Title: Dualities for chiral algebras from S-duality

Date: Jun 01, 2017 03:20 PM

URL: http://pirsa.org/17060026

Abstract: During past few decades, string theory has been used as a source of conjectural dualities in various areas of physics and mathematics. We have extended these applications of string dualities to the study of chiral algebras in 2d CFT. In this talk, I will sketch how to use S-duality of D3-D5-NS5 systems to shed some light on already known dual constructions of chiral algebras and generate huge amount of new dualities.

Dualities for Chiral Algebras from S-duality

Miroslav Rapčák

Perimeter Institute

miroslav.rapcak@gmail.com

June 1, 2017

Introduction

Vertex Operator Algebras (VOA) are algebras of chiral operators in 2d CFT with algebra structure given by OPE. Some **examples** are:

- Stress-energy tensor: T

$$T(z)T(w) \sim \frac{c/2}{(z-w)^2} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w}$$

- Kac-Moody algebra: J^a

$$J^a(z)J^b(w) \sim \frac{kg^{ab}}{(z-w)^2} + \sum_c \frac{if_c^{ab} J^c(w)}{z-w}$$

- Super-Virasoro: T, G
- W_N algebras: $J, T, W_3, W_4, \ldots W_N$

Introduction

Applications of extended VOAs:

- Superstrings
- Lattice models with Z_n symmetry
- AdS_3/CFT_2
- AGT correspondence
- Geometric Langlands program
- KdV hierarchies

Constructions:

- **Drinfeld-Sokolov (DS) reduction** (labeled as $W[G]$)
- **Coset** (labeled as G/H for $H \subset G$)

Introduction

There exist **three ways from $U(N)$ Kac-Moody algebra to W_N:**

$$
\begin{tikzcd}
W_N\left[U(N)_\Psi\right] \arrow[rr, leftrightarrow, "\text{Feigin-Frenkel duality}"] \arrow[dr, leftrightarrow, "?"'] & & W_N\left[U(N)_{\frac{1}{\Psi}}\right] \arrow[dl, leftrightarrow, "?"] \\
& \dfrac{U(N)_{\frac{1}{\Psi-1}} \times U(N)_{N+1}}{U(2)_{\frac{1}{\Psi-1}+1}} &
\end{tikzcd}
$$

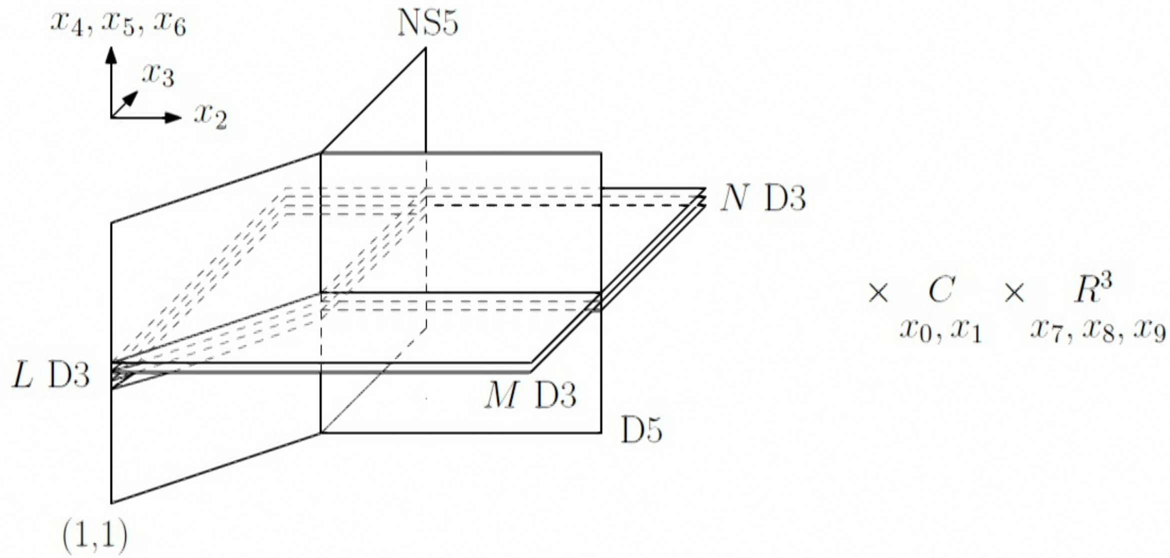
where we adopt notation $U(N)_\Psi = U(1)_{N\Psi} \times SU(N)_{\Psi-N}$.

Goals:

- **Interpret above dualities as a consequence of S-duality.**
- **Generalize to four-parameter family of Y-algebras.**

Y-algebras

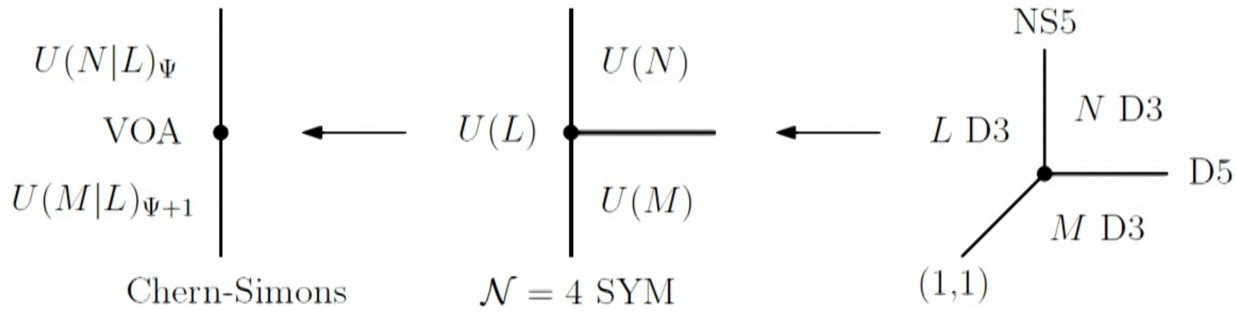
Consider D3-branes attached to the junction of D5, NS5 and (1,1) branes in type IIB string theory.



Low energy theory of N D3 branes can be identified with $\mathcal{N} = 4$ super Yang-Mills theory with $U(N)$ gauge group. From the point of view of 4d theory, fivebranes play the role of domain-walls between $U(L)$, $U(M)$, and $U(N)$ gauge theories.

Y-algebras

If we restrict to a particular class of supersymmetric operators, path integral of the 4d theory localizes to the path integral of $U(N|L)$ and $U(M|L)$ Chern-Simons theories connected by an interface.



Interfaces in Chern-Simons theory are known to lead to vertex operator algebras. Translating the gluing condition at the junction leads us to the identification of corresponding vertex operator algebra with a combination of coset and DS-deruction

$$\boxed{Y_{L,M,N}[\Psi] = \frac{W_{N-M}[U(N|L)_\Psi]}{U(M|L)_{\Psi-1}}, \quad \text{for } N > M}.$$

Y-algebras

S-duality of type IIB is an $SL(2,Z)$ action on (p,q) branes and parameter Ψ generated by

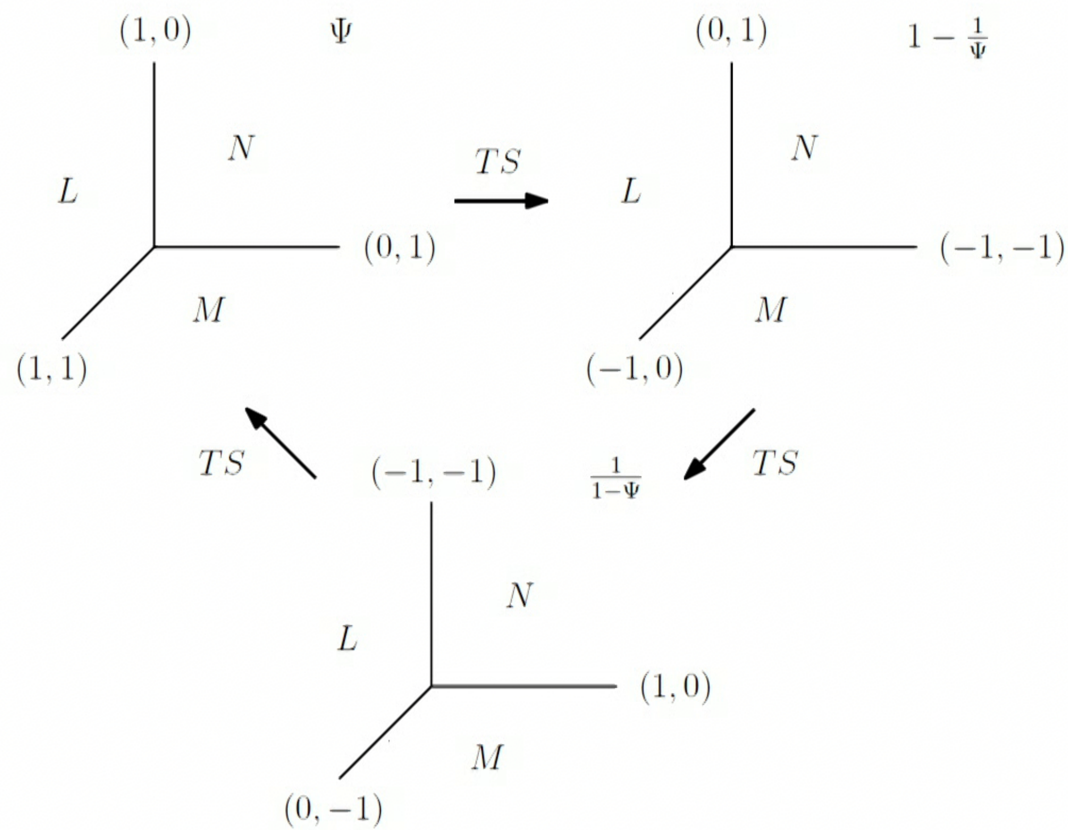
$$T: \quad \Psi \to \Psi + 1, \quad (m,n) \to (m, n+m)$$
$$S: \quad \Psi \to -\frac{1}{\Psi}, \quad (m,n) \to (-n, m)$$

S_3 **subgroup of the S-duality group** preserves the trivalent junction and acts as a permutation of the branes accomplanied with a transformation of Ψ. This induces S_3 **duality action on Y-algebras**:

$$\boxed{Y_{L,M,N}[\Psi] = Y_{N,L,M}\left[\frac{1}{1-\Psi}\right] = Y_{M,N,L}\left[1-\frac{1}{\Psi}\right]}$$

Y-algebras

Diagrammatically, the triality action is given by transformations:



Y-algebras

Examples:

- Three realizations of W_N $(Y_{0,0,N}[\Psi])$:

$$W_N[U(N)_\Psi] \quad \leftrightarrow \quad W_N[U(N)_{\frac{1}{\Psi}}] \quad \leftrightarrow \quad \frac{U(N)_{\frac{1}{\Psi-1}} \times Ff^{U(N)}}{U(N)_{\frac{1}{\Psi-1}-1}}$$

- Three realizations of parafermions $(Y_{0,1,2}[\Psi])$:

$$\frac{U(2)_\Psi}{U(1)_{\Psi-1}} \quad \leftrightarrow \quad \frac{W_2[U(2|1)_{\frac{1}{\Psi}}]}{U(1)_{1-\frac{1}{\Psi}}} \quad \leftrightarrow \quad \frac{U(2|1)_{\frac{1}{\Psi-1}}}{U(2)_{\frac{1}{\Psi-1}-1}}$$

- Many new dualities. . .

Summary

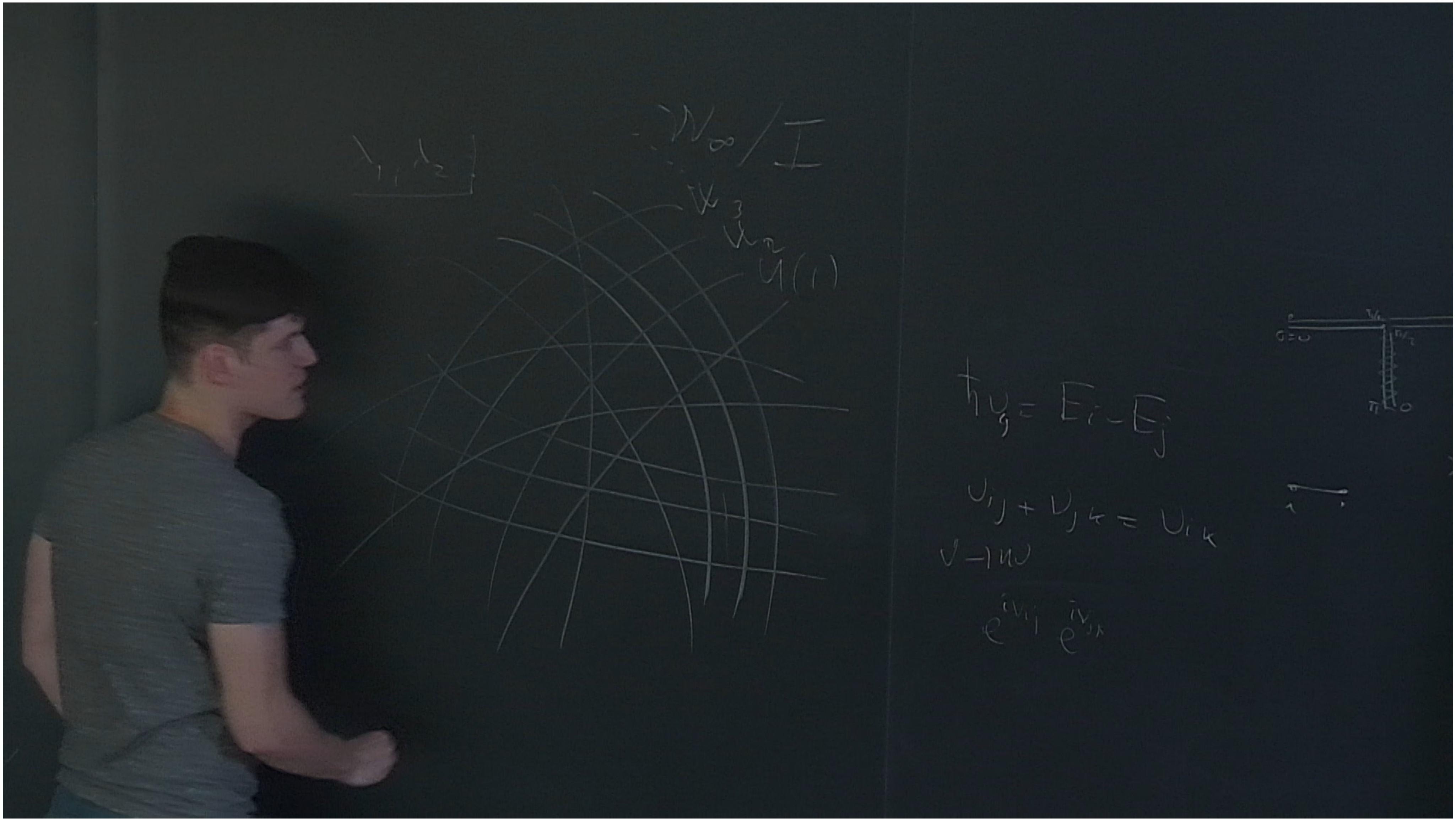
We have explained that:

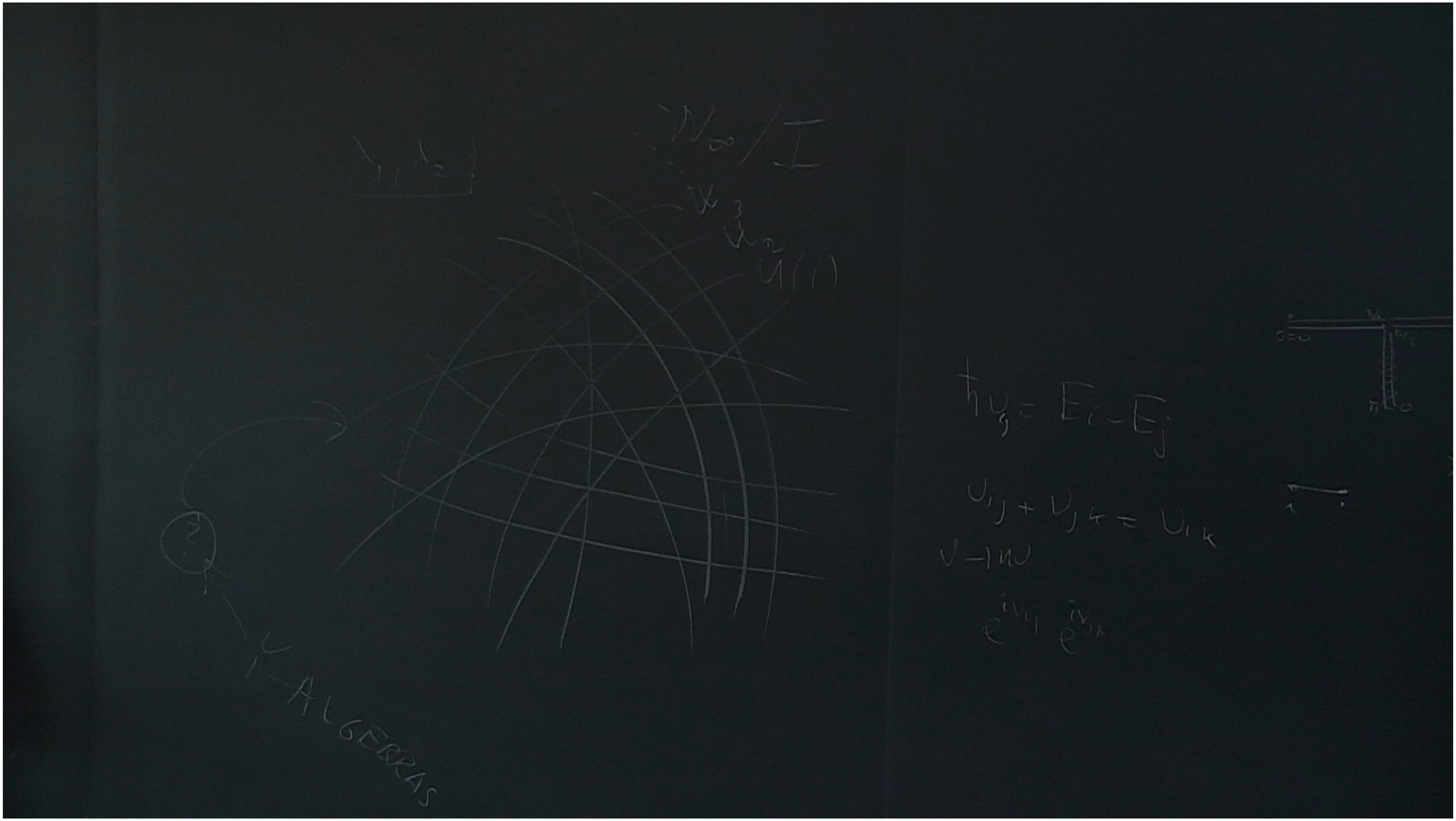
- Mysterious triality of W_N is a consequence of S-duality.
- Generalization to four-parameter family of Y-algebras exists.

Further developments:

- Generalization to more complicated webs of fivebranes (e.g. X-algebras with $Z_2 \times Z_2$ duality)
- Exploration of relations to other probles of physics and mathematics such as AGT correspondence, counting of D0-D2-D4 bound stated in Calabi-Yau manifolds, geometric Langlands program, $\mathrm{AdS}_3/\mathrm{CFT}_2, \ldots$


λ_1, λ_2
W_∞ / I
$\hbar\nu_{ij} = E_i - E_j$
$+ \nu_{jk} = \nu_{ik}$
$e^{i\nu_{ik}}$


$\hbar \nu_{ij} = E_i - E_j$
$\nu_{ij} + \nu_{jk} = \nu_{ik}$
$e^{i\nu_{ij}}\ e^{i\nu_{jk}}$



hν_ij = E_i − E_j
ν_ij + ν_jk = ν_ik