

Title: Frustrating quantum spin ice: a tale of three spin liquids and hidden order

Date: Jun 08, 2017 03:45 PM

URL: <http://pirsa.org/17060054>

Abstract: "Quantum spin ice" materials have been widely discussed in terms of an XXZ model on a pyrochlore lattice, which is accessible to quantum Monte Carlo simulation for unfrustrated interactions $J_{\pm} > 0$. Here we argue that the properties of this model may become even more interesting once it is "frustrated". Using a combination of large-scale classical Monte Carlo simulation, semi-classical molecular dynamics, symmetry analysis and analytic field theory we explore the new phases which arise for $J_{\pm} < 0$. We find that the model supports not one, but three distinct forms of spin liquid: spin ice, a U(1) spin liquid; a disguised version of the U(1) x U(1) x U(1) spin-liquid found in the Heisenberg antiferromagnet on a pyrochlore lattice; and another entirely new form of spin liquid described by a U(1) x U(1) gauge group. At low temperatures this novel spin liquid undergoes a thermodynamic phase transition into a ground state with hidden, spin-nematic order. We present explicit predictions for inelastic neutron scattering experiments carried out on the three different spin liquids [M. Taillefumier et al., arXiv:1705.00148].

Frustrating quantum spin ice : a tale of three spin liquids (and hidden order...)

nic shannon

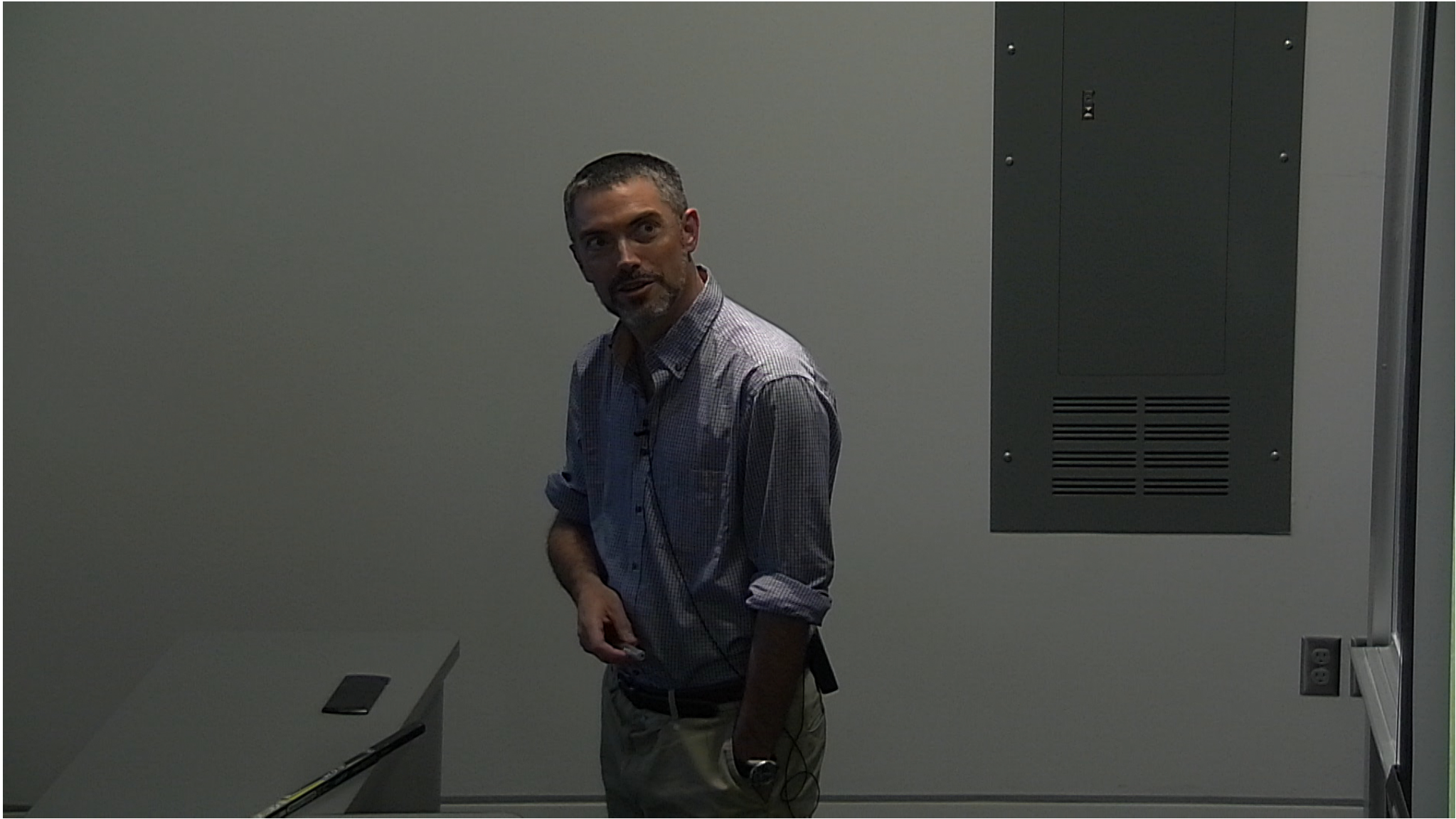


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“All good things come in threes.”



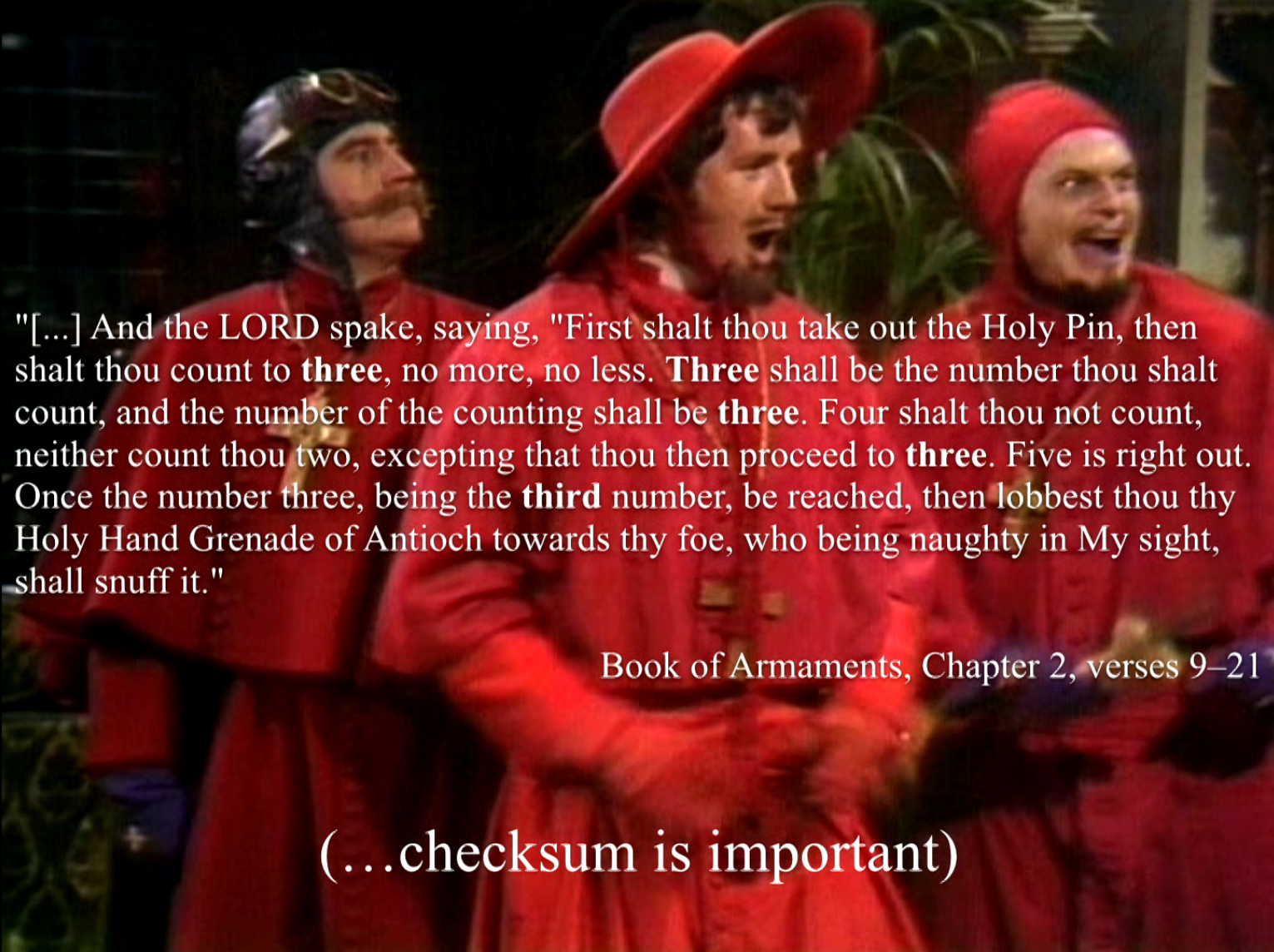








(...checksum is important)



"[...] And the LORD spake, saying, "First shalt thou take out the Holy Pin, then shalt thou count to **three**, no more, no less. **Three** shall be the number thou shalt count, and the number of the counting shall be **three**. Four shalt thou not count, neither count thou two, excepting that thou then proceed to **three**. Five is right out. Once the number three, being the **third** number, be reached, then lobbest thou thy Holy Hand Grenade of Antioch towards thy foe, who being naughty in My sight, shall snuff it."

Book of Armaments, Chapter 2, verses 9–21

(...checksum is important)

spin ice is a good thing...

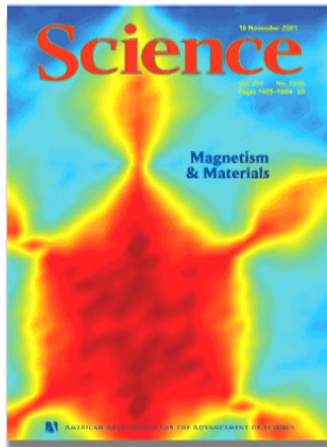


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9

spin ice is a good thing...



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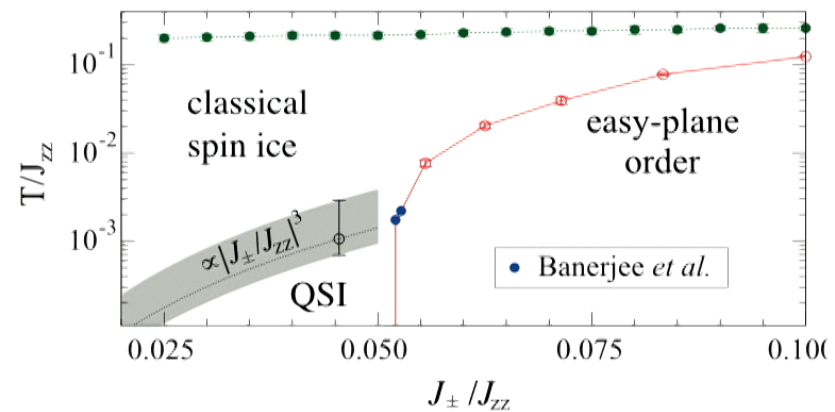
story for today

minimal model of quantum spin ice

$$\mathcal{H}_{\text{QSI}} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z - J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+)$$

phase diagram from quantum
Monte Carlo simulation

$$(J_{\pm} > 0)$$



Y. Kato and S. Onoda, Phys. Rev. Lett. **115**, 077202 (2015)
[cf. A. Banerjee *et al.* Phys. Rev. Lett. **100**, 047208 (2008)]



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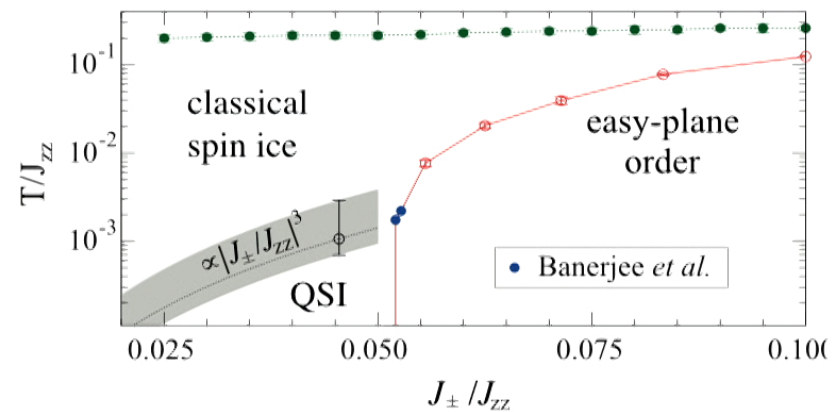
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story for today

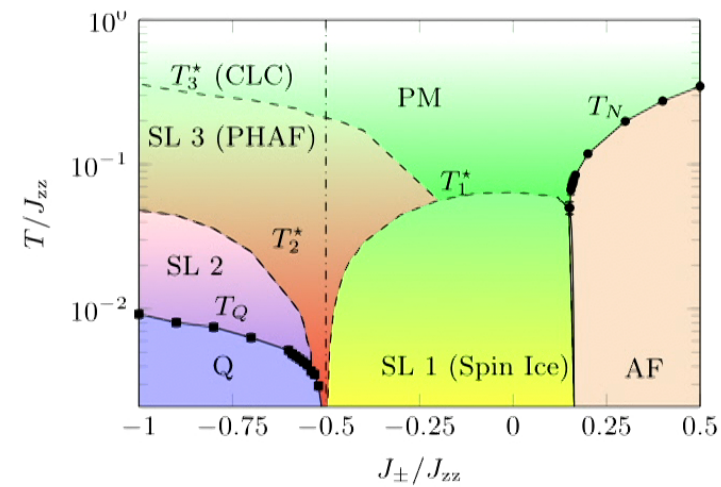
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change sign of J_{\pm}

phase diagram from classical
Monte Carlo simulation



M. Taillefumier *et al.*, arXiv:1705.00148



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story for today

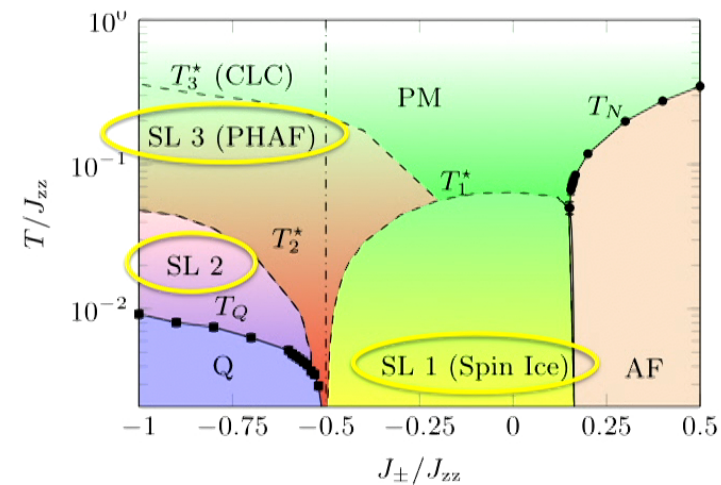
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change sign of J_{\pm}

3 spin liquids
+ hidden spin-nematic order !

phase diagram from classical
Monte Carlo simulation



M. Taillefumier *et al.*, arXiv:1705.00148



wouldn't have happened without...



Mathieu Taillefumier
(OIST → ETH)



Owen Benton
(Bristol/OIST
→ RIKEN)



Han Yan
(OIST)



Ludovic Jaubert
(OIST → CNRS)

minimal model of a QSI



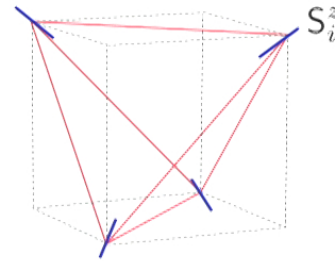
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minimal model of a QSI

general model of nearest-neighbor exchange in local basis (Kramers ion) :

$$\mathcal{H}_{\text{ex}} = \sum_{\langle ij \rangle} \left\{ J_{zz} S_i^z S_j^z - J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+) + J_{\pm\pm} [\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-] + J_{z\pm} [S_i^z (\zeta_{ij} S_j^+ + \zeta_{ij}^* S_j^-) + i \leftrightarrow j] \right\}$$



local S^z
axis
rotates
from site
to site



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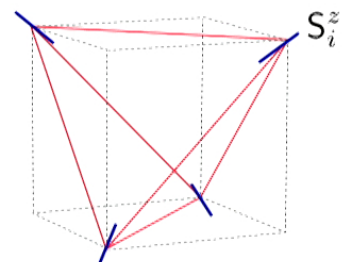
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general model of nearest-neighbor exchange in local basis (Kramers ion) :

$$\mathcal{H}_{\text{ex}} = \sum_{\langle ij \rangle} \left\{ \underbrace{J_{zz} S_i^z S_j^z}_{\text{favors spin-ice}} - \underbrace{J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+)}_{\text{introduces dynamics}} + J_{\pm\pm} [\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-] + J_{z\pm} [S_i^z (\zeta_{ij} S_j^+ + \zeta_{ij}^* S_j^-) + i \leftrightarrow j] \right\}$$

favors spin-ice
($J_{zz} > 0$)

introduces dynamics



local S^z axis
rotates from site to site

cf. most of the talks in this workshop !

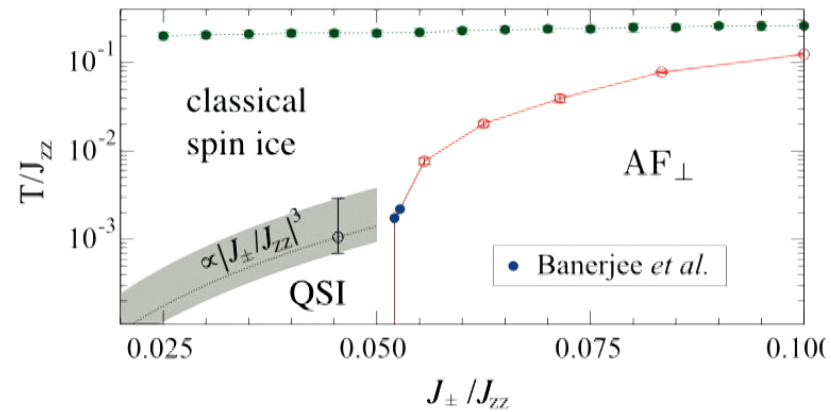
many authors (in many notations), with papers including :

S. Curnoe, Phys. Rev. B **78**, 094418 (2007); McClarty et al., J. Phys. Conf. Series **14**, 012032 (2009)
 J. D. Thompson et al., Phys. Rev. Lett. **106**, 187202 (2011); Ross et al., Phys. Rev. X **1**, 021002 (2011);
 S. Onoda et al., Phys. Rev. B **83**, 094411 (2011); L. Savary and L. Balents, Phys. Rev. Lett **108**, 037202 (2012);



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what happens for $J_{\pm} > 0$?

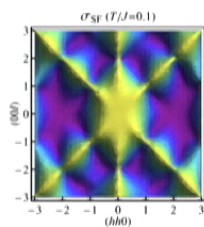


QMC results taken from : Y. Kato and S. Onoda, Phys. Rev. Lett. **115**, 077202 (2015)
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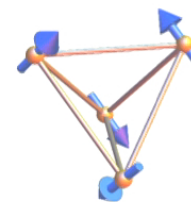
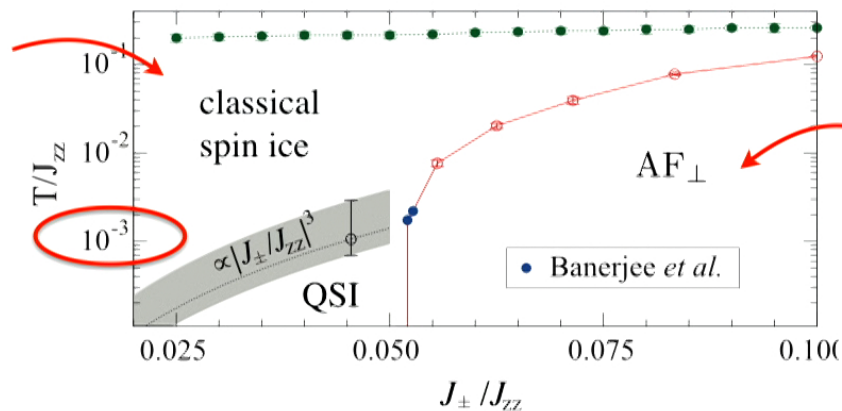


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what happens for $J_{\pm} > 0$?



pinch points in $S(\mathbf{q})$



easy-plane AF order

QMC results taken from : Y. Kato and S. Onoda, Phys. Rev. Lett. **115**, 077202 (2015)
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what happens for $J_{\pm} < 0$?



QMC “frustrated”, i.e. suffers from (severe) sign problems

what do we know ?

1. quantum spin ice ground state perturbatively stable

M. Hermele *et al.*, Phys. Rev. B **69**, 064404 (2004)



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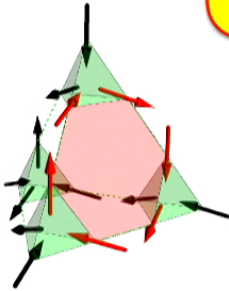


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$$\mathcal{H}_{\text{tunneling}} = -g \sum_{\text{tet}} |\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| \quad g = \frac{12J_{\pm}^3}{J_{zz}^2}$$



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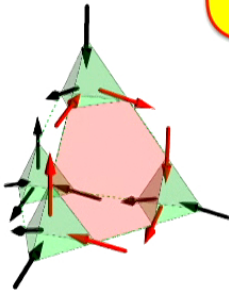


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$$\mathcal{H}_{\text{tunneling}} = -g \sum_{\diamond} |\circ\rangle\langle\circ| + |\circ\rangle\langle\circ| \quad g = \frac{12J_{\pm}^3}{J_{zz}^2}$$

can swap sign using “gauge” transformation

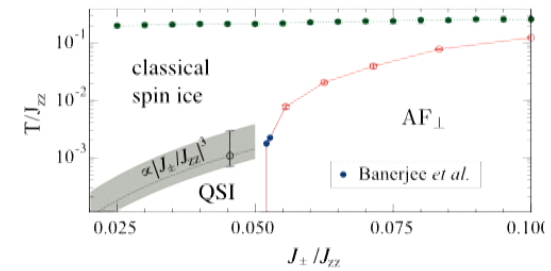
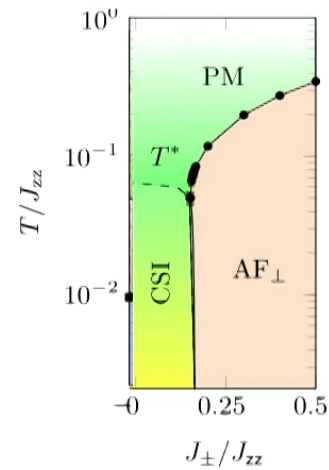
2. spinon dispersion is modified; gauge MFT predicts broad QSI regime

S.-B. Lee *et al.*, Phys. Rev. B **86**, 104412 (2012)



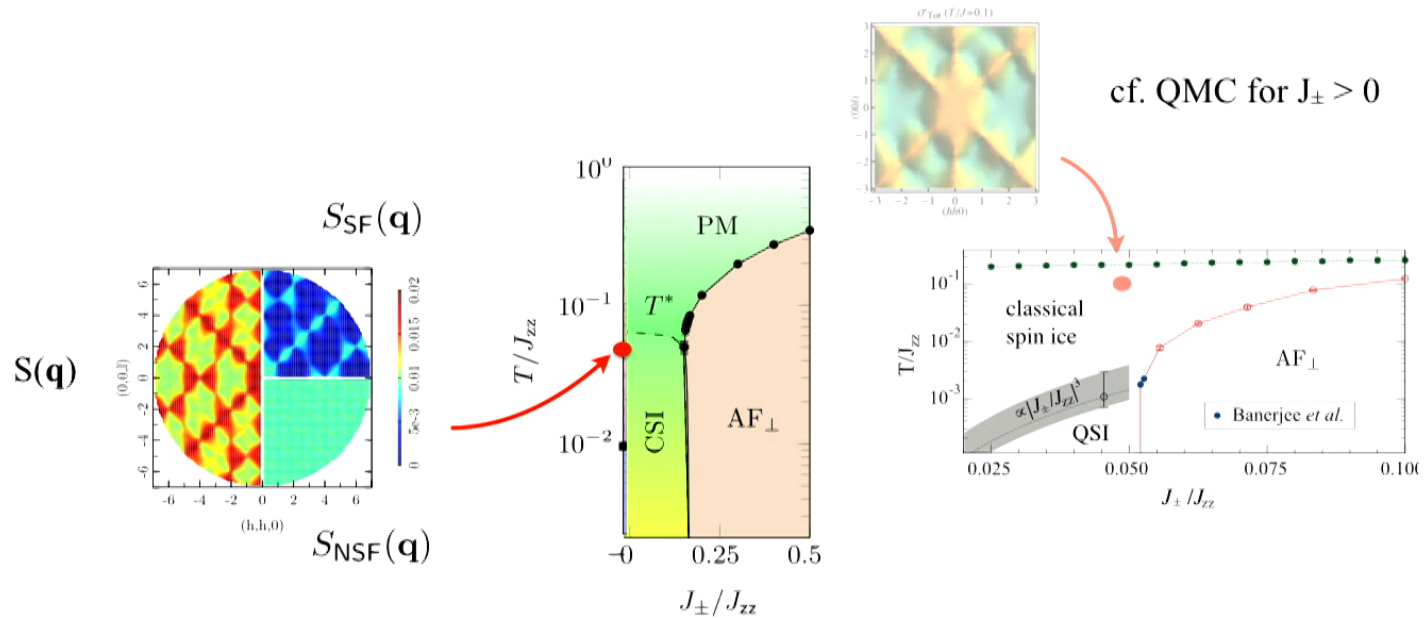
can classical simulations help ?

cf. QMC for $J_{\pm} > 0$

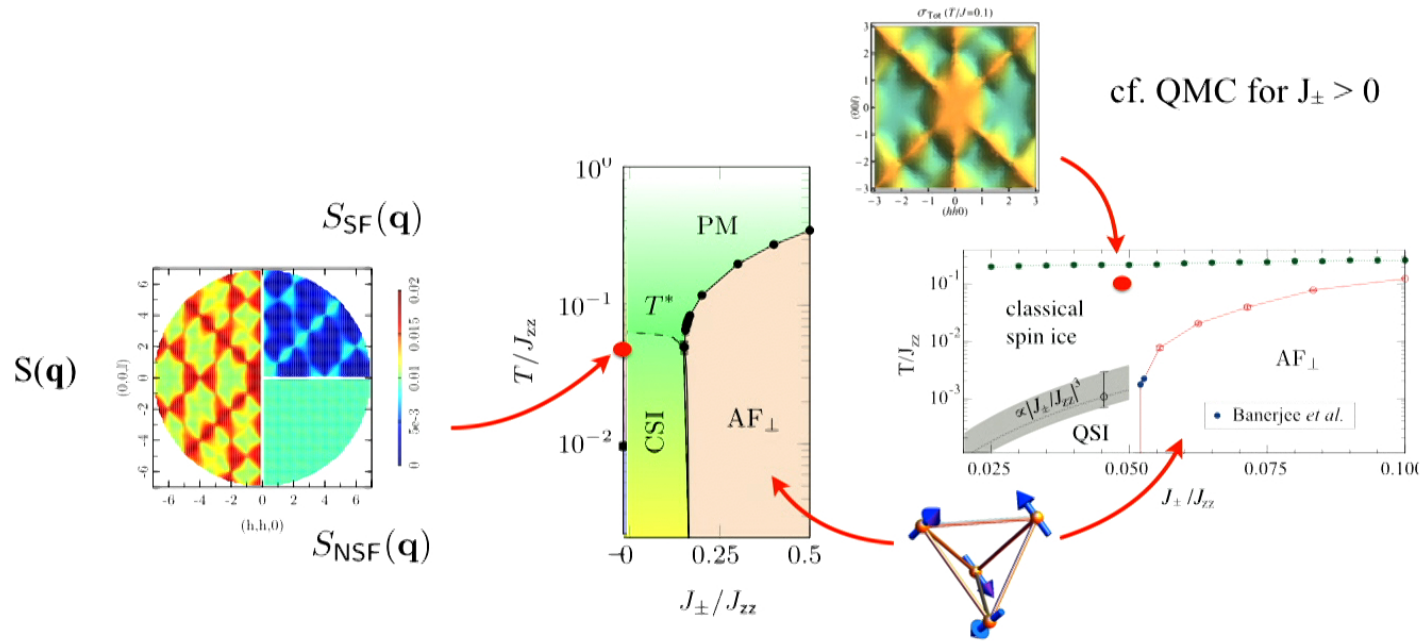


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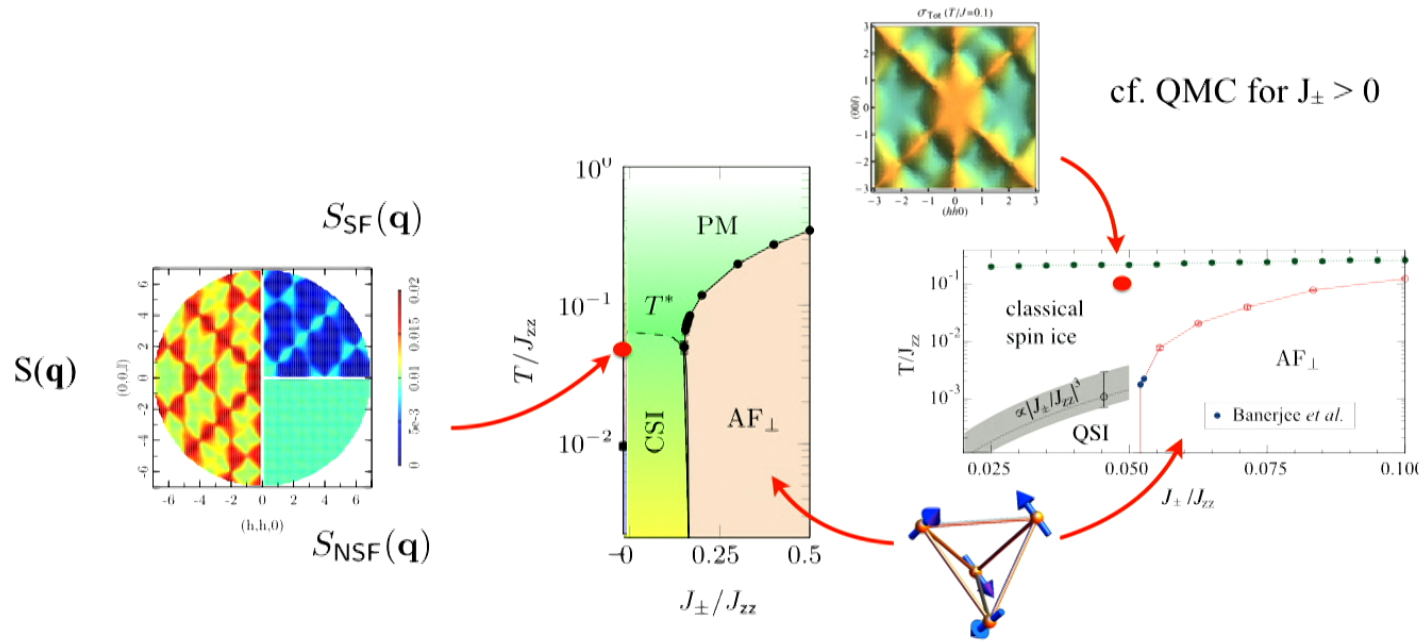
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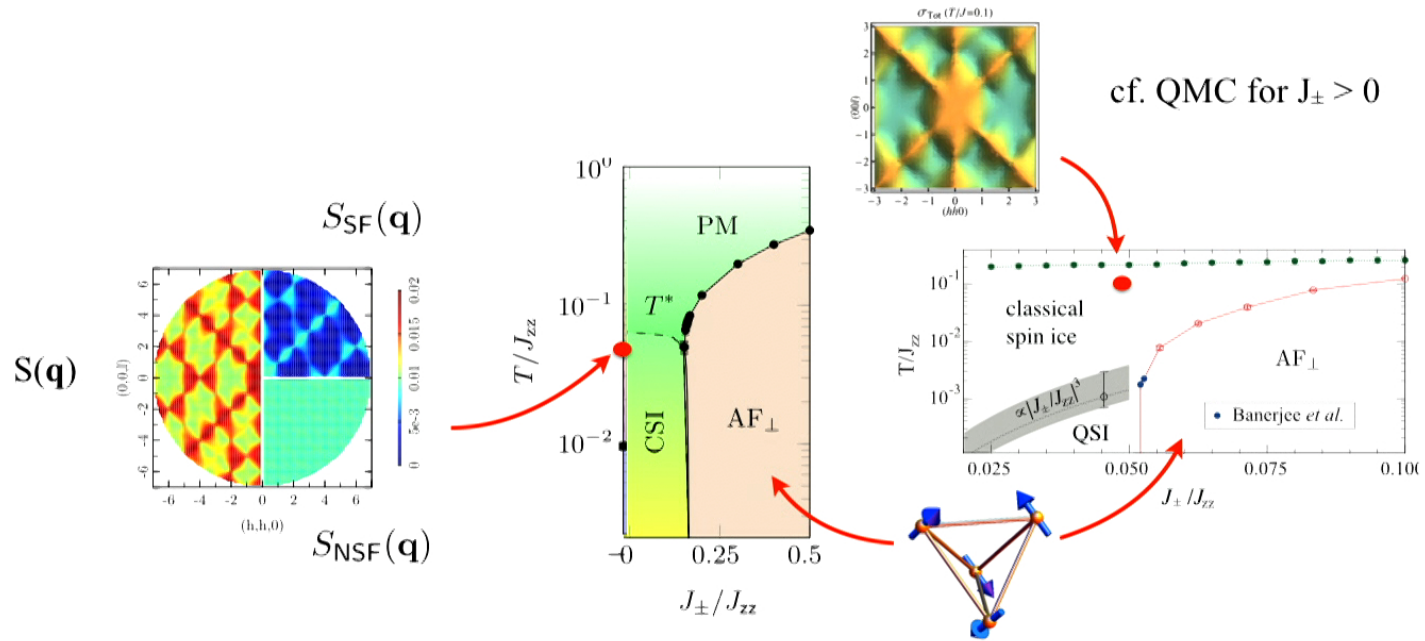
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can classical simulations help ?



can classical simulations help ?



qualitatively identical phase diagram (apart from absence of low-T QSI regime)

quantitative differences in T^* , T_N , and critical value of J_{\pm}/J_{zz}



classical ground states ?

$$\mathcal{H}_{\text{ex}} = \sum_{\langle ij \rangle} \left\{ J_{zz} S_i^z S_j^z - J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+) + J_{\pm\pm} [\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-] + J_{z\pm} [S_i^z (\zeta_{ij} S_j^+ + \zeta_{ij}^* S_j^-) + i \leftrightarrow j] \right\}$$

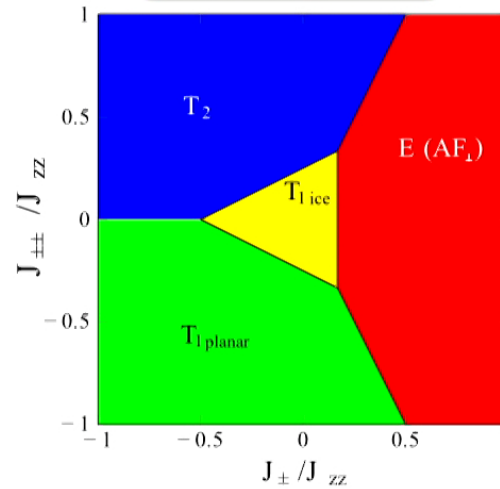


$$\mathcal{H}_{\text{ex}} = \frac{1}{2} \sum_{\text{tet}, \lambda} a_{\lambda} \mathbf{m}_{\lambda}^2$$

rewrite in terms of irreps of T_d

$$\{\mathbf{m}_{\lambda}\} = \{m_{A_2}, \mathbf{m}_E, \mathbf{m}_{T_1 \text{ice}}, \mathbf{m}_{T_1 \text{planar}}, \mathbf{m}_{T_2}\}$$

spin ice is surrounded by competing forms of easy-plane AF order

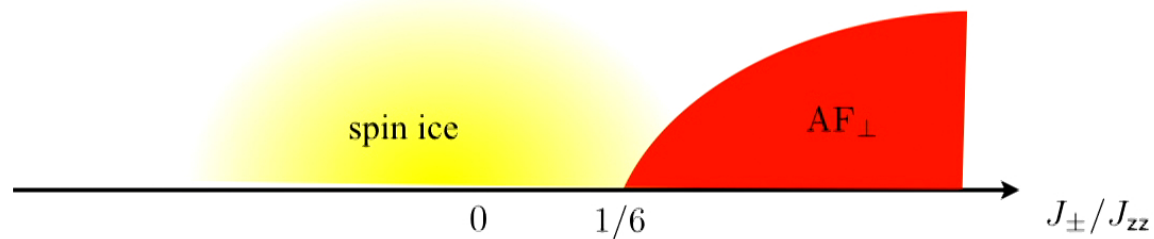


Owen Benton, PhD Thesis
H. Yan *et al.*, arXiv:1311.3501
H. Yan *et al.*, Phys. Rev. B **95**, 094422 (2017)

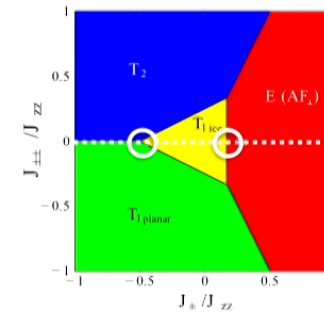


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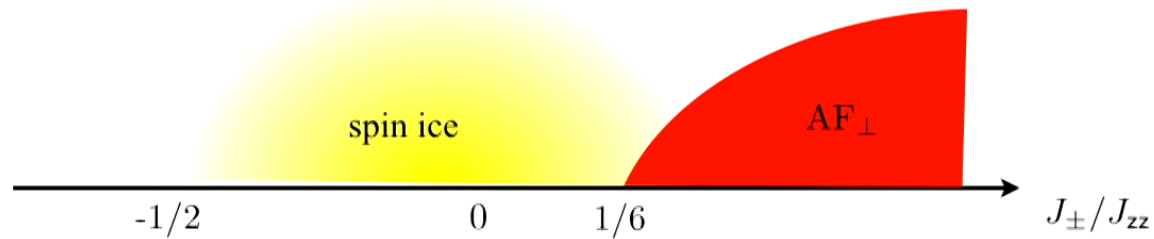
what do we expect at finite T ?



T=0 classical phase boundary

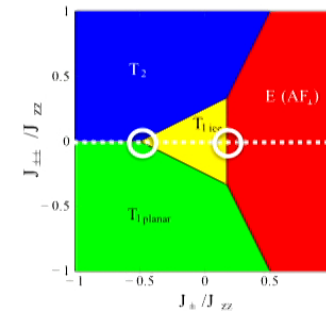


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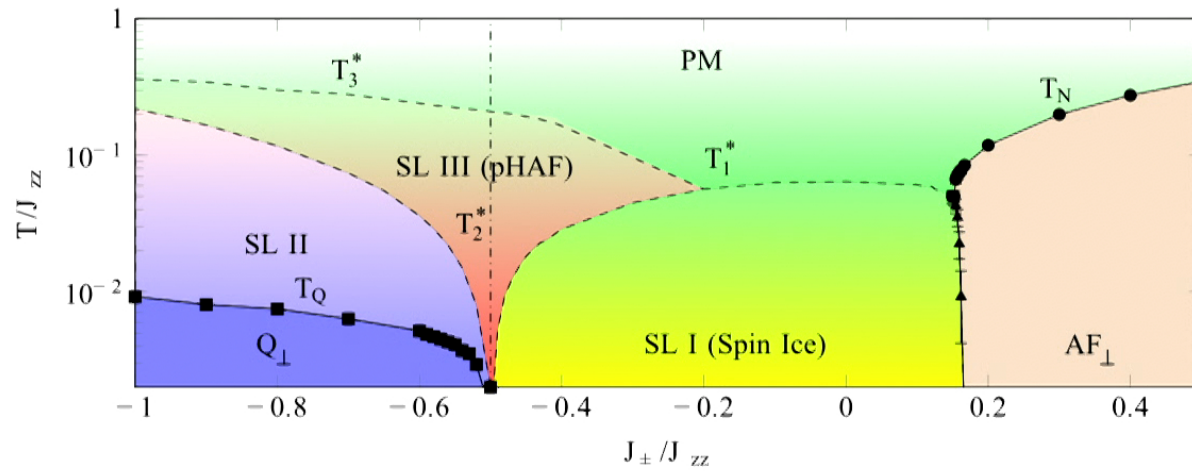


$$\mathcal{H}_{\text{QSI}} = J_{zz} \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

T=0 classical phase boundary



what do we find ?



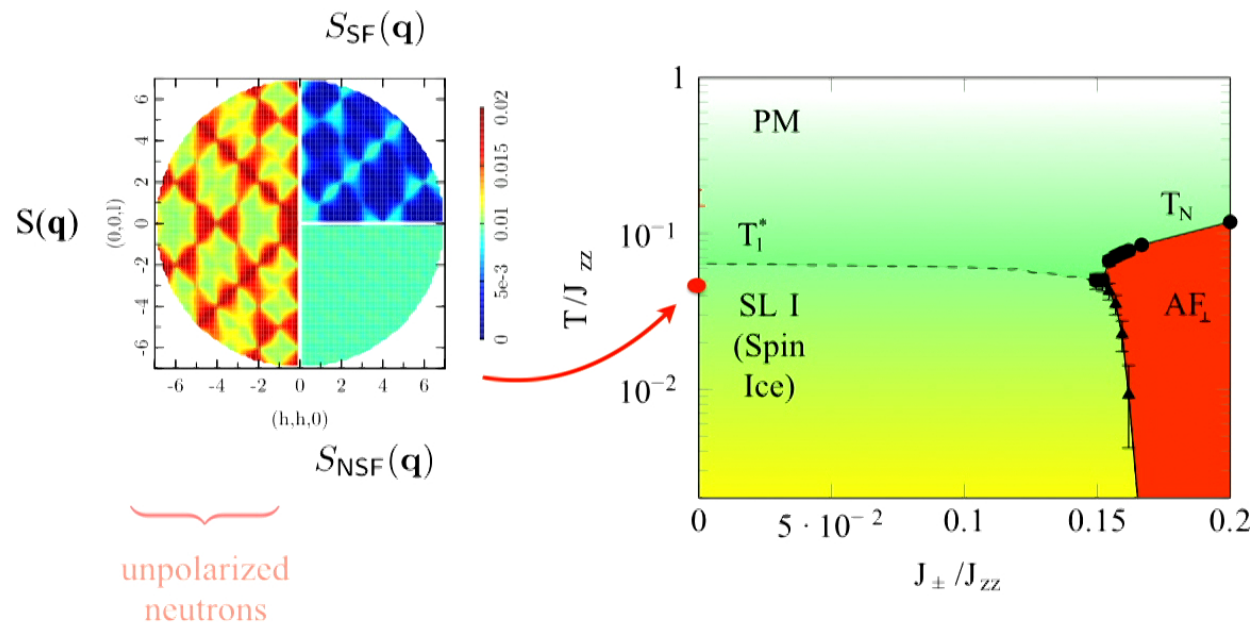
M. Taillefumier *et al.*, arXiv:1705.00148



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SL 1 (spin ice)

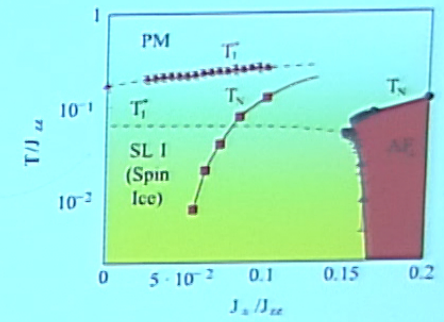


M. Taillefumier *et al.*, arXiv:1705.00148; M. Taillefumier *et al.*, in preparation.



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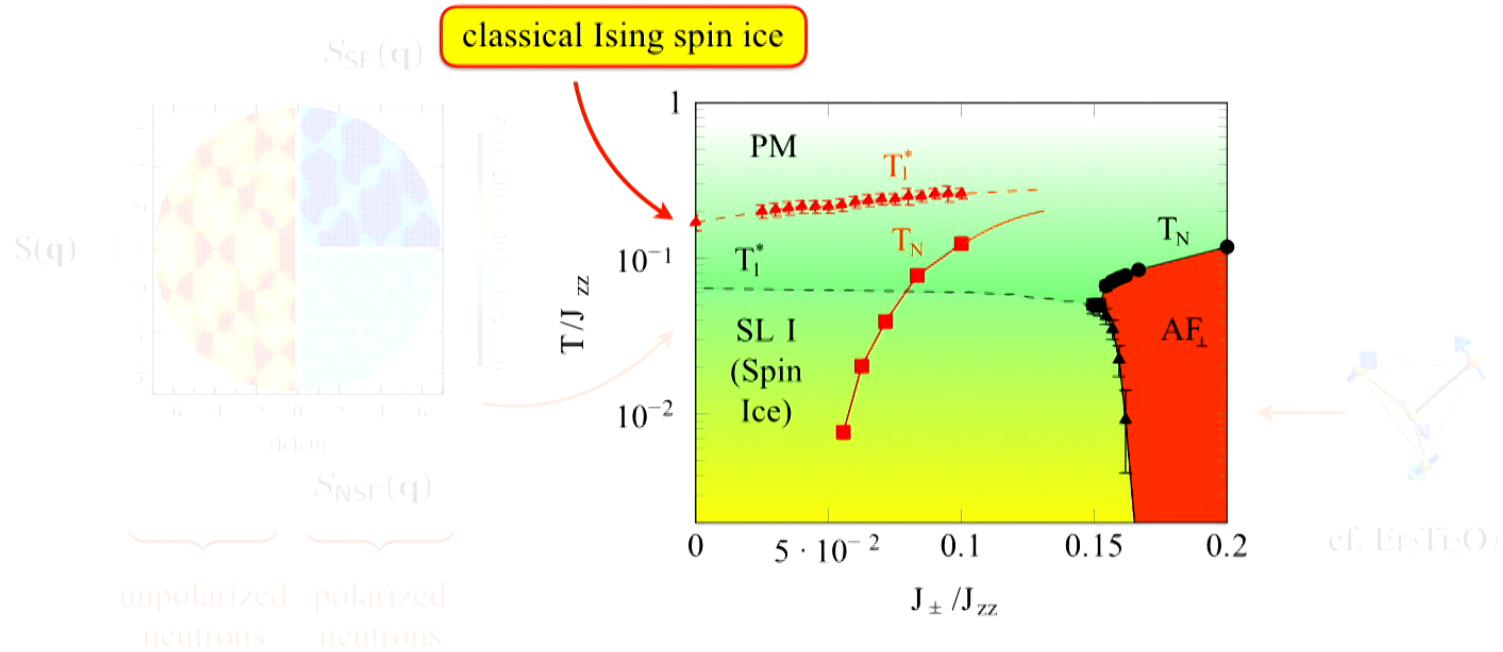
[QMC results taken from: Y. Kato and S. Onoda, Phys. Rev. Lett. 115, 077202 (2015)]



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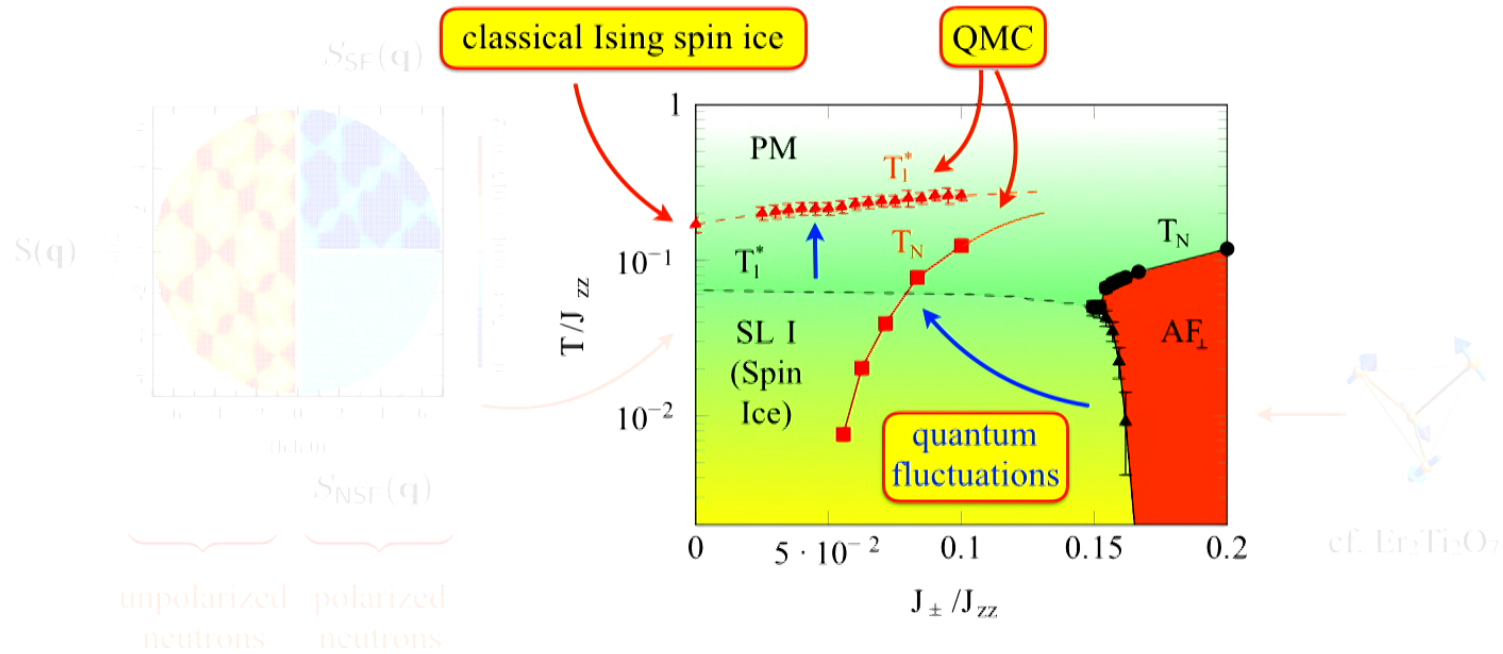
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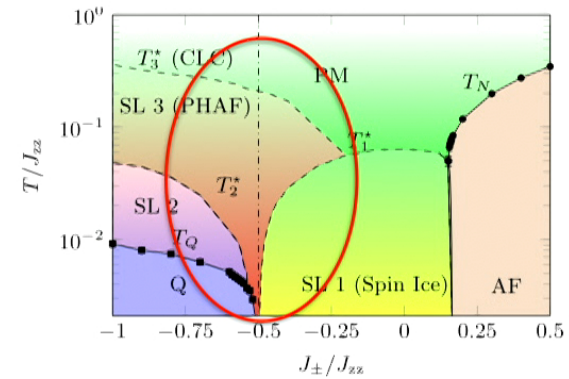


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SL 3 (“pseudo-Heisenberg AF”)

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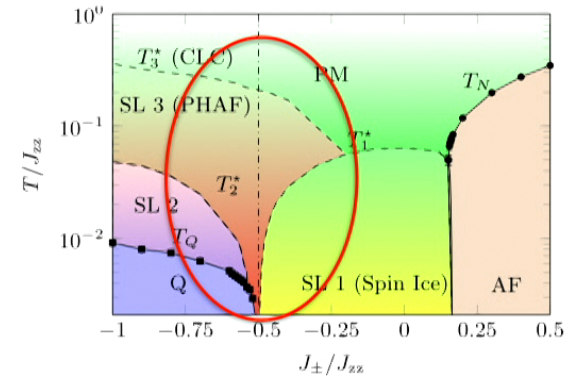
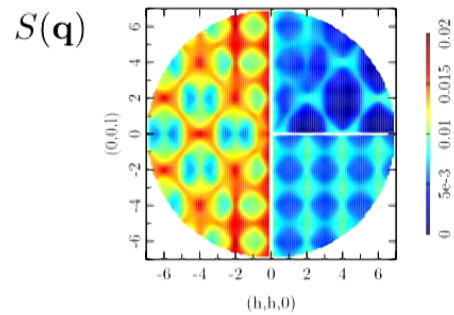
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$$J_{\pm} / J_{zz} = -0.5$$

$$\mathcal{H}_{\text{pHAF}} = J \sum_{\langle ij \rangle} S_i \cdot S_j$$



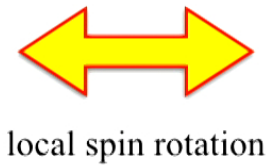
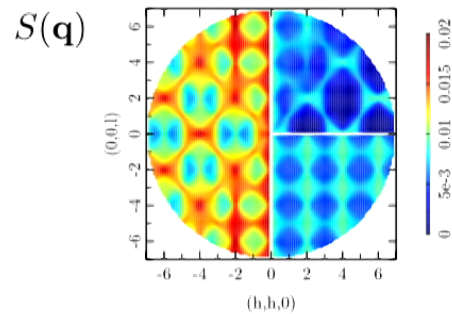
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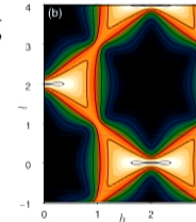
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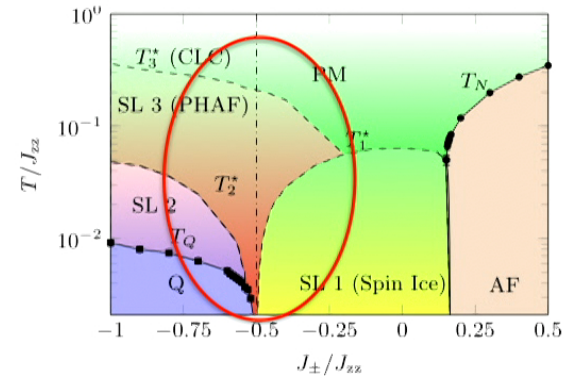


local spin rotation

Heisenberg
AF :



P.H. Conon and J. Chalker,
Phys. Rev B **81**, 224413 (2010)



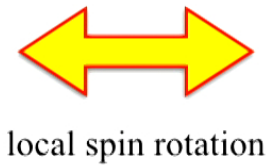
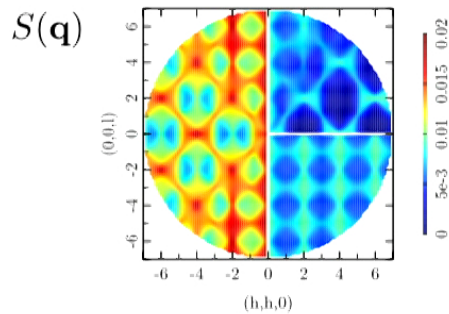
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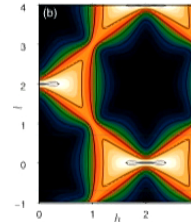
$$J_{\pm}/J_{zz} = -0.5$$

$$\mathcal{H}_{\text{pHAF}} = J \sum_{\langle ij \rangle} S_i \cdot S_j$$

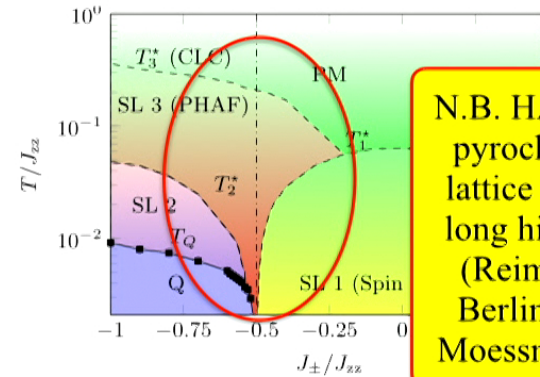


local spin rotation

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P.H. Conon and J. Chalker,
Phys. Rev B **81**, 224413 (2010)



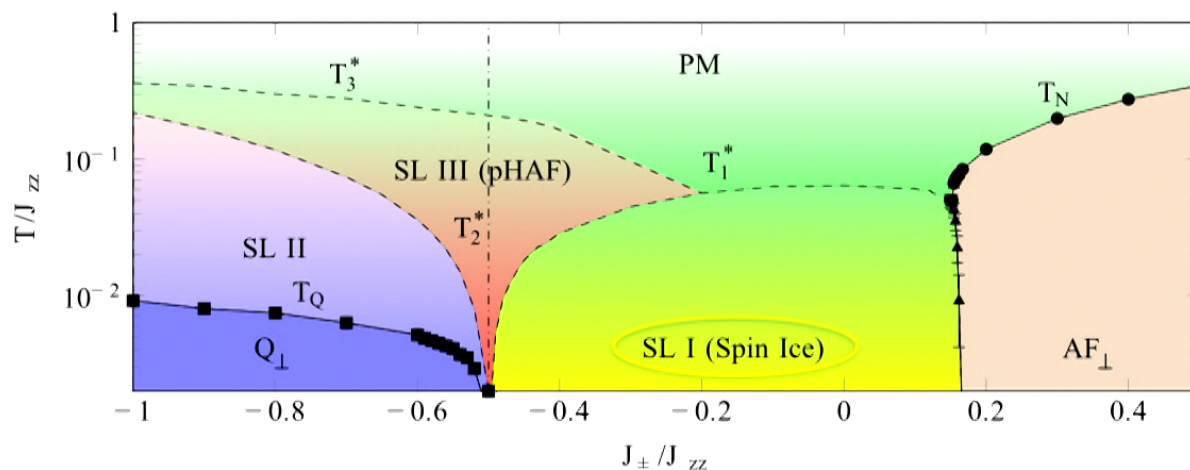
N.B. HAF on
pyrochlore
lattice has a
long history
(Reimers,
Berlinsky,
Moessner...)

C. Henley, Phys. Rev B **71**, 014424 (2005) : $U(1) \times U(1) \times U(1)$ gauge theory

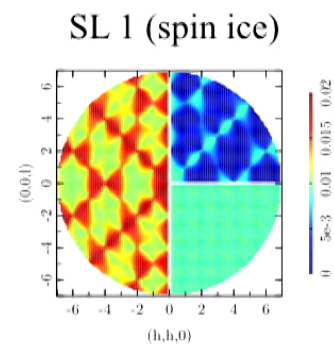
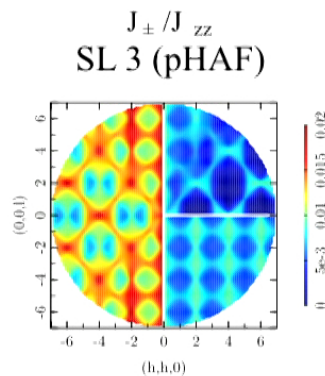
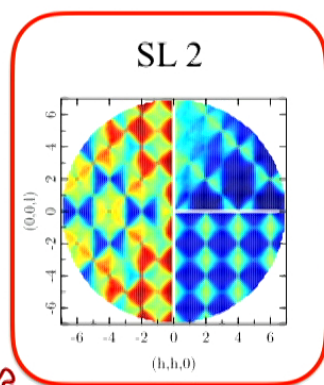
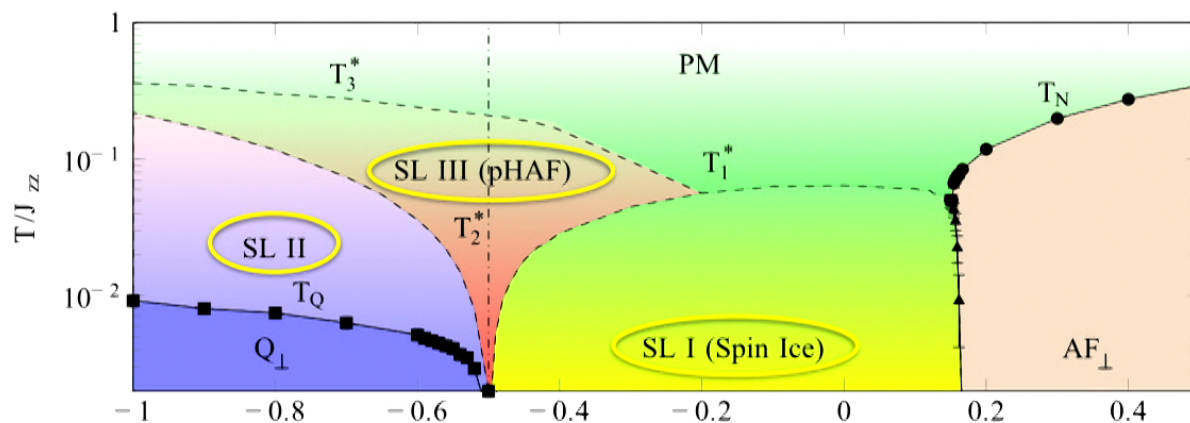


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SL 2 (something completely different)

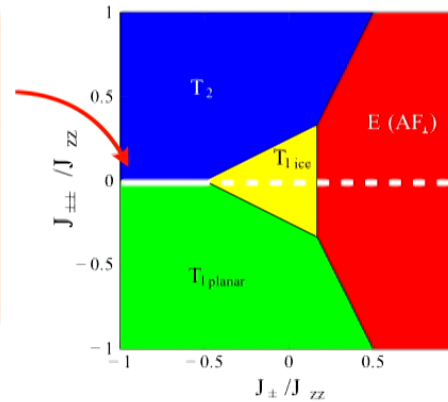


SL 2 (something completely different)



where does SL 2 come from ?

highly-degenerate manifold formed of combination of easy-plane states with T_1 and T_2 symmetry



minimal model QSI

O. Benton, PhD Thesis; M. Taillefumier *et al.*, arXiv:1705.00148

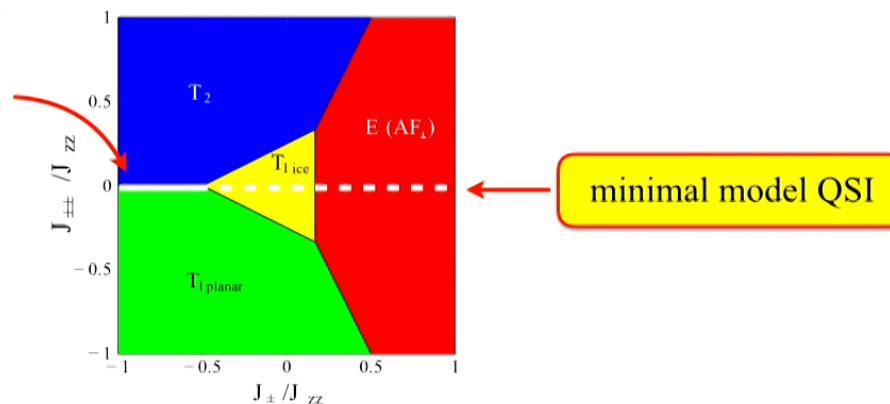


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| | Definition in terms of spins within tetrahedron |
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| $\mathbf{m}_{T_{1,planar}}$ | $\begin{pmatrix} \frac{1}{2}(S_0^x + S_1^x - S_2^x - S_3^x) \\ \frac{1}{4}(-S_0^x + \sqrt{3}S_0^y + S_1^x - \sqrt{3}S_1^y - S_2^x + \sqrt{3}S_2^y + S_3^x - \sqrt{3}S_3^y) \\ \frac{1}{4}(-S_0^x - \sqrt{3}S_0^y + S_1^x + \sqrt{3}S_1^y + S_2^x + \sqrt{3}S_2^y - S_3^x - \sqrt{3}S_3^y) \end{pmatrix}$ |
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O. Benton, PhD Thesis; M. Taillefumier *et al.*, arXiv:1705.00148



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gauge structure ?



combine $\mathbf{m}_{T_1, \text{planar}}$ and \mathbf{m}_{T_2}
into two fields satisfying zero-divergence condition

$$\mathbf{B}_1 = \frac{1}{2}(2m_{T_1, \text{planar}}^x, -\sqrt{3}m_{T_2}^y - m_{T_1, \text{planar}}^y, \sqrt{3}m_{T_2}^z - m_{T_1, \text{planar}}^z) \quad \nabla \cdot \mathbf{B}_1 = 0$$

$$\mathbf{B}_2 = \frac{1}{2}(2m_{T_1, \text{planar}}^x, -m_{T_2}^y + \sqrt{3}m_{T_1, \text{planar}}^y, -m_{T_2}^z - \sqrt{3}m_{T_1, \text{planar}}^z) \quad \nabla \cdot \mathbf{B}_2 = 0$$

can describe spin liquid SL 2 by
introducing two U(1) gauge
fields \mathbf{A}_1 and \mathbf{A}_2

$$\mathcal{F}_{\text{SL2}} = \frac{T}{V} \int d^3r \lambda \left[\underbrace{(\nabla \times \mathbf{A}_1)^2}_{\mathbf{B}_1} + \underbrace{(\nabla \times \mathbf{A}_2)^2}_{\mathbf{B}_2} \right]$$

\Rightarrow pinch points in \mathbf{B}_1 and \mathbf{B}_2

M. Taillefer *et al.*, arXiv:1705.00148



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gauge structure ?



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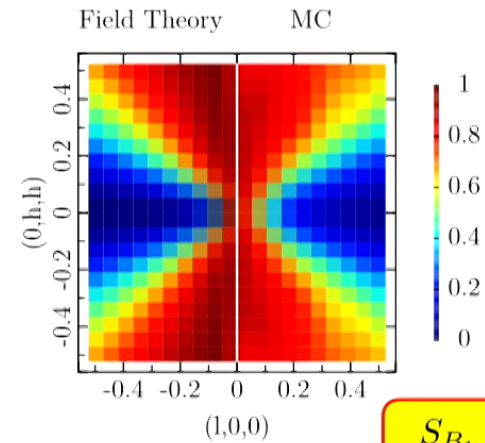
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$$\mathbf{B}_2 = \frac{1}{2}(2m_{T_1, \text{planar}}^x, -m_{T_2}^y + \sqrt{3}m_{T_1, \text{planar}}^y, -m_{T_2}^z - \sqrt{3}m_{T_1, \text{planar}}^z) \quad \nabla \cdot \mathbf{B}_2 = 0$$

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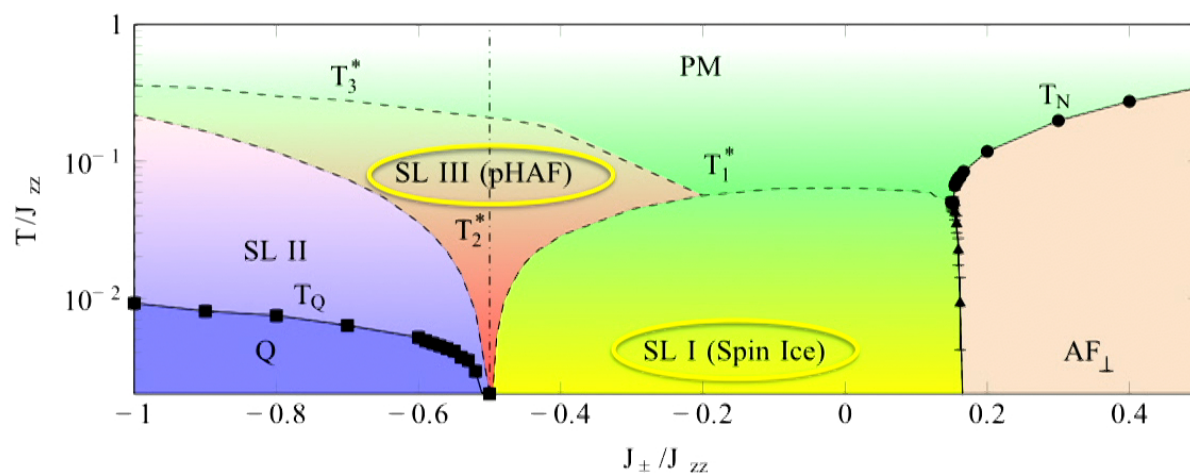
$$S_{B_{1,x}}(\mathbf{q})$$

M. Taillefumier *et al.*, arXiv:1705.00148

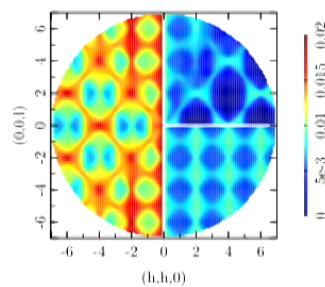


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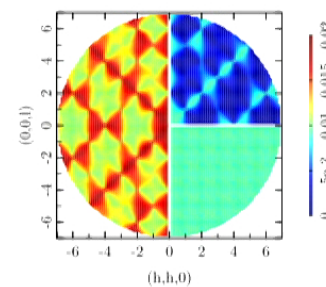
a new form of spin liquid...



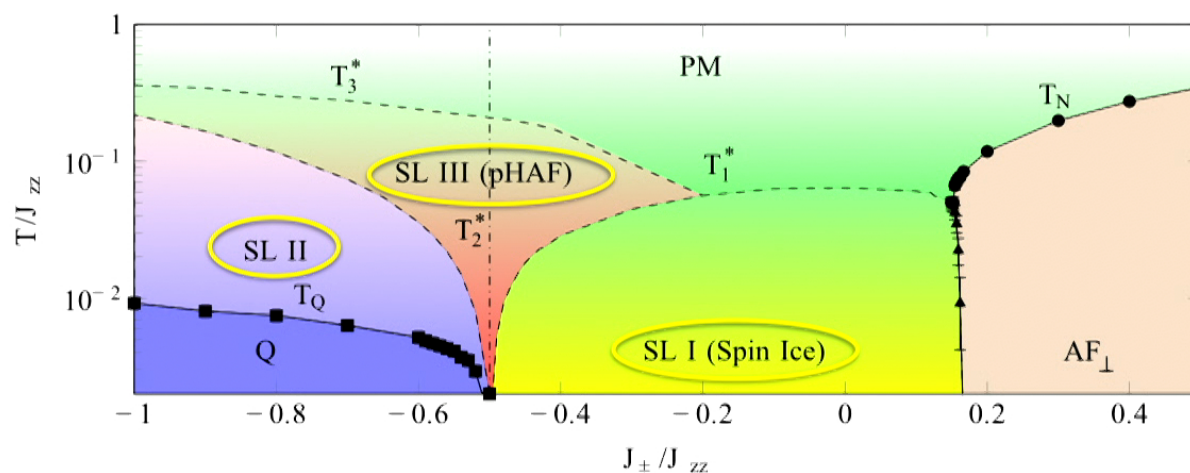
SL 3 (pHAF)



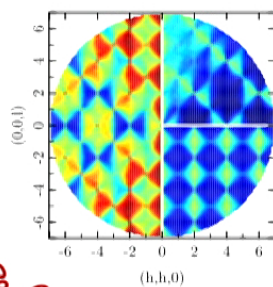
SL 1 (spin ice)



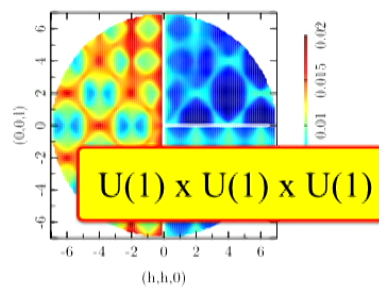
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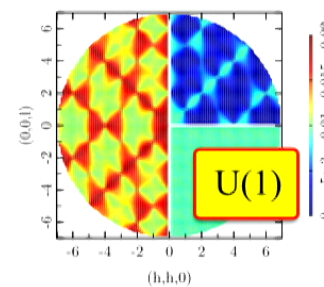
SL 2



SL 3 (pHAF)



SL 1 (spin ice)



is that it ?

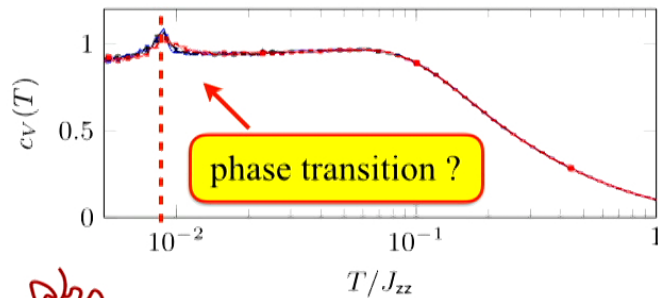
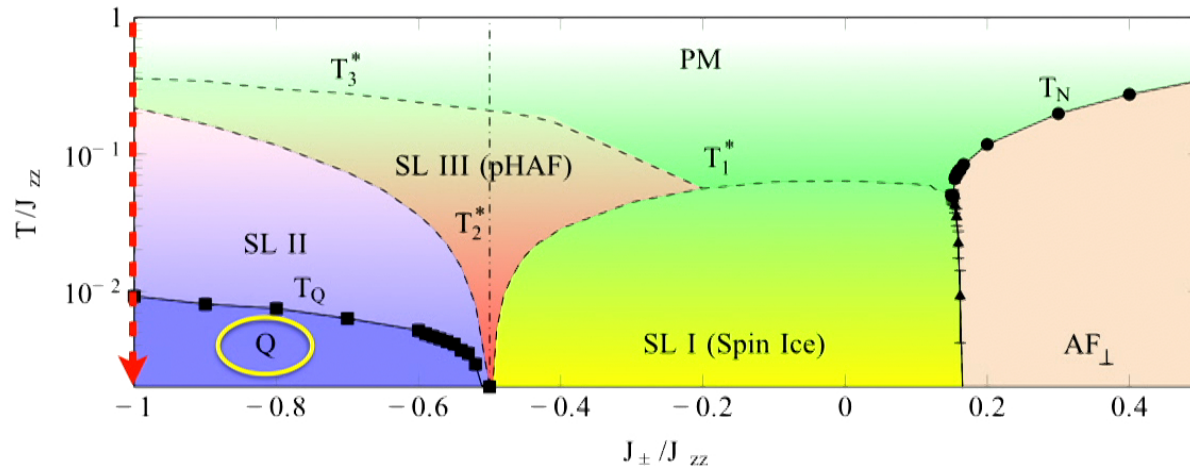


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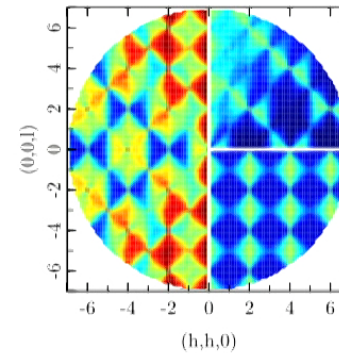
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staged loss
of entropy

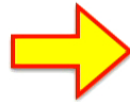
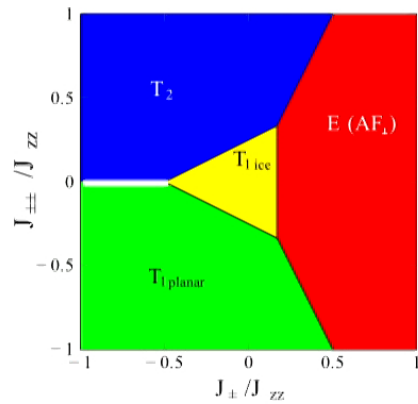
is that it ?



spin correlations
unchanged
 \Rightarrow
hidden order



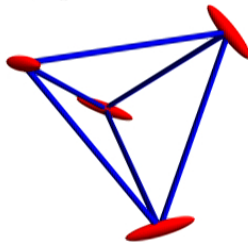
hidden order...



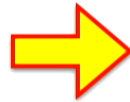
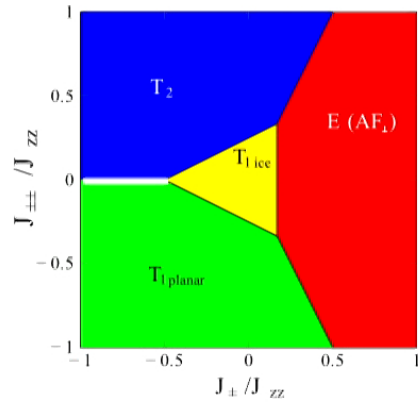
| | Definition in terms of spins within tetrahedron |
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candidate for hidden order
is spin-nematic state
with director in easy-plane

$$\mathbf{Q}_\perp = \begin{pmatrix} S^{x^2} - S^{y^2} \\ 2S^x S^y \end{pmatrix}$$



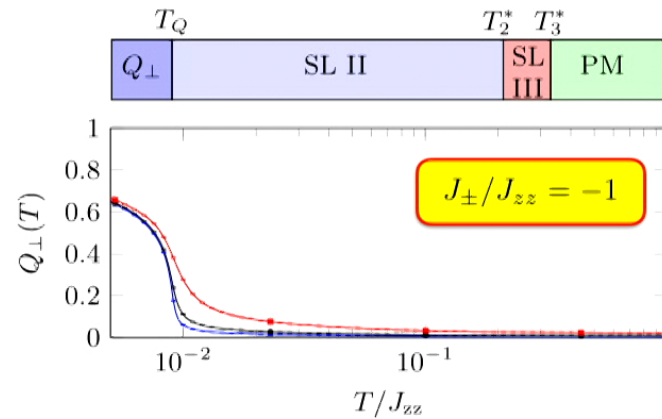
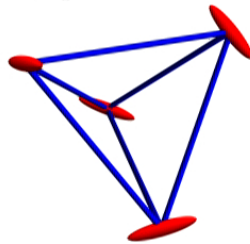
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M. Taillefumier *et al.*, arXiv:1705.00148



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30

what about spin excitations ?

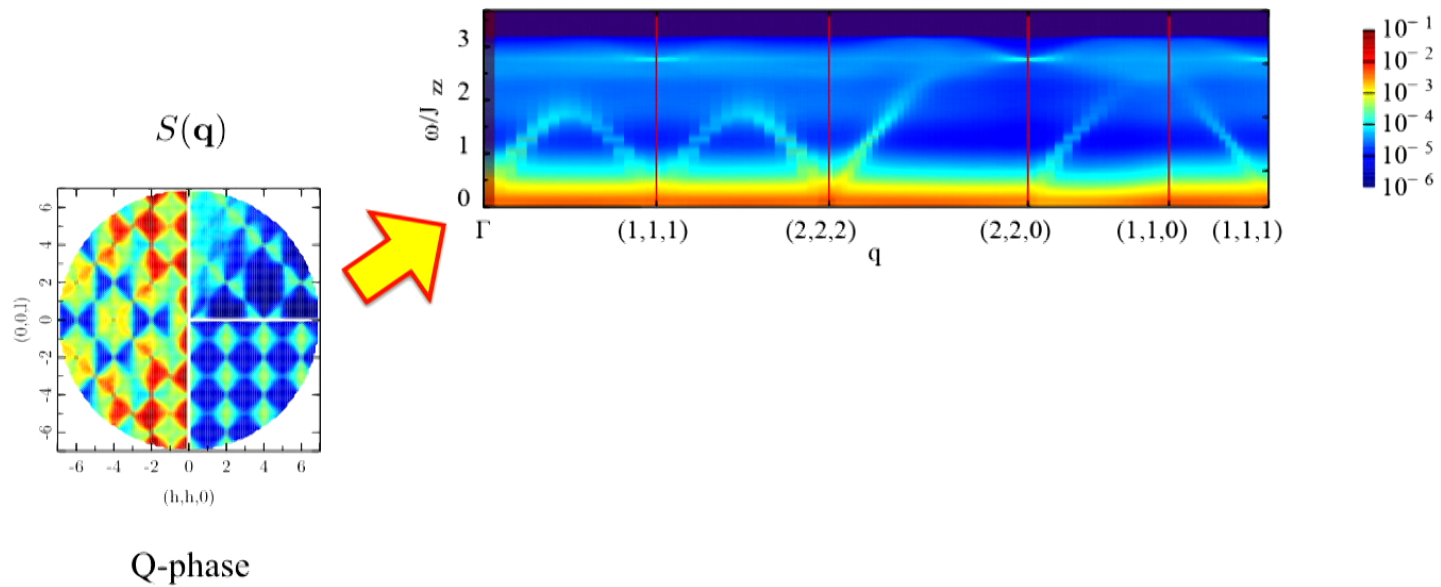


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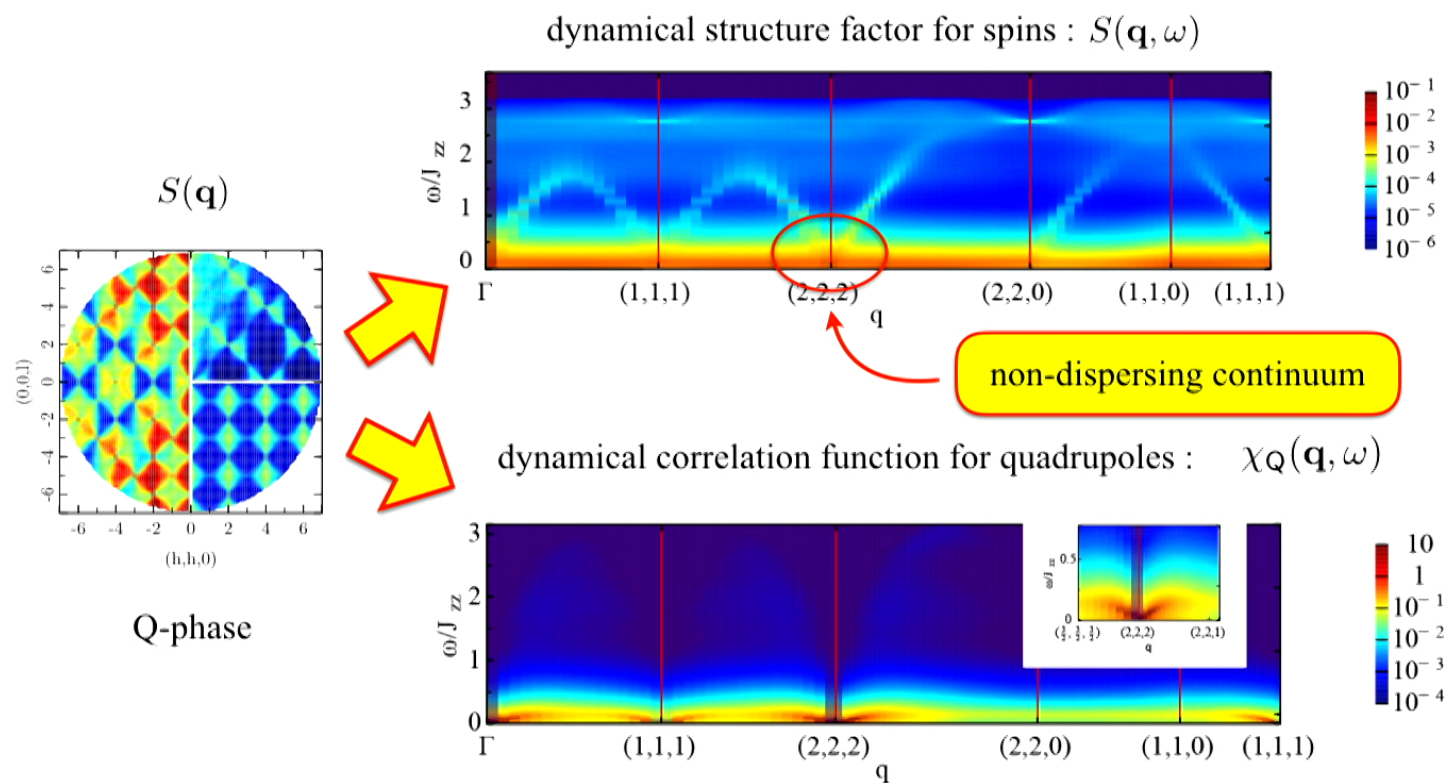
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what about spin excitations ?

dynamical structure factor for spins : $S(\mathbf{q}, \omega)$

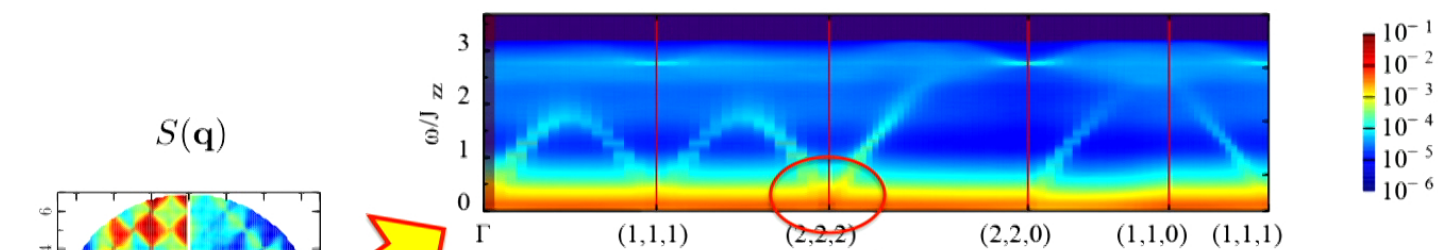


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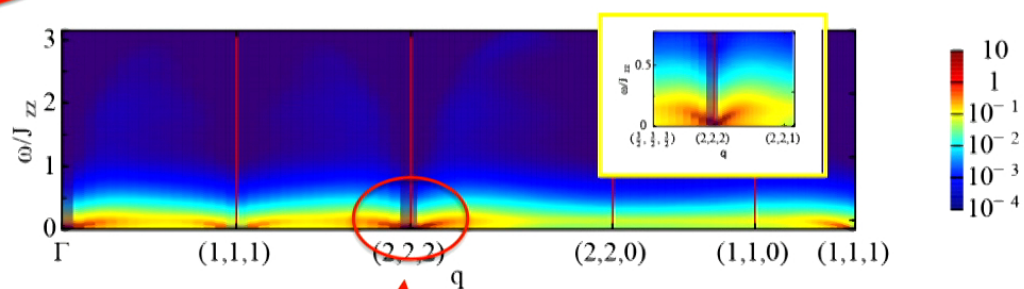
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non-dispersing continuum

dynamical correlation function for quadrupoles : $\chi_Q(\mathbf{q}, \omega)$

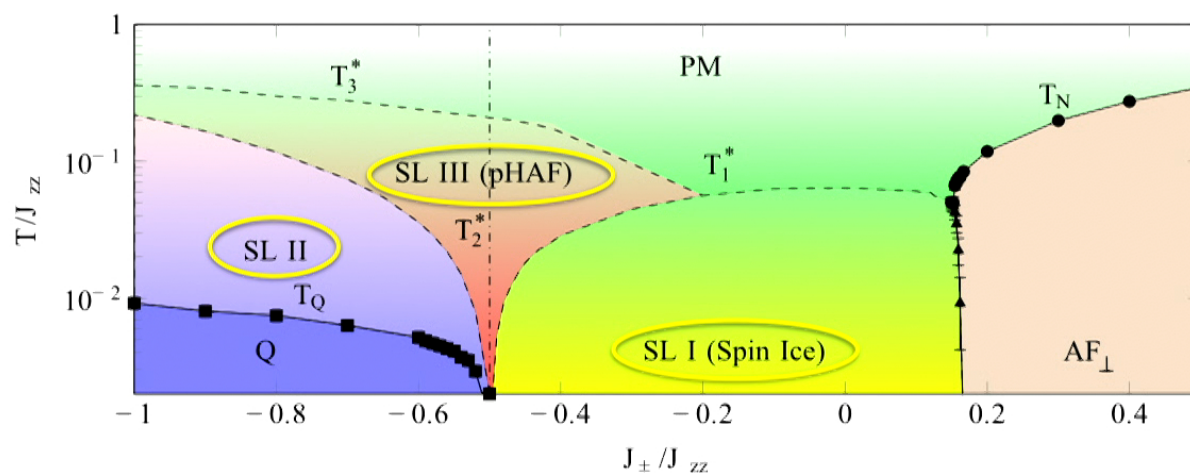


linearly-dispersing Goldstone mode

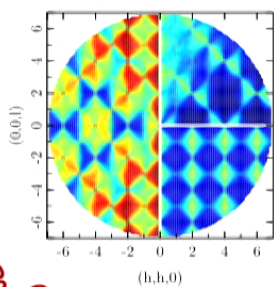


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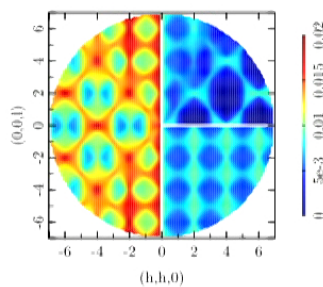
what have we learned ? (theory)



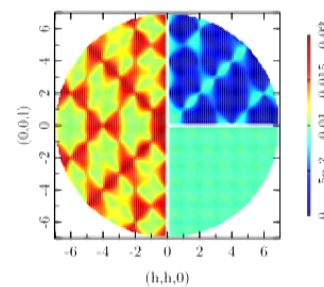
SL 2



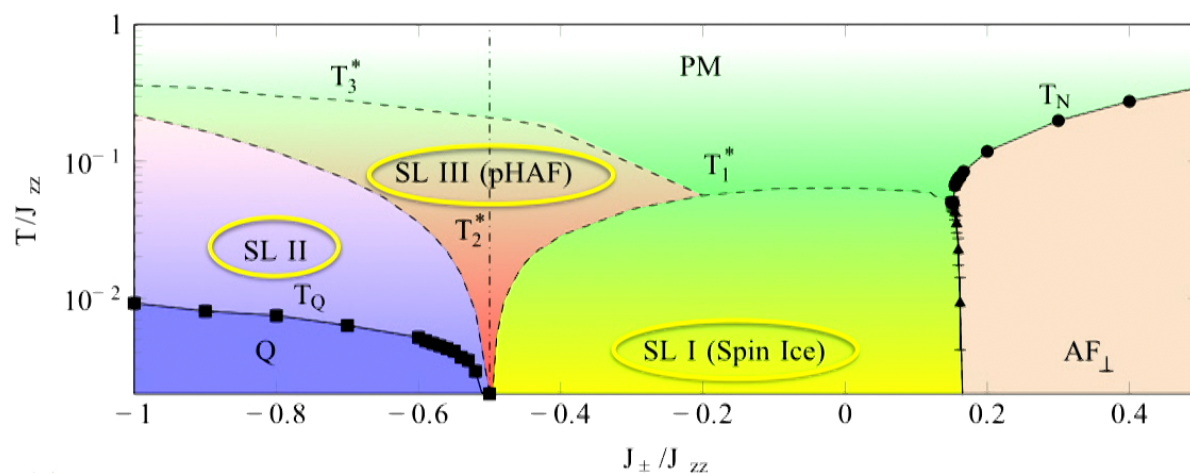
SL 3 (pHAF)



SL 1 (spin ice)

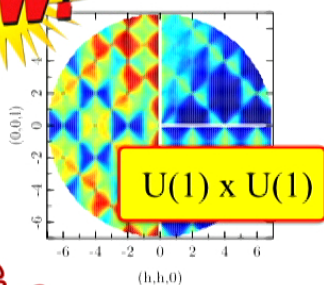


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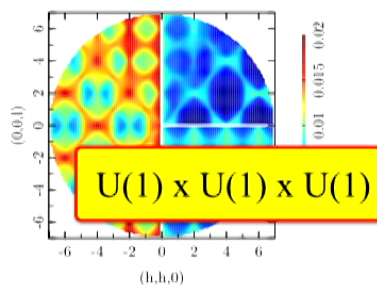


NEW!

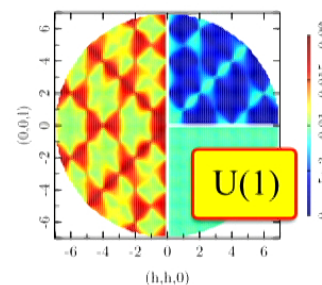
SL 2



SL 3 (pHAF)

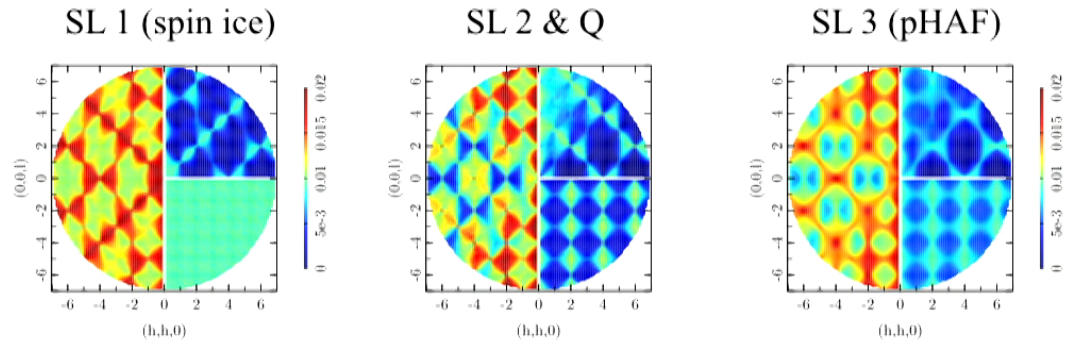


SL 1 (spin ice)



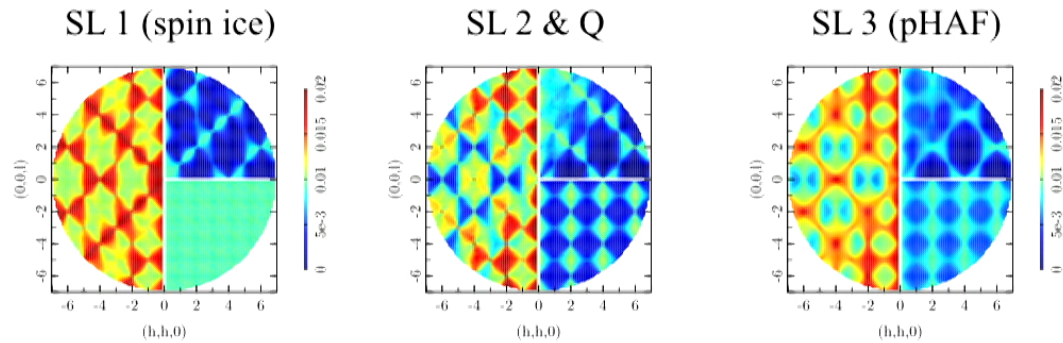
what have we learned ? (experiment)

pinch points \Rightarrow spin ice

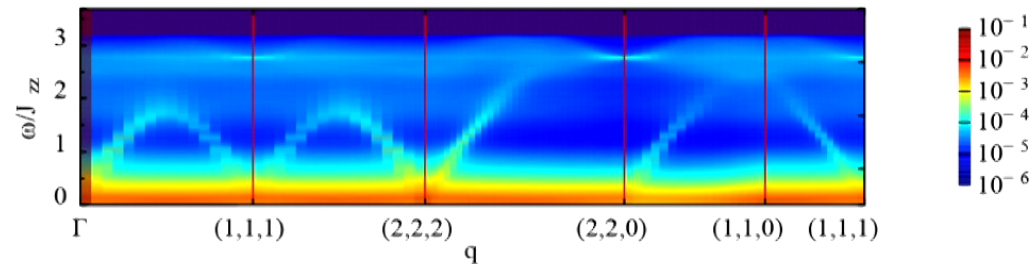


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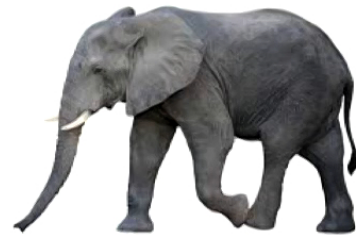
pinch points \Rightarrow spin ice



non-dispersing continua may hide many secrets...



what's left ?



quantum effects !!!



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the story so far...

minimal model of quantum spin ice

$$\mathcal{H}_{\text{QSI}} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z - J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+)$$

change sign of J_{\pm}

3 spin liquids
+ hidden spin-nematic order !



the story so far...

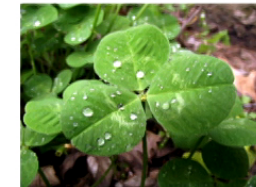
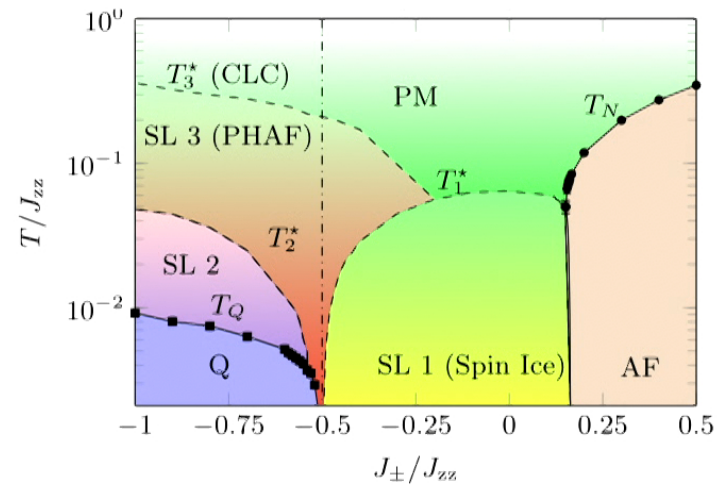
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change sign of J_{\pm}

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phase diagram from classical
Monte Carlo simulation



omne trium perfectum !!!



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