Title: Entanglement entropy of scalar fields in causal set theory

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Abstract: Entanglement entropy is now widely accepted as having deep connections with quantum gravity. It is therefore desirable to understand it in the context of causal sets, especially since they provide in a natural and covariant manner the UV cutoff needed to render entanglement entropy finite. Defining entropy in a causal set is not straightforward because the usual hypersurface data on which definitions of entanglement typically rely is not available. Instead, we appeal to a more global expression which, for a gaussian scalar field, expresses the entropy of a spacetime region in terms of the fieldâ€™s correlation function within that region. In this talk I will present results from evaluating this entropy for causal sets sprinkled into a 1 + 1-dimensional causal diamond in flat spacetime, and specifically for a smaller causal diamond within a larger concentric one. In the first instance we find an entropy that obeys a (spacetime) volume law instead of the expected (spatial) area law. We find, however, that one can obtain the expected area law by following a prescription for truncating the eigenvalues of a certain â€œPauli-Jordanâ€• operator and the projections of their eigenfunctions on the Wightman function that enters into the entropy formula.

Entanglement Entropy of Scalar Fields in Causal Set Theory

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Work with Rafael Sorkin
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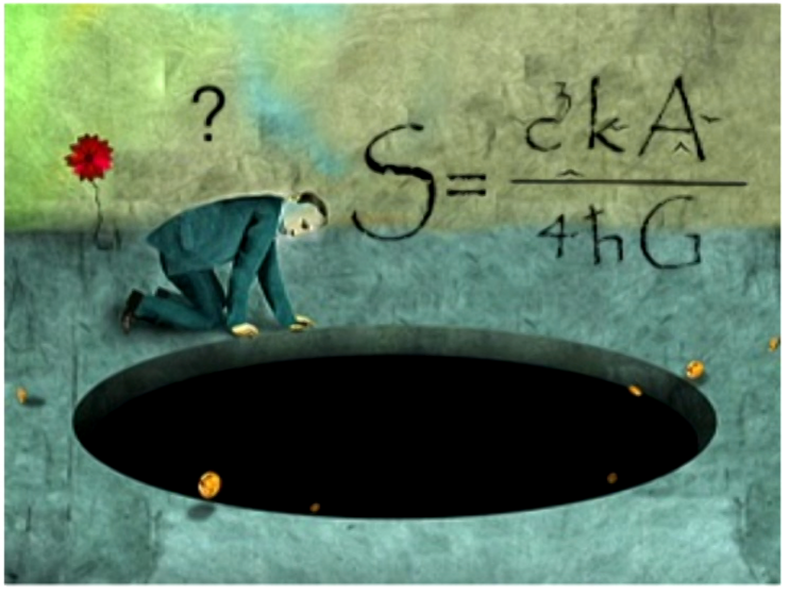
Outline

- Entanglement entropy and causal set theory.
- A spacetime definition of entanglement entropy which is meaningful both in causal set theory and in the continuum.
- Direct application of the definition leads to a spacetime-volume law in the causal set.
- We can recover the usual spatial area law in the causal set after applying a suitable truncation scheme.

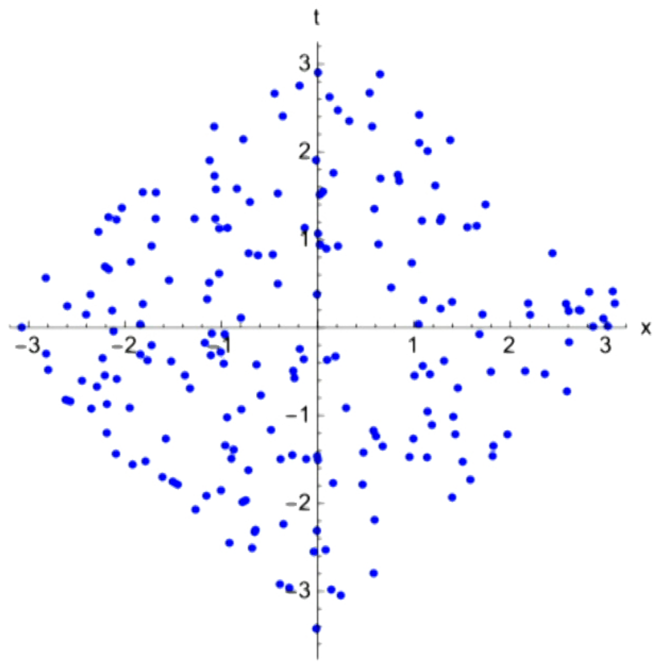
Is Black Hole Entropy Entanglement Entropy?

- R. D. Sorkin, *On the Entropy of the Vacuum outside a Horizon*, (1983), arXiv:1402.3589.
- L. Bombelli, R. Koul, J. Lee, and R. Sorkin, *Quantum Source of Entropy for Black Holes*, PRD 34 (1986) 373383.
- M. Srednicki, *Entropy and Area*, PRL 71 (1993) 666–669.



Causal Set Theory [1]

A Causal Set is a locally finite partially ordered set. It is a set $\mathcal{C}$ along with an ordering relation $\preceq$ that satisfy some conditions.



[1] Bombelli, L., Lee, J. H., Meyer, D. and Sorkin, R. D., 1987, Space-Time as a Causal Set, Phys. Rev. Lett. 59, 521.

Entanglement Entropy: The Usual Story

In conventional treatments, entropy is identified with

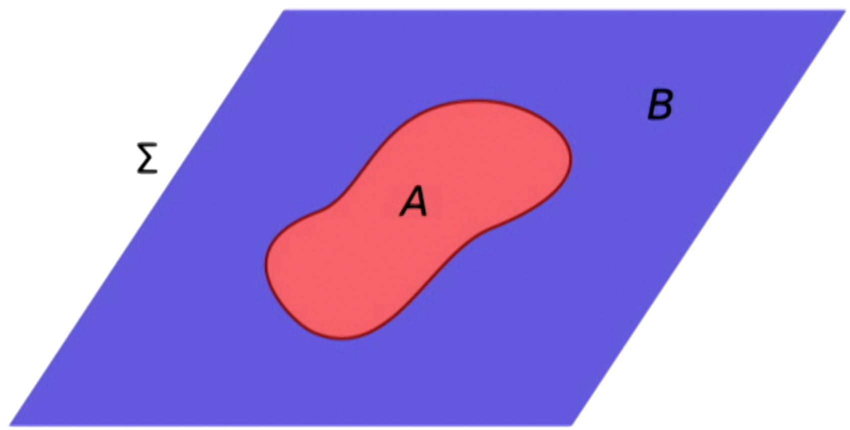
$$S = \mathrm{Tr}\rho \ln \rho^{-1} \tag{1}$$

where $\rho(\Sigma)$ is a density matrix and Σ is a spatial hypersurface. If Σ is divided into complementary subregions A and B, then the reduced density matrix for subregion A is

$$\rho_A = \mathrm{Tr}_B \rho \tag{2}$$

and its entropy is

$$S_A = -\mathrm{Tr}\rho_A \ln \rho_A \ . \tag{3}$$



Spacetime Definition of S (R. Sorkin, arXiv:1205.2953)

Express S directly in terms of the spacetime correlation functions.

The entropy can be expressed as a sum over the solutions λ of the generalized eigenvalue problem

$$W\,v = i\lambda\,\Delta\,v, \qquad (\Delta v \neq 0) \tag{4}$$

as

$$S = \sum \lambda \ln|\lambda| \; . \tag{5}$$

W and Δ are the "Wightman" and "Pauli-Jordan" matrices [2].

[2] In the field theory $W(x,x') = \langle 0|\phi(x)\phi(x')|0\rangle$ & $i\Delta(x,x') = [\phi(x),\phi(x')]$.

The Sorkin-Johnston Vacuum [3]

Write $i\Delta(X,X')$ in terms of its positive (u_k) and negative (v_k) eigenfunctions:

$$i\Delta(X,X') = \sum_k \left[\lambda_k u_k(X) u_k^\dagger(X') - \lambda_k v_k(X) v_k^\dagger(X')\right] \tag{6}$$

Restrict to positive eigenspace: $W_{SJ}(X,X') \equiv \sum_k \lambda_k u_k(X) u_k^\dagger(X')$

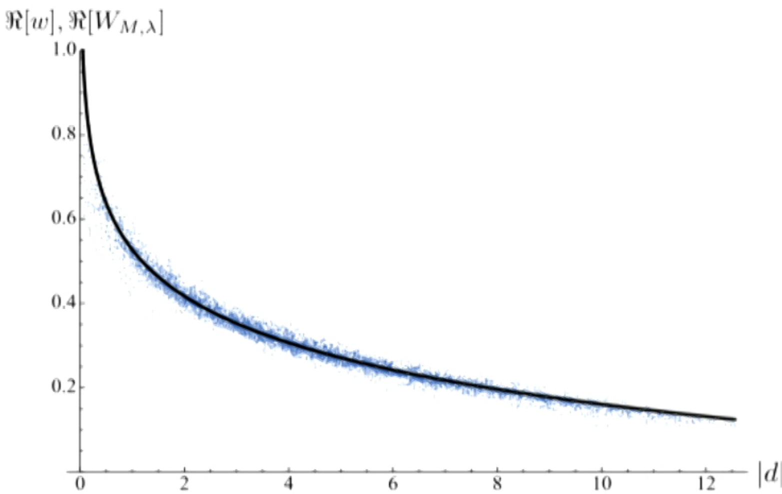
[3] R.D. Sorkin, J. Phys. Conf. Ser. 306 (2011) 012017 [arXiv:1107.0698]

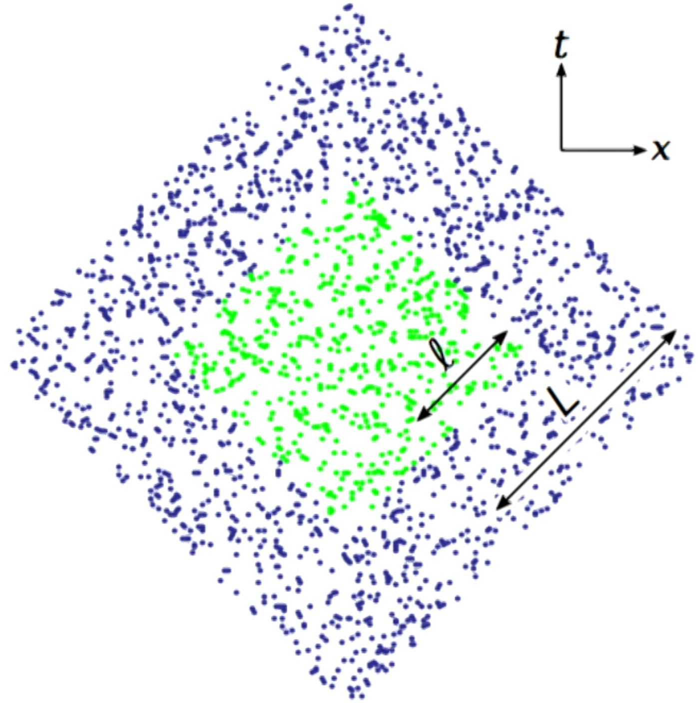
Causal Set Setup

$$\Delta(X, X') := G_R(X, X') - G_R(X', X)$$

For $m = 0$, we have that $G_R = \frac{1}{2}C$, where C is the causal matrix: $C_{xy} := \begin{cases} 1, & \text{if } x \preceq y. \\ 0, & \text{otherwise} \end{cases}$

For W, we choose W_{SJ}.





Causal Set Results

Instead of the expected spatial area law (logarithmic scaling with cutoff in 1+1), we obtain a spacetime volume law!

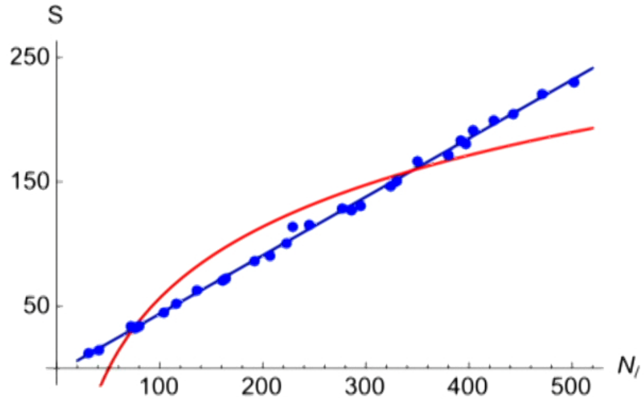


Figure: S vs. N_ℓ, for $\ell/L = 1/4$. Fits $S = aN + b$ with $a = 0.46$ & $b = -3.20$.

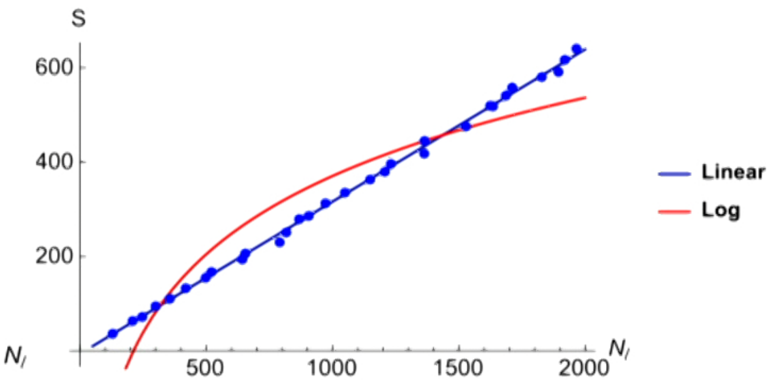


Figure: S vs. N_ℓ, for $\ell/L = 1/2$. Fits $S = aN + b$ with $a = 0.32$ & $b = -6.64$.

$\Delta v \neq 0$ Condition: Truncating the spectrum of Δ

The extra entropy comes from the near-zero part of the spectrum of Δ. Large contribution since: 1) eigenvalues are numerous, and 2) Δ is inverted in $-i\Delta^{-1}W$.

Need to truncate Δ such that $\tilde{\lambda}_{min} \sim \sqrt{N}/4\pi$, in agreement with the continuum:

$$\begin{aligned}
\tilde{\lambda}^{cont} &= \tilde{\lambda}^{cs}/\rho \\
\tilde{\lambda}^{cont}_{min} &= \frac{\ell}{k_{max}} = \frac{\ell^2}{\pi n_{max}} \\
\tilde{\lambda}^{cs}_{min} &\sim \sqrt{N_\ell}/4\pi \rightarrow \tilde{\lambda}^{cs}_{min}/\rho = \frac{\ell^2}{\pi\sqrt{N_\ell}} = \tilde{\lambda}^{cont}_{min}
\end{aligned} \tag{7}$$

when we identify $\sqrt{N} \sim n_{max}$, and $\rho = N_\ell/4\ell^2$.

$\Delta v \neq 0$ Condition: Truncating the Spectrum of Δ

We need to truncate Δ twice:

1) First truncate both Δ and W in the larger diamond.
Then restrict both matrices to the smaller diamond. Call these restricted matrices W^R and Δ^R.

2) Then do a second truncation on W^R and Δ^R, based on the spectrum of Δ^R.

Recovery of the Area Law

The S from the truncated Δ & W fits $S = b\ln(\sqrt{N_\ell}/4\pi) + c$ with $b = 0.346 \pm 0.028$ and $c = 1.883 \pm 0.035$. This is the usual result[4].

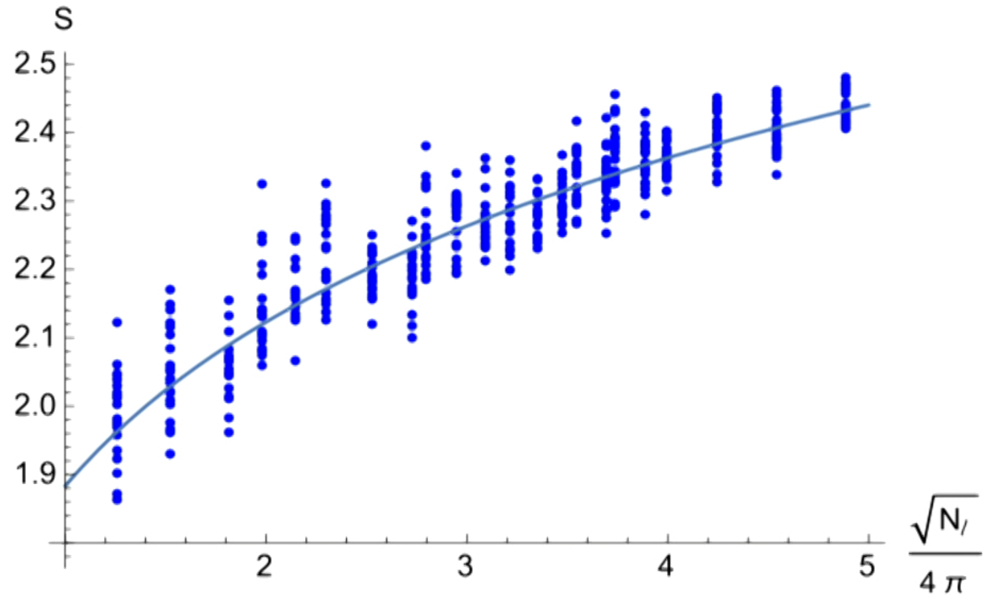


Figure: S vs. $\sqrt{N_\ell}/4\pi$, after Δ and W have been truncated. $\ell/L = 1/2$ in this example.

[4] Calabrese and Cardy, J. Phys A: Math. and Theor. 42 2009, no. 50

Conclusions and Future Directions

- The spacetime entropy definition in causal sets sprinkled in 2d flat spacetimes yields a spacetime volume law instead of the expected spatial area law.

- We can recover the area law by changing the kernel condition $\Delta v \neq 0$ to $\Delta v \neq \lambda_0 v$ (where $\lambda_0 < \sqrt{N}/4\pi$) and also projecting out these eigenvalues' contribution to W in the causal set.

- Opens the door to addressing many other interesting problems: eg. 4d flat space, de Sitter and black hole entropy in causal set theory, and entropy with nonlocal operators.