Title: The holographic dual to general covariance

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Abstract: One of the defining features of holography is the geometerization of the renormalization group scale. This means that when a quantum field theory is holographically dual to a bulk gravity theory, then the direction normal to the boundary in the bulk (the `radial' direction) is to be interpreted as the energy scale of the dual quantum field theory. So this direction can be seen to be `emergent', and the evolution of bulk fields along this direction is equated with the renormalization group flow of sources or couplings of boundary operators. Given that gravitational theories are generally covariant, this emergent direction must be treated on equal footing as those of the space on which the boundary field theory lives. I will describe the precise integrability condition the renormalization group flow need satisfy which encodes this peculiar response of the quantum field theory under coarse graining so as to respect this property of covariance. In other words, this condition is `dual' to general covariance itself.

The holographic dual to general covariance

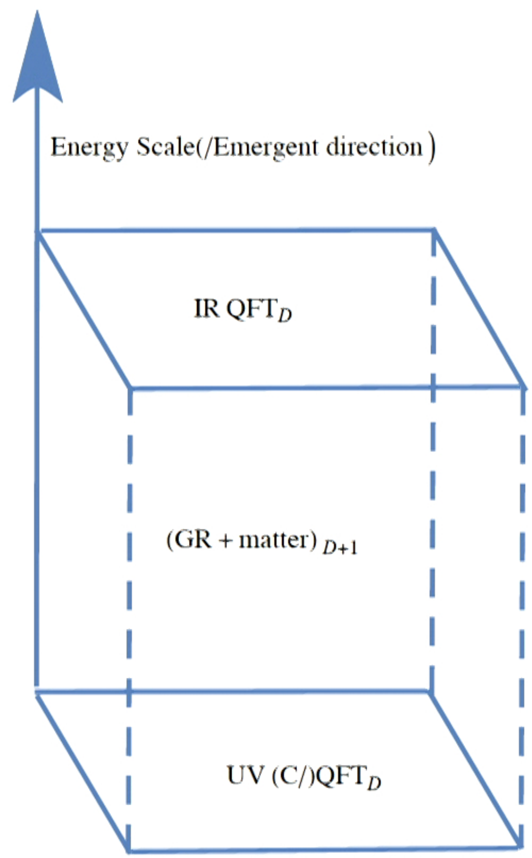
Vasudev Shyam

PI Day

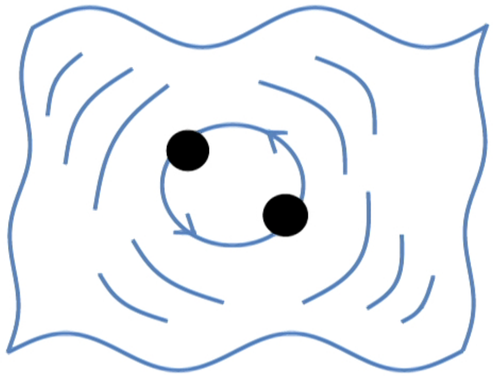
1st June 2017

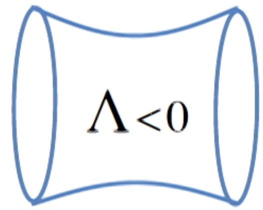
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

A Sketch of Holography



General Relativity



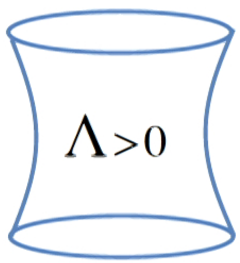


Matter

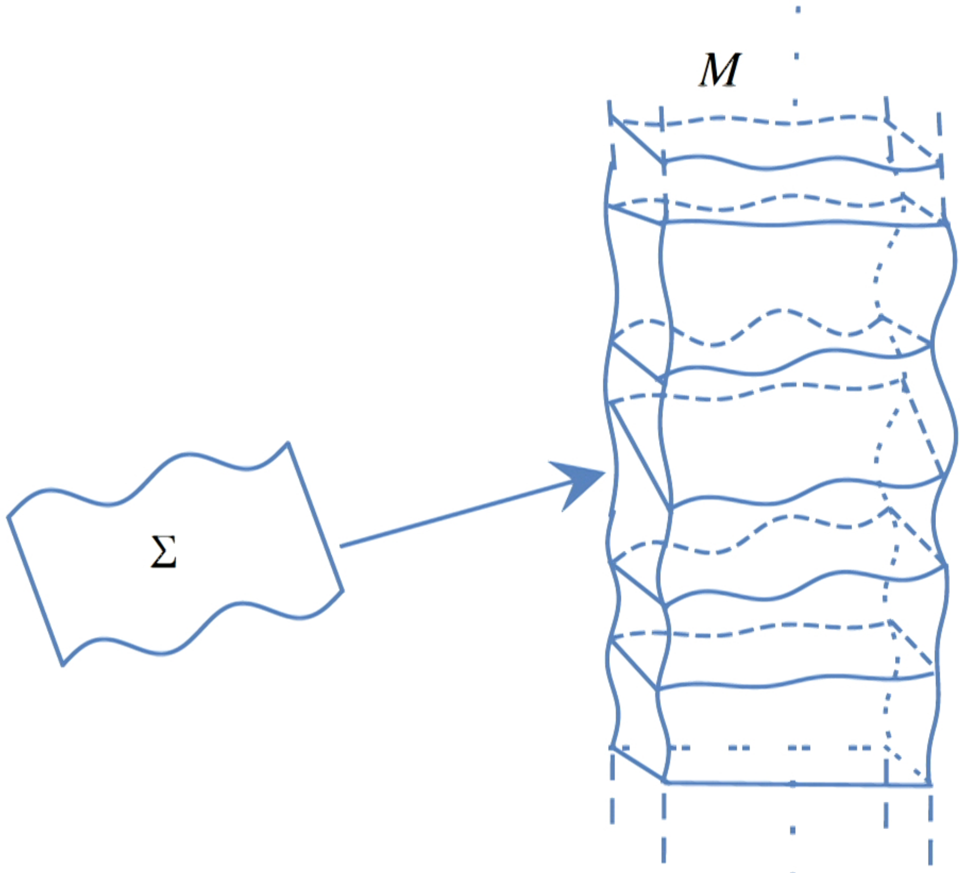
$$R_{\mu\nu} - (1/2)\, g_{\mu\nu} R = 8\pi G_N T_{\mu\nu} - \Lambda g_{\mu\nu}$$

Geometry of Spacetime

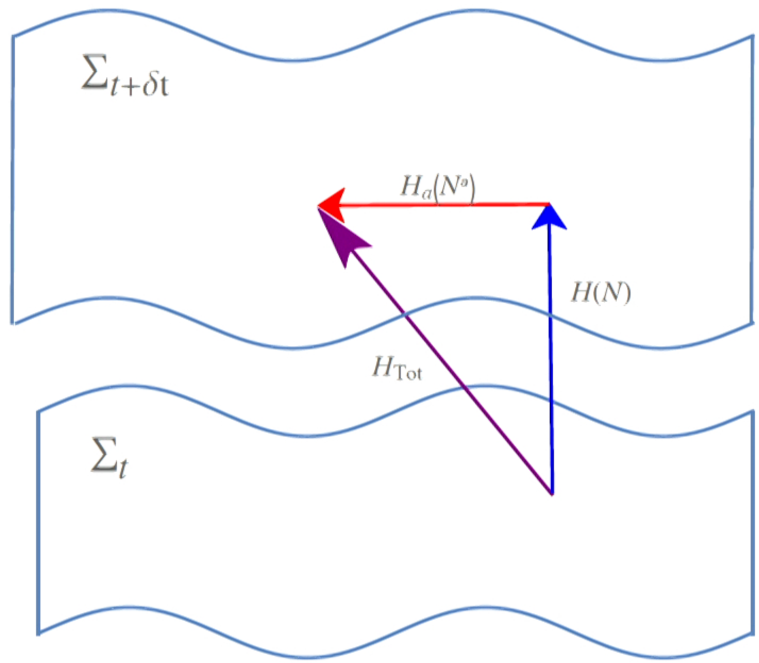
Cosmological Constant



Splitting spacetime into Space and Time



How GR Evolves embedded hypersurfaces



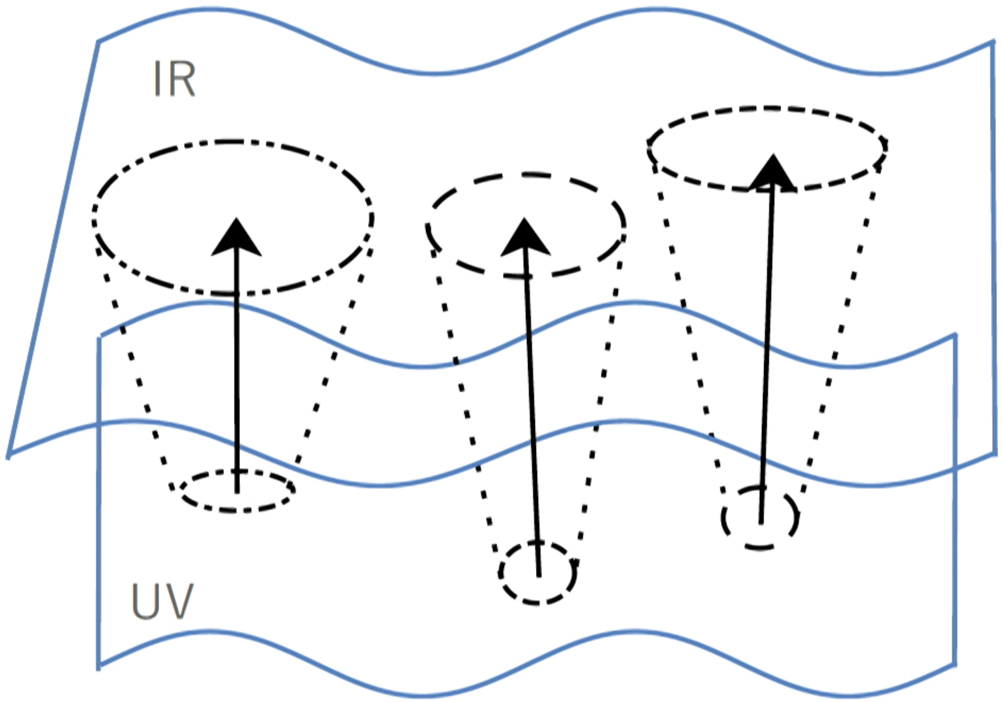
The blocking transformation and freedom in choosing it

- Blocking transformation = Field redefinition $\Phi \to \Phi' = \Phi + \Psi(\Phi)\delta z$

$$Z = \int \mathcal{D}\Phi e^{iS(\Phi)} = \int \mathcal{D}\Phi' e^{iS(\Phi')} = \int \mathcal{D}\Phi e^{iS(\Phi) - \delta z \int_x \frac{\delta S}{\delta \Phi} \cdot \Psi - \frac{\delta \Psi}{\delta \Phi}}$$

- One can perform additional field redefinition $\Phi \to \Phi + \theta(\Phi)$, so that $\Psi \to \Psi + \frac{\mathrm{d}}{\mathrm{d}z}\theta + \left(\Psi \cdot \frac{\delta\theta}{\delta\Phi} - (\Psi \leftrightarrow \theta)\right)$
- How to choose the 'right' blocking transformation to reproduce the action of $H(N), H_a(N^a)$?
- Answer: Fix composition law

Local Holographic RG



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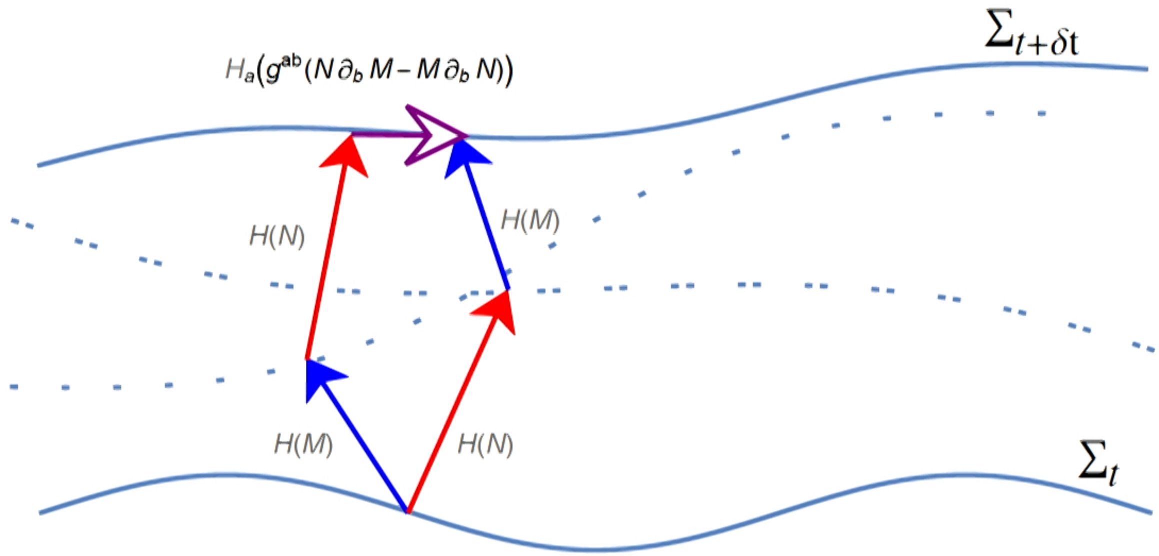
Algebra of Hypersurface Deformations



- Get blocking transformations to mimic this.
- QFTs responding to blocking in this way possess holographic duals.

What defines a Holographic Theory?

- The blocking functional of interest $\Psi_\sigma(\Phi)$ is fixed by

$$[\Psi_\sigma, \Psi_{\sigma'}] = V_{f^\mu(\sigma,\sigma')},$$

where $V_{f^\mu(\sigma,\sigma')}$ generates D dimensional diffeomorphisms, and $f^\mu(\sigma,\sigma') = g^{\mu\nu}(\sigma\partial_\nu\sigma' - \sigma'\partial_\nu\sigma)$ is the generator of said diffeomorphism.

- In terms of partition function Z, the condition we are after is:

$$[\Delta_\sigma, \Delta_{\sigma'}]\ln Z[g] = \int \mathrm{d}^D x\sqrt{g}g^{\mu\nu}(\sigma\partial_\nu\sigma' - \sigma'\partial_\nu\sigma)\langle\nabla_\alpha T^\alpha_\mu\rangle.$$

- A holographic dual can be constructed for any theory which satisfies this property, in addition to possessing a large number of degrees of freedom and a sparse spectrum.

Conclusion

- Effective field theory logic tells us why we should care about general covariance/diffeomorphism invariance if we are interested in theories of dynamical geometry.
- It can be encoded in a composition law for the renormalization group transformations of the dual theory.
- Conversely, the quantum field theories that respond to renormalization group transformations to respect aforementioned composition laws (in addition to satisfying some other conditions) possess holographic duals.

Thank you for your attention!