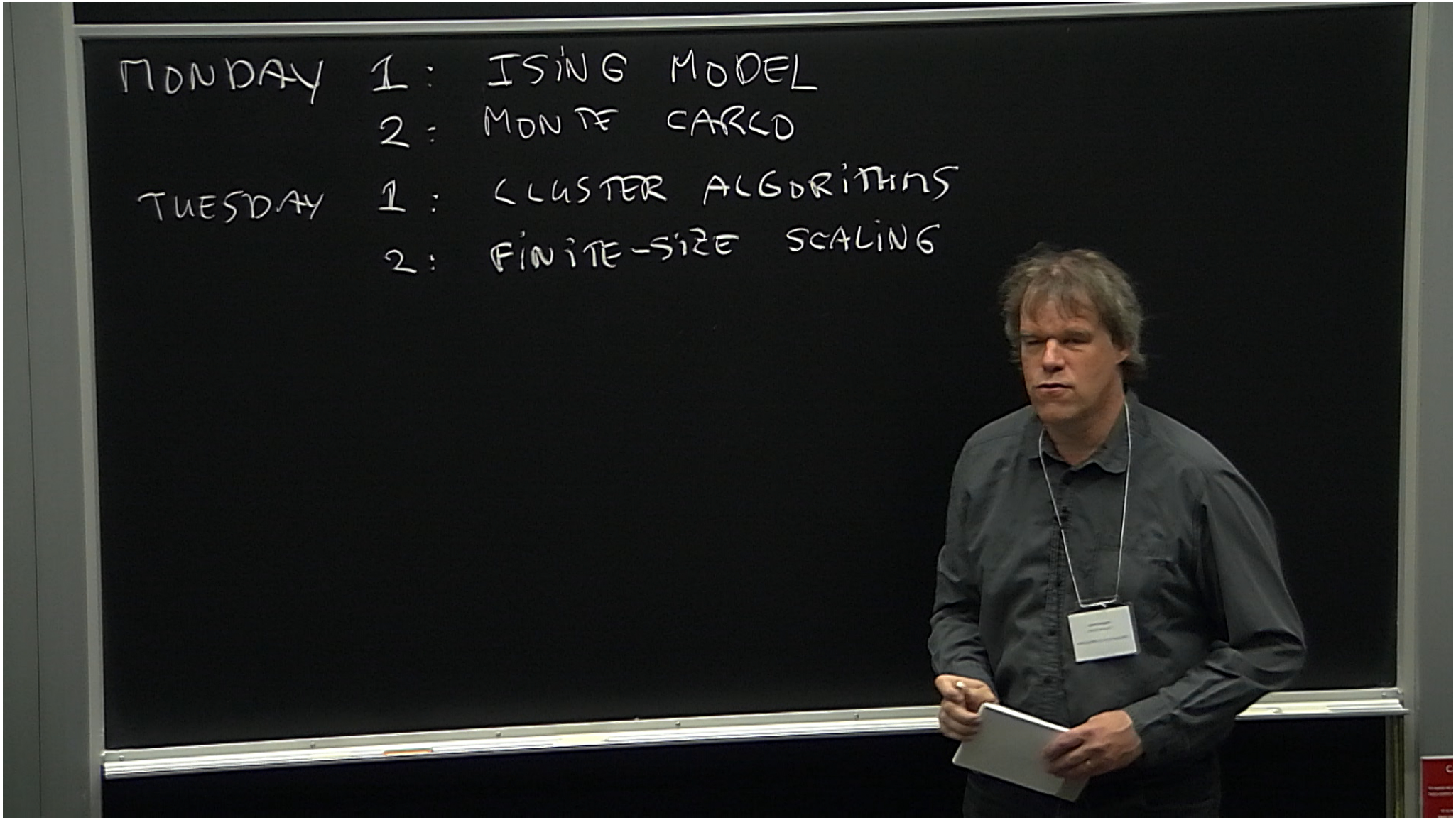
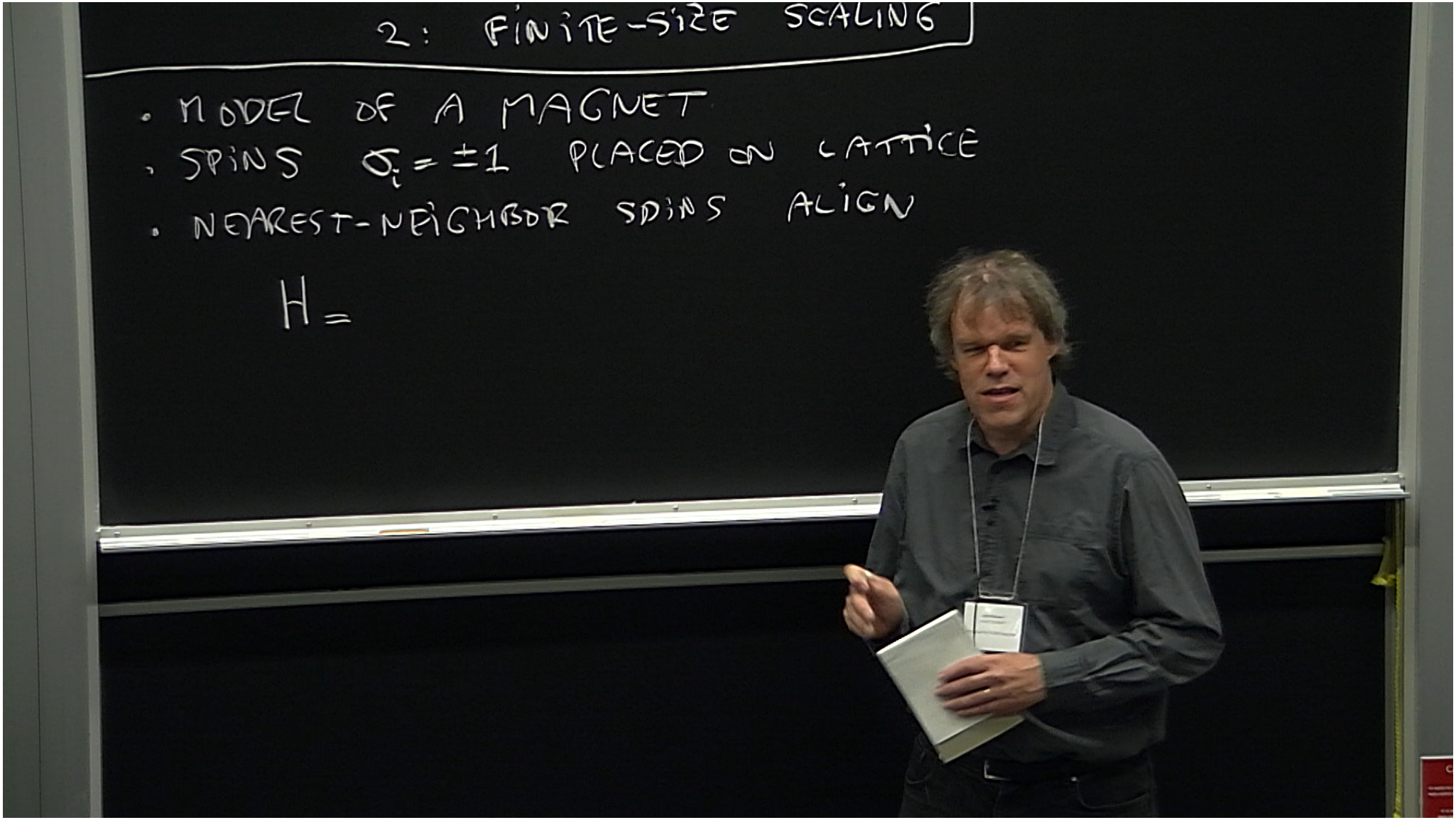
Title: Introduction to Monte Carlo methods - 1

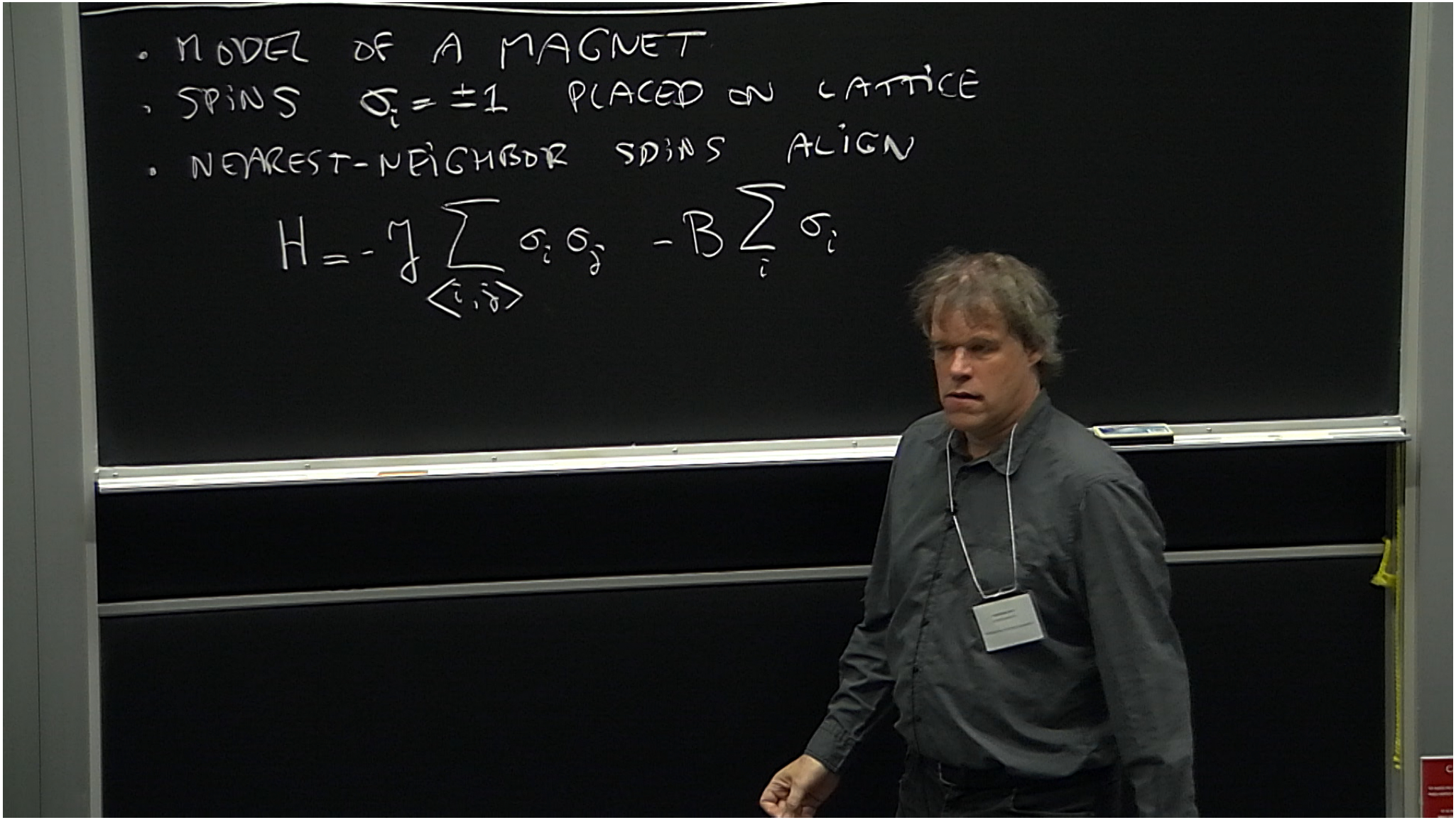
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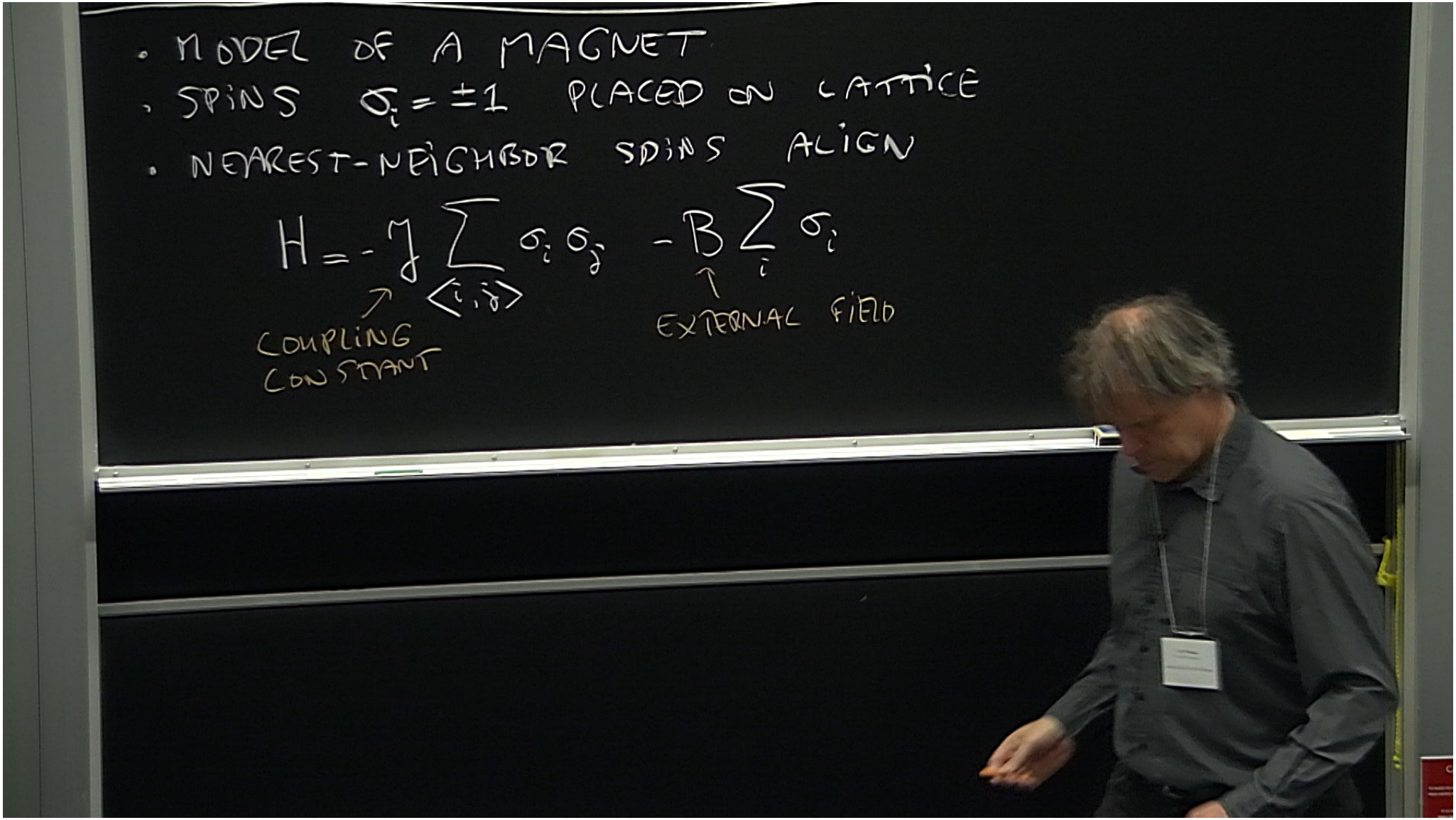
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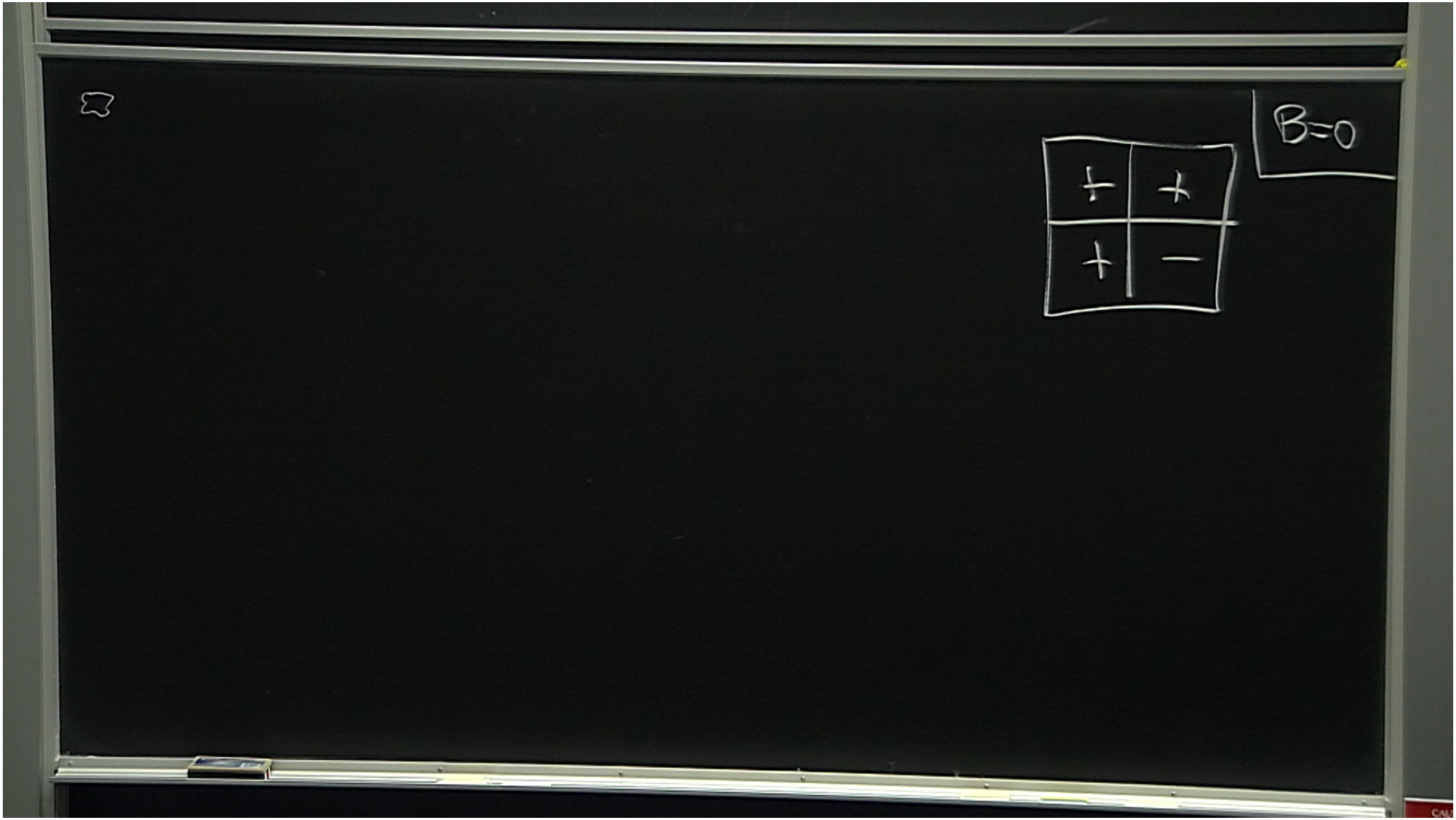
Abstract:

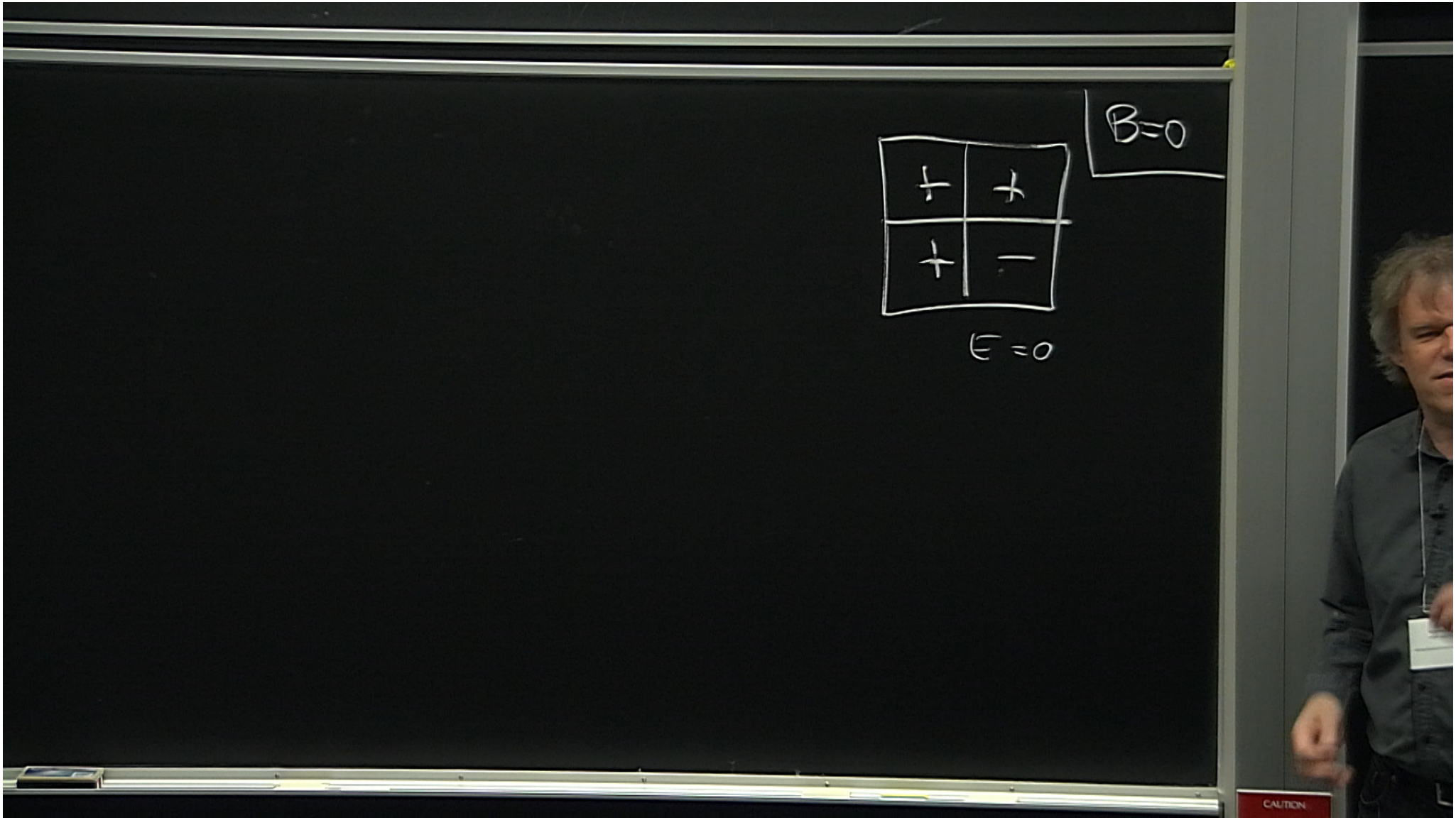

MONDAY 1: ISING MODEL
2: MONTE CARLO
TUESDAY 1: CLUSTER ALGORITHMS
2: FINITE-SIZE SCALING

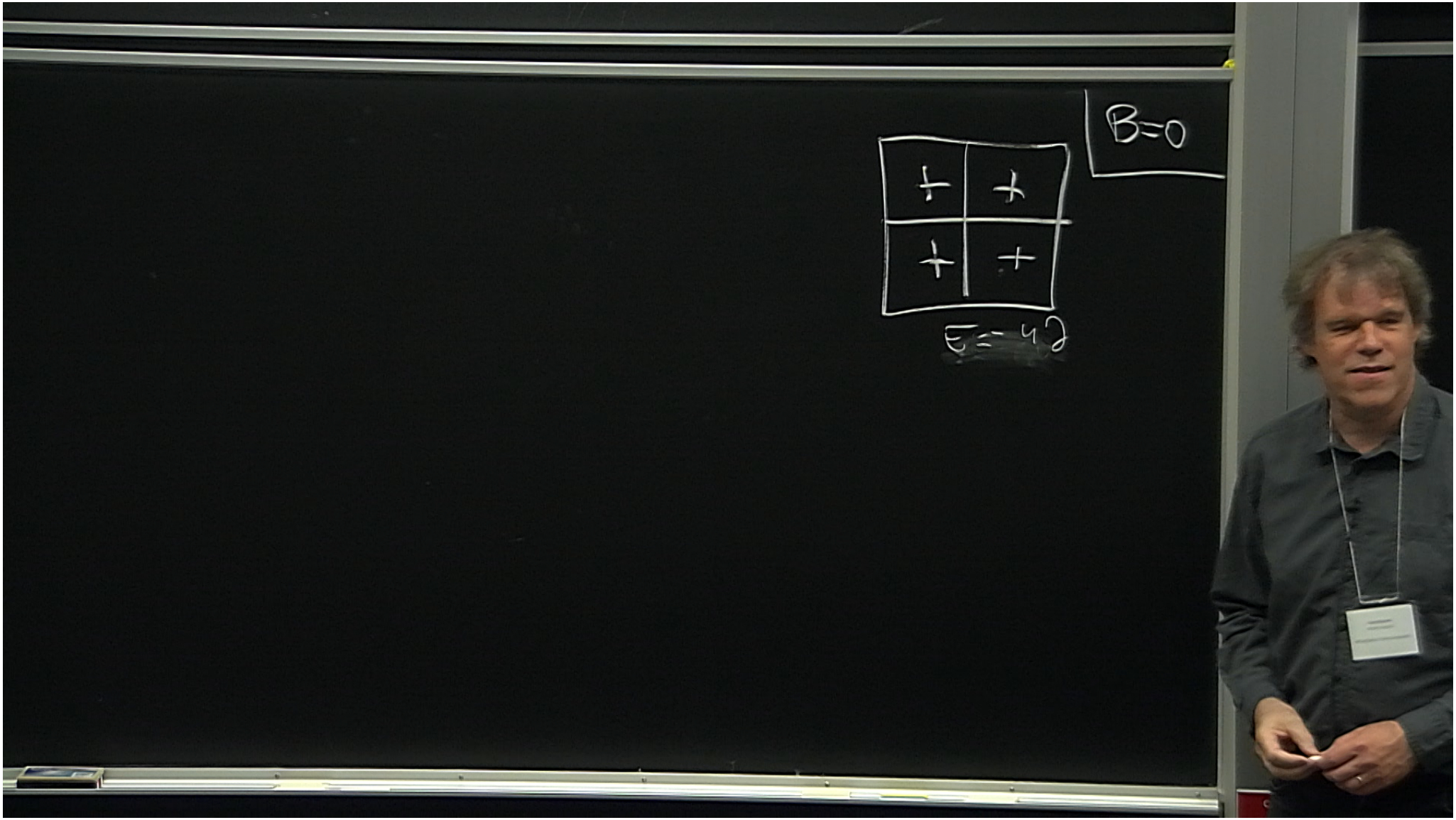

2: FINITE-SIZE SCALING
• MODEL OF A MAGNET
• SPINS $\sigma_i = \pm 1$ PLACED ON LATTICE
• NEAREST-NEIGHBOR SPINS ALIGN
H =

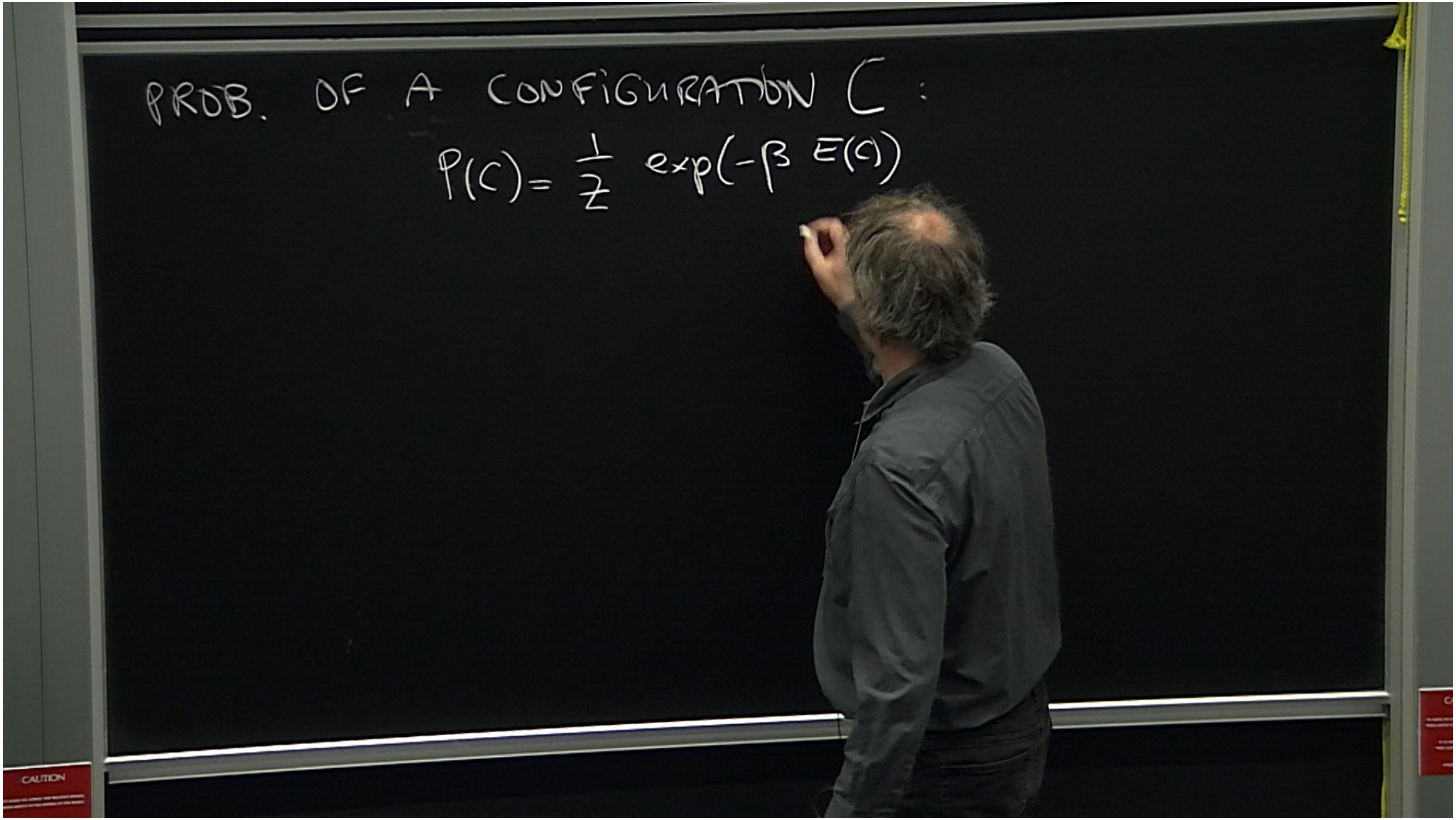

• MODEL OF A MAGNET
• SPINS $\sigma_i = \pm 1$ PLACED ON LATTICE
• NEAREST-NEIGHBOR SPINS ALIGN
$H = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j - B \sum_i \sigma_i$

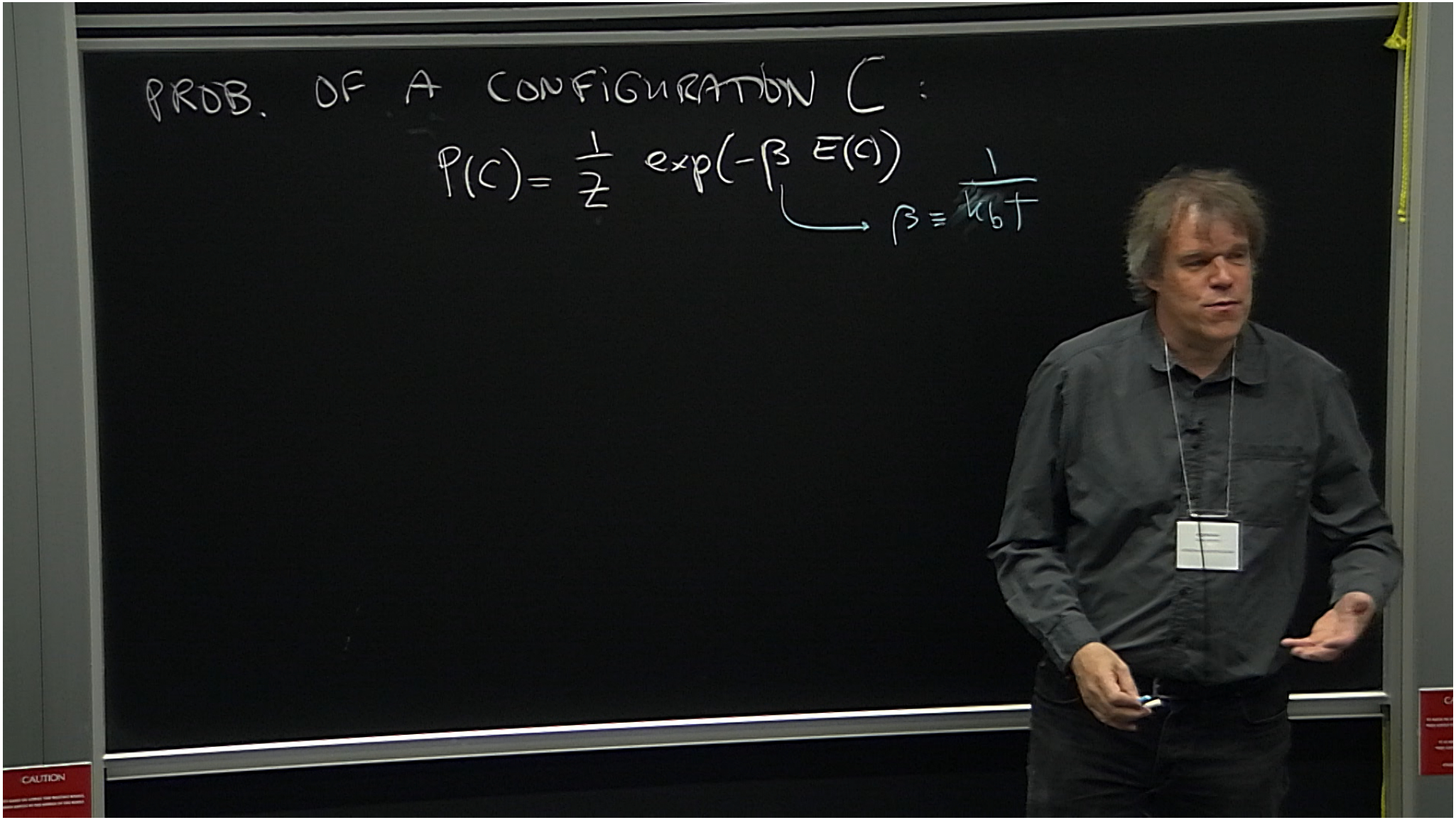

• MODEL OF A MAGNET
• SPINS $\sigma_i = \pm 1$ PLACED ON LATTICE
• NEAREST-NEIGHBOR SPINS ALIGN
$H = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j - B \sum_i \sigma_i$
COUPLING CONSTANT
EXTERNAL FIELD

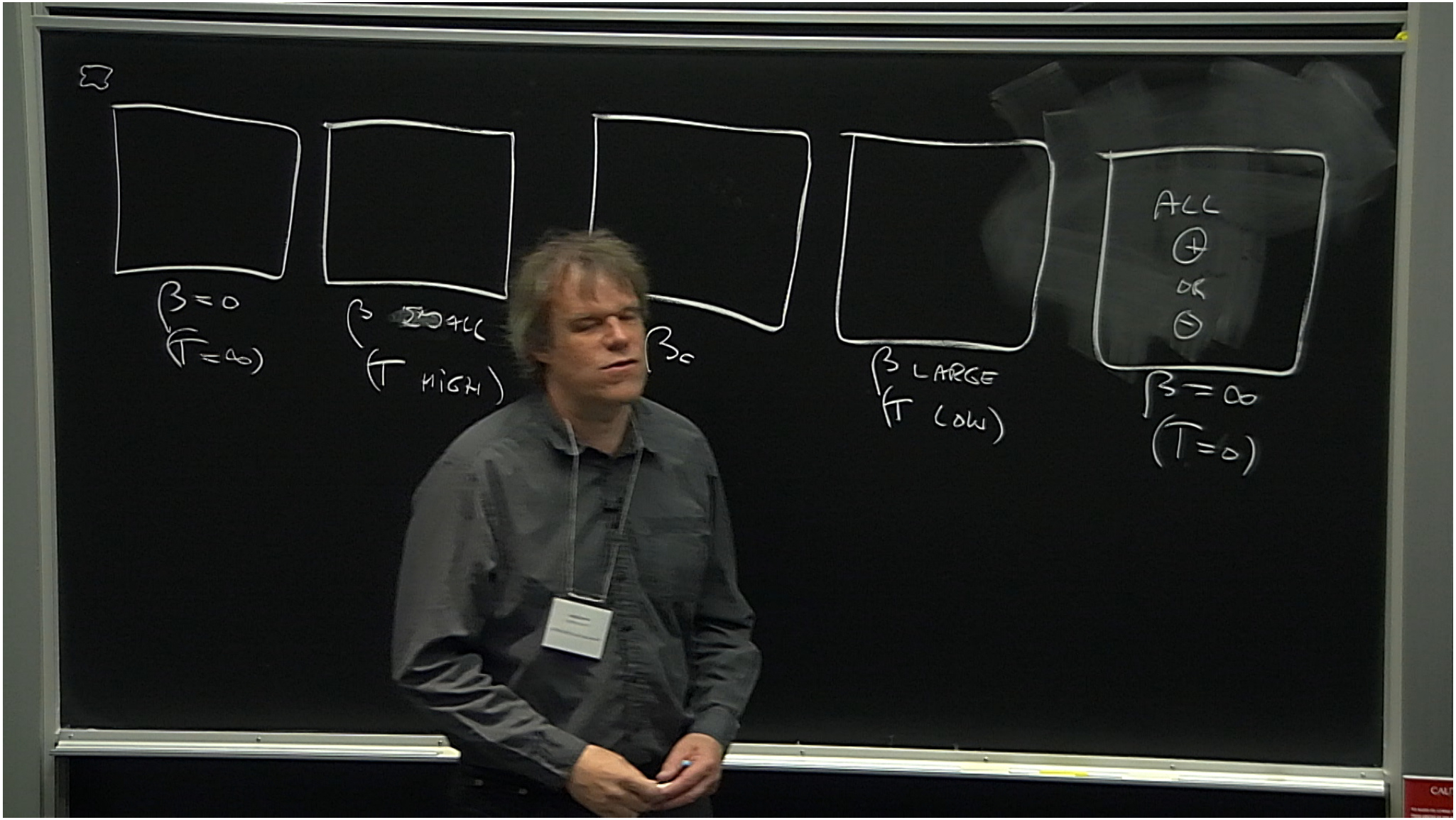

B=0
+
+
+
−

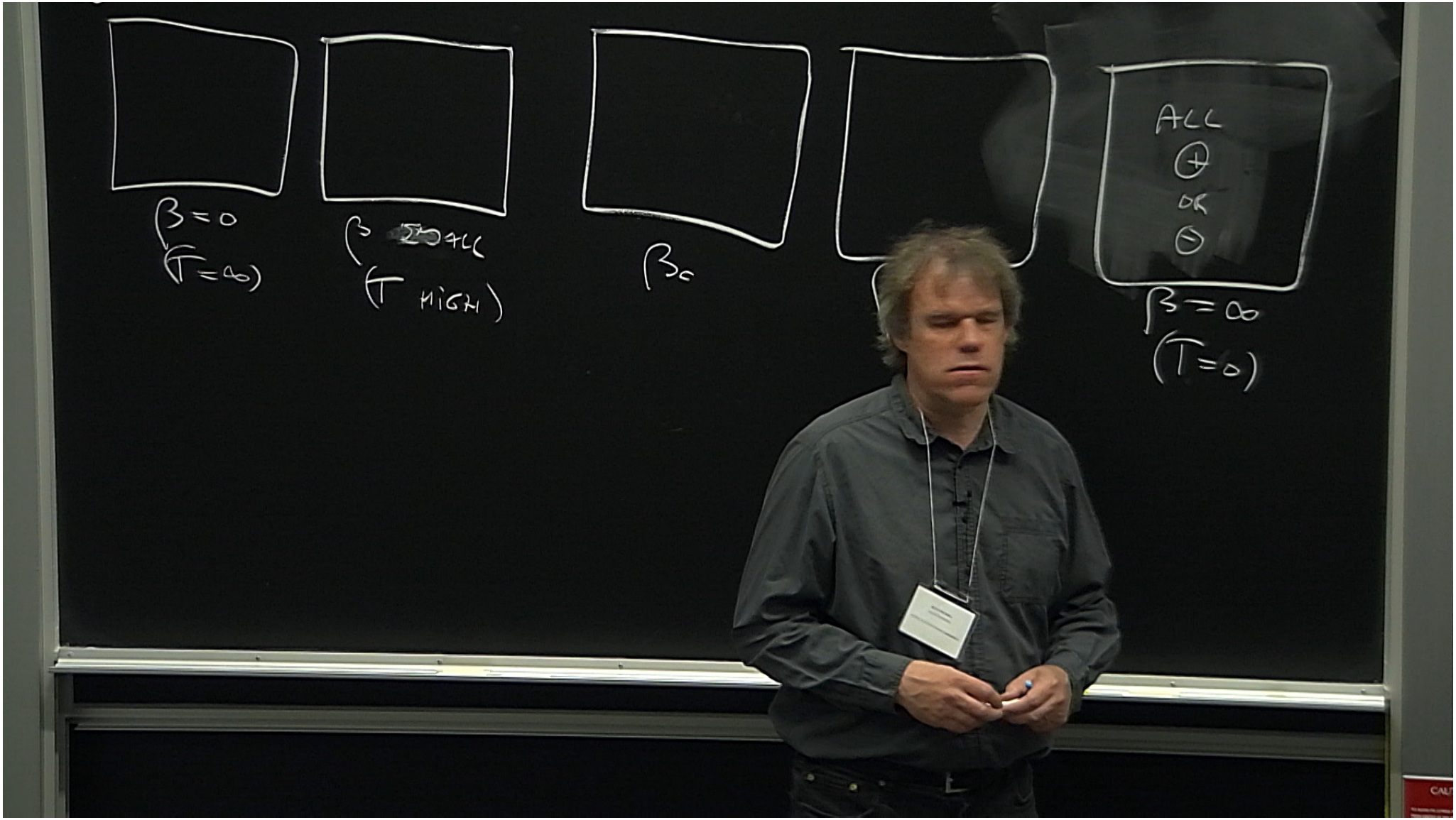

B=0
+
+
+
−
E=0
CAUTION

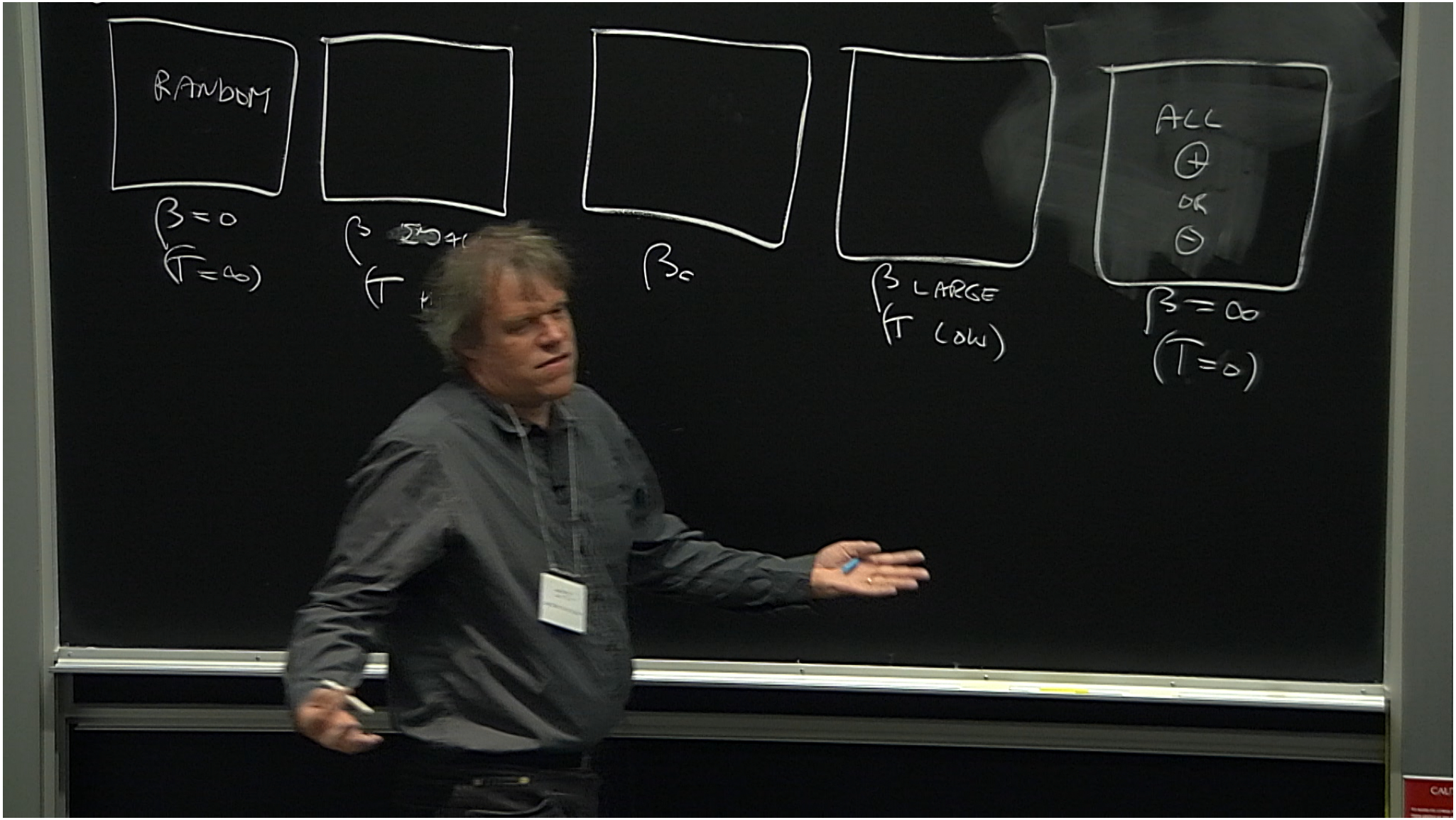

B=0
+
+
+
+
E=4J

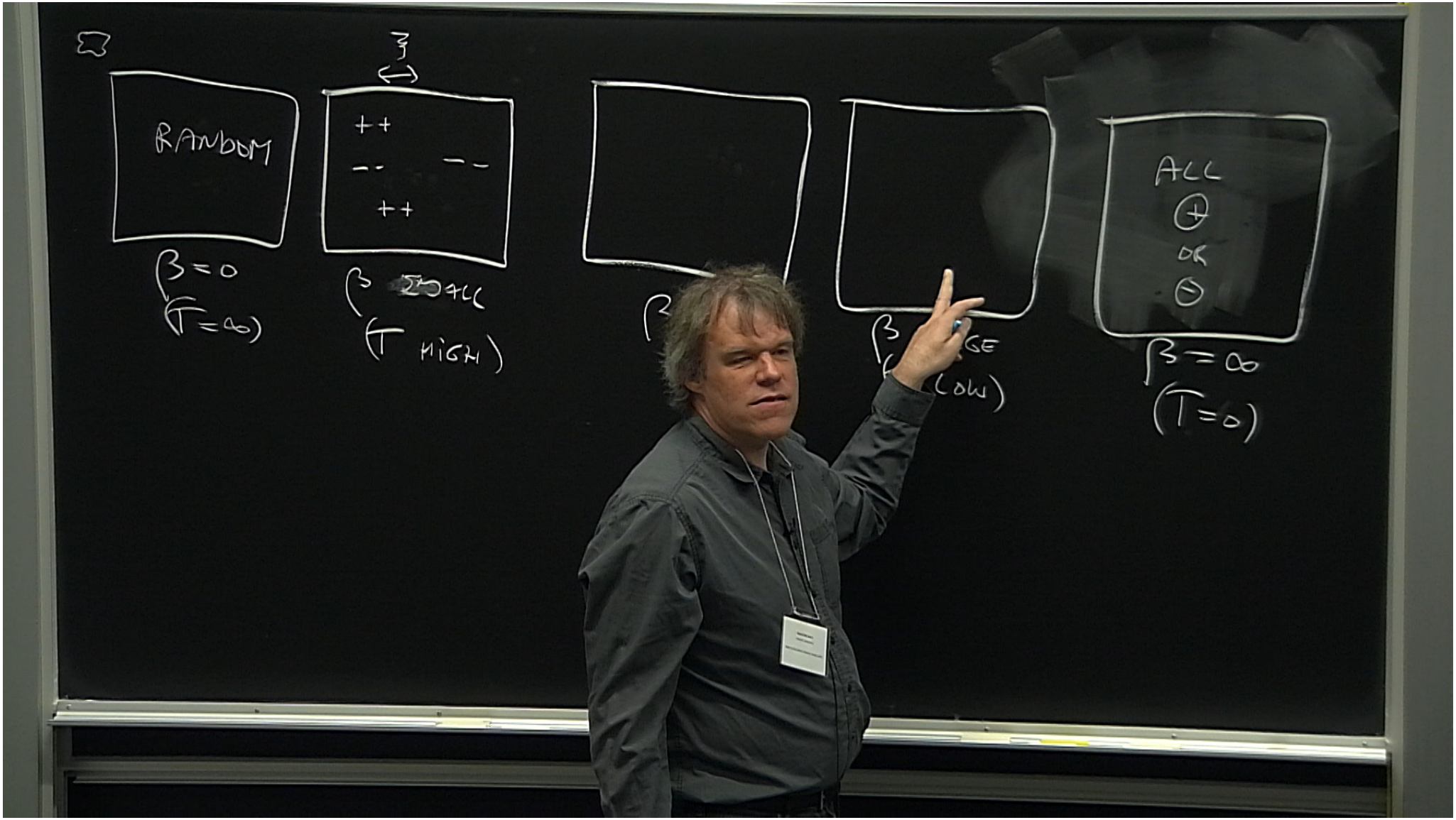

PROB. OF A CONFIGURATION C:
$P(C) = \frac{1}{Z} \exp(-\beta E(C))$
CAUTION

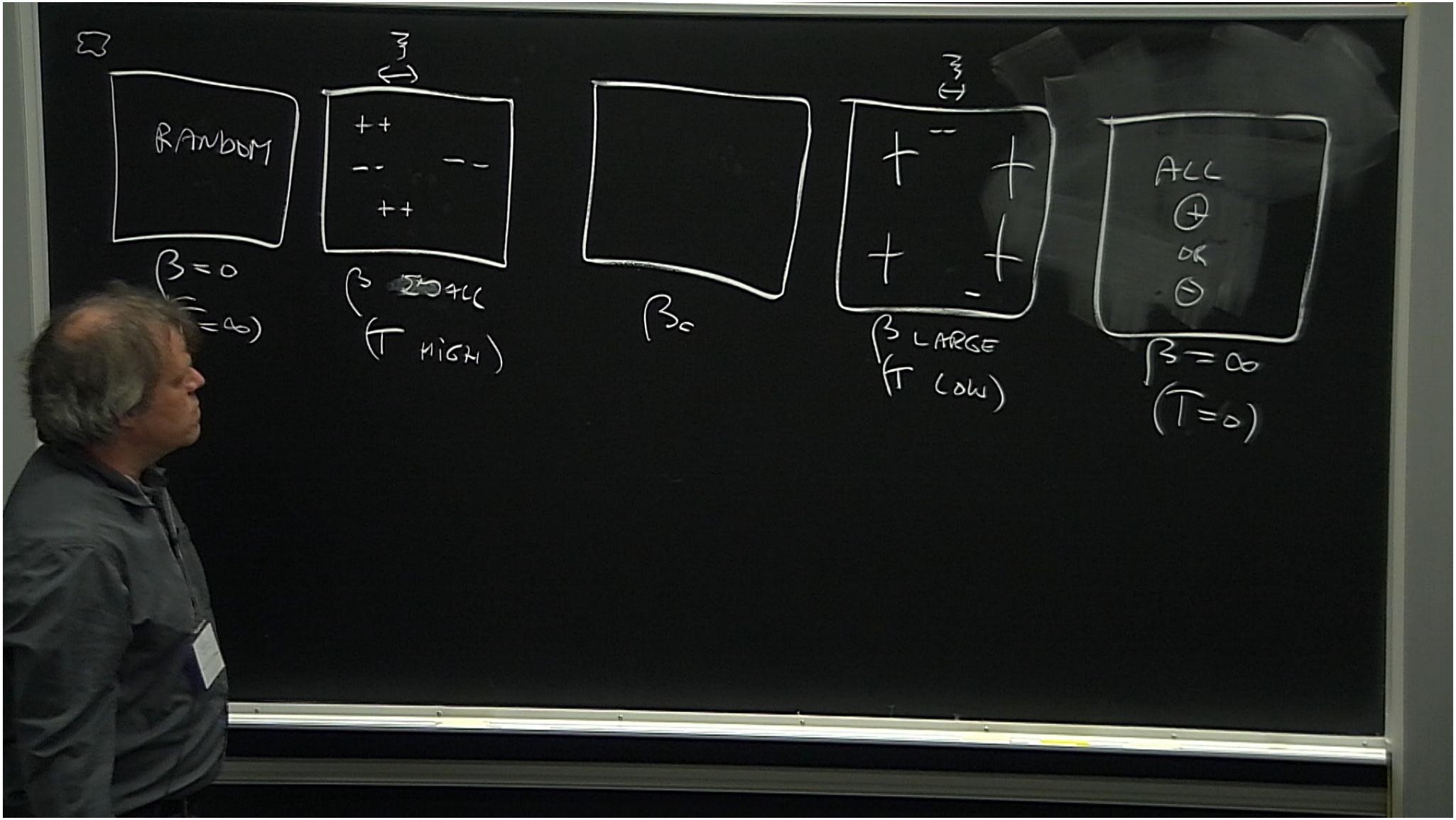

PROB. OF A CONFIGURATION C:
$P(C) = \frac{1}{Z} \exp(-\beta E(C))$
$\beta \equiv \frac{1}{k_b T}$
CAUTION

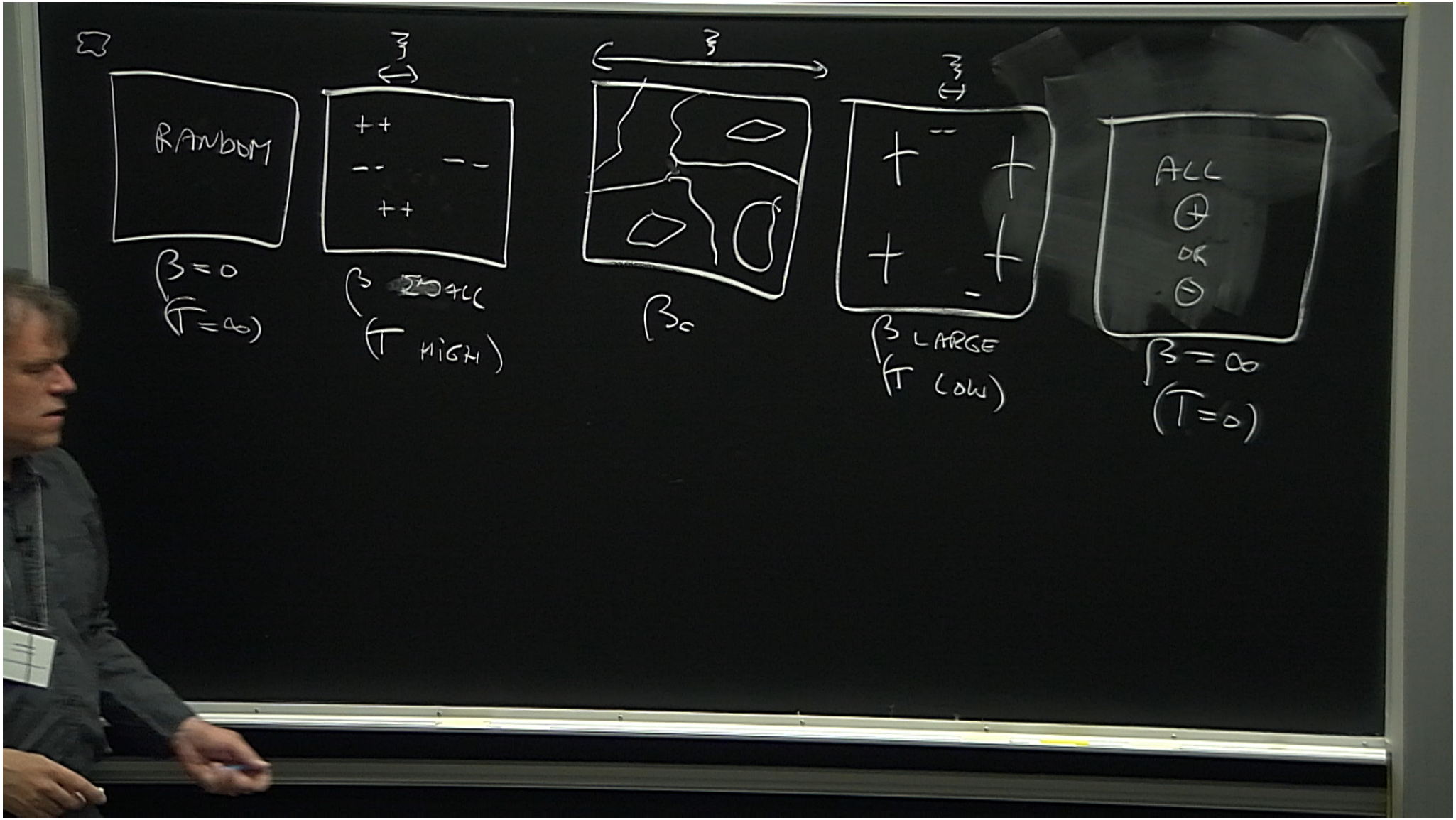

β=0
(T=∞)
(T HIGH)
βc
β LARGE
(T LOW)
ALL
+
OR
−
β=∞
(T=0)

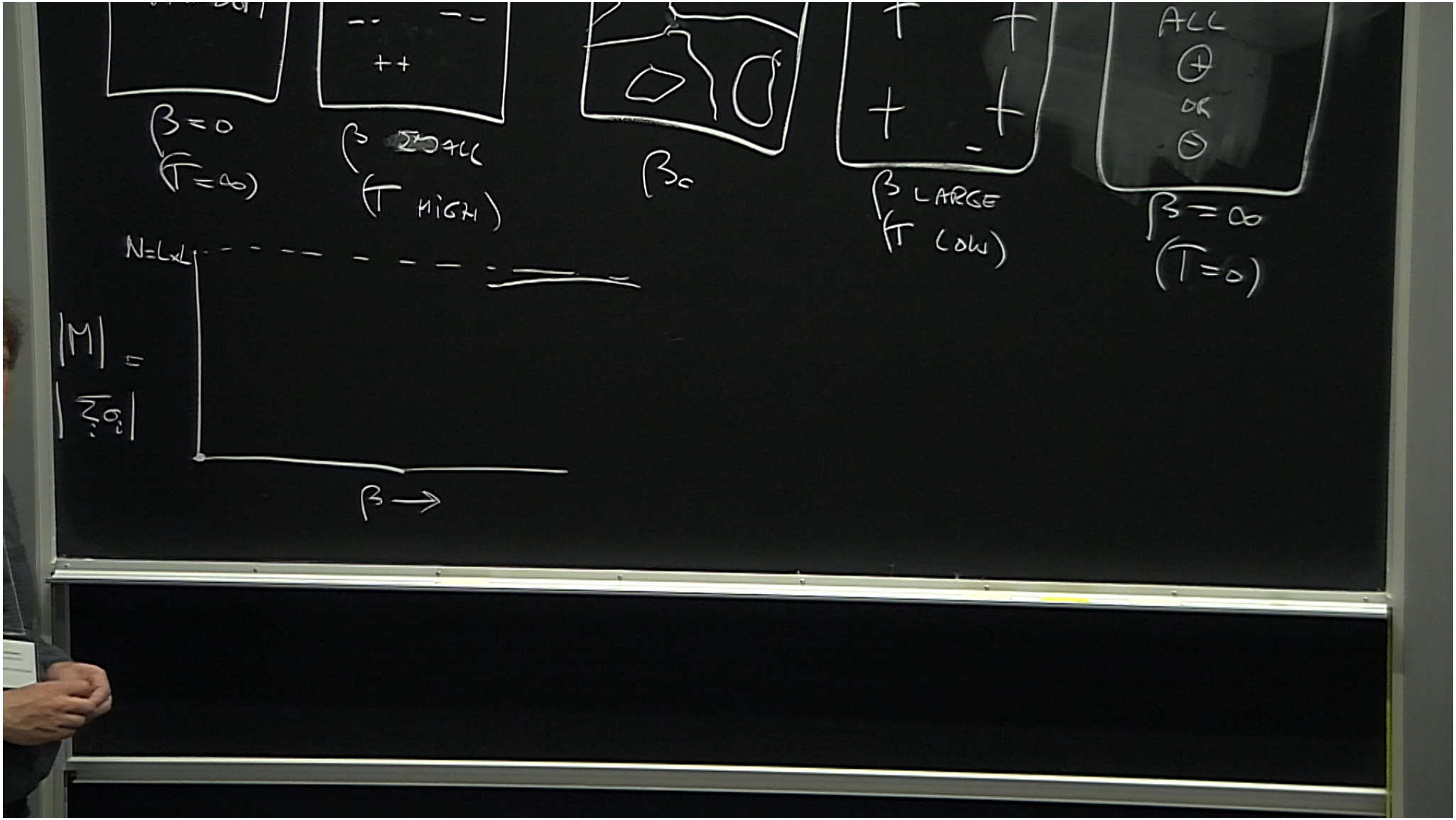

β = 0
(T = 0)
(T HIGH)
$\beta_c$
ALL
⊕
OR
⊖
β = ∞
(T = 0)

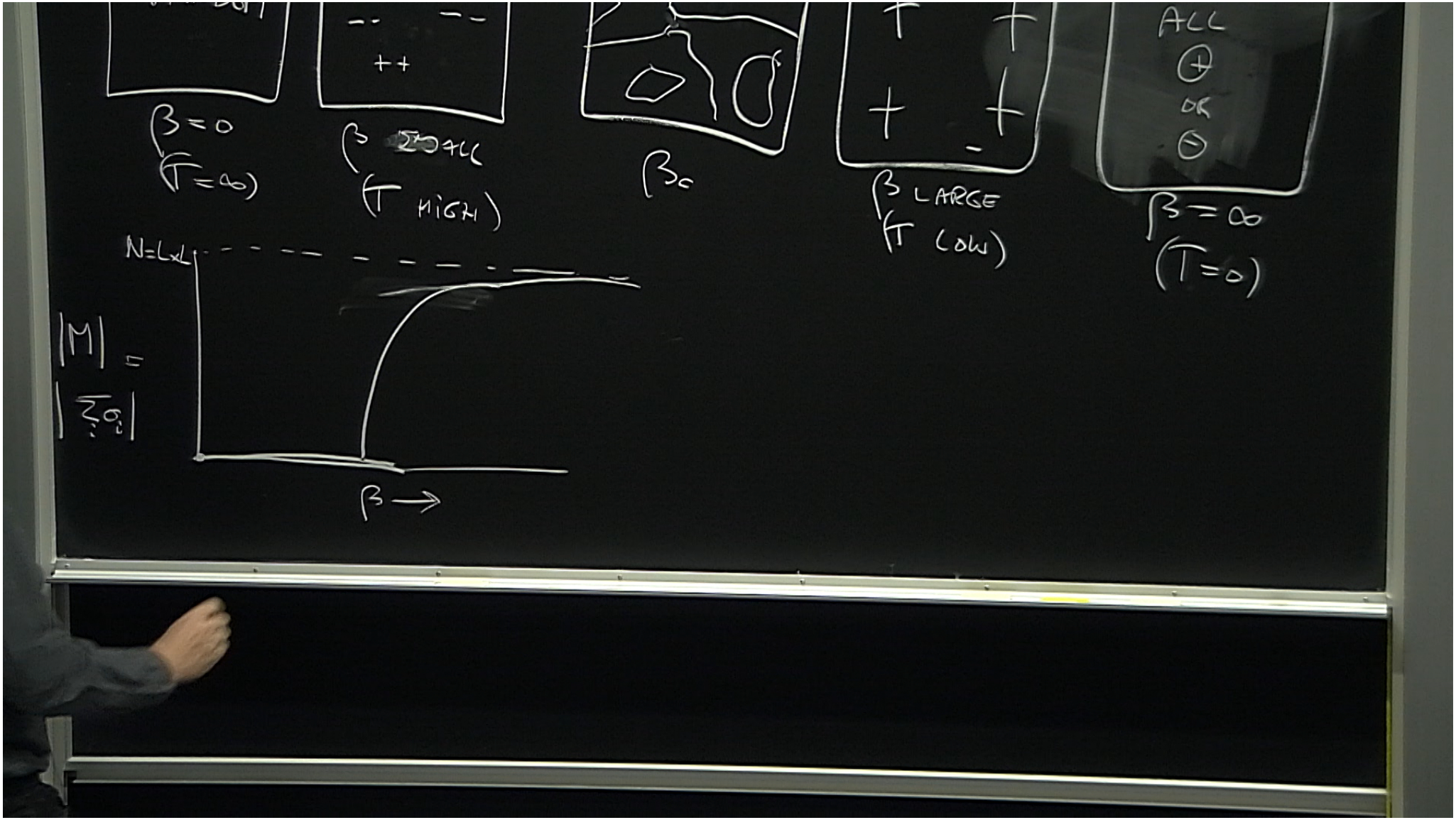

RANDOM
β = 0
(T = ∞)
β = ∞
(T = 0)
ALL
+
OR
−

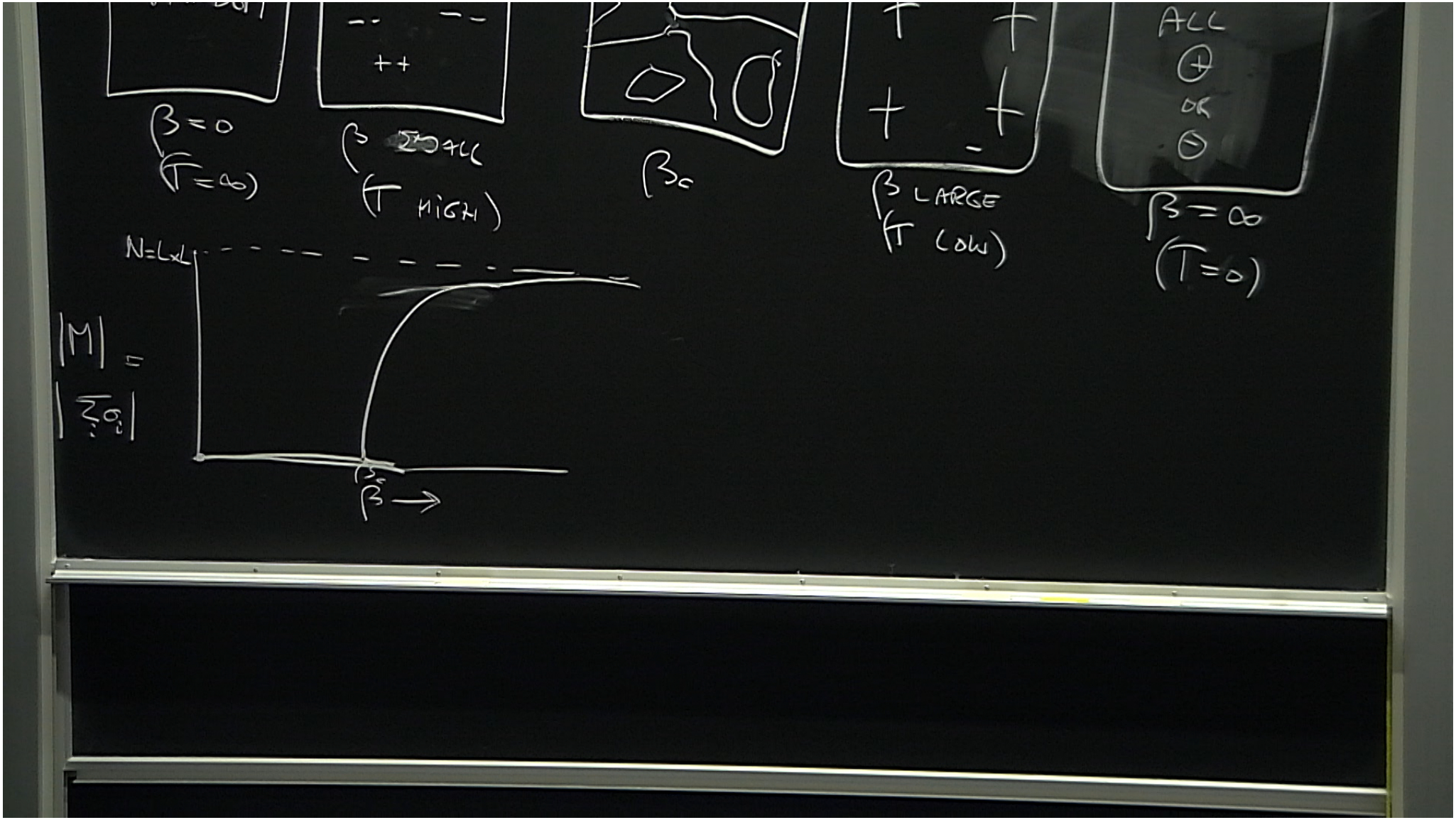

RANDOM
β=0
(T=∞)
++
--
--
++
ALL
(T HIGH)
ALL
⊕
OR
⊖
β=∞
(T=0)

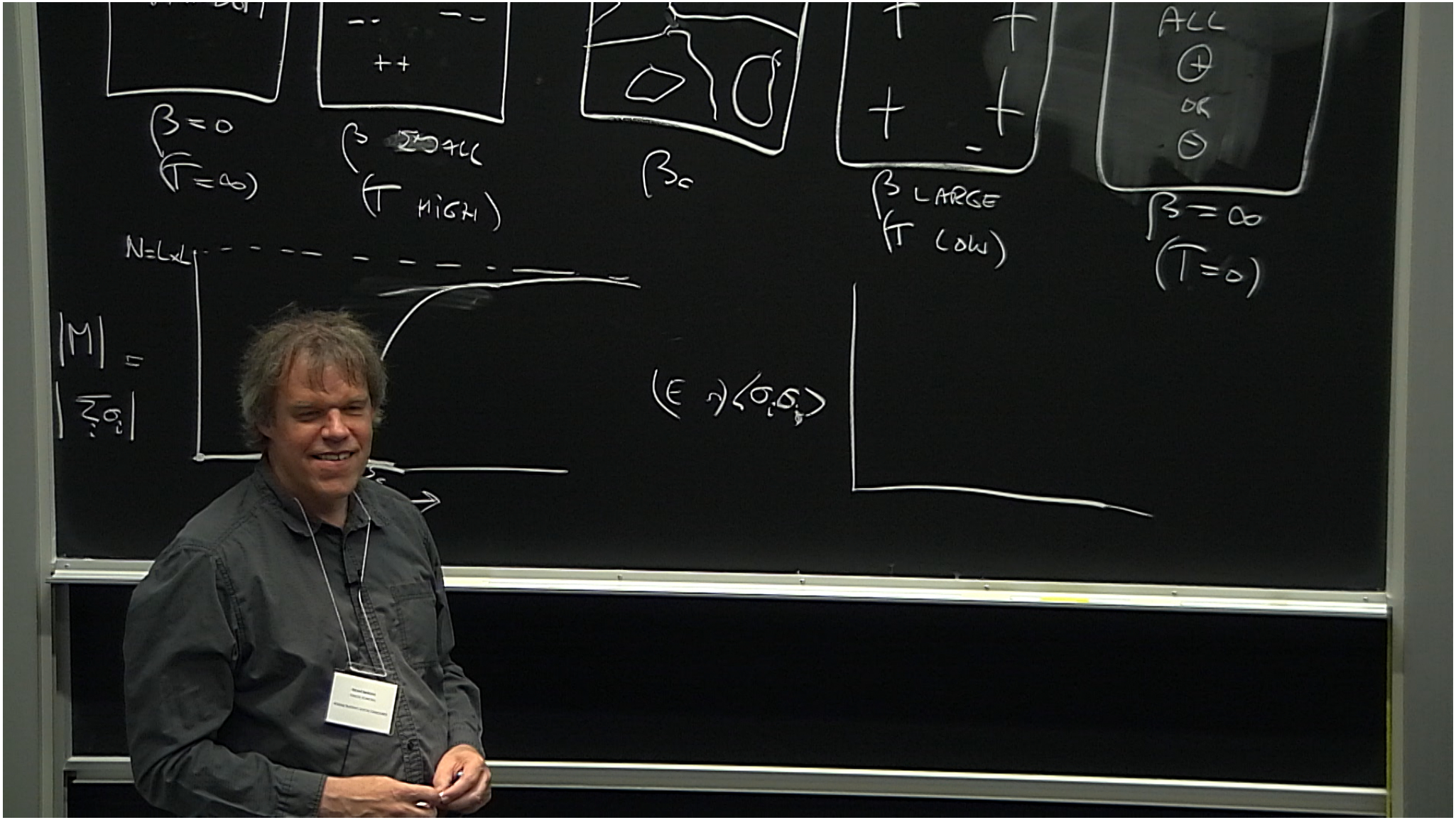

RANDOM
β = 0
(T = ∞)
++
--
--
++
(T HIGH)
β_c
+
+
+
+
−
β LARGE
(T LOW)
ALL
⊕
OR
⊖
β = ∞
(T = 0)

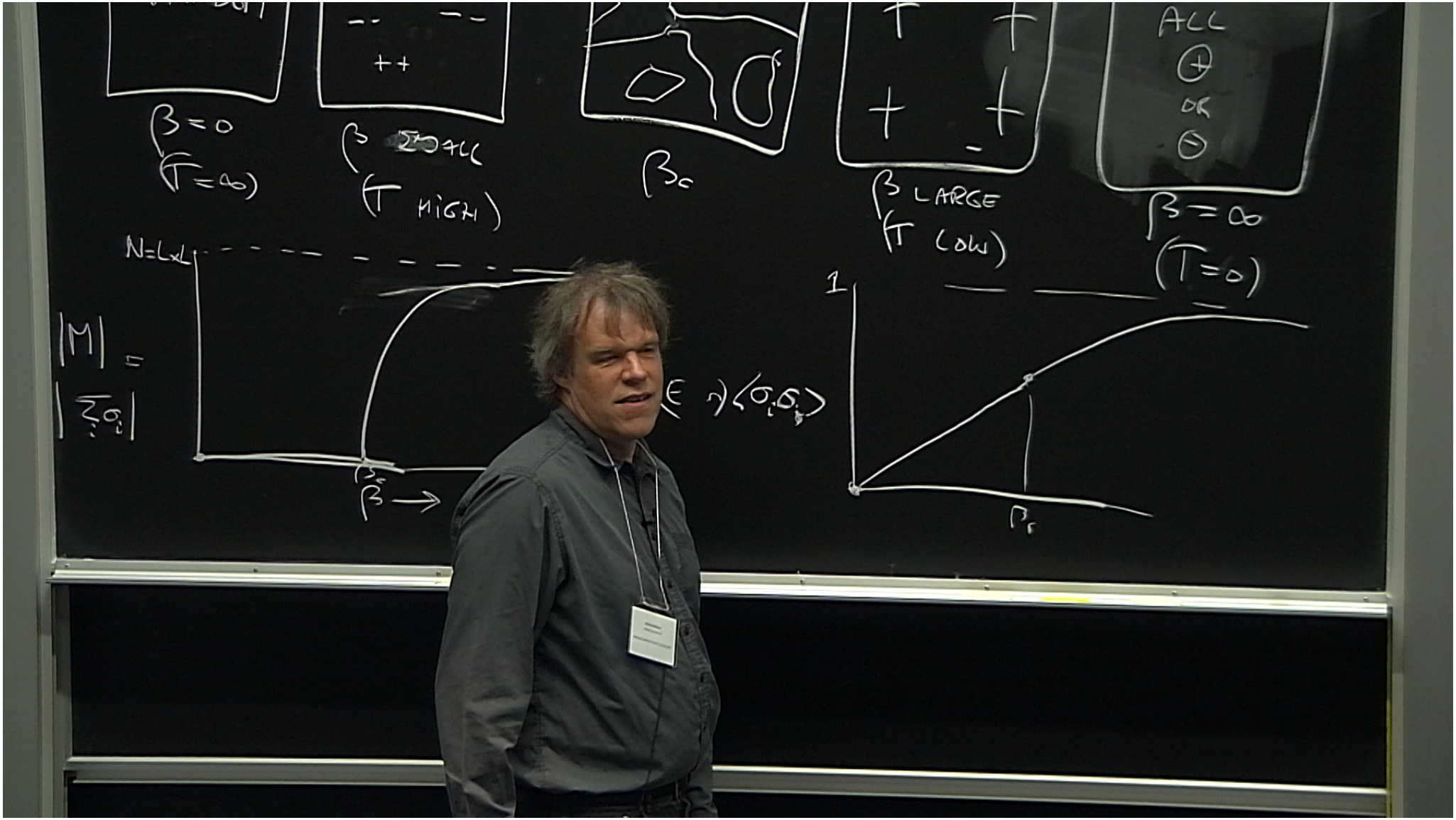

RANDOM
β = 0
(T = ∞)
++
--
--
++
β ALL
(T HIGH)
β_c
β LARGE
(T LOW)
ALL
⊕
OR
⊖
β = ∞
(T = 0)

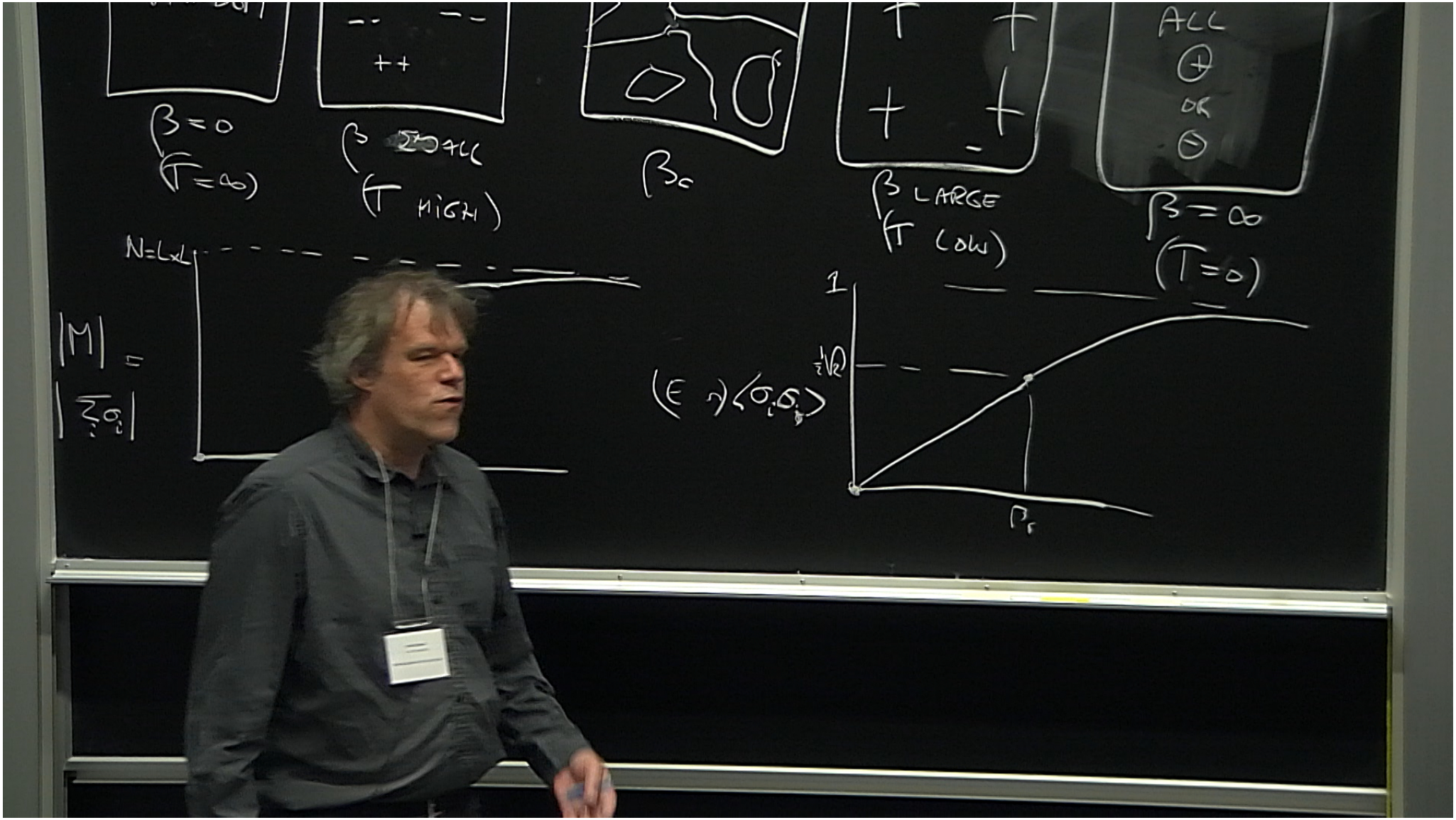

β=0
(T=∞)
++
(T HIGH)
$\beta_c$
β LARGE
(T LOW)
ALL
⊕
OR
⊖
β=∞
(T=0)
N=L×L
|M| =
$|\sum \sigma_i|$
β →

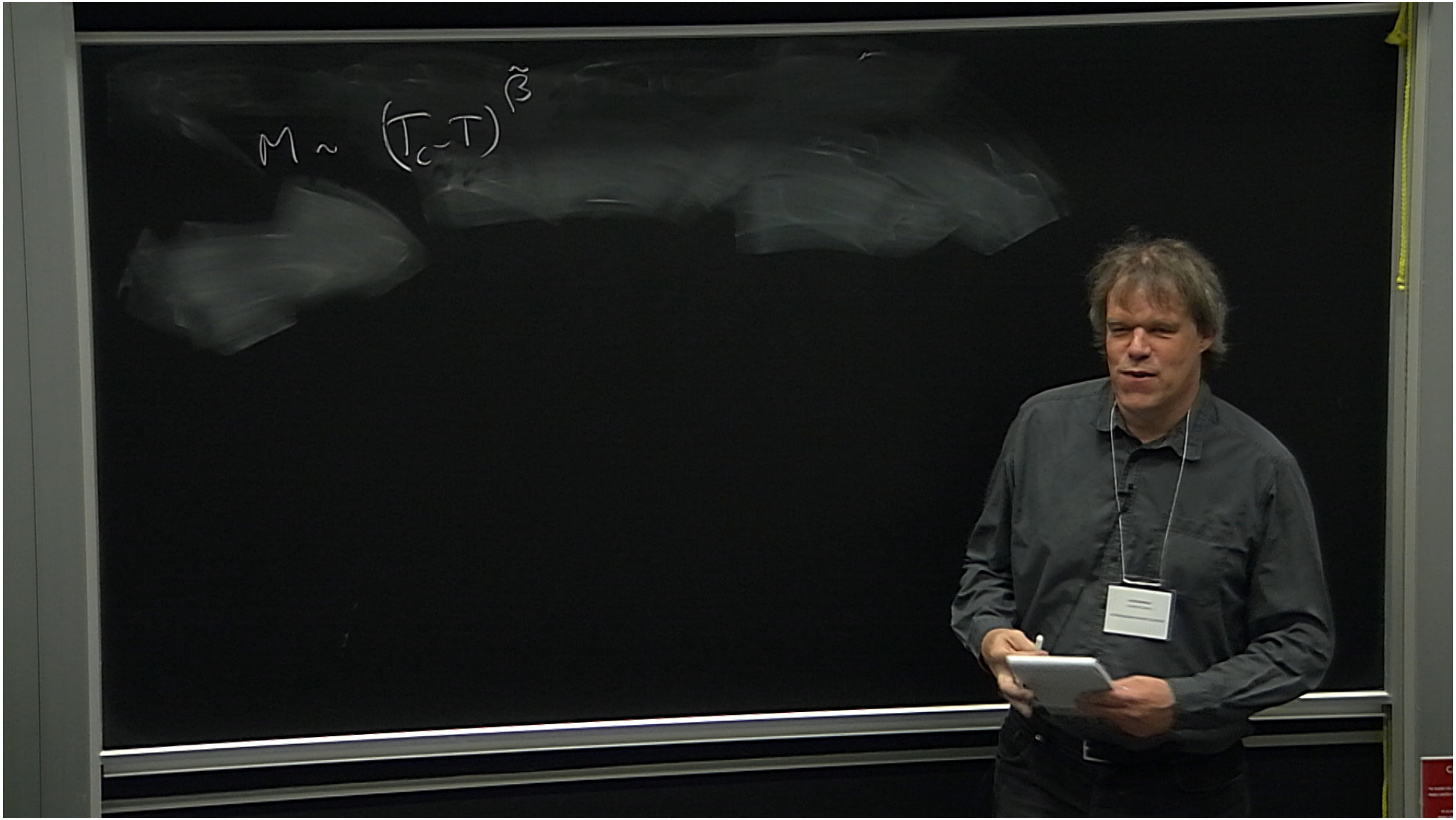

β = 0
(T = ∞)
(T HIGH)
β_c
β LARGE
(T LOW)
ALL
OR
β = ∞
(T = 0)
N = L × L
|M| = |Σ σ_i|
β →

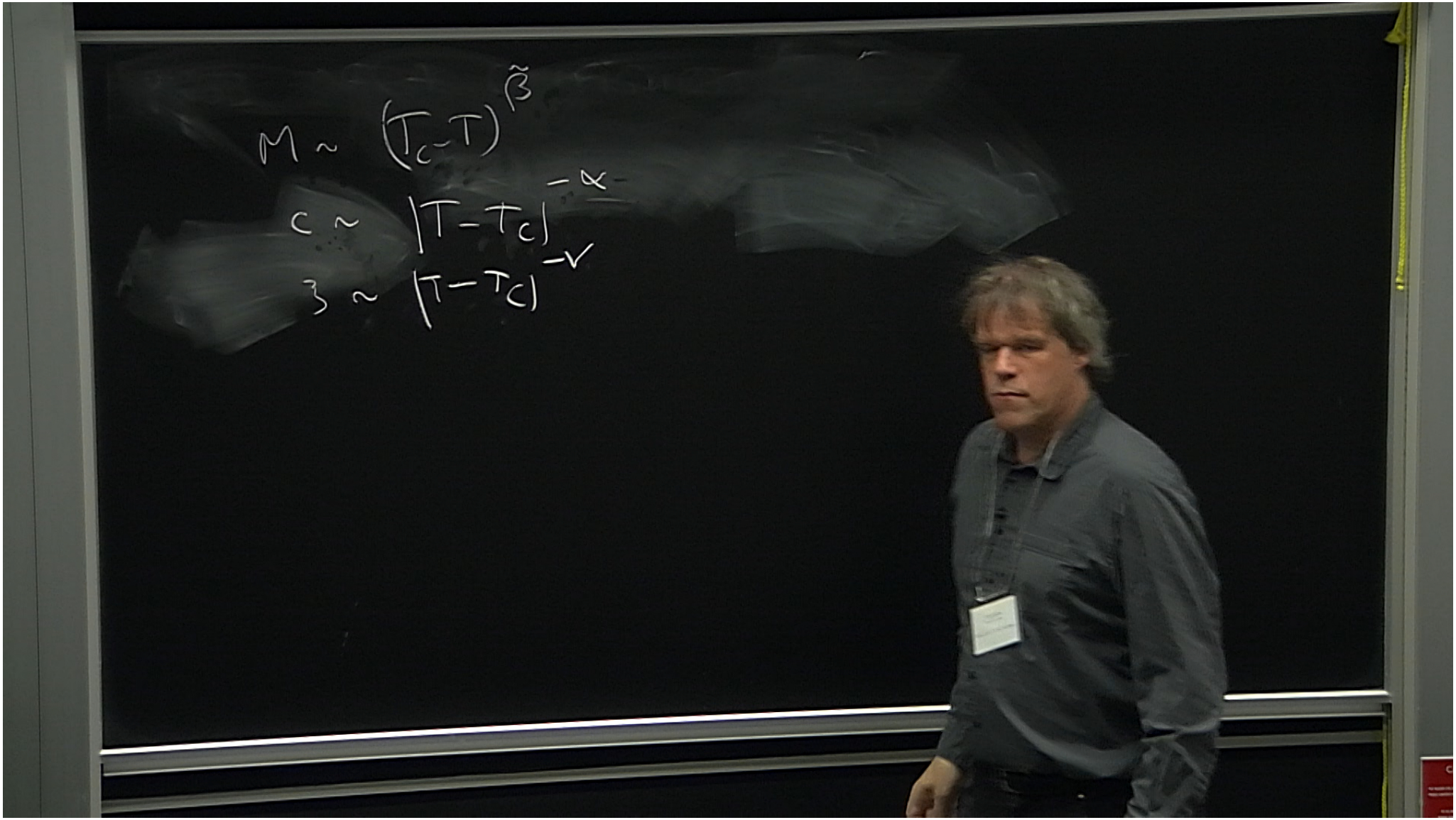

β = 0
(T = ∞)
(T HIGH)
β_c
β LARGE
(T LOW)
ALL
+
OR
−
β = ∞
(T = 0)
N = L × L
|M| = |Σ σ_i|
β_c
β →

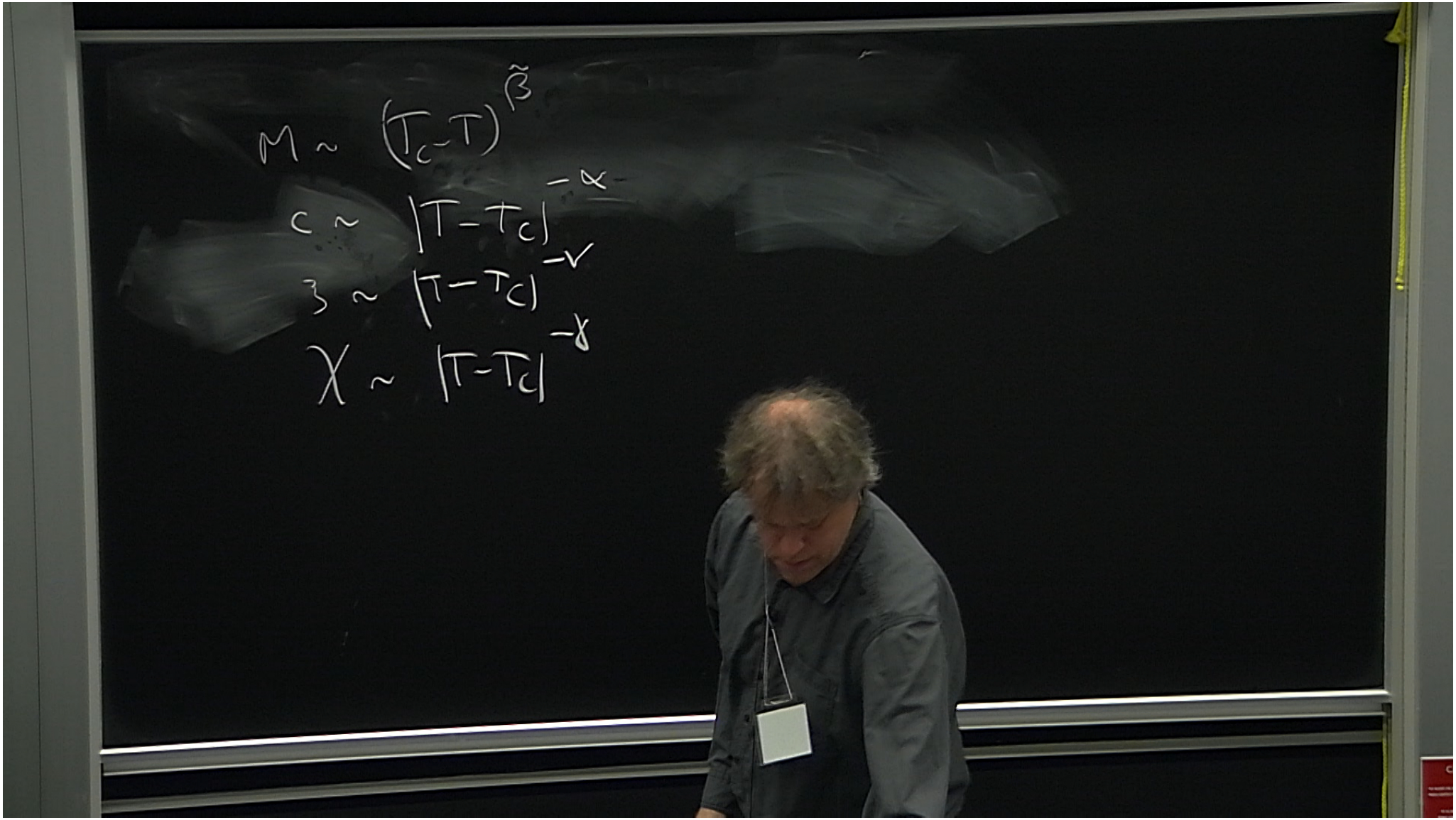

β=0
(T=∞)
(T HIGH)
β_c
β LARGE
(T LOW)
ALL
OR
β=∞
(T=0)
N=L×L
|M| =
|Σ σ_i|
⟨ε⟩⟨σ_i σ_j⟩

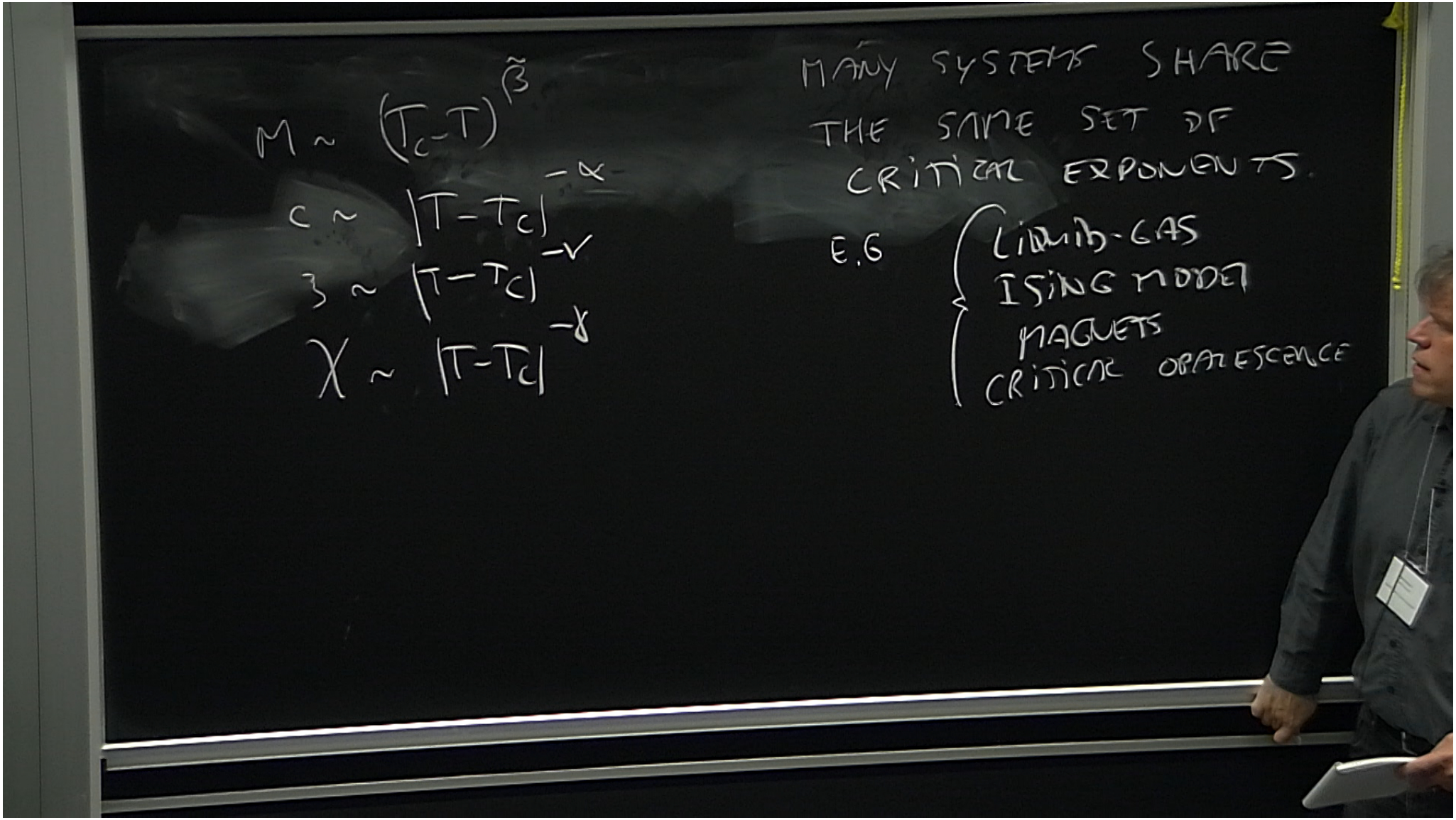

β=0
(T=∞)
(T HIGH)
$\beta_c$
β LARGE
(T LOW)
ALL
OR
β=∞
(T=0)
N=L×L
|M| =
$|\sum \sigma_i|$
$\beta_c$
β →
1
$P_\infty$

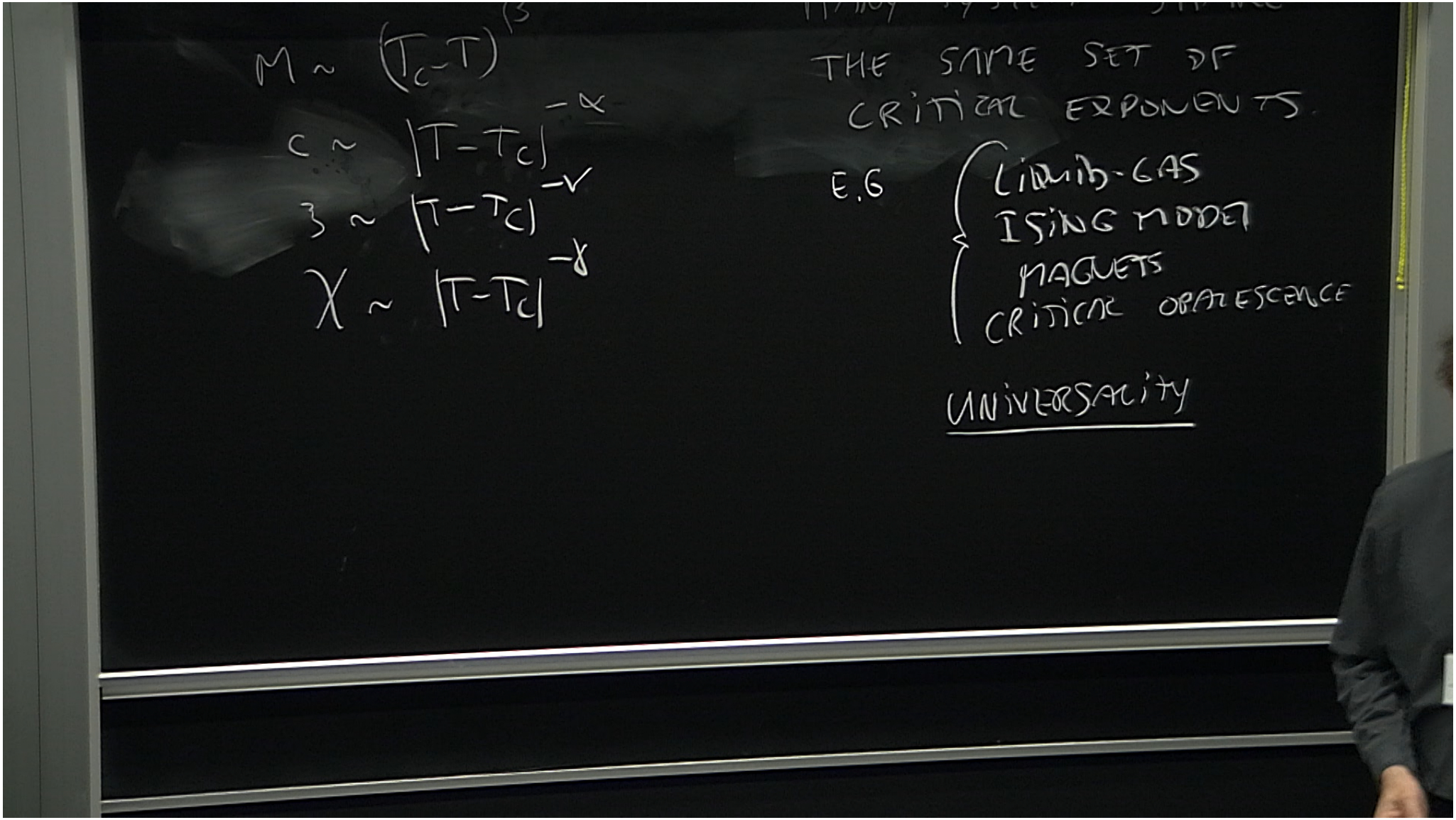

β=0
(T=∞)
(T HIGH)
$\beta_c$
β LARGE
(T LOW)
ALL
⊕
OR
⊖
β=∞
(T=0)
N=L×L
|M| =
$|\sum_i \sigma_i|$
$\langle \epsilon \rangle \langle \sigma_i \sigma_j \rangle$
1
$\frac{1}{2}\sqrt{2}$
$\beta_c$

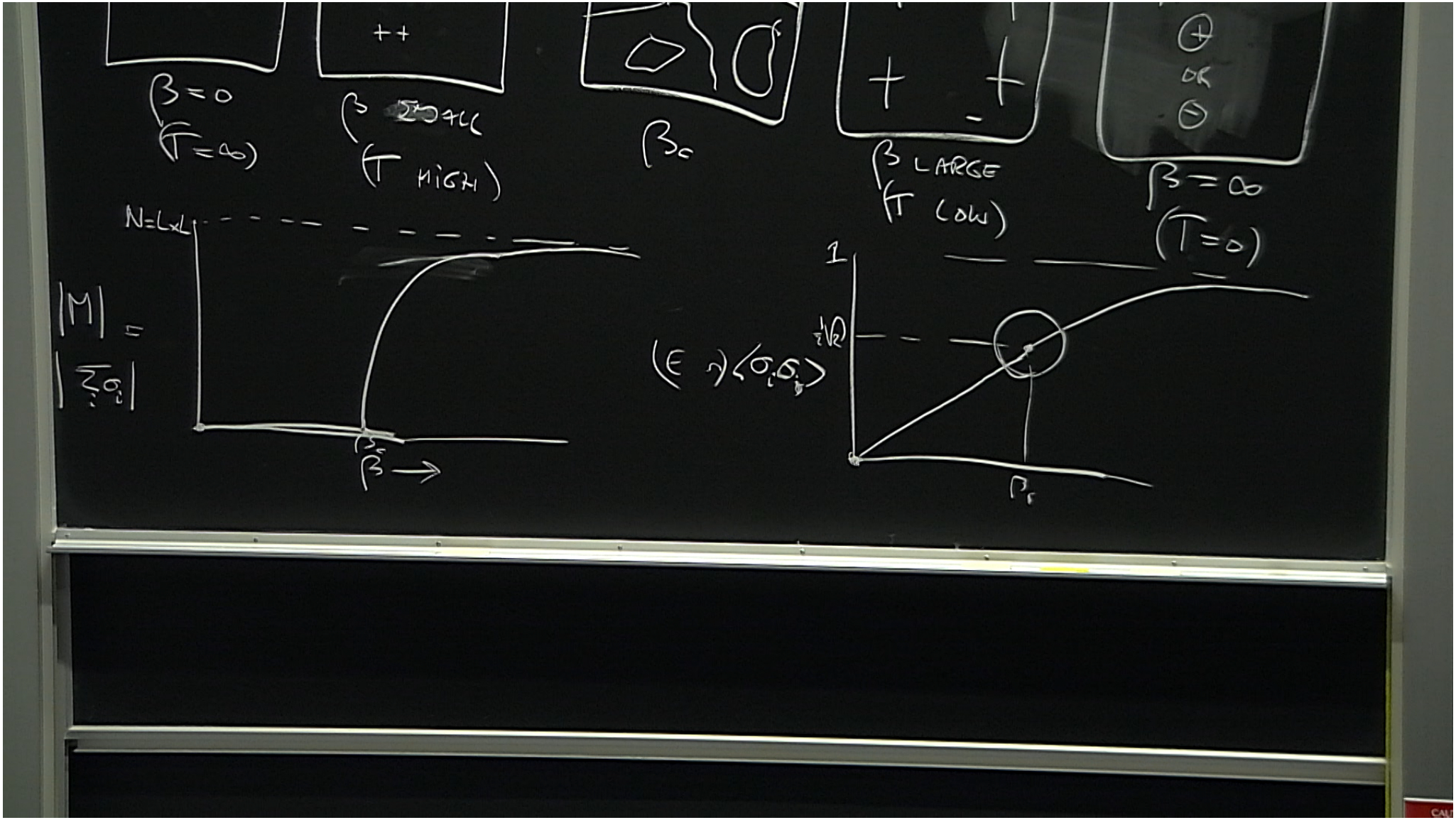

M ~ (T_c - T)^β̃

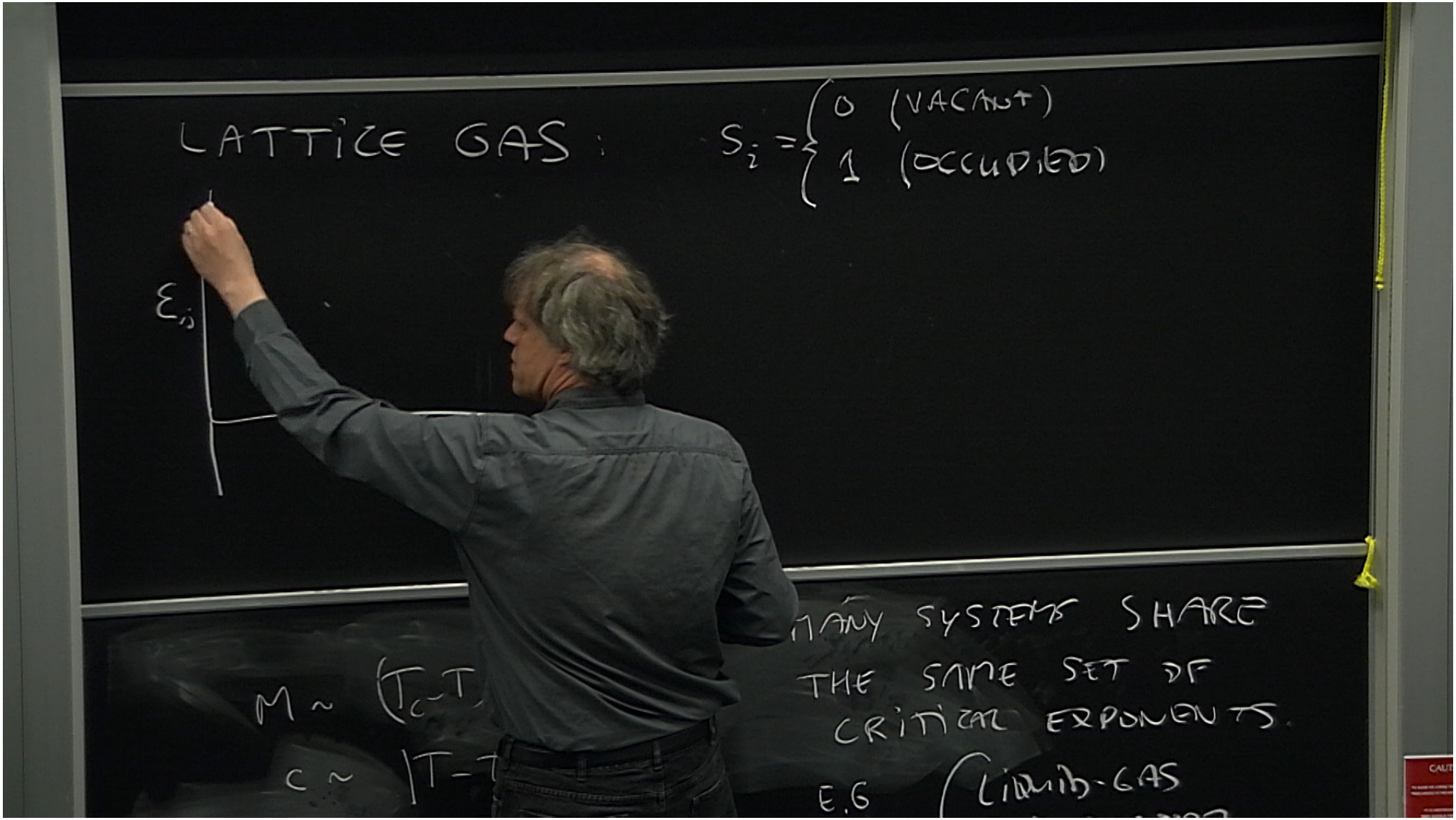

$M \sim (T_c - T)^{\tilde{\beta}}$
$c \sim |T - T_c|^{-\alpha}$
$\xi \sim |T - T_c|^{-\nu}$

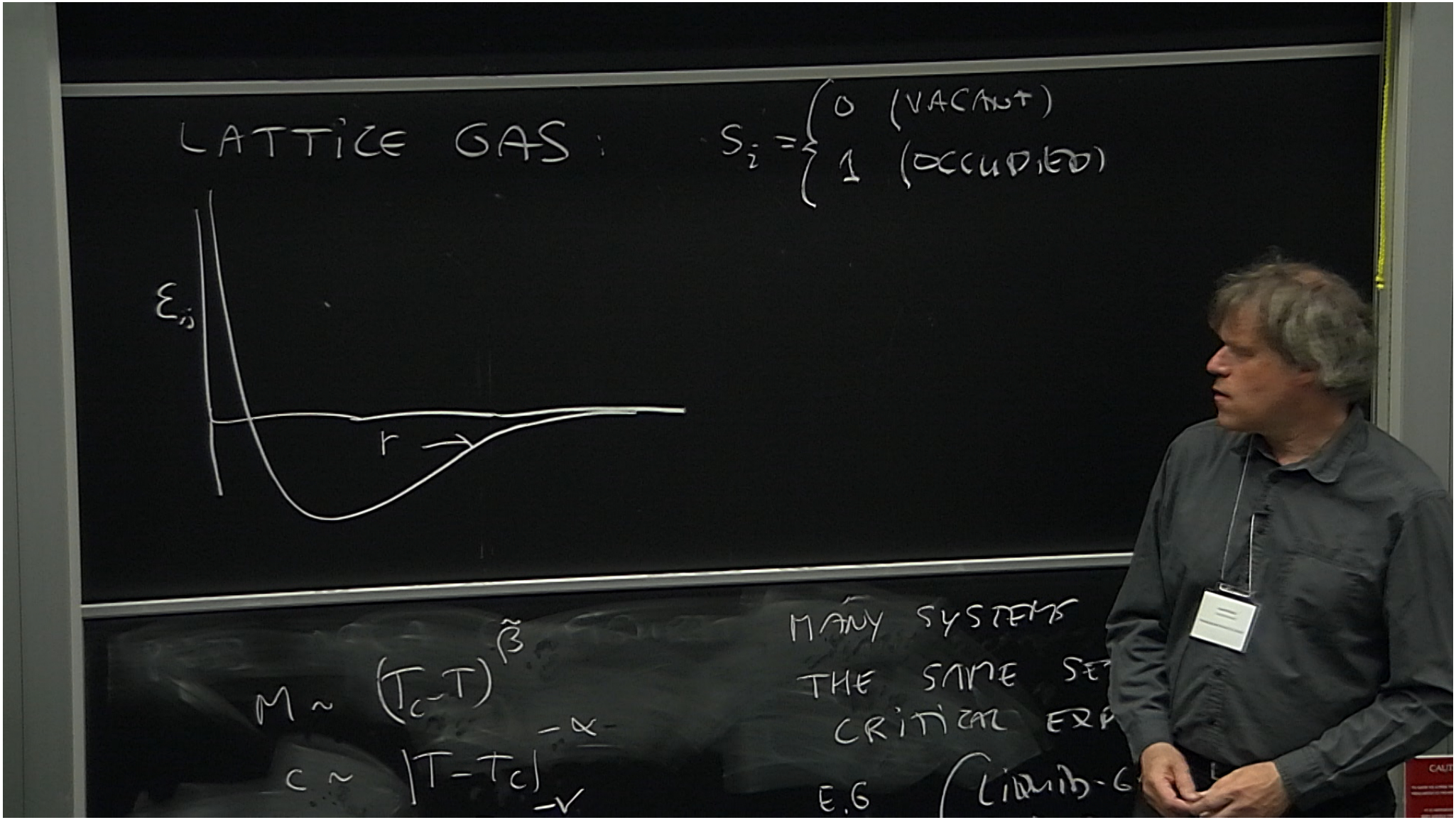

$M \sim (T_c - T)^{\tilde{\beta}}$
$c \sim |T - T_c|^{-\alpha}$
$\xi \sim |T - T_c|^{-\nu}$
$\chi \sim |T - T_c|^{-\gamma}$

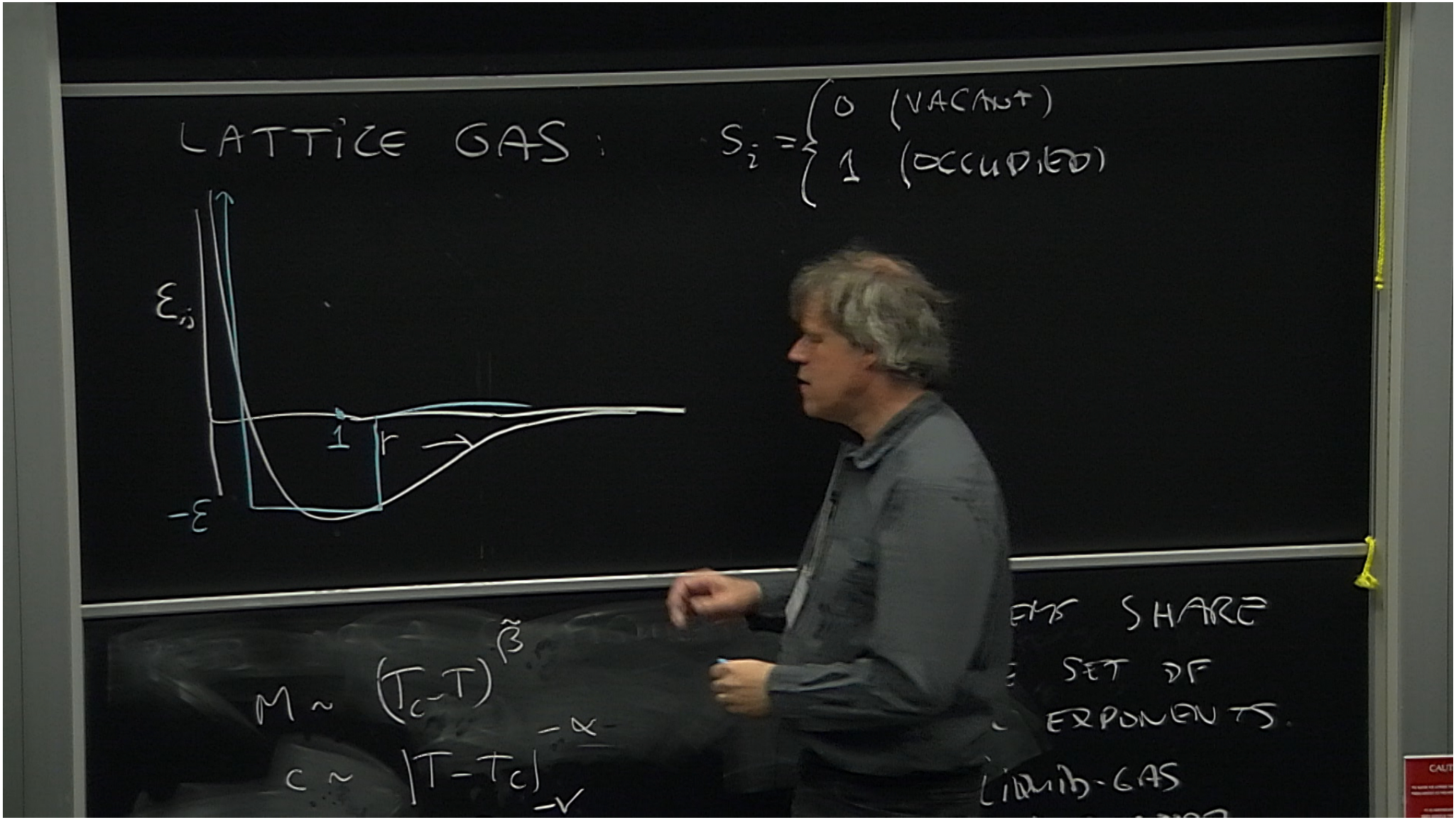

$M \sim (T_c - T)^{\tilde{\beta}}$
$c \sim |T - T_c|^{-\alpha}$
$\xi \sim |T - T_c|^{-\nu}$
$\chi \sim |T - T_c|^{-\gamma}$
MANY SYSTEMS SHARE THE SAME SET OF CRITICAL EXPONENTS.
E.G
LIQUID-GAS
ISING MODEL
MAGNETS
CRITICAL OPALESCENCE

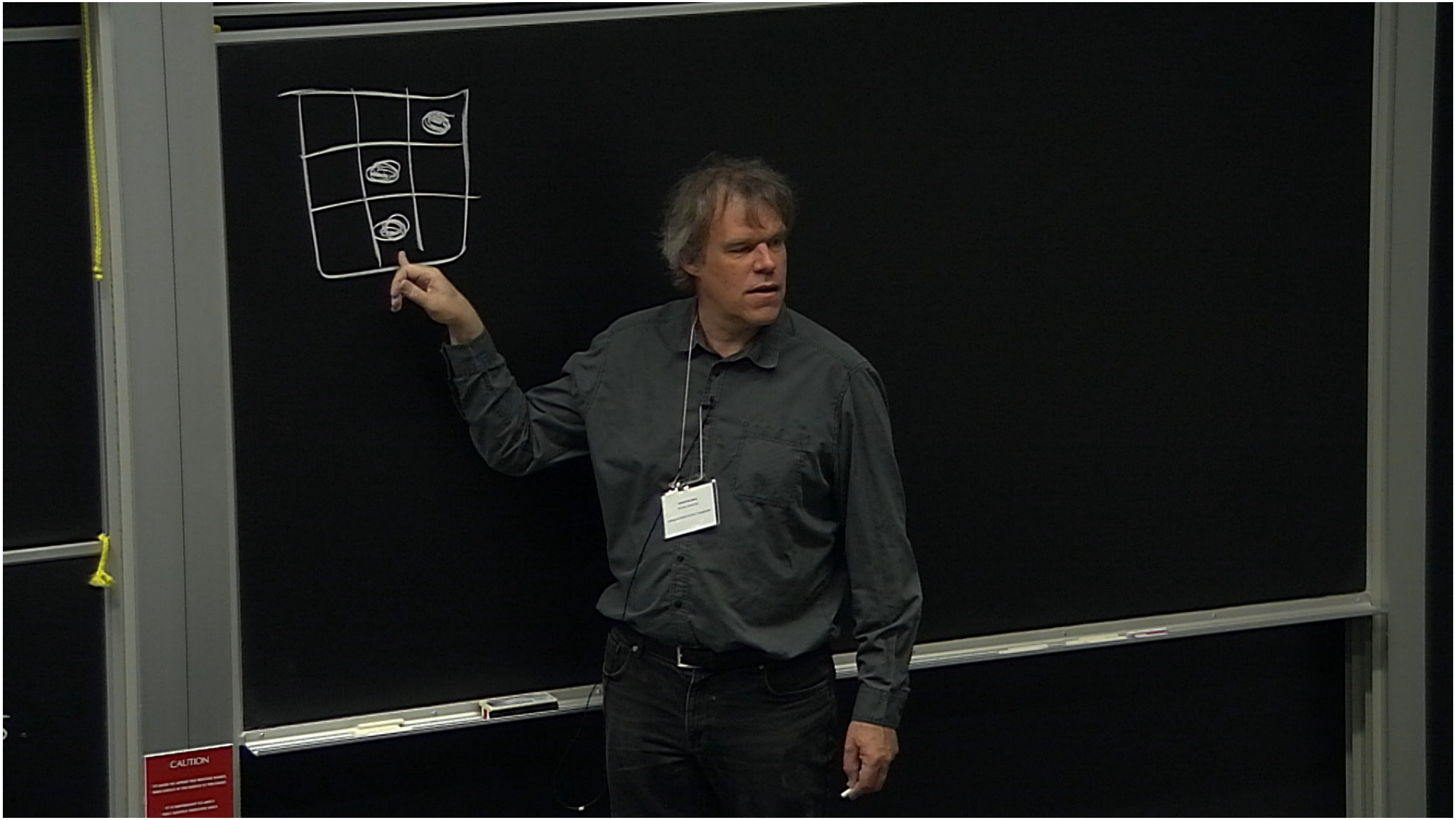

$M \sim (T_c - T)^{\beta}$
$c \sim |T - T_c|^{-\alpha}$
$\xi \sim |T - T_c|^{-\nu}$
$\chi \sim |T - T_c|^{-\gamma}$
THE SAME SET OF CRITICAL EXPONENTS
E.G
LIQUID-GAS
ISING MODEL
MAGNETS
CRITICAL OPALESCENCE
UNIVERSALITY


β = 0
(T = ∞)
++
β SMALL
(T HIGH)
β_c
β LARGE
(T LOW)
⊕
OR
⊖
β = ∞
(T = 0)
N = L × L
|M| = |Σ σ_i|
β_c
β →
⟨ε⟩ ⟨σ_i σ_j⟩
1
½√2
β_c


LATTICE GAS:
$S_i = \begin{cases} 0 & \text{(VACANT)} \\ 1 & \text{(OCCUPIED)} \end{cases}$
$\varepsilon_i$
MANY SYSTEMS SHARE
THE SAME SET OF
CRITICAL EXPONENTS.
E.G

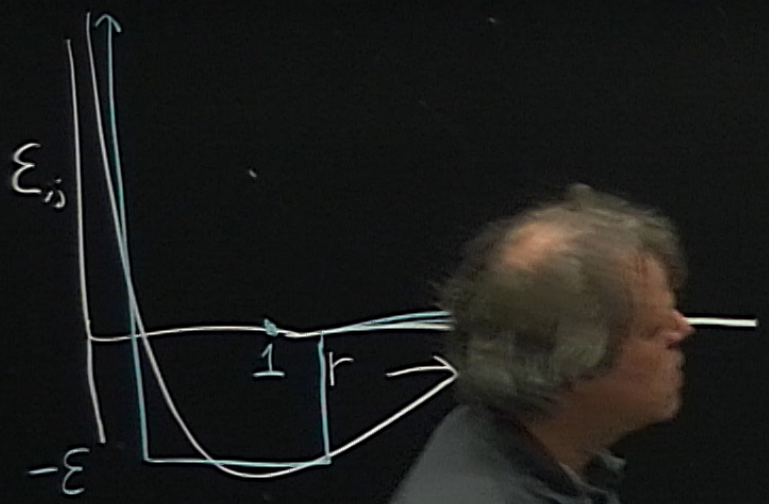

LATTICE GAS:
$S_i = \begin{cases} 0 & \text{(VACANT)} \\ 1 & \text{(OCCUPIED)} \end{cases}$
$\varepsilon_{ij}$
r
$M \sim (T_c - T)^{\tilde{\beta}}$
$c \sim |T - T_c|^{-\alpha}$
$-\nu$
MANY SYSTEMS
THE SAME
CRITICAL
E.G


LATTICE GAS:
$S_i = \begin{cases} 0 & \text{(VACANT)} \\ 1 & \text{(OCCUPIED)} \end{cases}$
$\varepsilon_{ij}$
1
r
$-\varepsilon$
$M \sim (T_c - T)^{\tilde{\beta}}$
$c \sim |T - T_c|^{-\alpha}$
$-\nu$
SHARE
SET OF
EXPONENTS.


CAUTION

LATTICE GAS:

$$S_i = \begin{cases} 0 & \text{(VACANT)} \\ 1 & \text{(OCCUPIED)} \end{cases}$$



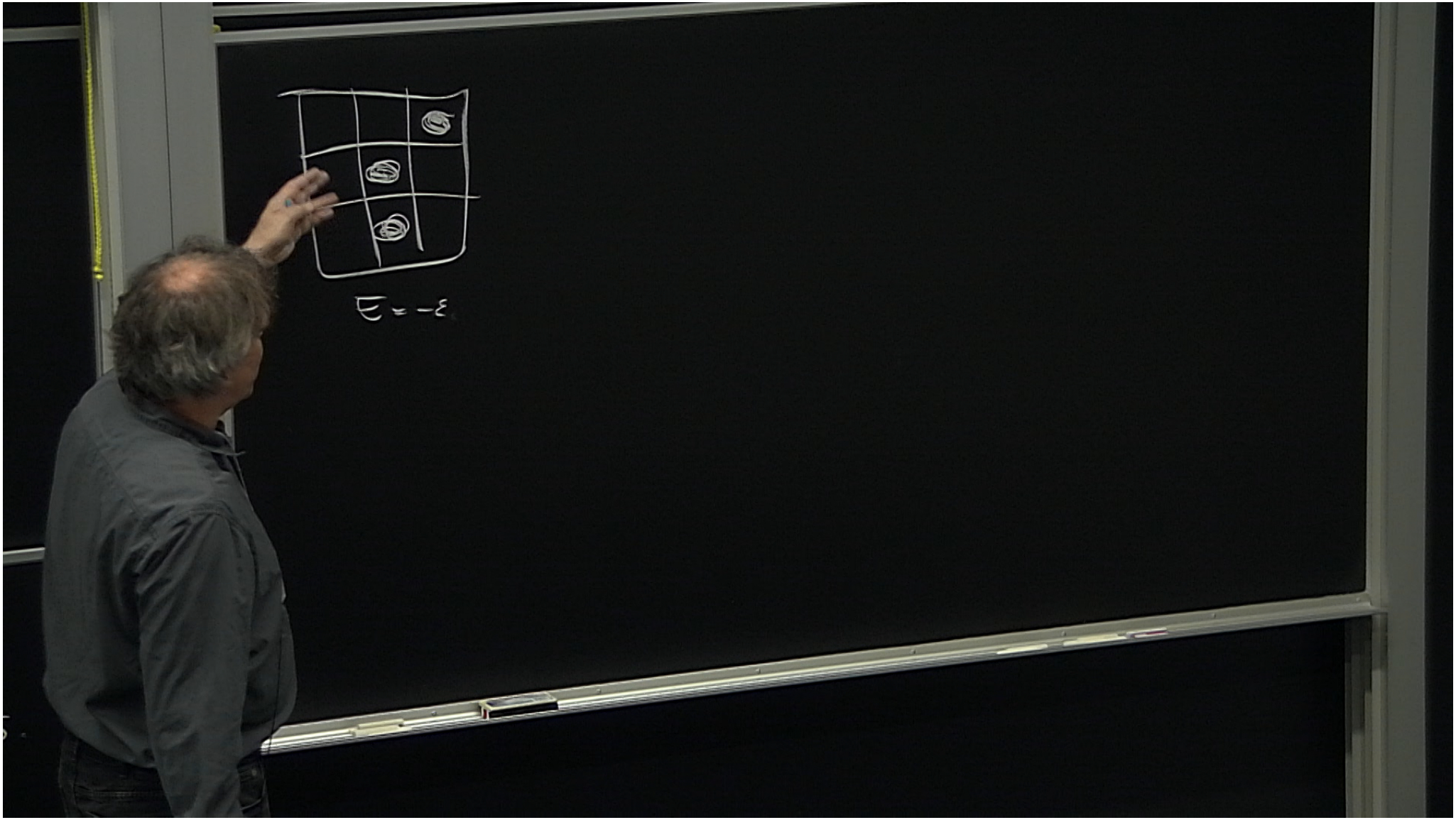

$$H = -\varepsilon \sum_{\langle i,j \rangle} S_i S_j$$

$M \sim$ (

$C \sim$

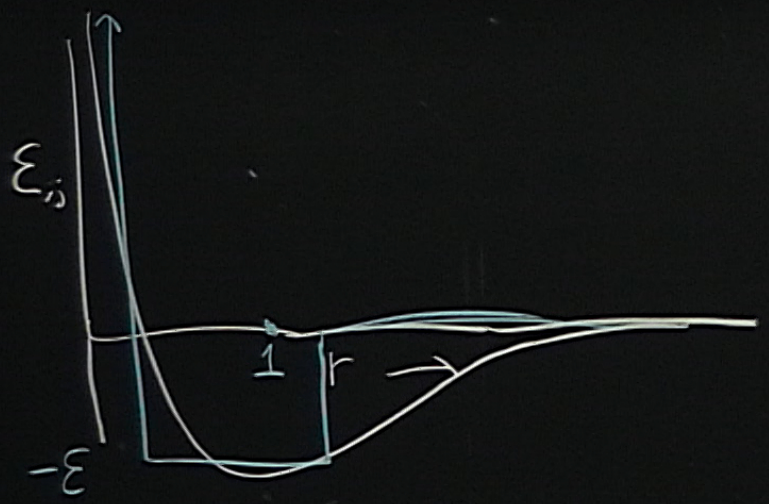
MANY SYSTEMS SHARE THE SAME SET OF CRITICAL EXPONENTS.

E.G (LIQUID-GAS


E = -ε

LATTICE GAS:

$$S_i = \begin{cases} 0 & \text{(VACANT)} \\ 1 & \text{(OCCUPIED)} \end{cases}$$




$$H = -\varepsilon \sum_{\langle i,j \rangle} S_i S_j$$

TAKE $\sigma_i = 2S_i - 1$ (=)

$S_i = \frac{1}{2}\sigma_i + \frac{1}{2}$

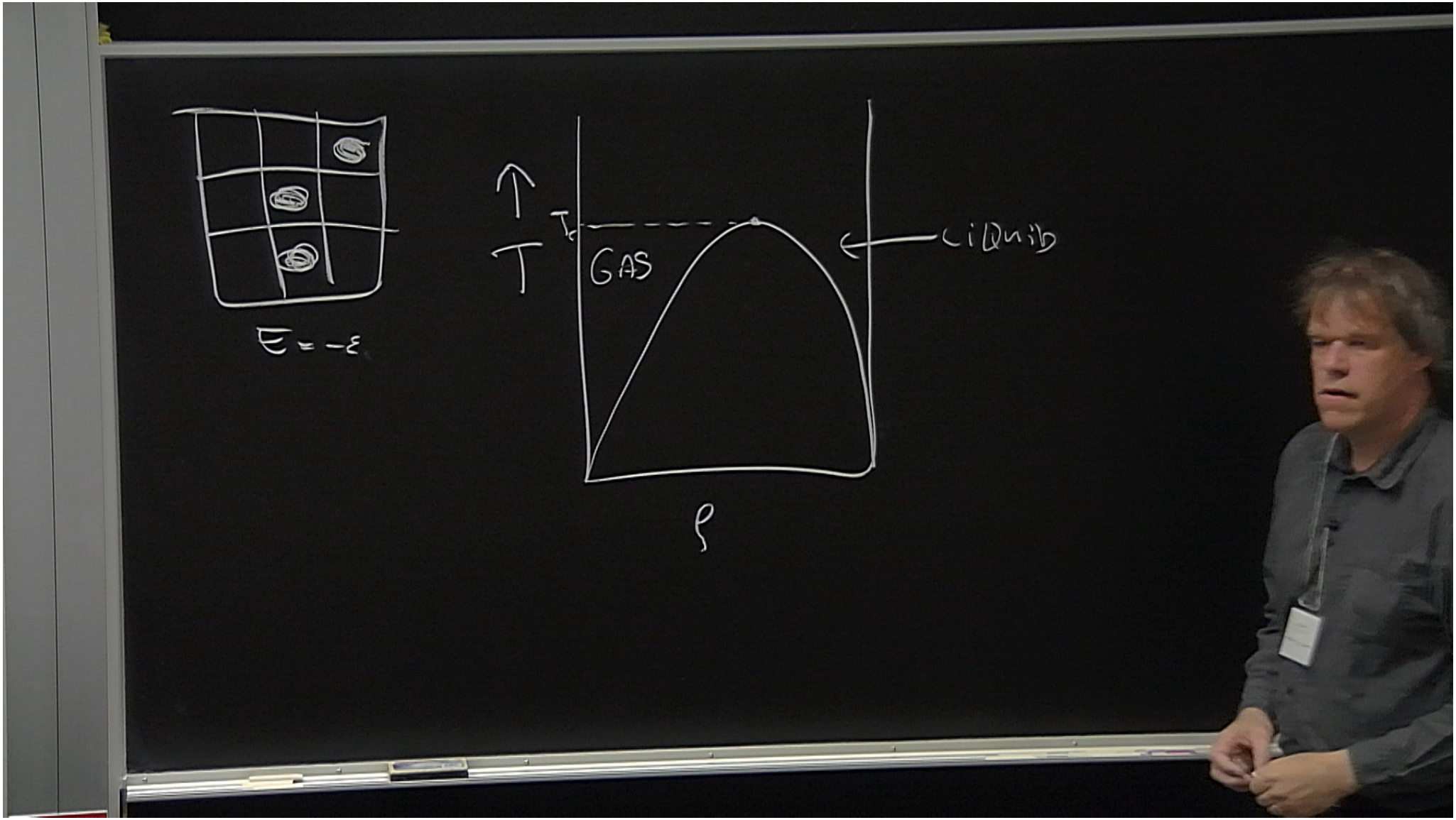
$$M \sim (T_c - T)^{\tilde{\beta}}$$

$$C \sim |T - T_c|^{-\alpha}$$

$$-\nu$$

MANY SYSTEMS SHARE THE SAME SET OF CRITICAL EXPONENTS.

E.G (Liquid-GAS


E = -ε
T
GAS
liquid
ρ