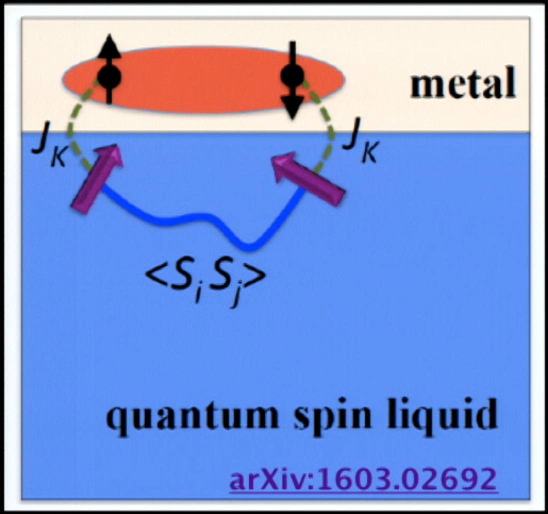
Title: Topological superconductivity in metal/quantum-spin-ice heterostructures

Date: Jun 08, 2017 02:00 PM

URL: http://pirsa.org/17060045

Abstract: Superconductivity research has traditionally been discovery driven. Of course, Tc is a non-universal quantity that cannot be predicted, hence off-limits to theorists. Nevertheless, it must be possible to reach intelligent predictions for superconductors that are interesting for reasons other than high Tc per se. Of particular interest are topological superconductors under pursuit as a platform for quantum computing. Here, I will present the strategy of using the spin-spin correlation of quantum spin ice to achieve topological superconductivity at the interface between metal and quantum spin ice.

“Topological superconductivity in metal/quantum-spin-ice heterostructures”



Eun-Ah Kim (Cornell)

Perimeter 6.8.2017

Majorana bound state in odd-parity SC

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- Vortices of p+ip SF ➡ zero modes at the core

  Kopnin and Salomaa PRB (1991)

- Zero modes are Majorana

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  - non-local q-bit: $c = \gamma_1 + i\gamma_2$

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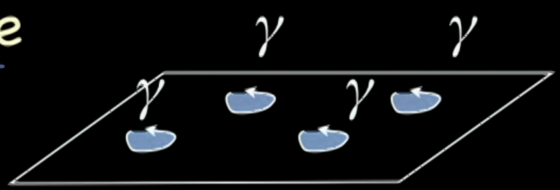
- Majorana + vortex composite

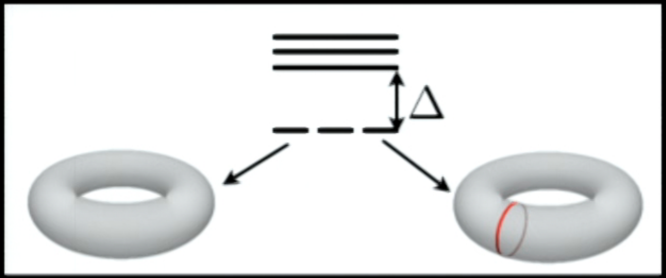
  ➡ non-Abelian statistics



fractionalization and Topo degeneracy

- Fractional charge $e^{*}=e/q$

  $N_g=q^g$ e.g., $N_1=3$



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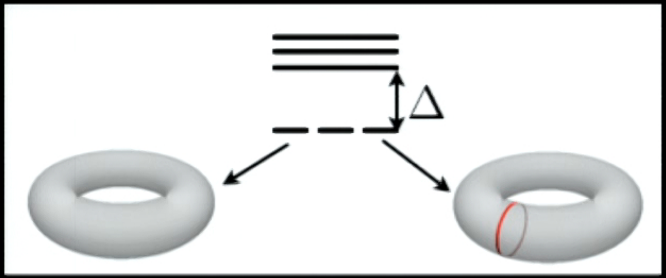
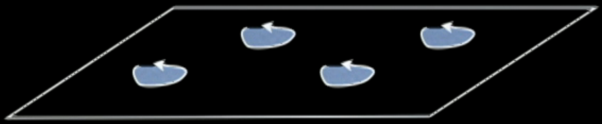


- 2n Non-abelian vortices



  $N_{2n}=2^{n-1}$ for MR state or p+ip SF

Anyons

- Fractional charge e^*
- Fractional statistics (spin) θ
- n- nonabelian qp state ⇨set of Qubits

$$\Psi(x_1, \cdots, x_n) = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_{d(n)} \end{pmatrix}$$

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$$\Psi(x_1, \cdots, x_n) = \begin{pmatrix} \psi_1 \\ \vdots \\ \underline{\underline{\psi_{d(n)}}} \end{pmatrix}$$

→ exchange of qp's: rotation

in *d(n)* dim

Hilbert space

$$\Psi(x_1 \leftrightarrow x_3) = \underline{\underline{M}}\,\Psi(x_1, \cdots, x_n)$$

$$\Psi(x_1 \leftrightarrow x_2) = \underline{\underline{N}}\,\Psi(x_1, \cdots, x_n)$$

Q. Topological Superconductor material?

Review:
Kallin & Berlinsky, Rep. Prog, Phys. (2016),
Alicea, Rep. Prog. Phys (2012)

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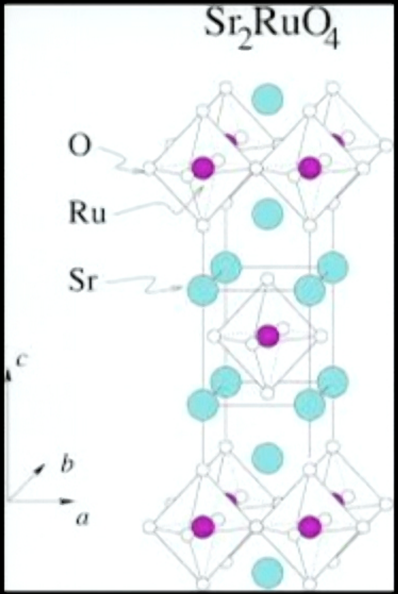
Bulk



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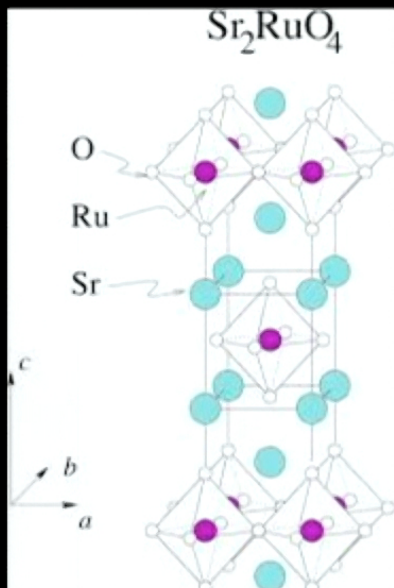
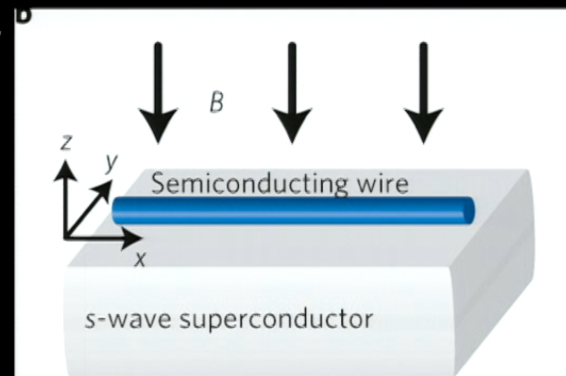


1D proximity



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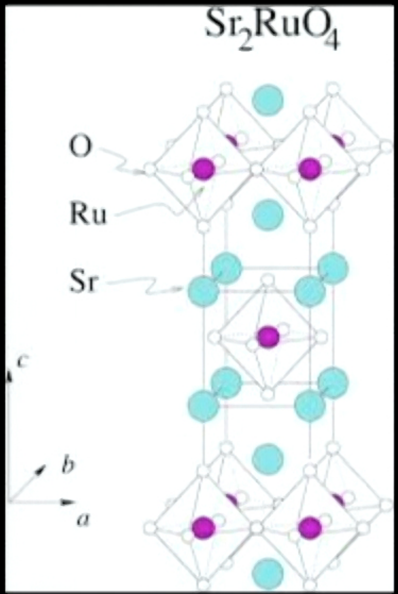

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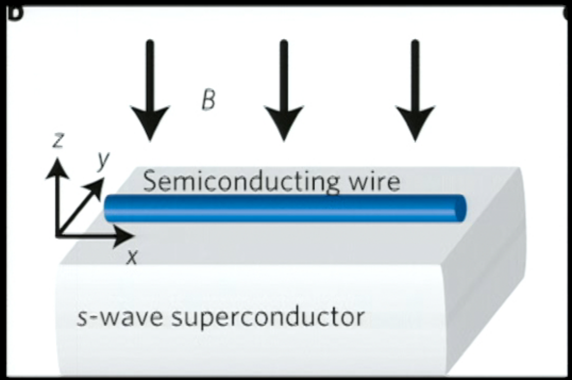
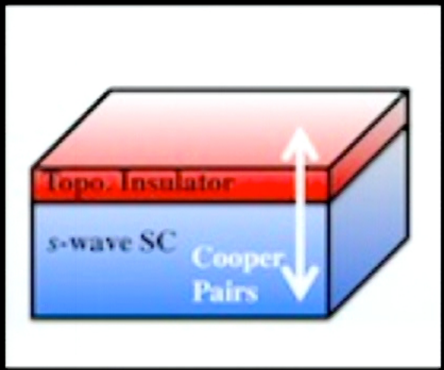


2D proximity?



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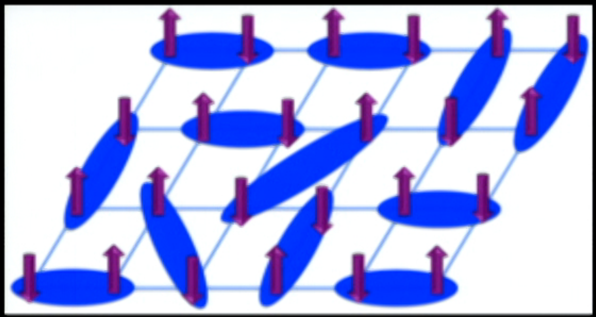
Design Strategy for intrinsic odd-parity superconductor?

- Manipulate the pairing interaction: target non-phononic mechanism

Wanted: non-phononic mechanism

Dope a Quantum spin liquid

P.W.Anderson



RVB singlet

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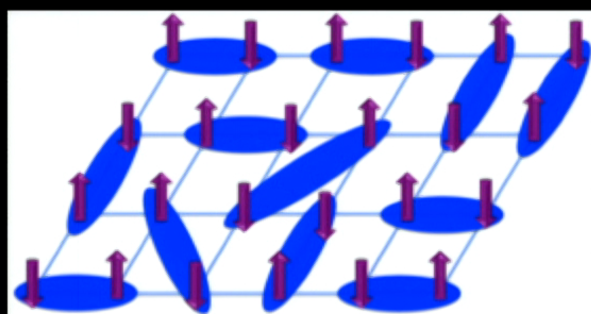
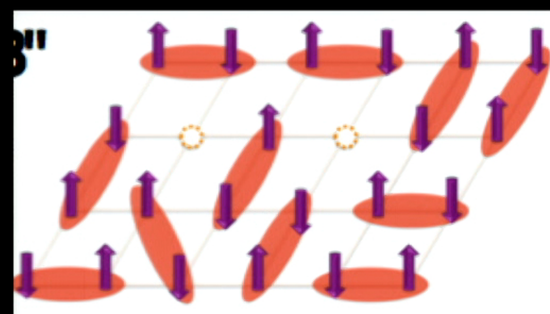
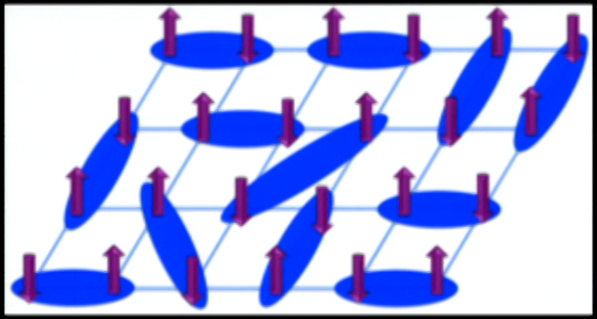




RVB singlet   Cooper pair singlet

Wanted: non-phononic mechanism

Use Quantum paramagnet



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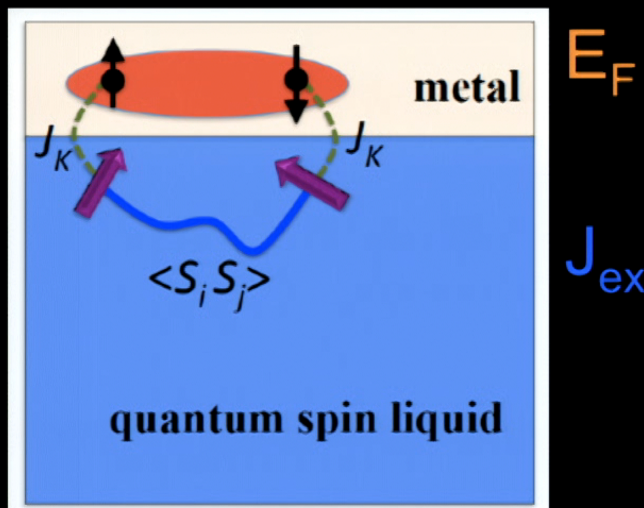

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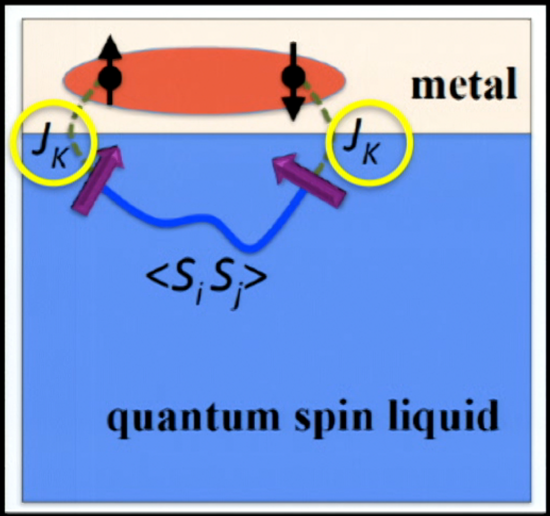


E_F

J_{ex}

- Characteristic energy scales:

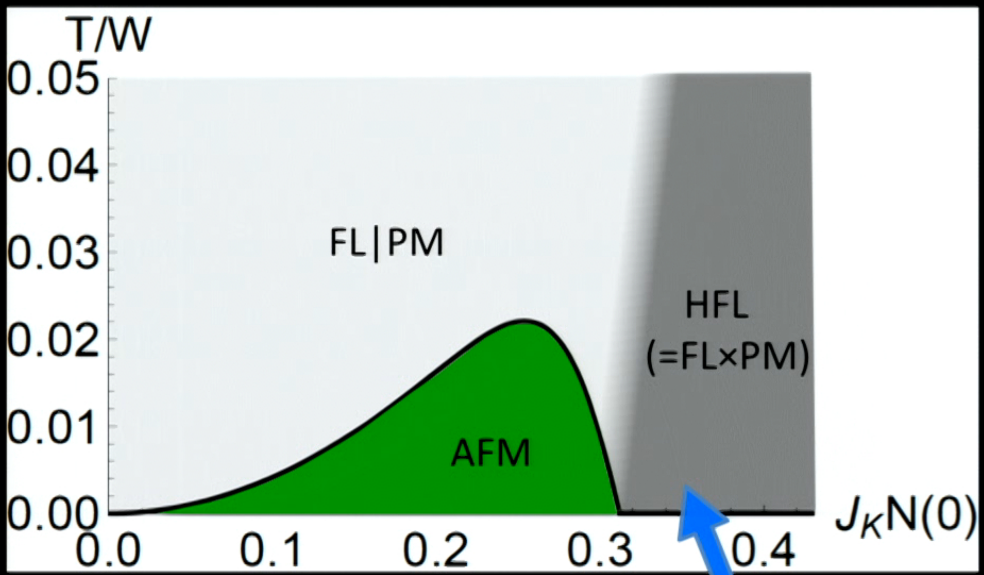
  $E_{F,}$ $J_{ex,}$ J_K

- Perturbative limit:

  $J_K / E_F << 1$

- Spin-fermion model

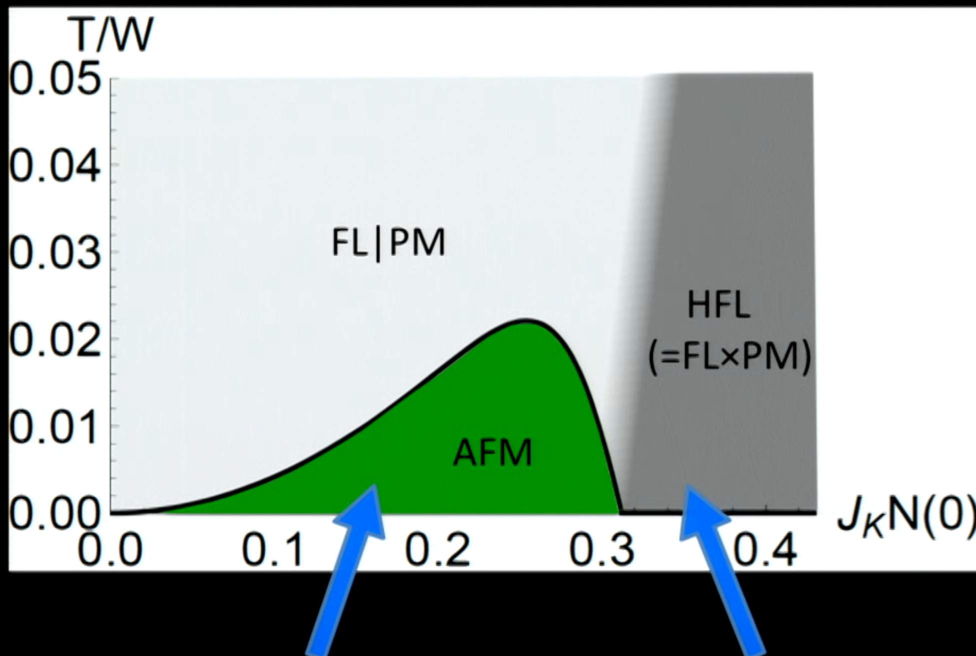
Spin-fermion model for J_{ex}=0



Kondo-Singlet

Doniach (1977)

Spin-fermion model for J_{ex}=0



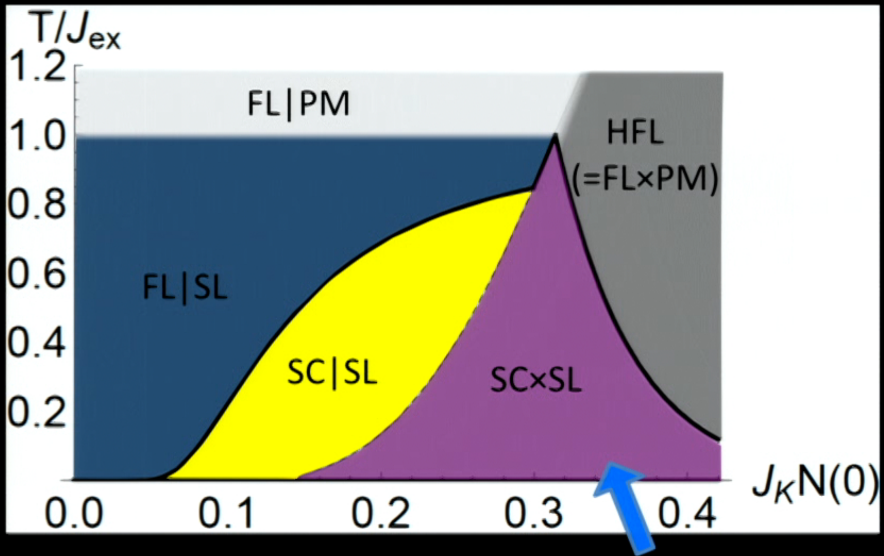
RKKY interaction

Kondo-Singlet

Doniach (1977)

Spin-fermion model for J_{ex}+ Frustration

For $J_{RKKY} \sim J_K^2 N(0) < J_{ex}$ no AFM

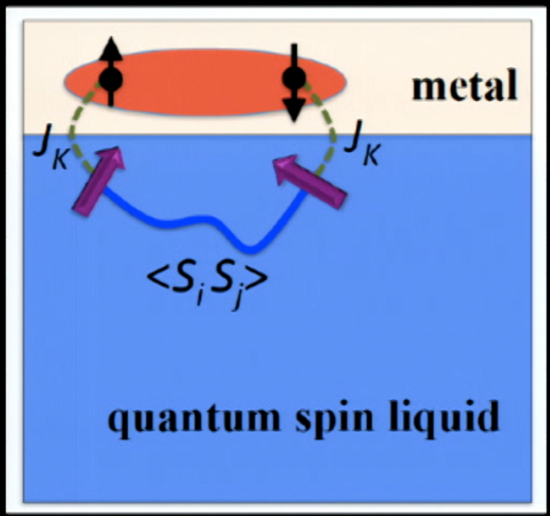


Kondo-Singlet + RVB singlet+Cooper pair

Coleman & Andrei (1989)

Senthil, Vojta, Sachdev (2003)

How to predictively materialize SC|QPM ?



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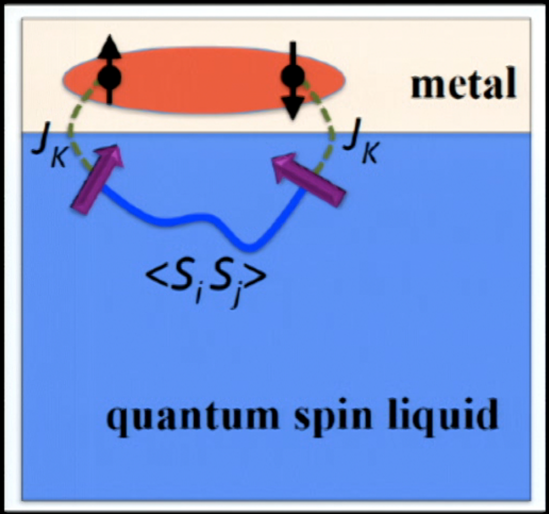


Simple isotropic metal

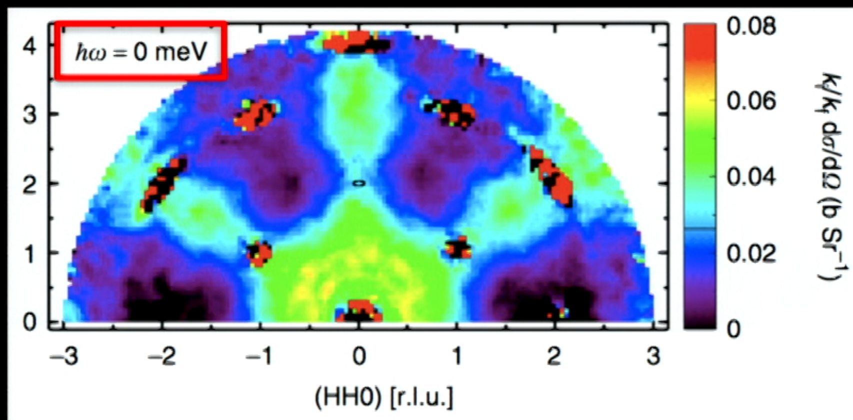
1. $<S>=0$
2. Dynamic spin fluctuation $<S_i S_j>$
3. Well understood

Emergent Vector Field in Spin Ice

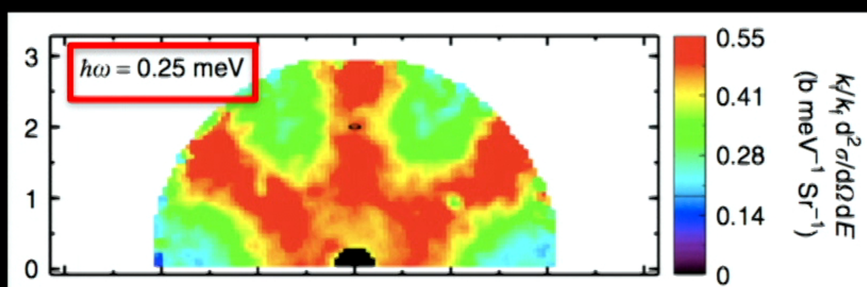
Received 20 Sep 2012 | Accepted 18 Apr 2013 | Published 17 Jun 2013 DOI: 10.1038/ncomms2914

Quantum fluctuations in spin-ice-like $Pr_2Zr_2O_7$

K. Kimura[1], S. Nakatsuji[1,2], J.-J. Wen[3], C. Broholm[3,4,5], M.B. Stone[5], E. Nishibori[6] & H. Sawa[6]

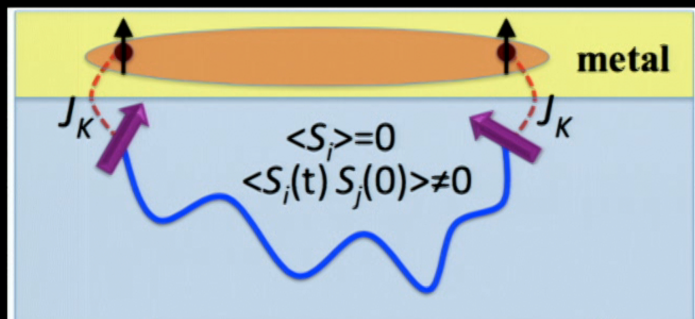


- Elastic neutron: pinch points (spin-ice like)



- Inelastic neutron: over 90% weight

Effective Continuum Theory



$$H_c = \sum_{\boldsymbol{k}\alpha} \left(\frac{\hbar^2 k^2}{2m} - E_F \right) \psi^\dagger_\alpha(\boldsymbol{k}) \psi_\alpha(\boldsymbol{k})$$

$$H_K(t) = J_K v_{\text{cell}} \sum_{a\alpha\beta} \int d^2\boldsymbol{r}\, \psi^\dagger_\alpha(\boldsymbol{r}) \sigma^a_{\alpha\beta} \psi_\beta(\boldsymbol{r}) S_a(\boldsymbol{r}_\perp = \boldsymbol{r}, z = 0, t)$$

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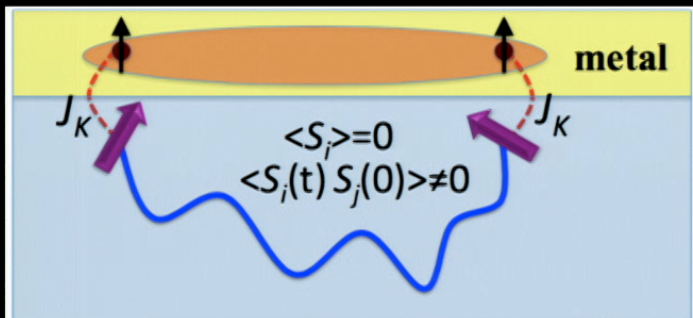

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- Integrate out spins >> Effective e-e interaction

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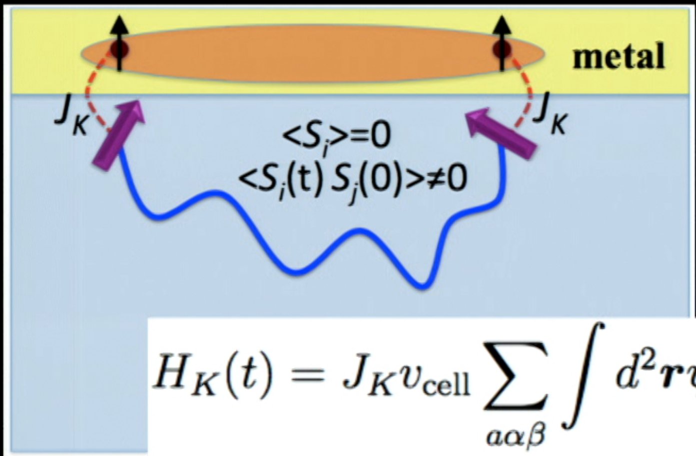

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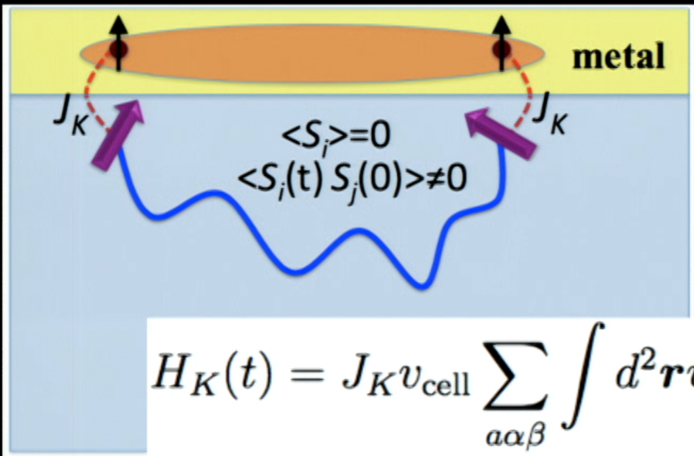
Effective Continuum Theory



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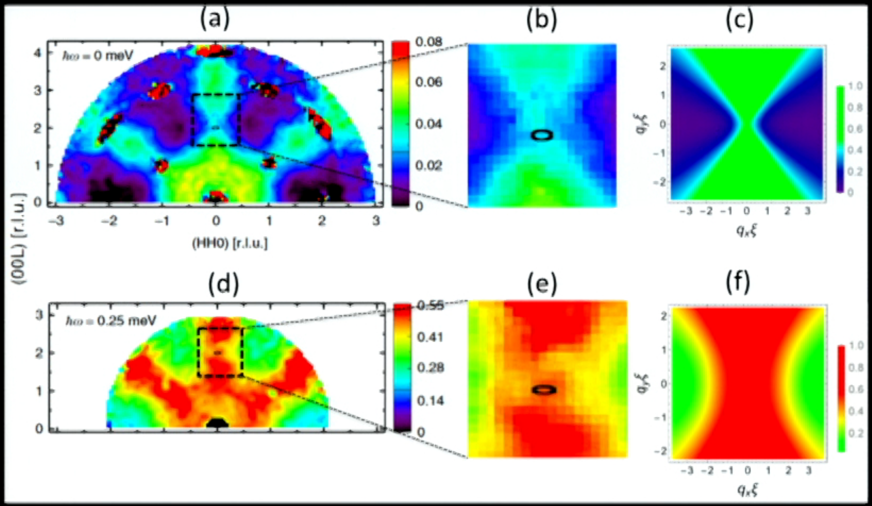
$$s_a(\boldsymbol{r}, t) \ = \ \sum_{\alpha\beta} \psi^\dagger_\alpha(\boldsymbol{r}, t) \sigma^a_{\alpha\beta} \psi_\beta(\boldsymbol{r}, t)$$

Hydrodynamic model for dynamic susceptibility

$$\chi_{ab}(\boldsymbol{q},\omega) = \frac{\chi_0^T}{1 - i\omega/\Omega}\left(\delta_{ab} - \frac{q_a q_b}{q^2}\right) + \frac{\chi_0^L}{1 - i\omega/\Omega + q^2\xi^2}\frac{q_a q_b}{q^2}$$



Unusual Gauge-Matter Coupling

- Minimal Coupling

$$\vec{j}(\boldsymbol{q}) \cdot \vec{A}(\boldsymbol{q}) = e \sum_{\boldsymbol{k}} \sum_{\alpha} \vec{A}(\boldsymbol{q}) \cdot \frac{\boldsymbol{k}}{m} \psi^{\dagger}_{\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\alpha} \psi_{\boldsymbol{k}-\frac{\boldsymbol{q}}{2},\alpha}$$

- Repulsion against Cooper pairing

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- Repulsion against Cooper pairing

$$-\sum_{\boldsymbol{p}_1 \boldsymbol{p}_2 \boldsymbol{q} \alpha} D(q) \frac{(\boldsymbol{p}_1 \times \hat{\boldsymbol{q}}) \cdot (\boldsymbol{p}_2 \times \hat{\boldsymbol{q}})}{m^2} \psi^{\dagger}_{\boldsymbol{p}_1+\boldsymbol{q},\alpha} \psi_{\boldsymbol{p}_1,\alpha} \psi^{\dagger}_{\boldsymbol{p}_2-\boldsymbol{q},\beta} \psi_{\boldsymbol{p}_2,\beta}$$

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- Spin-ice/electron

$$J_K \sum_{\boldsymbol{r}\alpha\beta} \psi^{\dagger}_{\boldsymbol{r}\alpha} \vec{\sigma}_{\alpha\beta} \psi_{\boldsymbol{r}\beta} \cdot \left[\vec{\nabla} \times \vec{A}(\boldsymbol{r}) \right]$$

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Electrons are not magnetic monopoles

- Attractive equal-spin interaction!

$$-J_K^2 D(q) \left(\vec{\sigma}_{\alpha\beta} \times \hat{\boldsymbol{q}} \right) \cdot \left(\vec{\sigma}_{\alpha'\beta'} \times \hat{\boldsymbol{q}} \right)$$

Selection Rule Dictated Odd-Parity

- Pair binding problem with dipole-dipole interaction

$$V_{\rm dd} = \frac{1}{r^3}[\vec{S}_1 \cdot \vec{S}_2 - 3(\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r})] \propto \mathcal{R}^{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2) \cdot \mathcal{S}^{(2)}(\boldsymbol{s}_1, \boldsymbol{s}_2)$$

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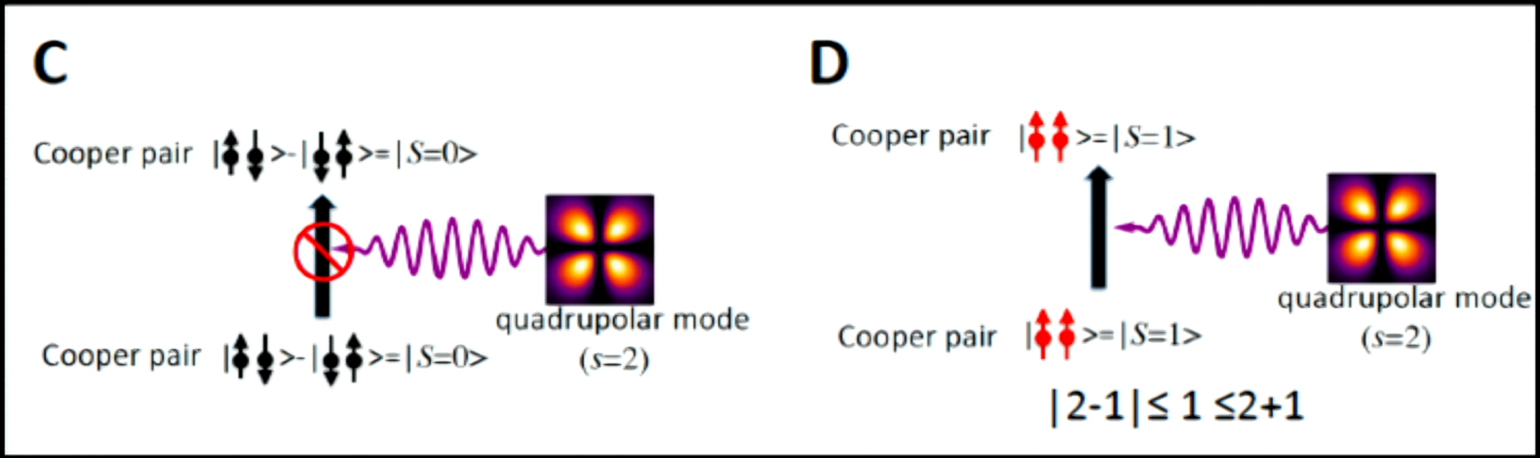

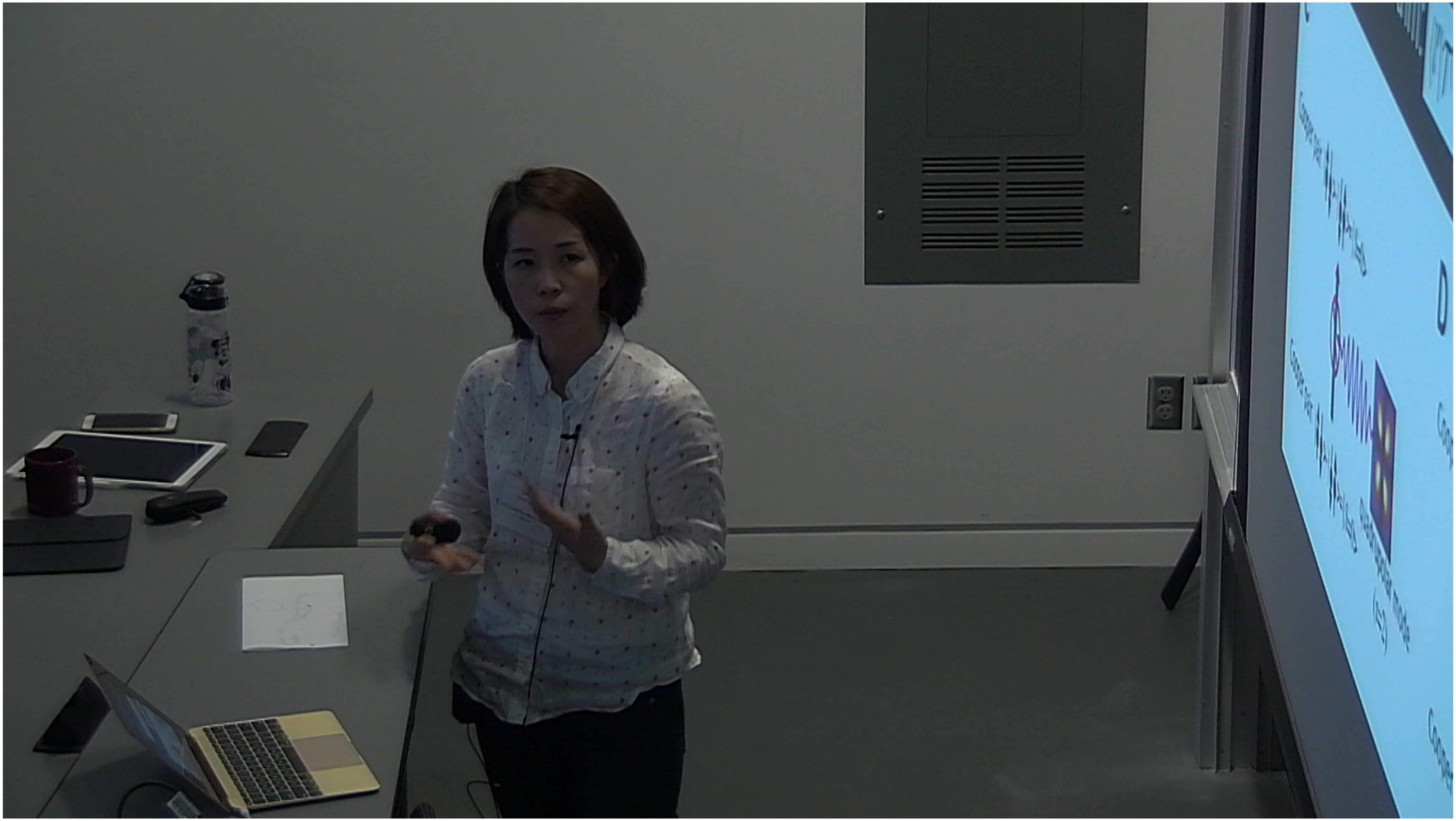

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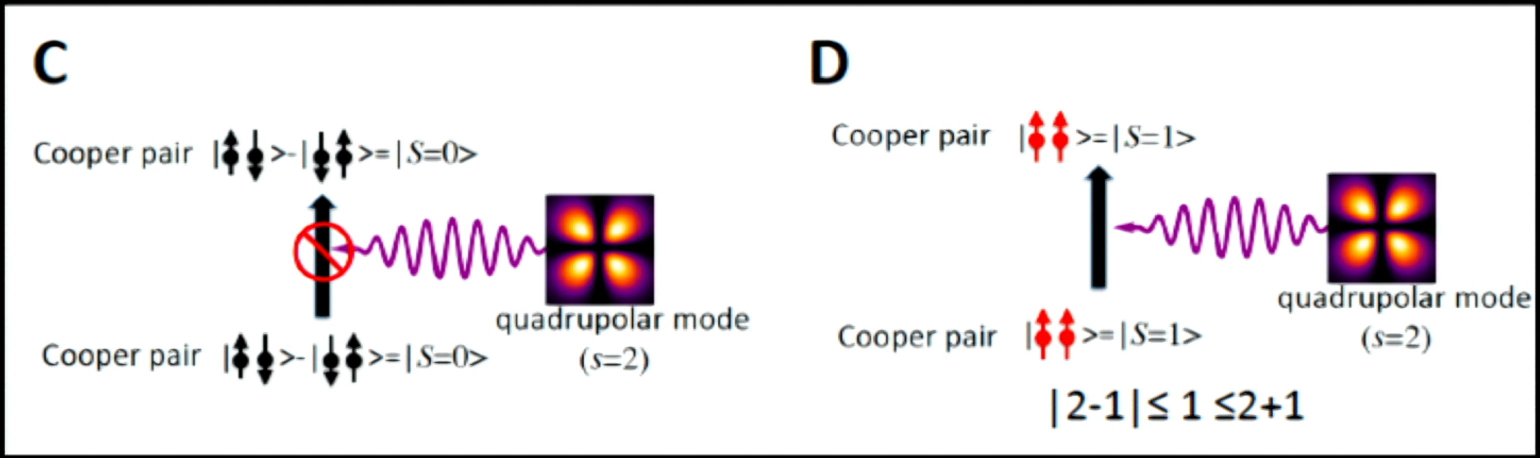

Dealing with interacting electrons?

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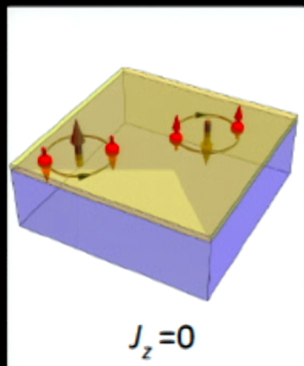
- Separation of scale: $\omega_S/E_F \ll 1$
  ➡"Migdal theorem"
- Dimensionless ratio: $\lambda \sim N(0)V \sim J_K^2 N(0)/J_{\rm ex} < 1$
- Full problem ≈
  solving the BCS mean-field theory

Dealing with interacting electrons?

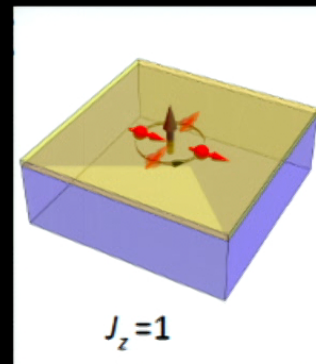
$$H_{\rm int}(t) = -(J_K^2 v_{\rm cell}^2/2\hbar) \sum_{ab} \int dt' \int d^2\boldsymbol{r} d^2\boldsymbol{r}' s_a(\boldsymbol{r},t) \langle S_a(\boldsymbol{r},0,t) S_b(\boldsymbol{r}',0,t')\rangle s_b(\boldsymbol{r}',t')$$

- Separation of scale: ω_S/E_F <<1
  ➡"Migdal theorem"
- Dimensionless ratio: $\lambda \sim N(0)V \sim J_K^2 N(0)/J_{\rm ex} < 1$
- Full problem ≈
  solving the BCS mean-field theory $T_c \sim \omega_s e^{-1/\lambda}$

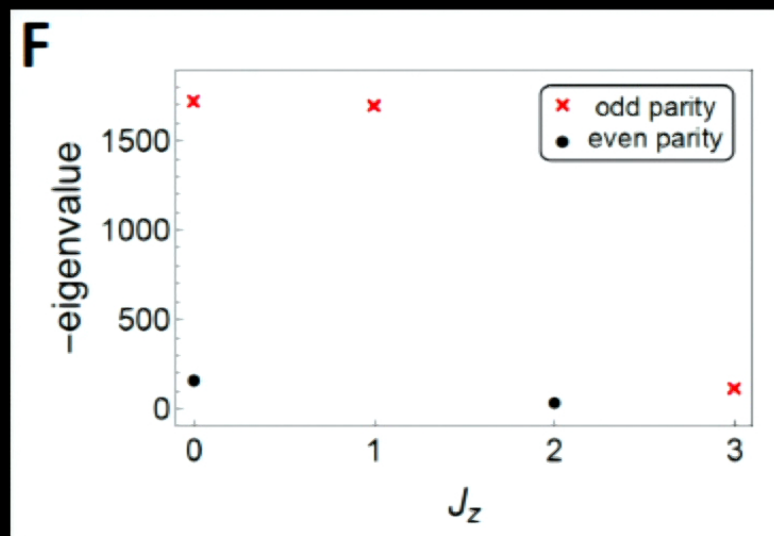
Leading channels



$$(k_x + ik_y)|\downarrow\downarrow\rangle + (k_x - ik_y)|\uparrow\uparrow\rangle$$



$$(k_x \pm ik_y)\frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}$$



Criteria for Metal

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- Structural
  - Lattice match
    - ➡ $A_2B_2O_7$
  - No orphan bonds

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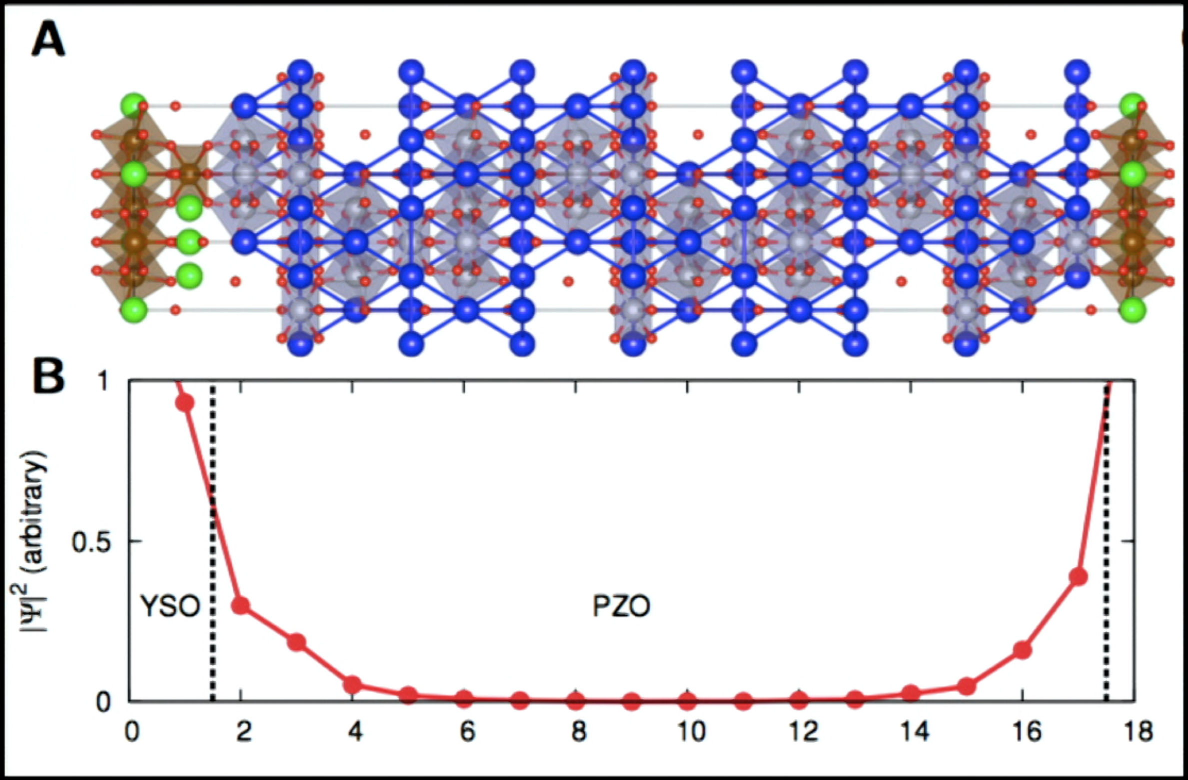
Wave function penetration



Band structure for the Proposal

$Pr_2Zr_2O_7/Y_2Sn_{2-x}Sb_xO_7$ (111)

x=0.2

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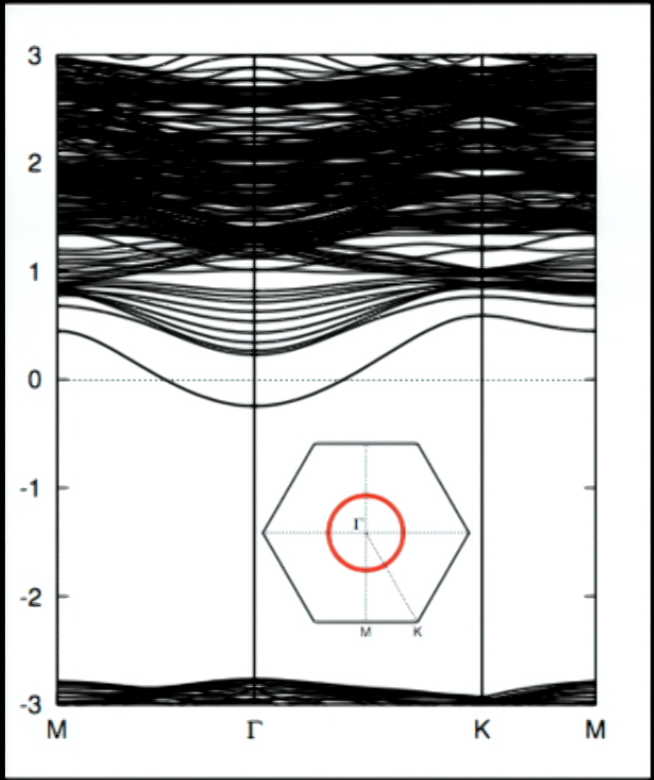
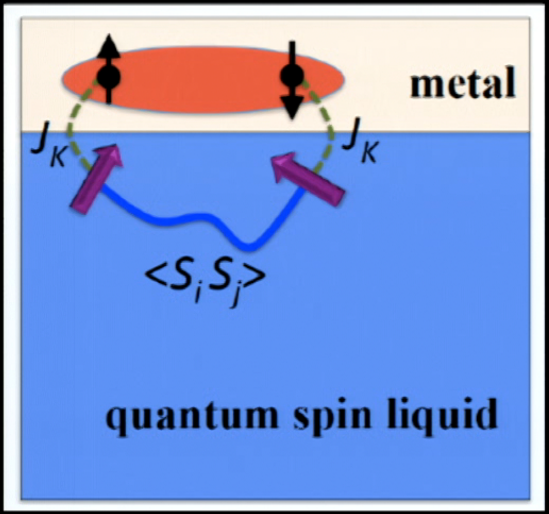


TSC in Metal/Quantum-Spin-Ice Heterostructures



- Topological superconductor riding on QSL

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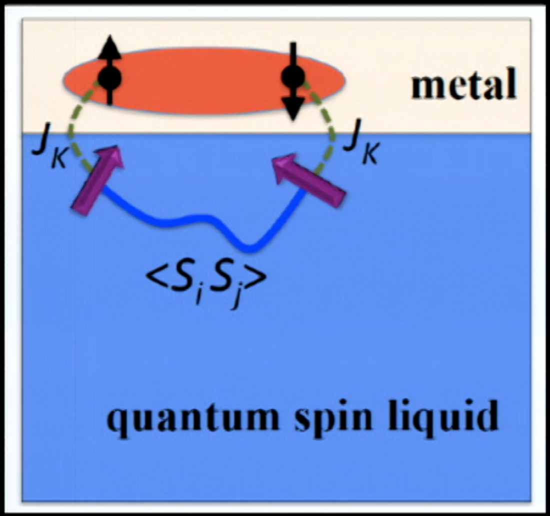
TSC in Metal/Quantum-Spin-Ice Heterostructures



- Topological superconductor riding on QSL
- Selection Rule Dictated Intrinsic Topo SC.

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