

Title: The thick sandwich problem in (2+1)-dimensional causal dynamical triangulations

Date: Jun 01, 2017 04:20 PM

URL: <http://pirsa.org/17060028>

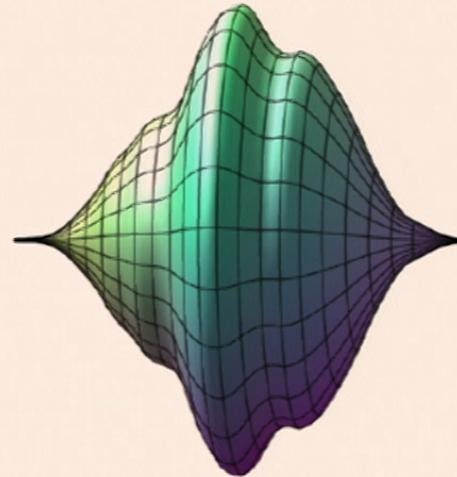
Abstract: Causal dynamical triangulations (CDT) is a sum-over-histories approach to quantum gravity which leverages the techniques developed in lattice quantum field theory. In this talk, I discuss the thick sandwich problem in CDT: Given initial and final spacelike hypersurfaces, each with a fixed geometry, what is the transition amplitude for one transitioning into the other? And what geometries dominate the associated path integral? I discuss preliminary studies performed in this direction. I also highlight open problems and interesting directions for future research.

# The Thick Sandwich Problem in (2+1)-dimensional Causal Dynamical Triangulations

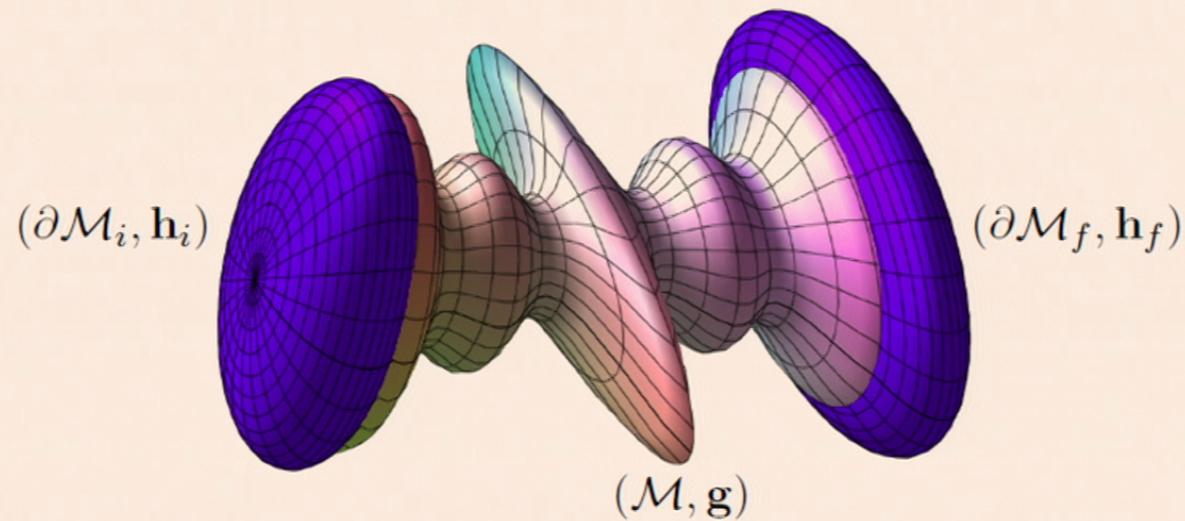
Jonah M. Miller

In Collaboration With: J. H. Cooperman, E. Schnetter, K. Lee

PI Day, 2017

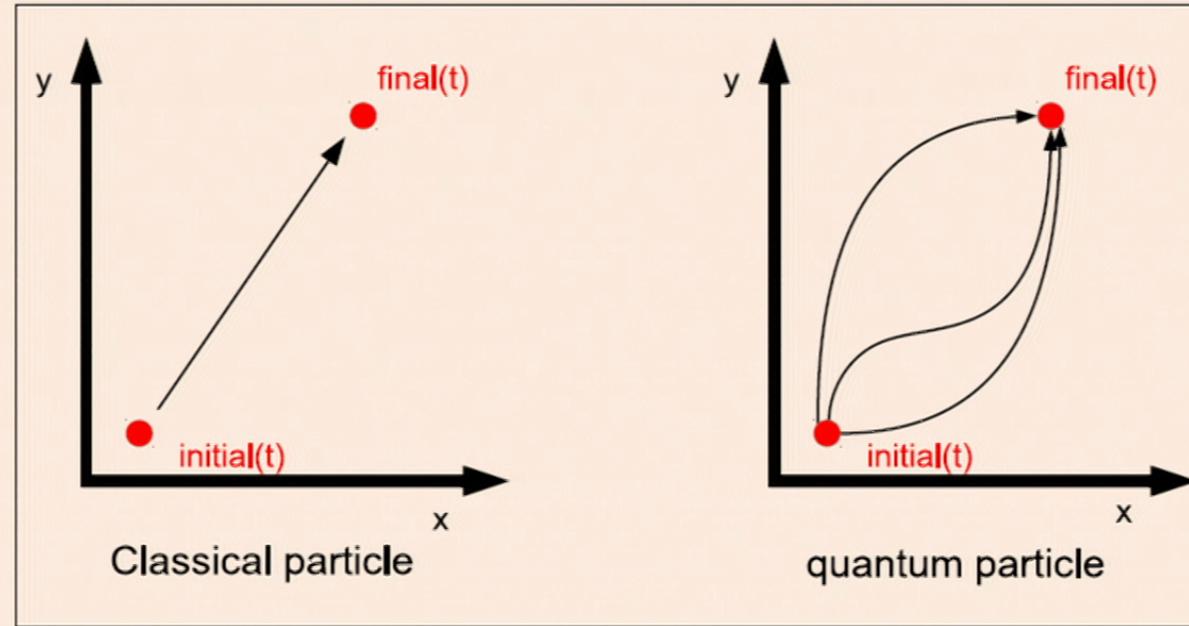


# The Thick Sandwich Problem

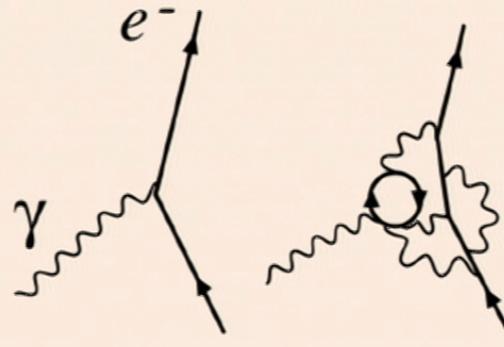


# The Feynman Path Integral

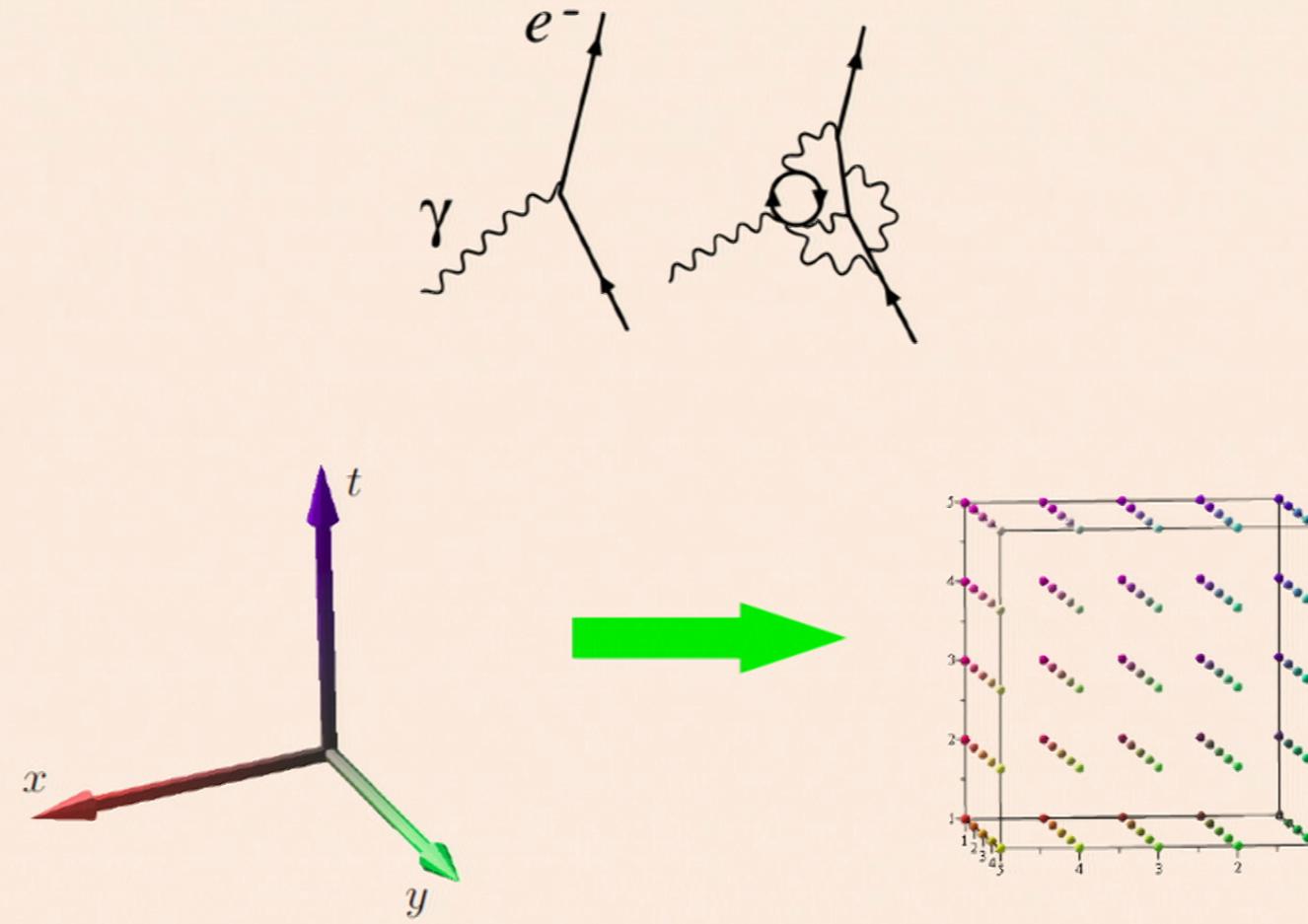
$$\frac{dL}{dp} = \frac{d}{dx} \frac{dL}{dq} \longrightarrow \mathcal{A} = \int \mathcal{D}g e^{iS}$$



# Non-Perturbative Integration



# Non-Perturbative Integration



J. Miller (PI)

CDT

PI Day

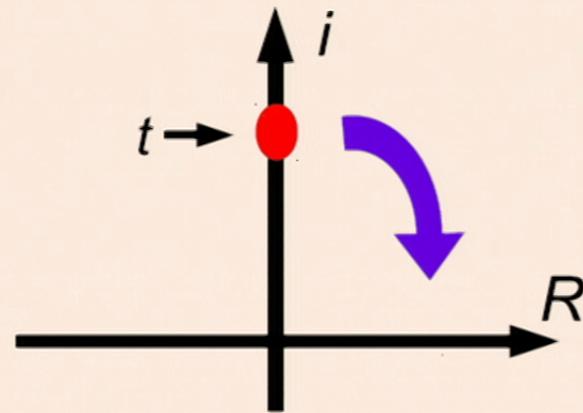
4 / 19

# Taming the Path Integral

$$\mathcal{A} = \int \mathcal{D}g e^{iS[g]} \rightarrow \mathcal{A}_{lattice} = \sum_{\mathcal{G}} \mu(\mathcal{G}) e^{iS[\mathcal{G}]} = ...?$$

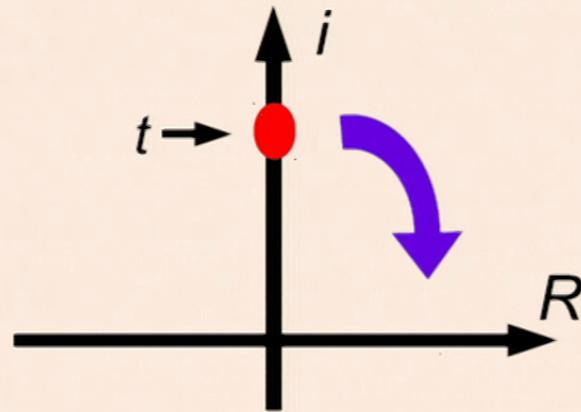
# Taming the Path Integral

$$\mathcal{A} = \int \mathcal{D}g e^{iS[g]} \rightarrow \mathcal{A}_{lattice} = \sum_{\mathcal{G}} \mu(\mathcal{G}) e^{iS[\mathcal{G}]} = \dots ?$$



# Taming the Path Integral

$$\mathcal{A} = \int \mathcal{D}g e^{iS[g]} \rightarrow \mathcal{A}_{lattice} = \sum_{\mathcal{G}} \mu(\mathcal{G}) e^{iS[\mathcal{G}]} = \dots ?$$



$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$



$$ds^2 = d\tau^2 + dx^2 + dy^2 + dz^2$$

$$\Rightarrow \mathcal{A}_{lattice} = \sum_{\mathcal{G}} \mu(\mathcal{G}) e^{-S_{Euc}[\mathcal{G}]}$$

# Monte Carlo Integration

Path integral → Boltzmann distribution

$$\Rightarrow P[\mathcal{G}] = \mu(\mathcal{G}) e^{-S_{Euc}[\mathcal{G}]}$$

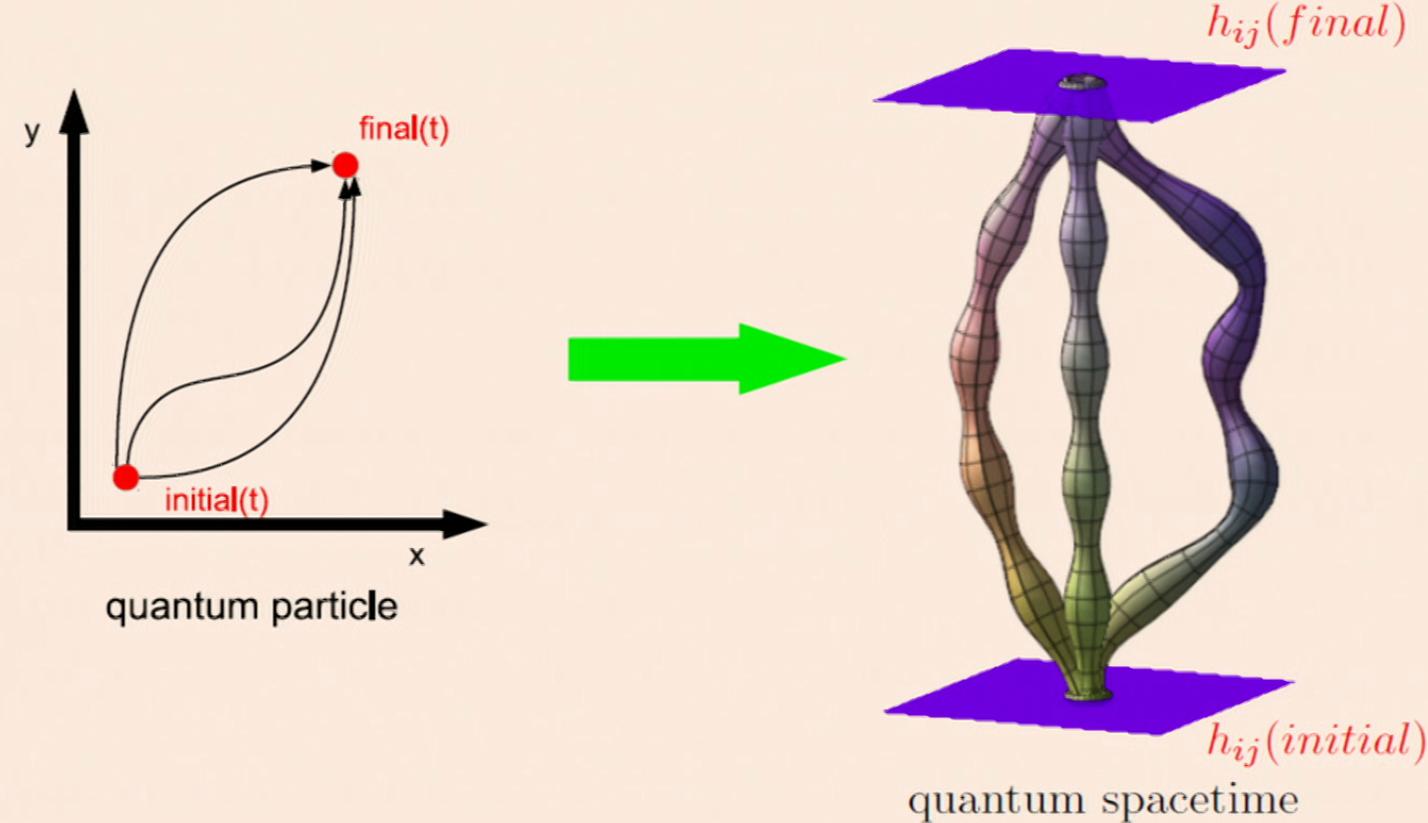
## The Metropolis-Hastings Algorithm

- Measure  $P[\mathcal{G}_{old}]$
- Make a small change to  $\mathcal{G}$
- Calculate  $\mathcal{R} = \frac{P[\mathcal{G}_{new}]}{P[\mathcal{G}_{old}]} = e^{-(S_{Euc}[\mathcal{G}_2] - S_{Euc}[\mathcal{G}_1])}$ .
- If  $\mathcal{R} > 1$ , accept the change. Otherwise reject it.
- Repeat until probable geometry is reached.

## Build ensemble of probable geometries

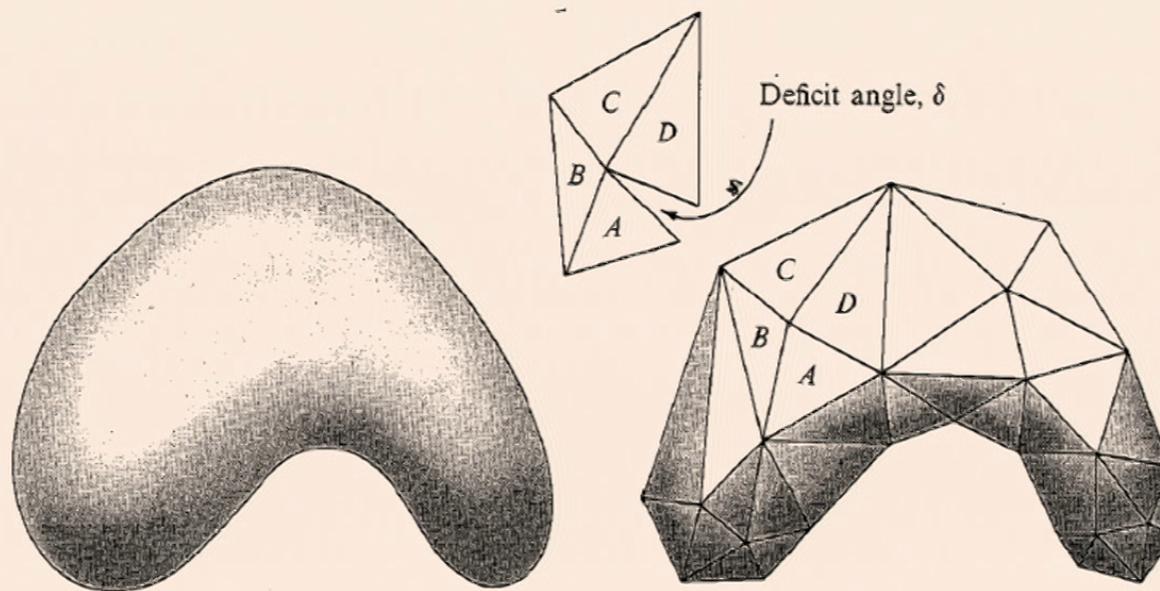
- Observables can be computed as ensemble averages.
- Sum over the most probable geometries to evaluate the path integral

## Sum-Over-Histories



# Discretizing Spacetime

Regge Calculus:

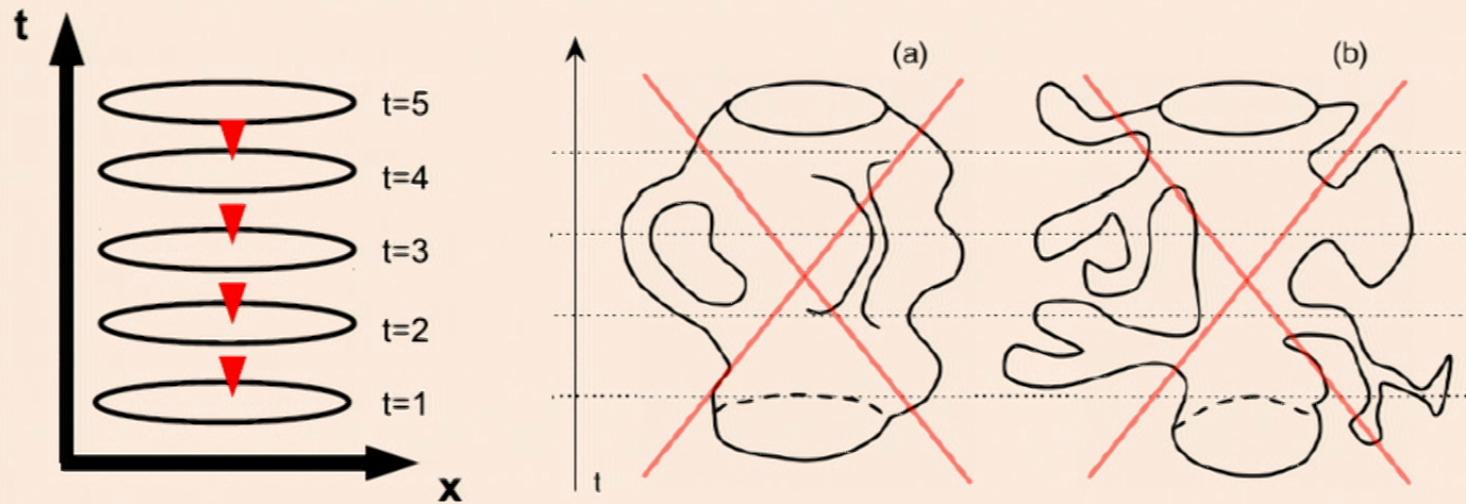


$$\sum_{\text{discrete geometry}} A_i \delta_i = \frac{1}{2} \int_{\text{smooth geometry}} R(|g|)^{1/2} d^n x$$

Image due to Misner, Thorne, and Wheeler

## The Causality Conditions

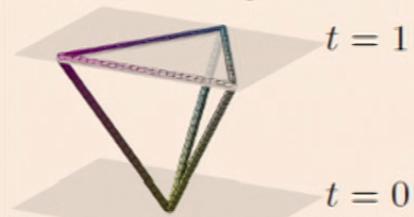
Discrete pieces of the manifold must start Lorentzian!



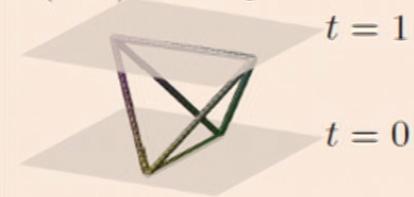
J. Ambørn and R. Loll

# Causal Triangulations in 2+1 Dimensions

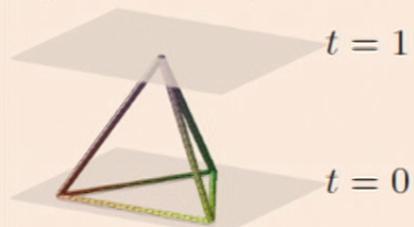
(1, 3) 3-simplex



(2, 2) 3-simplex



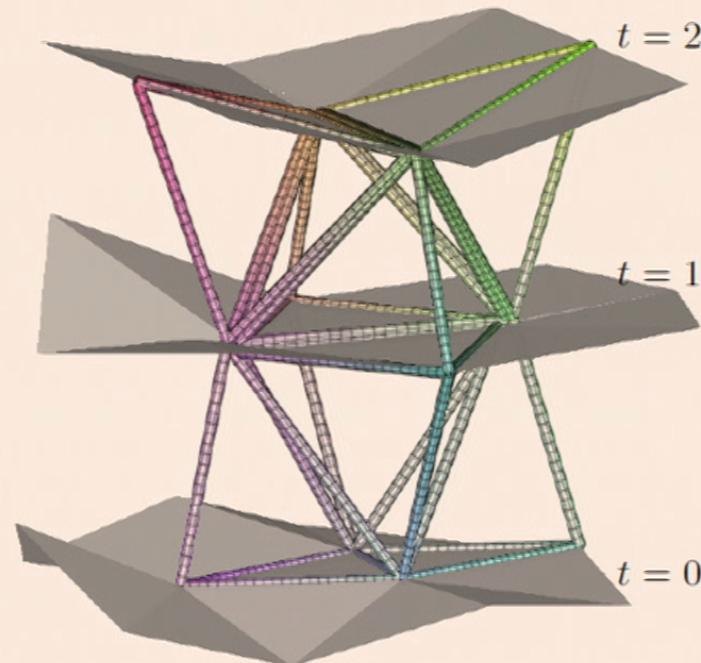
(3, 1) 3-simplex



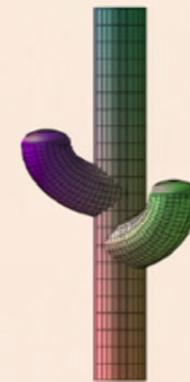
$$l_{SL}^2 = a^2$$

$$l_{TL}^2 = -\alpha a^2$$

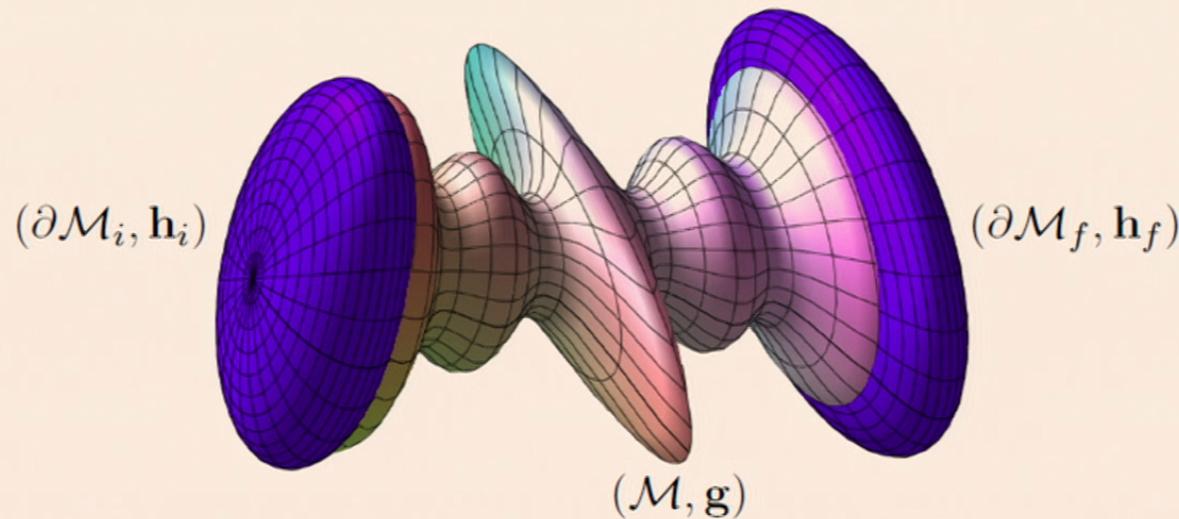
Segment of a causal triangulation



Forbidden  
spacetimes



# The Simulation Process

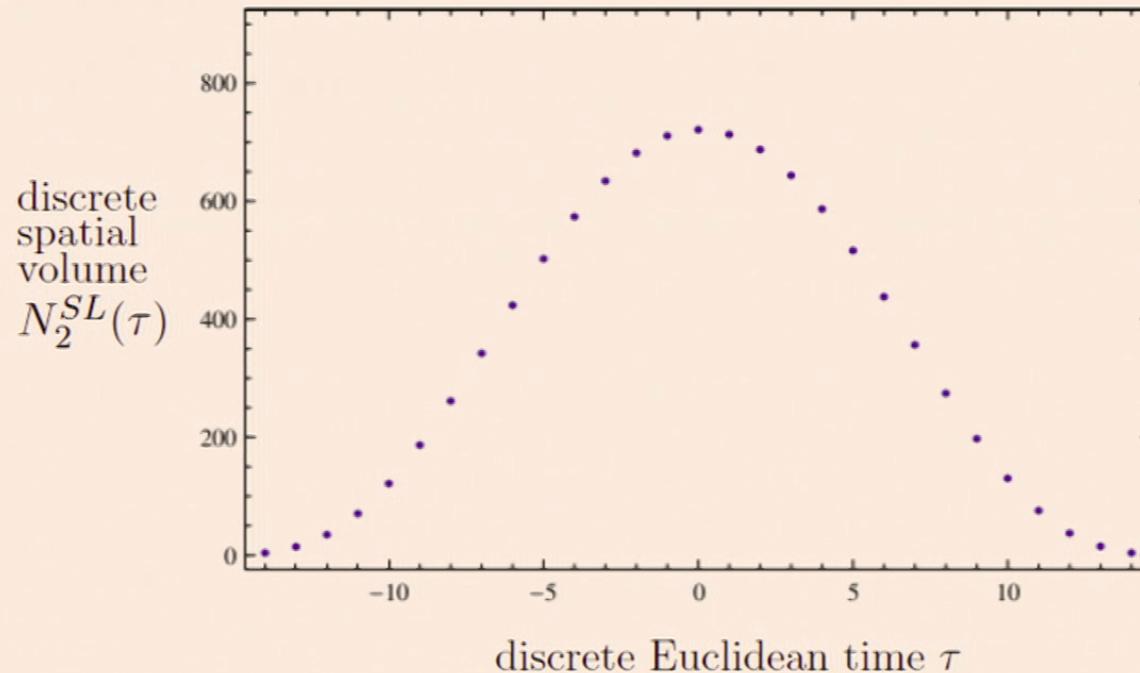


- Constrain system
  - Fix topology (spatial hypersurfaces and timelike direction)
  - Fix number of discrete time slices  $T$
  - Fix spacetime volume (or number of 3-dimensional tetrahedra),  $N_3$
  - Fix geometries of initial and final spacelike hypersurfaces  $\mathbf{h}_i$  and  $\mathbf{h}_f$
- Generate ensemble of most likely spacetimes based on partition function  $Z_{CDT}$
- Measure observables

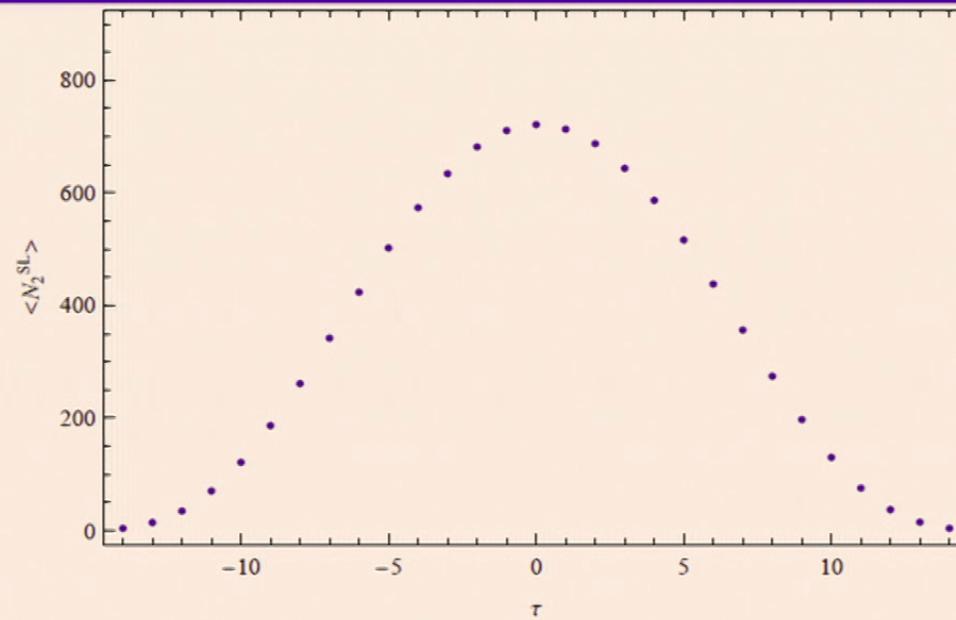
# Expected Spacetimes

Quantization of Einstein gravity for spacetime topology  $\mathcal{S}^2 \times \mathcal{I}$

- Observable  $N_2^{SL}(\tau)$ , spacelike surface area as a function of time  
Ensemble Average  $\langle N_2^{SL}(\tau) \rangle$



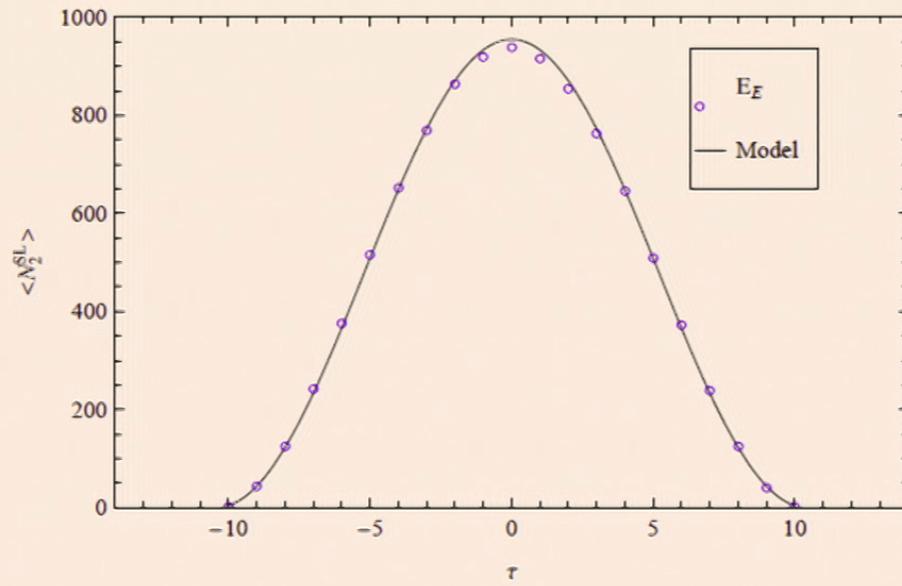
## Making the Analytic Connection



Guess a minisuperspace effective action:

$$ds^2 = d\tau^2 + a^2(\tau)(d\theta^2 + \sin^2(\theta)d\phi^2)$$

## Making the Analytic Connection



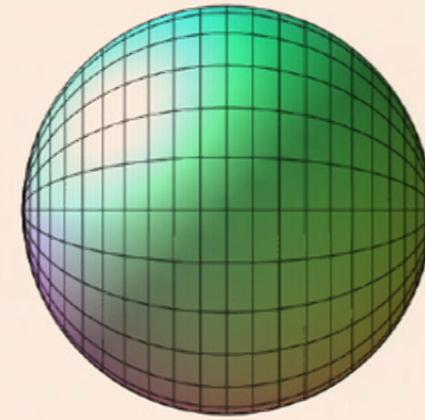
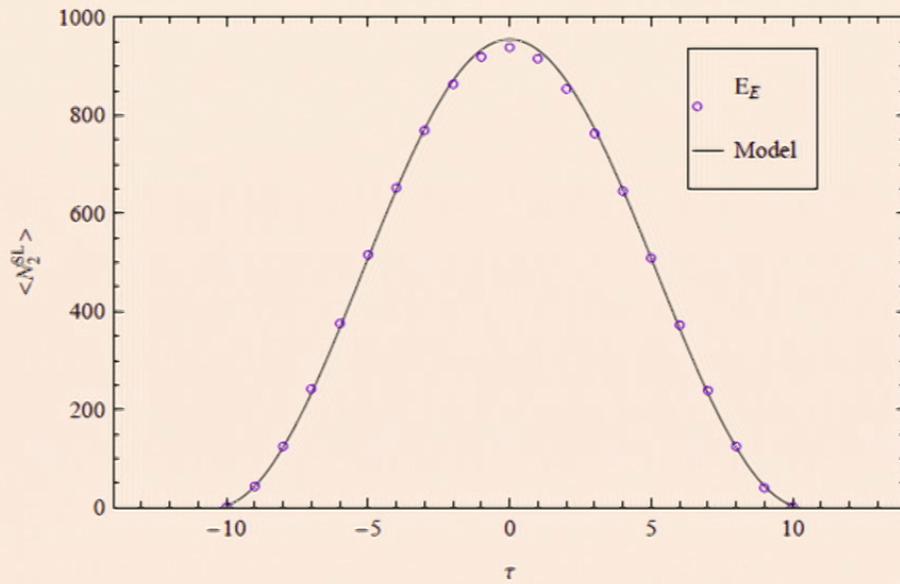
Guess a minisuperspace effective action:

$$ds^2 = d\tau^2 + a^2(\tau)(d\theta^2 + \sin^2(\theta)d\phi^2)$$

$$S_{model}[a] = \frac{\omega}{2G} \int_{\tau_i}^{\tau_f} \left[ \frac{\dot{a}^2(\tau)}{\omega^2} - \Lambda a^2(\tau) \right] d\tau$$

$$a(\tau) \sim \cos(\omega\tau/l_{dS})$$

# Making the Analytic Connection



Guess a minisuperspace effective action:

$$ds^2 = d\tau^2 + a^2(\tau)(d\theta^2 + \sin^2(\theta)d\phi^2)$$

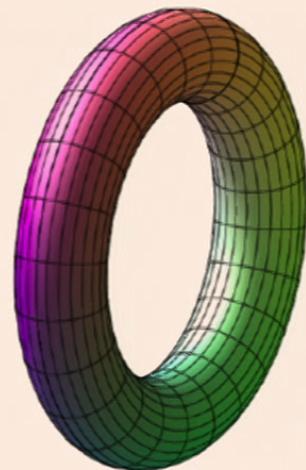
$$S_{model}[a] = \frac{\omega}{2G} \int_{\tau_i}^{\tau_f} \left[ \frac{\dot{a}^2(\tau)}{\omega^2} - \Lambda a^2(\tau) \right] d\tau$$

$a(\tau) \sim \cos(\omega\tau/l_{dS})$  : Wick-Rotated de Sitter Space!

## My Work

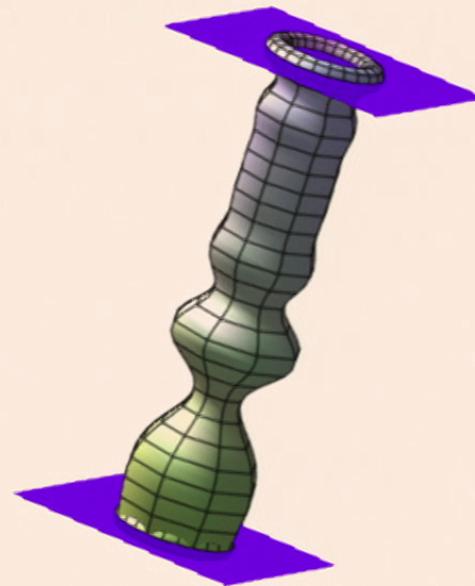
Standard Way

$$\tau = S^2 \times S^1 \equiv T^3$$



My Way

$$\tau = S^2 \times \mathcal{I}$$



# Semiclassical expectations for transition amplitudes

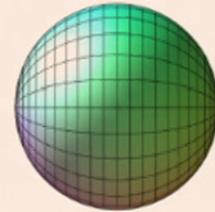
$$\mathcal{A}[\mathbf{h}_f] = \int^{\mathbf{g}|_{\partial\mathcal{M}_f} = \mathbf{h}_f} \mathcal{D}\mathbf{g} e^{iS[\mathbf{g}]} \longrightarrow \mathcal{A}[\mathbf{h}_f] = \int_{\mathbf{g}|_{\partial\mathcal{M}_i} = \emptyset}^{\mathbf{g}|_{\partial\mathcal{M}_f} = \mathbf{h}_f} \mathcal{D}\mathbf{g} e^{-S^{(E)}[\mathbf{g}]}$$

- Minisuperspace truncation

$$ds^2 = d\tau^2 + a^2(\tau)(d\theta^2 + \sin^2 \theta d\phi^2)$$

(Approximate) extrema  $a_{\text{cl}}(\tau)$  of the action  $S^{(E)}[a(\tau)]$

Case 1:  $a_i = 0, a_f = 0$

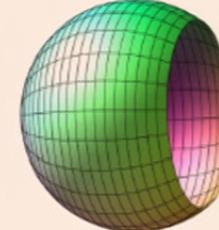


arXiv:1305.2932

Case 2:  $a_i = 0, a_f > 0$

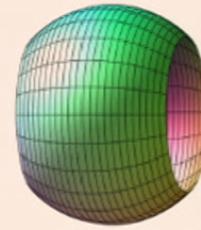


with  $0 < a_f \leq l_{dS}$



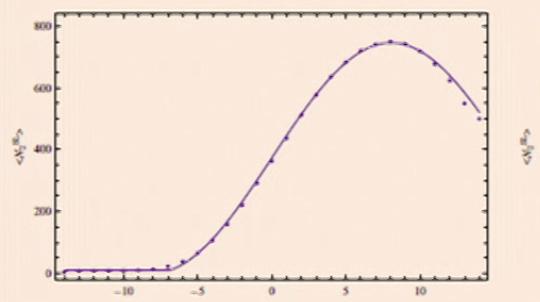
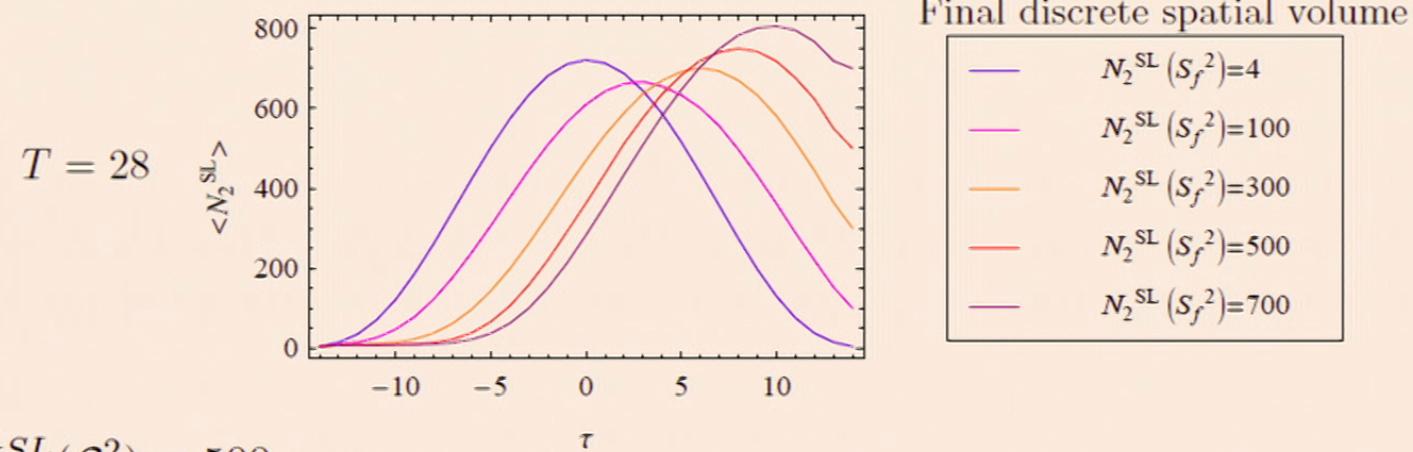
Case 3:  $a_i > 0, a_f > 0$

with  $a_i \leq l_{dS}, a_f \leq l_{dS}$

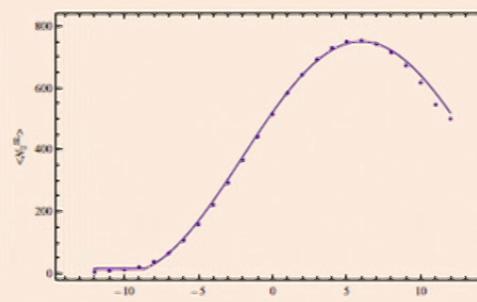


## Case 2: Minimal initial to nonminimal final boundary

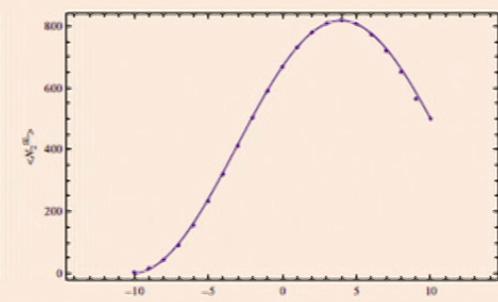
2-sphere spatial topology, finite interval in time



$T = 28$

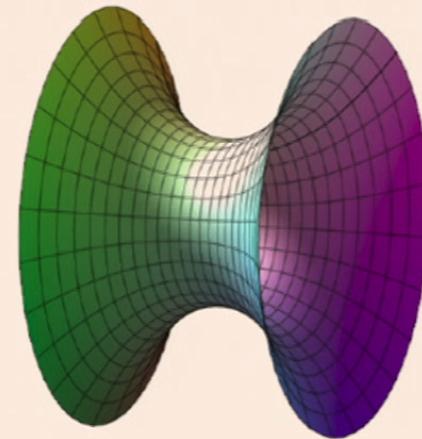
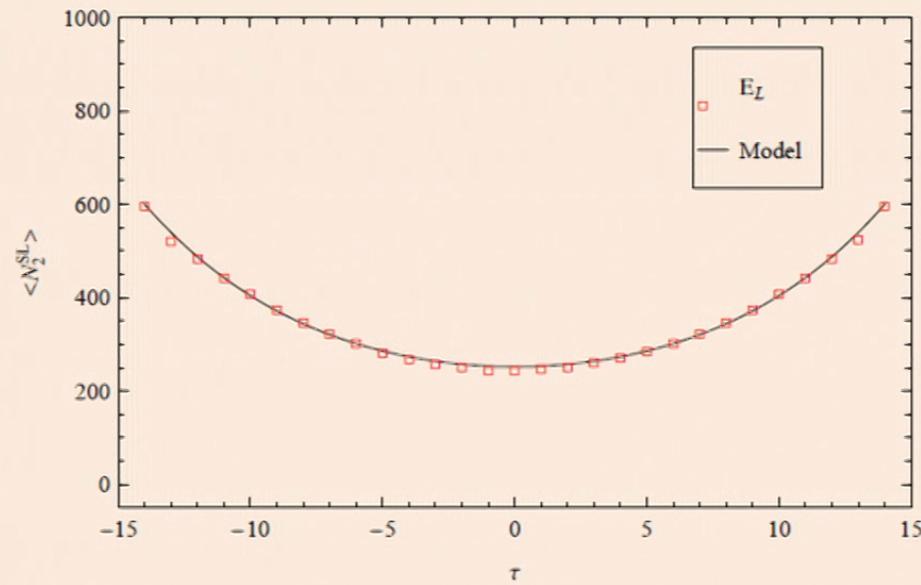


$T = 24$



$T = 20$

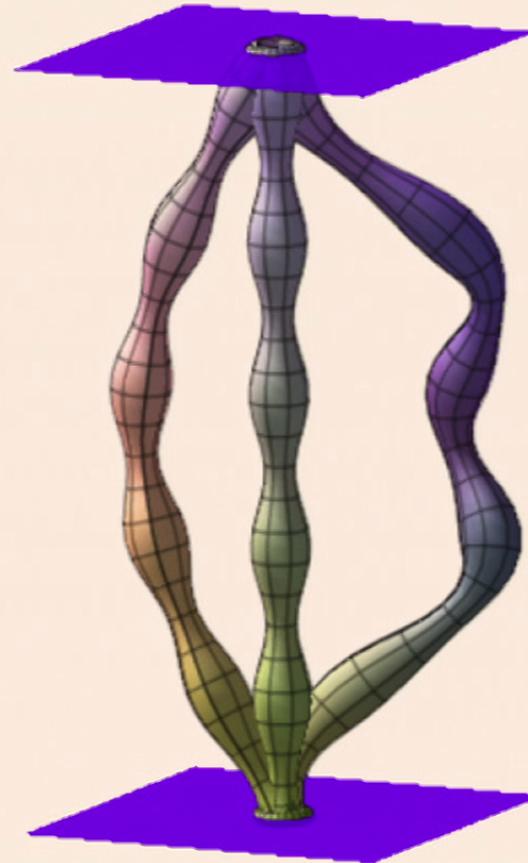
## Case 3: Nonminimal initial to final boundary



To Be Continued in [ArXiv:1610.02408](https://arxiv.org/abs/1610.02408)

## Summary

- CDT is a [sum-over-histories](#) approach to quantum gravity, which regularizes the path integral via lattice field theory
- Early evidence indicates a [good classical limit](#) and interesting quantum behaviour
- The thick sandwich problem in CDT is a fruitful and interesting topic
- For more, see:
  - [arXiv:1305.2932](#)
  - [arXiv:1610.02408](#)



## Exciting Open Questions

- What is the continuum limit of CDT?
- Which analytic minisuperspace quantization corresponds to CDT?
- Can we observe effects beyond the minisuperspace truncation by imposing nonspherically symmetric boundary geometries?
- Is there gauge redundancy in the number  $T$  of time slices of a causal triangulation?
- Is an effective signature change possible? (See arXiv:1502.04320 and redXiv:1503.08580)
- What is an appropriate measure of distance in CDT?
- Can a unitful measure of distance be imposed for renormalization?