Title: Gravity degrees of freedom on a null surface

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Abstract: A canonical analysis for general relativity is performed on a null surface without fixing the diffeomorphism gauge, and the canonical pairs of configuration and momentum variables are derived. Next to the well-known spin-2 pair, also spin-1 and spin-0 pairs are identified. The boundary action for a null boundary segment of spacetime is obtained, including terms on codimension two corners.
FH, Laurent Freidel arXiv:1611.03096, Phys. Rev. D 95, 104006 (2017)

Gravity Degrees of Freedom on a Null Surface

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$\hat{\text{PI}}$ Day

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FH and L. Freidel, "Gravity Degrees of Freedom on a Null Surface,"
Phys. Rev. D **95**, no. 10, 104006 (2017) [arXiv:1611.03096 [gr-qc]].

Why Study Degrees of Freedom on Null Surface?

Hamiltonian particle mechanics: canonical pairs (q^i, p_i) associated to instant of time.
Hamiltonian GR: $(Q^i(x), P_i(x))$ associated to 3D hypersurface B.

Here: B null hypersurface. What are (Q, P) in terms of the metric?

= Analogue of the ADM pair $(h_{ab}, K^{ab} - h^{ab}K)$ for null.
$\rightarrow$ Why null? Flow of information, simple. Initial value problem, black holes, horizons, null infinity

Here: Don't fix diffeo gauge.
$\rightarrow$ Why not? Black hole hair/BMS: non–gauge diffeos. Geometrical intuition.

Here: B has a 2D boundary, keep all terms.
$\rightarrow$ Why? Charges (=symmetry generators) live in 2D $\rightarrow$ black hole thermo, null surface thermo, boundary & corner action.

Outline

- Method: Covariant Hamiltonian Formalism, Geometry
- Null Canonical Pairs
- Null Boundary Action
- Outlook & Summary

Covariant Hamiltonian Field Theory

- **Hamiltonian GR**: $(Q(x), P(x))$ with null hypersurface B and $x \in B$. What are (Q, P) in terms of the metric and its derivatives?
- How to get Q's and P's from Lagrangian?
  Pre–symplectic potential Θ defined through $\delta L = -E + \mathrm{d}\Theta$.

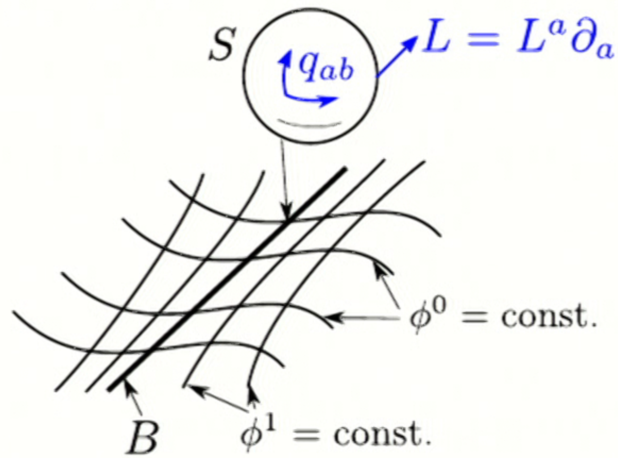
  Θ is schematically:

$$\int_B \Theta = \underbrace{\int_B P\delta Q}_{\text{Bulk canonical structure}} + \underbrace{\int_{\partial B} p\delta q}_{\text{Corner can. structure}} + \delta\Big(\underbrace{\int_B A + \int_{\partial B} a}_{\text{Boundary action}} \Big)$$

- Make **choice of polarization**: Q, q contain no derivatives of the metric (Dirichlet boundary conditions).

Geometry

Metric on B is degenerate, no inverse.
$\rightarrow$ Foliate B into spacelike surfaces S.



- Null normal (=tangential) vector field L to B, normalized as $L^a \partial_a \phi^0 = 1$.
- Induced geometry of B is captured by (q_{ab}, L^a).
- Only restriction on metric: B is null. **No gauge fixing**.

Strategy

Symplectic potential for vacuum Einstein gravity with cosmological constant:

$$\int_B \Theta = \int_B \Big(L^a \nabla^b \delta g_{ab} - \nabla_L (g^{ab}\delta g_{ab})\Big) \mathrm{d}B = \int_B P \delta Q + \ldots$$

Problem: Want to read off (Q, P), but $\int_B \Theta$ contains *derivatives of variations*.

Goal: Get rid of derivatives of variations.

Strategy:

- *Split derivatives* of variations into tangential and transverse to B
- *Integrate by parts* to absorb tangential derivatives $\rightarrow$ terms on ∂B
- **Vary by parts** to absorb transverse derivatives $\rightarrow$ total variation

Null Canonical Pairs: Results

Spin 2:

Conformal metric on S: — *Conformal shear density*

$\tilde{q}_{ab}$ — $\tilde{\sigma}^{ab}$

2D shape — change of shape along L

Known from null initial data, radiative modes

(Sachs 1962, Ashtekar Streubel 1981, Reisenberger).

Spin 1:

Normal vector to B — *Twist density*

L^a — $\bar{\eta}_a$

Displacement of S — change of orientation of S along L

$\bar{\eta}$ appears as momentum in membrane paradigm (Damour 82)

Related to angular momentum of isolated horizons (Ashtekar et al)

Spin 0:

Volume element on S — *Expansion and surface gravity dens.*

$\sqrt{q}$ — $\frac{D-3}{D-2}\theta + \kappa$

Area — change of area and acceleration of L

Non–expanding ($\theta = 0$): black hole thermo $T\delta S$. Parikh, Wilczek 1998

Corner: (suppressing some)

Volume element on S:	*Redshift factor*
$\sqrt{q}$	$1 + h$
Area	size of normal geometry
	$h = \ln(\sqrt{g}/\sqrt{q})$

Reisenberger 2013, Booth 2001

We find: Complete list of null pairs from canonical analysis, no gauge fixing.

Gravity on a null surface has
spin 2, spin 1, spin 0 and boundary DoF!

(Previous related work: Parattu et al. 2016, I. Booth 2001)

Null Boundary Action

What is the action for a space–time with a null boundary segment?

Strategy: We decomposed

$$\int_B \Theta = \int_B P\delta Q + \int_{\partial B} p\delta q + \delta\Big(\int_B A + \int_{\partial B} a\Big).$$

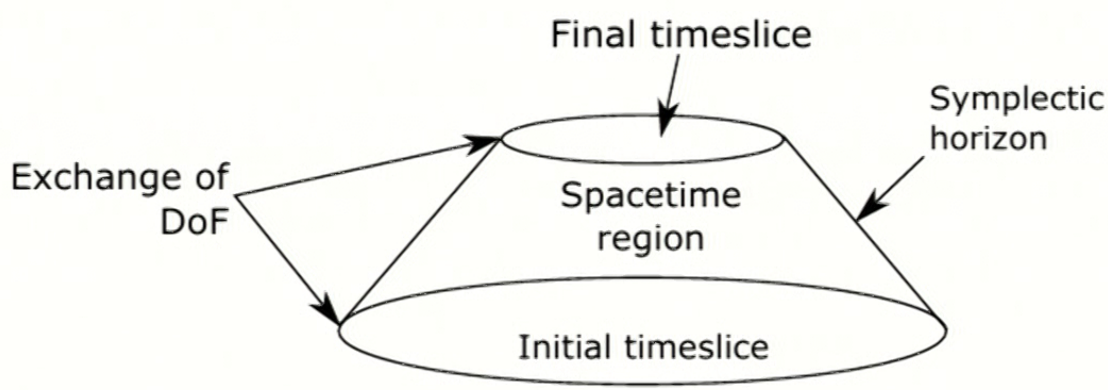
$\delta(\ )$ can be cancelled by setting $S = \int_M L - \int_B A - \int_{\partial B} a$.

Result: $S = \int_M L \boxed{-\int_B \kappa \, \mathrm{d}B - \frac{1}{2}\int_{\partial B} (1-h)\mathrm{d}S}$

Surface gravity κ = acceleration of L
Redshift factor h = size of normal geometry
Agrees with Lehner et al 2016 from a different analysis

Outlook: Boundary Degrees of Freedom

- **Gauge fixing:** $\rightarrow$ Pick up spin-1 and spin-0 diffeo DoF on corner ∂B. They neatly combine with metric corner DoF!
  $\rightarrow$ soft gravitons?
- Define **symplectic horizons**: All degrees of freedom live on ∂B.
  $\rightarrow$ expanding horizons *à la* isolated horizons, their thermodynamics?



Summary

- We found the **null canonical pairs of gravity**
- They contain **spin 2** (well–known), **spin 1** (horizon–fluid analogy), **scalar** (black hole thermodynamics) and **corner** degrees of freedom
- We obtained the **null boundary and corner action**

Thank you!

Canonical Pairs:

Spin 2:	*Conformal metric* on S: $\tilde{q}_{ab}$	*Conformal shear* $\tilde{\sigma}^{ab}$
Spin 1:	*Normal vector* to B L^a	*Twist* $-\bar{\eta}_a$
Spin 0:	*Volume element* on S $\sqrt{q}$	*Expansion and surface gravity* $-\left(\frac{D-3}{D-2}\theta + \kappa\right)$
Corner:	*Volume element* on S $\sqrt{q}$	*Redshift factor* $1 + h$

Boundary Action:

$$S = \int_M L - \int_B \kappa \mathrm{d}B - \frac{1}{2} \int_{\partial B} (1-h) L^a \mathrm{d}_a S$$