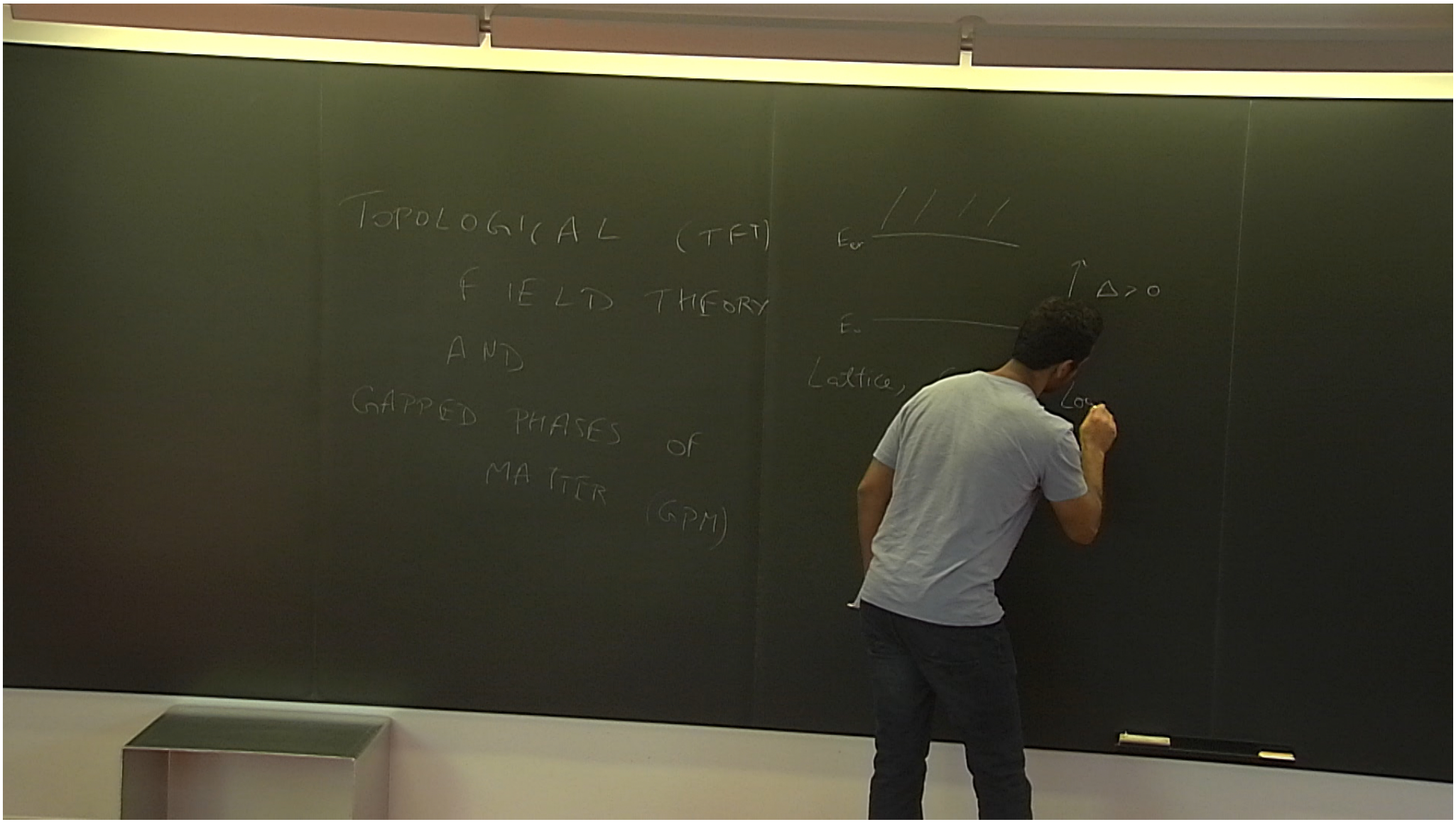
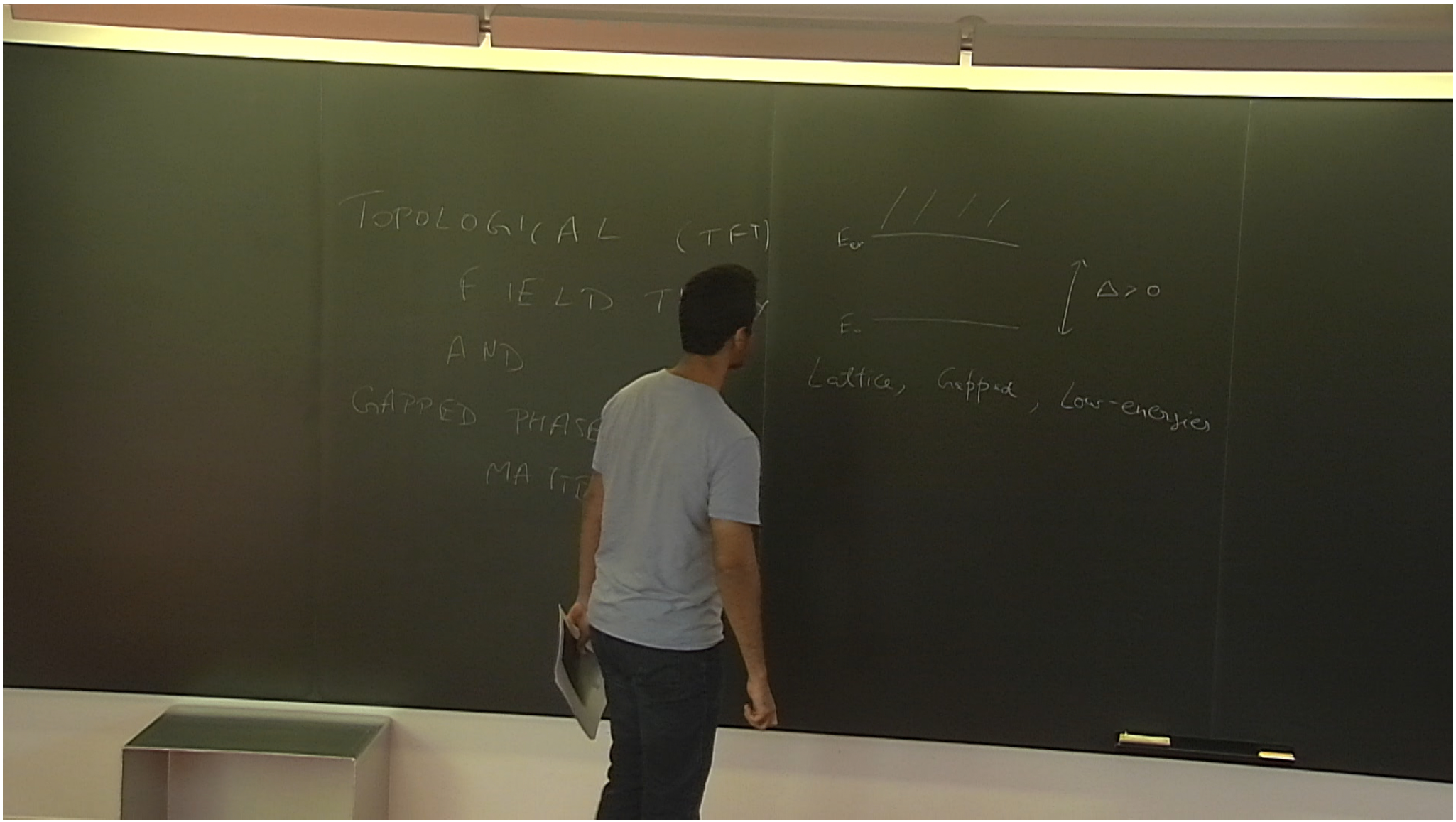
Title: Topological field theories and gapped phases of matter

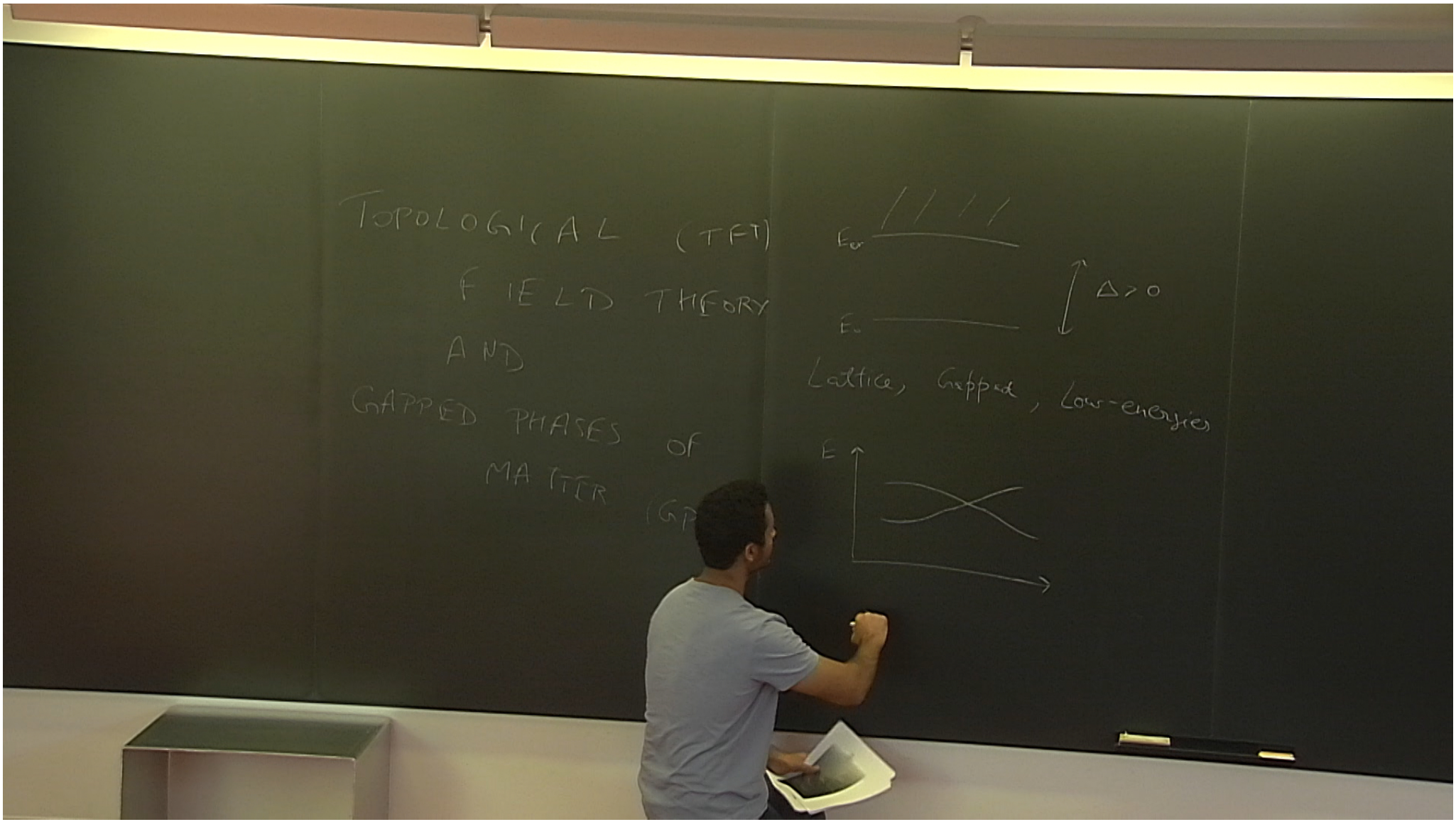
Date: Jun 01, 2017 11:20 AM

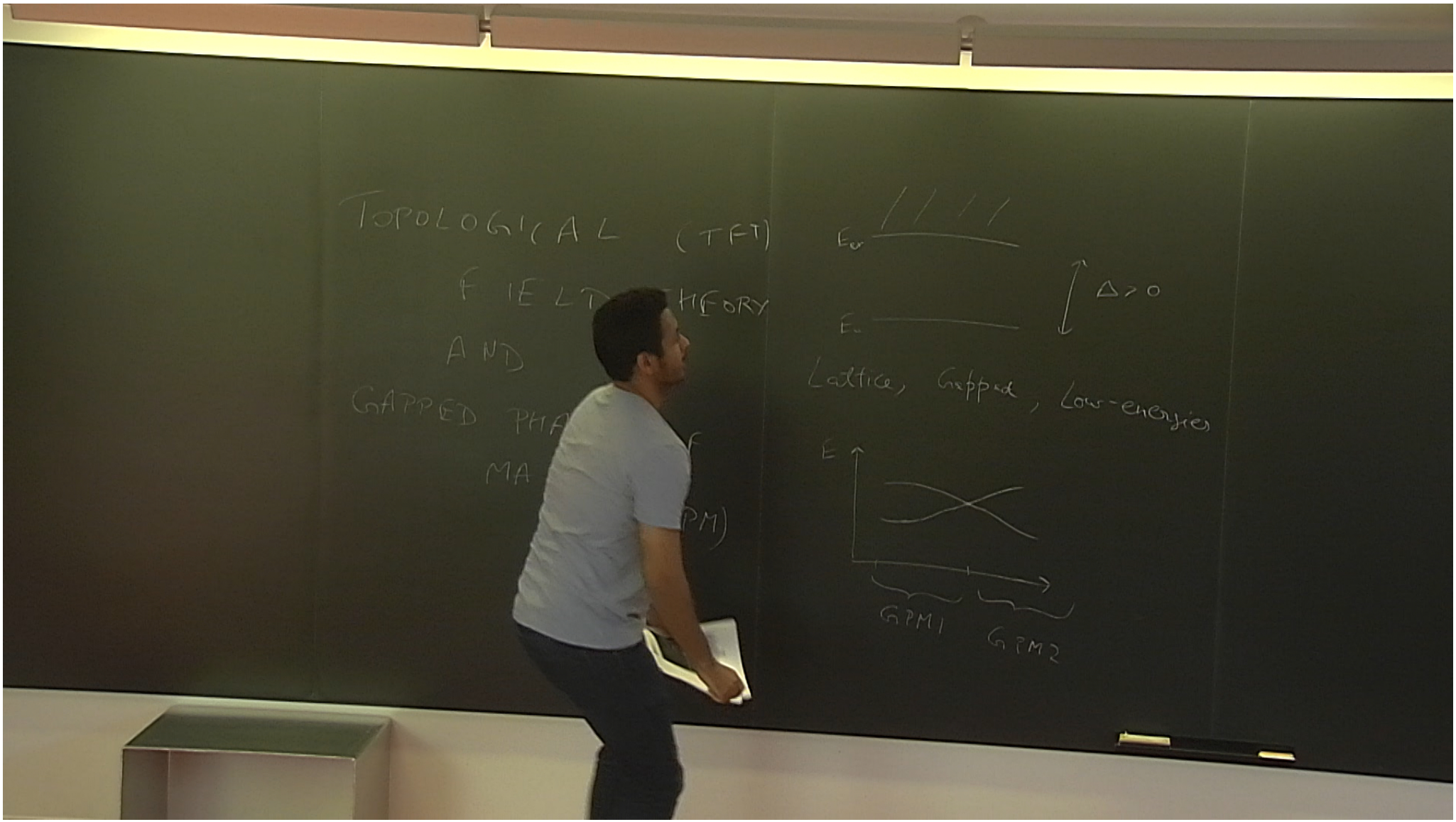
URL: http://pirsa.org/17060021

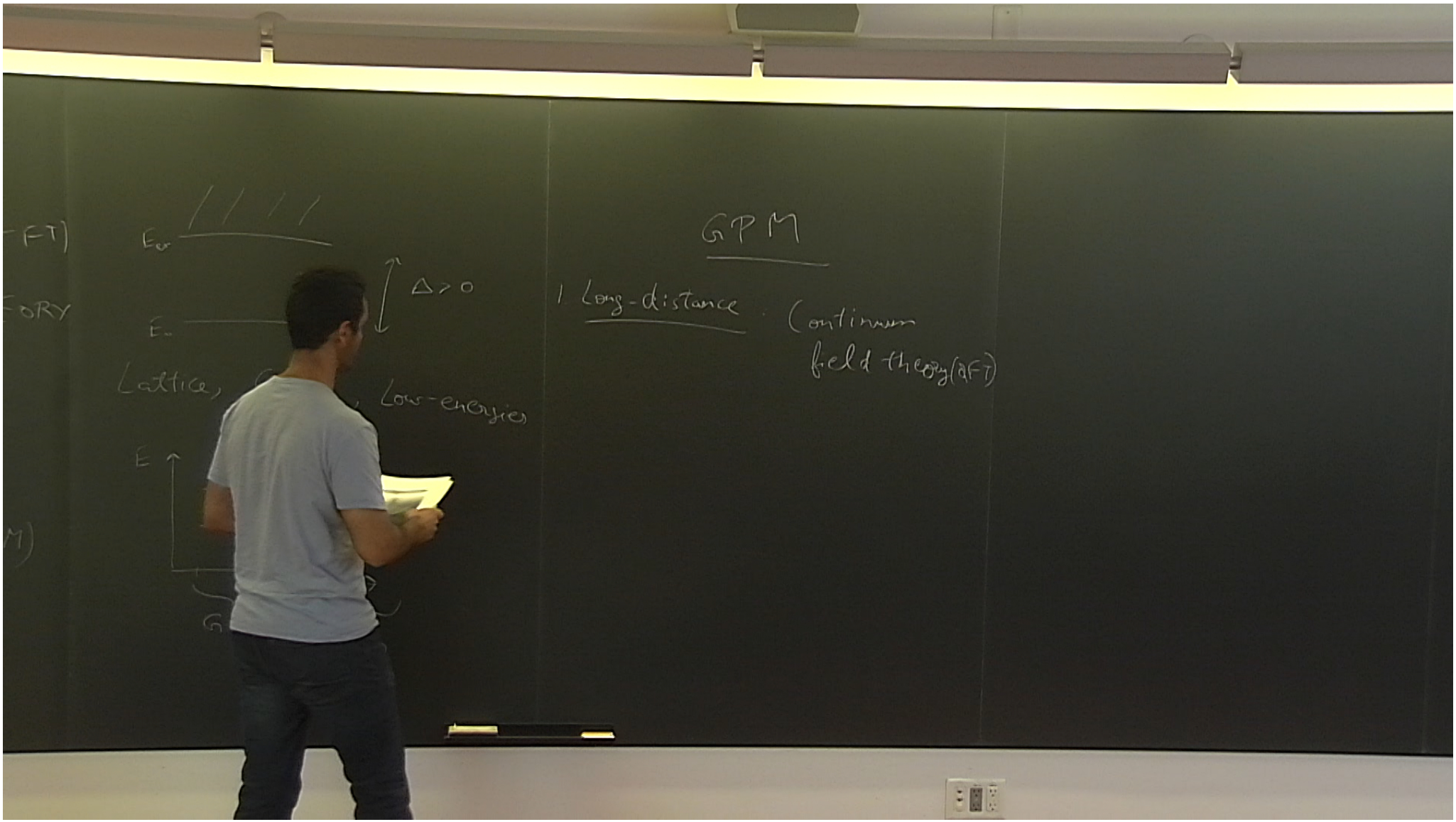
Abstract: I will introduce the idea that topological field theories describe the low-energy properties of gapped local quantum systems. This idea has proved fruitful in recent studies of gapped phases of matter.

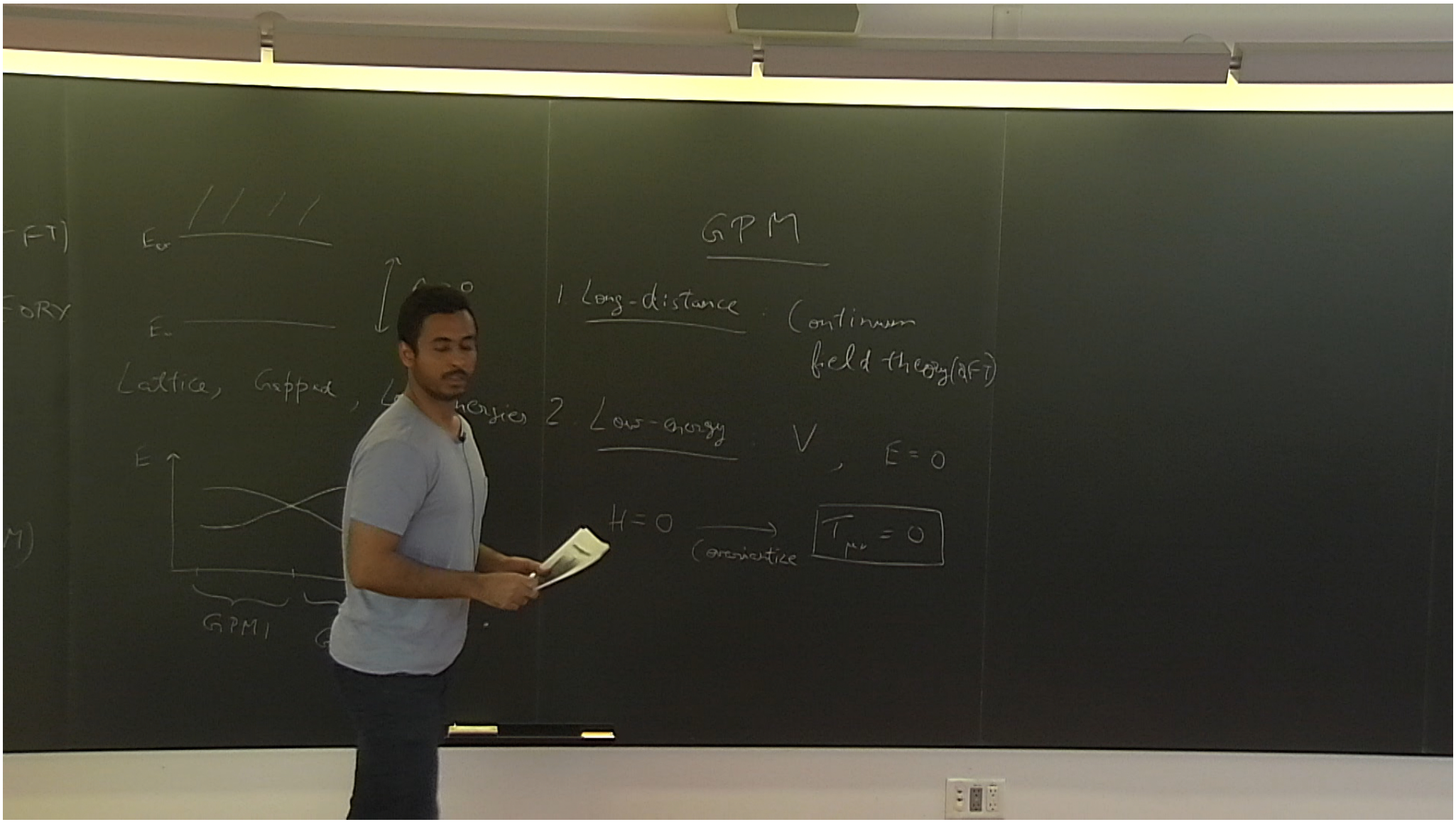

TOPOLOGICAL (TFT)
FIELD THEORY
AND
GAPPED PHASES of
MATTER (GPM)
Δ > 0
Lattice,

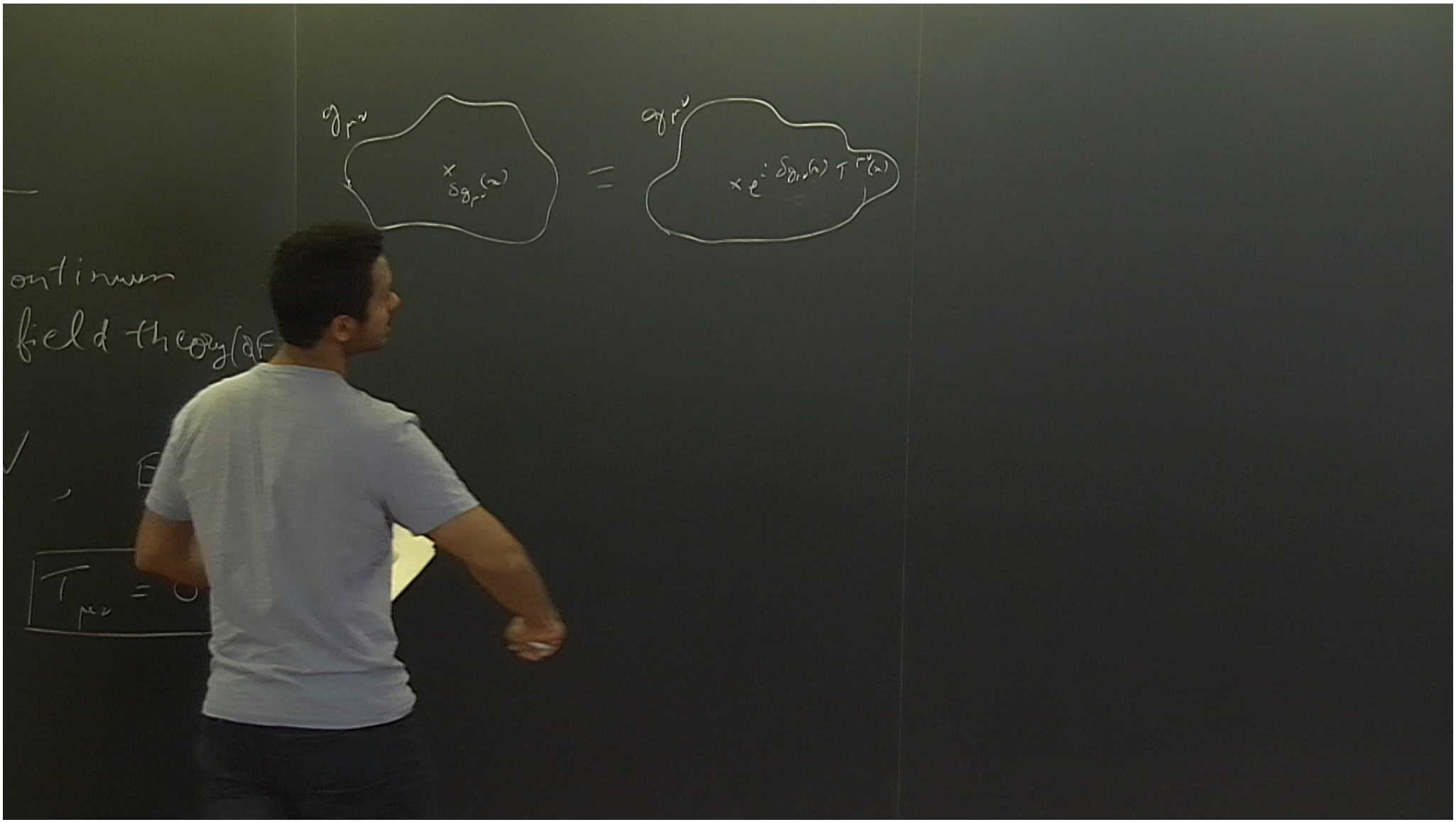

TOPOLOGICAL (TFT)
FIELD T
AND
GAPPED PHASE
MATT
Δ > 0
Lattice, Gapped, Low-energies

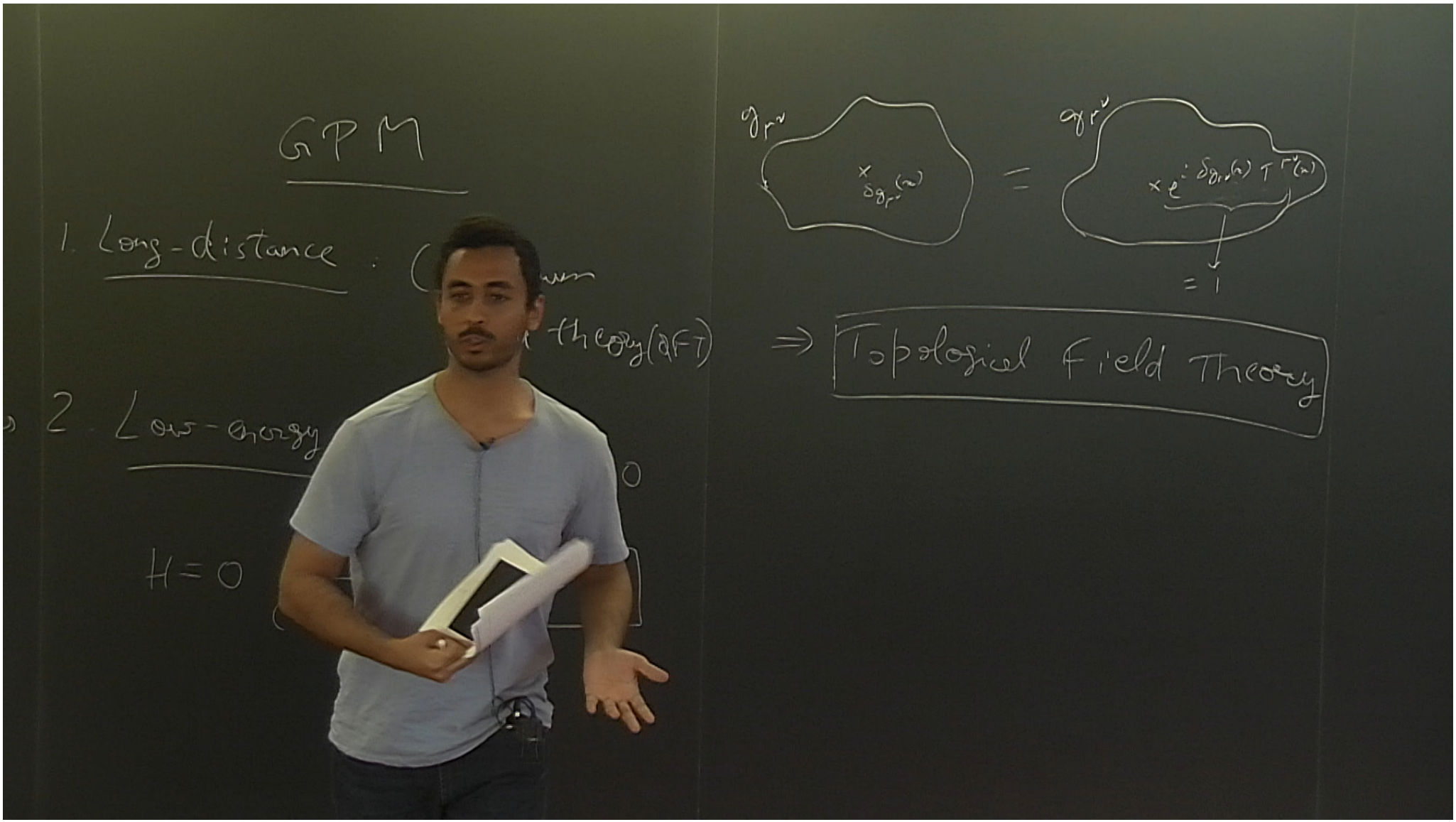

TOPOLOGICAL (TFT)
FIELD THEORY
AND
GAPPED PHASES of
MATTER (GP
Δ > 0
Lattice, Gapped, Low-energies
E


TOPOLOGICAL (TFT)
FIELD THEORY
AND
GAPPED PHA
MA
PM)
Δ > 0
Lattice, Gapped, Low-energies
E
GPM1
GPM2


GPM
1. Long-distance
Continuum
field theory (QFT)
Δ > 0
Lattice,
Low-energies

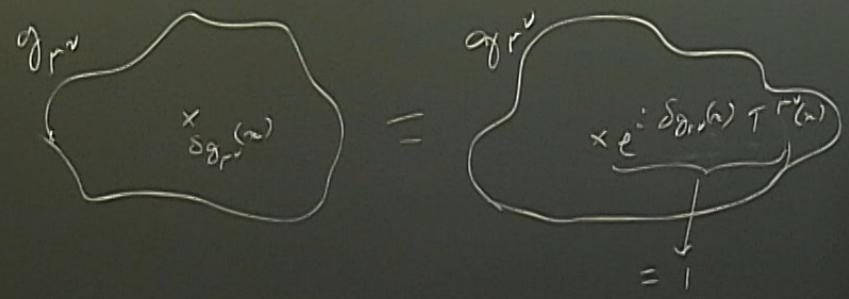

GPM
1. Long-distance : Continuum field theory (QFT)
2. Low-energy : V , E = 0
H = 0
Covariantize
$T_{\mu\nu} = 0$
Lattice, Gapped
GPM1


$g_{\mu\nu}$
$\times\ \delta g_{\mu\nu}(x)$
=
$g_{\mu\nu}$
$\times\ e^{i\,\delta g_{\mu\nu}(x)\,T^{\mu\nu}(x)}$
ontinuum
field theory (QF
$T_{\mu\nu} = 0$


GPM
1. Long-distance
theory (QFT)
2. Low-energy
H = 0
Topological Field Theory

GPM

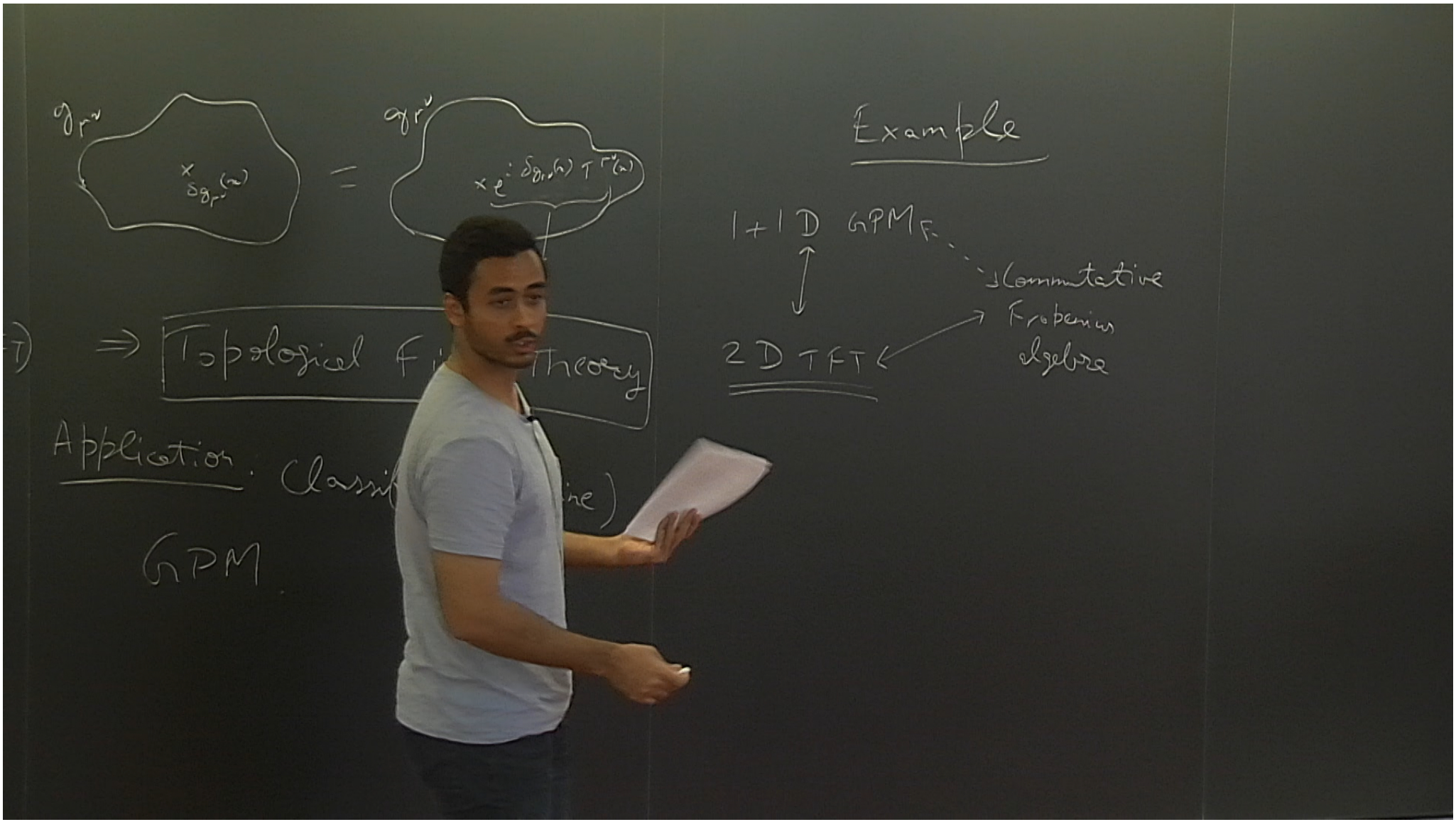
1. Long-distance : Continuum field theory (QFT) $\Rightarrow$ $\boxed{\text{Topological Field Theory}}$

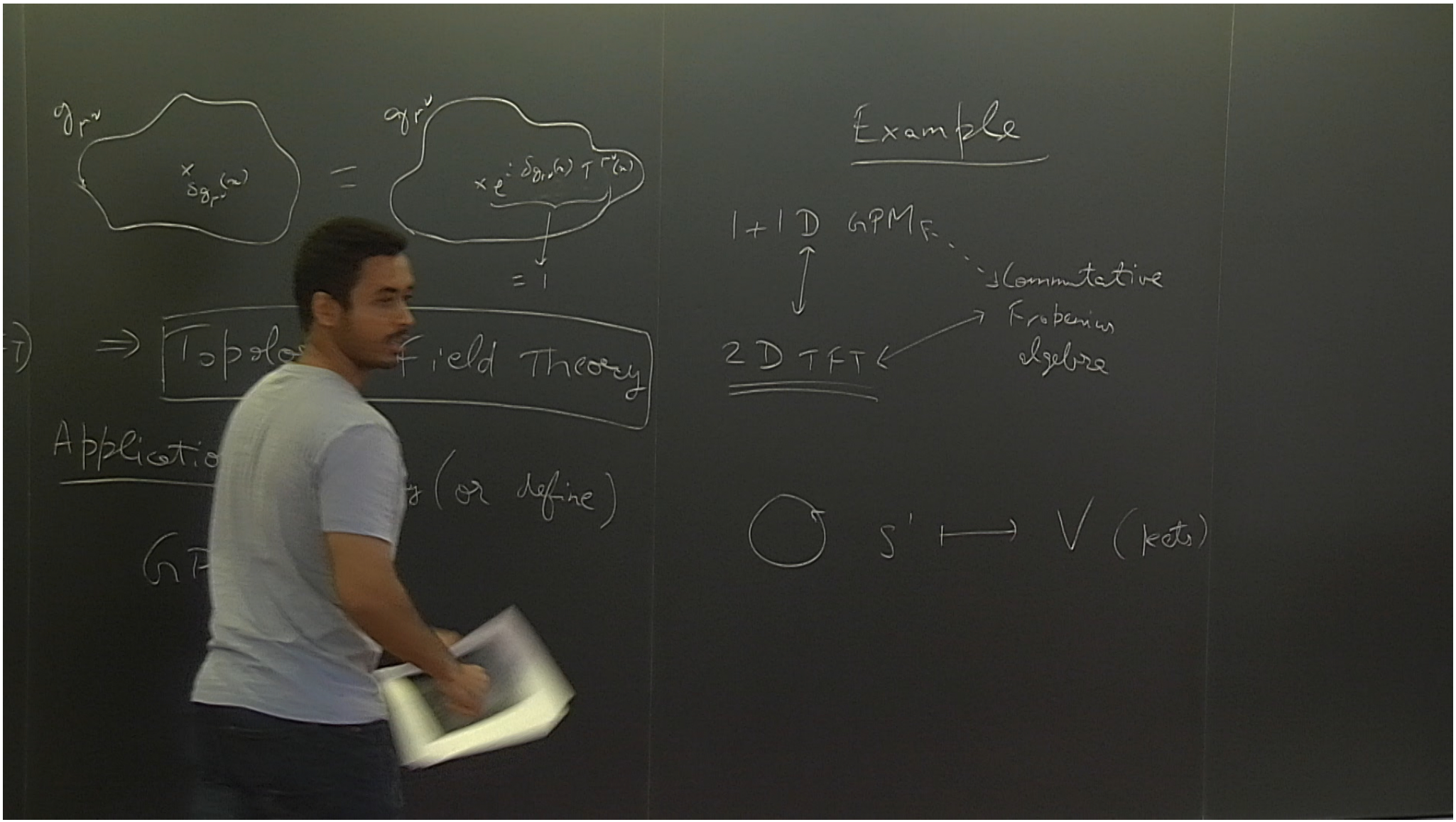


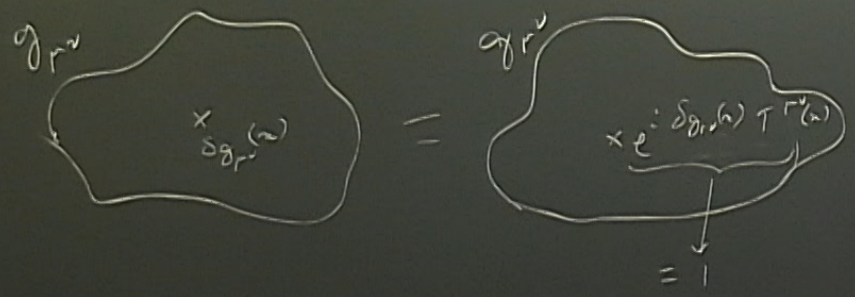
2. Low-energy : V, $E = 0$

$H = 0 \xrightarrow[\text{Covariantize}]{} \boxed{T_{\mu\nu} = 0}$

Application : Classify (or define) GPM.


Example
1+1 D GPM_F
commutative Frobenius algebra
2D TFT
Topological Fi Theory
Application: Classif ine)
GPM


Example
1+1 D GPM_F
commutative Frobenius algebra
2D TFT
S^1 ⟼ V (kets)
Topolo Field Theory
Applicati
(or define)



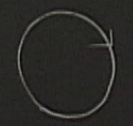
$\Rightarrow$ $\boxed{\text{Topological Field Theory}}$

Application: Classify (or define) GPM

Example

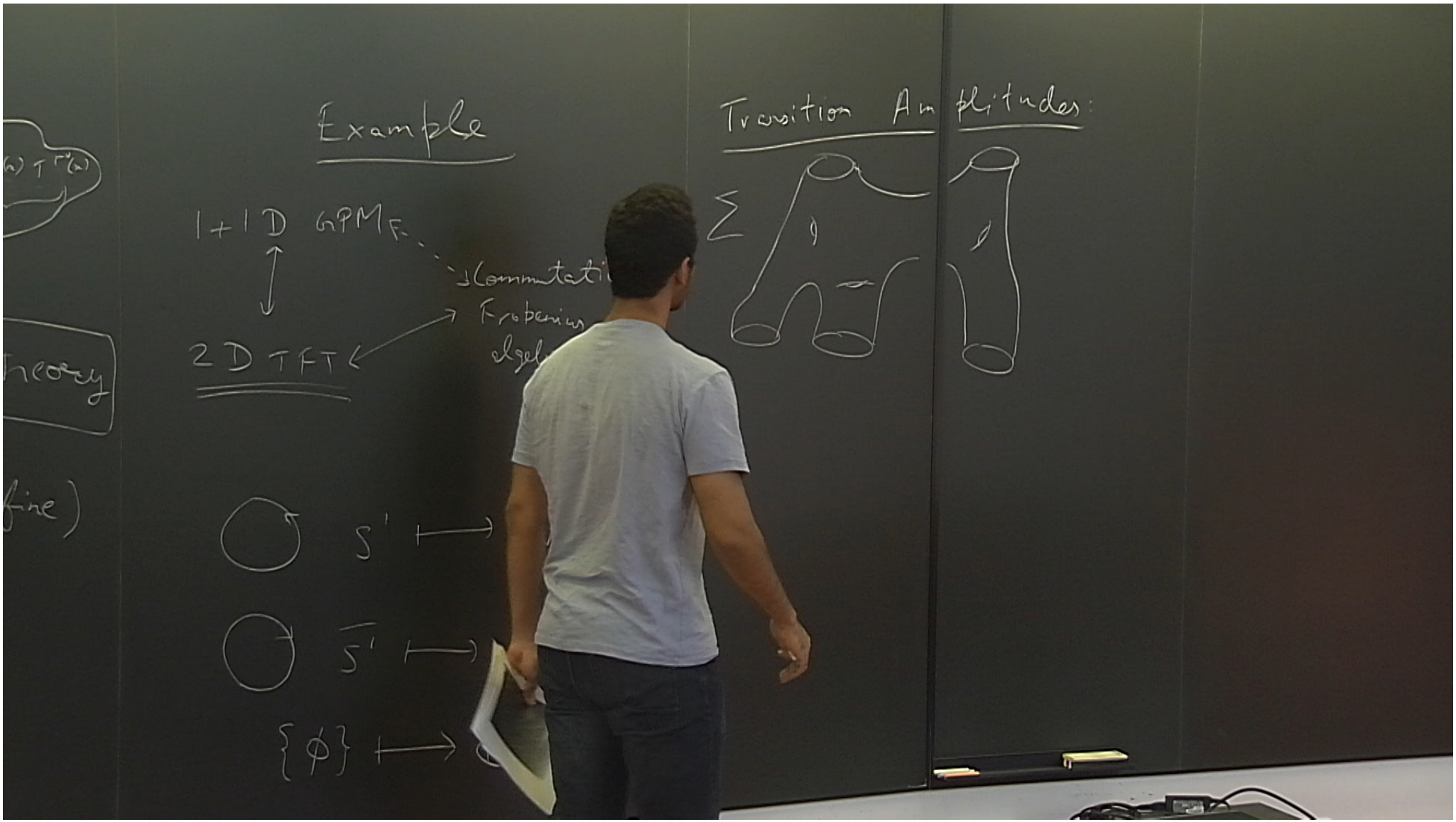
1+1 D GPM$_F$ $\longleftrightarrow$ 2D TFT $\longleftrightarrow$ Commutative Frobenius algebra

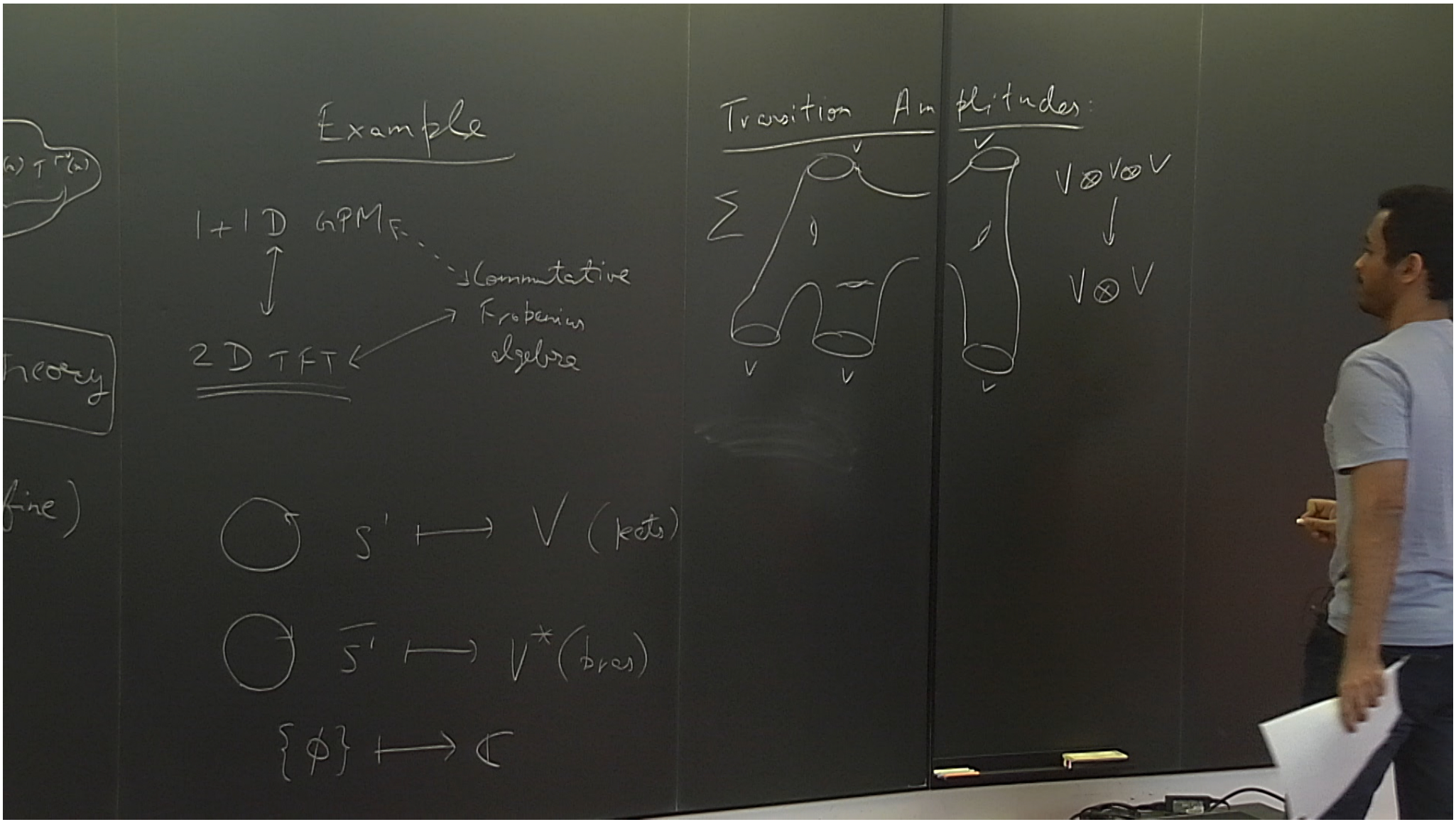
$S^1 \longmapsto V$ (kets)



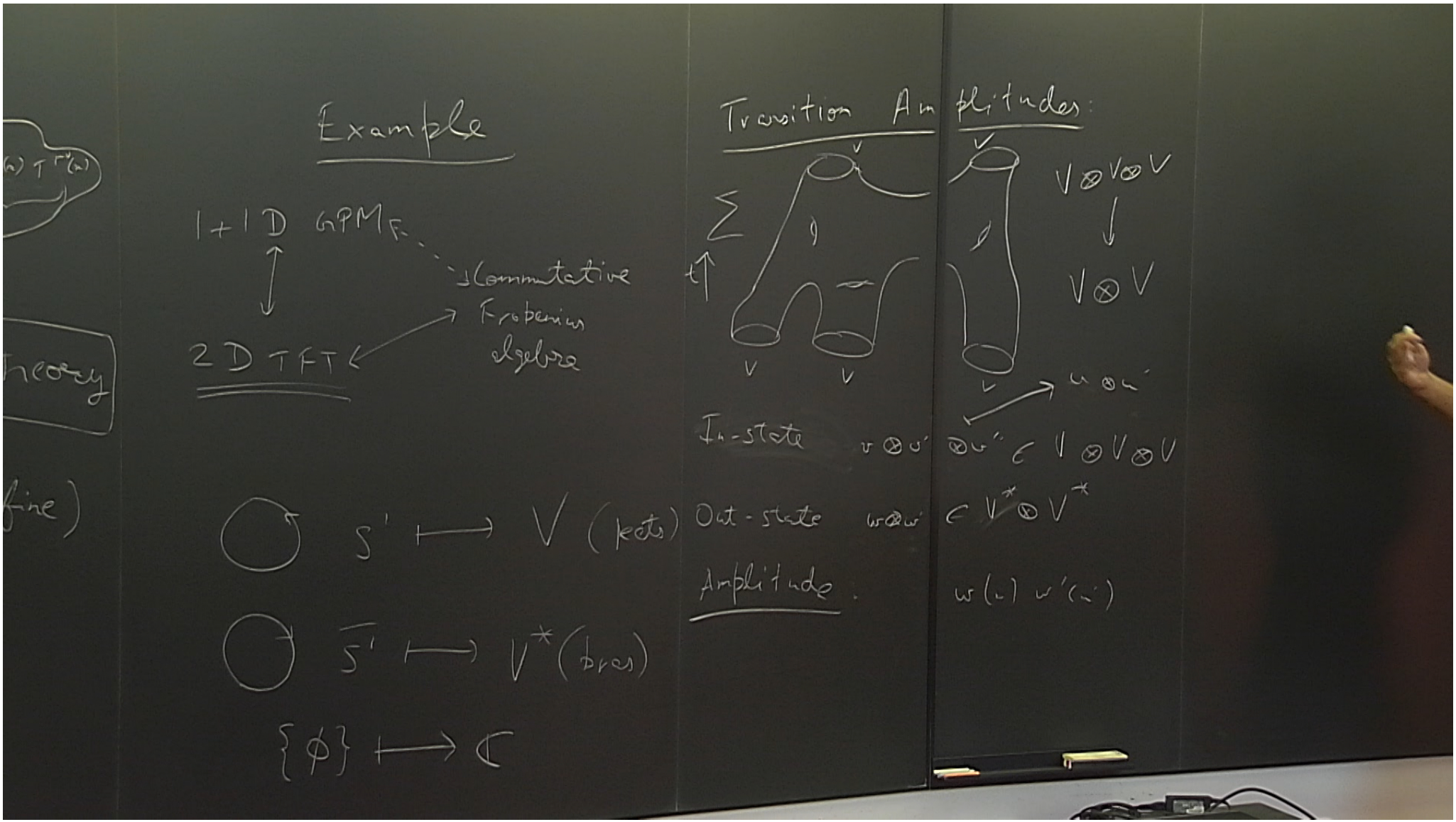
$\bar{S}^1 \longmapsto V^*$ (bras)

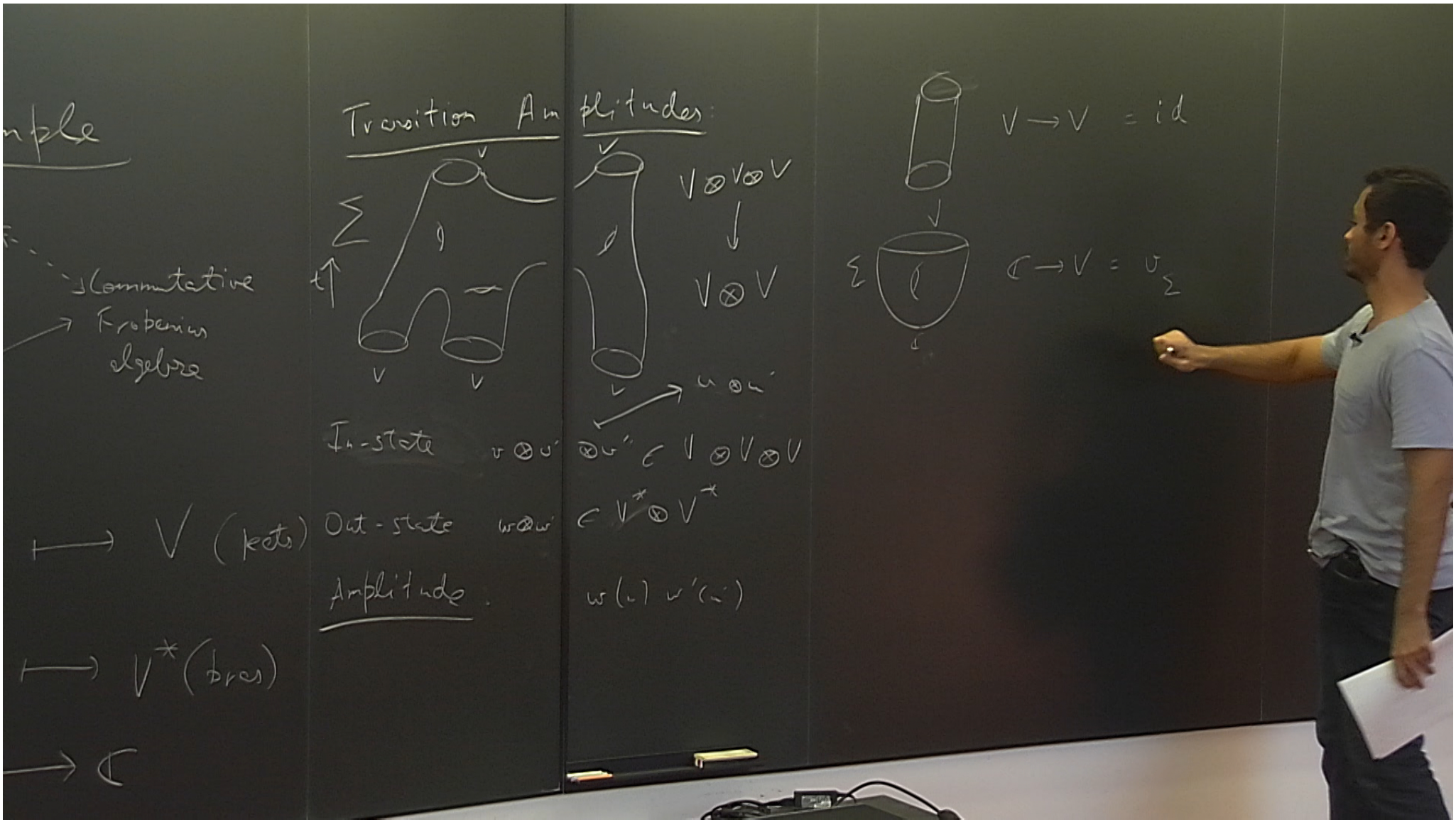
$\{\phi\} \longmapsto \mathbb{C}$

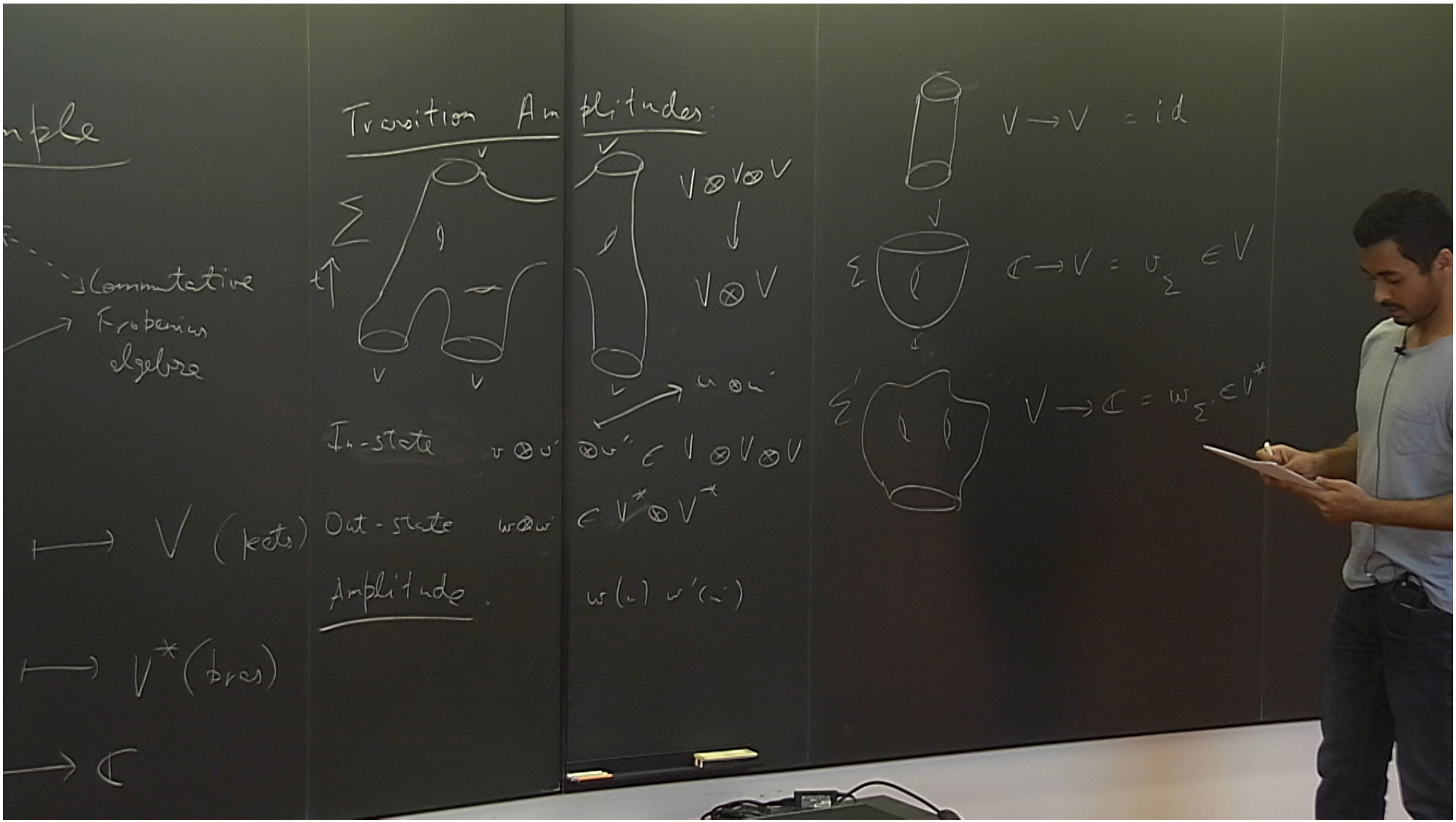

Example
1+1D GPM
2D TFT
Transition Amplitudes:
Σ
S'
{φ}

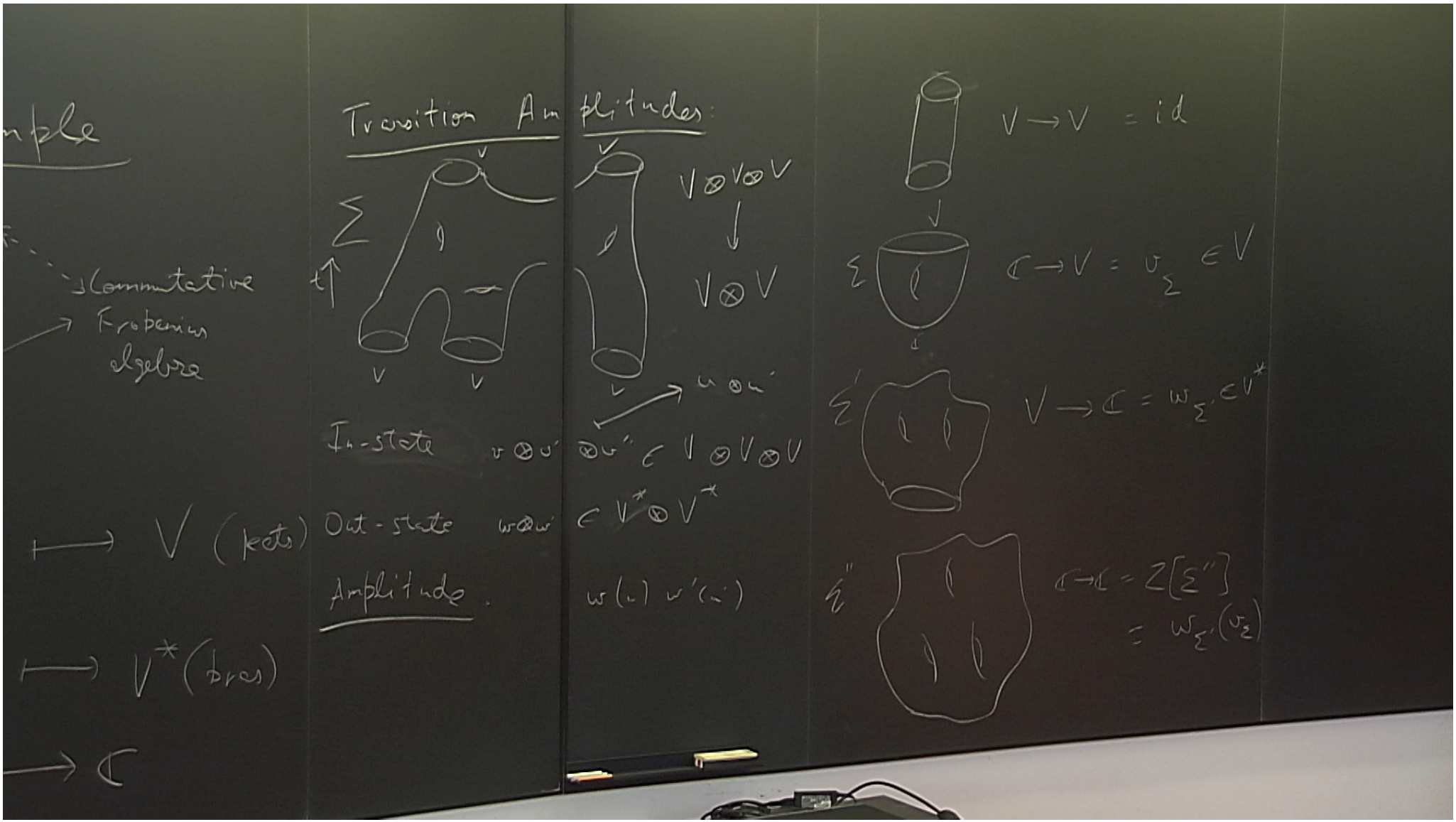

Example
1+1 D GPM_F
commutative Frobenius algebra
2 D TFT
S^1 ⟼ V (kets)
S̄^1 ⟼ V^* (bras)
{∅} ⟼ ℂ
Transition Amplitudes:
Σ
V
V ⊗ V ⊗ V
V ⊗ V

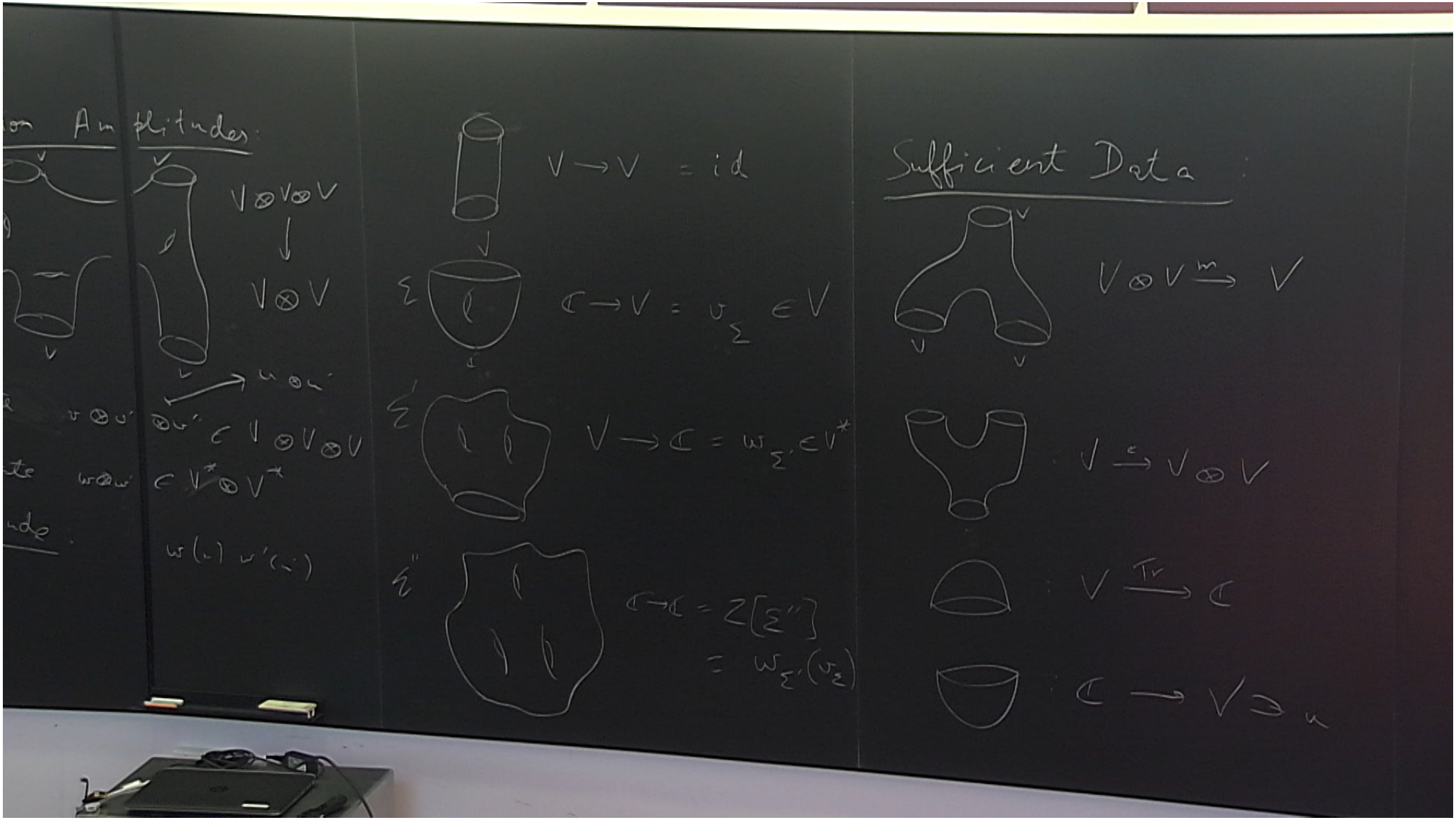
Example
1+1D GPM
2D TFT
Commutativ
Frobenius
Transition Amplitudes:
V⊗V⊗V
V⊗V
In-state
v⊗v'⊗v'' ∈ V⊗V⊗V
S'
S̄'
{φ} ⟼ ℂ

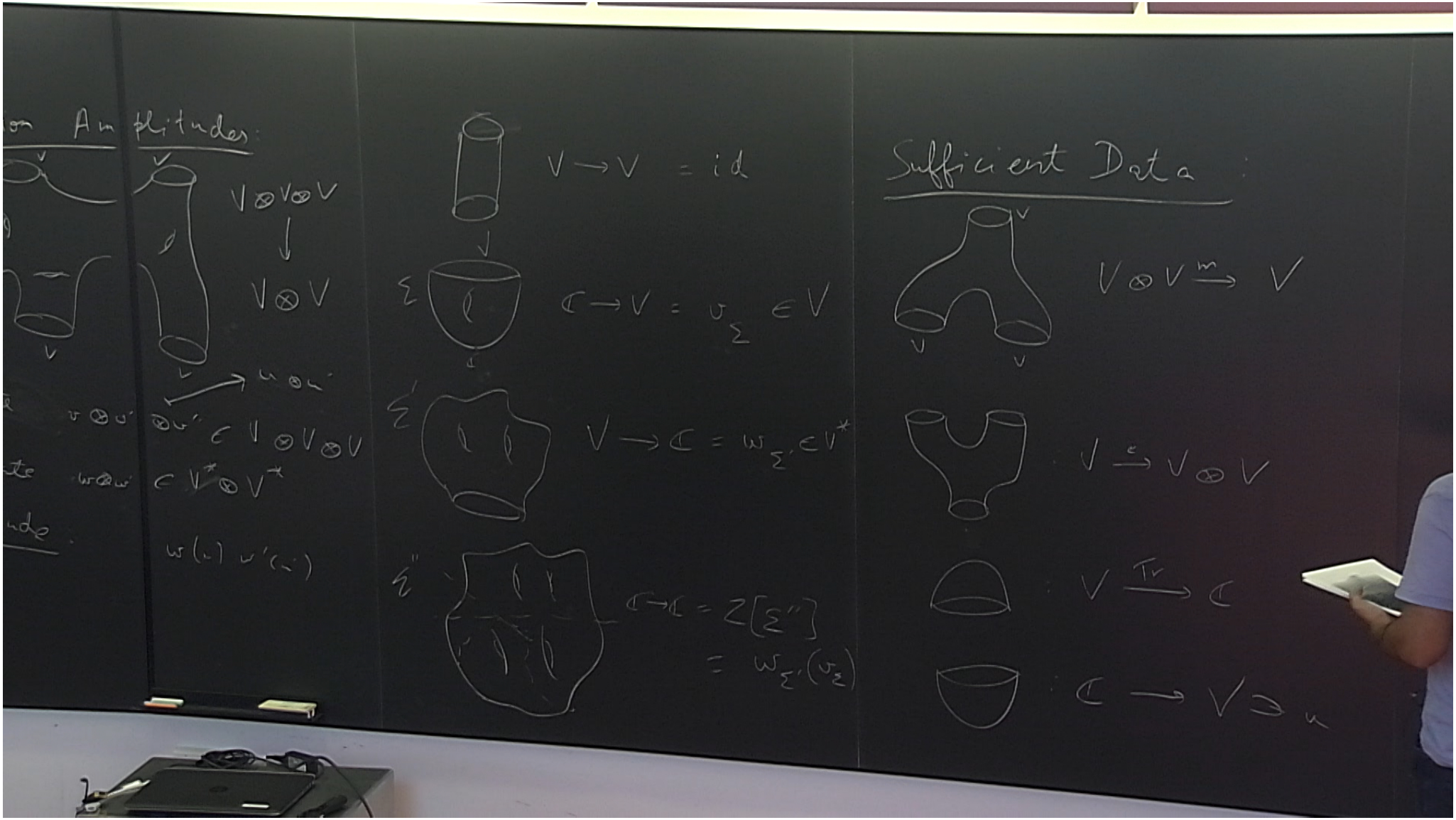

Example
1+1D GPM
Commutative Frobenius algebra
2D TFT
S¹ ⟼ V (kets)
S̄¹ ⟼ V* (bras)
{φ} ⟼ ℂ
Transition Amplitudes:
Σ
t
V
V ⊗ V ⊗ V
V ⊗ V
u ⊗ u'
In-state v ⊗ v' ⊗ v'' ∈ V ⊗ V ⊗ V
Out-state w ⊗ w' ∈ V* ⊗ V*
Amplitude: w(u) w'(u')

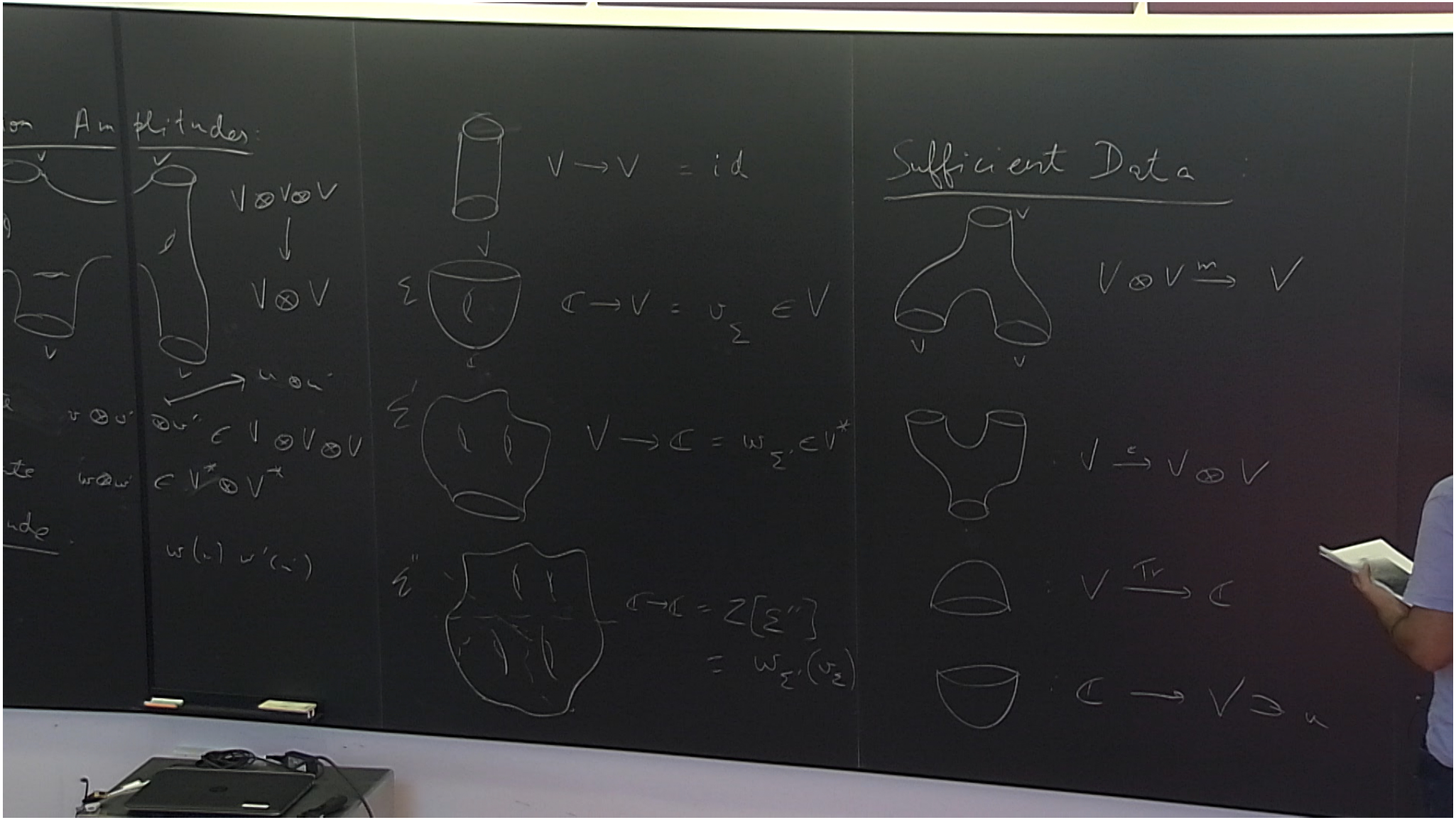

Transition Amplitudes:
Commutative
Frobenius
algebra
V (kets)
V* (bras)
V⊗V⊗V
V⊗V
In-state
Out-state
Amplitude
V → V = id
ℂ → V = v_Σ

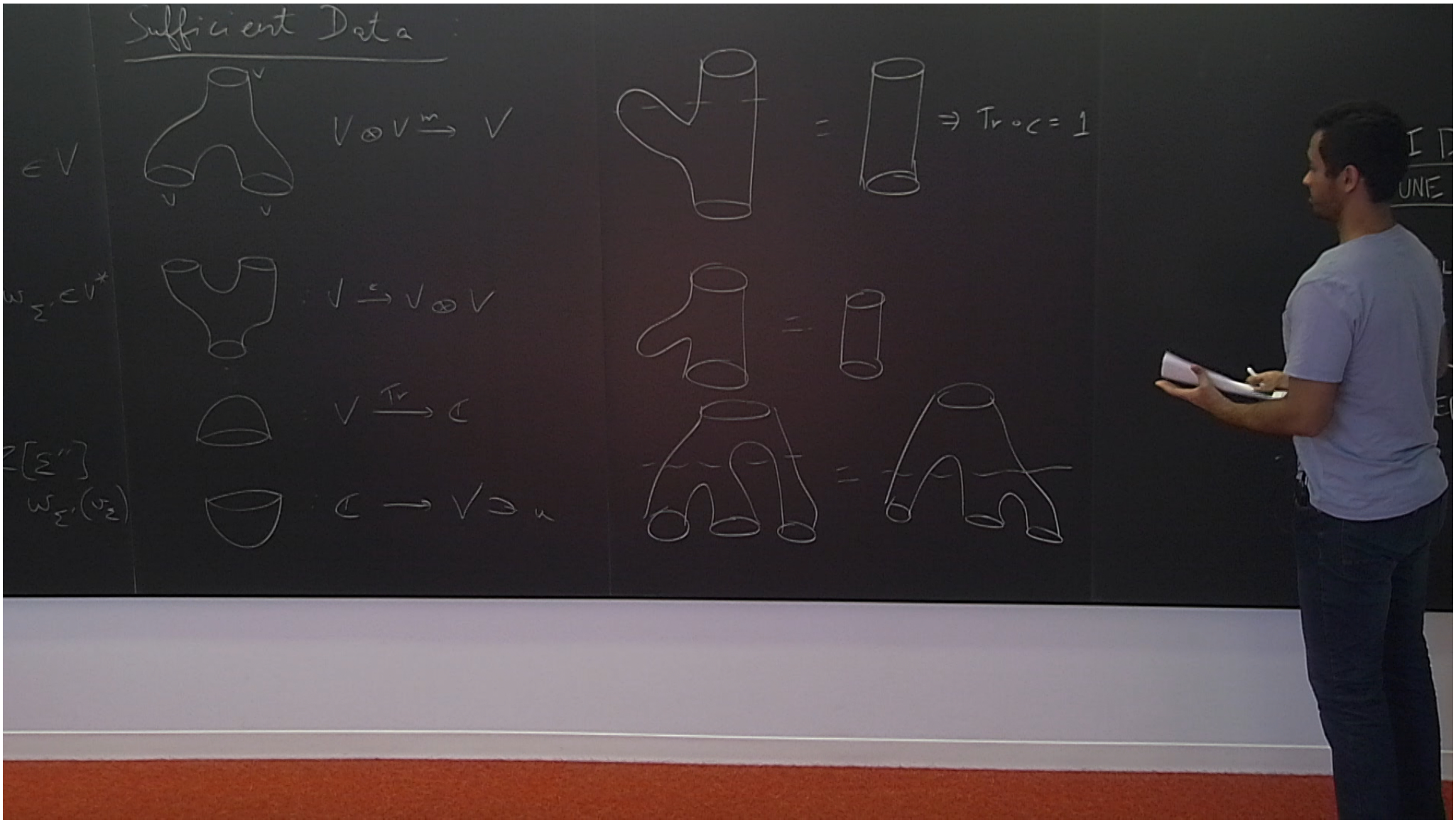

Commutative
Frobenius
algebra
$\longmapsto V$ (kets)
$\longmapsto V^*$ (bras)
$\longrightarrow \mathbb{C}$
Transition Amplitudes:
Σ
t
V
$V \otimes V \otimes V$
$V \otimes V$
$v \otimes v'$
In-state $v \otimes v' \otimes v'' \in V \otimes V \otimes V$
Out-state $w \otimes w' \in V^* \otimes V^*$
Amplitude: $w(v)\, w'(v')$
$V \to V = id$
$\mathbb{C} \to V = v_\Sigma \in V$
$V \to \mathbb{C} = w_{\Sigma'} \in V^*$

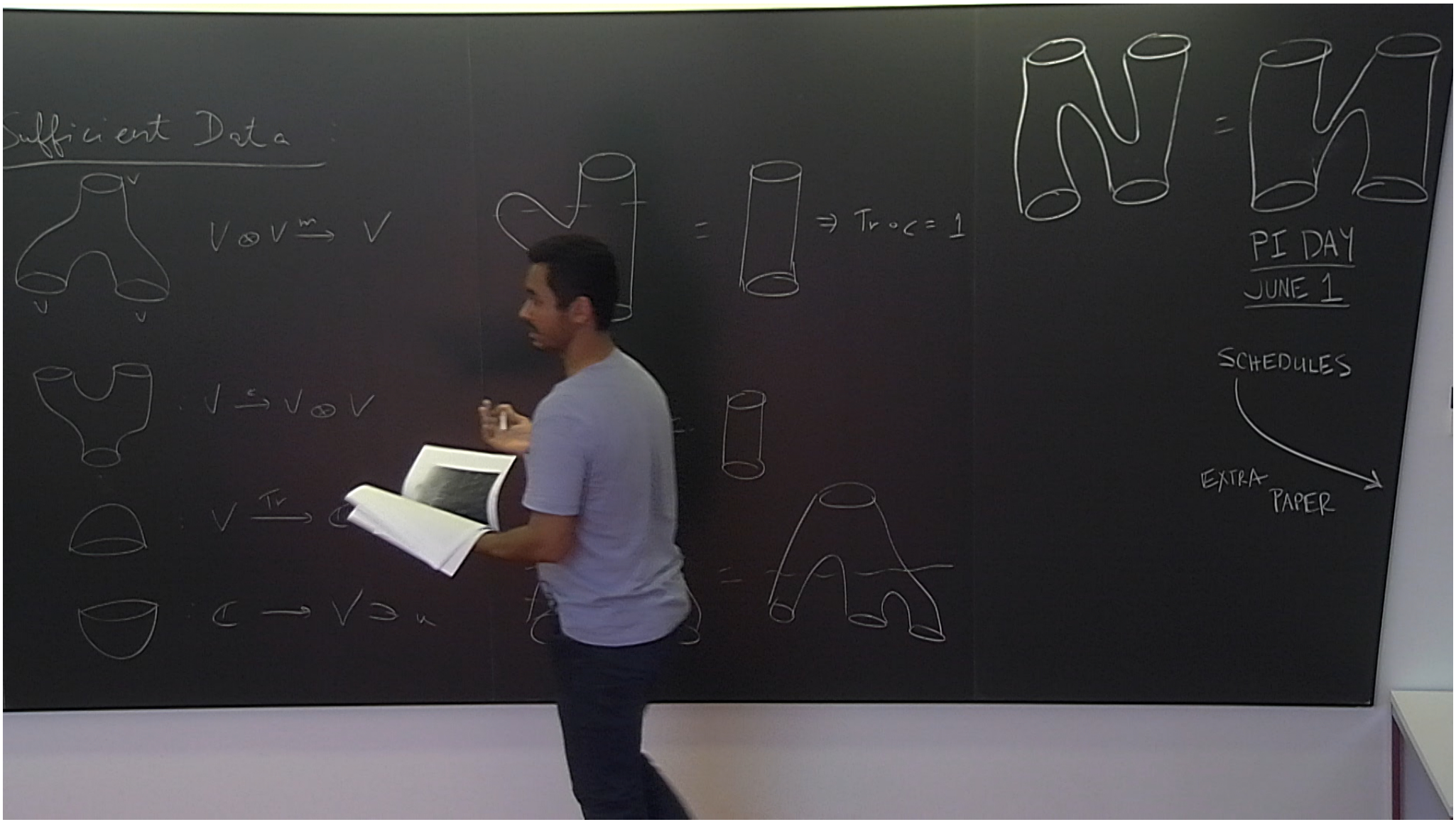

mple
Commutative
Frobenius
algebra
$\longmapsto V$ (kets)
$\longmapsto V^*$ (bras)
$\longrightarrow \mathbb{C}$
Transition Amplitudes:
Σ
t
V
$V \otimes V \otimes V$
$V \otimes V$
$u \otimes u'$
In-state $v \otimes v' \otimes v'' \in V \otimes V \otimes V$
Out-state $w \otimes w' \in V^* \otimes V^*$
Amplitude $w(u)\, w'(u')$
$V \to V = id$
Σ
$\mathbb{C} \to V = v_\Sigma \in V$
Σ'
$V \to \mathbb{C} = w_{\Sigma'} \in V^*$
Σ''
$\mathbb{C} \to \mathbb{C} = Z[\Sigma'']$
$= w_{\Sigma'}(v_\Sigma)$

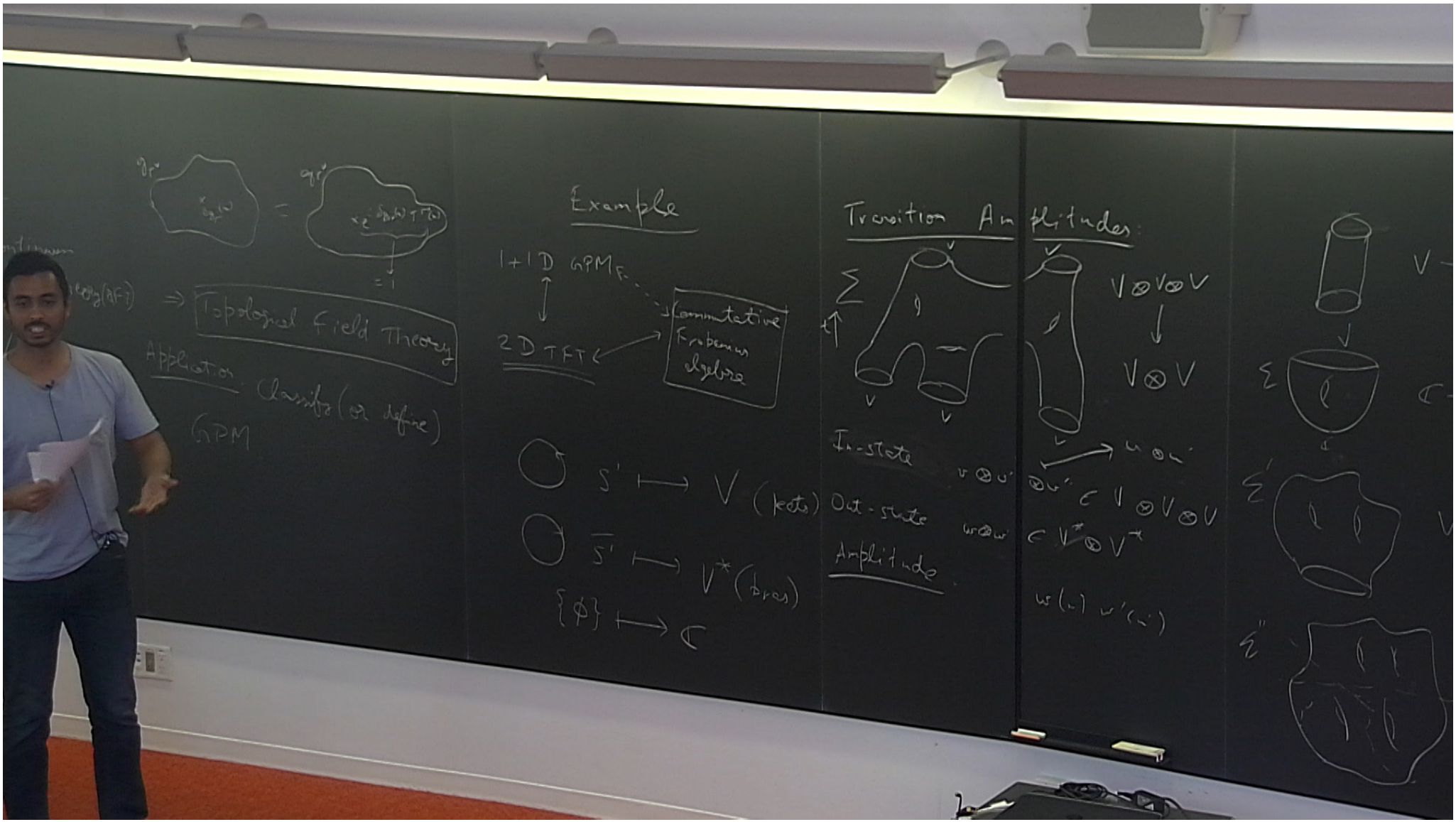

Amplitudes
V ⊗ V ⊗ V
V ⊗ V
u ⊗ u'
v ⊗ v'
⊗ w' ∈ V ⊗ V ⊗ V
w ⊗ w' ∈ V* ⊗ V*
w(·) v'(·)
V → V = id
Σ
ℂ → V = v_Σ ∈ V
Σ'
V → ℂ = w_Σ' ∈ V*
Σ''
ℂ → ℂ = Z[Σ'']
= w_Σ'(v_Σ)
Sufficient Data
V ⊗ V → V
V → V ⊗ V
V → ℂ
ℂ → V ∋ u


Amplitudes:
V ⊗ V ⊗ V
V ⊗ V
u ⊗ u'
v ⊗ v' ⊗ w' ∈ V ⊗ V ⊗ V
w ⊗ w' ∈ V* ⊗ V*
w(u) w'(u')
V → V = id
Σ
ℂ → V = v_Σ ∈ V
Σ'
V → ℂ = w_Σ' ∈ V*
Σ''
ℂ → ℂ = Z[Σ'']
= w_Σ'(v_Σ)
Sufficient Data:
V ⊗ V → V
V → V ⊗ V
V → ℂ
ℂ → V ∋ u


Amplitudes:
V ⊗ V ⊗ V
V ⊗ V
u ⊗ u'
v ⊗ v'
⊗ w' ∈ V ⊗ V ⊗ V
w ⊗ w' ∈ V* ⊗ V*
w(·) w'(·)
V → V = id
Σ
ℂ → V = v_Σ ∈ V
Σ'
V → ℂ = w_Σ' ∈ V*
Σ''
ℂ → ℂ = Z[Σ'']
= w_Σ'(v_Σ)
Sufficient Data:
V ⊗ V → V
V → V ⊗ V
V → ℂ
ℂ → V ∋ u


Sufficient Data
$V \otimes V \xrightarrow{m} V$
$\Rightarrow Tr \circ c = 1$
$V \xrightarrow{c} V \otimes V$
$V \xrightarrow{Tr} \mathbb{C}$
$\mathbb{C} \longrightarrow V \ni u$


Sufficient Data
V ⊗ V → V
V → V ⊗ V
V → ℂ
ℂ → V ∋ u
⇒ Tr ∘ c = 1
PI DAY
JUNE 1
SCHEDULES
EXTRA PAPER


Example
1+1 D GPM
2D TFT
Commutative Frobenius algebra
Topological Field Theory
Application: Classify (or define) GPM
Transition Amplitudes
In-state
Out-state
Amplitude