Title: Entanglement structure and UV regularization in cMERA

Date: Jun 01, 2017 01:30 PM

URL: http://pirsa.org/17060023

Abstract: We give an introduction to cMERA, a continuous tensor networks ansatz for ground states of QFTs. We also explore a particular feature of it: an intrinsic length scale that acts as an ultraviolet cutoff. We provide evidence for the existence of this cutoff based on the entanglement structure of a particular family of cMERA states, namely Gaussian states optimized for free bosonic and fermionic CFTs. Our findings reflect that short distance entanglement is not fully present in the ansatz states, thus hinting at ultraviolet regularization.

Entanglement structure and UV regularization in cMERA

Adrián Franco Rubio

Perimeter Institute for Theoretical Physics
University of Waterloo

PI Day
June 1st, 2017

Outline

1. Intro to cMERA
2. UV regularization
3. Conclusions

Joint work with Guifré Vidal (PI)

What are (discrete) tensor networks?

Ansatz states → Approximations to **quantum many-body states**

Provide:

- An **efficient** representation (with less parameters)

  e.g. $O(N)$ vs. $O(\exp N)$ ($N =$ number of d.o.f)

What are (discrete) tensor networks?

Ansatz states → Approximations to **quantum many-body states**

Provide:

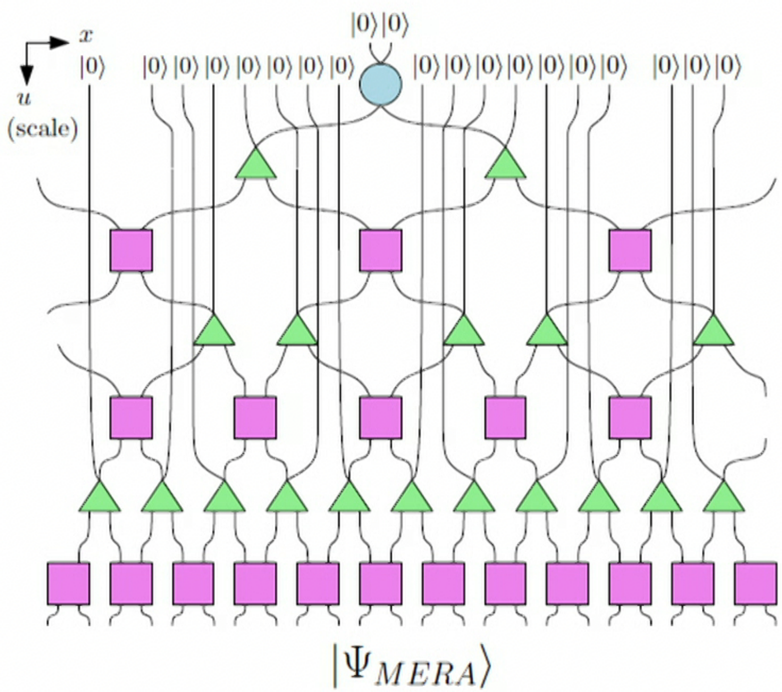
- An **efficient** representation (with less parameters)

  e.g. $O(N)$ vs. $O(\exp N)$ ($N =$ number of d.o.f)

- An **insightful** representation:
  - Diagrammatic notation
  - Clues about entanglement structure
  - Encoded RG flow

The multi-scale entanglement renormalization ansatz (MERA)

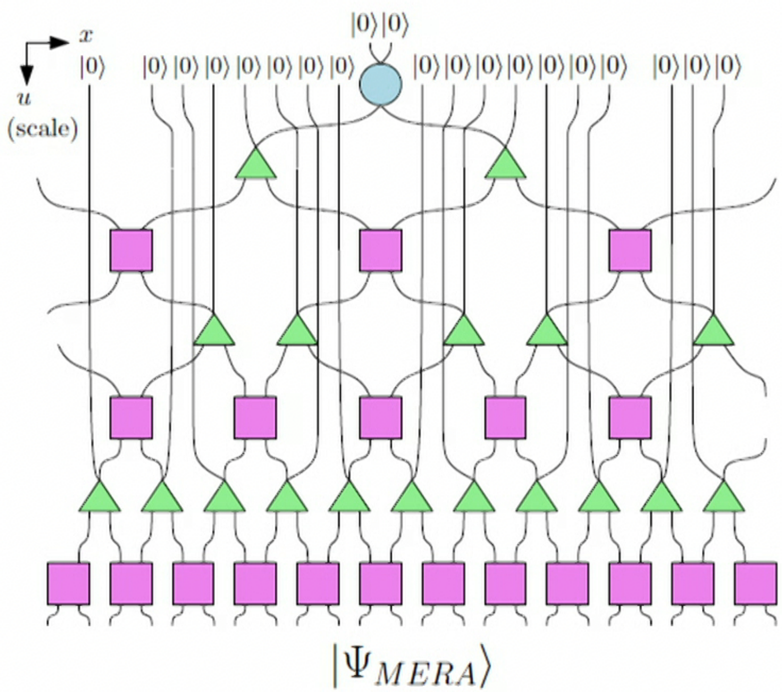
[Vidal, 06]



- Ground states of critical Hamiltonians

The multi-scale entanglement renormalization ansatz (MERA)

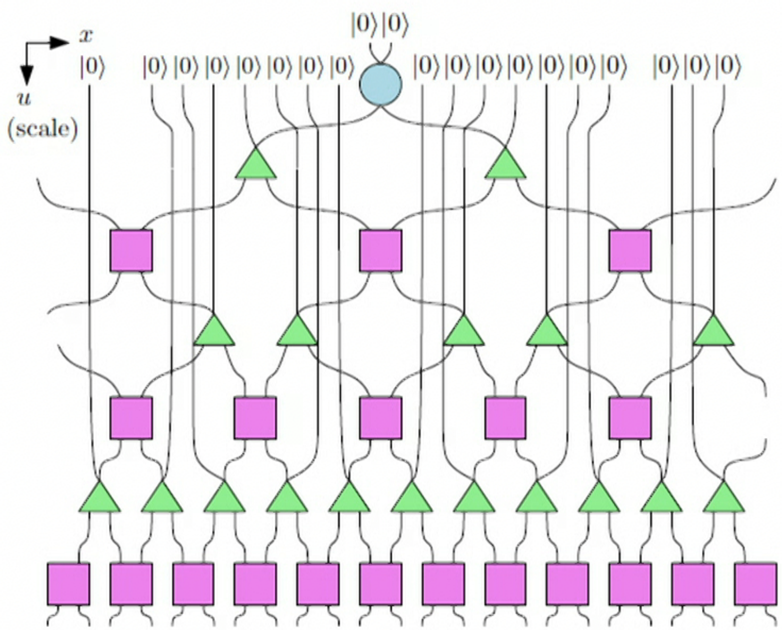
[Vidal, 06]



- Ground states of critical Hamiltonians
- Entangling evolution in scale

The multi-scale entanglement renormalization ansatz (MERA)

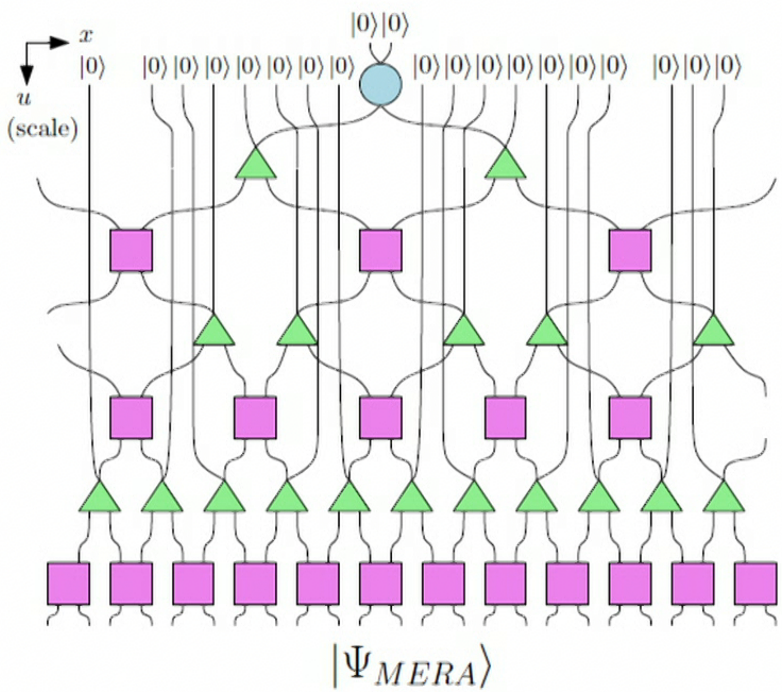
[Vidal, 06]



$|\Psi_{MERA}\rangle$

- Ground states of critical Hamiltonians
- Entangling evolution in scale
- Variational parameters: gates

The multi-scale entanglement renormalization ansatz (MERA)



[Vidal, 06]

- Ground states of critical Hamiltonians
- Entangling evolution in scale
- Variational parameters: gates
- Allows for recovery of CFT data!

What does it mean to have a continuous tensor network?

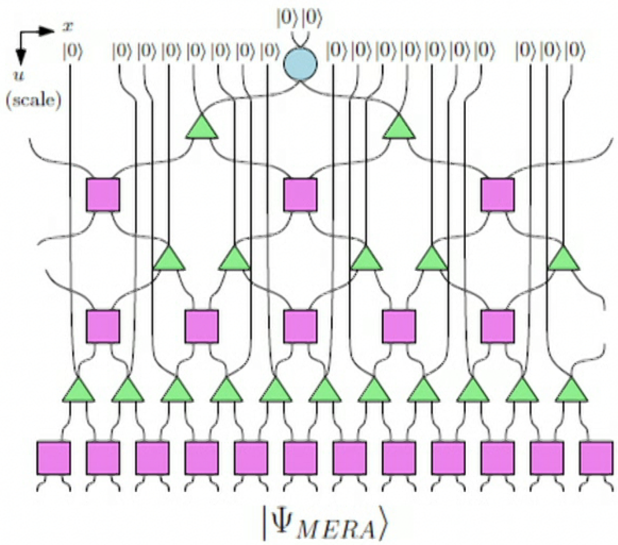
- Quantum lattice system $\rightarrow$ Quantum field theory

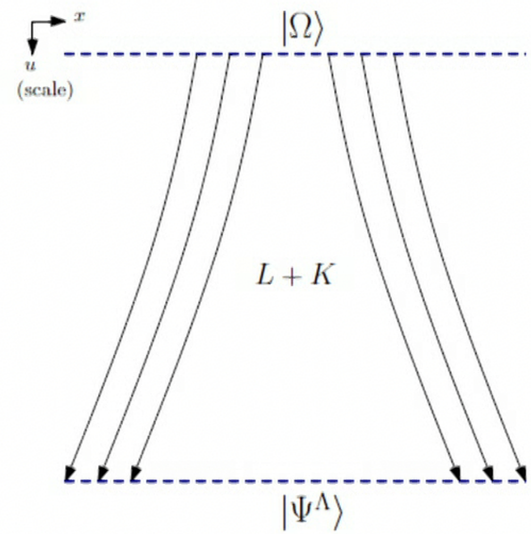
$$(\mathbb{Z} \rightarrow \mathbb{R})$$

- Again: ansatz dependent on a number of parameters.
- Provides approximation to a target state $|\Psi\rangle$. For us: the ground state of a CFT.
- Our chances of making nice diagrams decrease dramatically :(

MERA and cMERA

[Haegeman et al., 11]





$$|\Psi^\Lambda\rangle = U(0,-\infty)\,|\Omega\rangle \qquad U(u_2,u_1) = e^{-i(L+K)u}$$

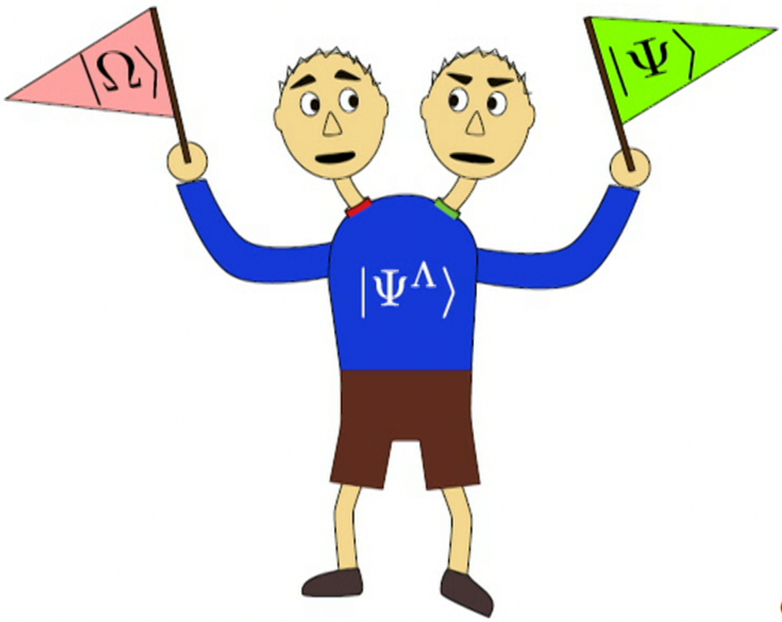
$1/\Lambda$ is a length scale associated with K below which *it doesn't entangle*

Our claim

cMERA states (like $|\Psi^{\Lambda}\rangle$) **interpolate** between the target state $|\Psi\rangle$ (at long distances) and an uncorrelated product state $|\Omega\rangle$ (at short distances), where *long* and *short* should be taken w.r.t. $1/\Lambda$. Hence Λ acts here as an **intrinsic UV cutoff**.

Our claim

cMERA states (like $|\Psi^\Lambda\rangle$) **interpolate** between the target state $|\Psi\rangle$ (at long distances) and an uncorrelated product state $|\Omega\rangle$ (at short distances), where *long* and *short* should be taken w.r.t. $1/\Lambda$. Hence Λ acts here as an **intrinsic UV cutoff**.



cMERAs are chimeras!

An example

[Haegeman et al., 11]

1+1 dimensional free Dirac fermion

$$H = -i \int_{-\infty}^{\infty} dx \left(\psi_1(x)^\dagger \partial_x \psi_2(x) - \psi_2(x)^\dagger \partial_x \psi_1(x)\right)$$

An example

[Haegeman et al., 11]

1+1 dimensional free Dirac fermion

$$H = -i \int_{-\infty}^{\infty} dx\, (\psi_1(x)^\dagger \partial_x \psi_2(x) - \psi_2(x)^\dagger \partial_x \psi_1(x))$$

Entangler:

$$K = -i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx\, dy\, \mu(x-y)[\psi_1(x)^\dagger \psi_2(y) - \psi_2(x)^\dagger \psi_1(y)]$$

$$\mu(x) = \frac{-i\Lambda^2\pi}{4} x e^{-\frac{(\Lambda x)^2}{4}}$$

An example

[Haegeman et al., 11]

1+1 dimensional free Dirac fermion

$$H = -i \int_{-\infty}^{\infty} dx \, (\psi_1(x)^\dagger \partial_x \psi_2(x) - \psi_2(x)^\dagger \partial_x \psi_1(x))$$

Entangler:

$$K = -i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx \, dy \, \mu(x-y)[\psi_1(x)^\dagger \psi_2(y) - \psi_2(x)^\dagger \psi_1(y)]$$

$$\mu(x) = \frac{-i\Lambda^2 \pi}{4} x e^{-\frac{(\Lambda x)^2}{4}}$$

Let's look for evidence of UV regularization in the cMERA!

Gaussian cMERA

- Gaussian states can be characterized by a set of annihilation operators (linear constraints).

Gaussian cMERA

- Gaussian states can be characterized by a set of annihilation operators (linear constraints).

- For our three states ($|\chi\rangle = |\Omega\rangle, |\Psi^\Lambda\rangle, |\Psi\rangle$),

$$[\cos\theta(k)\ \psi_1(k) + \sin\theta(k)\ \psi_2(k)]\ |\chi\rangle = 0$$

$$[-\sin\theta(k)\ \psi_1(k) + \cos\theta(k)\ \psi_2(k)]^\dagger\ |\chi\rangle = 0$$

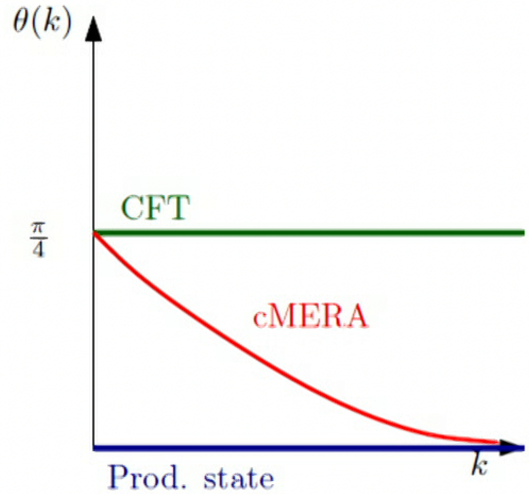
Gaussian cMERA

- Gaussian states can be characterized by a set of annihilation operators (linear constraints).
- For our three states ($|\chi\rangle = |\Omega\rangle, |\Psi^{\Lambda}\rangle, |\Psi\rangle$),

$$[\cos\theta(k)\ \psi_1(k) + \sin\theta(k)\ \psi_2(k)]\ |\chi\rangle = 0$$
$$[-\sin\theta(k)\ \psi_1(k) + \cos\theta(k)\ \psi_2(k)]^{\dagger}\ |\chi\rangle = 0$$

We can parameterize the states by just one function $\theta(k)$!

cMERA for 1+1 Dirac fermion

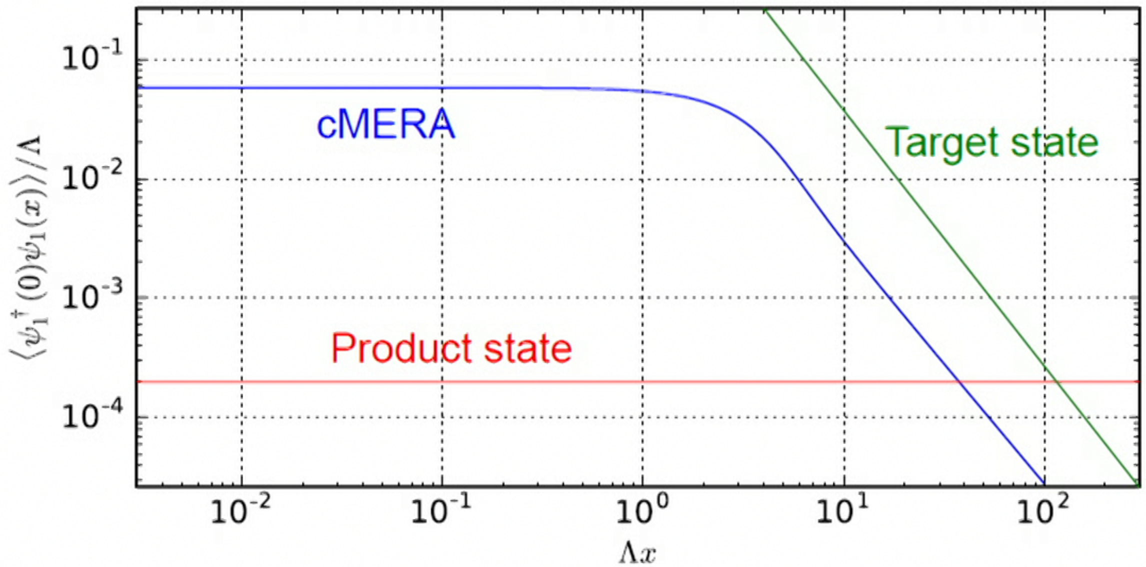


cMERA constraints mimic those of $|\Omega\rangle$ at large momenta, and those of $|\Psi\rangle$ at small momenta!

$$\theta_{CFT}(k) = \pi/4 \qquad \theta_{\Omega}(k) = 0 \qquad \theta(k) = \frac{\pi}{4}\left[1 - \text{erf}\left(\frac{k}{\Lambda}\right)\right]$$

Correlation measures: 2-point function

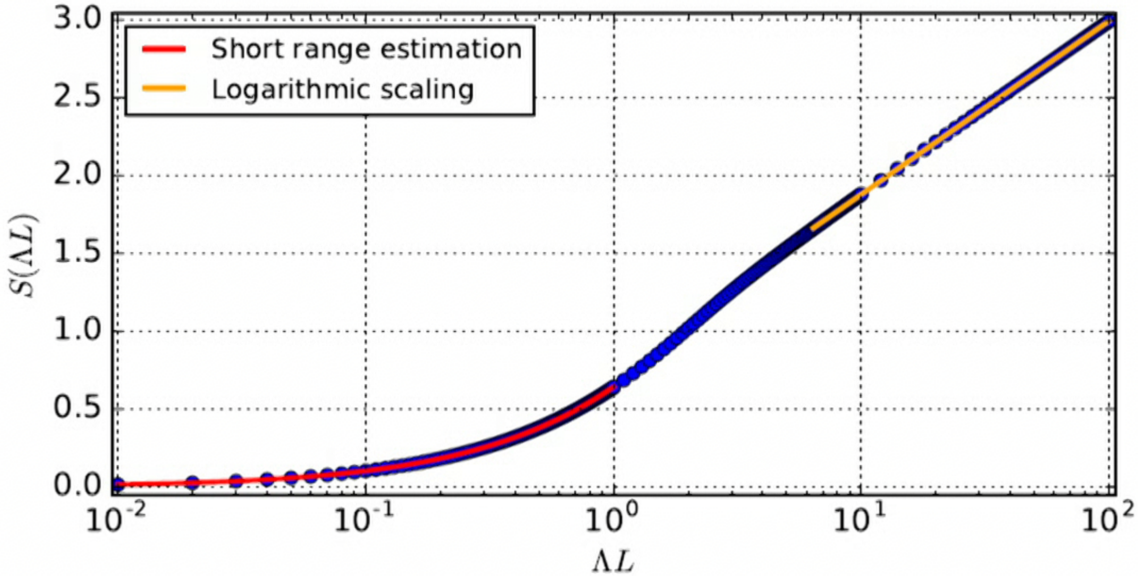
[AFR, Vidal, 17]



Two clearly different regimes: separated by the intrinsic cutoff $1/\Lambda$!
Also: UV divergence removed!

Correlation measures: Entanglement entropy

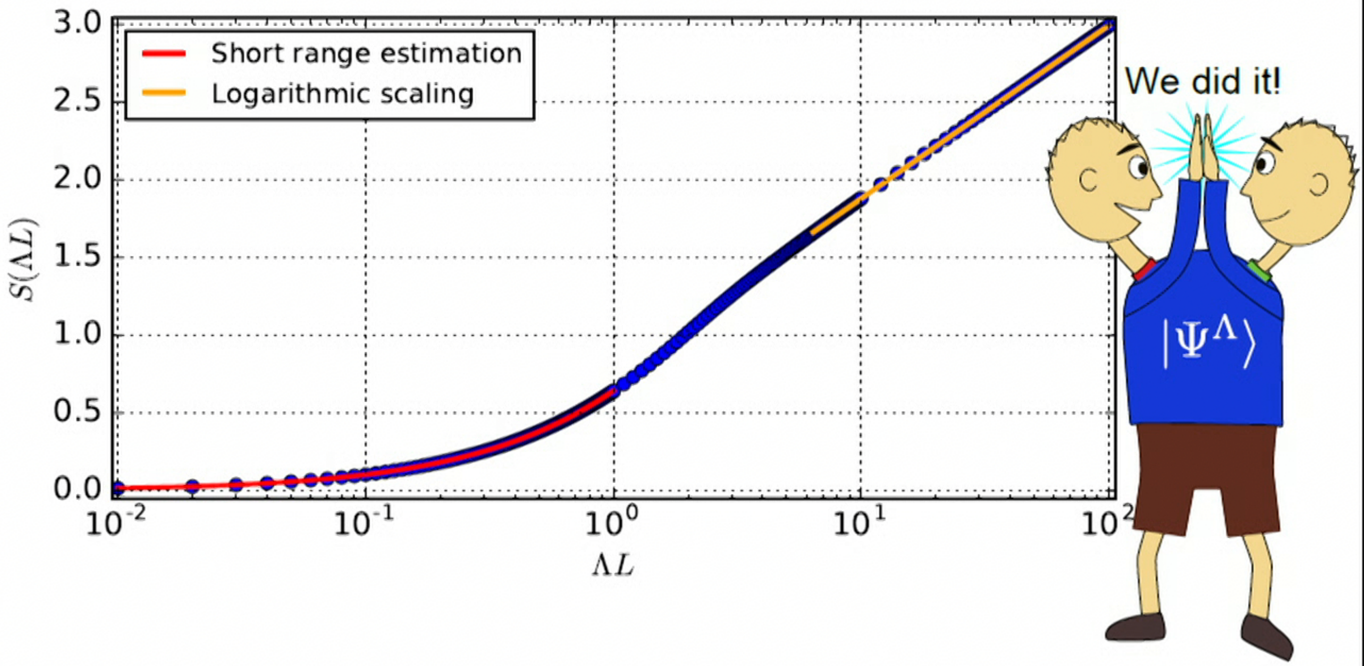
[AFR, Vidal, 17]



Again two different regimes separated by the intrinsic cutoff $1/\Lambda$!
Also: UV divergence removed!

Correlation measures: Entanglement entropy

[AFR, Vidal, 17]



Again two different regimes separated by the intrinsic cutoff $1/\Lambda$!
Also: UV divergence removed!

Summary and outlook

- cMERA states are tensor network inspired variational class for approximations to target QFT states.

Summary and outlook

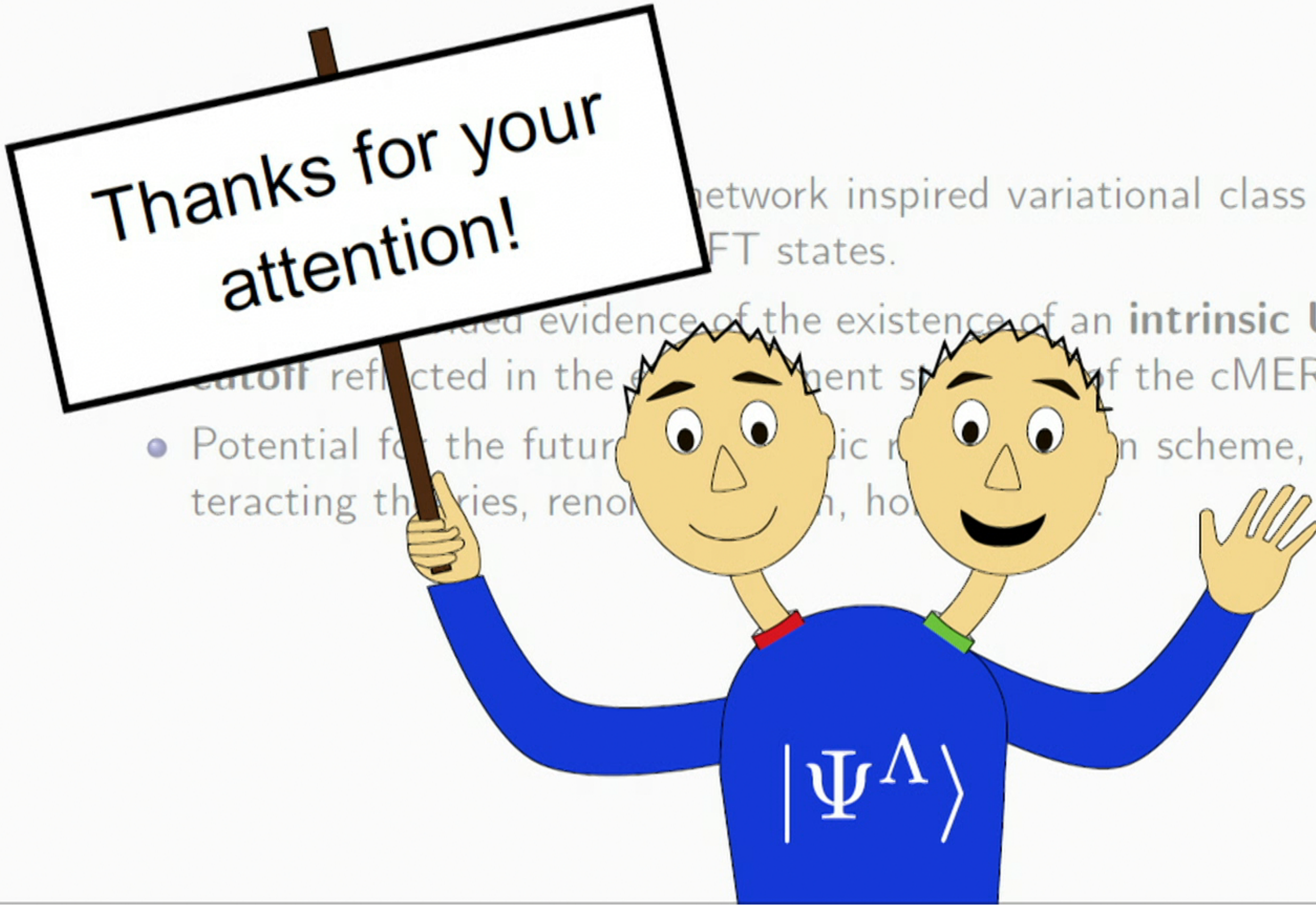
- cMERA states are tensor network inspired variational class for approximations to target QFT states.
- We have provided evidence of the existence of an **intrinsic UV cutoff** reflected in the entanglement structure of the cMERA.

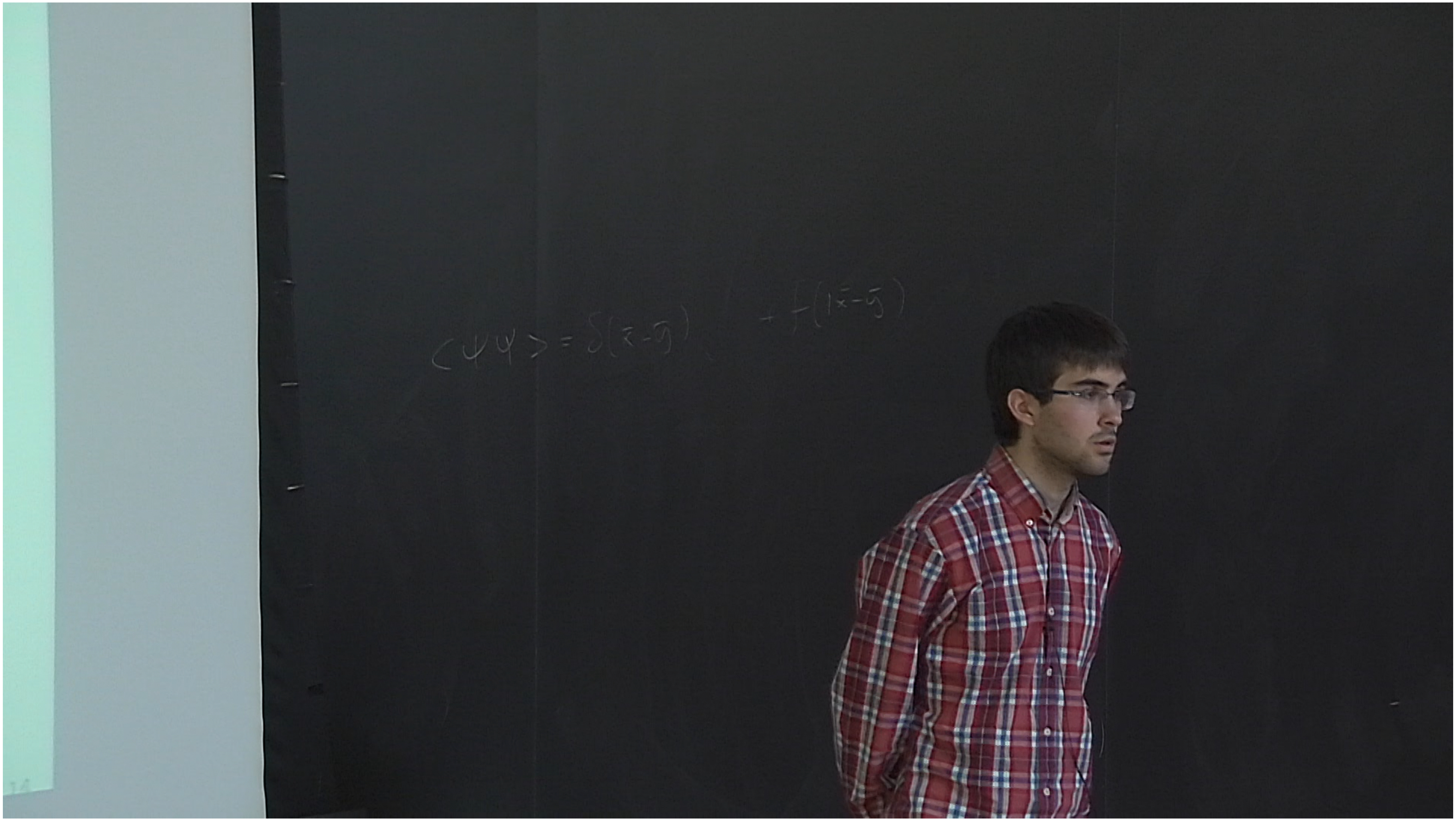
Summary and outlook

- cMERA states are tensor network inspired variational class for approximations to target QFT states.
- We have provided evidence of the existence of an **intrinsic UV cutoff** reflected in the entanglement structure of the cMERA.
- Potential for the future: systematic regularization scheme, interacting theories, renormalization, holography...

Summary and outlook

- …etwork inspired variational class for …FT states.
- …ded evidence of the existence of an **intrinsic UV cutoff** refl…cted in the e… …ent s… …f the cMERA.
- Potential f… the futur… …ic r… …n scheme, interacting th…ries, reno… …n, ho…




<ψψ> = δ(x̄ - ȳ) + f(|x̄ - ȳ|)