Title: Lightning review on emergent quantum electrodynamics in quantum spin ice

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Abstract: We aim to provide a concise review on theoretical background on emergent quantum electrodynamics in pyrochlore quantum spin ice. We first introduce elementary excitations in quantum spin ice using a simple model and then extend the discussion to more realistic systems. Implications to experiments are also discussed.

Lightning Review of Emergent Electrodynamics in Quantum Spin Ice

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University of Toronto

Quantum Spin Ice Conference
Perimeter Institute, June 7, 2017

NSERC CRSNG

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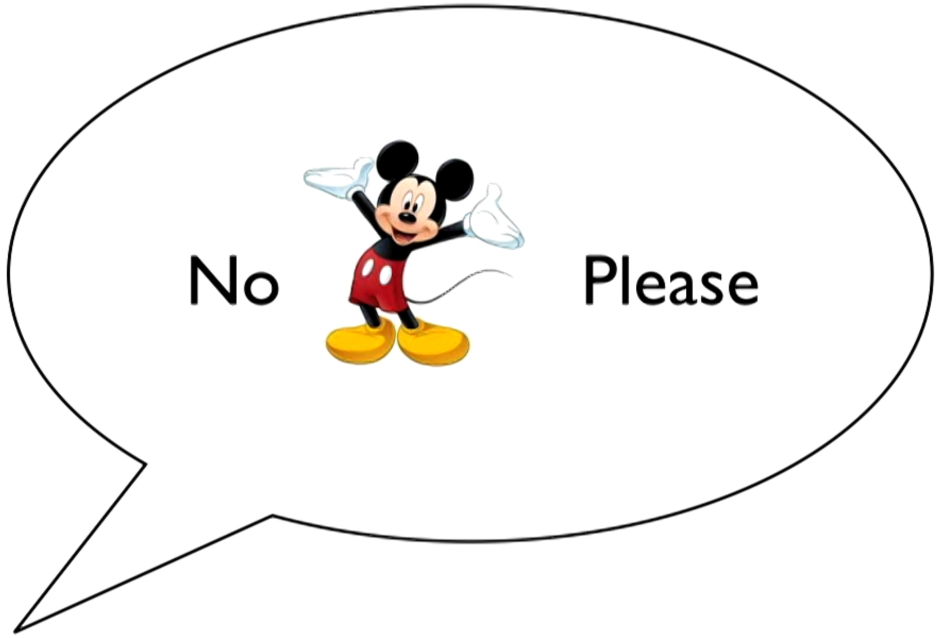
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Outline

I. Classical Spin Ice: Electrostatics

II. Quantum Spin Ice: Quantum Electrodynamics
XXZ Model

III. Electromagnetic Duality

IV. Excitations

V. Signatures in experiments

VI. Numerics

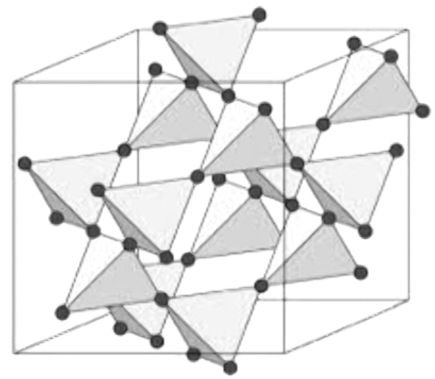
VII. Realistic Model

Minimal Model: Quantum Spin Ice

$$\mathcal{H} = \mathcal{H}_I + \mathcal{H}'$$

$$\mathcal{H}_I = \frac{J_z}{2} \sum_t (S_t^z)^2 \qquad S_t^z = \sum_{i \in t} S_i^z$$

$$\mathcal{H}' = -\frac{J_\perp}{2} \sum_{\langle ij \rangle} (S_i^+ S_j^- + h.c.)$$



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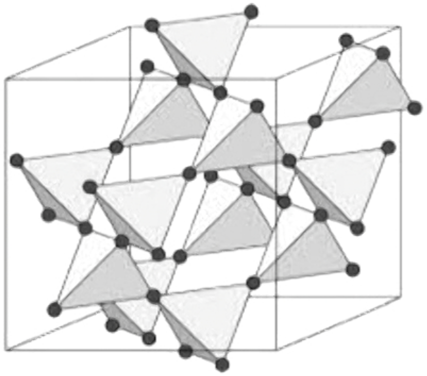


Electrostatics for Classical Spin Ice

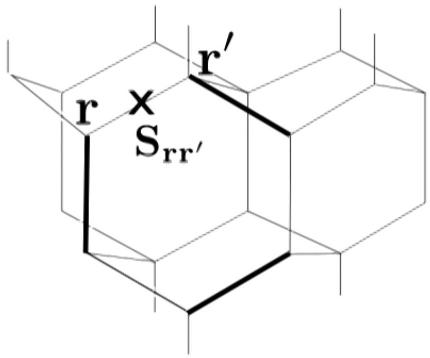
$$\mathcal{H}_I = \frac{J_z}{2}\sum_t (S_t^z)^2 \qquad S_t^z = \sum_{i\in t} S_i^z$$

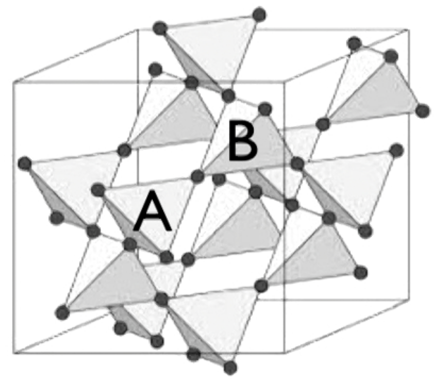
i sites on pyrochlore lattice

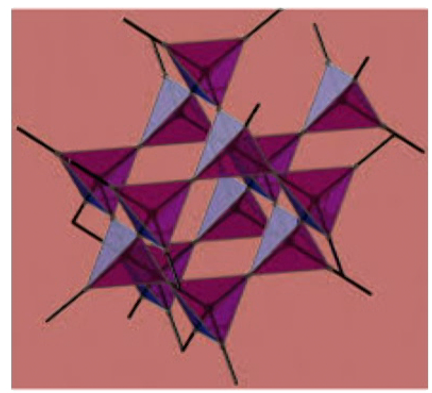
$\mathbf{r}$ sites on diamond lattice

$\mathbf{S}_i = \mathbf{S}_{\mathbf{rr}'}$

$\mathbf{rr}'$ link connecting two diamond lattice sites







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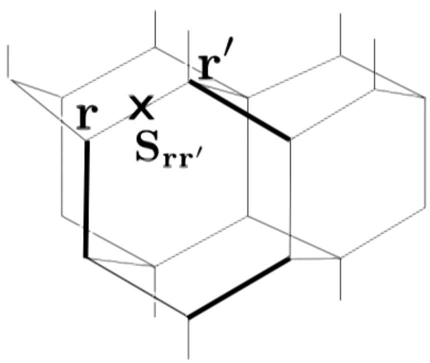

oriented vector field

$$e_{\mathbf{r}\mathbf{r}'} = \pm S^z_{\mathbf{r}\mathbf{r}'}$$

$$\pm \quad \mathbf{r} \in \mathrm{A/B}$$

$$e_{\mathbf{r}\mathbf{r}'} = -e_{\mathbf{r}'\mathbf{r}}$$

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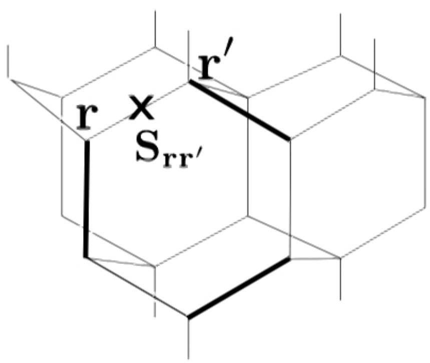

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$$(\mathrm{div}\, e)_{\mathbf{r}} = \sum_{\mathbf{r}'\leftarrow\mathbf{r}} e_{\mathbf{r}\mathbf{r}'} = \pm S_t^z$$

$$(\mathrm{div}\ e)_{\mathbf{r}} = 0$$

classical spin ice ground state

Electrostatics for Classical Spin Ice

$$e_{\mathbf{rr'}} = \pm S^z_{\mathbf{rr'}} \qquad \pm \;\; \mathbf{r} \in \mathrm{A/B}$$

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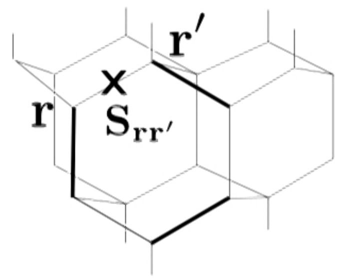

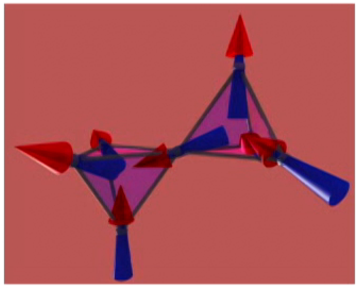

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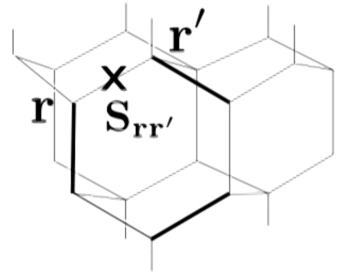

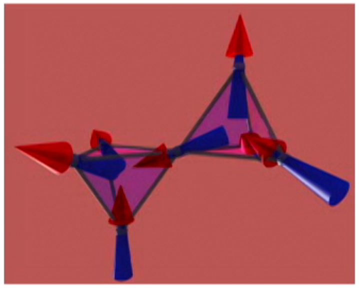

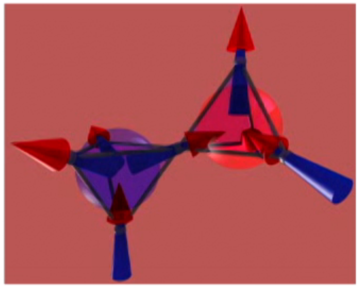

electric monopole-antimonopole pair

(magnetic monopole if we use $e_{\mathbf{r}\mathbf{r}'} \to b_{\mathbf{r}\mathbf{r}'}$)

Electrostatics for Classical Spin Ice

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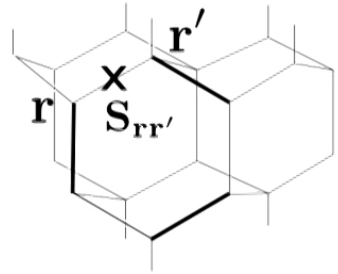

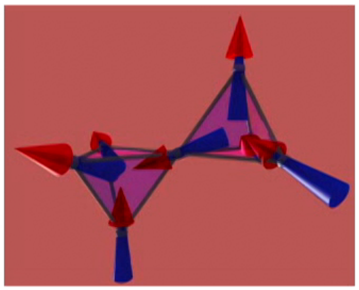

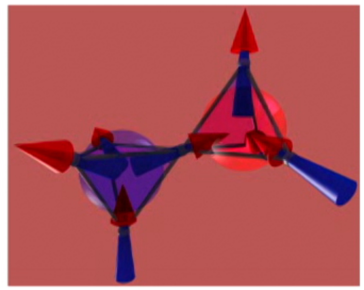

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Quantum Spin Ice

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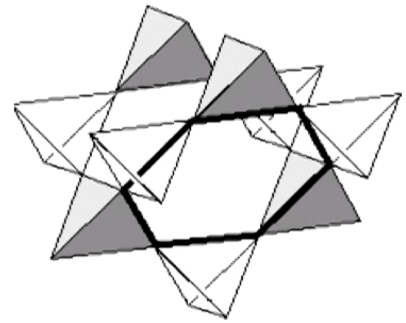

Hermele, Balents, Fisher (2003)

Quantum Spin Ice

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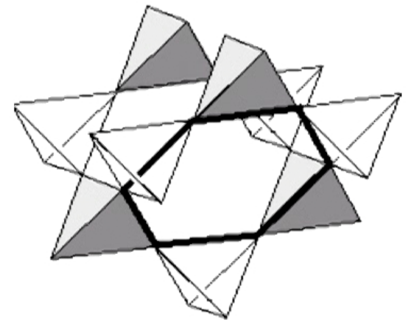
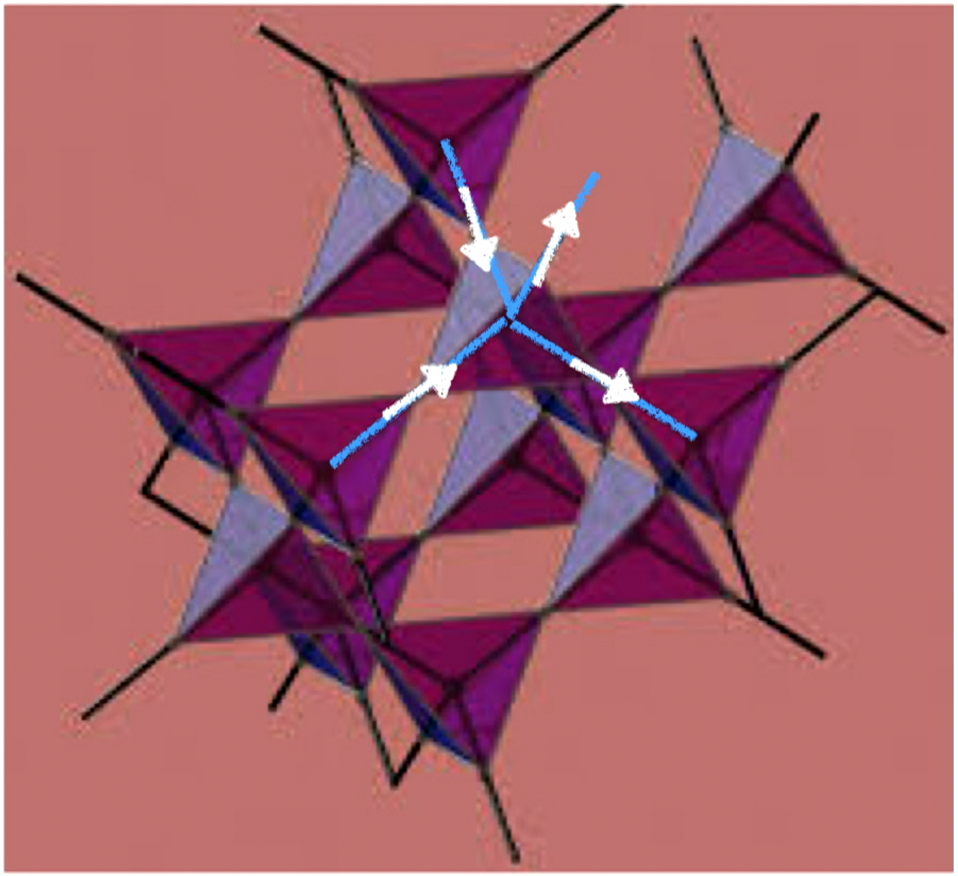


$J_z \gg J_\perp$ **degenerate perturbation theory**

$$\mathcal{H}_{eff} = -J_{ring}\sum_{\hexagon}(S_1^+ S_2^- S_3^+ S_4^- S_5^+ S_6^- + h.c.)$$

$$J_{ring} = 12 J_\perp^3 / J_z^2$$

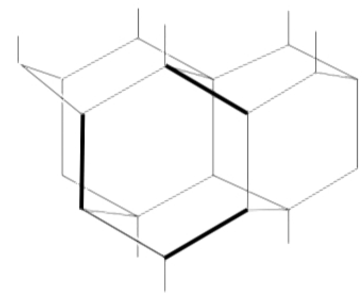
Hermele, Balents, Fisher (2003)



Connection to Quantum Dimer Model

$$\mathcal{H}_p = -J_{ring} \sum_{\hexagon} (S_1^+ S_2^- S_3^+ S_4^- S_5^+ S_6^- + h.c.)$$ dimer kinetic energy on diamond lattice

two dimers touch each site on diamond lattice
[2-in/2-out on pyrochlore lattice]



"flippable" hexagon

Hermele, Balents, Fisher (2003)

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two dimers touch each site on diamond lattice
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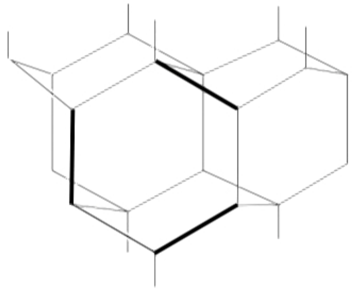
add dimer potential term by hand

$$\mathcal{H}_V = V N_f$$

exactly solvable point: RK model

$$V = J_{ring}$$



"flippable" hexagon

Spin liquid: equal superposition of all dimer configurations

All in $J_z \gg J_\perp$ limit !

Hermele, Balents, Fisher (2003)

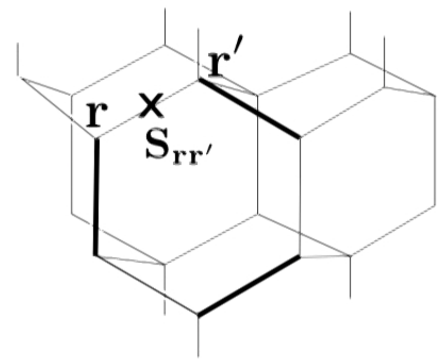
Quantum Electrodynamics

Quantum rotor variables $n_{\mathbf{rr'}} \in \mathbb{Z} \quad \phi_{\mathbf{rr'}} \in [-\pi, \pi)$

$$[\phi_{\mathbf{r}i}, n_{\mathbf{r'}j}] = i\delta_{ij}\delta_{\mathbf{rr'}}$$

$S^z_{\mathbf{rr'}} = n_{\mathbf{rr'}} - 1/2 \qquad n_{\mathbf{rr'}} = 0, 1$

$S^+_{\mathbf{rr'}} = e^{i\phi_{\mathbf{rr'}}}$ hard core boson mapping



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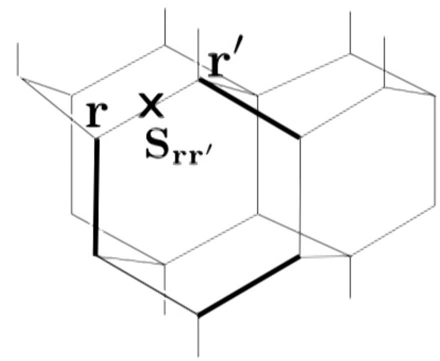

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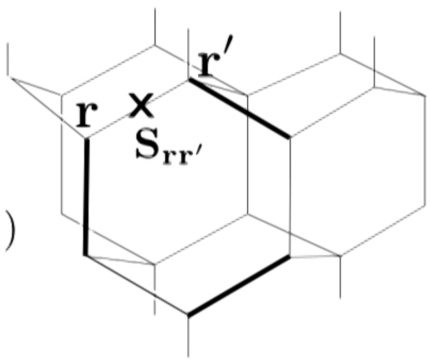

$U/K \to \infty$ hard core boson limit

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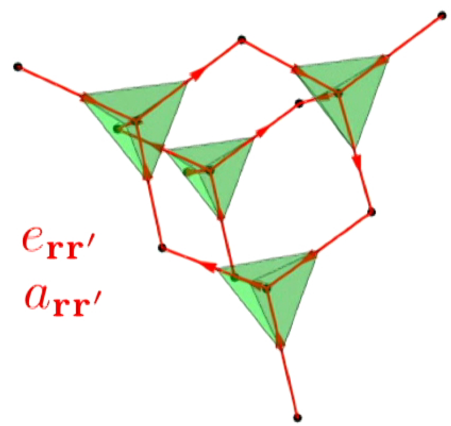
electric field:
half-integer values !



defined in "direct"
diamond lattice

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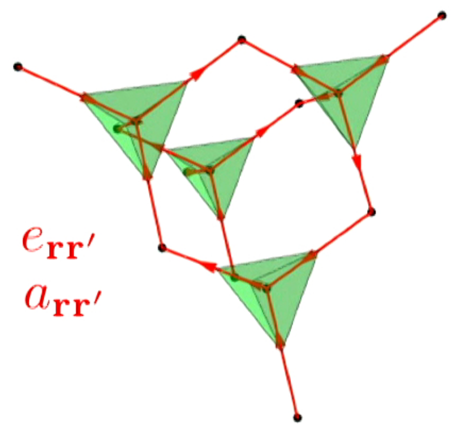

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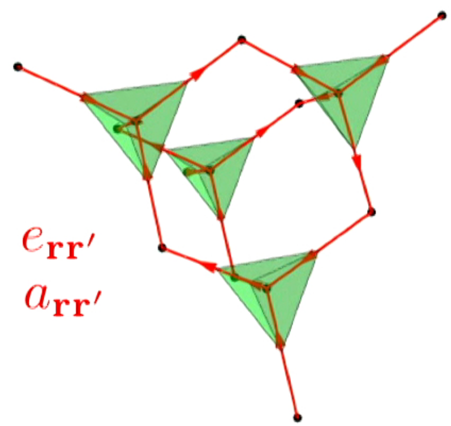

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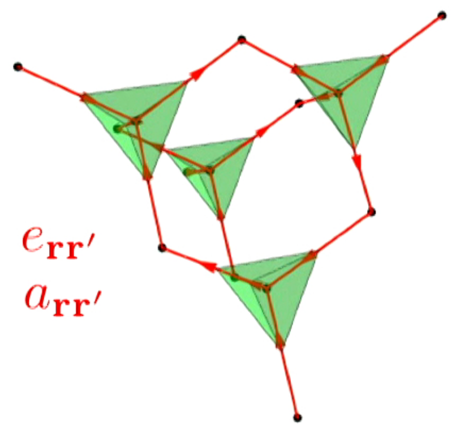
$$(\mathrm{curl}\, a)_{\hexagon} = \sum_{\mathbf{rr'}\in\hexagon}^{\circlearrowleft} a_{\mathbf{rr'}}$$

$$(\mathrm{div}\, e)_{\mathbf{r}} = \sum_{\mathbf{r'}\leftarrow\mathbf{r}} e_{\mathbf{rr'}} = \pm S^z_t$$

$$a_{\mathbf{rr'}} \rightarrow a_{\mathbf{rr'}} + \chi_{\mathbf{r'}} - \chi_{\mathbf{r}}$$

gauge invariance



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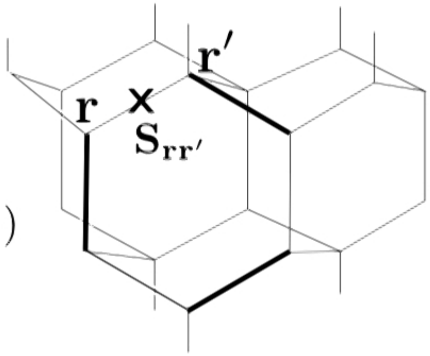

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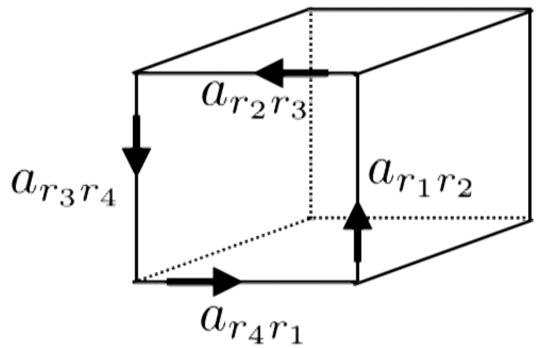
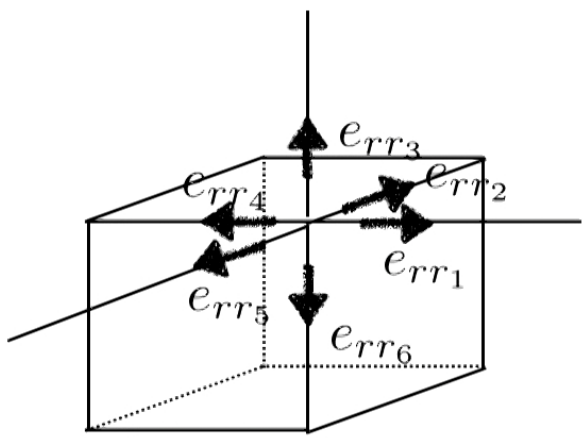
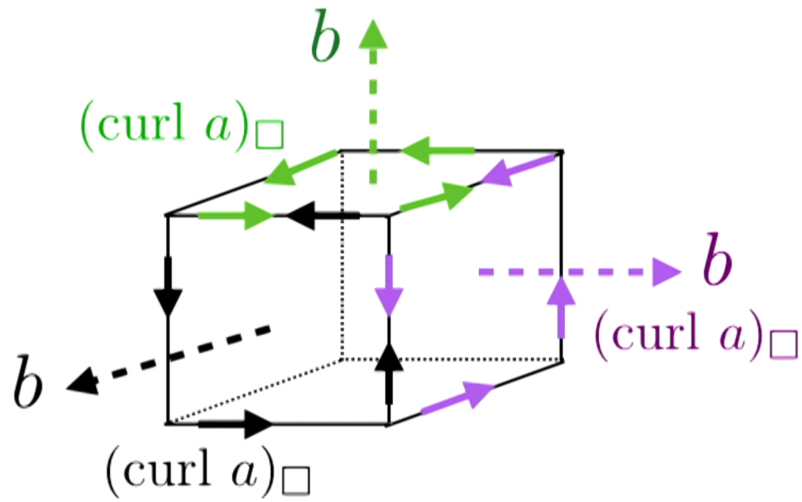


$$(\text{curl } a)_{\square} \sim \sum_{rr' \in \square} a_{rr'}$$



$$(\text{div } e)_r \sim \sum_{r' \leftarrow r} e_{rr'}$$



$b_{\mathsf{rr}'} \sim (\text{curl } a)_\square$

rr' link connecting centers r and r' of two nearby cubes

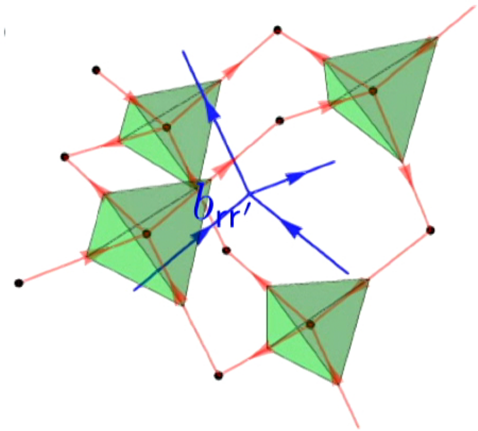
r defines the dual cubic lattice

rr' links penetrate the centers of square surfaces

What about "magnetic" field ?

$$\pi b_{\mathbf{rr'}} \;=\; (\mathrm{curl}\, a)_{\hexagon}$$ r "dual" diamond lattice sites

$$\mathcal{H}_p = \frac{U}{2} \sum_{\langle \mathbf{rr'} \rangle} e^2_{\mathbf{rr'}} - K \sum_{\hexagon} \cos\Big((\mathrm{curl}\, a)_{\hexagon} \Big)$$



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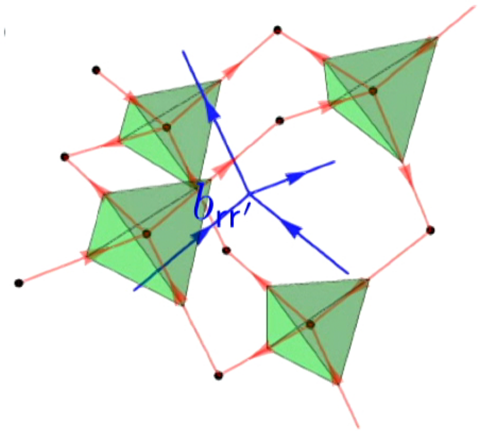
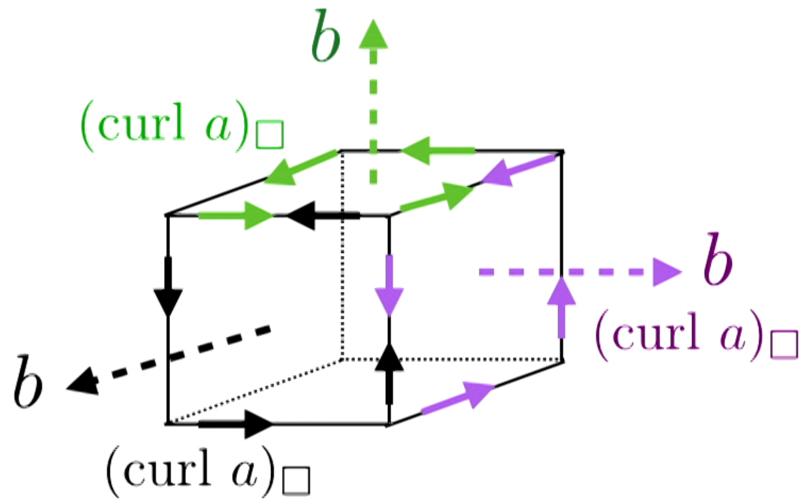
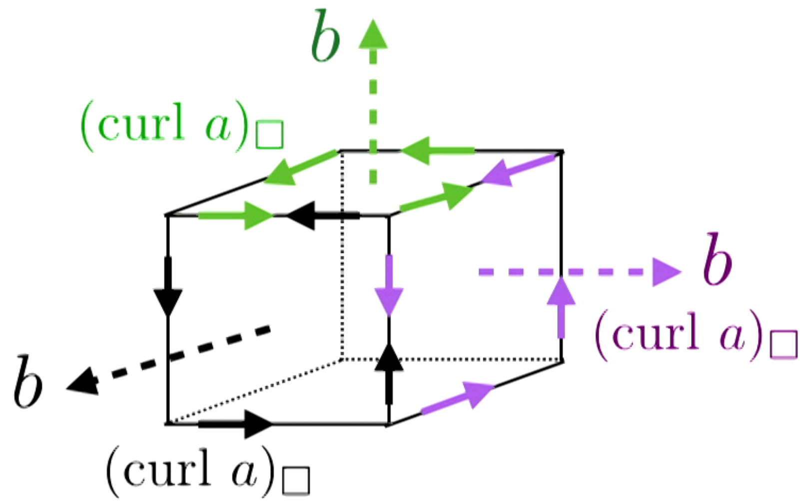


$$b \to b + 2 \qquad b_{\mathsf{rr'}} \in [-1, 1)$$

$$(\mathrm{div}\ b)_{\mathsf{r}} = ?$$



$$b \sim (\text{curl } a)_\square \qquad (\text{div } b)_{\text{cube}} \sim \sum_\square (\text{curl } a)_\square$$



$b \sim (\text{curl } a)_\square$ $\quad\quad$ $(\text{div } b)_{\text{cube}} \sim \sum_\square (\text{curl } a)_\square$

This is naively zero as all "a" would cancel out

But "b"⟶"b+2" and the net sum is generally an even integer

What about "magnetic" field ?

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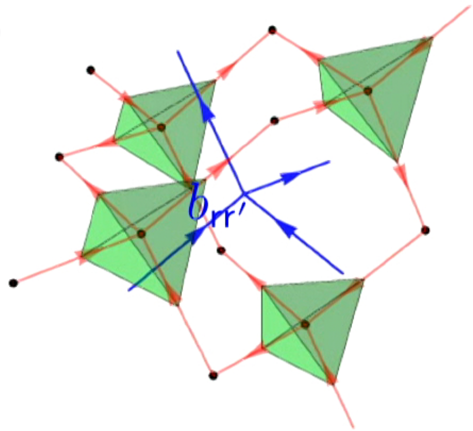

$$b \longrightarrow b + 2 \qquad b_{\mathsf{rr}'} \in [-1, 1)$$

$$(\mathrm{div}\, b)_{\mathsf{r}} = 2n_{\mathsf{r}} \quad n_{\mathsf{r}} = 0, \pm 1$$

gauge theory is compact = periodic in "magnetic" flux

"magnetic charge" is quantized

"magnetic" monopole is allowed

Confinement ?

$$\mathcal{H}_p = \frac{U}{2}\sum_{\langle \mathbf{r}\mathbf{r}'\rangle} e_{\mathbf{r}\mathbf{r}'}^2 - K\sum_{\hexagon} \cos\left((\mathrm{curl}\, a)_{\hexagon}\right)$$

Confinement ?

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if U/K is small — electric charge is deconfined (Coulomb phase)

$$\cos\left((\mathrm{curl}\, a)_{\hexagon} \right) \approx 1 - \frac{1}{2} \left((\mathrm{curl}\, a)_{\hexagon} \right)^2$$

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if $U/K \to \infty$

If $e_{\mathbf{r}\mathbf{r}'} = \pm n_{\mathbf{r}\mathbf{r}'}$

the vacuum (ground state) would have been trivial $e = 0$ confinement !

Confinement ?

$$\mathcal{H}_p = \frac{U}{2} \sum_{\langle \mathbf{rr}' \rangle} e_{\mathbf{rr}'}^2 - K \sum_{\hexagon} \cos\Big((\mathrm{curl}\, a)_{\hexagon} \Big)$$

if U/K is small — electric charge is deconfined (Coulomb phase)

$$\cos\big((\mathrm{curl}\, a)_{\hexagon} \big)$$

$$\approx 1 - \frac{1}{2}\big((\mathrm{curl}\, a)_{\hexagon} \big)^2$$

if $U/K \to \infty$

$e_{\mathbf{rr}'} = \pm(n_{\mathbf{rr}'} - 1/2)$ — $e = \pm 1/2$ frustrated vacuum

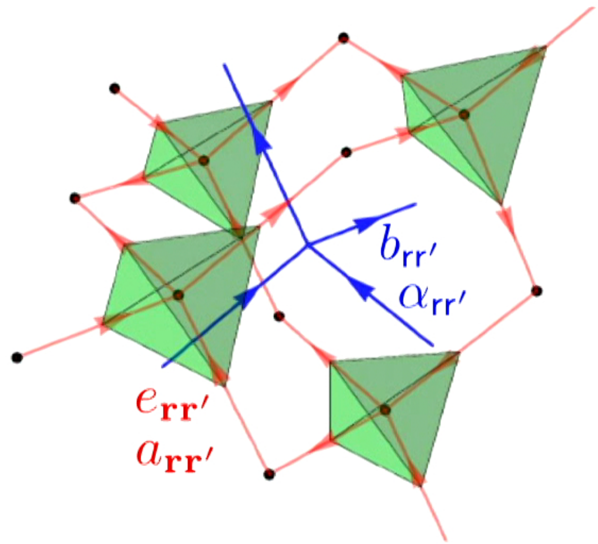
quantum spin ice — Non-trivial problem !

Electromagnetic Duality

dual vector potential $\alpha_{\mathsf{rr'}} \in \pi\mathbb{Z}$ $[b, \alpha] = i$

$\pi b_{\mathsf{rr'}} = (\mathrm{curl}\, a)_{\hexagon}$ $b \to b + 2$

$(\mathrm{div}\, b)_{\mathsf{r}} = 2n_{\mathsf{r}}$ $n_{\mathsf{r}} = 0, \pm 1$



Electromagnetic Duality

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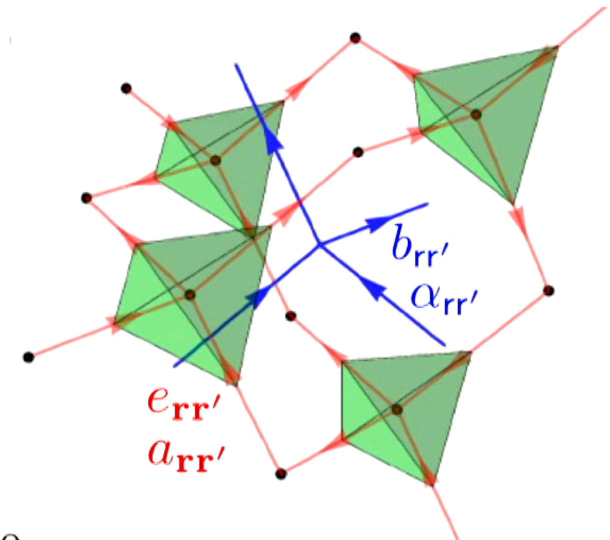

$$\boxed{\pi(e_{\mathbf{rr}'} - e^0_{\mathbf{rr}'}) \;=\; (\mathrm{curl}\, \alpha)_{\hexagon^*}}$$

$$e^0_{\mathbf{rr}'} = \pm 1/2 \qquad e_{\mathbf{rr}'} \in \mathbb{Z} + 1/2$$

$$\mathrm{div}\, e \;=\; \mathrm{div}(e^0 + \mathrm{curl}\, \alpha/\pi) \;=\; 0$$

$$[a_{\mathbf{rr}'}, e_{\mathbf{rr}'}] = i$$

Electromagnetic Duality

$$\mathcal{H}_p = \frac{U}{2}\sum_{\langle \mathbf{r}\mathbf{r}'\rangle} e^2_{\mathbf{r}\mathbf{r}'} - K\sum_{\hexagon} \cos\Big((\mathrm{curl}\, a)_{\hexagon}\Big) \qquad [a_{\mathbf{r}\mathbf{r}'}, e_{\mathbf{r}\mathbf{r}'}] = i$$

$$\mathcal{H}_{dual} = \frac{U}{2\pi^2}\sum_{\hexagon^*}((\mathrm{curl}\,\alpha)_{\hexagon^*} + \pi e^0_{\mathbf{r}\mathbf{r}'})^2 - K\sum_{\langle \mathbf{r}\mathbf{r}'\rangle} \cos(\pi b_{\mathbf{r}\mathbf{r}'}) \qquad [b, \alpha] = i$$

Electromagnetic Duality

$$\mathcal{H}_p = \frac{U}{2}\sum_{\langle \mathbf{rr}'\rangle} e^2_{\mathbf{rr}'} - K\sum_{\hexagon}\cos\Big((\mathrm{curl}\, a)_{\hexagon}\Big) \qquad [a_{\mathbf{rr}'}, e_{\mathbf{rr}'}] = i$$

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$$\mathrm{div}\, b \in 2\mathbb{Z} \qquad \pi b_{\mathsf{rr}'} = (\mathrm{curl}\, a)_{\hexagon}$$

$$\mathrm{div}\ e = 0 \qquad \pi(e_{\mathbf{rr}'} - e^0_{\mathbf{rr}'}) = (\mathrm{curl}\,\alpha)_{\hexagon^*} \qquad e^0_{\mathbf{rr}'} = \pm 1/2$$

Coulomb Phase (deconfined phase)

dual vector potential $\pi e = \text{curl}\ \tilde{\alpha}$

$$\cos(\pi b) \approx 1 - \frac{1}{2}\pi^2 b^2$$ small fluctuations

$$\mathcal{H}^0 = \frac{\mathcal{K}}{2}\sum_{\langle \mathsf{rr}' \rangle} \tilde{b}^2_{\mathsf{rr}'} + \frac{\mathcal{U}}{2}\sum_{\hexagon^*}(\text{curl}\,\tilde{\alpha})^2$$

electric (magnetic) charges are interacting each other via 1/r

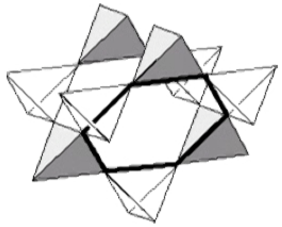
Excitations

electric monopoles (or "magnetic monopole"):
spinons

$$2\Delta_{\rm spinon} \sim J_z$$

magnetic monopoles (or "electric monopole")

$$\Delta_{\rm mon} \sim J_{\pm}^3/J_z^2 \qquad S^+_{\mathbf{rr}'} = e^{i\phi_{\mathbf{rr}'}}$$



$$\pi b_{\mathsf{rr}'} = ({\rm curl}\, a)_{\hexagon} \sim \phi_1 - \phi_2 + \phi_3 - \phi_4 + \phi_5 - \phi_6$$

Excitations

electric monopoles (or "magnetic monopole"):
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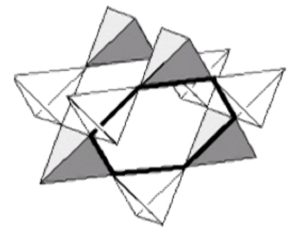
$$2\Delta_{\text{spinon}} \sim J_z$$

magnetic monopoles (or "electric monopole")



$$\Delta_{\text{mon}} \sim J_{\pm}^3/J_z^2 \qquad S^+_{\mathbf{rr}'} = e^{i\phi_{\mathbf{rr}'}}$$

$$\pi b_{\mathsf{rr}'} = (\text{curl}\, a)_{\hexagon} \sim \phi_1 - \phi_2 + \phi_3 - \phi_4 + \phi_5 - \phi_6$$

emergent photons $\omega(\mathbf{k}) \approx c|\mathbf{k}|$ $\qquad C(T) \propto \frac{1}{c^3}T^3$

$$c \propto \sqrt{UK}a_0/\hbar$$

Spin Structure Factor

$$\langle S_i^z(\mathbf{r}) S_j^z(\mathbf{r}')\rangle = (\mathbf{u}_i)_k(\mathbf{u}_j)_l \mathcal{C}^e_{kl}(\mathbf{r}-\mathbf{r}')$$

$$\mathcal{C}^e_{ij}(\mathbf{r}-\mathbf{r}') = \langle e_i(\mathbf{r}) e_j(\mathbf{r}')\rangle$$

Hermele, Balents, Fisher (2003)

$$\mathcal{C}^e_{ij}(\mathbf{r}) \propto \frac{K}{cr^4}\left(2\frac{r_i r_j}{r^2} - \delta_{ij}\right) \qquad c \propto \sqrt{UK}a_0/\hbar$$

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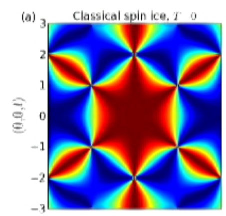
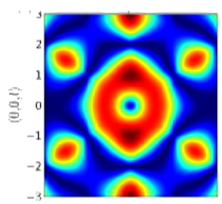
classical and quantum pinch point

$$\langle S_\mu(-\mathbf{k}) S_\nu(\mathbf{k})\rangle_{\text{classical}} \propto \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$



$$\langle S_\mu(-\mathbf{k}) S_\nu(\mathbf{k})\rangle_{\text{quantum}} \propto k\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$



Nic Shannon (2011,2012)

Quantum Monte Carlo (SSE)

$$\mathcal{H} = \mathcal{H}_I + \mathcal{H}'$$

$$\mathcal{H}_I = \frac{J_z}{2}\sum_t (S_t^z)^2 \qquad \mathcal{H}' = -\frac{J_\perp}{2}\sum_{\langle ij\rangle}(S_i^+ S_j^- + h.c.)$$

Banerjee,
Isakov,
Damle,
YBK
(2007)

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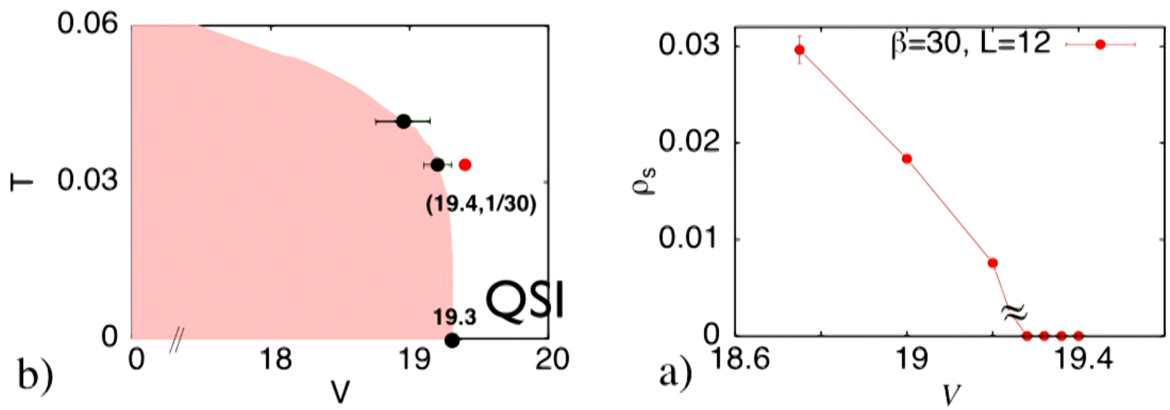
$$\mathcal{H}_{boson} = -t\sum_{\langle ij\rangle}\left(b_i^\dagger b_j + b_j^\dagger b_i\right) + V\sum_{\langle ij\rangle}\left(n_i - \frac{1}{2}\right)\left(n_j - \frac{1}{2}\right)$$

$$S_i^z = n_i - 1/2 \qquad J_z = V \qquad J_\perp = -2t$$

Banerjee, Isakov, Damle, YBK (2007)



Quantum Electrodynamics

$$\mathcal{H} = \frac{\gamma}{2}\sum_{<\mathbf{rr}'>} e_{rr'}^2 + \frac{\kappa}{2}\sum_{\hexagon}(\Delta_{rr'} \times a_{rr'})^2$$

$$S = \frac{1}{2}\int_0^{\beta\sqrt{\kappa\gamma}} d\tilde{\tau} \left[\sum_{\langle rr'\rangle}(\partial_{\tilde{\tau}}\tilde{a}_{rr'} - \Delta_{rr'}\tilde{a}_{\tau})^2 + \sum_{\hexagon}(\Delta_{rr'} \times \tilde{a}_{rr'})^2\right]$$

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$$S_i^z = n_i - 1/2$$

$C^{\alpha\alpha'}(\mathbf{q}, \tau = 0) = \langle n_\alpha(\mathbf{q}) n_{\alpha'}(-\mathbf{q})\rangle$ **equal time**

$S^{\alpha\alpha'}(\mathbf{q}, \omega_n = 0) = \int_0^\beta d\tau C^{\alpha\alpha'}(\mathbf{q}, \tau)$ **static**

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$C(q) \sim S_{\text{equal}}(\mathbf{q}) = \int d\omega \; S(\mathbf{q}, \omega)$ **inelastic contribution**

$S(q) \sim S_{\text{static}}(\mathbf{q}) = S(\mathbf{q}, \omega = 0)$ **elastic scattering**

Quantum Electrodynamics

$$\mathcal{H} = \frac{\gamma}{2}\sum_{<\mathbf{rr'}>} e^2_{rr'} + \frac{\kappa}{2}\sum_{\hexagon}(\Delta_{rr'} \times a_{rr'})^2$$

$$S \;=\; \frac{1}{2}\int_0^{\beta\sqrt{\kappa\gamma}} d\tilde{\tau}\left[\sum_{\langle rr'\rangle}(\partial_{\tilde{\tau}}\tilde{a}_{rr'} - \Delta_{rr'}\tilde{a}_{\tau})^2 + \sum_{\hexagon}(\Delta_{rr'} \times \tilde{a}_{rr'})^2\right]$$

$$S^z_i = n_i - 1/2$$

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$C^{\alpha\alpha'}(\mathbf{q},\tau = 0) = \langle n_\alpha(\mathbf{q})n_{\alpha'}(-\mathbf{q})\rangle$ **equal time**

$S^{\alpha\alpha'}(\mathbf{q},\omega_n = 0) = \int_0^\beta d\tau C^{\alpha\alpha'}(\mathbf{q},\tau)$ **static**

$C(\tau = 0,\mathbf{q}) = \epsilon^2\sqrt{v}f_{eq}(\beta\sqrt{\kappa\gamma},\mathbf{q})$

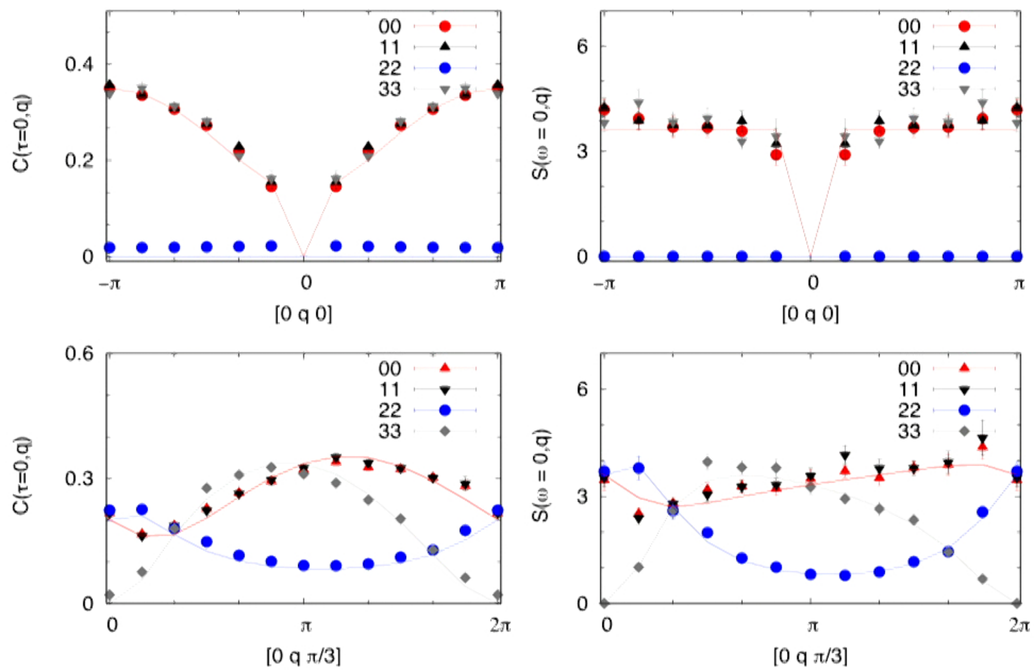
$S(\omega = 0,\mathbf{q}) = \epsilon^2\frac{\sqrt{v}}{\sqrt{\kappa\gamma}}f_{st}(\mathbf{q})$

Analytic expression of lattice correlators

Fit equal time correlator in different lattice directions, then using the same parameters, fit static correlator !

$\varepsilon^2(\kappa/\gamma)^{1/2} = 0.3 \pm 0.01$, $(\kappa\gamma)^{1/2} = 0.053 \pm 0.006$

equal time



static

Quantum-Classical Crossover

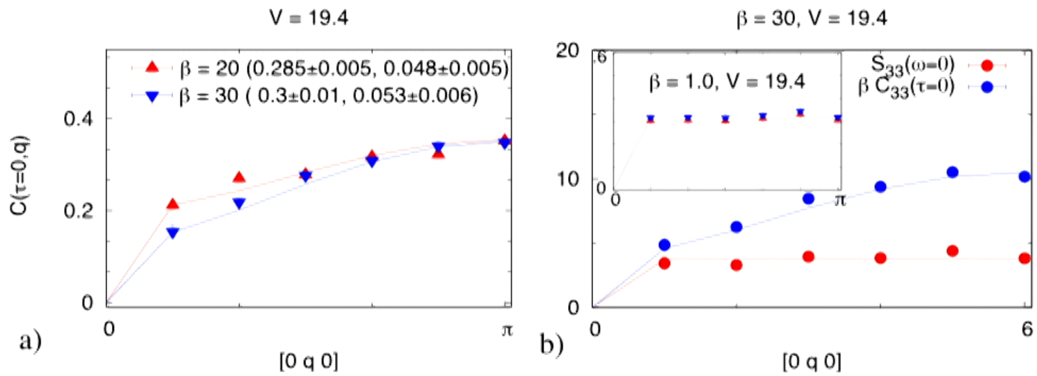


FIG. 4: (color online). a) The same values of $\sqrt{\kappa\gamma}$ and $\epsilon^2\sqrt{\kappa/\gamma}$ fit data at different temperatures for fixed V/t. b) $S^{\alpha\alpha}(\mathbf{q},\omega_n=0)$ and $\beta C^{\alpha\alpha}(\mathbf{q},\tau=0)$ for $V=19.4$ are essentially equal at $\beta=1$, but not at $\beta=30$.

Quantum Dimer Model

$$\mathcal{H}_\mu = \mathcal{H}_{\text{tunneling}} + \delta\mathcal{H}_\mu$$

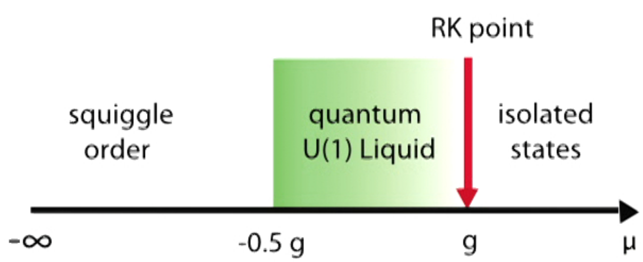
Nic Shannon (2011,2012)

$$\mathcal{H}_{\text{tunnelling}} = -g \sum_{\hexagon} \left[|\circlearrowright\rangle\langle\circlearrowleft| + |\circlearrowleft\rangle\langle\circlearrowright| \right]$$

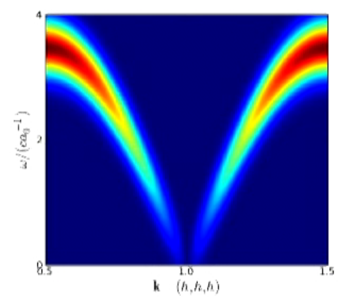
$$\delta\mathcal{H}_\mu = \mu \sum_{\hexagon} \left[|\circlearrowleft\rangle\langle\circlearrowleft| + |\circlearrowright\rangle\langle\circlearrowright| \right]$$

$g = \mu$ RK point

$$\mathcal{H}_p = -J_{ring} \sum_{\hexagon} (S_1^+ S_2^- S_3^+ S_4^- S_5^+ S_6^- + h.c.) \qquad J_z \gg J_\perp$$



$c \sim 0.3 \text{ meV Å} \sim 50 \text{ ms}^{-1}$

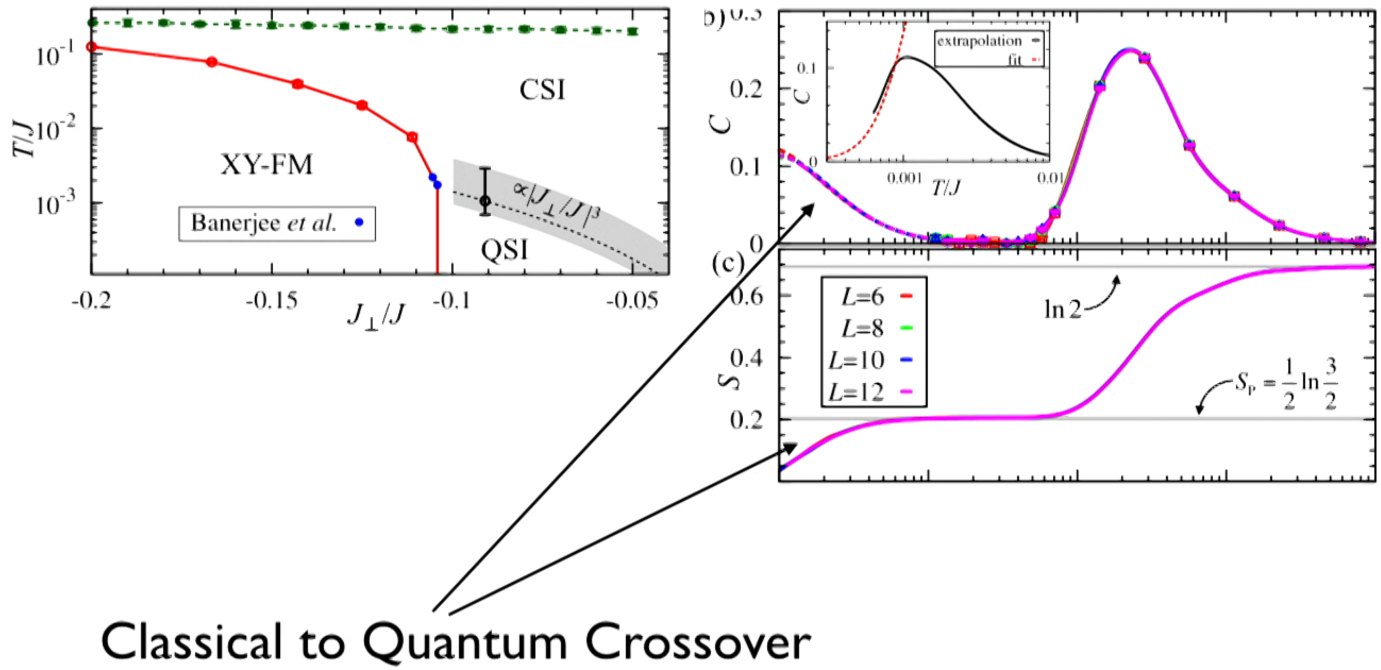


$c = (0.6 \pm 0.1)\; g\; a_0\; \hbar^{-1}$

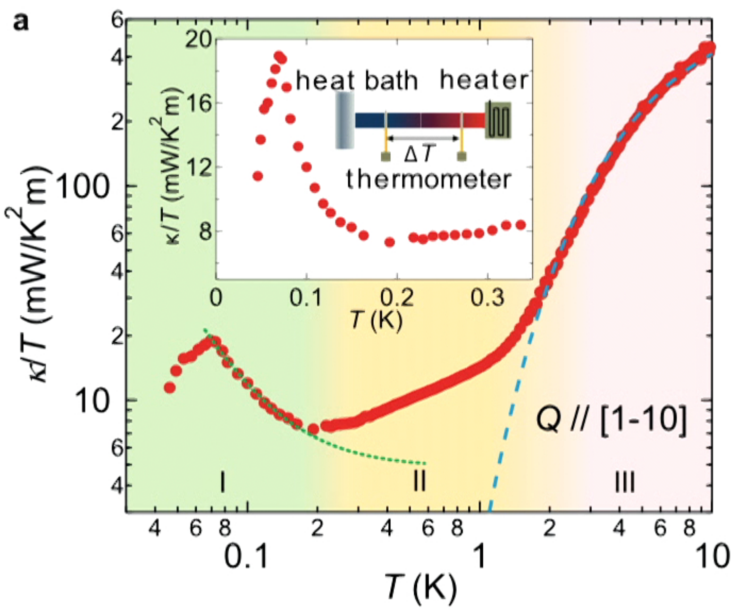
$c = (1.8 \pm 0.2)\; g\; a_0\; \hbar^{-1}$

Numerical evidence of quantum melting of spin ice: quantum-classical crossover

Yasuyuki Kato[1,2] and Shigeki Onoda[1,3]



thermal conductivity (Yuji Matsuda)



magnetic monopoles ("electric monopoles") ?

photons ?

Realistic Model

$$\mathcal{H}_{\mathsf{S}=1/2} = \sum_{\langle ij \rangle} \Big\{ J_{zz} \mathsf{S}_i^z \mathsf{S}_j^z - J_{\pm} (\mathsf{S}_i^+ \mathsf{S}_j^- + \mathsf{S}_i^- \mathsf{S}_j^+)$$
$$+ J_{\pm\pm} \left[\gamma_{ij} \mathsf{S}_i^+ \mathsf{S}_j^+ + \gamma_{ij}^* \mathsf{S}_i^- \mathsf{S}_j^- \right]$$
$$+ J_{z\pm} \left[\mathsf{S}_i^z (\zeta_{ij} \mathsf{S}_j^+ + \zeta_{ij}^* \mathsf{S}_j^-) + i \leftrightarrow j \right] \Big\}$$

$$J_{\pm\pm} = 0$$

$$\mathcal{H}_{\mathsf{tunnelling}} = -g \sum_{\hexagon} \left[\mathsf{S}_1^+ \mathsf{S}_2^- \mathsf{S}_3^+ \mathsf{S}_4^- \mathsf{S}_5^+ \mathsf{S}_6^- + h.c. \right] \qquad g = \frac{12 J_{\pm}^3}{J_{zz}^2}$$

$$\mathcal{H}_{\mathsf{J}_3} = -J_3 \sum_{\langle ij \rangle_3} \mathsf{S}_i^z \mathsf{S}_j^z \qquad J_3 = \frac{3 J_{z\pm}^2}{J_{zz}} > 0$$

Benton, Sikora, Shannon (2012)

Gauge Mean Field Theory (gMFT)

$$\Phi_{\mathbf{r}}^\dagger = e^{i\varphi_{\mathbf{r}}} \qquad [\varphi_{\mathbf{r}}, Q_{\mathbf{r}}] = i$$
$$\Phi_{\mathbf{r}} = e^{-i\varphi_{\mathbf{r}}} \qquad |\Phi_{\mathbf{r}}| = 1$$

"spinons" on the diamond lattice

$$Q_{\mathbf{r}} = \eta_{\mathbf{r}} \sum_\mu \mathsf{s}^z_{\mathbf{r},\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\mu}$$ constraint $\mathrm{I/II},\ \text{with}\ \eta_{\mathbf{r}} = \pm 1$

Introduce spinons explicitly using a constraint

$$\mathsf{S}^+_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu} = \Phi^\dagger_{\mathbf{r}}\, \mathsf{s}^+_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu}\, \Phi_{\mathbf{r}+\mathbf{e}_\mu}$$
$$\mathsf{S}^z_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu} = \mathsf{s}^z_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu}$$

$$\mathsf{s}^z_{\mathbf{r}\mathbf{r}'} = E_{\mathbf{r}\mathbf{r}'}$$
$$\mathsf{s}^\pm_{\mathbf{r}\mathbf{r}'} = e^{\pm i A_{\mathbf{r}\mathbf{r}'}}$$

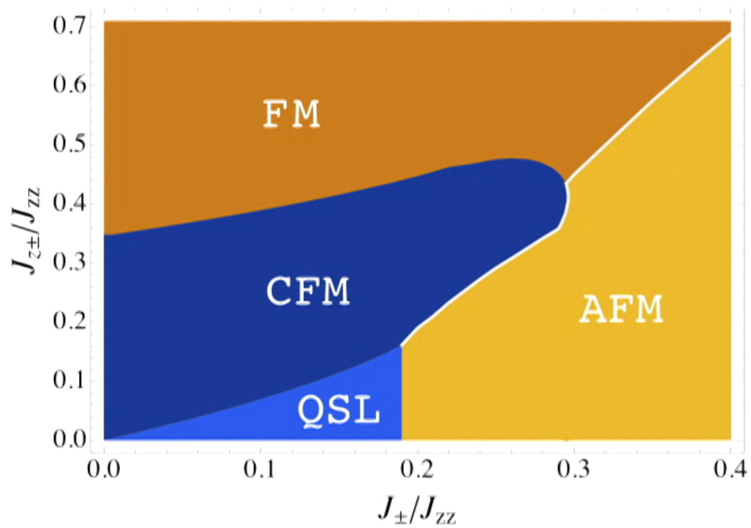
Savary, Balents

$\langle\Phi\rangle$	$\langle s^z\rangle$	$\langle s^\pm\rangle$	phase
0	0	$\neq 0$	QSL
0	$\neq 0$	$\neq 0$	CFM
$\neq 0$	$\neq 0$	$\neq 0$	FM
$\neq 0$	0	$\neq 0$	AFM

No confinement

$$\langle s^\pm\rangle = \langle e^{\pm iA}\rangle \neq 0$$

condensation of "Higgs" field(s)



Some Comments

This mean-field theory ignores the fluctuations of magnetic monopoles

Misses out confinement physics

Finite monopole density at finite temperature screens long-range interaction

Strictly speaking, the U(1) spin liquid (Coulomb phase) is well-defined only at T=0

Subtle … in finite temperature mean-field theory