

Title: Living on the edge: multiphase competition in  $\text{Yb}_2\text{Ti}_2\text{O}_7$  and monopole crystal in  $\text{Tb}_2\text{Ti}_2\text{O}_7$

Date: Jun 08, 2017 09:00 AM

URL: <http://pirsa.org/17060039>

Abstract:  $\text{Yb}_2\text{Ti}_2\text{O}_7$  and  $\text{Tb}_2\text{Ti}_2\text{O}_7$  share the common point to sit at the edge between different phases. This multiphase competition makes the characterization of their low-temperature physics a challenge and, more generally, brings another degree of complexity to rare-earth pyrochlores. In this talk, we will take two different approaches to study these materials. For  $\text{Yb}_2\text{Ti}_2\text{O}_7$ , we explicitly investigate the influence of thermal and quantum fluctuations in this competition, with finite-temperature order by disorder and quantum shifting of the phase boundaries. For  $\text{Tb}_2\text{Ti}_2\text{O}_7$ , we will step away from the puzzling zero-field physics, and explain the undisputed order observed experimentally in a high [110] field. This order strongly supports the presence of magneto-electric coupling in  $\text{Tb}_2\text{Ti}_2\text{O}_7$ , and offers a rare example of long-range order of magnetic monopoles.

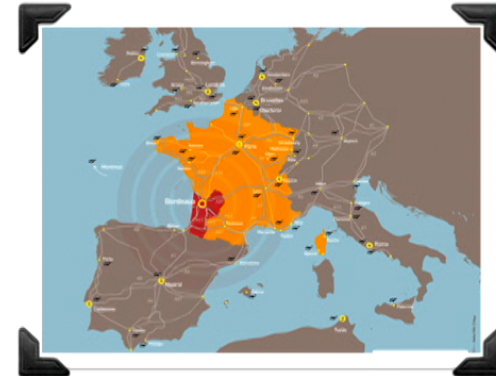
# Living on the edge

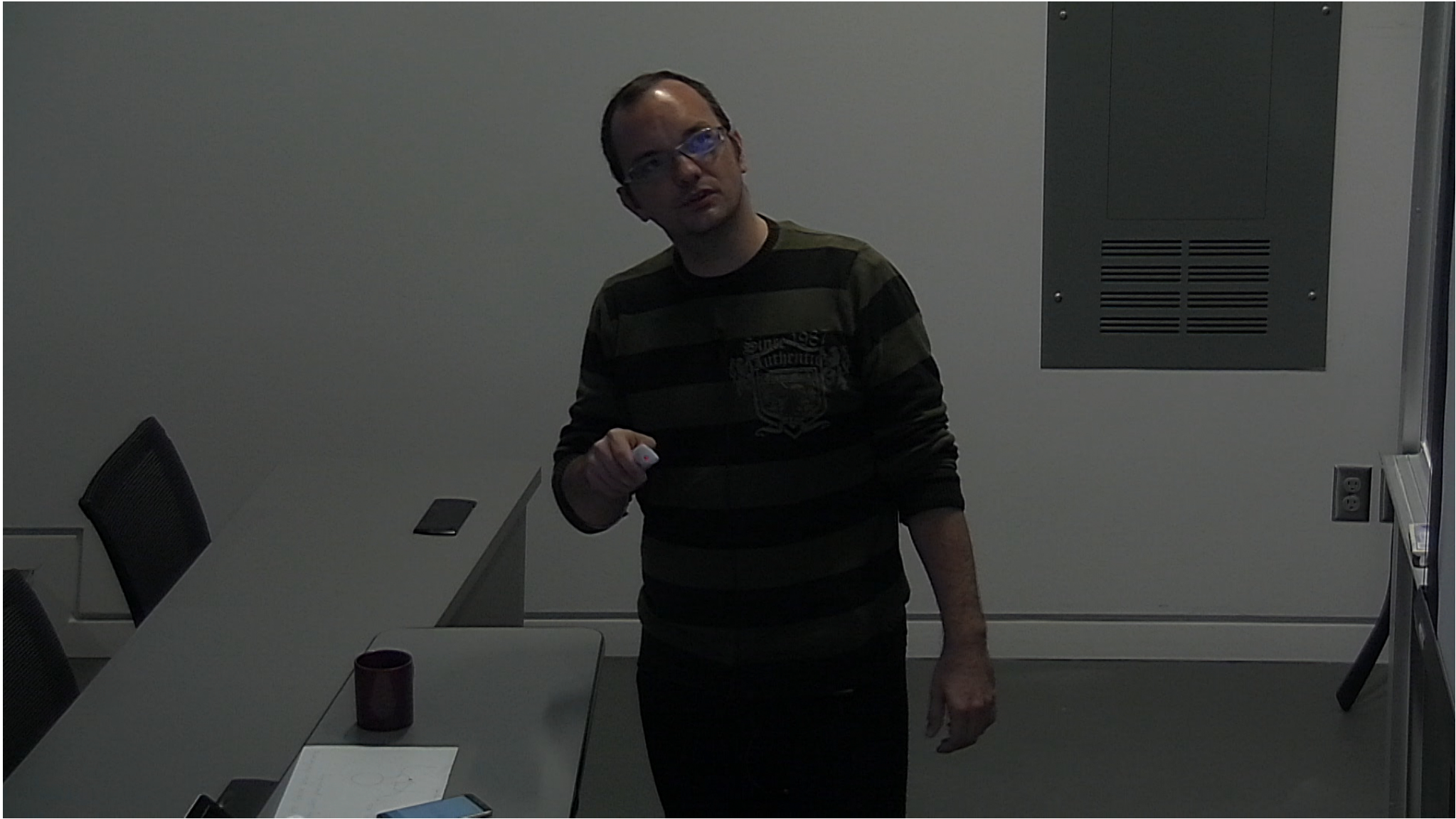
Multiphase competition in  $\text{Yb}_2\text{Ti}_2\text{O}_7$   
Monopole crystal in  $\text{Tb}_2\text{Ti}_2\text{O}_7$



QSI workshop  
Perimeter Institute  
June 8<sup>th</sup>, 2017

université  
de BORDEAUX





# Influence of boundaries in $\text{Yb}_2\text{Ti}_2\text{O}_7$

## Generic nearest-neighbour Hamiltonian

Curnoe PRB 2007

$$\mathcal{H} = \sum_{\langle ij \rangle} \vec{S}_i \vec{J}_{ij} \vec{S}_j \quad \text{with} \quad \vec{J} = \begin{pmatrix} J_2 & J_4 & J_4 \\ -J_4 & J_1 & J_3 \\ -J_4 & J_3 & J_1 \end{pmatrix}$$

$\text{Yb}_2\text{Ti}_2\text{O}_7$

Ross et al PRX 2011

Robert et al PRB 2015

Thompson et al arXiv 2017

$\text{Er}_2\text{Ti}_2\text{O}_7$

Savary et al PRL 2012

$\text{Er}_2\text{Sn}_2\text{O}_7$

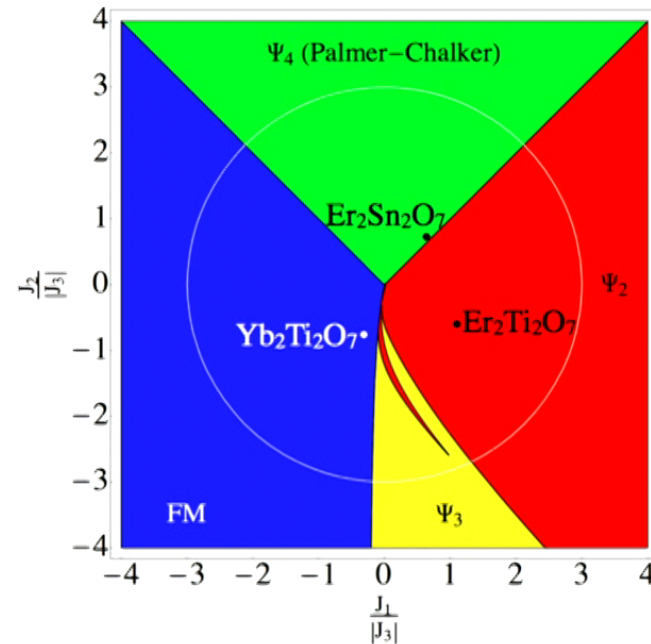
Guitteny et al PRB 2013

Petit et al arXiv 2017

## Phase diagram at $T = 0$

for classical Heisenberg spins

$J_3 < 0$  &  $J_4 = 0$



Yan, Benton, Jaubert, Shannon PRB 2017



# Influence of boundaries in $\text{Yb}_2\text{Ti}_2\text{O}_7$

Parameters of  $\text{Yb}_2\text{Ti}_2\text{O}_7$  (Ross et al PRX 2011)

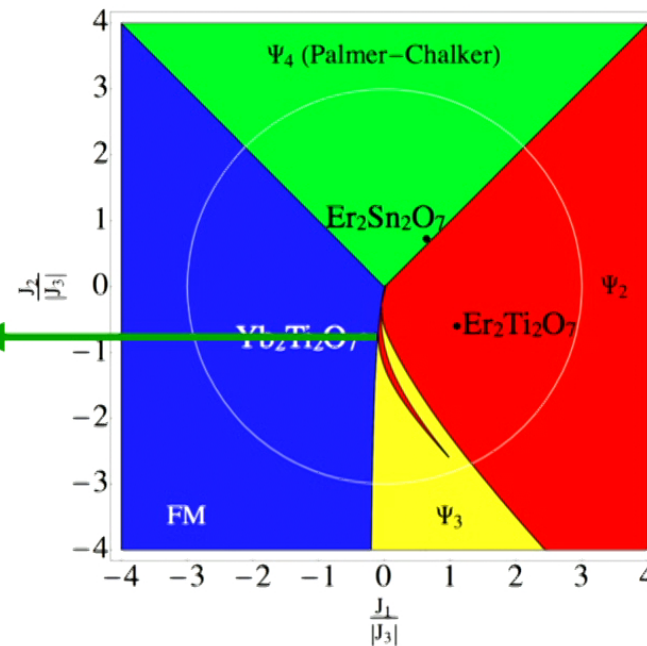
$$\{J_{i=1,2,3,4}\} = \{-0.09(3), -0.22(3), -0.29(2), 0.01(2)\} \text{ meV}$$

Jaubert, Benton, Rau, Oitmaa,  
Singh, Shannon & Gingras PRL 2015

We will consider

$$J_1 \in [-0.09 : 0] \quad \{J_{i=2,3,4}\} = \{-0.22, -0.29, 0\}$$

**Phase diagram at  $T = 0$**   
for classical Heisenberg spins  
 $J_3 < 0$  &  $J_4 = 0$



Yan, Benton, Jaubert, Shannon PRB 2017

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Parameters of  $\text{Yb}_2\text{Ti}_2\text{O}_7$  (Ross et al PRX 2011)

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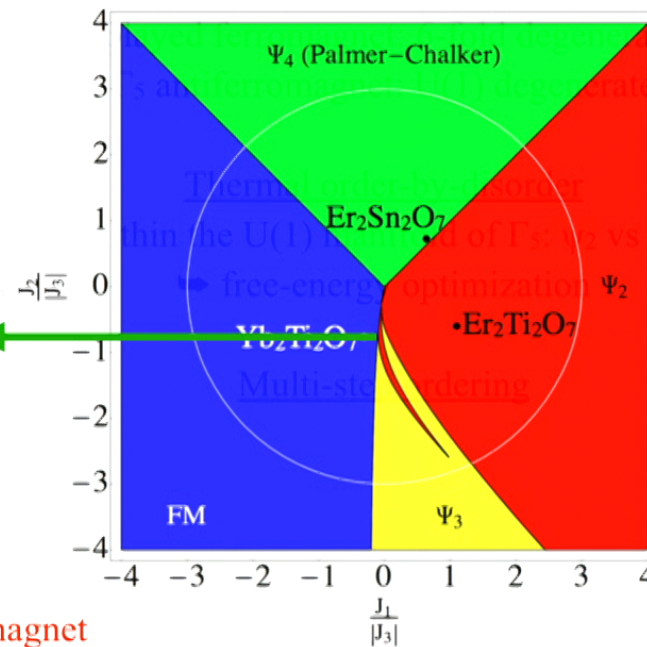
**Phase diagram at  $T = 0$**   
for classical Heisenberg spins  
 $J_3 < 0$  &  $J_4 = 0$

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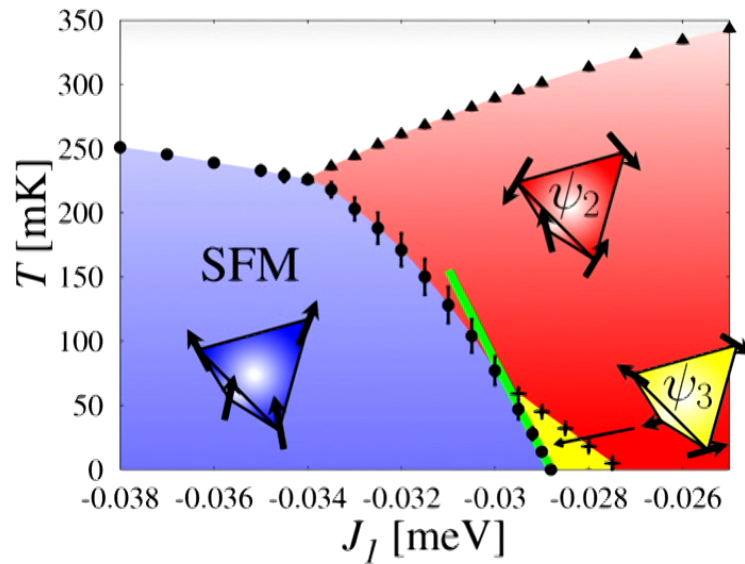
$$J_1 \in [-0.09 : 0] \quad \{J_{i=2,3,4}\} = \{-0.22, -0.29, 0\}$$

splayed ferromagnet (SFM) |  $\Gamma_5$  antiferromagnet



Yan, Benton, Jaubert, Shannon PRB 2017

# Classical phase diagram (Monte Carlo)



Multiphase competition  
splayed ferromagnet: 6-fold degenerate  
 $\Gamma_5$  antiferromagnet: U(1) degenerate

Thermal order-by-disorder  
within the U(1) manifold of  $\Gamma_5$ :  $\psi_2$  vs  $\psi_3$   
➔ free-energy optimization

Multi-step ordering

Jaubert, Benton, Rau, Oitmaa, Singh, Shannon & Gingras PRL 2015

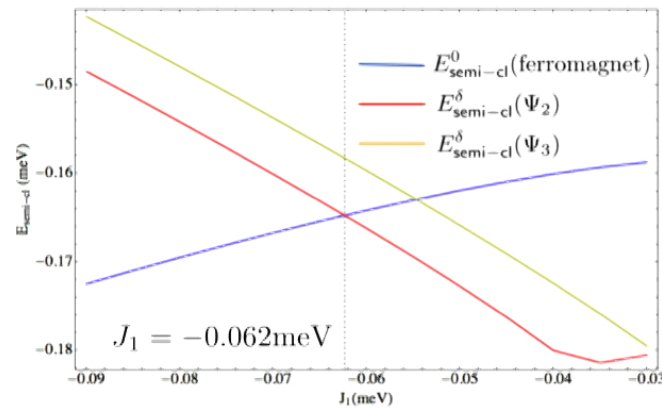
Robert, Lhotel, Remenyi, Sahling, Mirebeau, Decorse, Canals & Petit PRB 2015

# Quantum fluctuations at zero temperature

Semiclassical (linear spin wave theory - LSWT):

We compare the stability of the classical phases (SFM,  $\psi_2$ ,  $\psi_3$ ) using LSWT.

To study the stability of a classically *excited* state, we use the method of [Coletta et al PRB 2013] which provides an upper-bound of the semiclassical energy of the  $\psi_2$  and  $\psi_3$  states.





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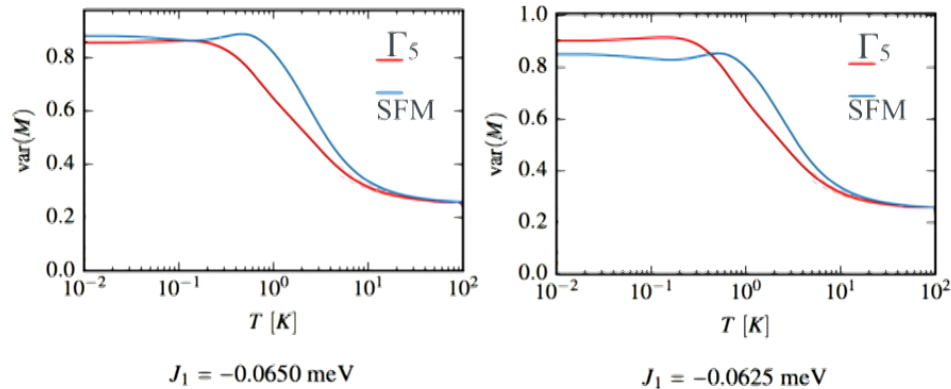
To study the stability of a classically *excited* state, we use the method of [Coletta et al PRB 2013] which provides an upper-bound of the semiclassical energy of the  $\psi_2$  and  $\psi_3$  states.

The frontier is at  $J_1 = -0.062$  meV.

Quantum (exact diagonalization with 4 and 16 sites - ED):

We compare the correlators (variance) of the corresponding order parameters.

The frontier is at  $J_1 = -0.064(2)$  meV.



# Quantum fluctuations at finite temperature

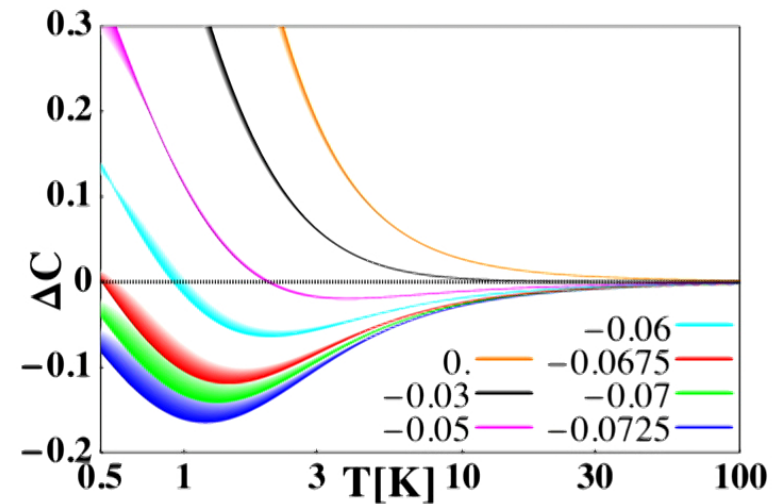
Numerical-linked-cluster calculations (NLC)

We compare the correlators (variance) of the corresponding order parameters:

$$\Delta C = C_{\Gamma_5} - C_{\text{SFM}}$$

At high temperature, we recover the classical frontier. Below  $\sim 3$  K, quantum fluctuations modify the classical physics (upturn of  $\Delta C$ ).

The frontier is at  $J_I = -0.070(3)$  meV.



# Quantum fluctuations at finite temperature

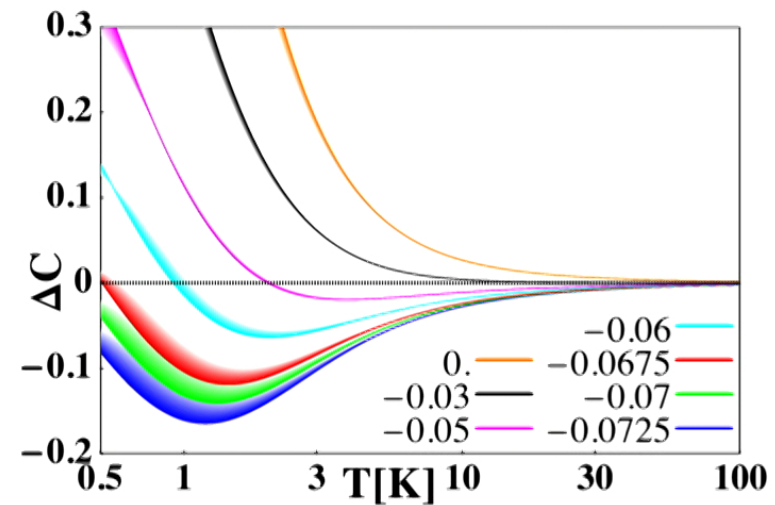
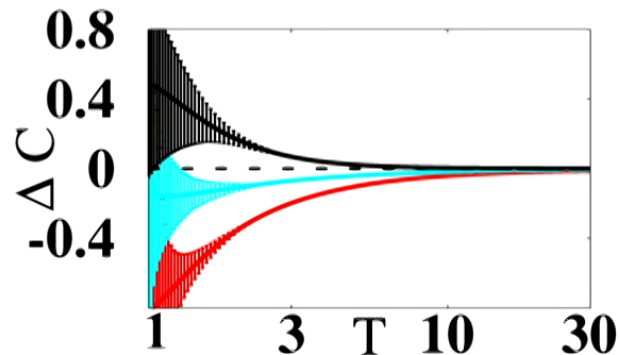
Numerical-linked-cluster calculations (NLC)

We compare the correlators (variance) of the corresponding order parameters:

$$\Delta C = C_{T5} - C_{SFM}$$

At high temperature, we recover the classical frontier. Below  $\sim 3$  K, quantum fluctuations modify the classical physics (upturn of  $\Delta C$ ).

The frontier is at  $J_I = -0.070(3)$  meV.

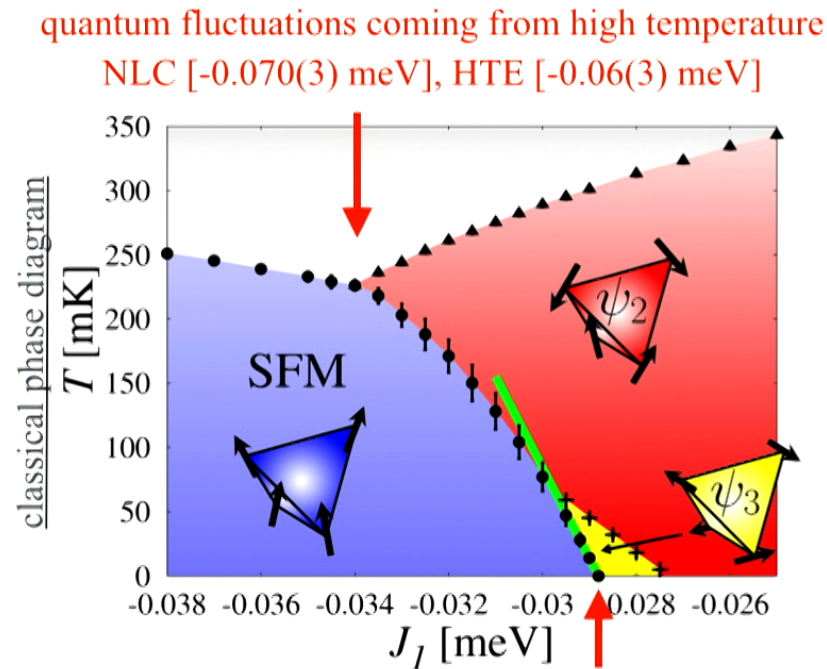


High-temperature-expansion (HTE)

The determination of the boundary is more difficult, but its quantum shifting is confirmed.

The frontier is at  $J_I = -0.06(3)$  meV.

# Quantum shifting of the phase diagram



quantum fluctuations at  $T = 0$ : LSWT [-0.062 meV], ED [-0.064(2) meV]

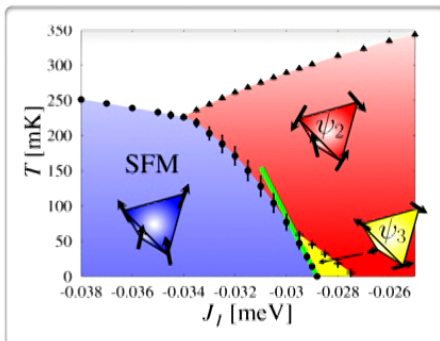
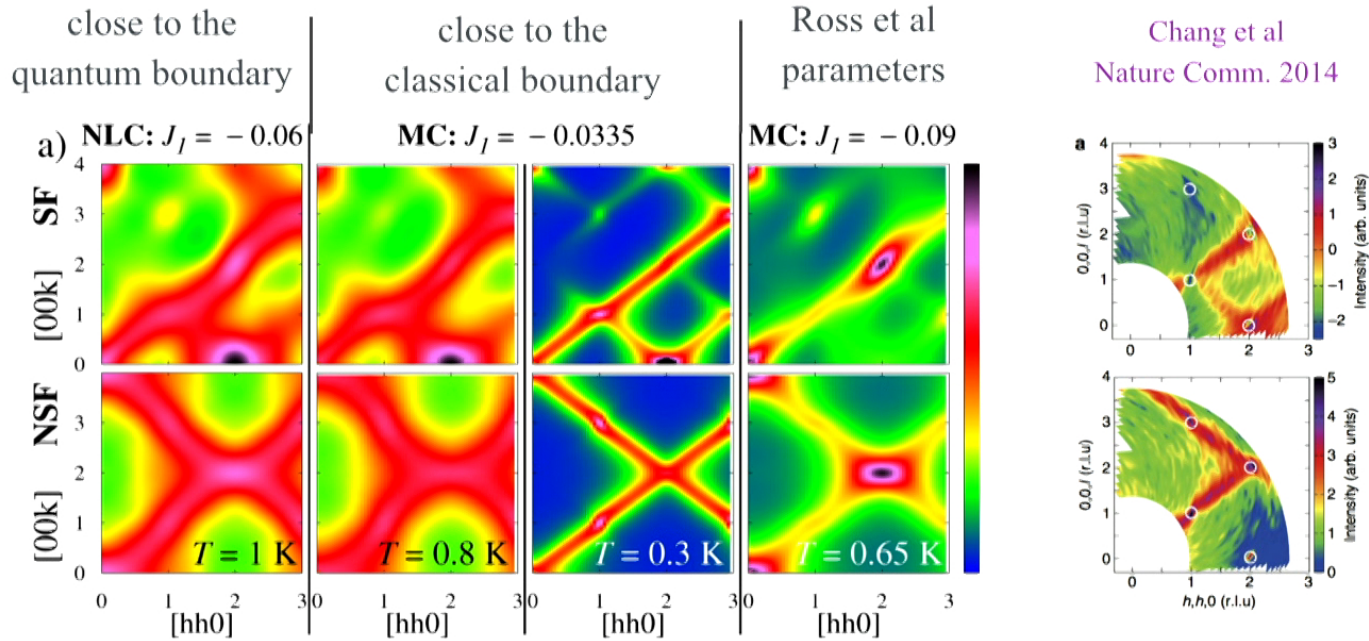
The  $\Gamma_5$  manifold is favored by quantum fluctuations.

➡ the classical boundary is shifted to more negative values of  $J_1$  by quantum fluctuations.

Our results suggest the same shape of the phase diagram.

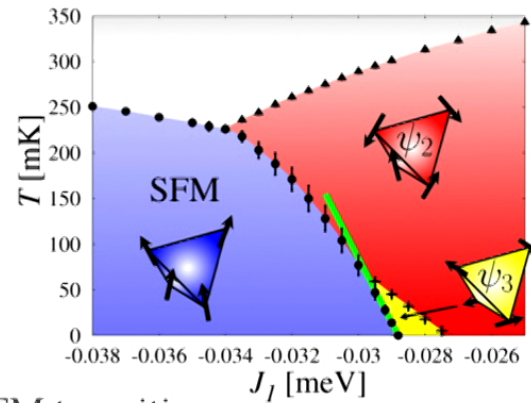


# Neutron scattering of $\text{Yb}_2\text{Ti}_2\text{O}_7$



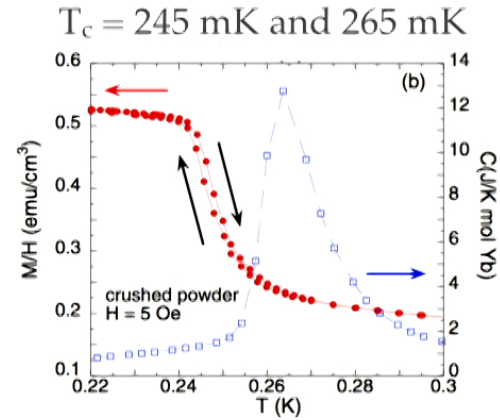
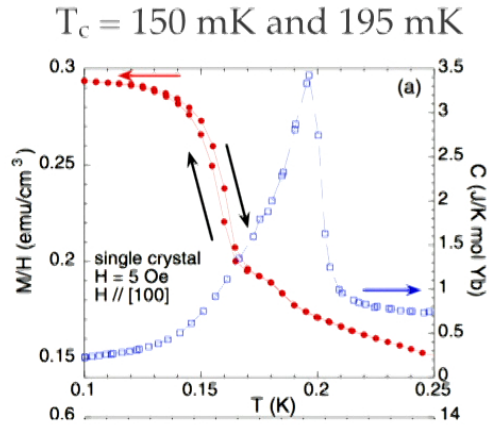
The boundary shifts with quantum fluctuations.  
 ➔ different theoretical methods might give slightly different coupling parameters near the boundary.

# Bulk measurements: double transition

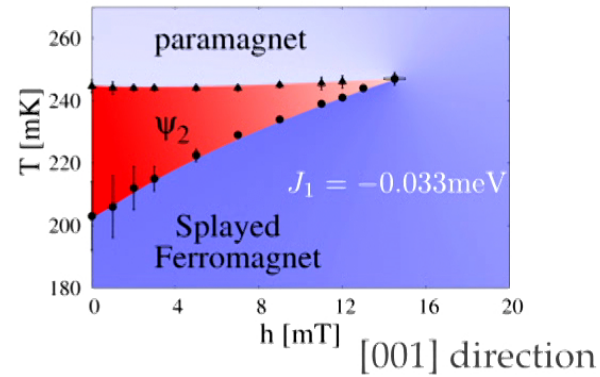
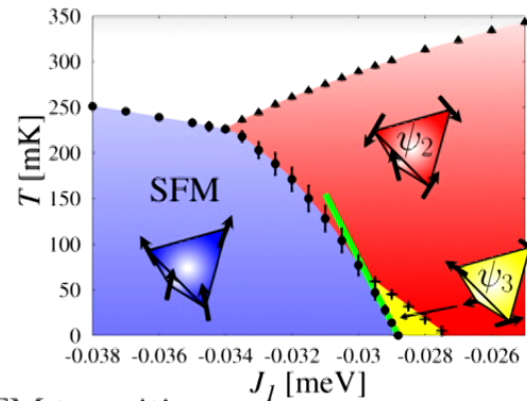


1<sup>st</sup> order SFM transition

Lhotel et al PRB 2014

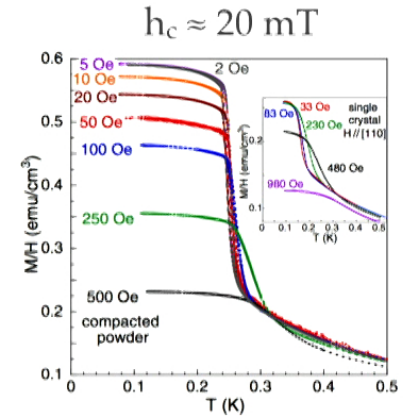
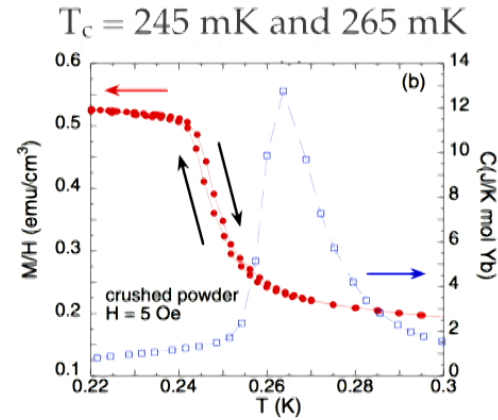
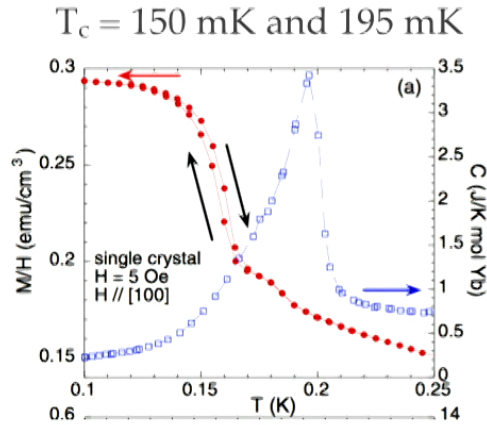


# Bulk measurements: double transition

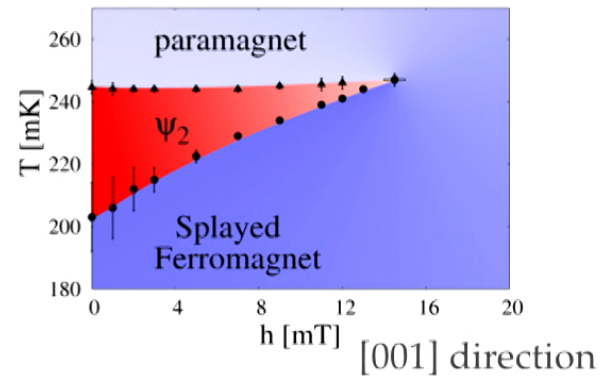
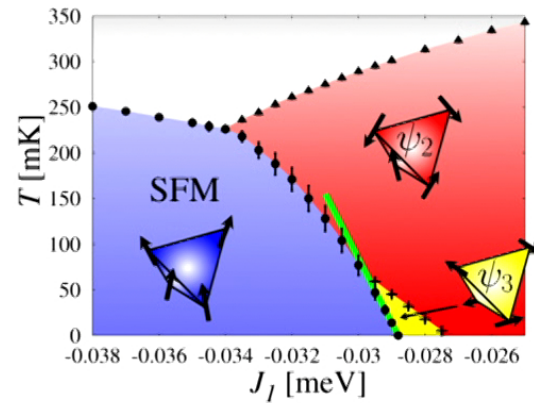


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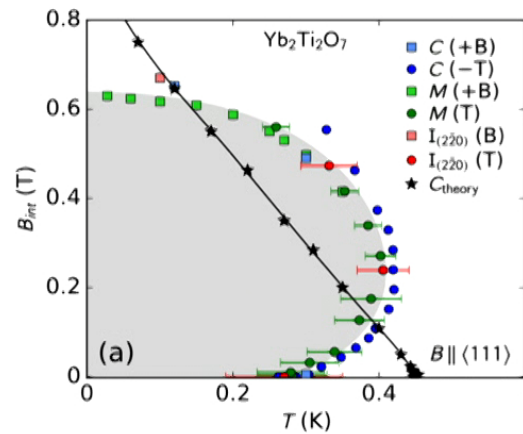
Lhotel et al PRB 2014



# Bulk measurements: reentrance in a field

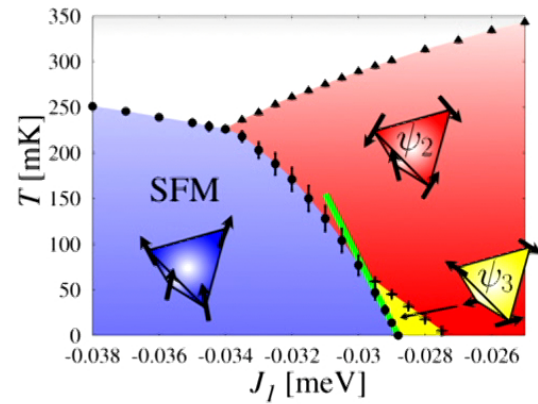


Scheie et al arXiv 2017

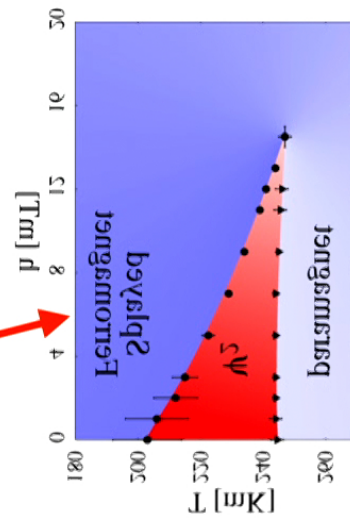
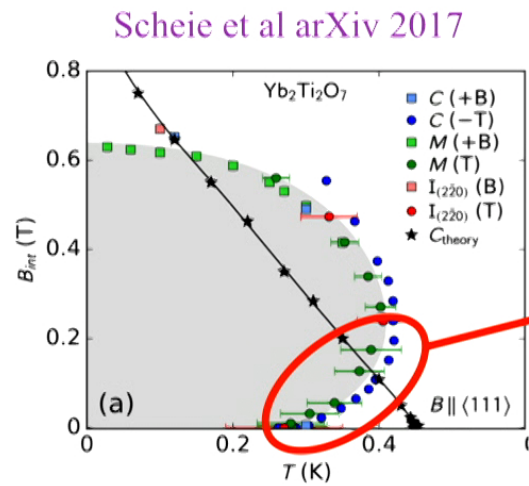




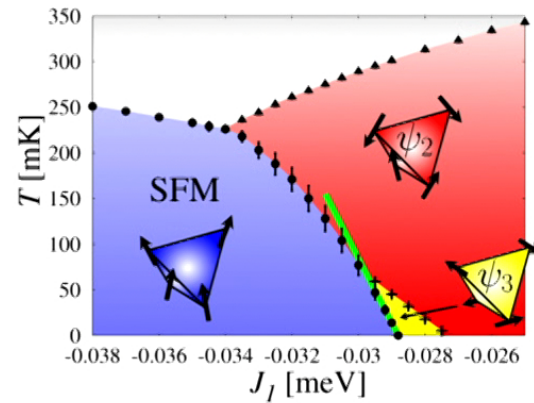
# Bulk measurements: reentrance in a field



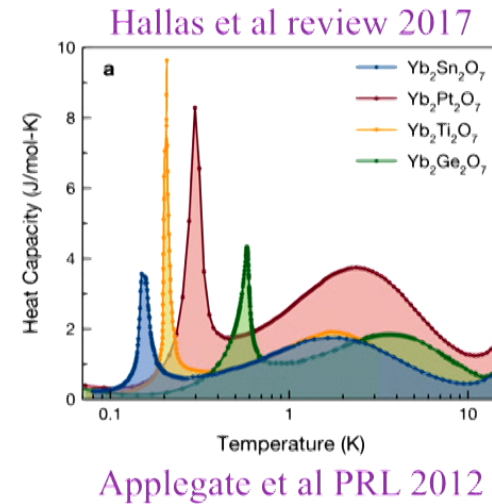
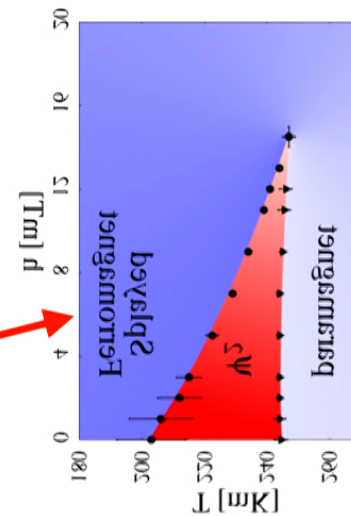
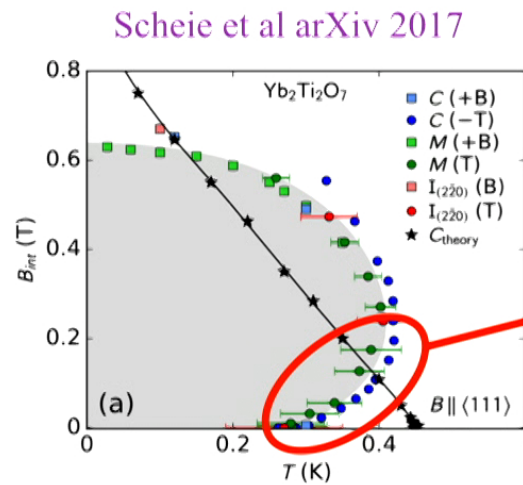
[001] direction



# Bulk measurements: reentrance in a field



[001] direction

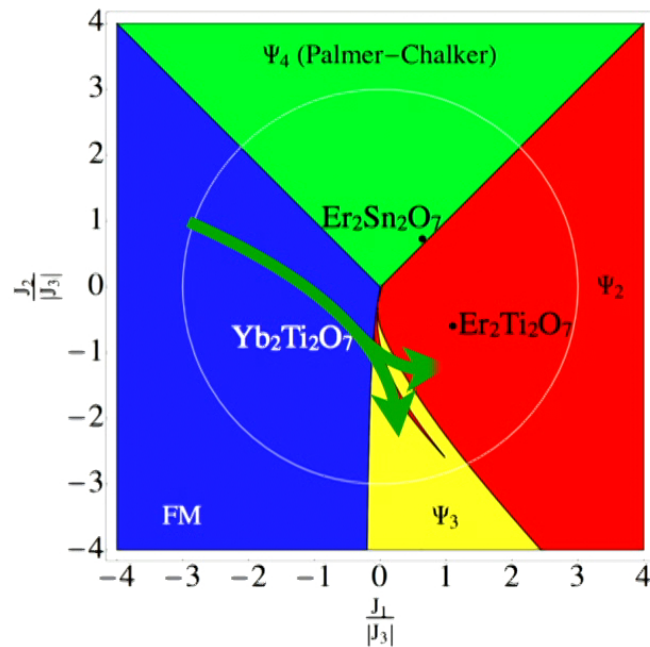


# Chemical pressure

$\text{Yb}_2\text{Sn}_2\text{O}_7$   
splayed ferromagnet  
Yaouane et al PRL 2013  
Dun et al PRB 2013

$\text{Yb}_2\text{Ti}_2\text{O}_7$

$\text{Yb}_2\text{Ge}_2\text{O}_7$   
 $\Gamma_5$  antiferromagnet  
Dun et al PRB 2014  
Dun et al PRB 2015  
Hallas et al PRB 2016



# Monopole crystal in $Tb_2Ti_2O_7$



Han  
Yan  
OIST  
Japan



Nic  
Shannon  
OIST  
Japan

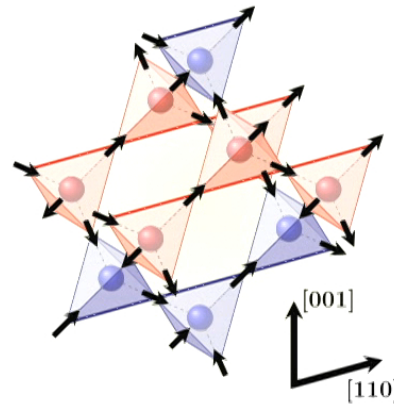
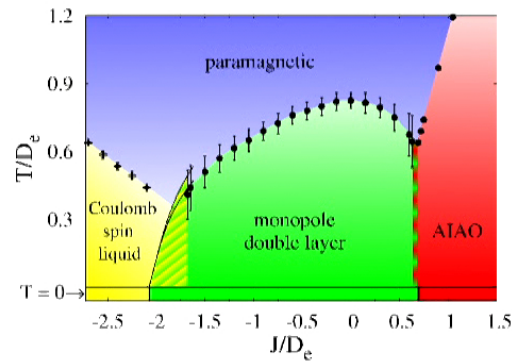


Owen  
Benton  
RIKEN  
Japan



Roderich  
Moessner  
MPI-PkS  
Dresden  
Germany

Jaubert & Moessner PRB 2015



Jeff  
Rau  
U. Waterloo  
Canada



Michel  
Gingras  
U. Waterloo  
Perimeter Inst.  
CIFAR, Canada



Rajiv  
Singh  
UC Davis  
USA



Jaan  
Oitmaa  
UNSW  
Australia

# Tb<sub>2</sub>Ti<sub>2</sub>O<sub>7</sub>

VOLUME 82, NUMBER 5

PHYSICAL REVIEW LETTERS

1 FEBRUARY 1999

## Cooperative Paramagnetism in the Geometrically Frustrated Pyrochlore Antiferromagnet Tb<sub>2</sub>Ti<sub>2</sub>O<sub>7</sub>

J. S. Gardner,<sup>1</sup> S. R. Dunsiger,<sup>2</sup> B. D. Gaulin,<sup>1</sup> M. J. P. Gingras,<sup>3</sup> J. E. Greedan,<sup>4</sup> R. F. Kiefl,<sup>2</sup> M. D. Lumsden,<sup>1</sup>  
W. A. MacFarlane,<sup>2</sup> N. P. Raju,<sup>4</sup> J. E. Sonier,<sup>2</sup> I. Swainson,<sup>5</sup> and Z. Tun<sup>5</sup>

Benton, Yan, Jaubert & Shannon, Nature Comm. 2016





# Tb<sub>2</sub>Ti<sub>2</sub>O<sub>7</sub> in a [110] magnetic field

PHYSICAL REVIEW B **82**, 100401(R) (2010)

## Superlattice correlations in Tb<sub>2</sub>Ti<sub>2</sub>O<sub>7</sub> under the application of [110] magnetic field

J. P. C. Ruff,<sup>1</sup> B. D. Gaulin,<sup>1,2,3</sup> K. C. Rule,<sup>4</sup> and J. S. Gardner<sup>5,6</sup>

<sup>1</sup>Department of Physics and Astronomy, McMaster University, Hamilton, Ontario, Canada L8S 4M1

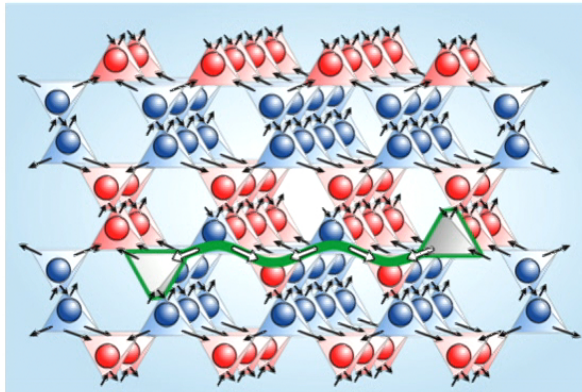
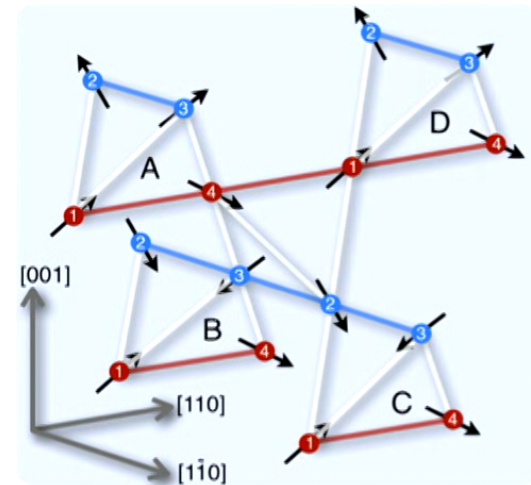
<sup>2</sup>Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada L8S 4M1

<sup>3</sup>Canadian Institute for Advanced Research, 180 Dundas Street West, Toronto, Ontario, Canada M5G 1Z8

<sup>4</sup>Helmholtz Zentrum Berlin für Materialien und Energie, D-14109 Berlin, Germany

<sup>5</sup>NIST Center for Neutron Research, National Institute of Standards and Technology, Gaithersburg, Maryland 20899-6102, USA

<sup>6</sup>Department of Physics, Indiana University, 2401 Milo B. Sampson Lane, Bloomington, Indiana 47408-1398, USA



PHYSICAL REVIEW B **85**, 214420 (2012)

## Double-layered monopolar order in the Tb<sub>2</sub>Ti<sub>2</sub>O<sub>7</sub> spin liquid

A. P. Sazonov,<sup>\*</sup> A. Gukasov, and I. Mirebeau

CEA, Centre de Saclay, DSM/IRAMIS/Laboratoire Léon Brillouin, F-91191 Gif-sur-Yvette, France

P. Bonville

CEA, Centre de Saclay, DSM/IRAMIS/Service de Physique de l'Etat Condensé, F-91191 Gif-Sur-Yvette, France

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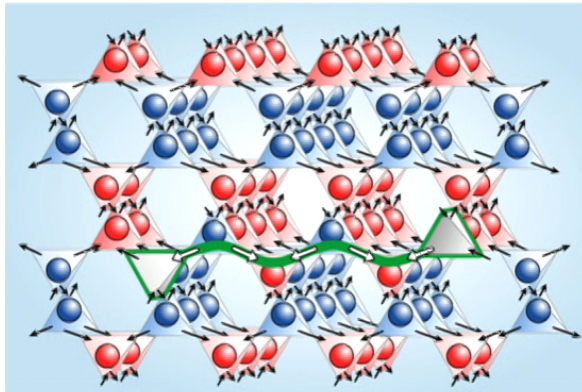
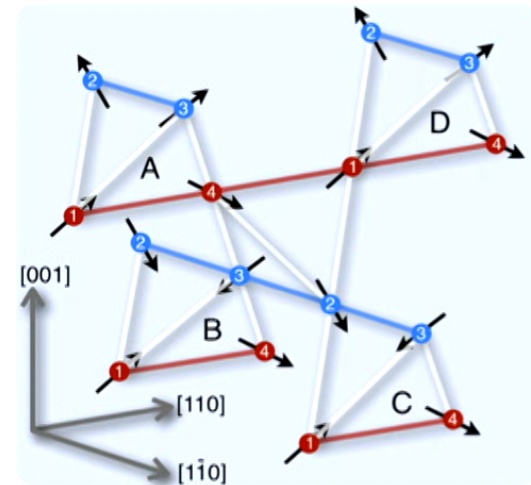
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<sup>3</sup>Canadian Institute for Advanced Research, 180 Dundas Street West, Toronto, Ontario, Canada M5G 1Z8

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PHYSICAL REVIEW B **85**, 214420 (2012)

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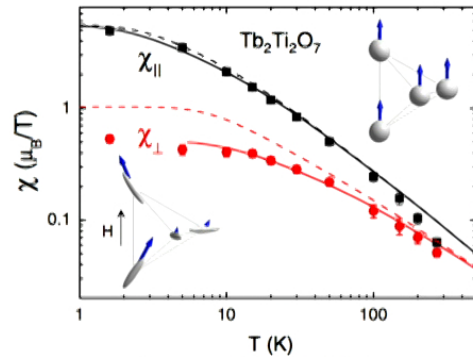
P. Bonville

CEA, Centre de Saclay, DSM/IRAMIS/Service de Physique de l'Etat Condensé, F-91191 Gif-Sur-Yvette, France

# Tb<sub>2</sub>Ti<sub>2</sub>O<sub>7</sub>: spin ice & magneto-elastic

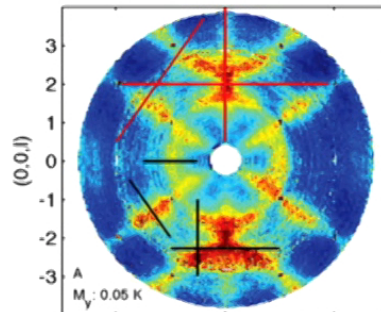
## Spin Ice physics

Ising, Coulomb phase



*Cao et al. PRL 2009*

*Gingras et al. PRB 2000 (CEF)*



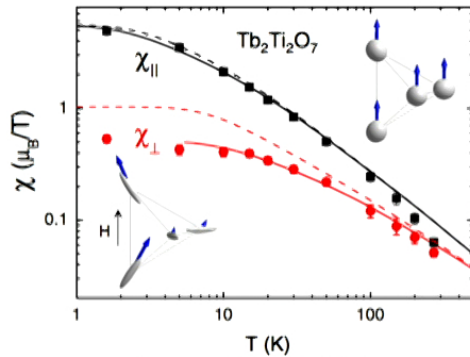
*Fennell et al. PRL 2012*

*Guitteny et al. PRL 2013*

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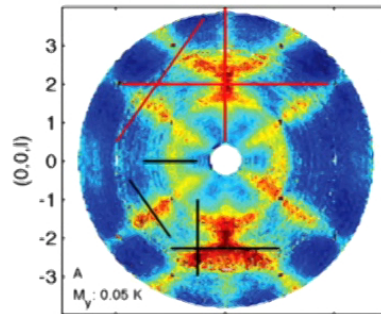
## Spin Ice physics

### Ising, Coulomb phase



*Cao et al. PRL 2009*

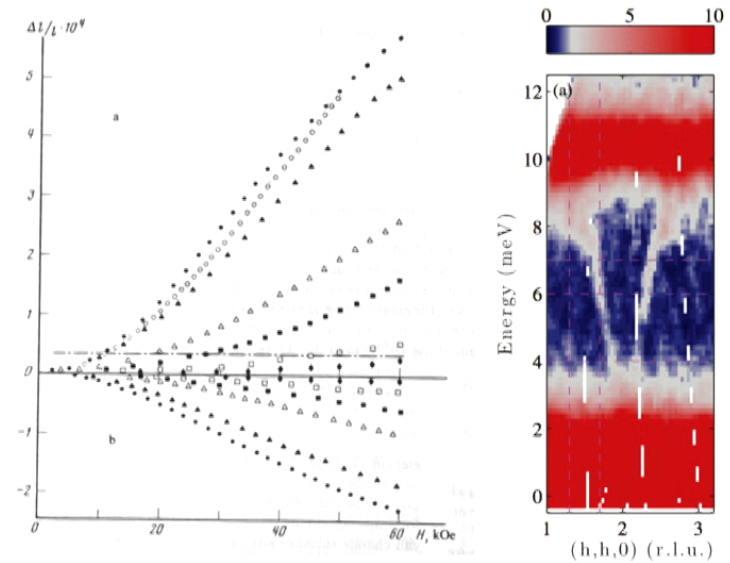
*Gingras et al. PRB 2000 (CEF)*



*Fennell et al. PRL 2012*

*Guitteny et al. PRL 2013*

## Magneto-striction



*Aleksandrov et al. JETP 1985*

*Rule et al. JPCS 2009*

*Ruff et al. PRL 2010*

*Klevkovkina et al. JPCS 2011*

*Fennell et al. PRL 2014*

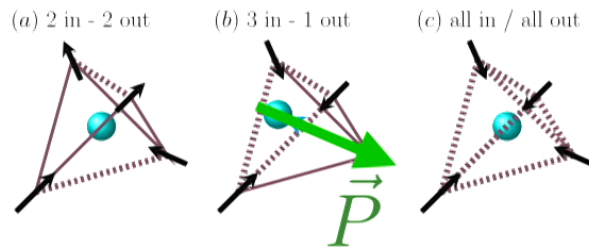
*Bonville et al. PRB 2014*

*Constable et al. PRB 2017*

# *Magneto-electric coupling in spin ice*

## Electric dipoles on magnetic monopoles

Khomskii Nature Comm. 2012



## Spin-current model

$$\vec{P} \propto \vec{e}_{ij} \times (\vec{S}_i \times \vec{S}_j)$$

Katsura *et al.* PRL 2005

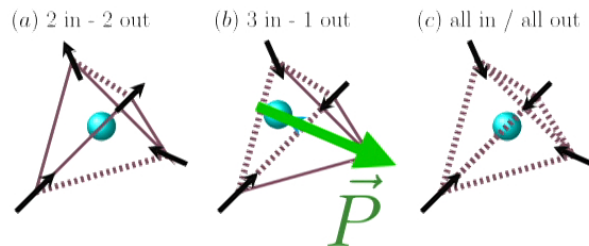
Sarkar *et al.* PRB 2014



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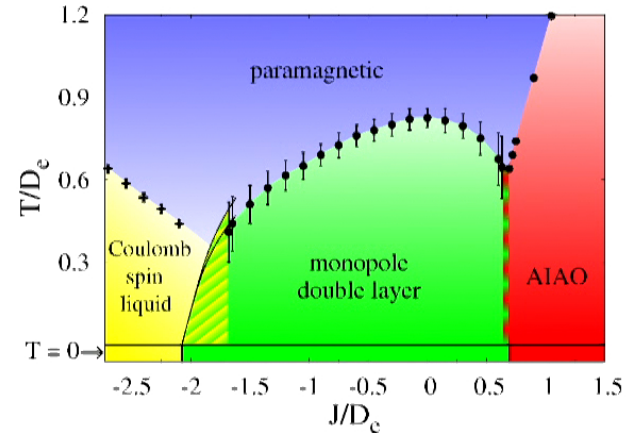
Sarkar *et al.* PRB 2014

$$\mathcal{H} = J \sum_{(ij)} \vec{S}_i \cdot \vec{S}_j - \sum_i \vec{S}_i \cdot \vec{e}_i$$

$$+ D_m r^3 \sum_{i>j} \frac{\vec{S}_i \cdot \vec{S}_j - 3(\vec{S}_i \cdot \vec{e}_{ij})(\vec{S}_j \cdot \vec{e}_{ij})}{r_{ij}^3}$$

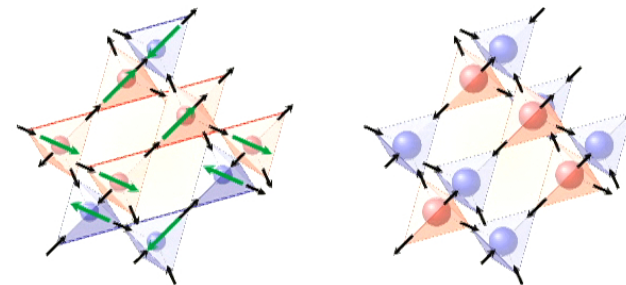
$$+ D_e r_e^3 \sum_{\alpha>\beta} \frac{\vec{P}_\alpha \cdot \vec{P}_\beta - 3(\vec{P}_\alpha \cdot \vec{e}_{\alpha\beta})(\vec{P}_\beta \cdot \vec{e}_{\alpha\beta})}{r_{\alpha\beta}^3}$$

nearest neighbour + electric dipolar interaction



double-layer

AIAO

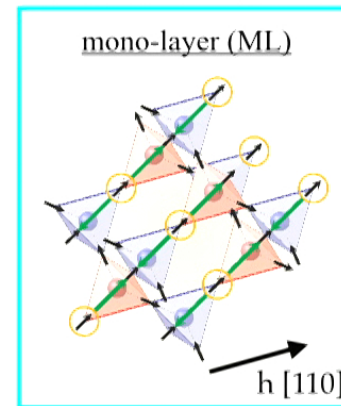
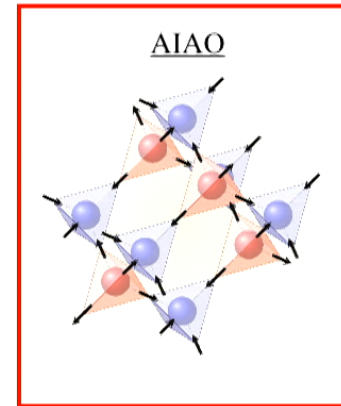
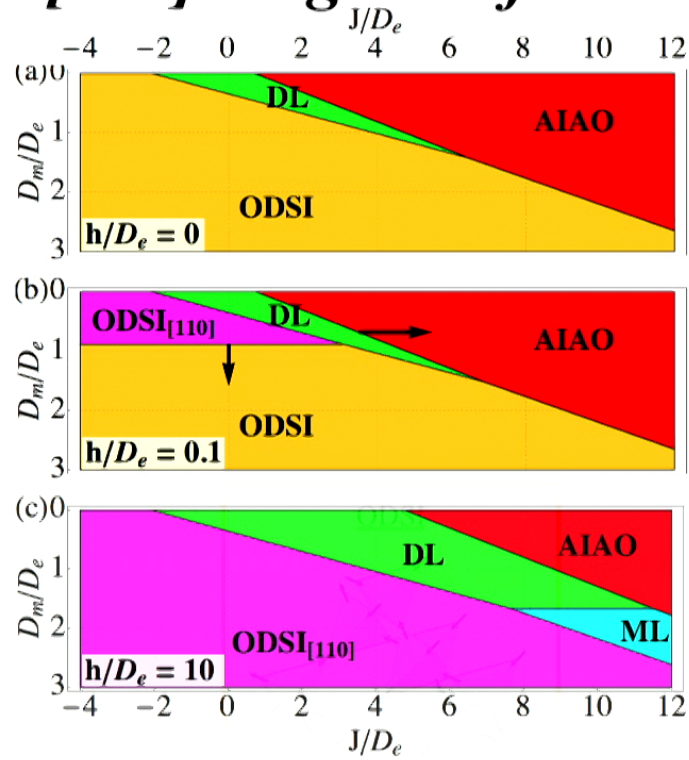
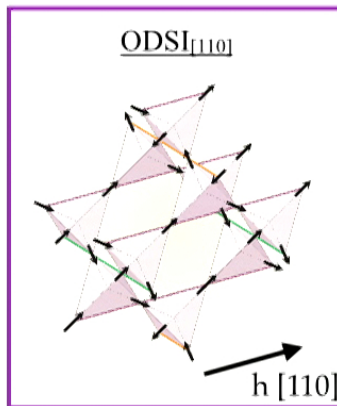
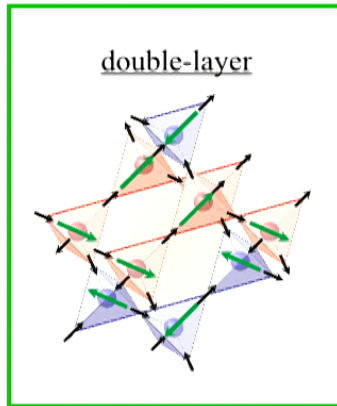


magnetic crystallography



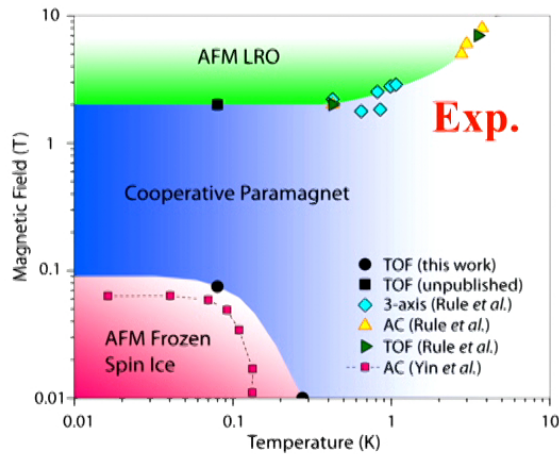
# Electric & Magnetic dipolar interactions

## [110] magnetic field



# Magneto-electric coupling in $Tb_2Ti_2O_7$

Double-layer observed in a  
[110] magnetic field

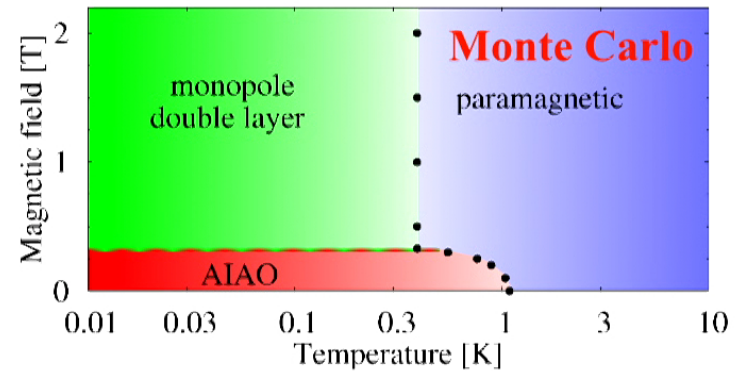


Ruff et al. PRB 2010  
Sazonov et al. PRB 2010  
Sazonov et al. PRB 2012  
Fritsch et al. PRB 2014

$$\mathcal{H} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - \vec{h} \cdot \sum_i \vec{S}_i$$

$$+ D_m r_m^3 \sum_{i>j} \frac{\vec{S}_i \cdot \vec{S}_j - 3(\vec{S}_i \cdot \vec{e}_{ij})(\vec{S}_j \cdot \vec{e}_{ij})}{r_{ij}^3}$$

$$+ D_e r_e^3 \sum_{\alpha>\beta} \frac{\vec{P}_\alpha \cdot \vec{P}_\beta - 3(\vec{P}_\alpha \cdot \vec{e}_{\alpha\beta})(\vec{P}_\beta \cdot \vec{e}_{\alpha\beta})}{r_{\alpha\beta}^3}$$



$$J = -2.7K$$

$$D_m = 0.48K$$

$$D_e = 0.32K$$

$$P \sim 2.10^{-31} \text{Cm}$$

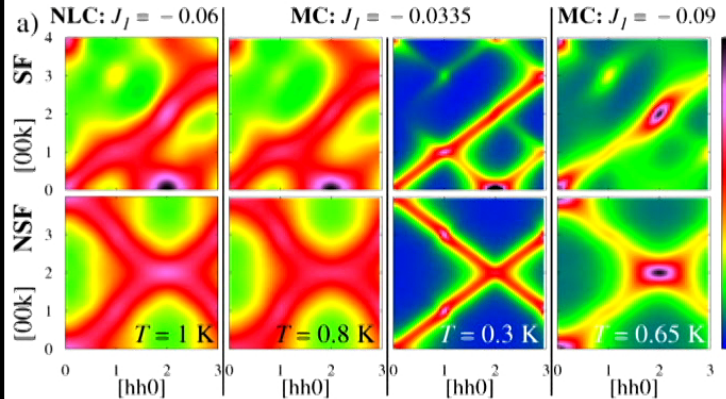
$$\delta r \sim 0.6 \text{ pm}$$

## Multiphase competition in $\text{Yb}_2\text{Ti}_2\text{O}_7$

Order-by-disorder responsible for multi-step ordering and field-reentrance in  $\text{Yb}_2\text{Ti}_2\text{O}_7$ .  
Thermal and quantum fluctuations work together to favour the  $\Gamma_5$  antiferromagnet.

Jaubert, Benton, Rau, Oitmaa, Singh,  
Shannon & Gingras PRL 2015

Yan, Benton, Jaubert & Shannon, PRB 2017



## Multiferroic spin ice

- crystal of magnetic charges(double-layer)
- enhanced by a  $[110]$  magnetic field
- observed in  $\text{Tb}_2\text{Ti}_2\text{O}_7$

Jaubert & Moessner PRB 2015

