Title: Black hole superradiance in a bathtub vortex

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Abstract: <p>I will discuss the possibilities to mimic black hole physics in fluid flows. The starting point is an analogy discovered by Unruh between the propagation of sound in a flowing fluid and waves around a black hole. In these analog setups, it is possible to test various black hole effects, and challenge their robustness. In a recent water wave experiment, we have shown how to exploit this analogy to observe superradiant scattering, that is, the amplification of waves by extraction of angular momentum from a rotating flow.</p>

Black hole superradiance in a bathtub vortex

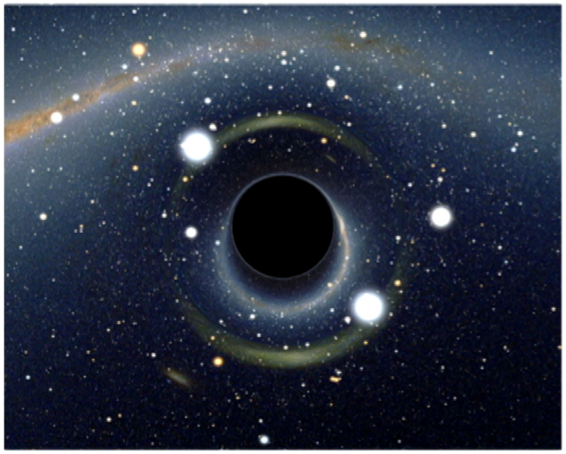
Antonin Coutant

15 June 2017

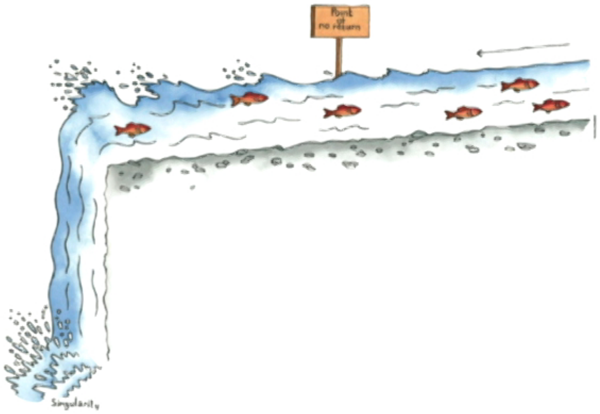
At: Perimeter Institute

From University of Nottingham,
supported by Marie Skłodowska-Curie Actions (Horizon2020)

The University of Nottingham



Black Hole simulation: Riazuelo, '07

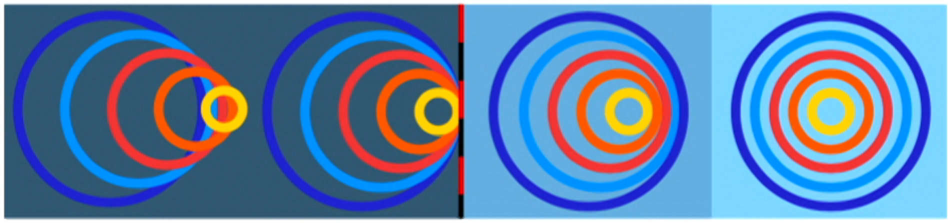


[Picture Credit: Yan Nascimbene]

Analogie

[Unruh 1981]

Wave propagation around a black hole	$\leftrightarrow$	Wave propagation in a moving medium

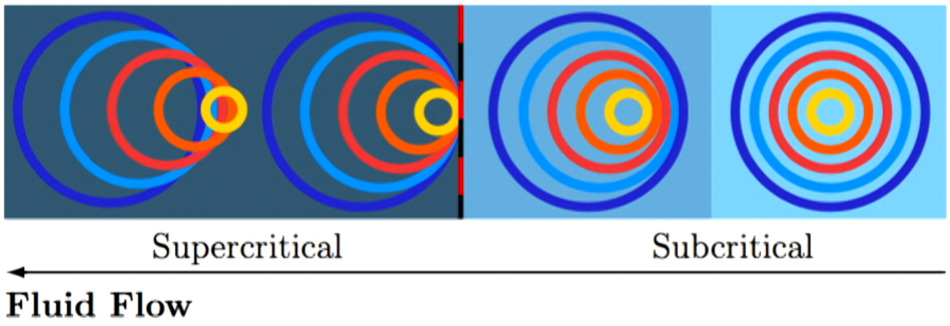


Hydrodynamical regime

- Sound waves of celerity c on moving fluid
- Current velocity $v(x)$
- Wave propagation

$$\left[(\partial_t + \partial_x v)(\partial_t + v\partial_x) - c^2\partial_x^2\right]\phi(x,t) = 0$$

- Geometry $ds^2 = c^2dt^2 - (dx - vdt)^2$
- **Acoustic Black Hole:** Horizon where $v = \pm c$



- Same in higher dimensions $\rightarrow$ possibility of rotation
- If $v_r^2 + v_\theta^2 > c^2 \rightarrow$ analog **ergoregion**
  $\rightarrow$ **waves must co-rotate**
- If $v_r^2 > c^2 \rightarrow$ analog **horizon**
  $\rightarrow$ **waves trapped**

Large class of analogue systems

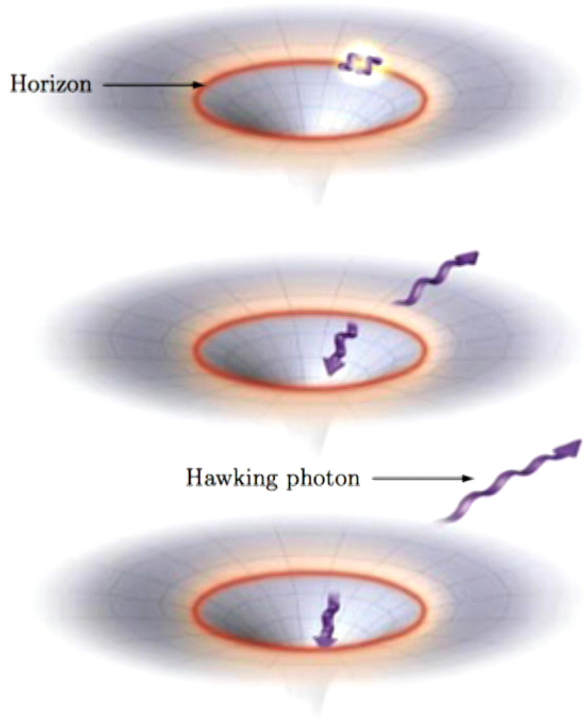
- Classical fluids
  - Sound in a flowing fluid [Unruh '81]
  - Surface waves in shallow water [Schutzhold, Unruh '02]
  - Acoustic waves in ducts [Aurégan *et al.* '15]
- Quantum fluids
  - Sound in a superfluid [Comer '92]
  - Sound in a Bose-Einstein condensate [Garay *et al.* '00]
  - Surface waves of a superfluid [Volovik '06]
  - Photons in nonlinear optical media [Leonhardt, Piwnicki '00]

Historical motivation

- Hawking radiation [Hawking '74]
- Quantum field in a black hole background
- Spontaneous emission of **thermal flux**

$$n_\omega = \frac{1}{e^{\omega/T_H} - 1}$$

- Consequences:
  - Black hole entropy
  - Black hole evaporation
  - Information paradox



(Image credit: Scientific American, 2007)

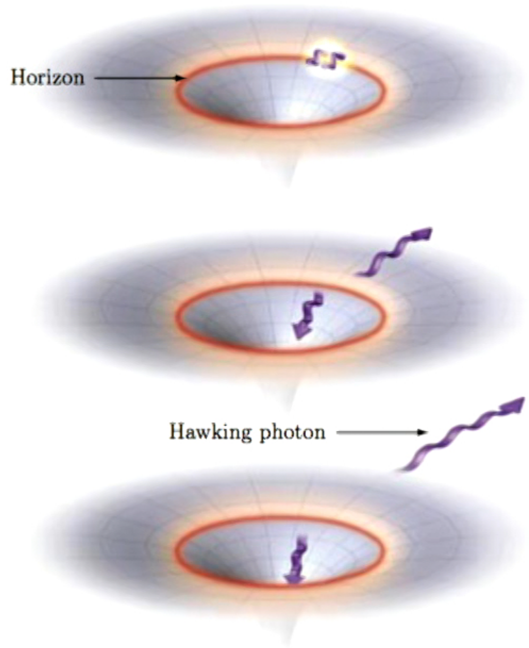
Historical motivation

But

- High redshift near horizon
- Low energy particles from high energy fluctuations
- Solar mass BH, 1s after formation

$$\frac{\Omega_{\text{in}}}{\Omega_{\text{out}}} = e^{10^5}$$

- **Transplackian problem**



(Image credit: Scientific American, 2007)

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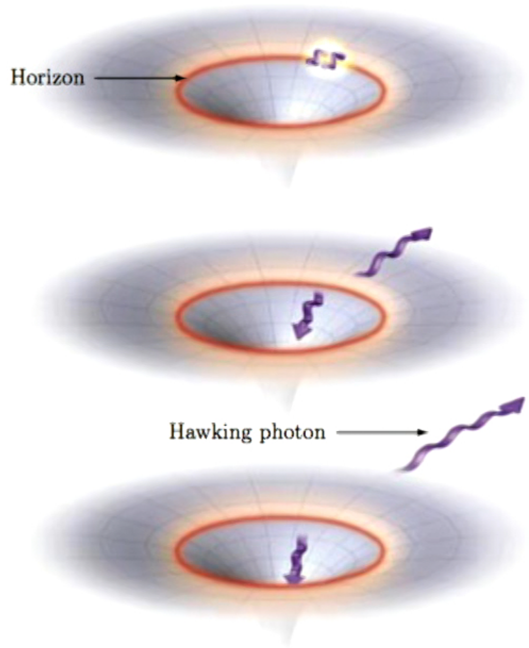
- **Transplackian problem**

Does Hawking radiation depends on the micro-physics?

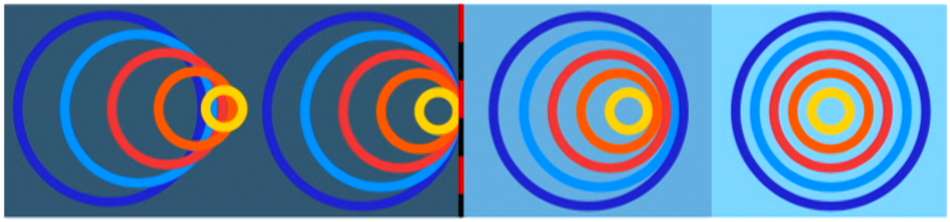
- In a fluid, we know the microscopic theory

[Unruh '81]



(Image credit: Scientific American, 2007)

What happens to the local wavelength?



- High redshift → **short wavelengths**
- **Short distance physics** → **dispersion** [Jacobson '91]

$$\Omega^2 = c^2k^2 \pm c^4k^4/\Lambda^2 + O(k^6)$$

Ω is the frequency in the **fluid frame**

Is the Hawking effect robust?

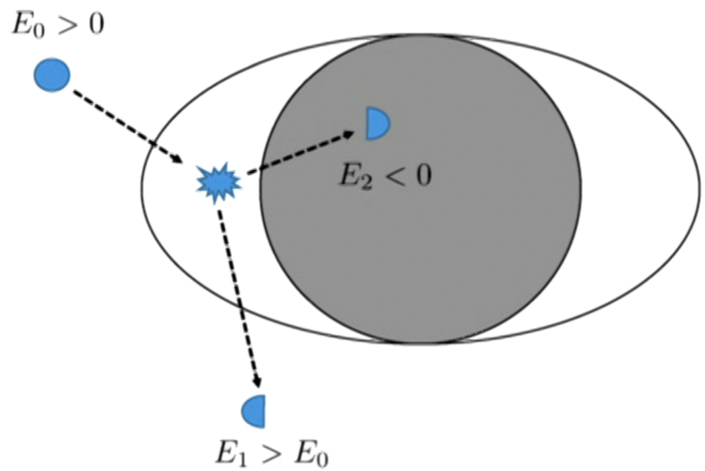
- **Yes:** many theoretical work (1995 → now)
  [Unruh, Jacobson, Corley, Brout, Massar, Parentani, AC and others]

What about experiments?

- **Classical Hawking effect** for water waves
  - Vancouver '10 [Weinfurtner, Tedford, Penrice, Unruh, Lawrence]
  - Poitiers '15 [Euvé, Michel, Parentani, Philbin, Rousseaux]
- **Quantum Hawking effect**
  - Optical fibers [Faccio *et al.* '10]
  - Bose Condensates [Steinhauer '14 and '16]
- **Many open questions**
  - Explain water waves spectrum (non-thermal) [Michel, Parentani '14] [AC, Weinfurtner '15]
  - Entanglement of produced pairs? [Steinhauer '16]

New experimental directions

- **Analog cosmological particle production**
  - Entanglement of pairs? [Jaskula *et al.* '12]
- **Higher dimensional flows**
  - Rotation $\to$ superradiance [Nottingham Lab. '17]
  - Necessary ingredient of the Hawking effect and thermodynamics
- **Black hole ring down** [Dolan, Oliveira, Crispino '11]



Superradiance is the wave pendant of this process

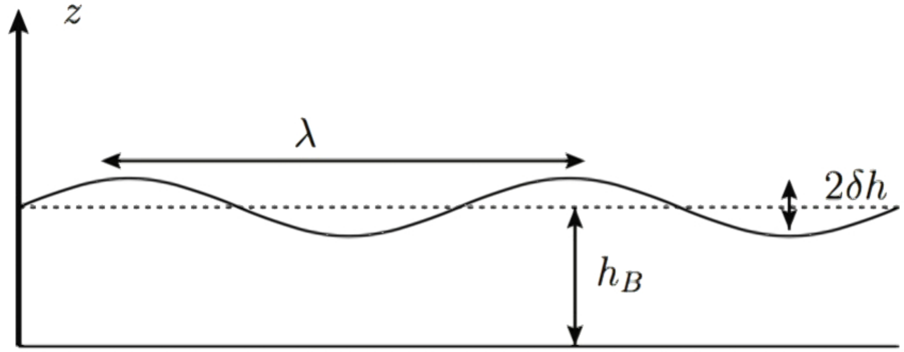
- Incoming wave of energy E_{in}
- Outgoing wave **amplified** $E_{\text{out}} > E_{\text{in}}$

Black hole superradiance under the condition

$$0 < \omega < m\Omega_{\mathcal{H}}$$

$\Omega_{\mathcal{H}}$ is angular velocity on horizon, m the azimuthal number

Water waves



- Dispersion relation

$$\omega^2 = gk \tanh(h_B k)$$

- More complicated propagation
- **Shallow water** $\rightarrow$ space-time analogy

$$\omega^2 = c^2 k^2 \qquad \text{with} \quad c^2 = g h_B$$

- Same effect should be present [Richartz, *et al.* '09] [Dolan '10] [Richartz, *et al.* '14]

- **Simple models** of an irrotational vortex [Dolan '10]

[Richartz, Prain, Liberati, Weinfurtner '14]

$$\vec{v} = -\frac{D}{r}\vec{e}_r + \frac{C}{r}\vec{e}_\theta$$

- Works for shallow water (no dispersion)
- **Horizon** at $r = D/c$
- **Ergoregion** $r < \sqrt{D^2 + C^2}/c$
- **Analogue superradiance** under the condition

$$0 < \omega < m\Omega_{\mathcal{H}} = \frac{mCc^2}{D^2}$$

- Many theoretical questions still to address
- E.g. superradiance can also be present if dispersion

[Richartz, Prain, Weinfurtner, Liberati '09]

- **General (2+1) theory:** wave scatter on a rotating flow
- **Circular symmetry:** different m are decoupled

$$\phi(t,r,\theta) = \varphi(r) e^{-i\omega t + im\theta}/\sqrt{r}$$

- Outside vortex core

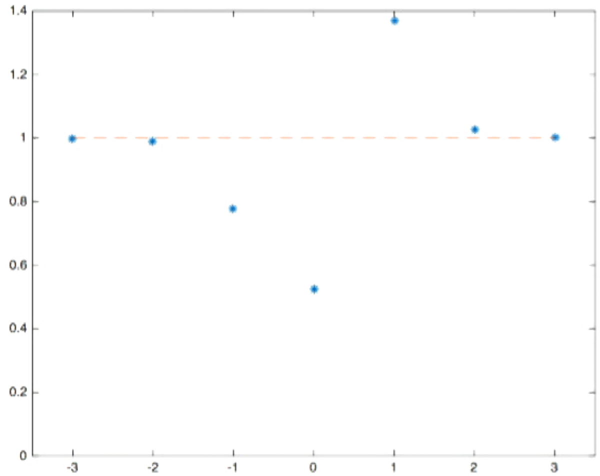
$$\varphi(r) \sim A_{\text{in}} e^{-ikr} + A_{\text{out}} e^{ikr}.$$

- **Reflection coefficient**

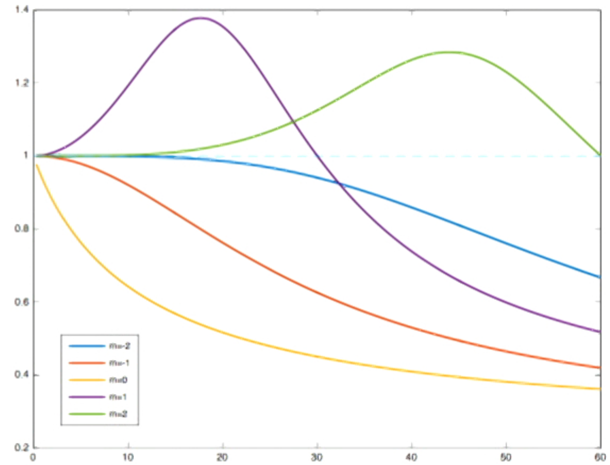
$$R = \frac{A_{\text{out}}}{A_{\text{in}}}$$

  - $|R| < 1 \rightarrow$ absorption
  - $|R| > 1 \rightarrow$ amplification

Amplification spectrum



Fixed background and ω
Varying m.

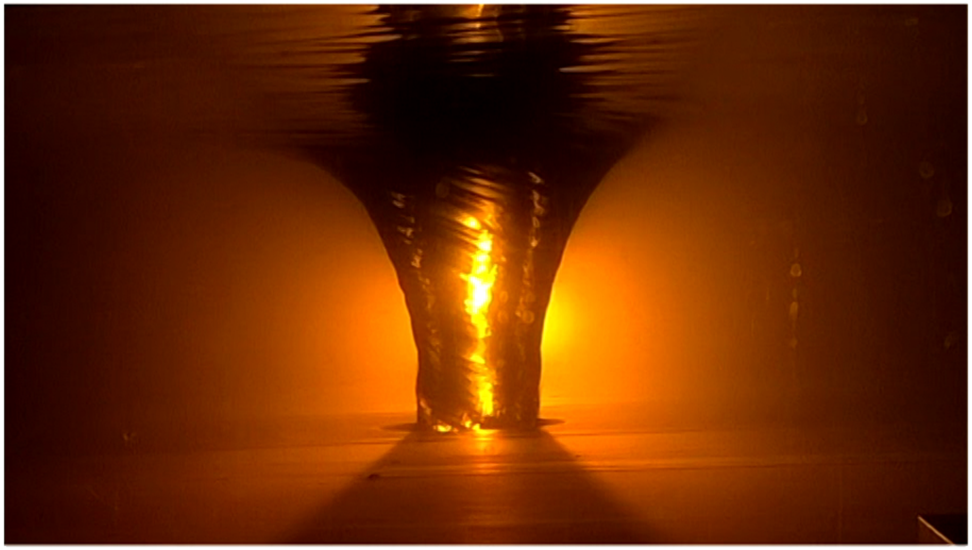


Fixed background
Varying ω.

Typical behavior

- Negative m's (counter-rotating) are **absorbed**
- Positive m's (co-rotating) are **amplified**

$$0 < \omega < m\Omega_{\mathcal{H}}$$

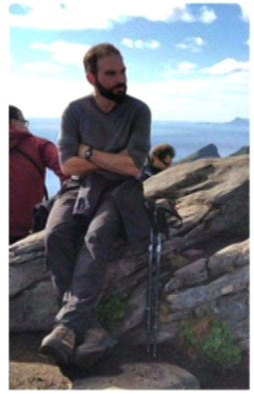


Can we observe this effect experimentally?

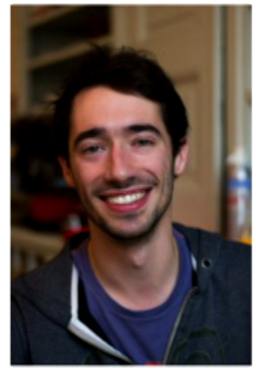
[Torres, Patrick, AC, Richartz, Tedford, Weinfurtner, *Nature Phys.* '17]

Zack Fifer

Sam Patrick



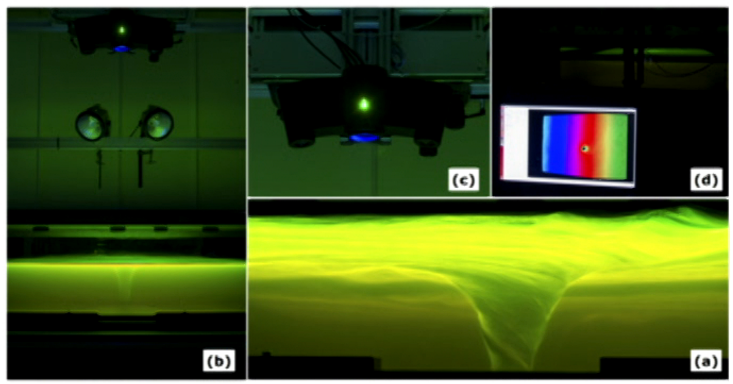
Théo Torres



Antonin Coutant

Silke Weinfurtner

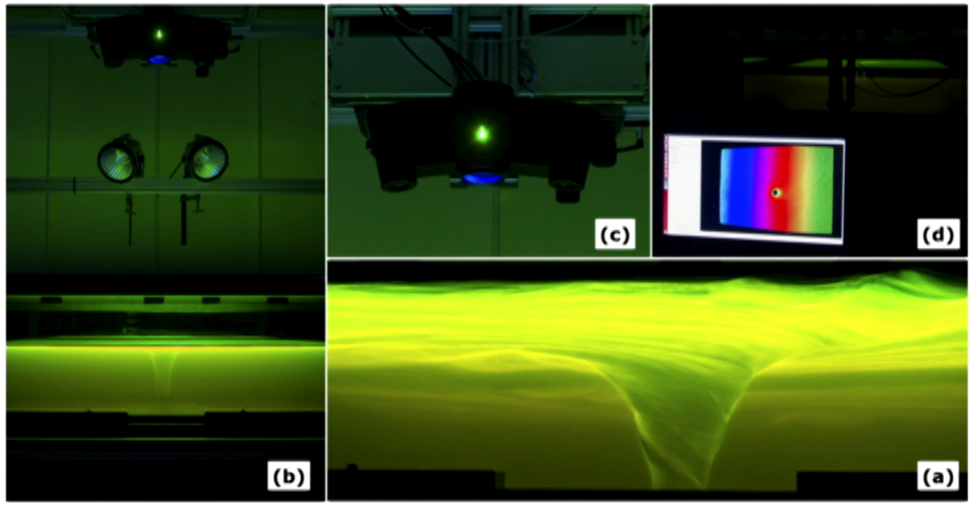
Mauricio Richartz (Sao Paulo)



Ted Tedford (Vancouver)

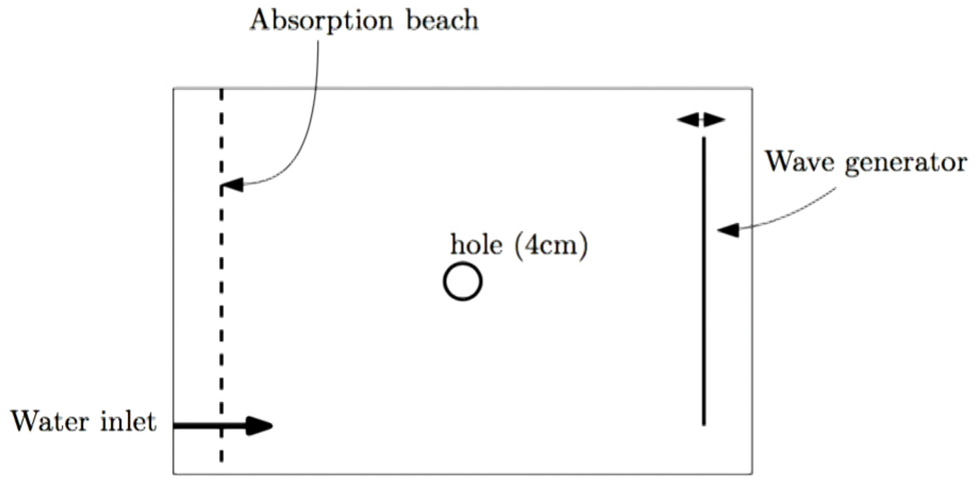
The Black Hole Laboratory

Experimental setup



- Water tank of 3m $\times$1.5m
- Closed circuit, water pumped from lower tank
  $\rightarrow$ **stationary rotating** flow
- Wave generator on one side
- Absorption beach on other side

Experimental setup



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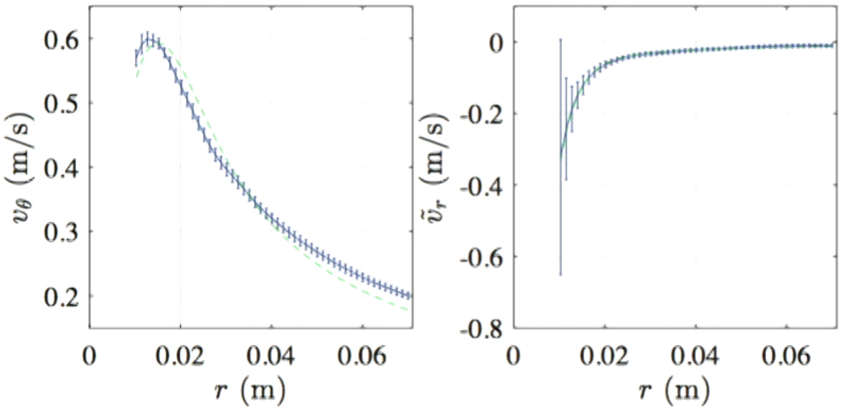
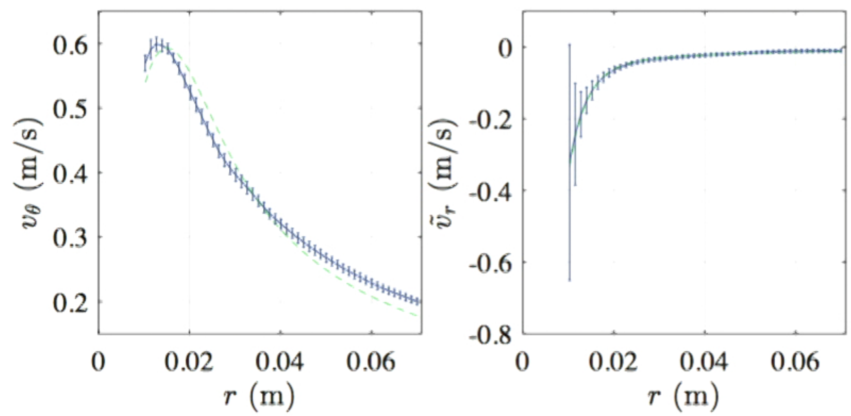
Background flow



- Method: Particle Imaging Velocimetry (PIV)
- Obtain radial $\tilde{v}_r$ and orthoradial velocity v_θ **at the free surface**
- $r \gtrsim 2\text{cm} \rightarrow$ ideal vortex

Background flow



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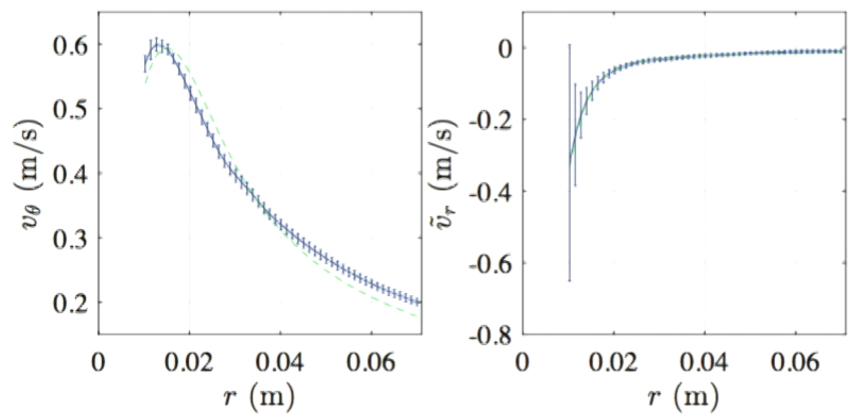
Core effects $\rightarrow$ **Fit to Lamb model**

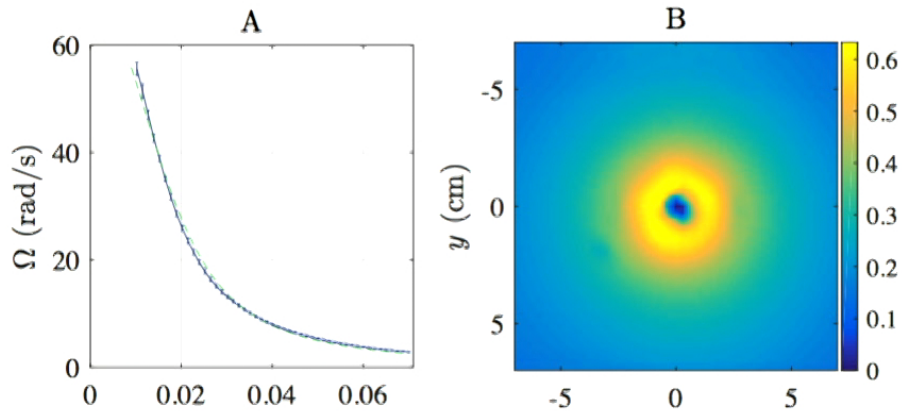
$$v_{r/\theta} = \frac{A}{r}\left(1 - e^{-r^2/r_c^2}\right)$$

Background flow



- Water depth 6.4cm
  → **Deep water regime** (dispersion)
- Notion of horizon **unclear** [AC, Parentani '14]
- First try: using group velocity $v_g \approx 0.2 - 0.3$ m/s
  - Ergoregion → ok
  - Horizon → in the core

Background flow



Left: angular velocity - Right: Velocity norm in (r, θ) plane

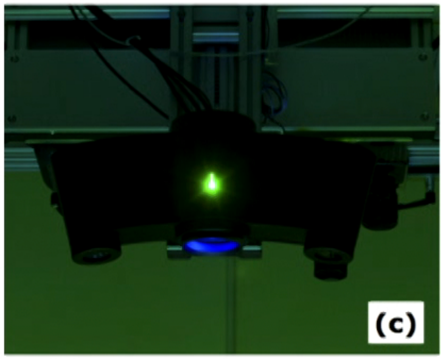
Two important conditions

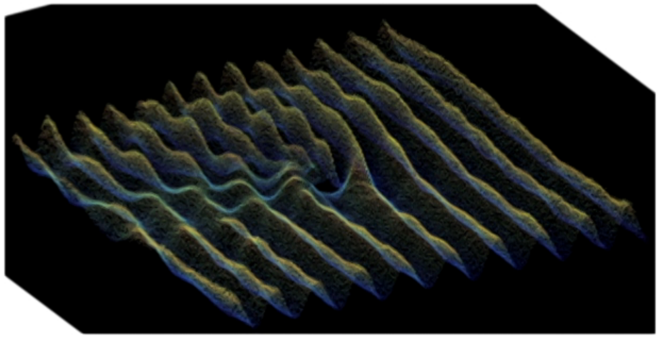
- **High angular velocities**

$$0 < \omega < m\Omega$$

  Necessary condition for superradiance
- Flow is **symmetric** $\Delta R \lesssim 3\%$
  $\rightarrow$ angular momentum of wave conserved

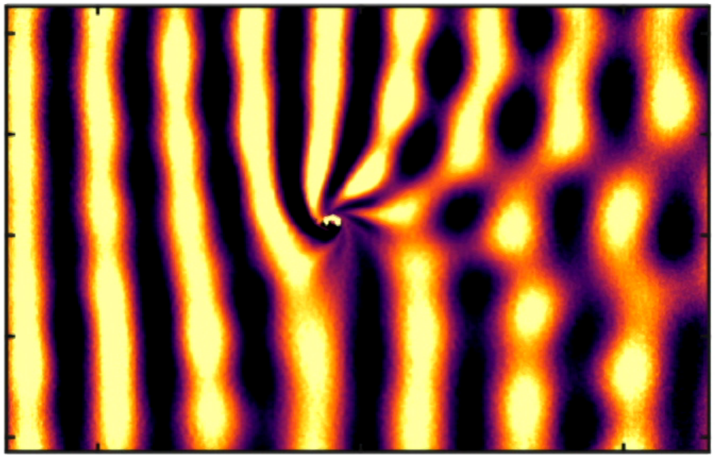
Scattering experiment





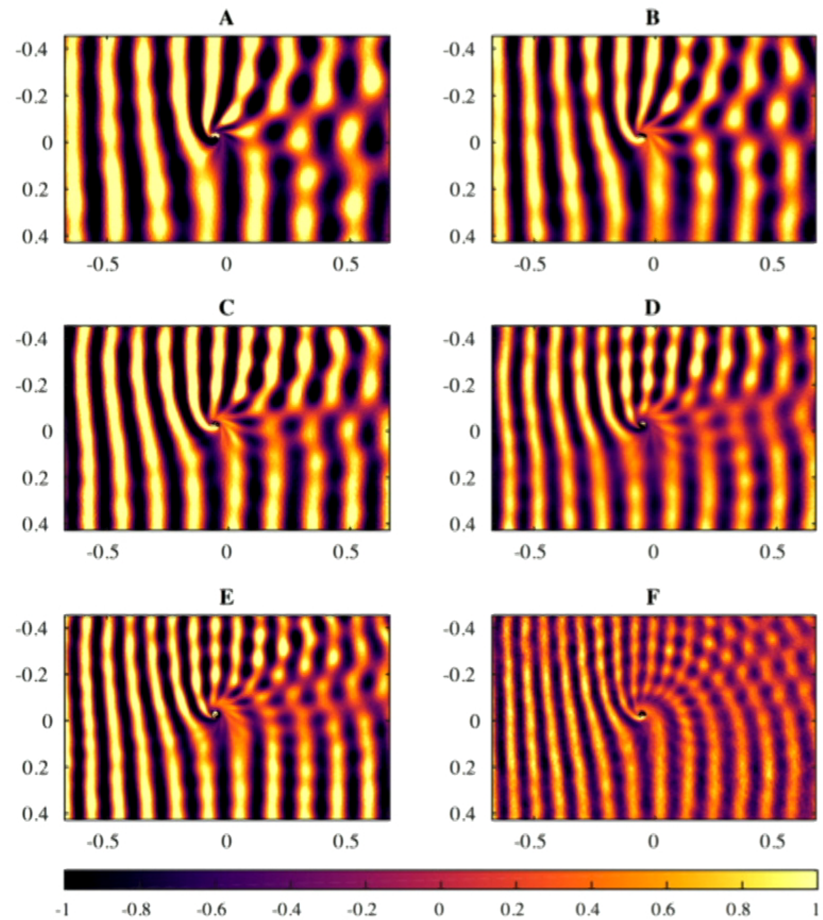
Detection method

- Stereographic imaging
- 3D reconstruction of the free surface in real time
- Joint invention between The University of Nottingham and EnShape GmbH (Patent application)



Scattering experiment

- We send a **plane wave**
- Plane wave propagate and scatter on vortex
- **Pattern** = interfering sum of **Incoming wave** + **Scattered wave**



First outcome:
analog Aharonov-Bohm effect
[Berry, Chambers, Large, Upstill, Walmsley '80]

Analog Aharonov-Bohm effect

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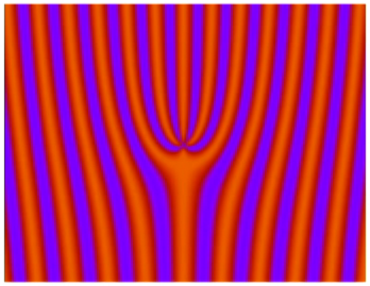


- Waves on rotating flow $\vec{v}$
- Propagate with group velocity v_g
- When $v \ll v_g$, phase shift

$$\vec{\nabla}\varphi = \vec{k} - \frac{k}{v_g}\vec{v}$$

- Analog of the Aharonov-Bohm phase

$$\frac{q\vec{A}}{\hbar} \leftrightarrow -\frac{k\vec{v}}{v_g}$$

- Long range flow $\vec{v} = \frac{C}{r}\vec{e}_\theta$

$$\oint \vec{\nabla}\varphi \cdot d\vec{l} \neq 0$$

How do we extract reflection coefficients?

Data analysis

Step 1: filter in frequency

- Extract **excitation frequency** f_0
- Eliminate background flow (at $f = 0$)
- Eliminate higher harmonics
- frequency range $2.9\text{Hz} \lesssim f_0 \lesssim 4.1\text{Hz}$

Page 48 sur 63

Data analysis

Step 2: Decompose into azimuthal waves

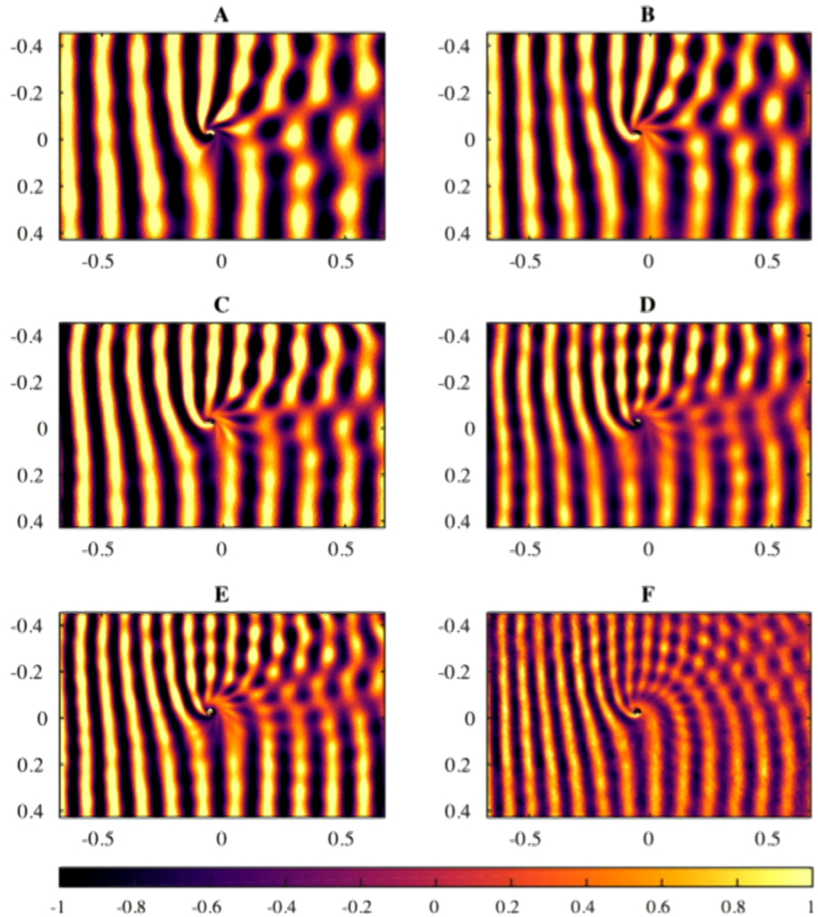
$$\varphi_m(r) = \frac{\sqrt{r}}{2\pi} \int_0^{2\pi} \phi(r,\theta) e^{-im\theta} d\theta$$

- For each m, radial profiles

$$\varphi_m(r) \sim A_m^{\text{in}} e^{-ikr} + A_m^{\text{out}} e^{ikr}$$

- Plane waves contain **all** azimuthal waves
- In practice $|m| \lesssim 3$

But



First outcome:
analog AB effect
[Berry, Chambers, Large, Upstill, Walmsley '80]

But significant scattering

Data analysis

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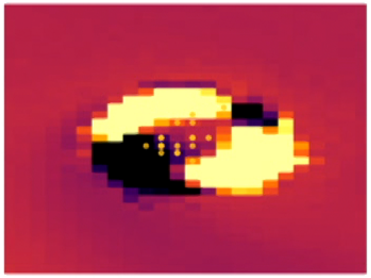
But

- Check that position of the center don't affect the result
- We look for amplification in **energy current**



Data analysis

- Reflection coefficient in **energy current**

$$|R_m|^2 = \frac{J_{\text{out}}}{J_{\text{in}}}$$

- **Energy current** of a wave

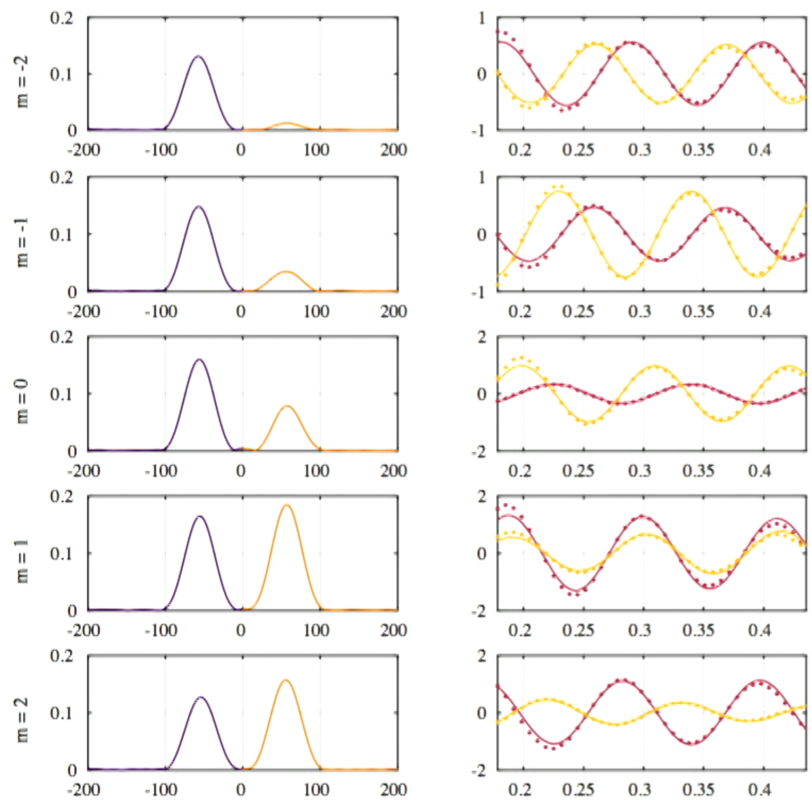
$$J_{\text{in/out}} = \left|\Omega^{-1} v_g\right| |A_{\text{in/out}}|^2$$

- $\Omega = \omega - \vec{v} \cdot \vec{k}$ co-moving frequency
- Flow $\to$ Doppler effect $\to$ $\Omega_{\text{in}} \neq \Omega_{\text{out}}$

Hence

$$|R_m|^2 = \left|\frac{A_{\text{out}}}{A_{\text{in}}}\right|^2 \times \left|\frac{\Omega_{\text{in}} v_g^{\text{out}}}{\Omega_{\text{out}} v_g^{\text{in}}}\right|$$

Radial profiles for different m



Right side

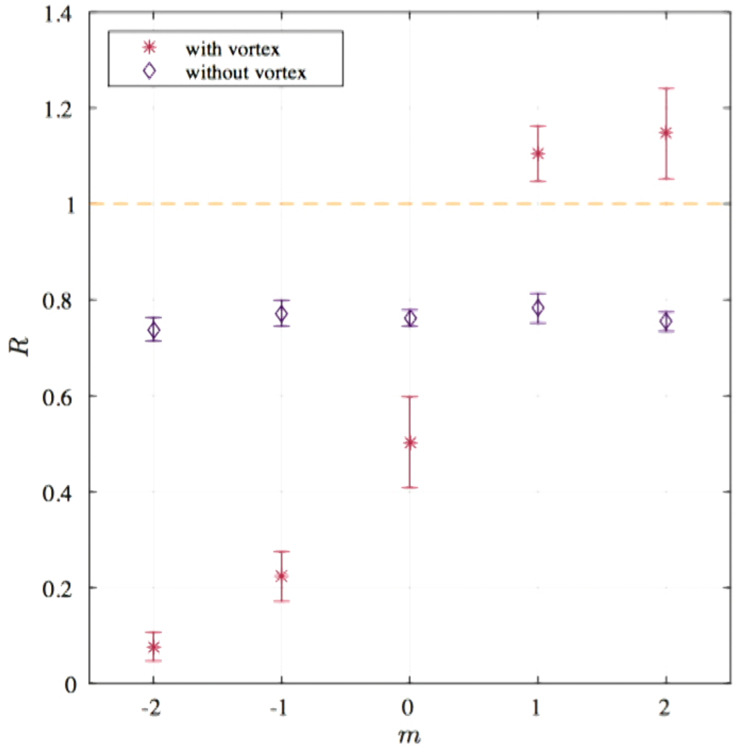
- Dots: signal
- Lines: fit to

$$A_{\text{in}} e^{-ikr} + A_{\text{out}} e^{ikr}$$

Left side

- Fourier profiles
- Relative peak amplitudes

$$|R_m(f)|^2 = \frac{J_{\text{out}}}{J_{\text{in}}}$$

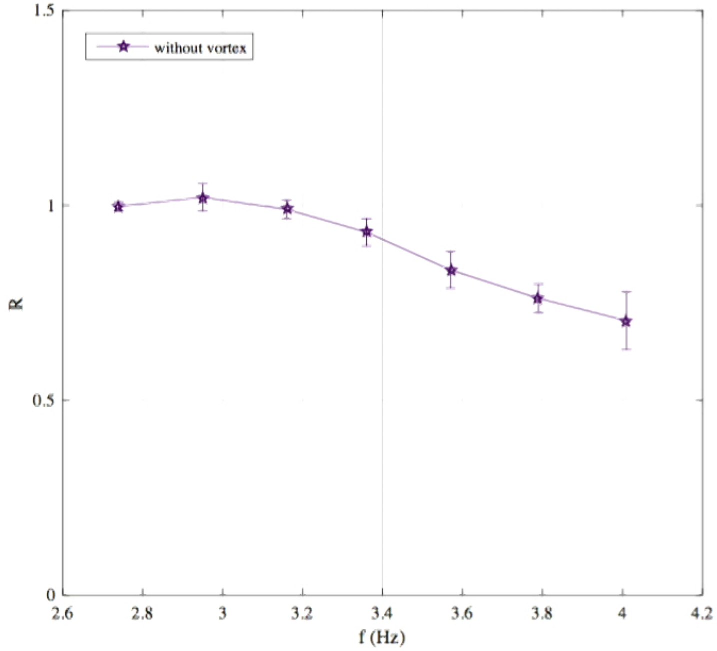


Best frequency $f = 3.7\text{Hz}$

Reflection coefficients

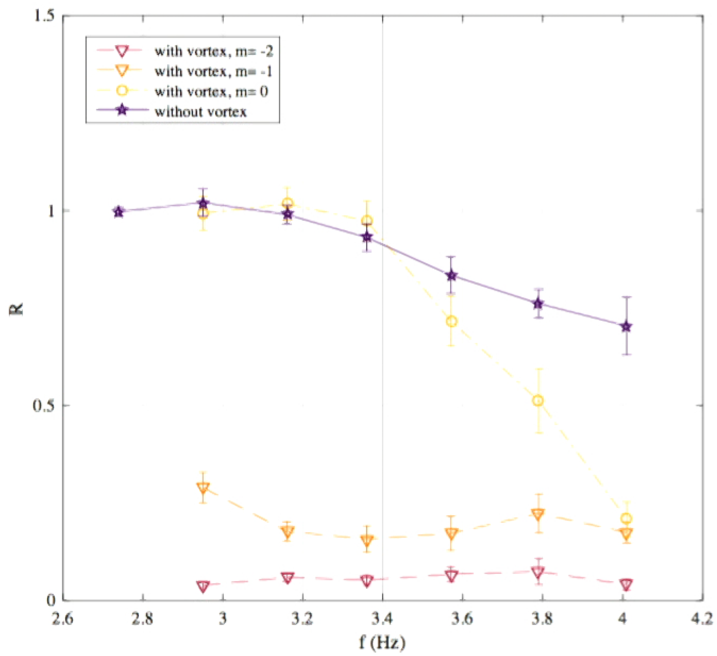
- **Wave on a vortex**
  - $m < 0$ **absorbed**
  - $m = 1$ **and** 2 **superradiant**
- **Plane wave** (no vortex) $\rightarrow$ all below 1

Reflection coefficients **spectrum**



- Only plane wave (no vortex)
- Significant dissipation (non pure water)

Reflection coefficients **spectrum**



- Only plane wave (no vortex)
- Significant dissipation (non pure water)
- Qualitative features are recovered
  - $m \leqslant 0$ absorption

36/37

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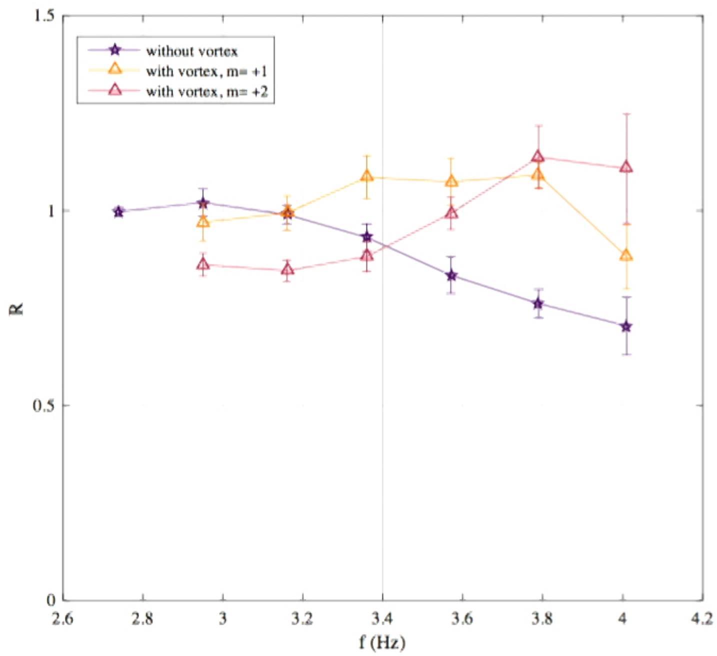
Reflection coefficients **spectrum**



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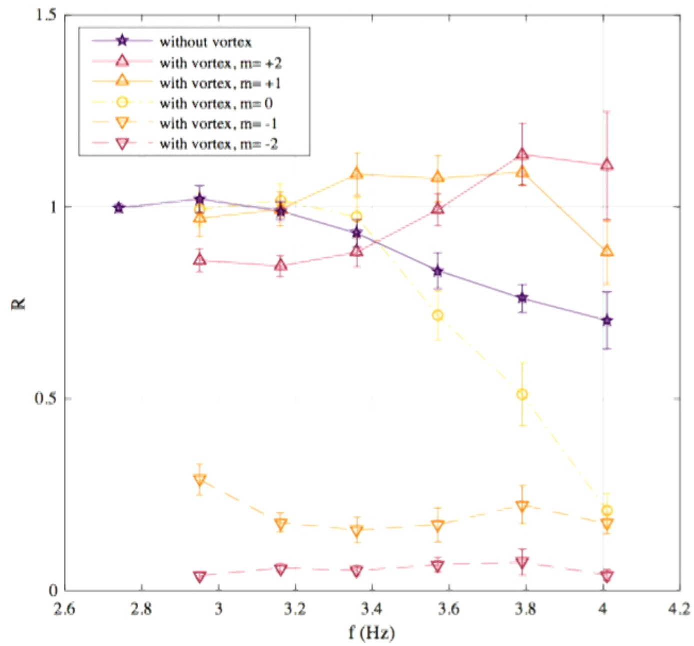
  - First $m = 1$ **amplified**, then $m = 2$

Reflection coefficients **spectrum**



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Successful observation of superradiant scattering

Conclusion

1. **Results**
   - **Experimental observation** of **superradiance**
     $\rightarrow$ extraction of angular momentum

   [Torres, Patrick, AC, Richartz, Tedford, Weinfurtner, *Nature Phys.* arXiv:1612.06180 (2017)]

2. **Perspectives**
   - Theory: better understanding of dispersive superradiance
   - Experiment: reduce dissipation