Title: The proton radius puzzle

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Abstract:

The Proton Radius Puzzle

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Charge radius from atomic physics

- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm
  more recently $r_E^p = 0.84087(39)$ fm [Antognini et al. Science **339**, 417 (2013)]
- CODATA value [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.87680(690)$ fm
  more recently $r_E^p = 0.87510(610)$ fm [Mohr et al. RMP **88**, 035009 (2016)]
  extracted mainly from (electronic) hydrogen
- **5σ discrepancy!**
- This is the proton radius puzzle

What could be the reason for the discrepancy?

- What could the reason for the discrepancy?

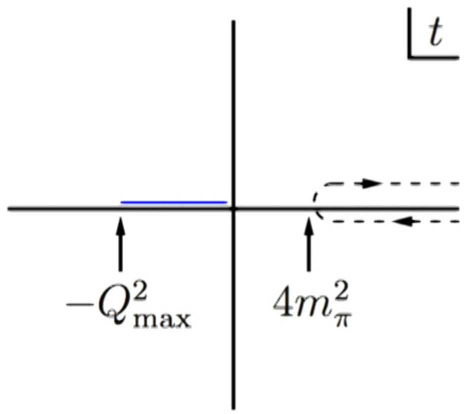
Outline

- Introduction: The proton radius puzzle
- Part 1: Proton radii from scattering
- Part 2: Hadronic Uncertainty?
- Part 3: Connecting muon-proton scattering and muonic hydrogen
- Conclusions and outlook

Form Factors: What we do know

- Analytic properties of $G_E^p(t)$ and $G_M^p(t)$ are known
- They are analytic outside a cut $t \in [4m_\pi^2, \infty]$

  [Federbush, Goldberger, Treiman, Phys. Rev. **112**, 642 (1958)]
- $e - p$ scattering data is in $t < 0$ region

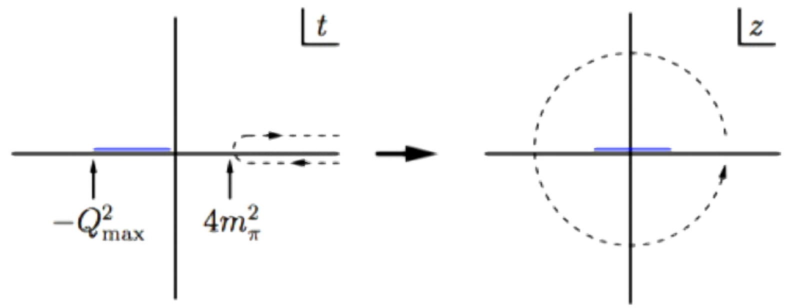


z expansion

- z expansion: map domain of analyticity onto unit circle

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

where $t_{\rm cut} = 4m_\pi^2$, $z(t_0, t_{\rm cut}, t_0) = 0$

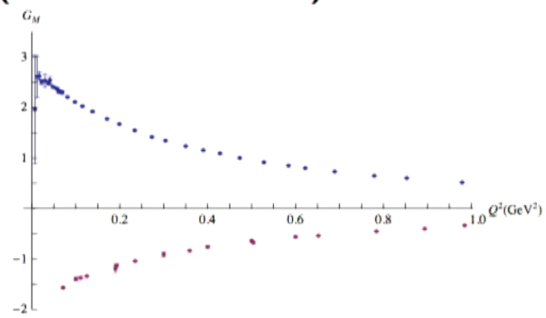


- Expand $G^p_{E,M}$ in a Taylor series in z: $G^p_{E,M}(q^2) = \sum_{k=0}^{\infty} a_k\, z(q^2)^k$

- **The** method for **meson** form factors
  [Flavor Lattice Averaging Group, EPJ C **74**, 2890 (2014)]

z expansion

- [Zachary Epstein, GP, Joydeep Roy PRD **90**, 074027 (2014)]
  $G_M(Q^2)$ for proton (blue, above axis) and neutron (red, below axis)



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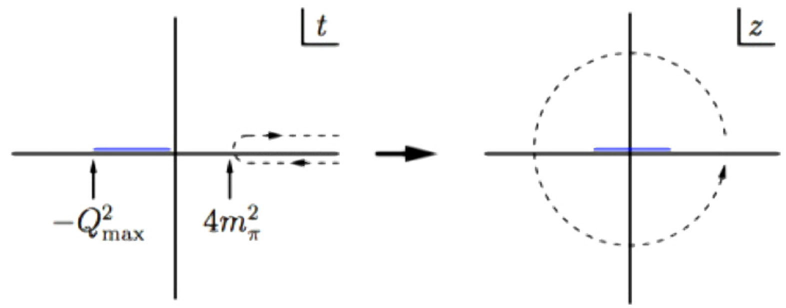

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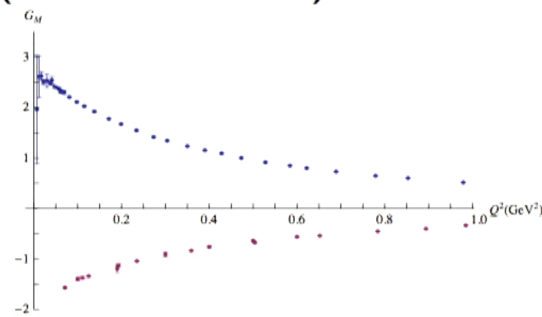

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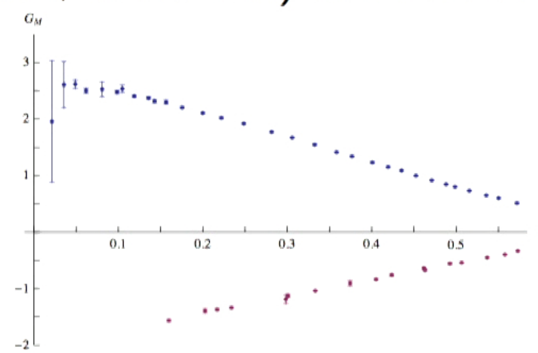

- See also R.J. Hill talk at FPCP 2006 [hep-ph/0606023]

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Extracting r_E^p using the z expansion

- First use of the z expansion to extract r_E^p
  [Richard J. Hill, GP PRD **82** 113005 (2010)]

- Proton: $Q^2 < 0.5$ GeV^2

$$r_E^p = 0.870 \pm 0.023 \pm 0.012\,\mathrm{fm}$$

- Proton and neutron data

$$r_E^p = 0.880^{+0.017}_{-0.020} \pm 0.007\,\mathrm{fm}$$

- Proton, neutron and $\pi\pi$ data

$$r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$$

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PDG 2016

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016)

p CHARGE RADIUS

This is the rms electric charge radius, $\sqrt{\langle r_E^2 \rangle}$.

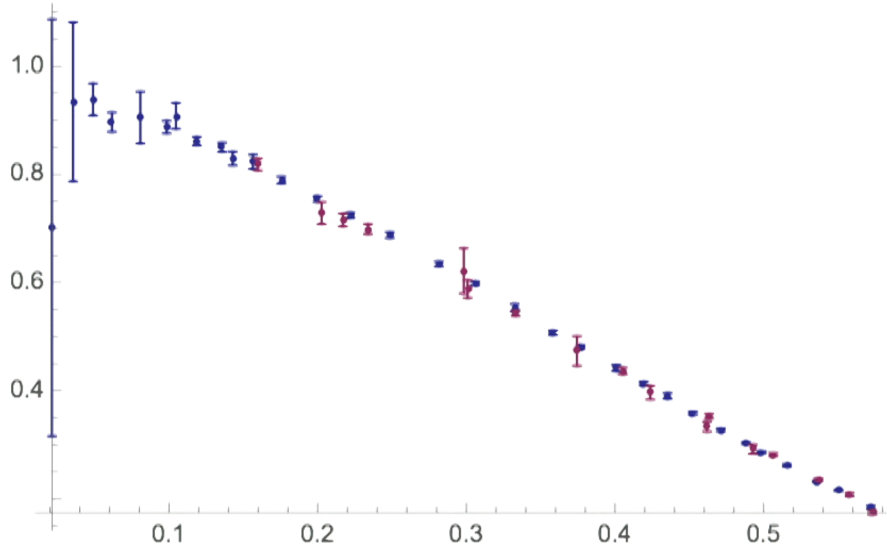
VALUE (fm)			*DOCUMENT ID*		*TECN*	*COMMENT*
0.8751	**±0.0061**		MOHR	16	RVUE	2014 CODATA value
0.84087	**±0.00026**	**±0.00029**	ANTOGNINI	13	LASR	μp-atom Lamb shift
• • • We do not use the following data for averages, fits, limits, etc. • • •						
0.895	±0.014	±0.014	[1] LEE	15	SPEC	Just 2010 Mainz data
0.916	±0.024		LEE	15	SPEC	World data, no Mainz
0.8775	±0.0051		MOHR	12	RVUE	2010 CODATA, ep data
0.875	±0.008	±0.006	ZHAN	11	SPEC	Recoil polarimetry
0.879	±0.005	±0.006	BERNAUER	10	SPEC	$ep \to ep$ form factor
0.912	±0.009	±0.007	BORISYUK	10		reanalyzes old ep data
0.871	±0.009	±0.003	HILL	10		z-expansion reanalysis
0.84184	±0.00036	±0.00056	POHL	10	LASR	See ANTOGNINI 13
0.8768	±0.0069		MOHR	08	RVUE	2006 CODATA value
0.844	$^{+0.008}_{-0.004}$		BELUSHKIN	07		Dispersion analysis
0.897	±0.018		BLUNDEN	05		SICK 03 + 2γ correction
0.8750	±0.0068		MOHR	05	RVUE	2002 CODATA value
0.895	±0.010	±0.013	SICK	03		$ep \to ep$ reanalysis

[Hill, GP PRD **82** 113005 (2010)]

Neutron and proton magnetic radii

[Epstein, GP, Roy PRD **90**, 074027 (2014)]

- $G_M(z)/G_M(0)$ for proton (blue) and neutron (red)



$$r_M^2 = 6 \frac{d}{dq^2} \frac{G_M(q^2)}{G_M(0)}\bigg|_{q^2=0}$$

Neutron and proton magnetic radii

- Comparing the proton and neutron magnetic radii
  $r_M^p = 0.87 \pm 0.02$ fm
  $r_M^n = 0.89 \pm 0.03$ fm

- They are equal within errors...

- Proton Magnetic moment/ Neutron Magnetic moment ratio can be explained by $SU(6)$ symmetry or quark model

- Is there a reason to the relation between the normalized slopes?

z expansion

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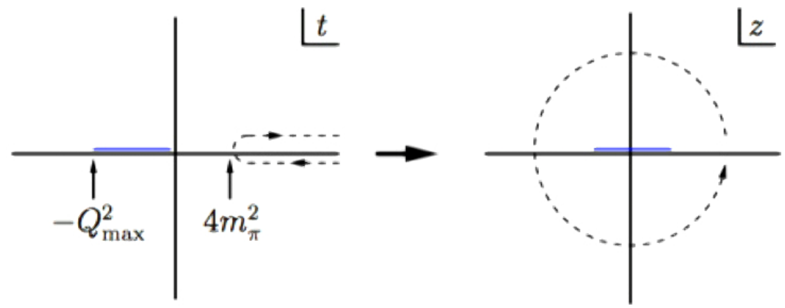
where $t_{\rm cut} = 4m_\pi^2$, $z(t_0, t_{\rm cut}, t_0) = 0$



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- **The** method for **meson** form factors
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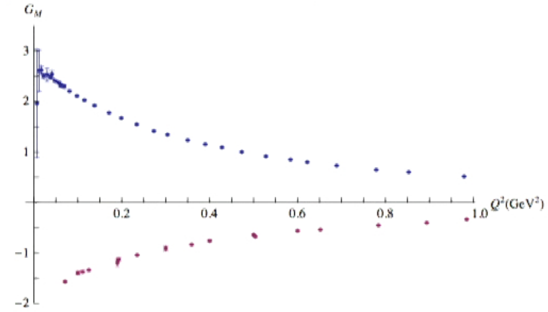
  $G_M(Q^2)$ for proton (blue, above axis) and neutron (red, below axis)



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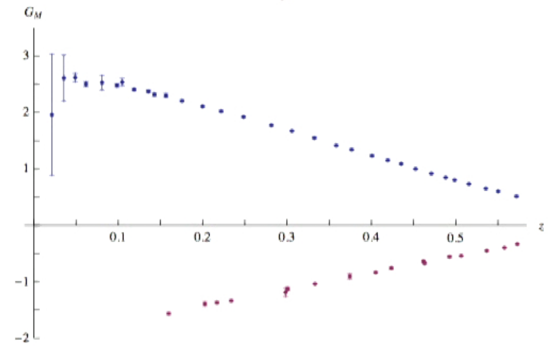

- See also R.J. Hill talk at FPCP 2006 [hep-ph/0606023]

Latest z expansion fit

- Most recent study using the z expansion
  [Gabriel Lee, J. R. Arrington, and R. J. Hill, PRD **92**, 013013 (2015)]
  Analyze the "Mainz" data set
  [J. C. Bernauer et al. PRL **105**, 242001 (2010)]
  and world data (excluding Mainz)

- World data

  [Lee, Arrington, Hill '15] $r_E^p = 0.918 \pm 0.024$ fm
  [Hill , GP '10] $r_E^p = 0.870 \pm 0.023 \pm 0.012$ fm

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- Mainz data
  $r_E^p = 0.895 \pm 0.020$ fm $r_M^p = 0.773 \pm 0.038$ fm

PDG 2016

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VALUE (fm)	*DOCUMENT ID*		*TECN*	*COMMENT*
0.776±0.034±0.017	[1] LEE	15	SPEC	Just 2010 Mainz data
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0.914±0.035	LEE	15	SPEC	World data, no Mainz
0.87 ±0.02	EPSTEIN	14		Using ep, en, $\pi\pi$ data
0.867±0.009±0.018	ZHAN	11	SPEC	Recoil polarimetry
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0.876±0.010±0.016	BORISYUK	10		Reanalyzes old $ep \to ep$ data
0.854±0.005	BELUSHKIN	07		Dispersion analysis

[1] Authors also provide values for a combination of all available data.

[Epstein, GP, Roy PRD **90**, 074027 (2014)]

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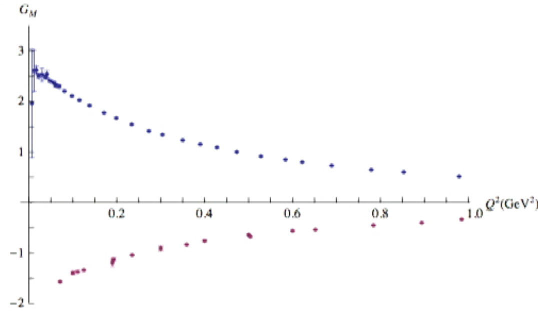

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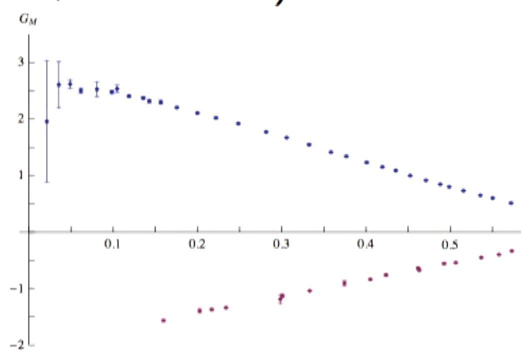

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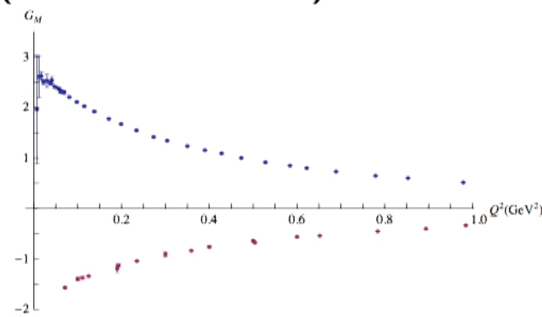

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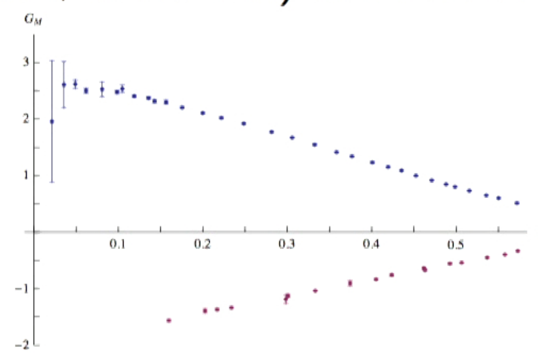

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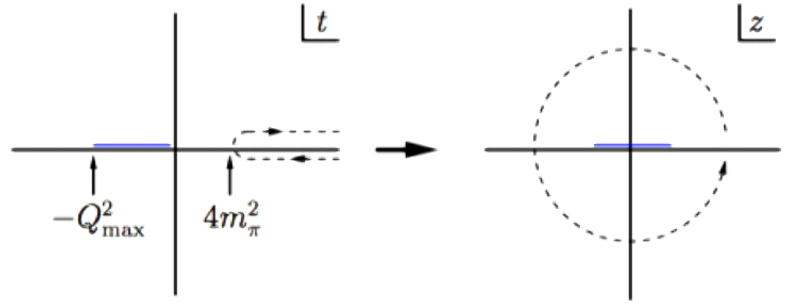

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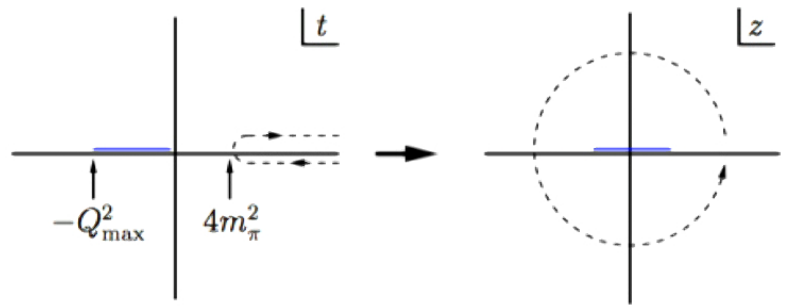

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Muonic hydrogen theory

- Is there a problem with muonic hydrogen *theory*?

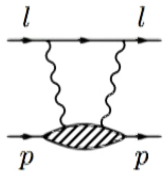
- Potentially yes!
  [Hill, GP PRL **107** 160402 (2011)]

- Muonic hydrogen measures ΔE and translates it to r_E^p

- [Pohl et al. Nature **466**, 213 (2010) Supplementary information]
  $\Delta E = 206.0573(45) - 5.2262(r_E^p)^2 + 0.0347(r_E^p)^3$ meV

- [Antognini et al. Science **339**, 417 (2013), Ann. of Phy. **331**, 127]
  $\Delta E = 206.0336(15) - 5.2275(10)(r_E^p)^2 + 0.0332(20)$ meV

- In both cases apart from r_E^p need two-photon exchange



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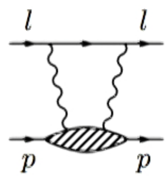

$$W^{\mu\nu} = \frac{1}{2}\sum_s i\int d^4x\, e^{iq\cdot x}\langle \boldsymbol{k},s|T\{J^\mu_{\rm e.m.}(x)J^\nu_{\rm e.m.}(0)\}|\boldsymbol{k},s\rangle$$

$$= \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)W_1 + \left(k^\mu - \frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu - \frac{k\cdot q\, q^\nu}{q^2}\right)W_2$$

Two photon exchange

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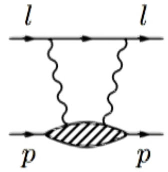

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$$= \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)W_1+\left(k^\mu-\frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu-\frac{k\cdot q\, q^\nu}{q^2}\right)W_2$$

- Dispersion relations ($\nu = 2k\cdot q,\ Q^2=-q^2$)

$$W_1(\nu,Q^2) = W_1(0,Q^2)+\frac{\nu^2}{\pi}\int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2\,\frac{{\rm Im}\,W_1(\nu',Q^2)}{\nu'^2(\nu'^2-\nu^2)}$$

$$W_2(\nu,Q^2) = \frac{1}{\pi}\int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2\,\frac{{\rm Im}\,W_2(\nu',Q^2)}{\nu'^2-\nu^2}$$

- W_1 requires subtraction...

NRQED

- You already know NRQED!

$$D_t = \frac{\partial}{\partial t} + ieA^0, \quad \boldsymbol{D} = \boldsymbol{\nabla} - ie\boldsymbol{A}$$

- Schrödinger equation: $iD_t + \frac{\boldsymbol{D}^2}{2m_p}$

- Hydrogen Fine Structure:
  - Spin-Orbit: $\boldsymbol{\sigma} \cdot \boldsymbol{B}$
  - Relativistic correction: $\boldsymbol{D}^4$
  - Darwin term: $\boldsymbol{\nabla} \cdot \boldsymbol{E}$

NRQED (NRQCD) Lagrangian

- The $1/m_p^2$ were given in Caswell, Lepage '86

$$\mathcal{L}_{\text{NRQED}} = \psi^\dagger \Big\{ iD_t + \frac{\boldsymbol{D}^2}{2m_p} + \frac{\boldsymbol{D}^4}{8m_p^3} + c_F g \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2m_p} + c_D g \frac{[\boldsymbol{\partial}\cdot\boldsymbol{E}]}{8m_p^2} + i c_S g \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E}-\boldsymbol{E}\times\boldsymbol{D})}{8m_p^2} + c_{W1} g \frac{\{\boldsymbol{D}^2, \boldsymbol{\sigma}\cdot\boldsymbol{B}\}}{8m_p^3} \Big\} \psi$$

NRQED (NRQCD) Lagrangian

- The $1/m_p^2$ were given in Caswell, Lepage '86

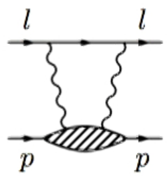
$$\mathcal{L}_{\text{NRQED}} = \psi^\dagger \left\{ iD_t + \frac{\boldsymbol{D}^2}{2m_p} + \frac{\boldsymbol{D}^4}{8m_p^3} + c_F g \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2m_p} + c_D g \frac{[\boldsymbol{\partial}\cdot\boldsymbol{E}]}{8m_p^2} + ic_S g \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E} - \boldsymbol{E}\times\boldsymbol{D})}{8m_p^2} + c_{W1} g \frac{\{\boldsymbol{D}^2, \boldsymbol{\sigma}\cdot\boldsymbol{B}\}}{8m_p^3} \right\} \psi$$

- The $1/m_p^3$ were given in Manohar '97

$$\mathcal{L}_{\text{NRQED}} = \psi^\dagger \left\{ \ldots - c_{W2} g \frac{D^i \boldsymbol{\sigma}\cdot\boldsymbol{B} D^i}{4m_p^3} + c_{p'p} g \frac{\boldsymbol{\sigma}\cdot\boldsymbol{D}\boldsymbol{B}\cdot\boldsymbol{D} + \boldsymbol{D}\cdot\boldsymbol{B}\boldsymbol{\sigma}\cdot\boldsymbol{D}}{8m_p^3} + ic_M g \frac{\{\boldsymbol{D}^i, [\boldsymbol{\partial}\times\boldsymbol{B}]^i\}}{8m_p^3} + c_{A1} g^2 \frac{\left(\boldsymbol{B}_a^i\boldsymbol{B}_b^i - \boldsymbol{E}_a^i\boldsymbol{E}_b^i\right) T^a T^b}{8M^3} - c_{A2} g^2 \frac{\boldsymbol{E}_a^i\boldsymbol{E}_b^i T^a T^b}{16M^3} + c_{A3} g^2 \frac{\left(\boldsymbol{B}_a^i\boldsymbol{B}_b^i - \boldsymbol{E}_a^i\boldsymbol{E}_b^i\right) \delta^{ab}}{8M^3} - c_{A4} g^2 \frac{\boldsymbol{E}_a^i\boldsymbol{E}_b^i \delta^{ab}}{16M^3} + - c_{B1} g^2 \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{B}_a\times\boldsymbol{B}_b - \boldsymbol{E}_a\times\boldsymbol{E}_b) f^{abc} T^c}{16M^3} + c_{B2} g^2 \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{E}_a\times\boldsymbol{E}_b) f^{abc} T^c}{16M^3} \right\} \psi$$

- Last line is non-zero only for Non-Relativistic Quantum Chromo Dynamics (NRQCD)

Two Photon exchange: small Q^2 limit



- Need poorly constrained non-perturbative function $W_1(0, Q^2)$
- Calculable in *small* Q^2 limit using NRQED
  [Hill, GP, PRL **107** 160402 (2011)]
  The photon sees the proton "almost" like an elementary particle

$$W_1(0, Q^2) = 2(c_F^2 - 1) + 2\frac{Q^2}{4m_p^2}\left(c_{A_1} + c_F^2 - 2c_F c_{W1} + 2c_M\right) + \mathcal{O}\left(Q^4\right)$$

- Matching

- Operators with one photon coupling:
  c_i given by $F_i^{(n)}(0)$
- Operators with only two photon couplings:
  c_{A_i} given by forward and backward Compton scattering

Two Photon exchange: small Q^2 limit

- *Small* Q^2 limit using NRQED [Hill, GP, PRL **107** 160402 (2011)]
  The photon sees the proton "almost" like an elementary particle

$$W_1(0,Q^2)=2a_p(2+a_p)+\frac{Q^2}{m_p^2}\left\{\frac{2m_p^3\bar{\beta}}{\alpha}-a_p-\frac{2}{3}\left[(1+a_p)^2m_p^2(r_M^p)^2-m_p^2(r_E^p)^2\right]\right\}+\mathcal{O}\left(Q^4\right)$$

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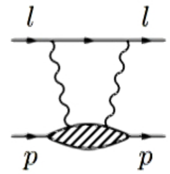
- $a_p = 1.793$, $\bar{\beta} = 2.5(4)\times 10^{-4}$ fm^3
- $r_M = 0.776(34)(17)$ fm,
- $r_E^H = 0.8751(61)$ fm or $r_E^{\mu H} = 0.84087(26)(29)$ fm

$$W_1(0,Q^2) = 13.6 + \frac{Q^2}{m_p^2}(-54\pm 7) + \mathcal{O}\left(Q^4\right)$$

Two Photon Exchange: large Q^2 limit



- Need poorly constrained non-perturbative function $W_1(0, Q^2)$
- Calculable in *large* Q^2 limit using Operator Product Expansion (OPE) [J. C. Collins, NPB **149**, 90 (1979)]
  The photon "sees" the quarks and gluons inside the proton

$$W_1(0, Q^2) = c/Q^2 + \mathcal{O}\left(1/Q^4\right)$$

- Result was used to estimate two photon exchange effects
- c calculated in [J. C. Collins, NPB **149**, 90 (1979)]

RENORMALIZATION OF THE COTTINGHAM FORMULA

John C. COLLINS *
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540, USA

Received 23 October 1978

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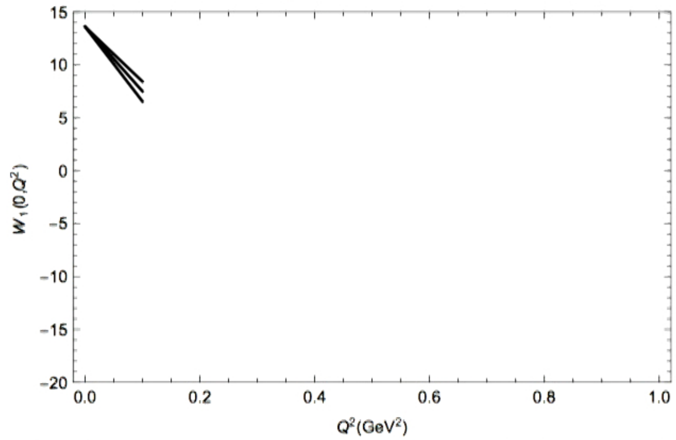
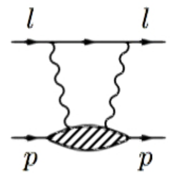


Two Photon Exchange: large Q^2 limit



$$W^{\mu\nu} = \frac{1}{2}\sum_s i\int d^4x\, e^{iq\cdot x}\langle \boldsymbol{k}, s|T\{J^\mu_{\rm e.m.}(x)J^\nu_{\rm e.m.}(0)\}|\boldsymbol{k}, s\rangle$$
$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)W_1 + \left(k^\mu - \frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu - \frac{k\cdot q\, q^\nu}{q^2}\right)W_2$$

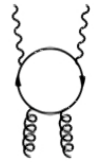
- $W_1(0, Q^2)$ is dimensionless

$$W_1 \sim \frac{\langle \text{Proton}|O|\text{Proton}\rangle}{Q^2} + \mathcal{O}\left(\frac{1}{Q^4}\right)$$

- O is a dimension 4 operator:

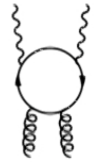
- Quarks: Spin 0: $m_q\bar{q}q$ Spin 2: $\bar{q}\left(iD^\mu\gamma^\nu + iD^\nu\gamma^\mu - \frac{1}{4}i\not{D}\, g^{\mu\nu}\right)q$
- Gluons: must be color singlet: $G_a^{\alpha\beta}G_a^{\rho\sigma}$
- What gluon operators can we have?

Gluon operators



- Gluons: must be color singlet $G_a^{\alpha\beta} G_a^{\rho\sigma}$
  A product of (E^i, B^i) and (E^j, B^j) has $7 \times 6/2 = 21$ components:
- 1 scalar: $G^{\mu\nu} G_{\mu\nu} = 2(\vec{B}^2 - \vec{E}^2)$
- 1 pseudo scalar: $\epsilon_{\alpha\beta\rho\sigma} G^{\alpha\beta} G^{\rho\sigma} = E \cdot B$: ruled out by parity

Gluon operators



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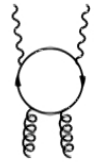
- 9 components of traceless symmetric tensor: $G^{\mu\alpha} G_{\alpha}^{\nu} - \frac{1}{4} G^{\alpha\beta} G_{\alpha\beta} g^{\mu\nu}$
  chromomagnetic stress-energy tensor
- What else? 10 components of

Gluon operators



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$$O^{\mu\alpha\nu\beta} = -\frac{1}{4}\left(\epsilon^{\mu\alpha\rho\sigma}\epsilon^{\nu\beta\kappa\lambda} + \epsilon^{\mu\beta\rho\sigma}\epsilon^{\nu\alpha\kappa\lambda}\right) G_{\rho\kappa} G_{\sigma\lambda} - \text{all possible traces}$$

  For example $O^{0123} = G^{01} G^{23} + G^{03} G^{21} = E^1 B^1 - E^3 B^3$
- For protons: $\langle \text{Proton} | O^{\mu\alpha\nu\beta} | \text{Proton} \rangle = 0$
  What about $\langle \text{Medium} | O^{\mu\alpha\nu\beta} | \text{Medium} \rangle$?
  Solution looking for a problem...

Large Q^2 behavior

- In 1978 Collins calculated EM corrections to the nucleon mass with an emphasis on $m_n - m_p$
  The mass only depends on spin-0 operators (q quark, $G^{\mu\nu}$ gluon)

$$\langle P|m_q\bar{q}q|P\rangle, \qquad \langle P|G^{\mu\nu}G_{\mu\nu}|P\rangle$$

	Quark	Gluon
Spin-0	Collins '78	Collins '78

- For $W_1(0, Q^2)$ you need also spin-2 operators

$$\langle P|\bar{q}\Big(iD^\mu\gamma^\nu + iD^\nu\gamma^\mu - \frac{1}{4}i\not{D}\, g^{\mu\nu}\Big)q|P\rangle, \qquad \langle P|G^{\mu\alpha}G^{\nu}_{\alpha} - \frac{1}{4}G^{\alpha\beta}G_{\alpha\beta}g^{\mu\nu}|P\rangle$$

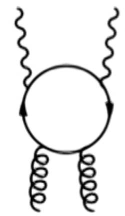
- Need to calculate the spin-2 contribution [Hill, GP arXiv:1611.09917]

	Quark	Gluon
Spin-0	Collins '78	Collins '78
Spin-2	Hill, GP '16	Hill, GP '16

- Collins's result is not enough for muonic hydrogen!

Large Q^2 behavior

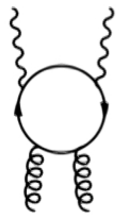
- Requires 1-loop calculation



- Doing that, we found a mistake in Collins spin-0 calculation from 1978...

Large Q^2 behavior

- Requires 1-loop calculation



- Doing that, we found a mistake in Collins spin-0 calculation from 1978...

- Collins didn't calculate the spin-0 gluon contribution directly
  He extracted it from another calculation

- For three light quark u, d, s
  Correct result: $\sum_q e_q^2 = (\frac{2}{3})^2 + (\frac{1}{3})^2 + (\frac{1}{3})^2 = \frac{2}{3}$
  Collins: $\sum_q = 3$
  Too large by a factor of 4.5...

Large Q^2 behavior

	Quark	Gluon
Spin-0	Collins '78	~~Collins '78~~ Hill, GP '16
Spin-2	Hill, GP '16	Hill, GP '16

- Even worse, quark spin-0 and gluon spin-0 come with opposite signs
  After correcting the mistake they largely cancel
  $W_1(0, Q^2)$ is **dominated** by spin-2 contribution

- Lesson: It is important to do a full calculation

Large Q^2 behavior

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Spin-0	Collins '78	~~Collins '78~~	Hill, GP '16
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  After correcting the mistake they largely cancel
  $W_1(0, Q^2)$ is **dominated** by spin-2 contribution

- Lesson: It is important to do a full calculation

- Some good news: The mistake has no effect on $m_n - m_p$
  since gluon contribution is the same at lowest order in isospin breaking

Large Q^2 behavior: Results

- The *correct* spin 0 result

$$\frac{Q^2}{2m_p^2}W_1^{(\mathrm{spin}-0)}(0,Q^2) = -2\sum_q e_q^2 f_q^{(0)} + \left(\sum_q e_q^2\right)\frac{\alpha_s}{12\pi}\tilde{f}_g^{(0)}$$

- $\langle N(k)|O_q^{(0)}|N(k)\rangle \equiv 2m_N^2 f_{q,N}^{(0)}, \quad \langle N(k)|O_g^{(0)}(\mu)|N(k)\rangle \equiv -2m_N^2 \tilde{f}_{g,N}^{(0)}(\mu)$

$$1 = (1-\gamma_m)\sum_q f_q^{(0)} - \frac{\beta}{2g}\tilde{f}_g^{(0)}$$

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$$1 = (1-\gamma_m)\sum_q f_q^{(0)} - \frac{\beta}{2g}\tilde{f}_g^{(0)}$$

- The *new* spin 2 result

$$\frac{Q^2}{2m_p^2}W_1^{(\mathrm{spin}-2)}(0,Q^2) = 2\sum_q e_q^2 f_q^{(2)}(\mu) + \left(\sum_q e_q^2\right)\frac{\alpha_s}{4\pi}\left(-\frac{5}{3}+\frac{4}{3}\log\frac{Q^2}{\mu^2}\right)f_g^{(2)}(\mu)$$

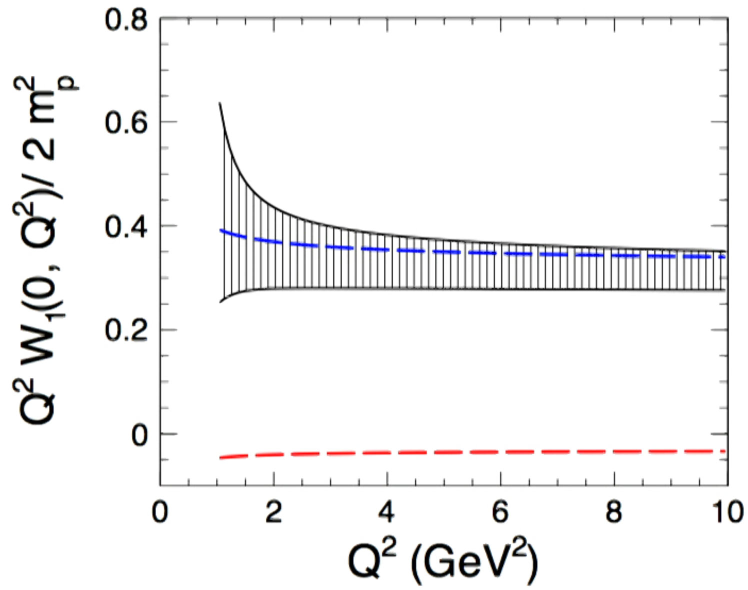
- $\langle N(k)|O_q^{(2)\mu\nu}(\mu)|N(k)\rangle \equiv 2\left(k^\mu k^\nu - \frac{g^{\mu\nu}}{4}m_N^2\right)f_{q,N}^{(2)}(\mu)$

  $\langle N(k)|O_g^{(2)\mu\nu}(\mu)|N(k)\rangle \equiv 2\left(k^\mu k^\nu - \frac{g^{\mu\nu}}{4}m_N^2\right)f_{g,N}^{(2)}(\mu)$

$$\sum_q f_q^{(2)}(\mu) + f_g^{(2)}(\mu) = 1$$

Large Q^2 behavior: Total contribution

- The total contribution



- Dashed red: spin 0
- Dashed blue: spin 2
- Vertical stripes: total contribution with perturbative and hadronic errors added in quadrature

Two Photon exchange: small Q^2 and large Q^2

- Using NRQED we have control over low Q^2

$$W_1(0,Q^2) = 2a_p(2+a_p) + \frac{Q^2}{m_p^2}\left\{\frac{2m_p^3\bar{\beta}}{\alpha} - a_p - \frac{2}{3}\left[(1+a_p)^2 m_p^2 (r_M^p)^2 - m_p^2 (r_E^p)^2\right]\right\} + \mathcal{O}\left(Q^4\right)$$

- Using OPE we *now* have control over the high Q^2

$$\frac{Q^2}{2m_p^2} W_1^{(\mathrm{spin}-0)}(0,Q^2) = -2\sum_q e_q^2 f_q^{(0)} + \left(\sum_q e_q^2\right)\frac{\alpha_s}{12\pi}\tilde{f}_g^{(0)}$$

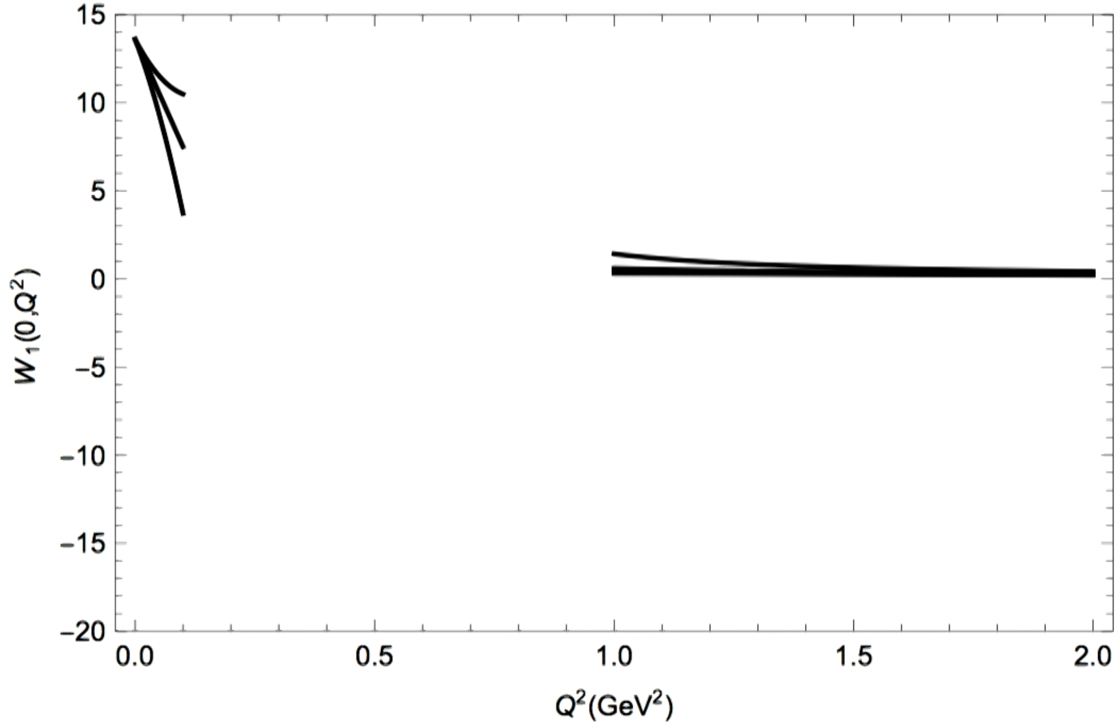
$$\frac{Q^2}{2m_p^2} W_1^{(\mathrm{spin}-2)}(0,Q^2) = 2\sum_q e_q^2 f_q^{(2)}(\mu) + \left(\sum_q e_q^2\right)\frac{\alpha_s}{4\pi}\left(-\frac{5}{3} + \frac{4}{3}\log\frac{Q^2}{\mu^2}\right) f_g^{(2)}(\mu)$$

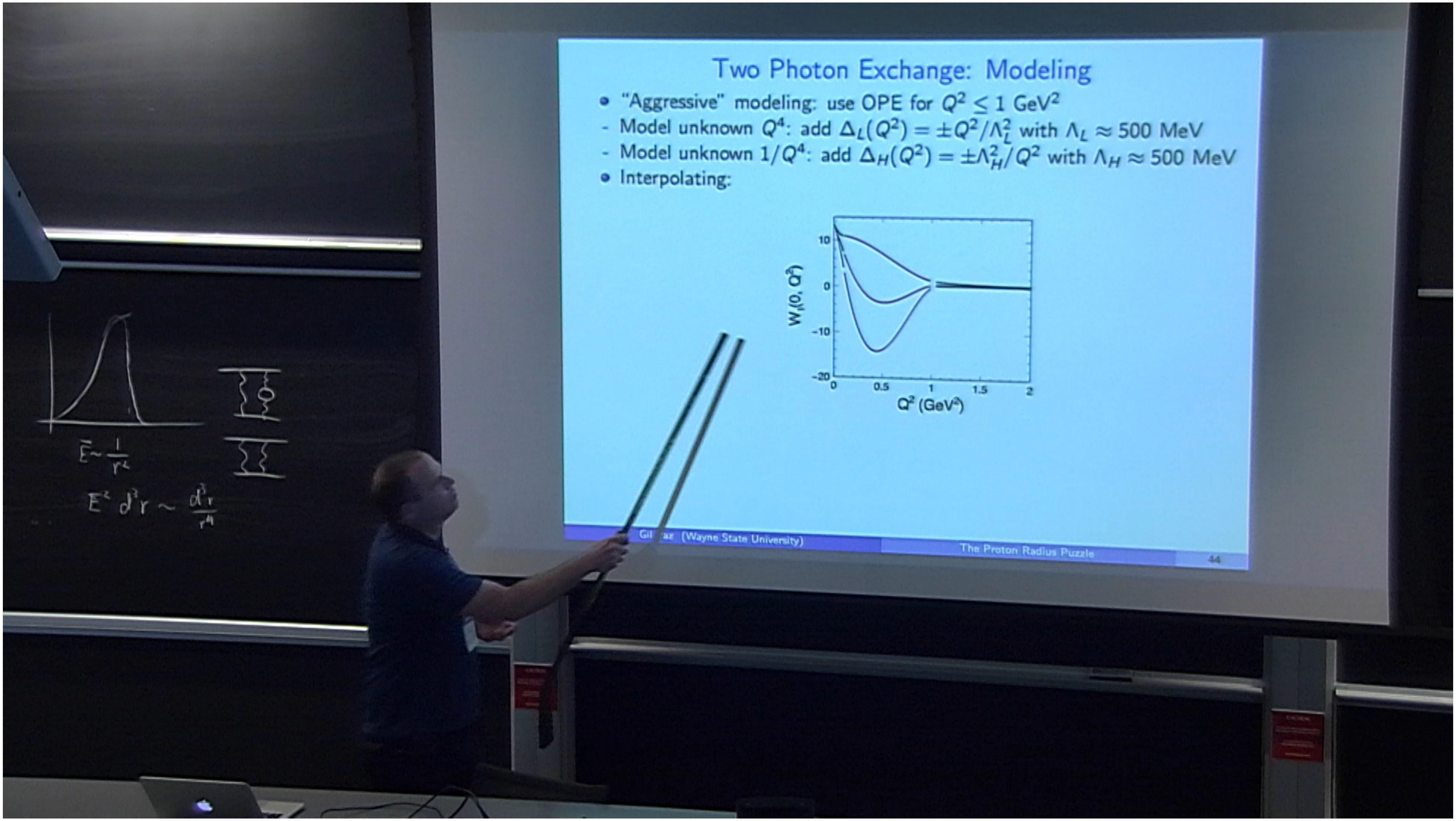
- The problem, like the joke, is how to make a whole fish from a head and a tail...

- Before this work we had only the low Q^2
  knowing the large Q^2 allows to connect the dots

Two Photon Exchange: Modeling

- "Aggressive" modeling: use OPE for $Q^2 \geq 1$ GeV^2
  - Model unknown Q^4: add $\Delta_L(Q^2) = \pm Q^2/\Lambda_L^2$ with $\Lambda_L \approx 500$ MeV
  - Model unknown $1/Q^4$: add $\Delta_H(Q^2) = \pm \Lambda_H^2/Q^2$ with $\Lambda_H \approx 500$ MeV
- How to connect the curves?

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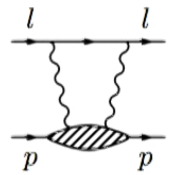
- Model unknown $1/Q^4$: add $\Delta_H(Q^2) = \pm \Lambda_H^2/Q^2$ with $\Lambda_H \approx 500$ MeV
• Interpolating:
$W_1(0, Q^2)$
10
0
-10
-20
0
0.5
1
1.5
2
Q^2 (GeV^2)
(Wayne State University)
The Proton Radius Puzzle
44
$E \sim \frac{1}{r^2}$
$E^2 d^3r \sim \frac{d^3r}{r^4}$

Two photon exchange

- In both cases apart from r_E^p we have two-photon exchange



$$W^{\mu\nu} = \frac{1}{2}\sum_s i\int d^4x\, e^{iq\cdot x}\langle \boldsymbol{k}, s|T\{J^\mu_{\rm e.m.}(x)J^\nu_{\rm e.m.}(0)\}|\boldsymbol{k}, s\rangle$$

$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1 + \left(k^\mu - \frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu - \frac{k\cdot q\, q^\nu}{q^2}\right) W_2$$

- Dispersion relations ($\nu = 2k\cdot q,\ Q^2 = -q^2$)

$$W_1(\nu, Q^2) = W_1(0, Q^2) + \frac{\nu^2}{\pi}\int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2\, \frac{{\rm Im}\, W_1(\nu', Q^2)}{\nu'^2(\nu'^2-\nu^2)}$$

$$W_2(\nu, Q^2) = \frac{1}{\pi}\int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2\, \frac{{\rm Im}\, W_2(\nu', Q^2)}{\nu'^2-\nu^2}$$

- W_1 requires subtraction...

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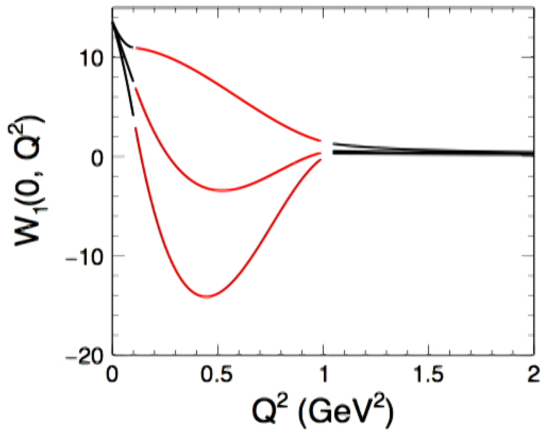
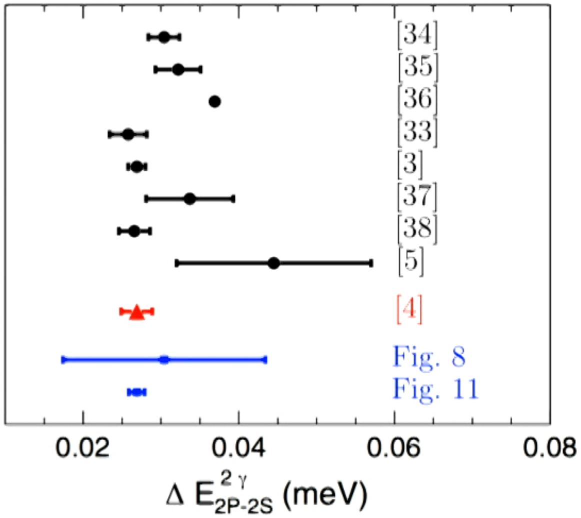
- Interpolating:



- Energy contribution: $\delta E(2S)^{W_1(0,Q^2)} \in [-0.046\,\text{meV}, -0.021\,\text{meV}]$
  To explain the puzzle need this to be ~ 0.3 meV
- Caveats: OPE might be only valid for larger Q^2
  $W_1(0, Q^2)$ might be different than the interpolated lines

Two Photon Exchange: Other approaches

- Similar results found by other groups



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[Fig. 8] Hill, GP PRD 95, 094017 (2017).

Experimental test

- How to test?
- New experiment: $\mu - p$ scattering
  MUSE (MUon proton Scattering Experiment) at PSI
  [R. Gilman et al. (MUSE Collaboration), arXiv:1303.2160]

- Need to connect muon-proton scattering and muonic hydrogen can use a new effective field theory: QED-NRQED
  [Hill, Lee, GP, Mikhail P. Solon, PRD **87** 053017 (2013)]
  [Steven P. Dye, Matthew Gonderinger, GP, PRD **94** 013006 (2016)]

MUSE

- Muonic hydrogen:

  Muon momentum $\sim m_\mu c\alpha \sim 1$ MeV

  Both proton and muon non-relativistic

MUSE

- Muonic hydrogen:
  Muon momentum $\sim m_\mu c\alpha \sim 1$ MeV
  Both proton and muon non-relativistic

- MUSE:
  Muon momentum $\sim m_\mu \sim 100$ MeV
  Muon is relativistic, proton is still non-relativistic

MUSE

- Muonic hydrogen:

  Muon momentum $\sim m_\mu c\alpha \sim 1$ MeV

  Both proton and muon non-relativistic

- MUSE:

  Muon momentum $\sim m_\mu \sim 100$ MeV

  Muon is relativistic, proton is still non-relativistic

- QED-NRQED effective theory:

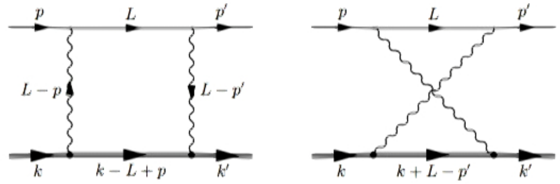
  - Use QED for muon
  - Use NRQED for proton

  $m_\mu/m_p \sim 0.1$ as expansion parameter

- A *new* effective field theory suggested in

  [Hill, Lee, GP, Mikhail P. Solon, PRD **87** 053017 (2013)]

QED-NRQED Effective Theory



- The amplitude

$$i\mathcal{M}(2\pi)^4\delta^4(k+p-k'-p') = Z^2e^4\int\frac{d^4L}{(2\pi)^4}\frac{1}{(L-p)^2L-p')^2}$$

$$\times\quad \bar{u}(p')\gamma^0\frac{i}{\not{L}-m}\gamma^0u(p)\chi^\dagger\chi(2\pi)\delta(L^0-p^0)\,(2\pi)\delta(L^0-p'^0)$$

$$\times\quad (2\pi)^3\delta^4(\vec{p}-\vec{p}'-\vec{k}')$$

- The cross section

$$\frac{d\sigma}{d\Omega}=\frac{Z^2\alpha^24E^2\left(1-v^2\sin^2\frac{\theta}{2}\right)}{\vec{q}^{\,4}}\left[1+\frac{Z\alpha\pi v\sin\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\right)}{1-v^2\sin^2\theta}\right]$$

$Z=1$, $E=$ muon energy, $v=|\vec{p}|/E$, $q=p'-p,\theta$ scattering angle

QED-NRQED Effective Theory

- QED-NRQED result

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2 4E^2\left(1 - v^2\sin^2\frac{\theta}{2}\right)}{\vec{q}^{\,4}}\left[1 + \frac{Z\alpha\pi v \sin\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\right)}{1 - v^2\sin^2\theta}\right]$$

- *Same result* as scattering relativistic lepton off static $1/r$ potential [Dalitz, Proc. Roy. Soc. Lond. **206**, 509 (1951)] reproduced in [Itzykson, Zuber, "Quantum Field Theory"]

- *Same result* as $m_p \to \infty$ of "point particle proton" QED scattering (For $m_p \to \infty$ only proton charge is relevant)

Connecting muon-proton scattering to muonic hydrogen

- Matching

QED, QCD $\qquad\qquad$ $G_{E,M}$, Structure func., $W_1(0, Q^2)$

Connecting muon-proton scattering to muonic hydrogen

- Matching

QED, QCD	$G_{E,M}$, Structure func., $W_1(0, Q^2)$
Scale: $m_p \sim 1$ GeV	$\Downarrow$
QED-NRQED: *MUSE*	r_E^p, $\bar{\mu}\gamma^0\mu\psi_p^\dagger\psi_p$

Connecting muon-proton scattering to muonic hydrogen

- Matching

QED, QCD	$G_{E,M}$, Structure func., $W_1(0, Q^2)$
Scale: $m_p \sim 1$ GeV	$\Downarrow$
QED-NRQED: *MUSE*	r_E^p, $\bar{\mu}\gamma^0\mu\psi_p^\dagger\psi_p$
Scale: $m_\mu \sim 0.1$ GeV	$\Downarrow$
NRQED-NRQED: *muonic H*	r_E^p, $\psi_\mu^\dagger\psi_\mu\psi_p^\dagger\psi_p$

- Need to match QED-NRQED contact interaction, e.g. $\bar{\mu}\gamma^0\mu\psi_p^\dagger\psi_p$ to NRQED-NRQED contact interaction, e.g. $\psi_\mu^\dagger\psi_\mu\psi_p^\dagger\psi_p$
[Dye, Gonderinger, GP *in progress*]

Connecting muon-proton scattering to muonic hydrogen

- To do list:

1) Relate QED-NRQED contact interactions
   to NRQED contact interactions and $W_1(0, Q^2)$

2) Calculate $d\sigma(\mu + p \to \mu + p)$ and asymmetry in terms of r_E^p and d_2

3) *Direct* relation between μ–p scattering and muonic H

Conclusions

- Proton radius puzzle: $> 5\sigma$ discrepancy between
  - r_E^p from muonic hydrogen
  - r_E^p from hydrogen and $e - p$ scattering
- Recent muonic deuterium results find similar discrepancies [Pohl et al. Science **353**, 669 (2016)]
- After more than 6 years the origin is still not clear
  1) Is it a problem with the electronic extraction?
  2) Is it a hadronic uncertainty?
  3) is it new physics?
- Motivates a reevaluation of our understanding of the proton

Conclusions

- Presented three topics:

1) Extraction of proton radii from scattering:
   Use an established tool of the z expansion
   Studies disfavor the muonic hydrogen value

2) The first *full* and *correct* evaluation of
   large Q^2 behavior of forward virtual Compton tensor
   Can improve the modeling of two photon exchange effects