

Title: Lattice QCD and neutrino physics

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Abstract:

Lattice QCD and Neutrino Physics

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University of Chicago/Fermilab

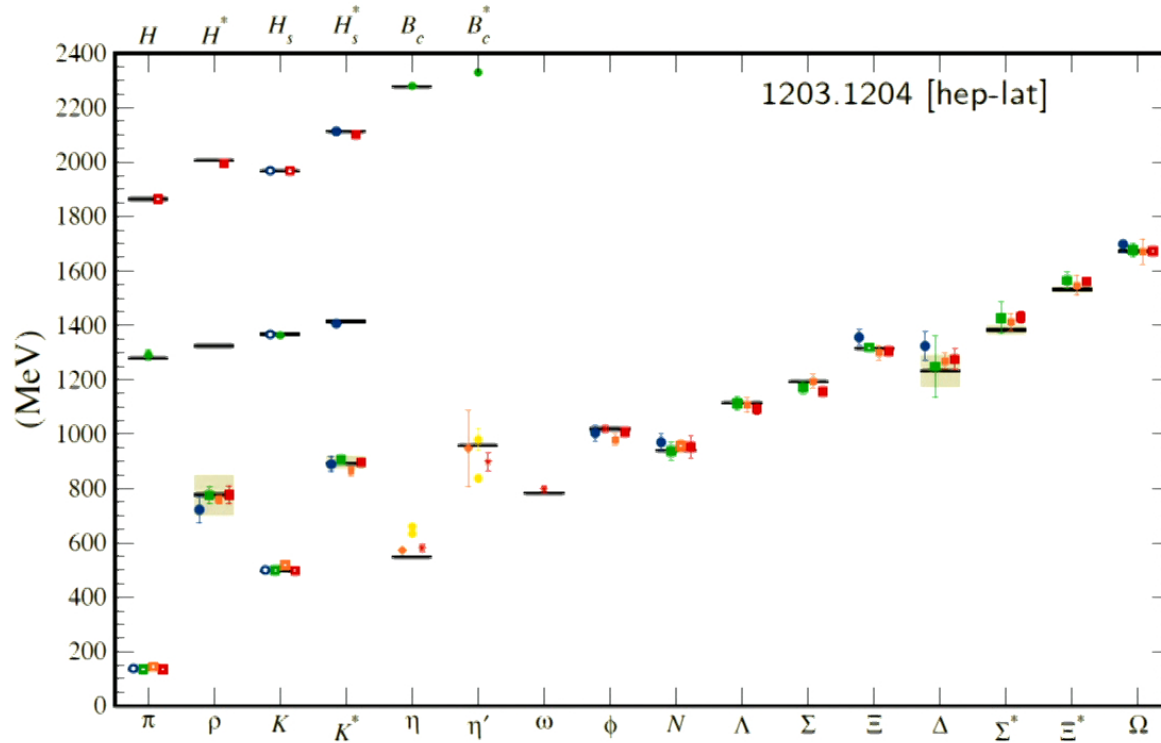
June 13, 2017

Radiative Corrections at the
Intensity Frontier of Particle Physics

Outline

- ▶ Introduction
 - ▶ Successes of LQCD
 - ▶ LQCD in neutrino physics
- ▶ Case Studies
 - ▶ z Expansion & CC F_A
 - ▶ NC F_A & CC F_P
 - ▶ $N \rightarrow \Delta$ Transitions
 - ▶ NN matrix elements
- ▶ Conclusions

Successes of Lattice QCD



Lattice QCD has been very successful in previous decade

- ▶ Spectrum calculations at sub-percent level precision, good agreement with experiment and consistency among many collaborations
- ▶ LQCD predictions cover vast number of observables with minimum input

Successes of Lattice QCD

Flavor physics makes use of LQCD to improve understanding of CKM matrix elements

Lattice computations of $\langle \mathcal{P} | \bar{q} \gamma_\mu \gamma_5 q' | 0 \rangle \propto f_{\mathcal{P}}$ may be used to disentangle decay width

$$\Gamma(\mathcal{P} \rightarrow \ell \bar{\nu}) = \frac{G_F}{8\pi} f_{\mathcal{P}}^2 m_\ell^2 M_{\mathcal{P}} \left(1 - \frac{m_\ell^2}{M_{\mathcal{P}}^2} \right)^2 |V_{qq'}|^2$$

Most precise PDG estimates of CKM elements use decay constants from LQCD:

$$\begin{array}{ll} |V_{ud}| = 0.9764(2)(127)(10) & |V_{us}| = 0.2255(3)(6)(3) \\ |V_{cd}| = 0.217(5)(1) & |V_{cs}| = 1.007(16)(5) \end{array}$$

(errors: experimental, decay constant, radiative corrections)

Successes of Lattice QCD

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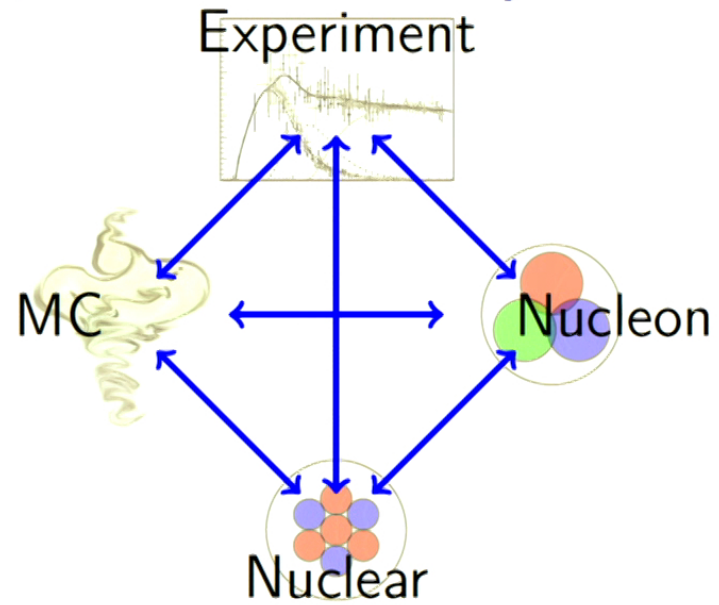
Flavor physics benefits from LQCD
 \implies neutrino physics should be able to benefit too

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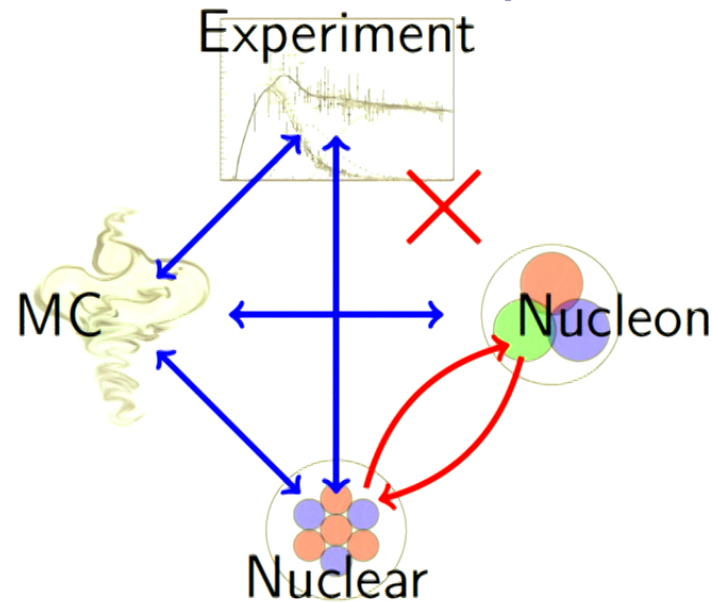
Utility of LQCD to Neutrino Physics



Neutrino oscillation analysis has many constituent parts

Ideally, lots of redundancy and checks between elements of analysis

Utility of LQCD to Neutrino Physics

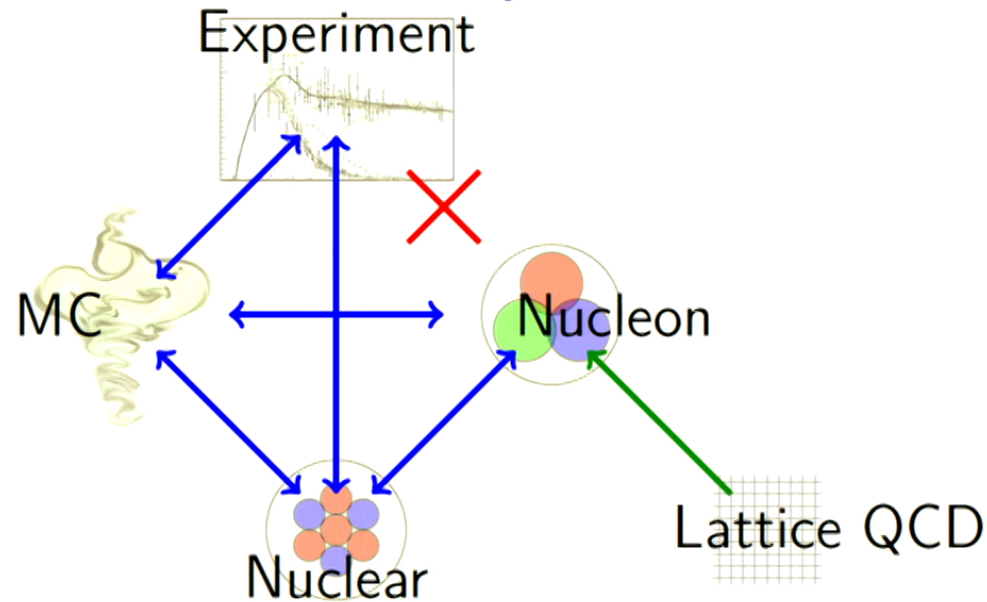


Neutrino oscillation analysis has many constituent parts

Ideally, lots of redundancy and checks between elements of analysis

If a piece is missing or not well determined,
missing element can cause ambiguity/degeneracy

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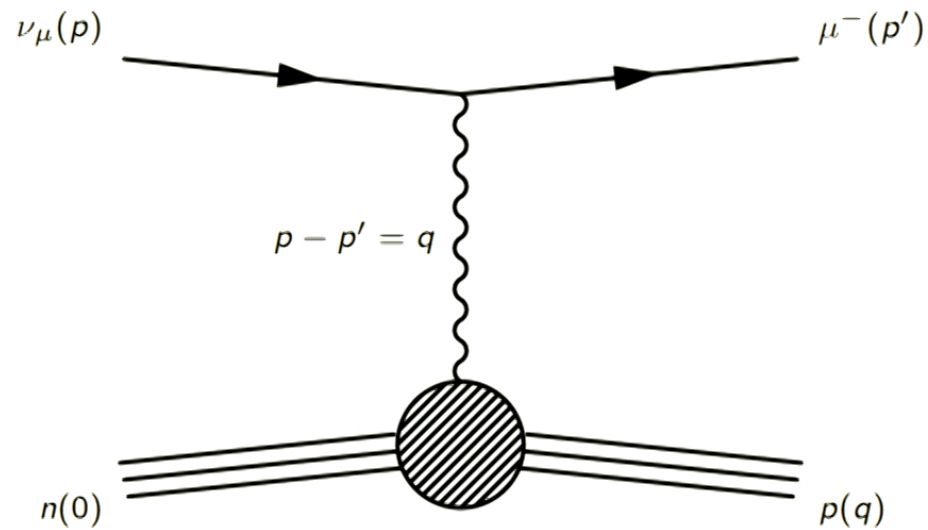
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Lattice QCD can restore missing pieces;
acts as a disruptive technology to break degeneracy

How Does Lattice Help?

Lattice is well suited to compute matrix elements:

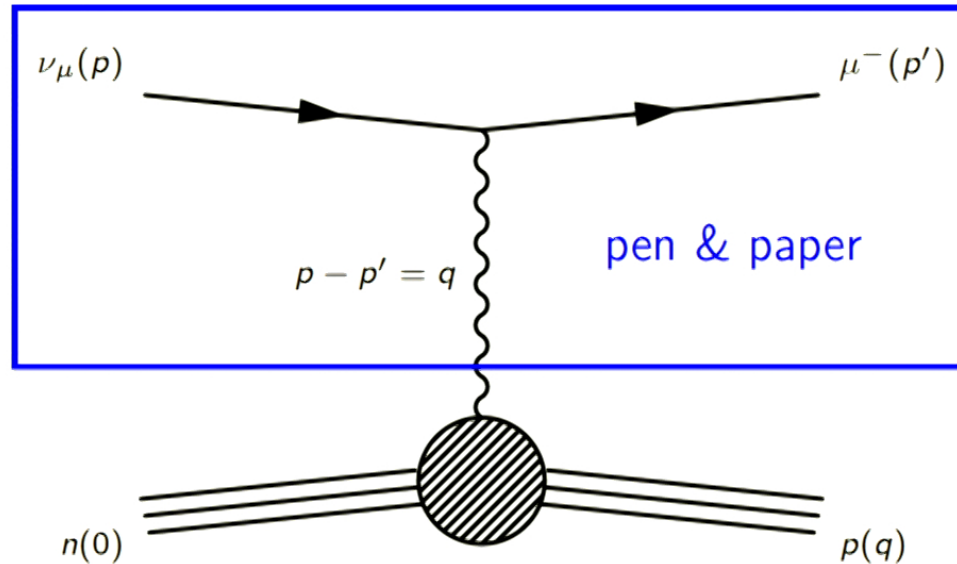
$$\mathcal{M}_{\nu_\mu n \rightarrow \mu p}(p, p') = \langle \mu(p') | (V_\mu - A_\mu) | \nu(p) \rangle \langle p(q) | (V_\mu - A_\mu) | n(0) \rangle$$



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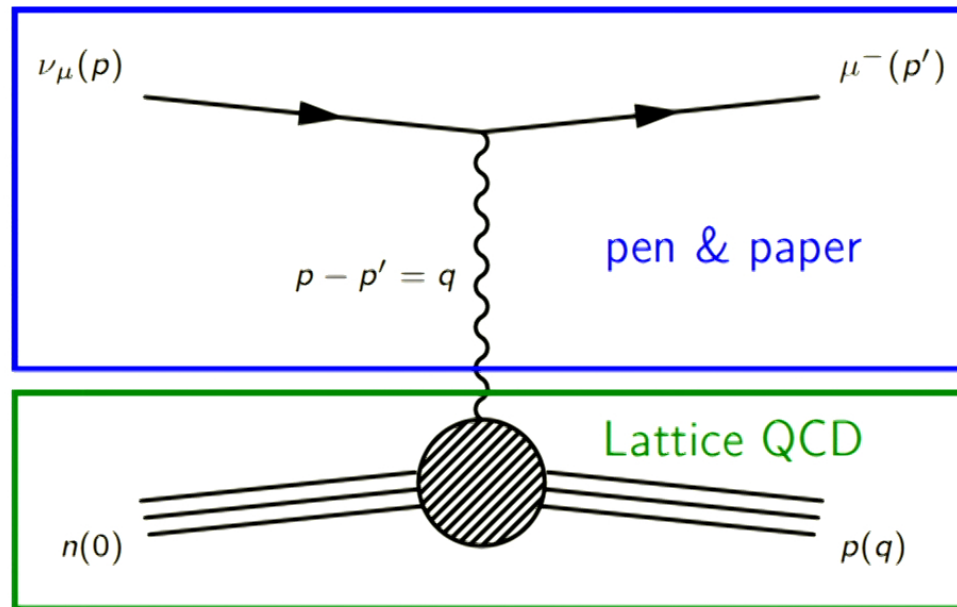
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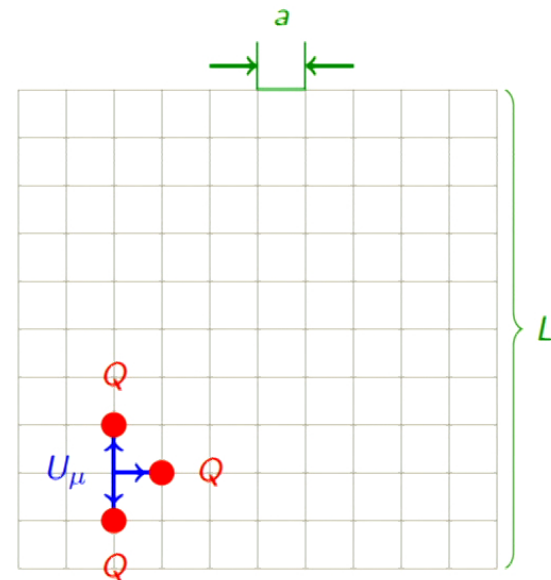


Lattice QCD: Formalism

- ▶ Numerical solution to path integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \exp(-S) \mathcal{O}_\psi [U]$$

- ▶ Discretize spacetime
⇒ #DOF finite
- ▶ Quark fields defined on sites
⇒ $Q(x)$
- ▶ Gauge fields defined between sites
⇒ $U_\mu(x)$
- ▶ Euclidean time
⇒ correlators $\propto e^{-Et}$
- ▶ Lattice spacing a provides UV cutoff
- ▶ Lattice size L provides IR cutoff

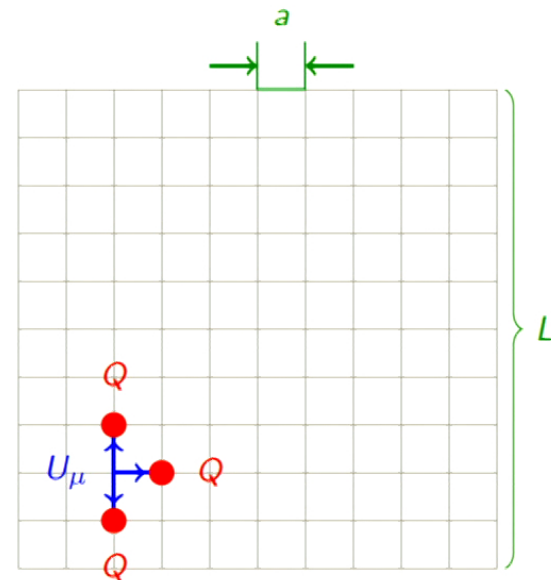


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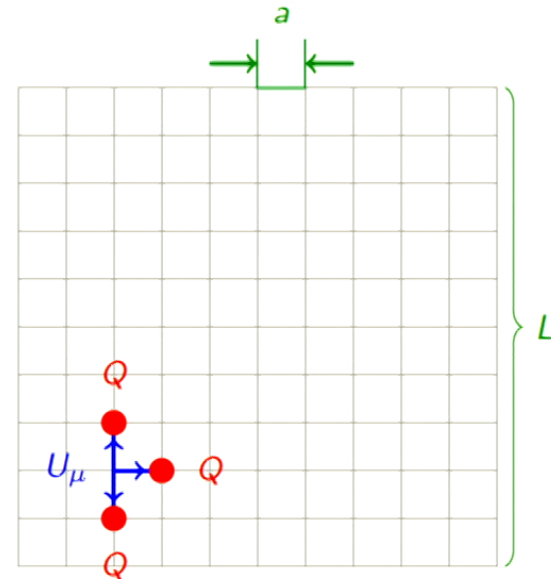
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Lattice QCD: Formalism

To get full control over all systematics, need at least:

- ▶ Multiple lattice spacings (“continuum” extrapolation)
- ▶ Multiple pion masses/physical pion mass (“chiral” extra/interpolation)
- ▶ Multiple volumes (“infinite volume” extrapolation)
- ▶ 3-4 quarks in virtual sea (2+1, 2+1+1, 1+1+1+1 flavors of sea quarks)
- ▶ Renormalization technique or scheme, usually nonperturbative



Baryon Systematics

Why are baryon matrix elements so difficult?

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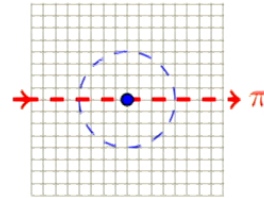
- ▶ Signal-to-Noise Grows Exponentially

- ▶ Signal $\propto \langle \overrightarrow{\Xi} \rangle \sim e^{-M_n t}$, noise² $\propto \langle | \overrightarrow{\Xi} |^2 \rangle = \langle \overrightarrow{\Xi} \overrightarrow{\Xi} \rangle \sim e^{-3M_\pi t}$
- ▶ Noise gets contribution from 3-pion term, worsens as M_π decreases

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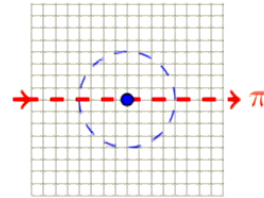
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- ▶ Finite size effects
 - ▶ self-interaction via π s which wrap around periodic BC



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 - ▶ Noise gets contribution from 3-pion term, worsens as M_π decreases
- ▶ Finite size effects
 - ▶ self-interaction via π s which wrap around periodic BC
- ▶ Excited state contamination
 - ▶ Operators couple to ground state + excited states
 - ▶ Requires fitting $\sum_n e^{-E_n t}$ for many n
 - ▶ Rotation/translation symmetry broken by lattice



Utility of LQCD

Lattice QCD is most useful as a tool when matrix element

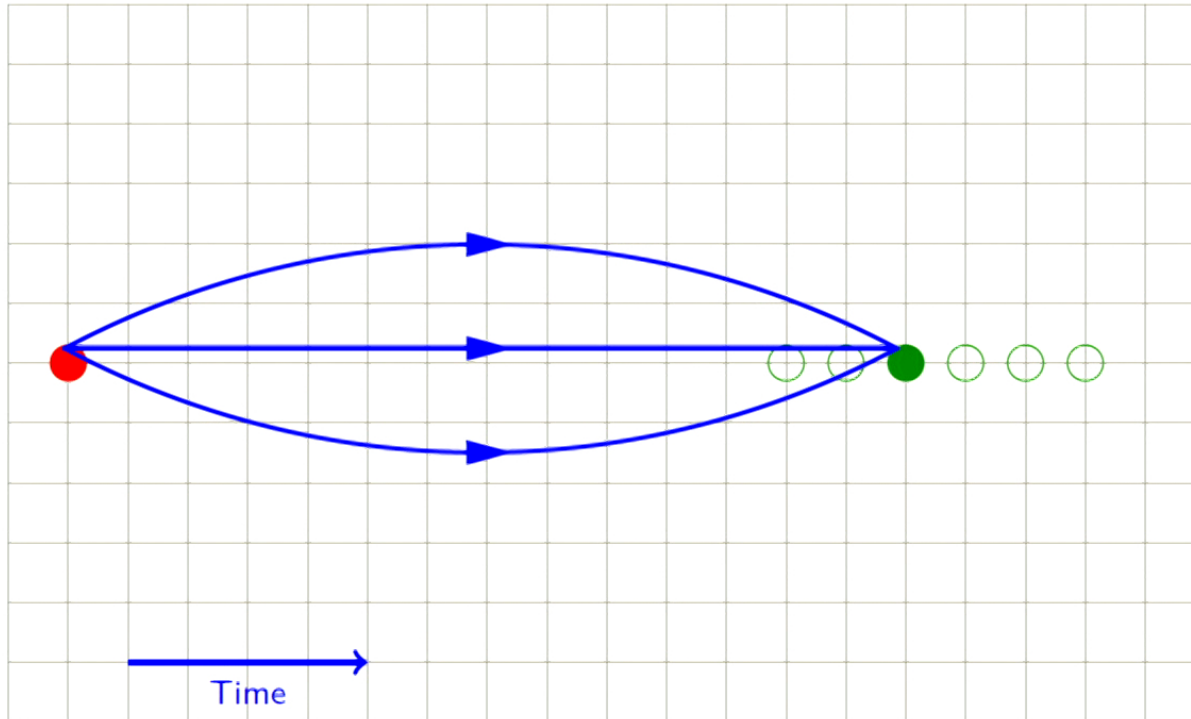
- ▶ is well-posed
- ▶ is difficult or impractical to measure in experiment
- ▶ gives sizeable contributions to systematic errors

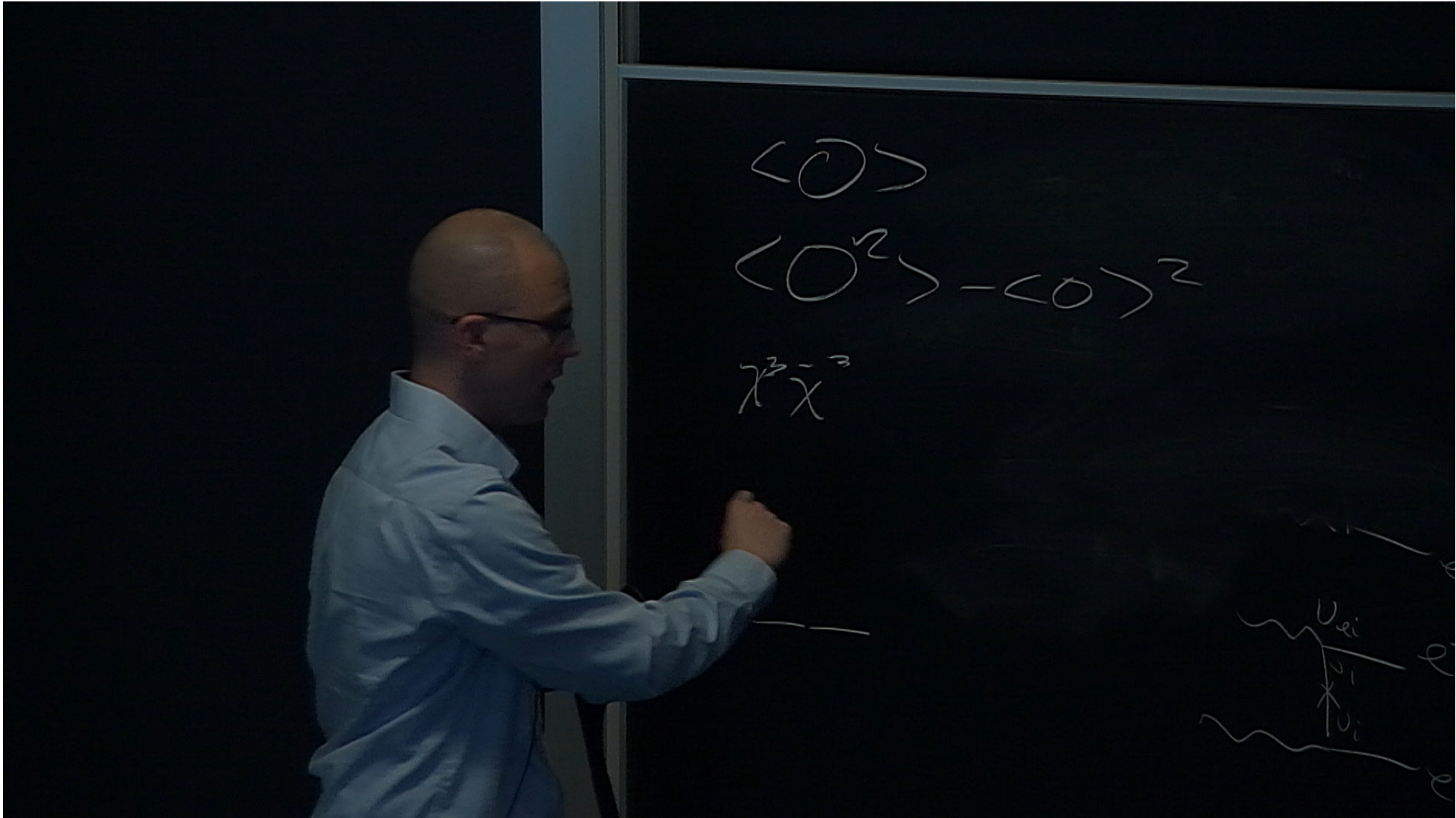
Computations have large overhead costs,
many studies come cheaply after paying overhead costs

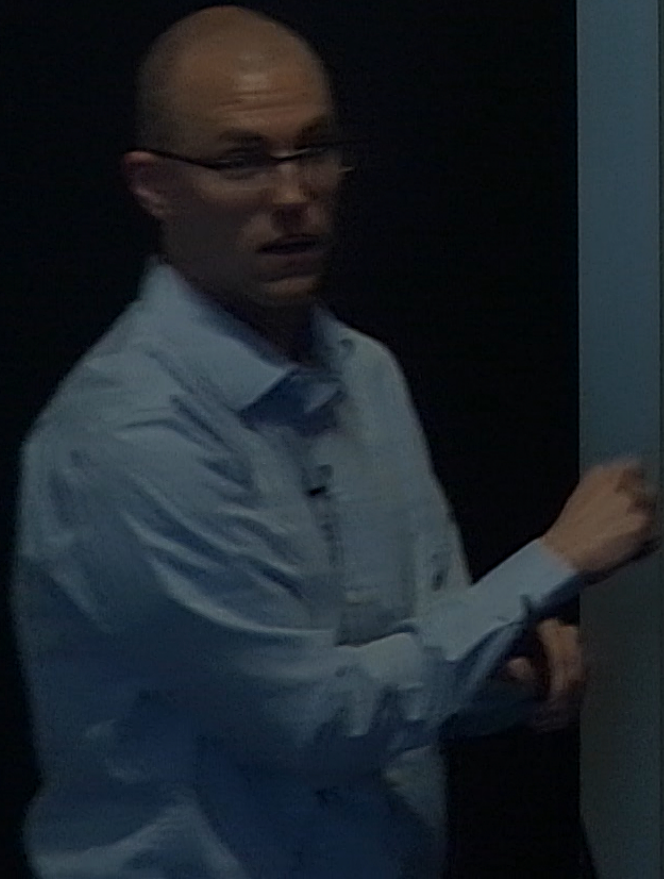
Two-point Correlation Functions

$$\sum_i \text{source (fixed)} \quad e^{-M_i t} \quad \text{sink (moved)} \quad = C_{ab}(t)$$

$$\langle N_i | \mathcal{O}_a^\dagger(0) | \Omega \rangle \quad \langle \Omega | \mathcal{O}_b(t) | N_i \rangle$$



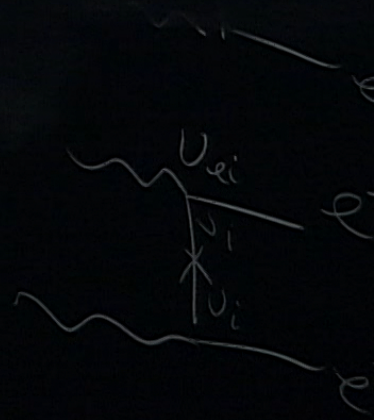




$$\langle 0 \rangle$$

$$\langle 0^2 \rangle - \langle 0 \rangle^2$$

$$\chi^3 \chi^3(0) \chi^3 \chi^2(\hbar)$$





$\langle 0 \rangle$

$\langle 0^2 \rangle - \langle 0 \rangle^2$

$\underbrace{\chi^3 \chi^3}_{3\pi} (0) \underbrace{\chi^3 \chi^2}_{\pi} (0)$

—

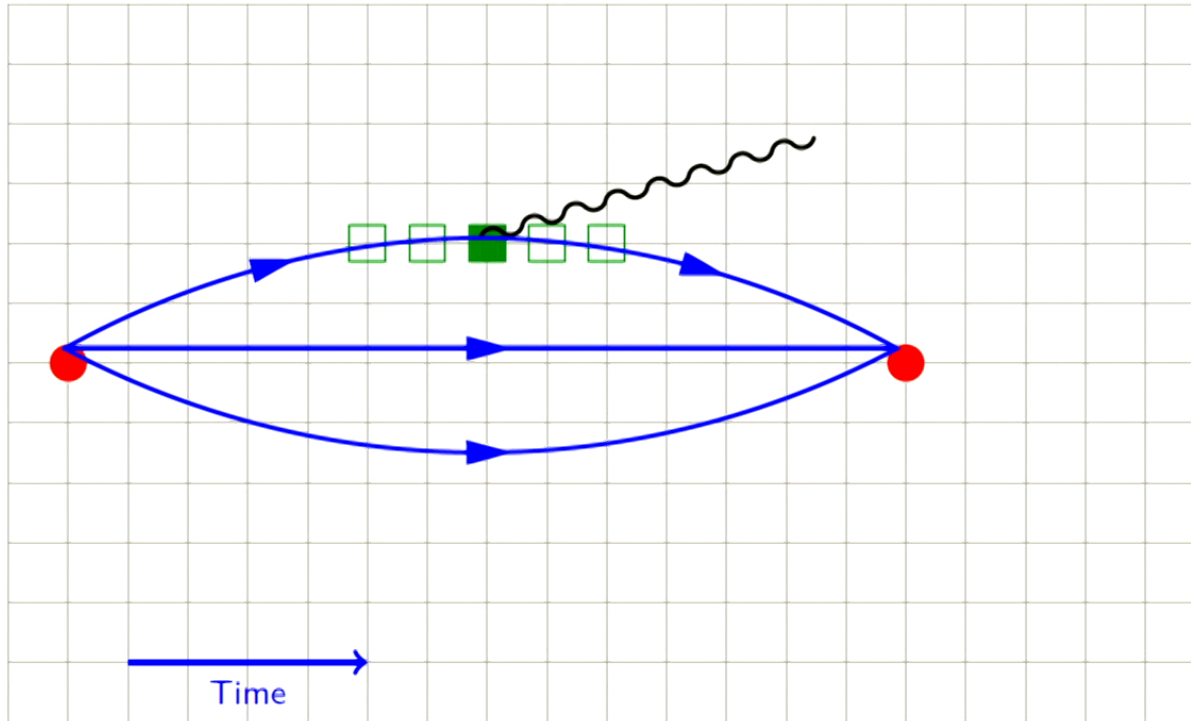
$\begin{array}{c} U_{ei} \\ \hline \chi_i \\ \hline U_{ei} \end{array}$

Three-point Correlation Functions

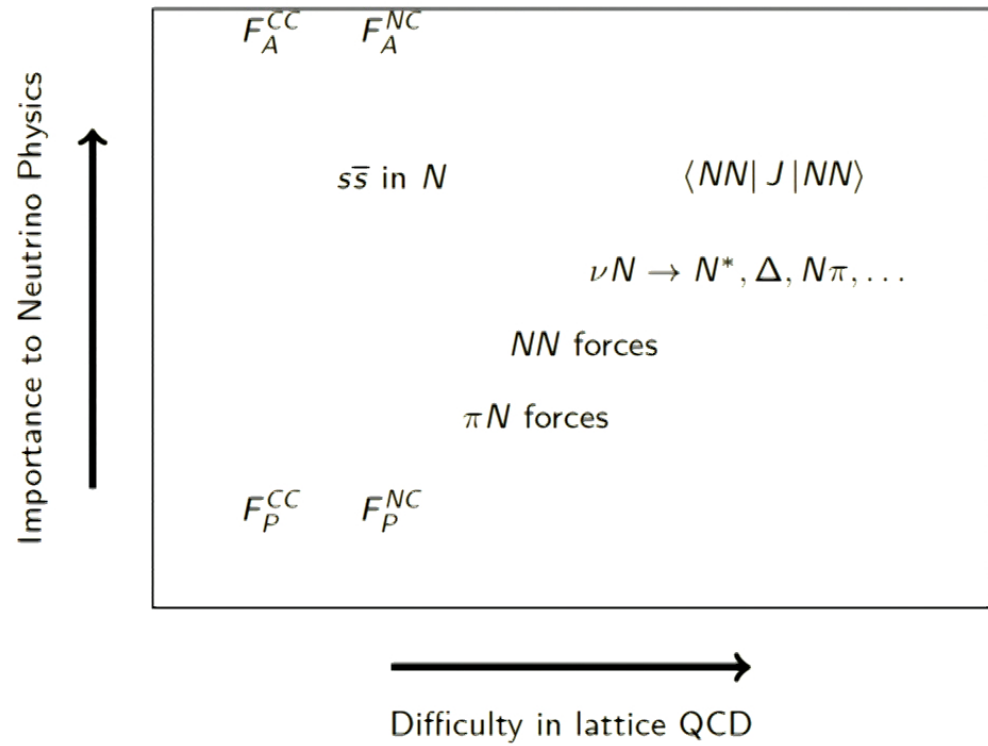
source (fixed) current (moved) sink (fixed)

$$\sum_{ij} \langle N_i | \mathcal{O}_a^\dagger(0) | \Omega \rangle \langle N_j | A_\mu(t) | N_i \rangle \langle \Omega | \mathcal{O}_b(T) | N_j \rangle$$

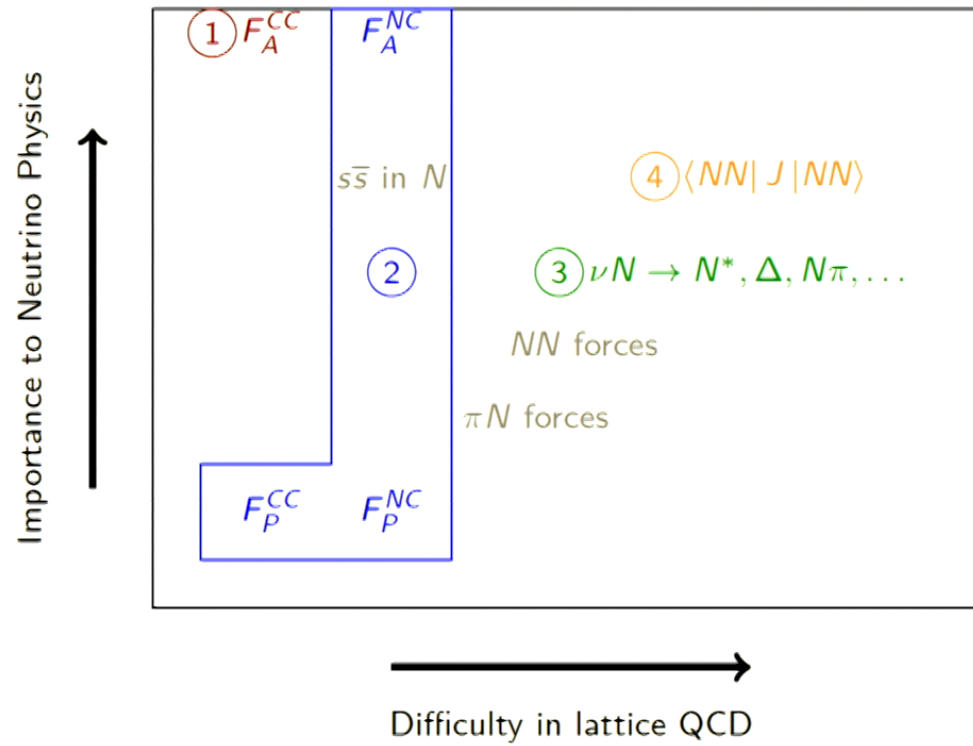
$$\times e^{-M_i t} e^{-M_j(T-t)} = C_{ab}^3(t, T)$$



Calculations of Interest



Calculations of Interest

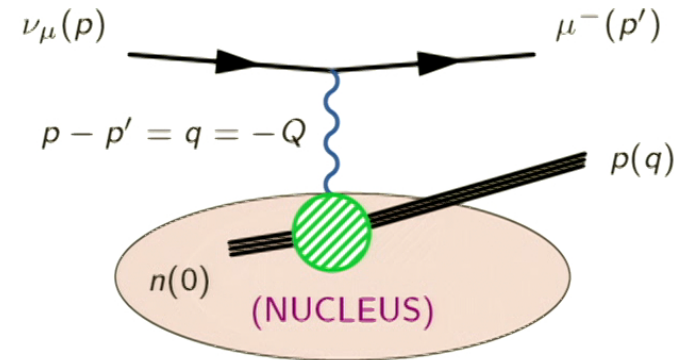


(Charged Current) Axial Form Factor

Quasielastic scattering

QE scattering is relatively easy measurement,
relatively theoretically clean:
 ν interacts with nearly-free nucleon

QE is **primary signal measurement process**
for neutrino oscillation experiments



Current Monte Carlo nuclear models assume gas of weakly bound nucleons
 \Rightarrow **free nucleon amplitudes** useful for determining nuclear matrix elements

QE matrix element involves many-body interactions
 \Rightarrow parametrized by form factors ($-q^2 = Q^2$):

$$\langle N(p') | A_\mu^a(q) | N(p) \rangle = \bar{u}_{p'} \left[\gamma_\mu \gamma_5 F_A(q^2) + \frac{q_\mu}{2M_n} F_P(q^2) \right] \frac{t^a}{2} u_p$$

Axial form factor $F_A(Q^2) \equiv G_A(Q^2)$ is important, not well-known

Dipole Form Factor

Most analyses assume the Dipole axial form factor
(Llewellyn-Smith, 1972):

$$F_A^{\text{dipole}}(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{m_A^2}\right)^2}$$

[Phys.Rept.3 (1972),261]

Dipole is an ansatz:

unmotivated in interesting energy region

⇒ uncontrolled systematics and therefore underestimated uncertainties

Large variation in m_A over many experiments
(dubbed the “axial mass problem”):

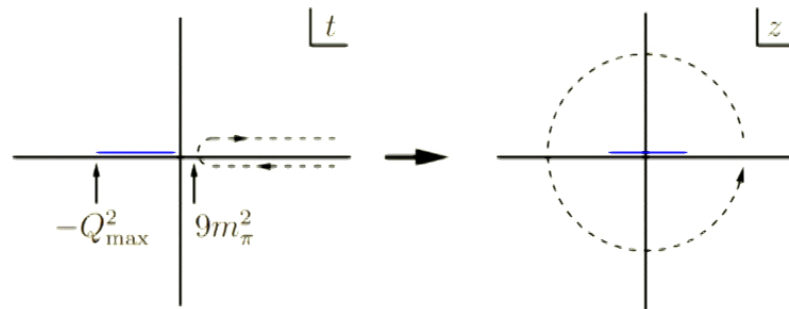
- ▶ $m_A = 1.026 \pm 0.021$ (Bernard *et al.*, [arXiv:00107088])
- ▶ $m_A^{\text{eff}} = 1.35 \pm 0.17$ (MiniBooNE, [arXiv:1002.2680])

Essential to use well-motivated parameterization of F_A

z Expansion

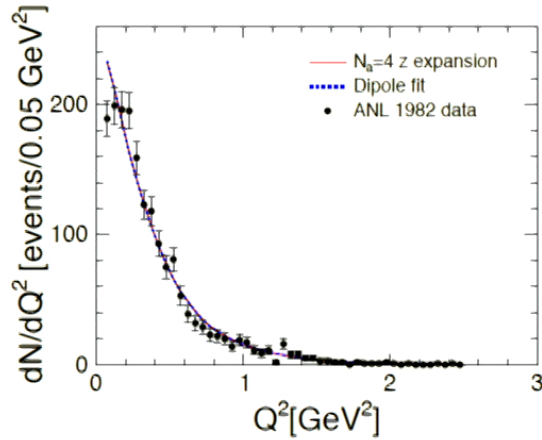
The z Expansion [arXiv:1108.0423] is a conformal mapping which takes kinematically allowed region ($t = -Q^2 \leq 0$) to within $|z| < 1$

$$z(t; t_0, t_c) = \frac{\sqrt{t_c - t} - \sqrt{t_c - t_0}}{\sqrt{t_c - t} + \sqrt{t_c - t_0}} \quad F_A(z) = \sum_{n=0}^{\infty} a_n z^n \quad t_c = 9m_\pi^2$$



- ▶ Motivated by analyticity arguments from QCD
- ▶ Only few parameters needed to get good description of form factor
- ▶ Sum rules regulate large- Q^2 behavior

Deuterium Fitting - Differential Cross Section

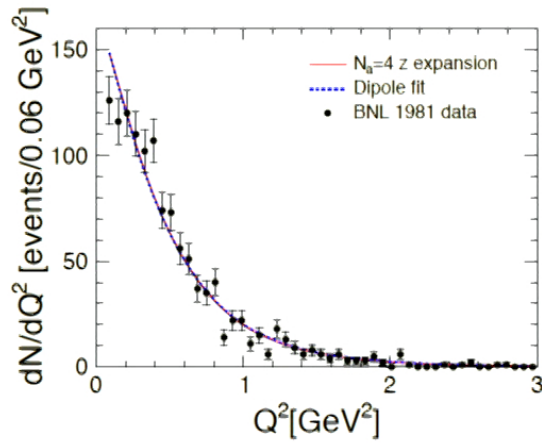


Dipole:

χ^2/N_{bins}	58.6/49
m_A	1.02(5)

z Expansion:

χ^2/N_{bins}	60.9/49
a_1	2.25(10)
a_2	0.2(0.9)
a_3	-4.9(2.4)
a_4	2.7(2.7)



Dipole:

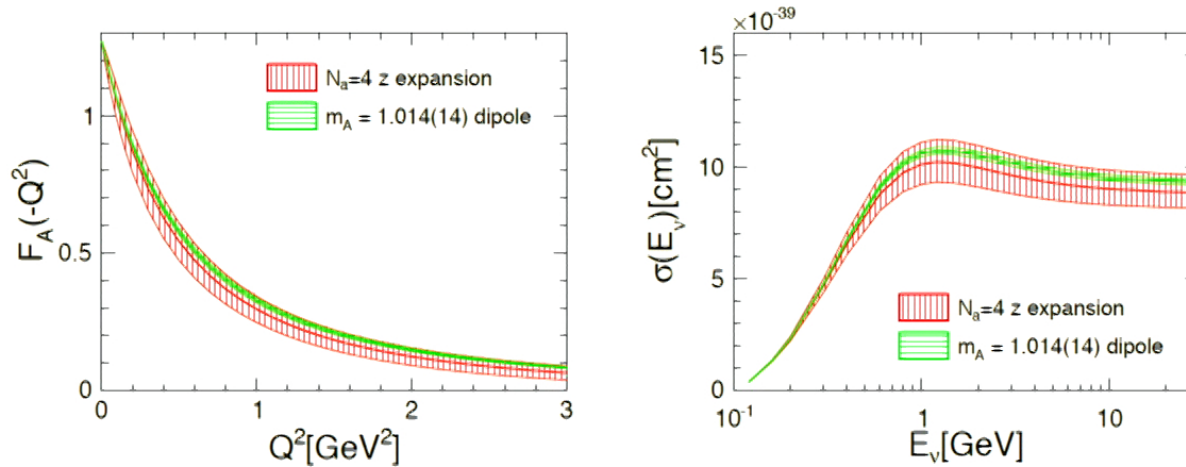
χ^2/N_{bins}	70.9/49
m_A	1.05(4)

z Expansion:

χ^2/N_{bins}	73.4/49
a_1	2.24(10)
a_2	0.6(1.0)
a_3	-5.4(2.4)
a_4	2.2(2.7)

Analysis in arXiv:1603.03048[hep-ph] ASM, M. Betancourt, R. Gran, R. Hill

Deuterium Fitting - Results Summary



$$\frac{1}{F_A(0)} \left. \frac{dF_A}{dQ^2} \right|_{Q^2=0} \equiv -\frac{1}{6} r_A^2$$

$$r_A^2 = 0.46(22) \text{ fm}^2, \quad \sigma_{\nu n \rightarrow \mu p}(E_\nu = 1 \text{ GeV}) = 10.1(0.9) \times 10^{-39} \text{ cm}^2$$

compared with Bodek *et al.* [Eur. Phys. J. C 53, 349]:

$$r_A^2 = 0.453(13) \text{ fm}^2, \quad \sigma_{\nu n \rightarrow \mu p}(E_\nu = 1 \text{ GeV}) = 10.63(0.14) \times 10^{-39} \text{ cm}^2$$

Dipole model significantly underestimates error from nucleon form factor

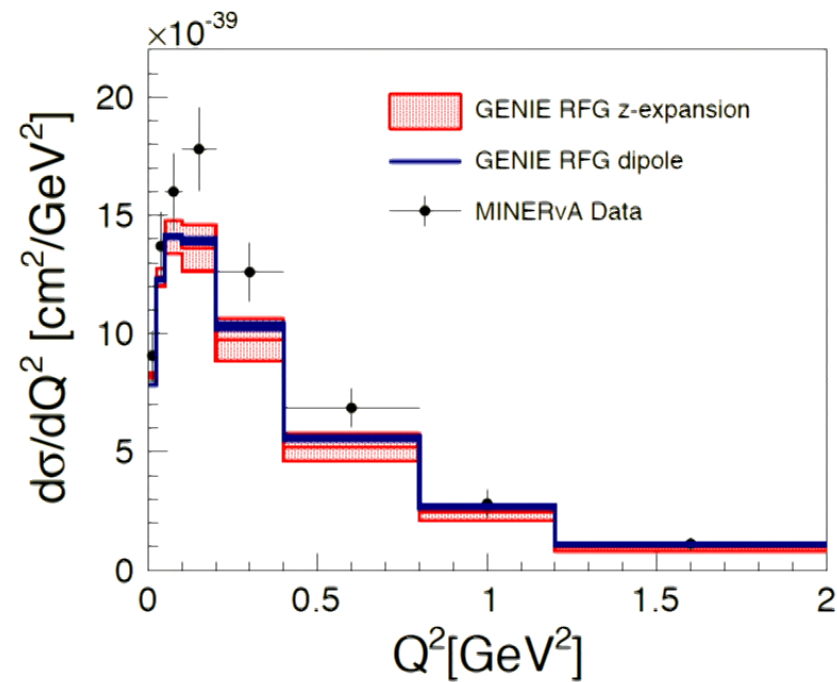
Most theoretically clean data do not constrain form factor precisely

z Expansion in GENIE

z expansion coded into GENIE - turned on with user switch

Officially released in production version 2.12

Uncertainties on free-nucleon cross section as large as data-theory discrepancy
⇒ need to improve F_A determination to make headway on nuclear effects



Nucleon axial form factor $G_A(Q^2)$

Previously, [Lin,0802.0863], [Yamazaki,0904.2039], [Bratt,1001.3620], [Bali,1412.7336]

Needed for neutrino oscillation experiments:

Charged current quasielastic (CCQE) neutrino-nucleus interaction must be known to high precision.

Connecting quark - nucleon level: $G_A(Q^2)$ form factor.
nucleon - nucleus level: nuclear model.

Traditionally: information on $G_A(Q^2)$ extracted from expt. using dipole fit:

$$G_A(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2} \quad \langle r_A^2 \rangle = \frac{12}{M_A^2}$$

World average (pre 1990) from ν scattering $M_A = 1.026(21)$ GeV.

Overconstrained form: different measurements, different M_A .

Lower energy expts: e.g. MiniBooNE: $M_A = 1.35(17)$ GeV

[Aguilar-Arevalo,1002.2680]

Systematics being explored including new analysis of old expt data:

$\langle r_A^2 \rangle = 0.46(22)$ fm² \rightarrow $M_A = 1.01(24)$ GeV from z-expansion [Meyer,1603.03048].

Nucle Four computations of $G_A(Q^2)$ appeared in response:

Previous

Needed

Charge

to high

Connec

LHPC 1703.06703 [hep-lat]

ETMC 1705.03399 [hep-lat]

CLS 1705.06186 [hep-lat]

PNDME 1705.06834 [hep-lat]

Additional g_A computation (CalLat): 1704.01114

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Feynman-Hellman (CalLat)

Perturb the action with a bilinear current density:

$$S_\lambda = \lambda \int d^4x j(x)$$

With this action, correlation functions are modified

$$C_\lambda(t) = \langle \lambda | N(t) N^\dagger(0) | \lambda \rangle$$

$$-\left. \frac{\partial C_\lambda(t)}{\partial \lambda} \right|_{\lambda=0} = -C_\lambda(t) \Big|_{\lambda=0} \int dt' \langle \Omega | J(t') | \Omega \rangle + \int dt' \langle \Omega | T \{ N(t) J(t') N^\dagger(0) \} | \Omega \rangle$$

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Vacuum matrix element vanishes for non-scalar currents

Derivative realized by replacing quark propagator with spacetime volume-averaged current insertion:

- ▶ Better volume averaging than RHS \implies better statistical properties
- ▶ LHS derivative evaluated analytically
- ▶ Computationally cheaper than RHS

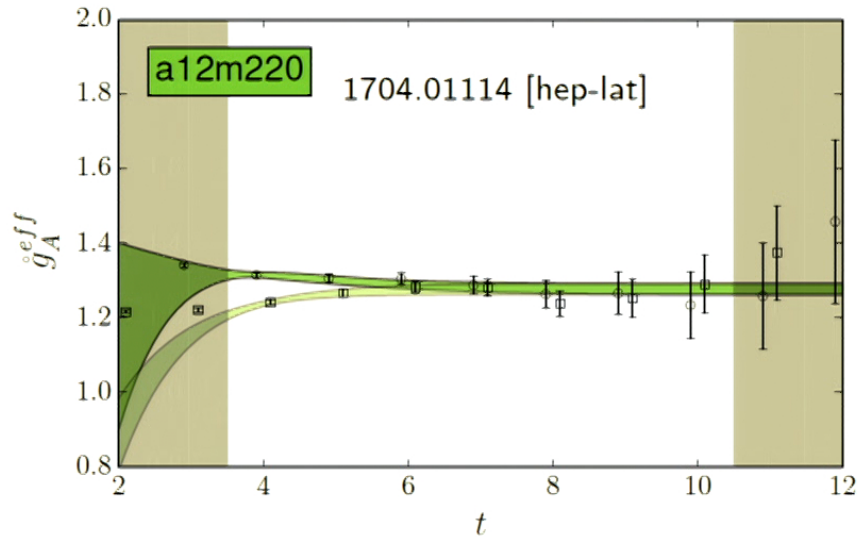
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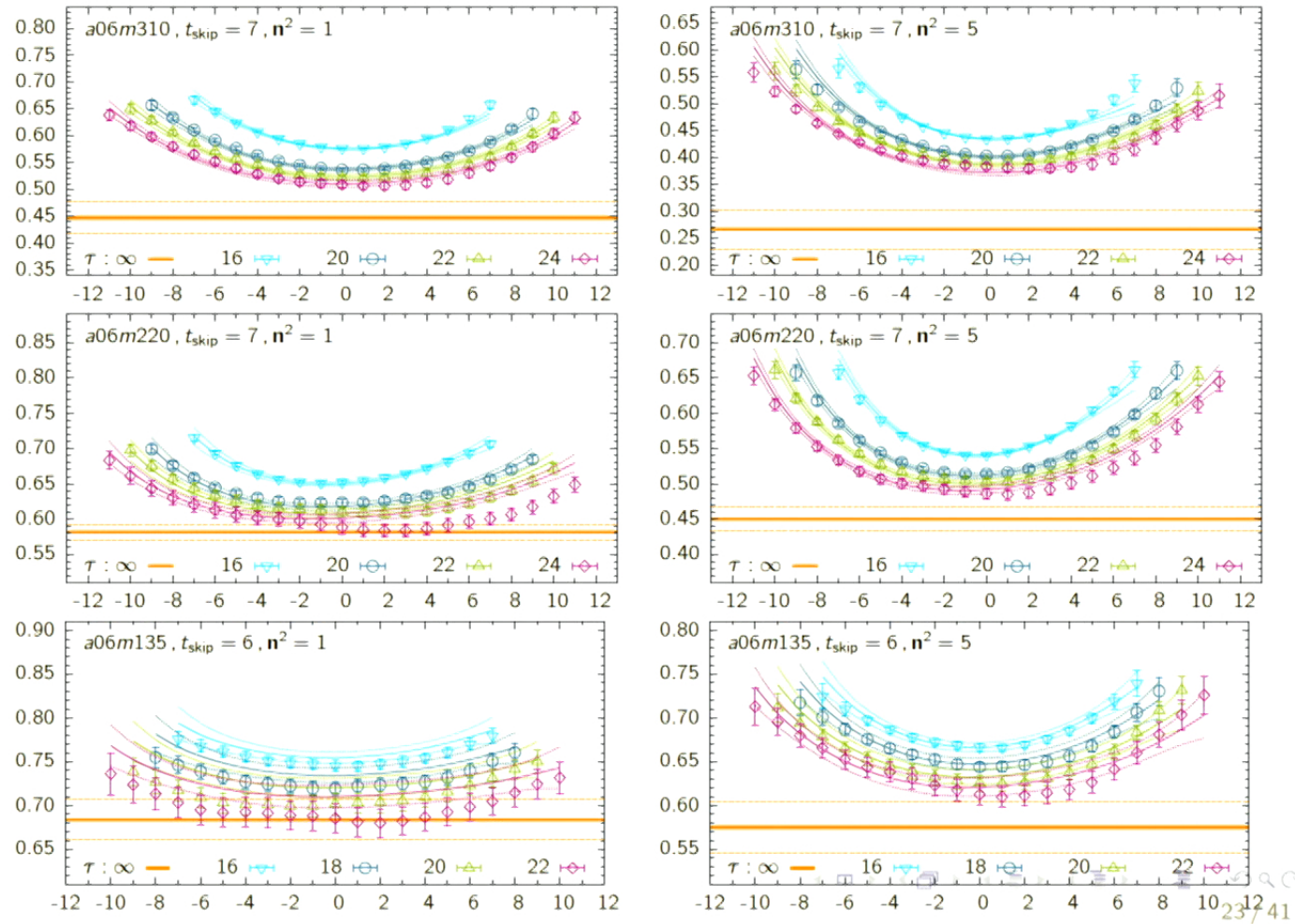


$$dt' \langle \Omega | T \{ N(t) J(t') N^\dagger(0) \} | \Omega \rangle$$

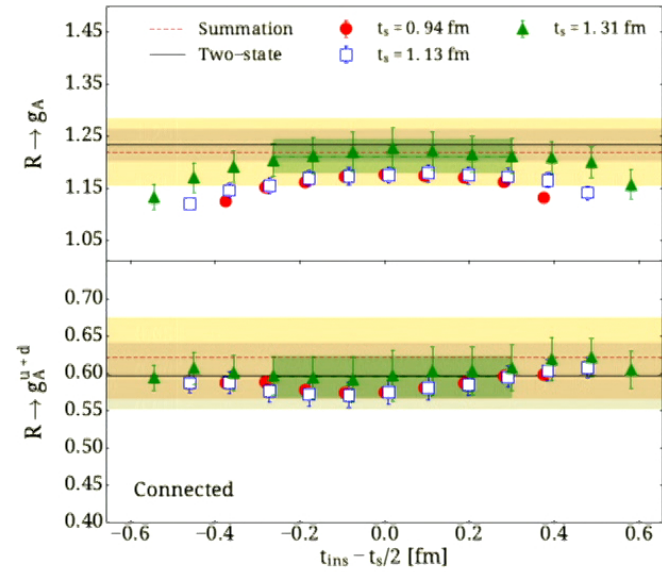
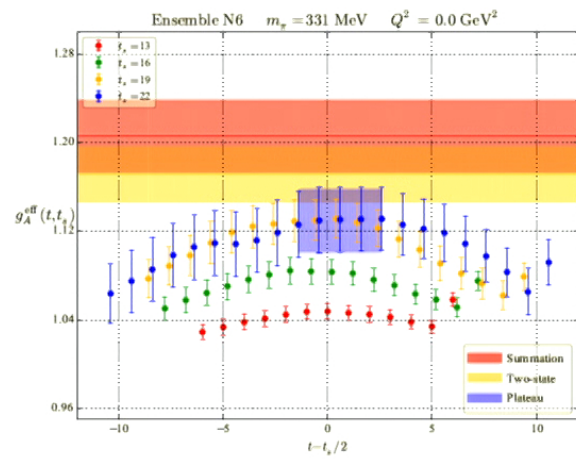
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statistical properties

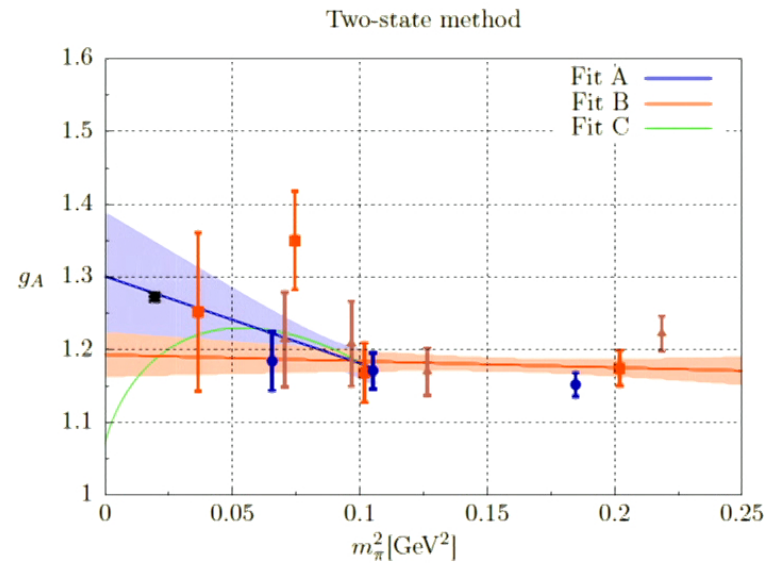
Excited State Contamination (PNDME)



Excited State Contamination (CLS,ETMC)



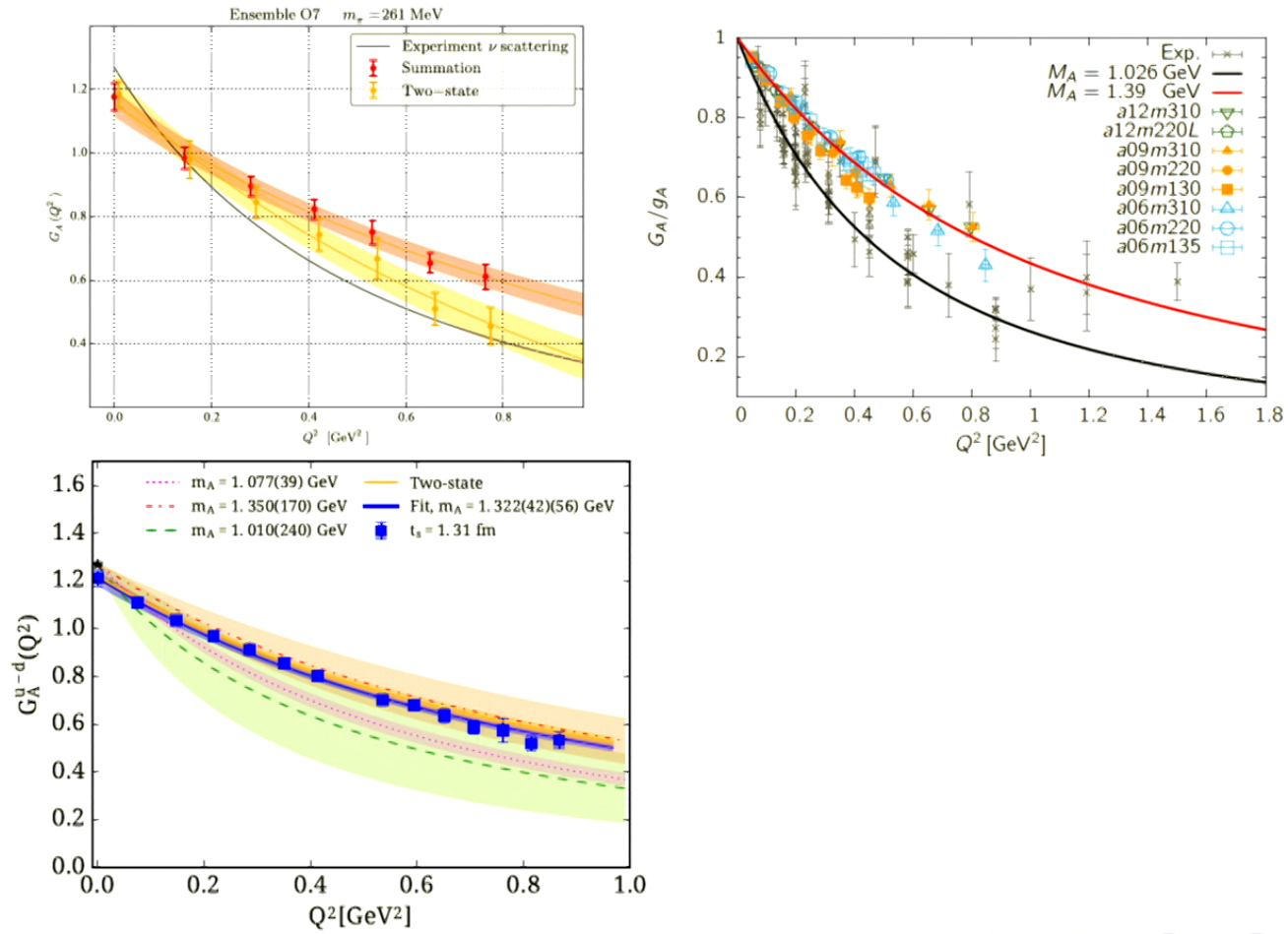
Extrapolations (CLS)



At NNLO (from 1704.01114 [hep-lat], CLS sets $c_3 = 0$):

$$g_A = \dot{g}_A + \epsilon_\pi^2 \left[c_2 - \left(\dot{g}_A + 2\dot{g}_A^3 \right) \log \epsilon_\pi^2 \right] + \epsilon_\pi^3 \dot{g}_A c_3 + \dots, \quad \epsilon_\pi = \frac{M_\pi}{4\pi f_\pi}$$

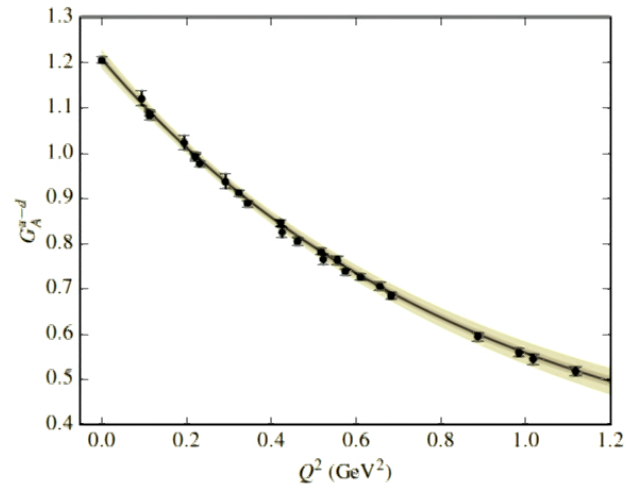
Form Factor Q^2 dependence (CLS,PNDME,ETMC)



Summary

Ref.	g_A	$\langle r_A^2 \rangle$ [fm ²]
LHPC	1.208(6)(16)(1)(10)	0.213(6)(13)(3)(0)
CalLat	1.278(21)(26)	—
ETMC	1.212(33)(22)	0.267(9)(11)
CLS	1.278(68) ⁽⁺⁰⁰⁾ ₍₋₈₇₎	0.360(36) ⁽⁺⁸⁰⁾ ₍₋₈₈₎
PNDME	1.195(33)(20)	0.22(7)(3)

CLS $F_A(Q^2)$:



NC Axial/Pseudoscalar Form Factors

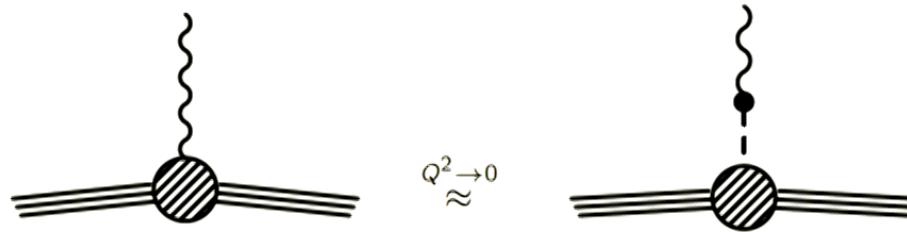
Pseudoscalar Form Factor

Pseudoscalar spinor factor vanishes at $Q^2 = 0$

\Rightarrow Form factor must be computed at $Q^2 \neq 0$, extrapolated back to $Q^2 = 0$

Otherwise, $F_P(Q^2)$ computation completely analogous to $F_A(Q^2)$

Can be related to axial form factor by pion pole dominance ansatz and PCAC:



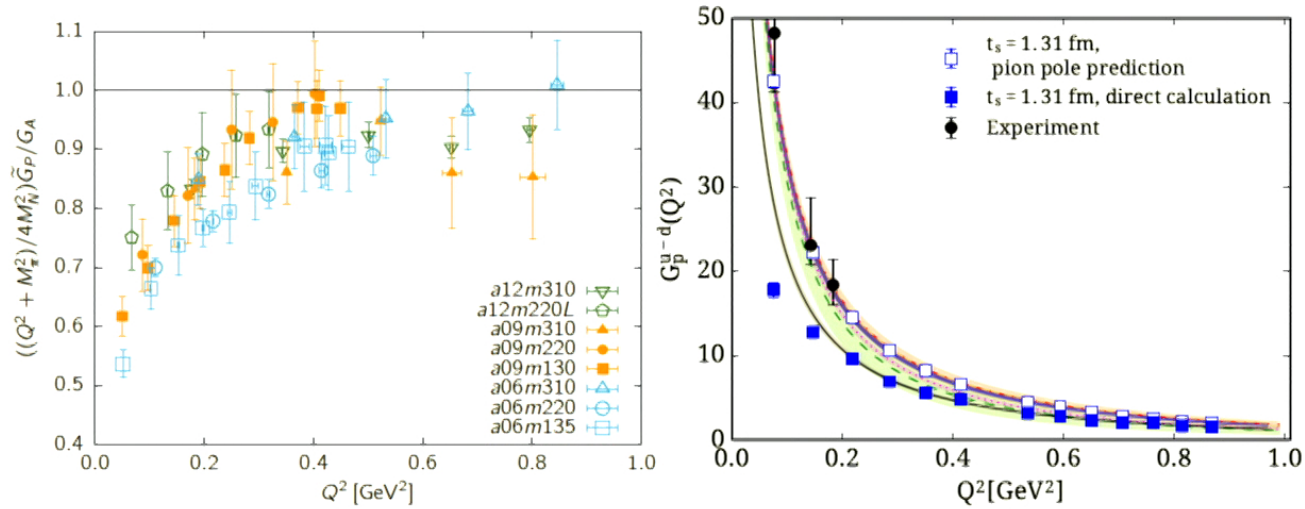
PCAC:

$$\partial^\mu \langle A_\mu^a(x) \rangle = 2m_q \langle P^a(x) \rangle$$

\Rightarrow

$$F_P(Q^2) = F_A(Q^2) \frac{4M_N^2}{Q^2 + M_\pi^2}$$

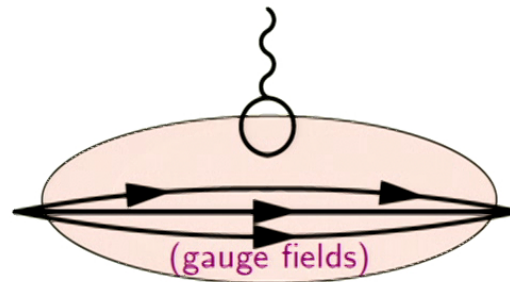
Departure from Pole Dominance (PNDME, ETMC)



With $g_P^* = \frac{m_\mu}{2M_N} G_P(Q^2 = 0.88m_\mu^2)$:

Ref.	g_P^*
LHPC	8.47(21)(87)(2)(7)
ETMC	—
CLS	7.7(1.8) $^{+0.8}_{-2.0}$
PNDME	4.49(19)

Disconnected Diagrams



Matrix element: $\langle N | \bar{Q} \gamma_\mu \gamma_5 Q | N \rangle$

Disconnected diagram: quarks connected by gauge field loops to all orders

Noisy, costly to compute:

- ▶ Sensitive to gauge fields/gauge configuration
- ▶ Stochastic sampling of correlation functions quickly saturate
- ▶ Many gauge configurations needed to improve convergence, costly to generate

Need lots of gauge configurations, lots of statistics to estimate well

More to gain from more efficient/smarter computation algorithms

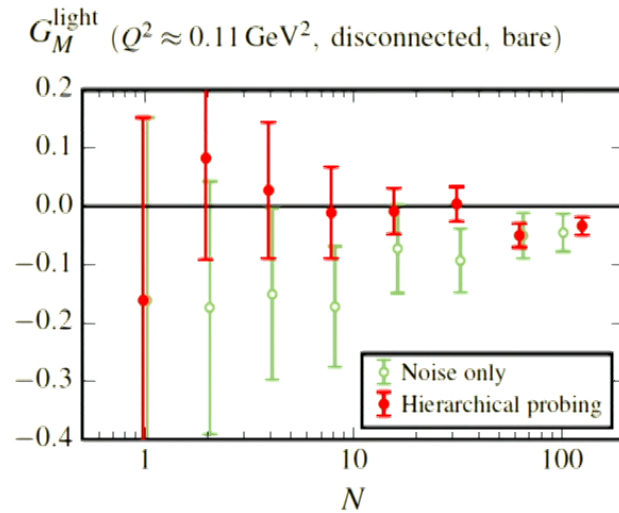
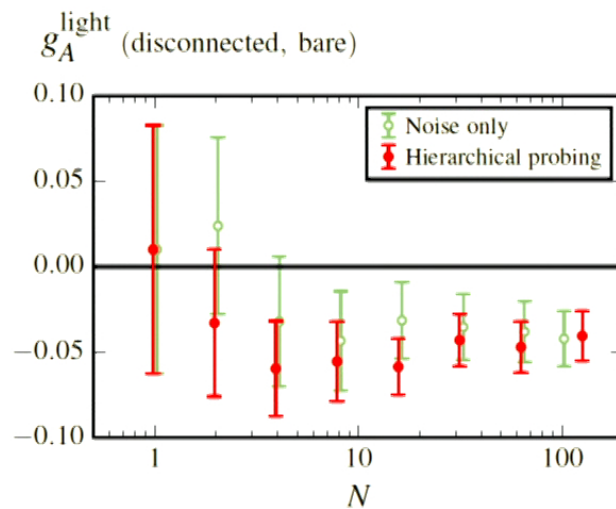
LHPC

Evaluation of quark loop contribution:

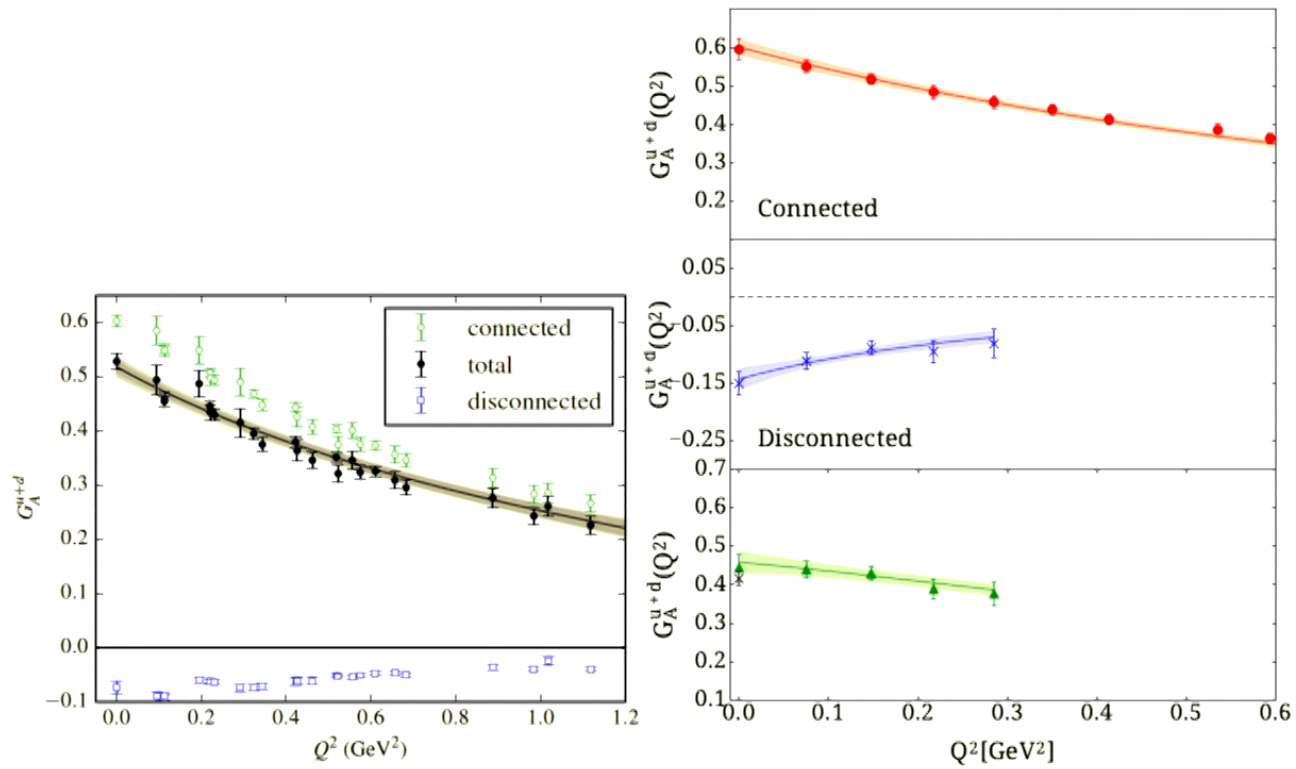
$$\langle N | \bar{Q} \Gamma Q | N \rangle \sim \text{Tr} \left[\Gamma (\not{D} + m)^{-1} \right]$$

Typical algorithms solve for $(\not{D} + m)^{-1}$, then perform trace

Hierarchical Probing: solve for trace directly + sample phase space more efficiently



Form Factor Q^2 dependence (CLS,PNDME,ETMC)



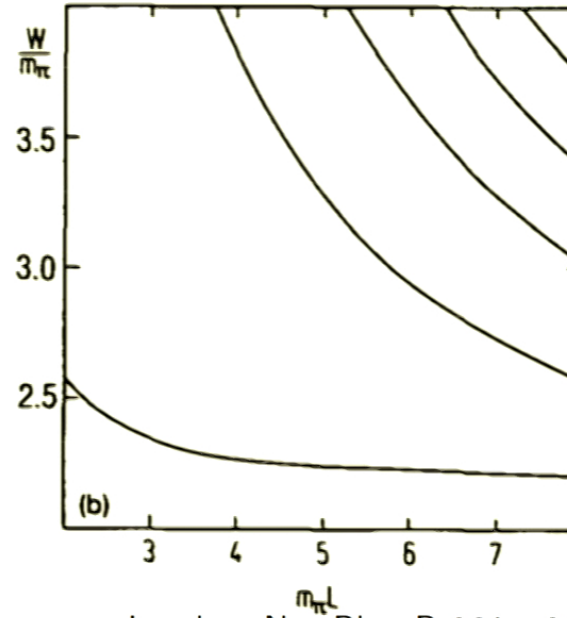
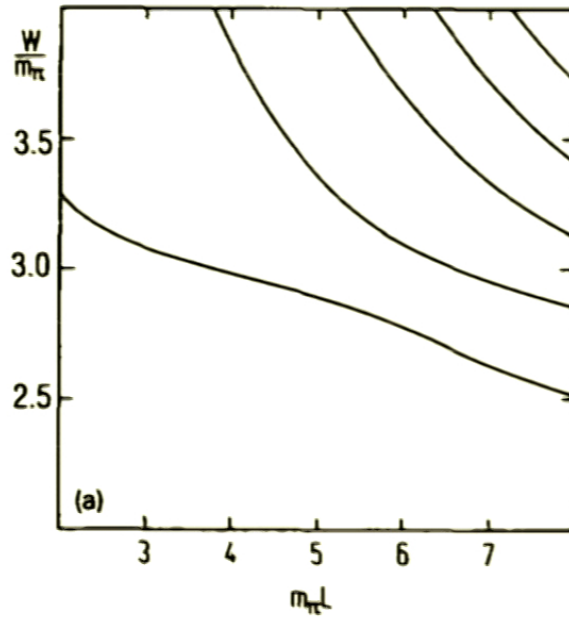
Transition Form Factors

LQCD with Resonances

For stable particles, interpretation of energy spectrum is cleaner

Unstable particles are not so straightforward:

- ▶ Resonances decay into two-particle states
- ▶ Two particle states are spatially large, impractical to isolate
- ▶ Need to enforce energy/momentum conservation with discretized momenta



Luscher, Nuc.Phys.B 364, p237.

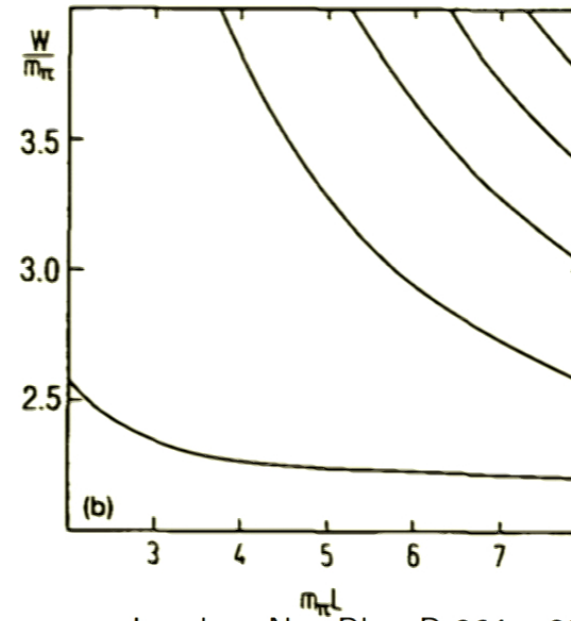
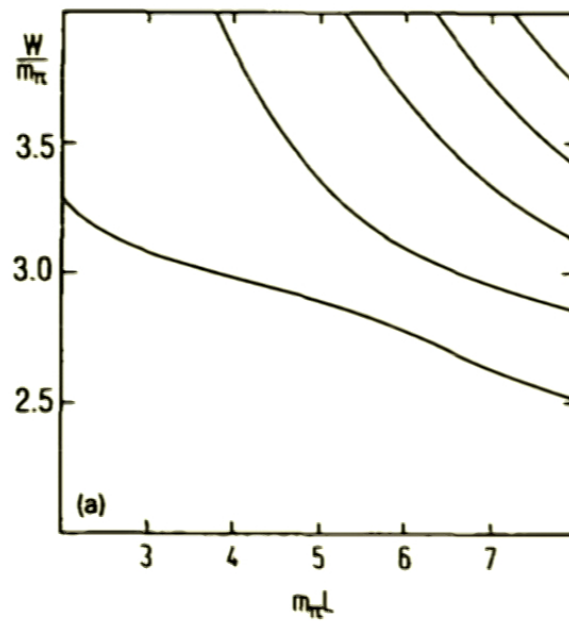
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LQCD with Resonances

Spectrum related to scattering phase shift, can be computed on multiple volumes

Costly to generate new ensembles, typically not done

⇒ WORKAROUND: keep M_π large ⇒ Δ baryon stable



Luscher, Nuc.Phys.B 364, p237.

Transition Form Factors (ETMC)

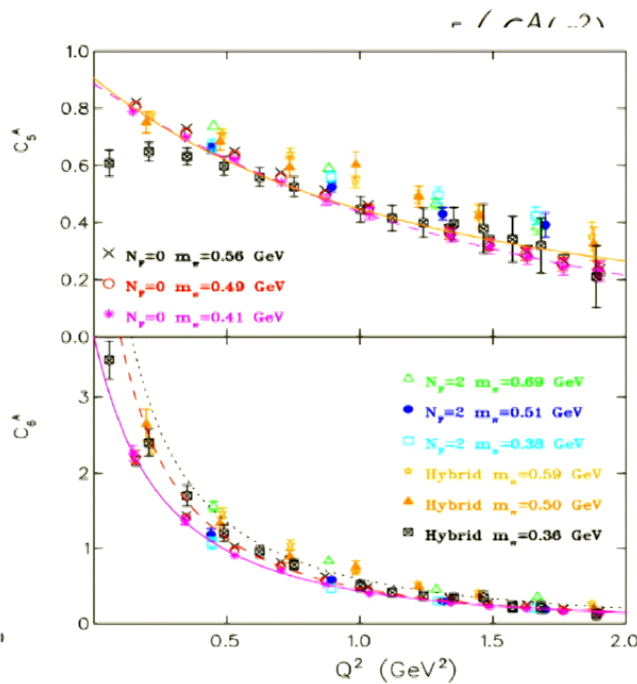
Used for determining $\nu N \rightarrow \Delta \rightarrow N\pi$ interactions

$$\begin{aligned} \langle \Delta(p', s') | A_\mu^3 | N(p, s) \rangle &= i \sqrt{\frac{2}{3}} \left(\frac{m_\Delta m_N}{E_\Delta(\mathbf{p}') E_N(\mathbf{p})} \right)^{1/2} \\ \times \bar{u}_{\Delta^+}^\lambda(p', s') &\left[\left(\frac{C_3^A(q^2)}{m_N} \gamma^\nu + \frac{C_4^A(q^2)}{m_N^2} p'^\nu \right) (g_{\lambda\mu} g_{\rho\nu} - g_{\lambda\rho} g_{\mu\nu}) q^\rho \right. \\ &\left. + C_5^A(q^2) g_{\lambda\mu} + \frac{C_6^A(q^2)}{m_N^2} q_\lambda q_\mu \right] u_P(p, s) \end{aligned}$$

Transition Form Factors (ETMC)

Used for determining $\nu N \rightarrow \Delta \rightarrow N\pi$ interactions

$$\langle \Delta(p', s') | A_\mu^3 | N(p, s) \rangle = i \sqrt{\frac{2}{3}} \left(\frac{m_\Delta m_N}{E_\Delta(\mathbf{p}') E_N(\mathbf{p})} \right)^{1/2}$$

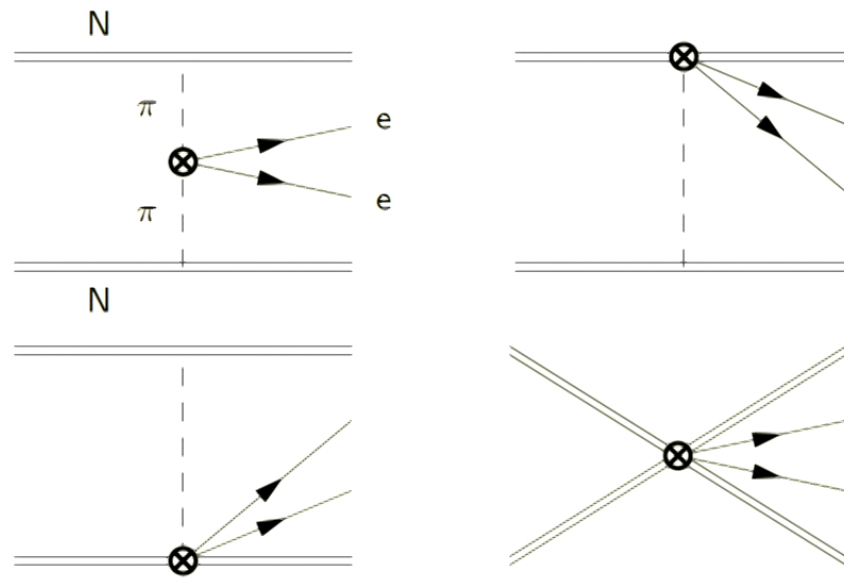


$$+ \frac{C_4^A(q^2)}{m_N^2} p'^\nu \left(g_{\lambda\mu} g_{\rho\nu} - g_{\lambda\rho} g_{\mu\nu} \right) q^\rho$$

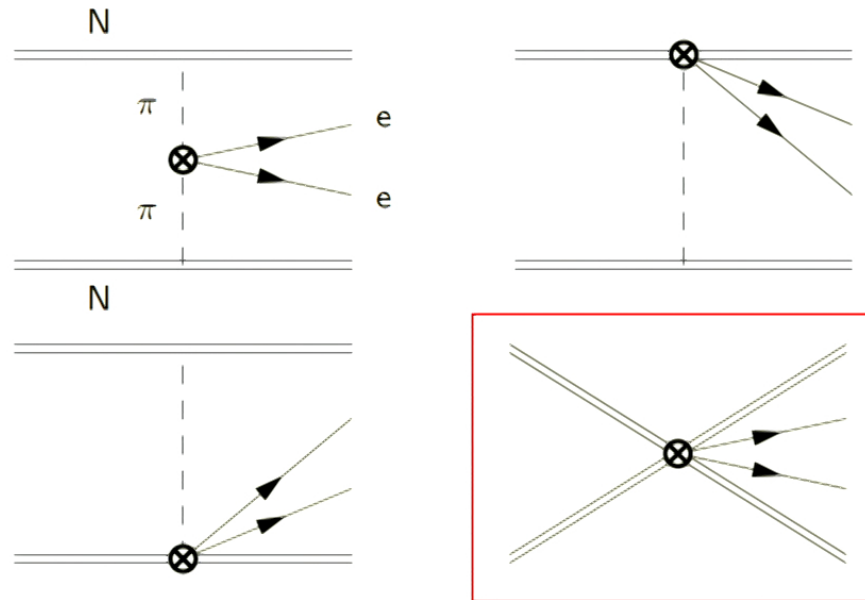
$$+ C_5^A(q^2) g_{\lambda\mu} + \frac{C_6^A(q^2)}{m_N^2} q_\lambda q_\mu \Big] u_P(p, s)$$

0710.2173 [hep-lat]

Neutrinoless Double Beta Decay



Neutrinoless Double Beta Decay

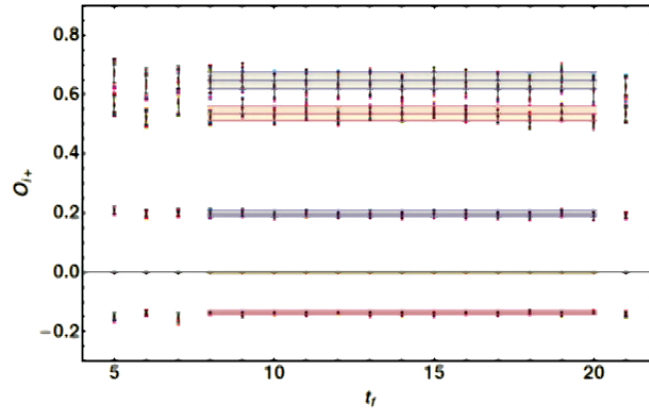


Difficult matrix element is short-range nucleon matrix element

Underway, but not complete yet!

Callat

Operators from the first diagram on previous slide $\implies \pi \rightarrow ee\pi$



1608.04793 [hep-lat]

$$\begin{aligned}
 \mathcal{O}_{1+}^{++} &= (\bar{q}_{LT} \tau^+ \gamma^\mu q_L) [\bar{q}_{RT} \tau^+ \gamma_\mu q_R] \\
 \mathcal{O}_{2+}^{++} &= (\bar{q}_{RT} \tau^+ q_L) [\bar{q}_{RT} \tau^+ q_L] + (\bar{q}_{LT} \tau^+ q_R) [\bar{q}_{LT} \tau^+ q_R] \\
 \mathcal{O}_{3+}^{++} &= (\bar{q}_{LT} \tau^+ \gamma^\mu q_L) [\bar{q}_{LT} \tau^+ \gamma_\mu q_L] + (\bar{q}_{RT} \tau^+ \gamma^\mu q_R) [\bar{q}_{RT} \tau^+ \gamma_\mu q_R] \\
 \mathcal{O}'_{1+}{}^{++} &= (\bar{q}_{LT} \tau^+ \gamma^\mu q_L) [\bar{q}_{RT} \tau^+ \gamma_\mu q_R] \\
 \mathcal{O}'_{2+}{}^{++} &= (\bar{q}_{LT} \tau^+ \gamma^\mu q_L) [\bar{q}_{LT} \tau^+ \gamma_\mu q_L] + (\bar{q}_{RT} \tau^+ \gamma^\mu q_R) [\bar{q}_{RT} \tau^+ \gamma_\mu q_R]
 \end{aligned}$$

Parentheses () and brackets [] denote terms with dotted color indices

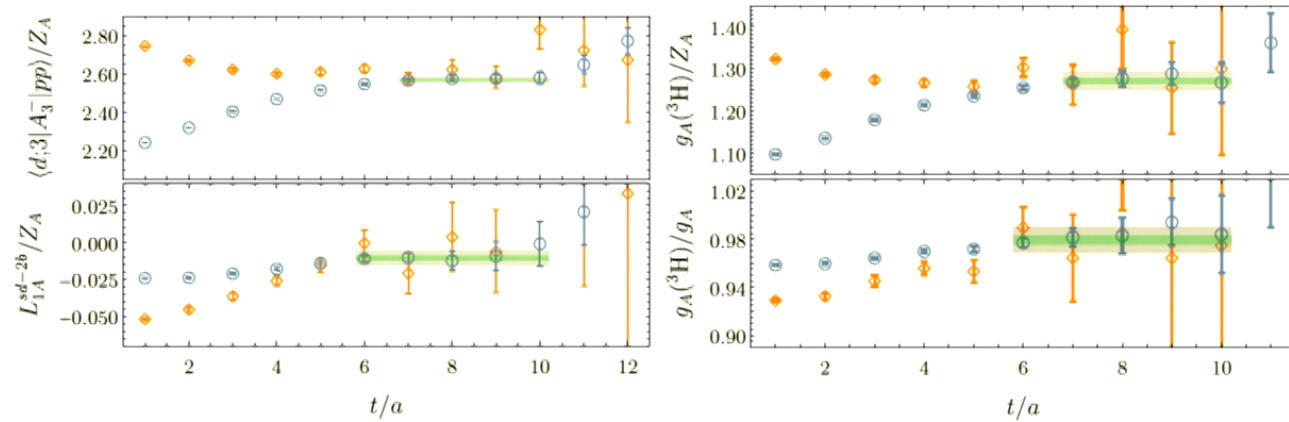
NPLQCD

Axial current computed on nucleus rather than nucleon

g_A “quenching”

Neutrinoless double beta decay sensitive to nuclear Gamow-Teller transition $\Rightarrow g_A^4$

Computations at $M_\pi = 800$ MeV!



1610.04545 [hep-lat]

Concluding Remarks

- ▶ Lattice QCD practitioners are thinking about neutrino physics
→ more dialogue with experimentalists is needed!
- ▶ Advances in computing power as well as computational strategies will allow for more precise and more sophisticated analyses
- ▶ Lattice QCD is an excellent tool for informing experiment and improving systematic uncertainties
- ▶ Baryon physics difficult to compute, but starting to probe multi-nucleon matrix elements

Thanks for listening!