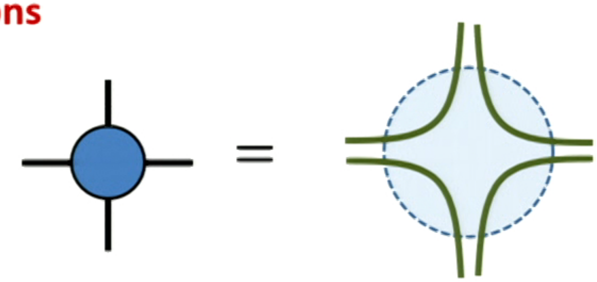

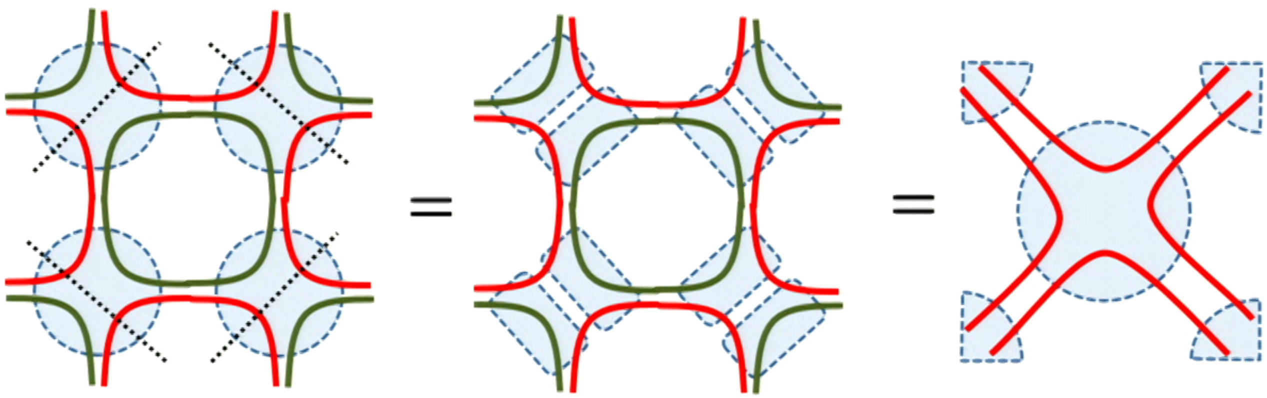

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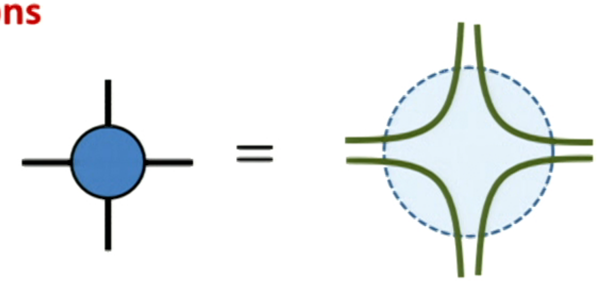

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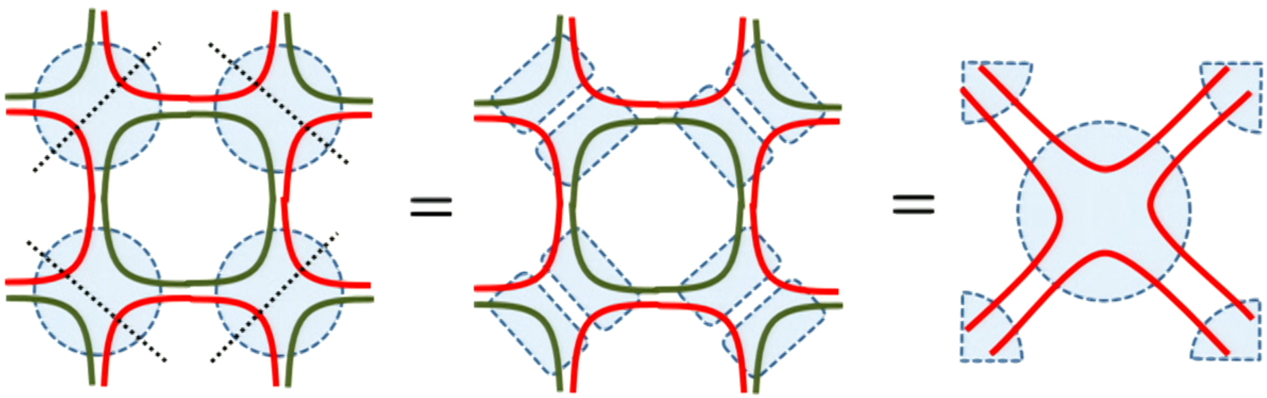

step 1:
SPLIT tensors
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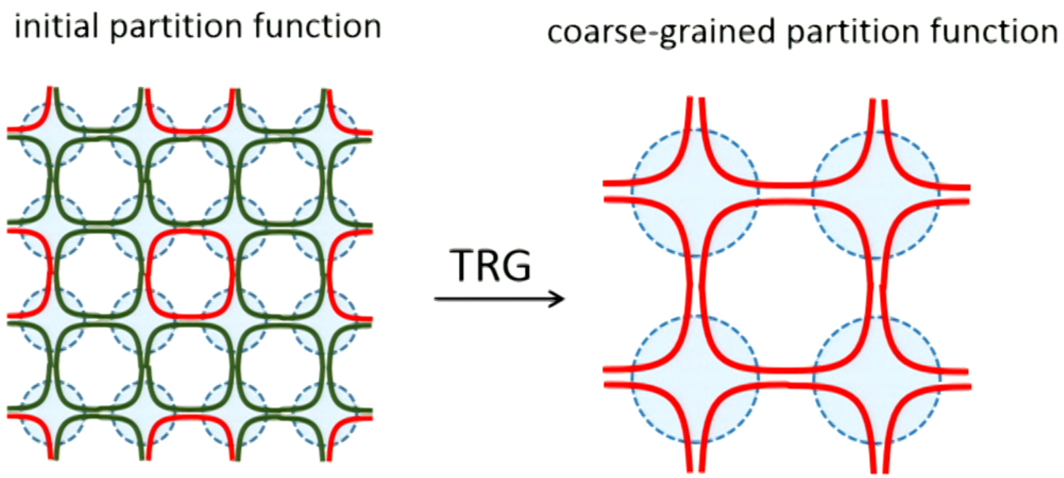
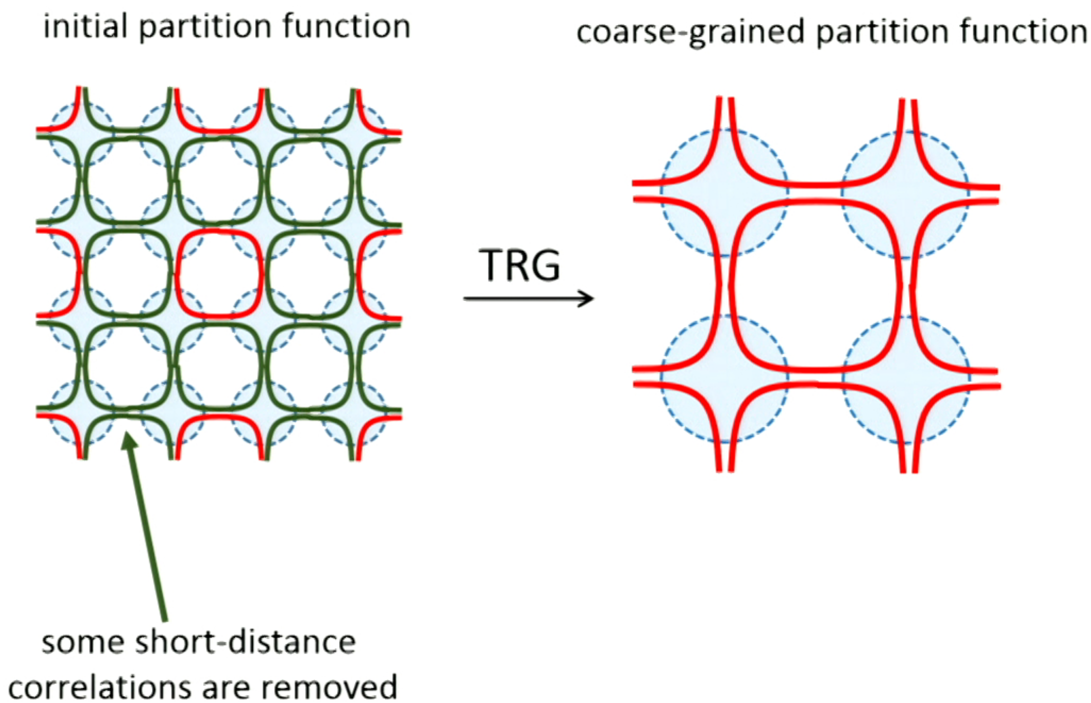
CDL tensors are a fixed-point of TRG !?!?!

(even though they have
short-ranged correlations)


initial partition function
coarse-grained partition function
TRG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some short-distance
correlations are removed

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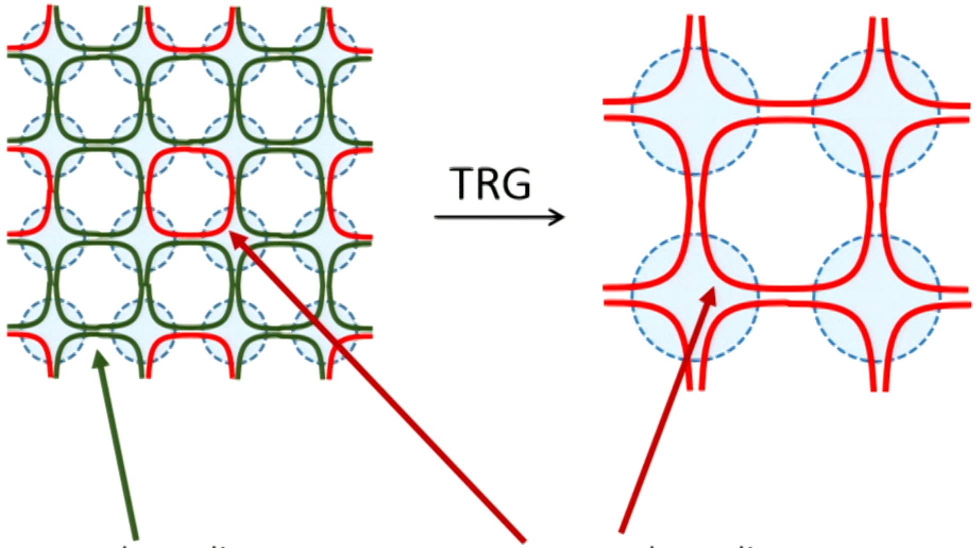

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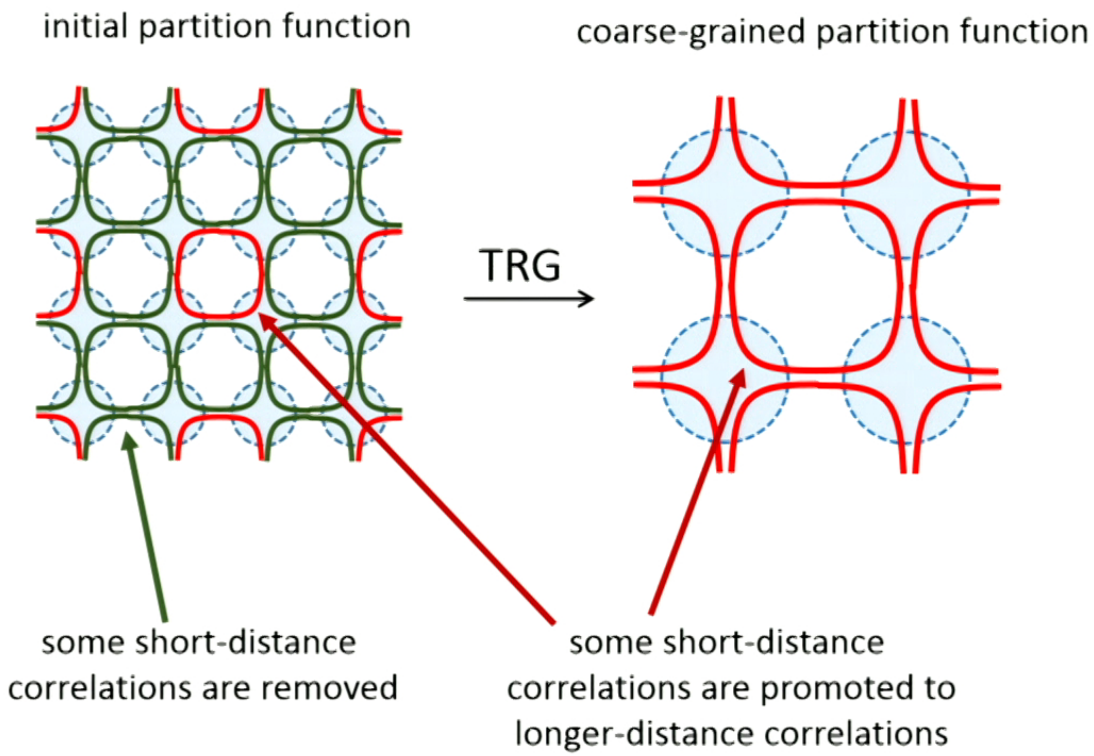
some short-distance correlations are promoted to longer-distance correlations



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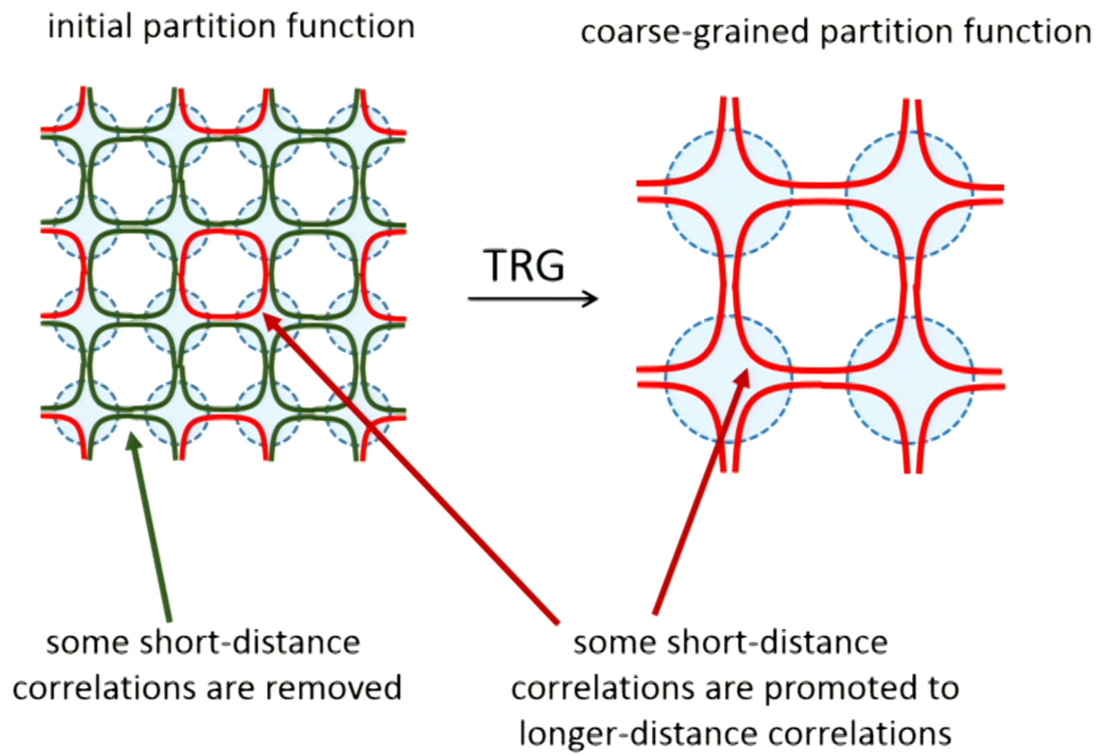
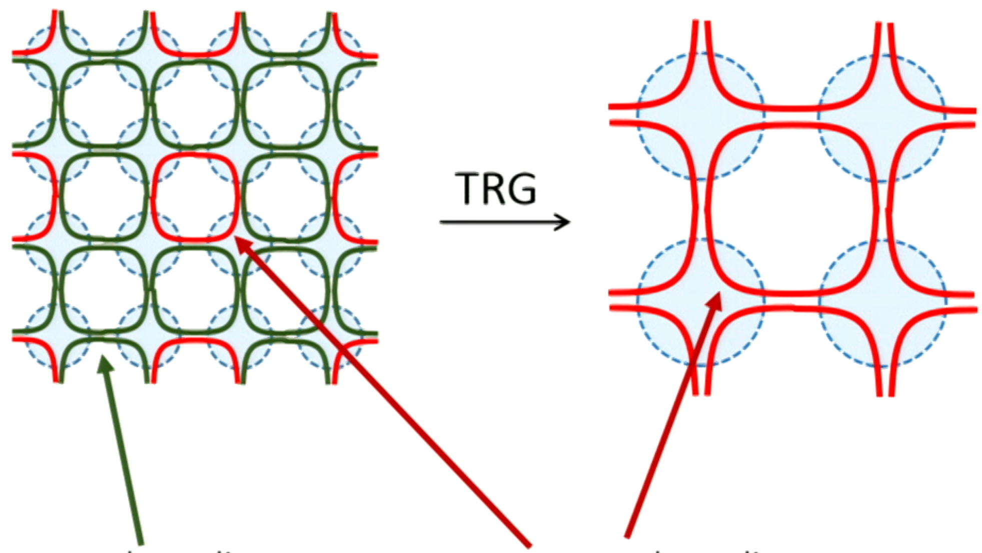


1) What did go wrong? using just SVDs is not enough to remove all short-distance correlations

2) Can we fix it?

initial partition function

coarse-grained partition function



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1) What did go wrong? using just SVDs is not enough to remove all short-distance correlations

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Tensor Network Renormalization (TNR)

(Glen Evenbly, GV, 2015)

Introduce *disentanglers* to remove all short-range correlations.

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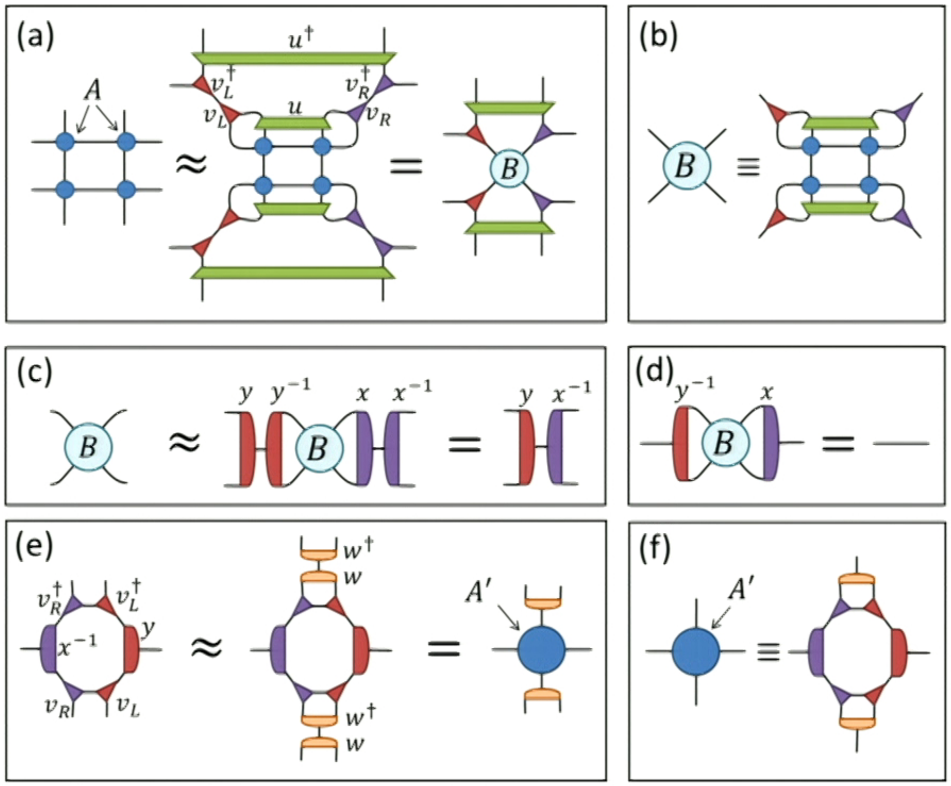
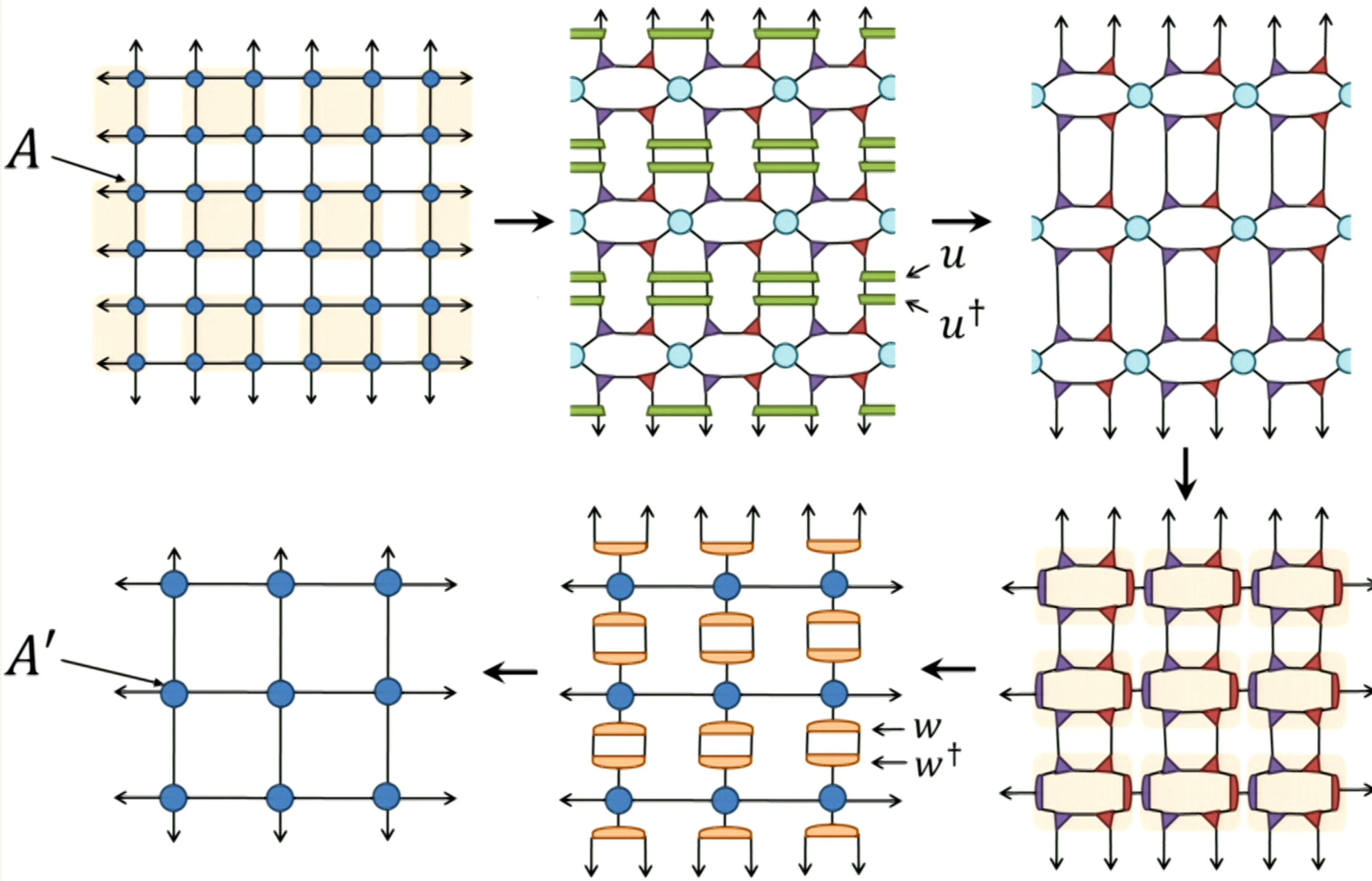
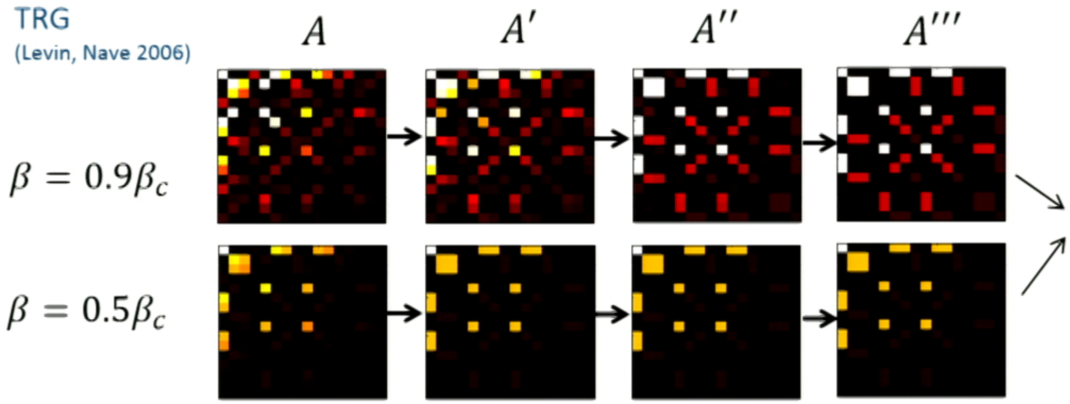



A
u
u†
A′
w
w†

Proper RG flow: 2D classical Ising

e.g. disordered phase $\beta < \beta_c$

TRG
(Levin, Nave 2006)



should converge to the same (trivial) fixed point, but they don't!

Proper RG flow: 2D classical Ising

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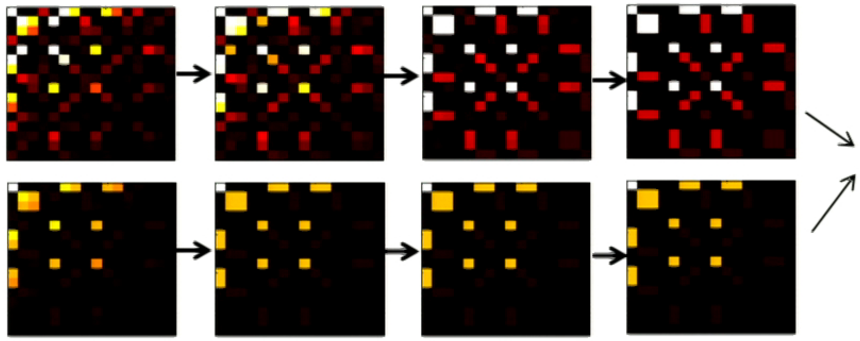
A A' A'' A'''

$\beta = 0.9\beta_c$

$\beta = 0.5\beta_c$



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TNR
(2015)

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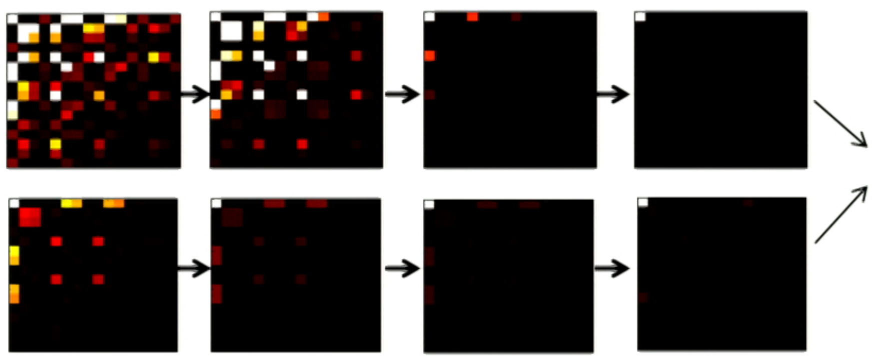

converge to the same fixed point (containing only universal information of the phase)

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TNR (2015):

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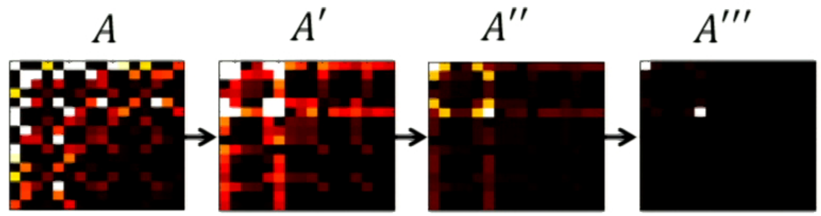
low temperature

$\beta = 1.1\,\beta_c$



ordered (Z2) fixed point

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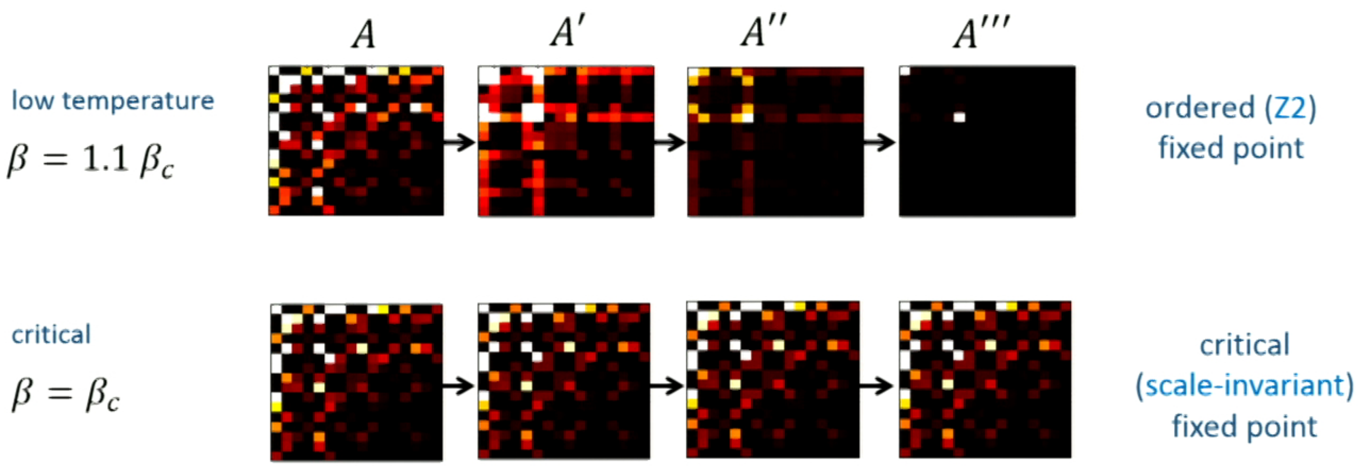

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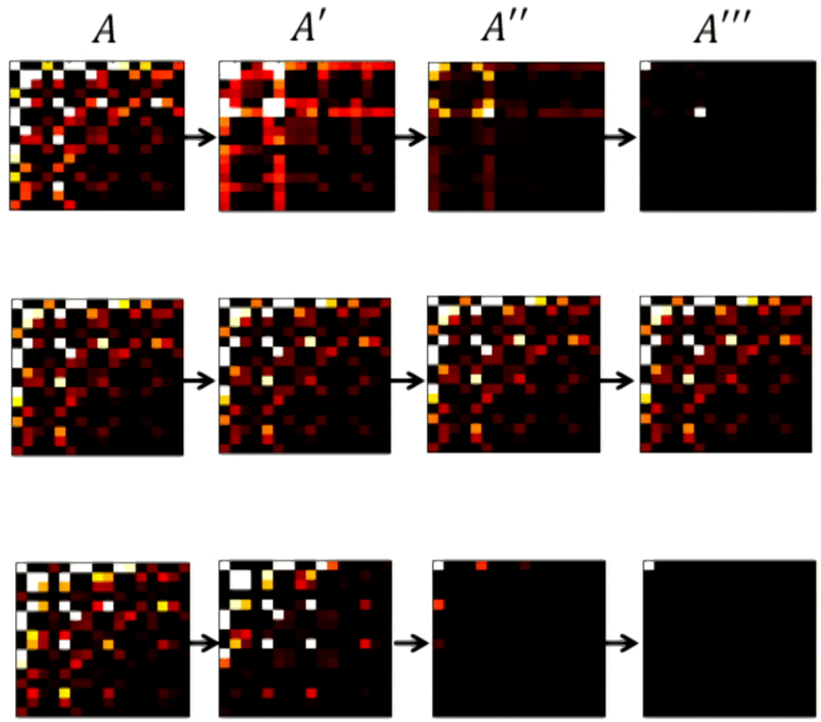


low temperature
$\beta = 1.1\,\beta_c$
ordered (Z2) fixed point

critical
$\beta = \beta_c$
critical (scale-invariant) fixed point

high temperature
$\beta = 0.9\,\beta_c$
disordered (trivial) fixed point

TRG at ciritcality

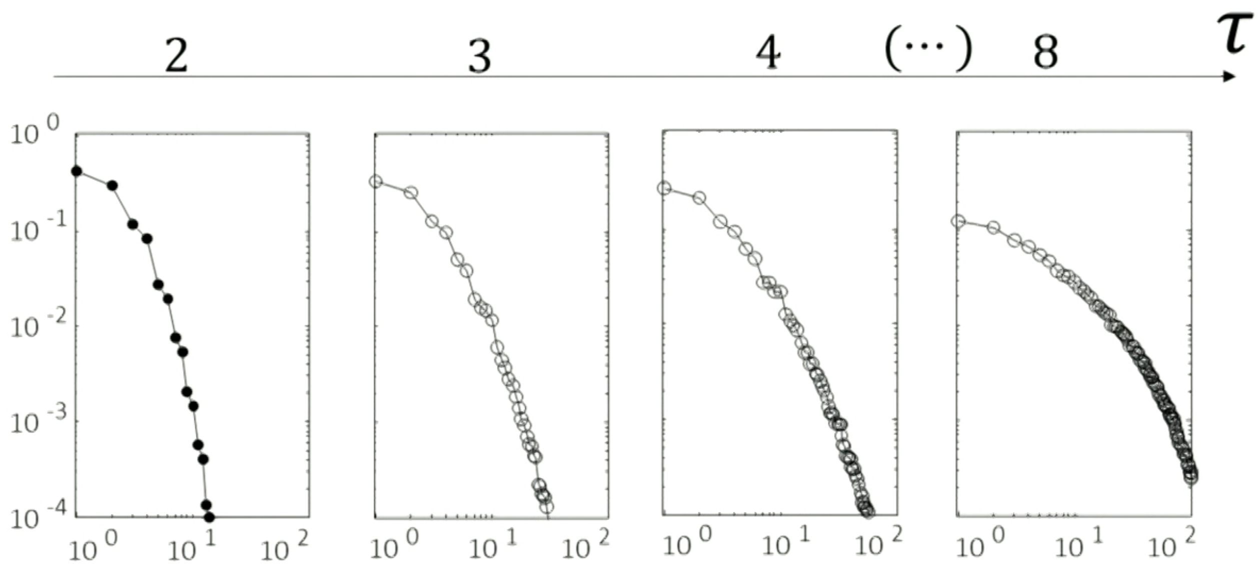
singular values of tensor $A^{(\tau)}$

critical temperature

$\beta = \beta_c$

$(\beta_c \approx 0.4406)$

coarse-graining steps



- singular values do not decay as fast (but spectrum is not flat!)
- spectrum does not converges to some fixed spectrum
- each iteration is more expensive (if we want to keep accuracy fixed, we need to increase bond dimension χ)

cost $\sim \chi^6$ $\chi \sim e^{\tau}$ cost $\sim \exp(\tau)$

TNR at ciritcality

singular values of tensor $A^{(\tau)}$

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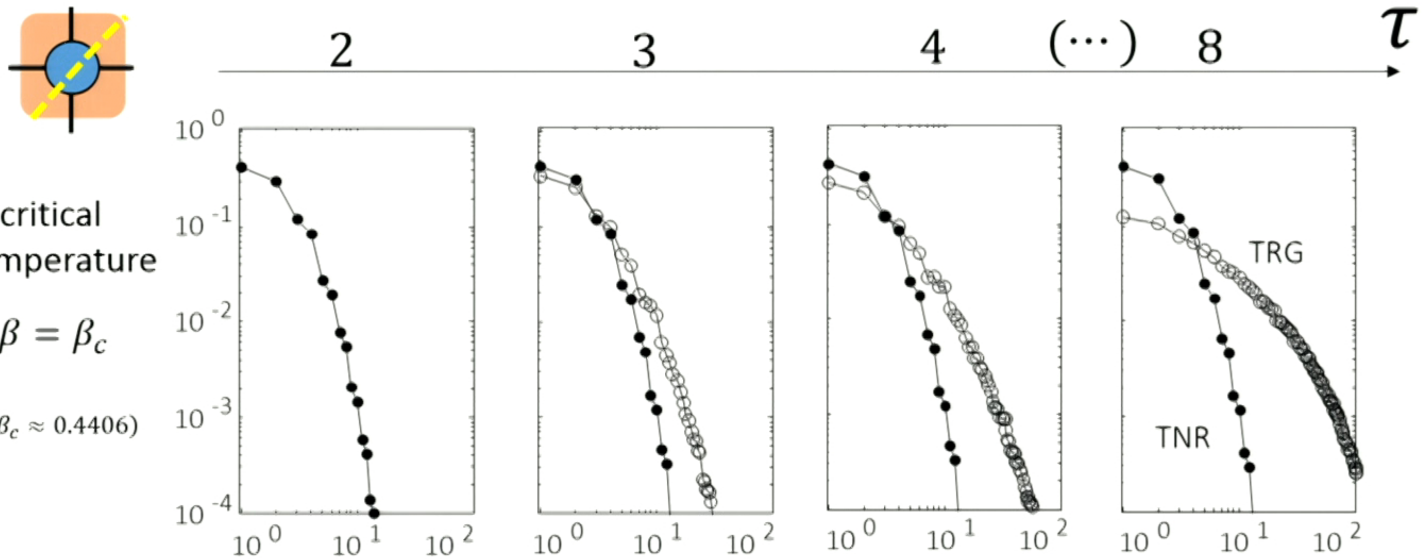

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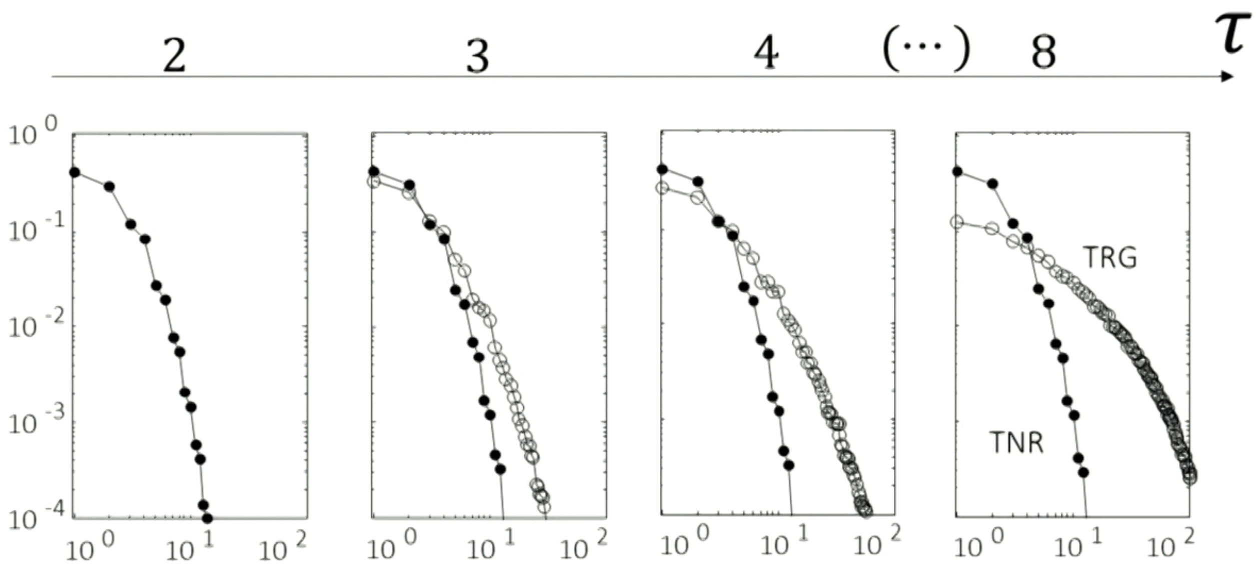

critical temperature

$\beta = \beta_c$

$(\beta_c \approx 0.4406)$

- singular values continue to decay fast
- spectrum quickly converges to a fixed-point spectrum
- each iteration has the same cost – we can iterate forever*!!!

TNR at ciritcality

critical
partition function

$\beta = \beta_c$

TNR bond
dimension:

$\chi = 4$

TNR at ciritcality

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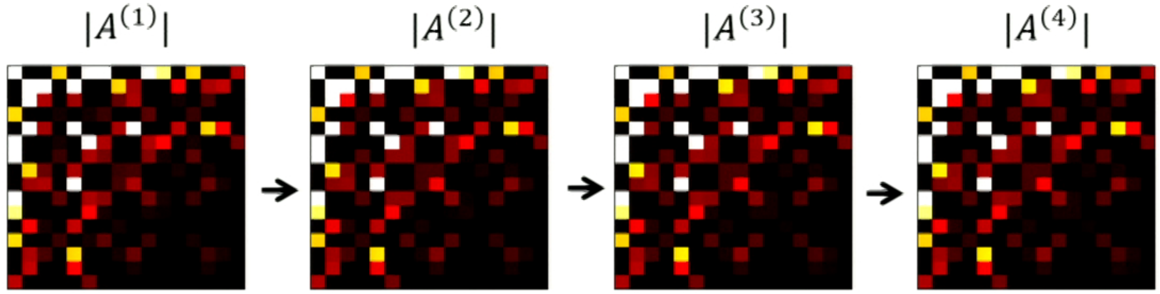

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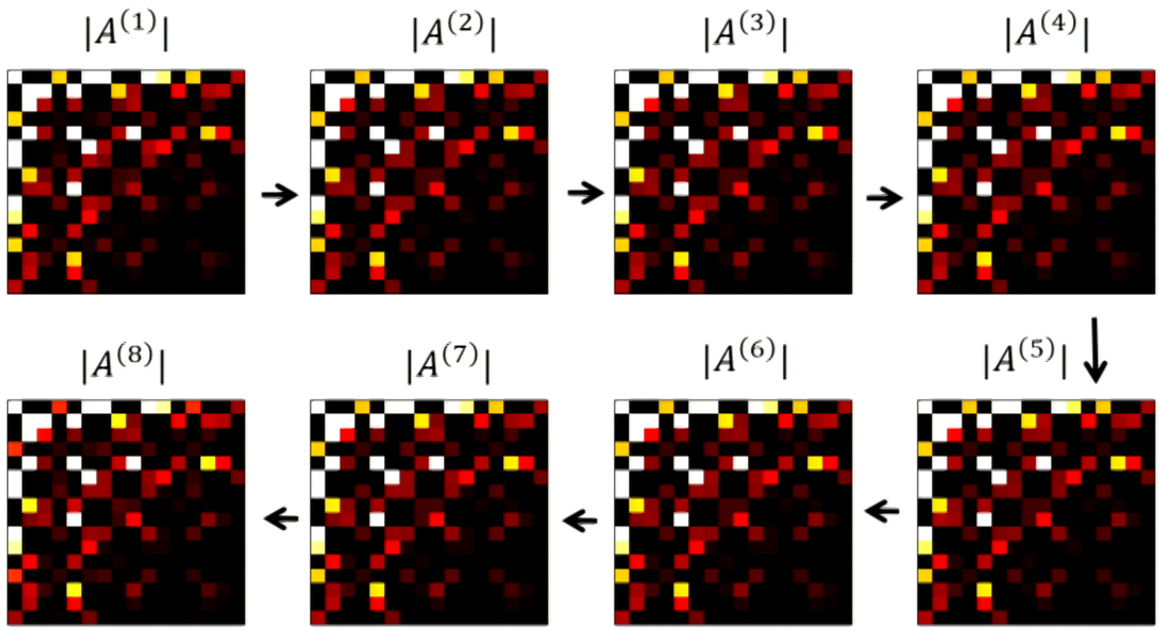

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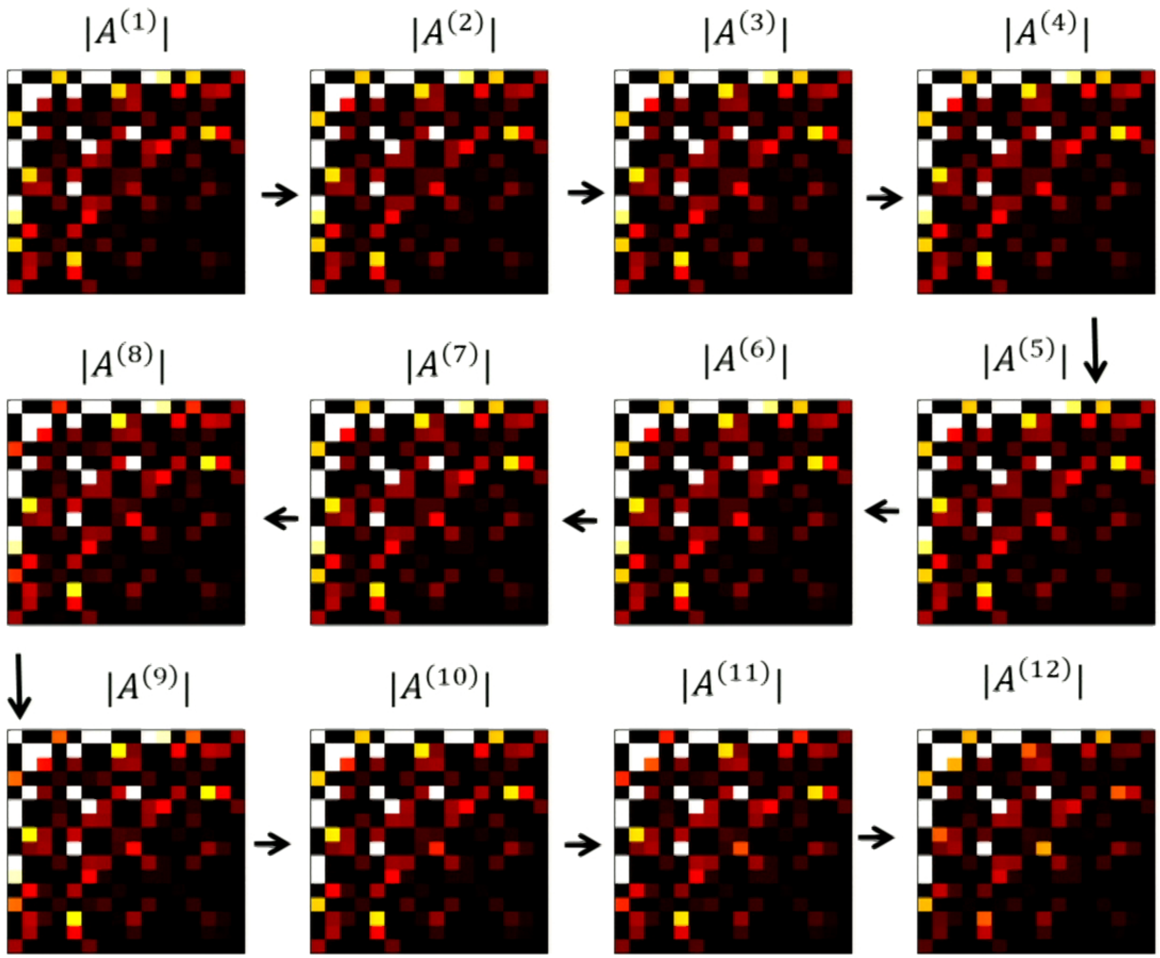

Proper RG flow: 2D classical Ising

almost-critical
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$\beta = 0.998\beta_c$

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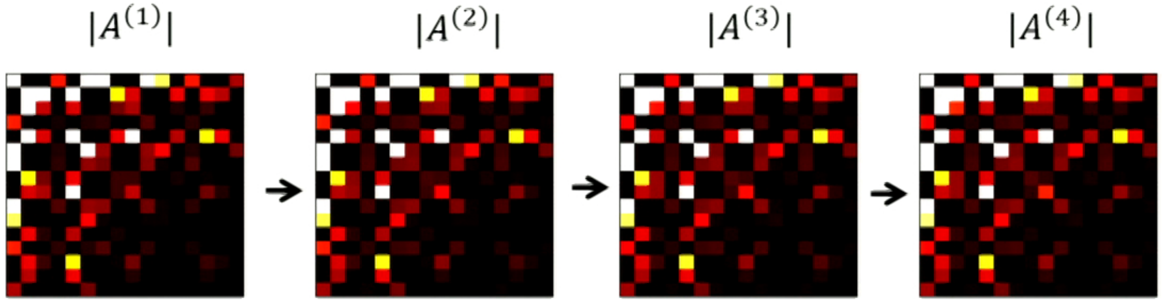

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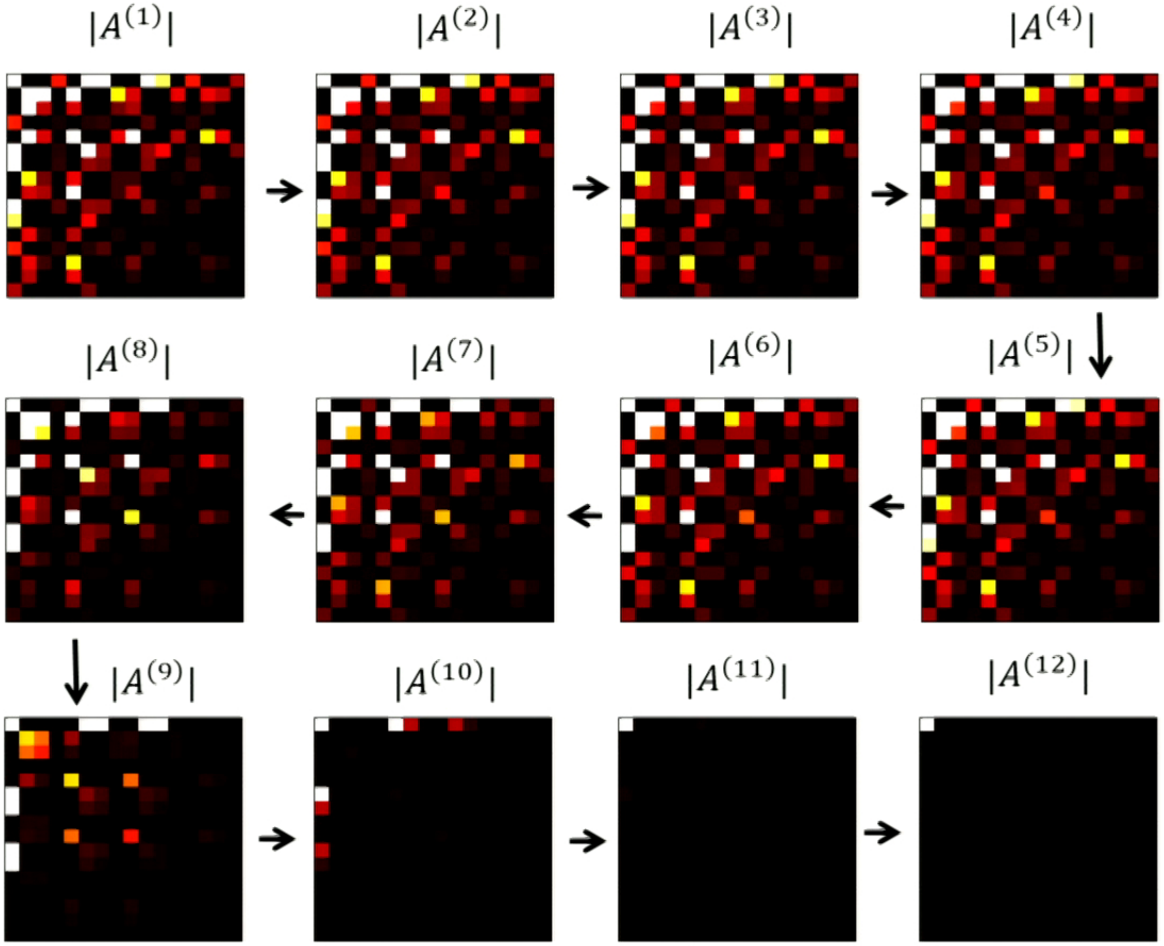
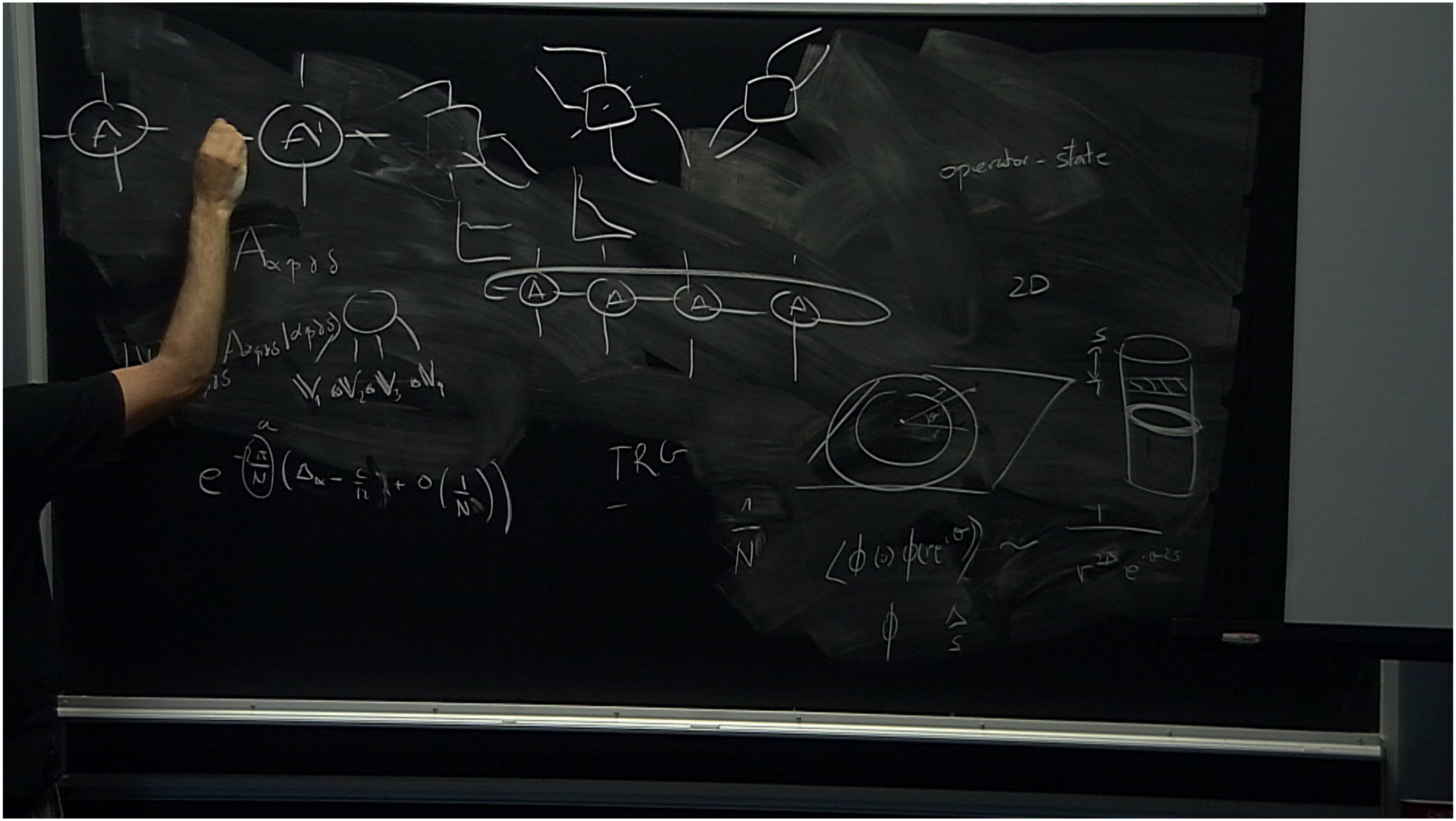
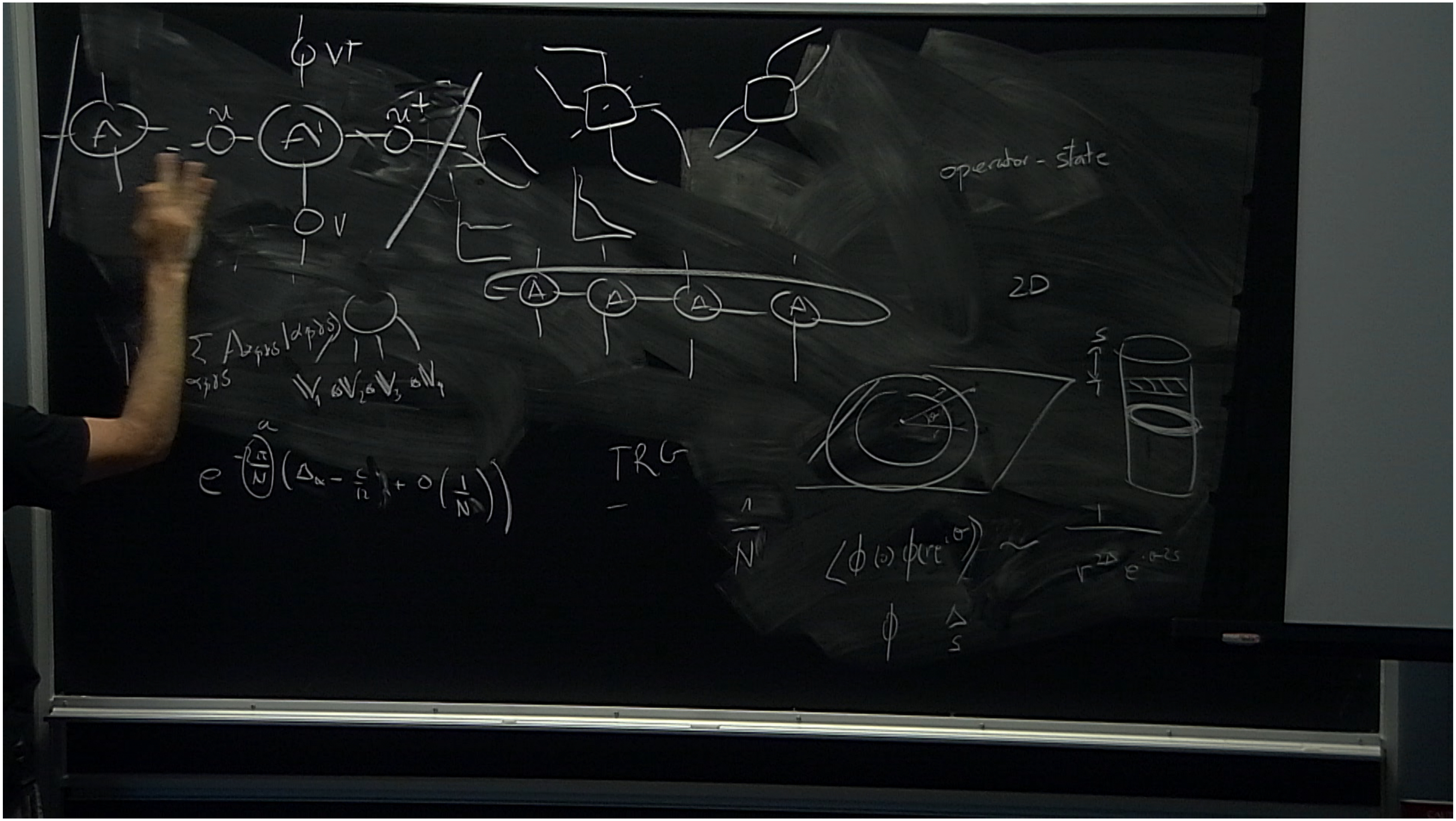



operator - state
2D
TRG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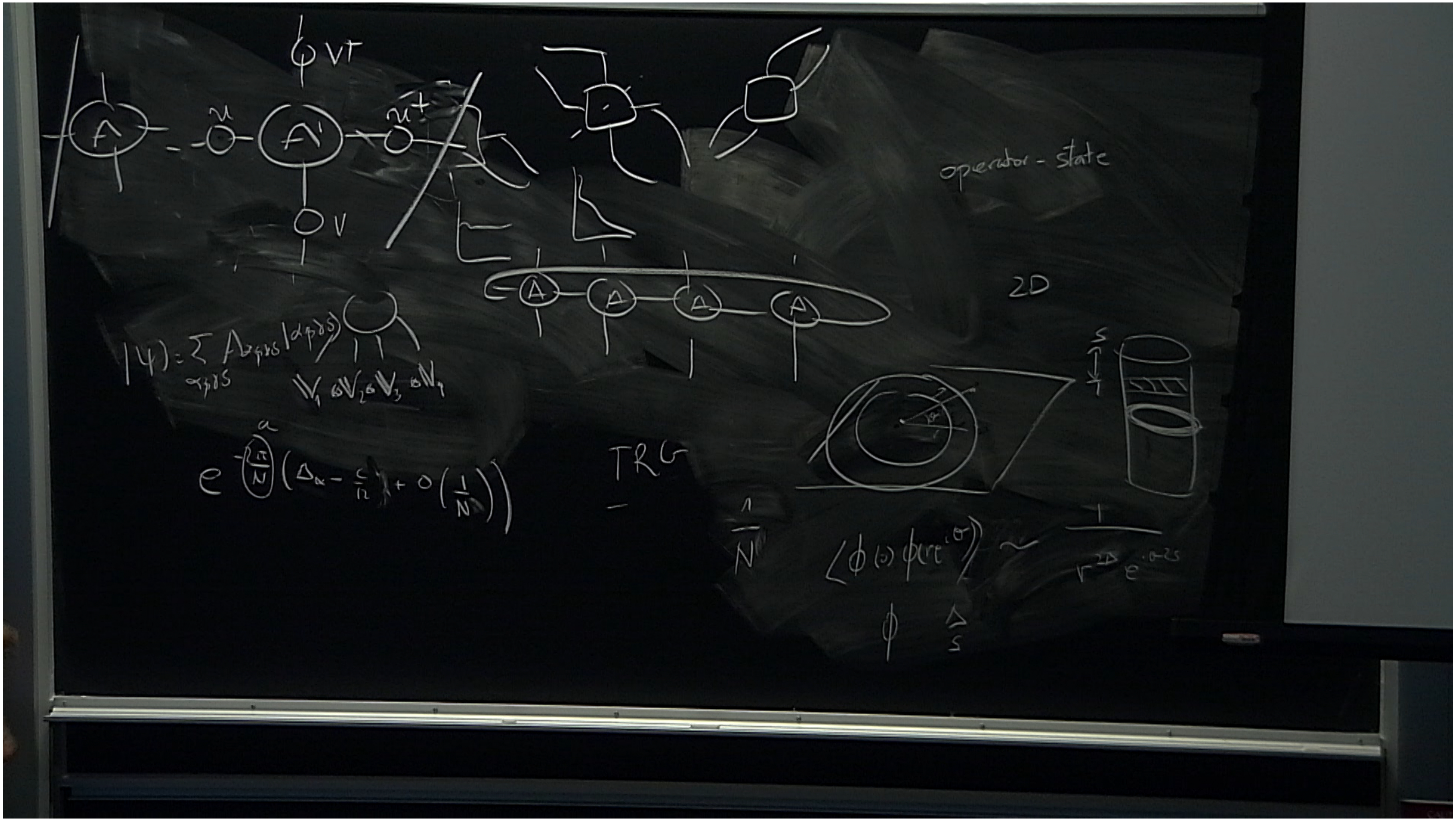
operator - state
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Introduction to Tensor Network techniques

Conclusions:

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quantum **ground state**
in d spatial dimension

$$S_{L^d} \sim L^{d-1}$$

Area law

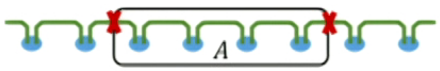
Conclusions:

quantum **ground state**
in d spatial dimension

$$S_{L^d} \sim L^{d-1}$$

Area law

example: d=1 dimensions



Conclusions:

quantum **ground state** in d spatial dimension	statistical **partition function** in D spatial dimension
$S_{L^d} \sim L^{d-1}$	$S_{L^D} \sim L^{D-2}$
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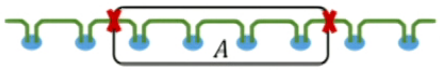

Introduction to Tensor Network techniques

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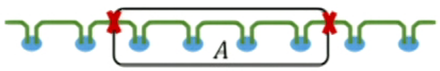

statistical **partition function** in D spatial dimension

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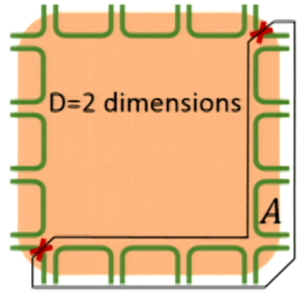

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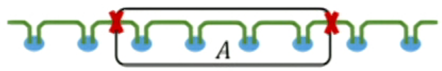

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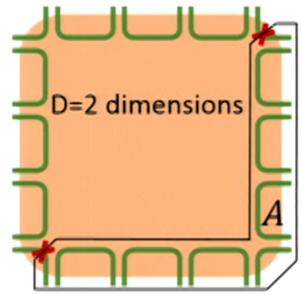
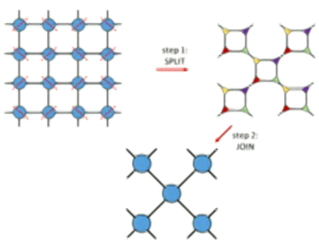


TRG [Levin,Nave 2006]



- computationally powerful (in D=2)
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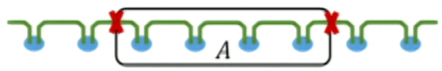

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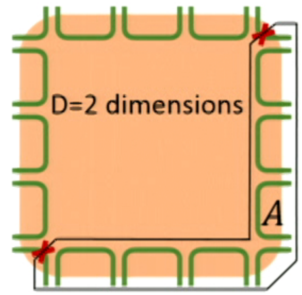

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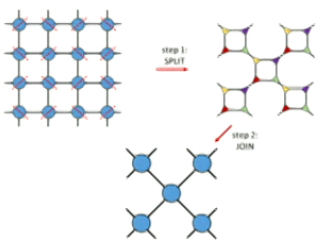

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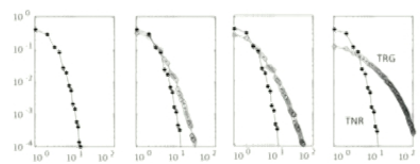
TNR [2015]

- correct RG flow
- works also at criticality



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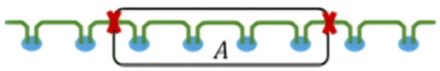

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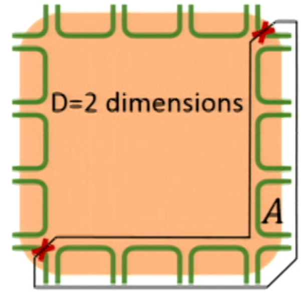

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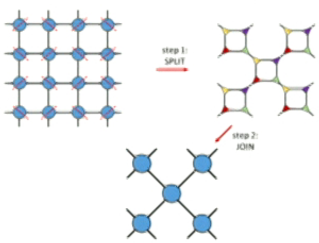

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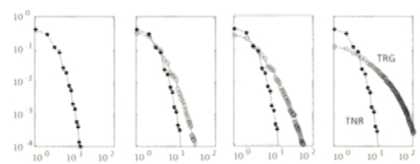

Big challenge in this field:
$D = 3{,}4$ dimensions

Introduction to Tensor Network techniques

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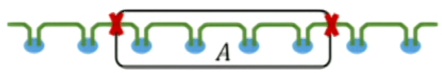

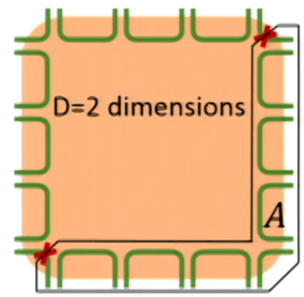

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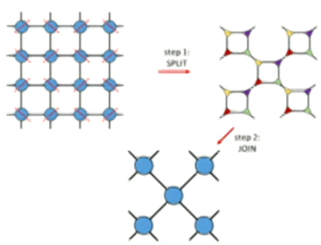

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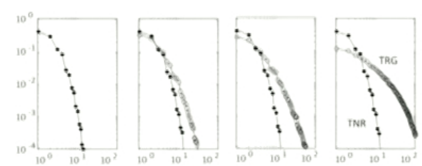

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How do you go from Ising model to Spin Foams?

Introduction to Tensor Network techniques

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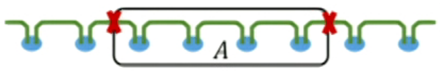

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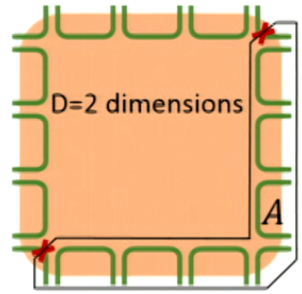

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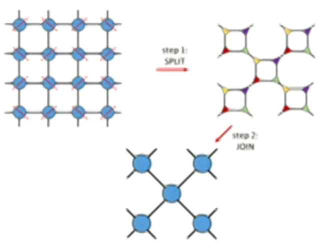

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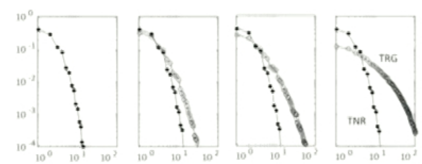

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How do you go from Ising model to Spin Foams?
Ask Benjamin and Bianca!