Title: Precision calculations for muonic bound states

Date: Jun 12, 2017 11:00 AM

URL: http://pirsa.org/17060003

Abstract:

Outline

Puzzles in muon physics

Muon decay in orbit
- context: search for muon-electron conversion
- energy regions and radiative corrections

g-factor of a bound electron
- best way to determine the atomic mass of the electron
- possible source of the fine structure constant

New era of experiments with muons

PSI (Switzerland):
- muonic atoms
- mu -> e + gamma
- mu + p scattering
- mu -> eee

Fermilab (USA):
- g-2
- Mu2e

J-PARC (Japan):
- g-2
- DeeMe
- COMET
- muonium HFS

Motivated by proton radius puzzle
See Gil Paz's talk

Muons are indeed a great tool for New Physics searches:
long-lived, just massive enough, easy to produce, with convenient spin properties.

They are also mysterious. Some precise measurements disagree with expectations:
g-2, proton radius, B-decays.

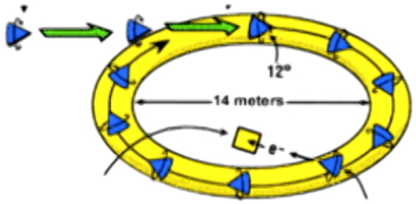
$$B^0 \to K^{*0}\mu^+\mu^-$$

$$R_K = \mathrm{BR}(B^+ \to K^+\mu^+\mu^-)/\ \mathrm{BR}(B^+ \to K^+e^+e^-)$$

Doug Bryman's talk

The puzzle of the muon magnetic moment

The 3.6 sigma discrepancy,



$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 287(80) \times 10^{-11}$$

PRD 86, 095009 (2012)

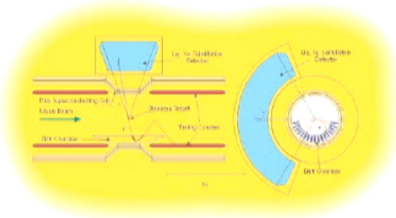
is rather large when compared with other bounds on New Physics.

Lepton flavor violation: μ ⟶ eγ

New bound (MEG @ Paul Scherrer Institute)

$$\mathrm{BR}\left(\mu \to e\gamma\right) < 4.2 \cdot 10^{-13}$$

arXiv:1605.05081



This corresponds to the transition dipole moment

$$d_{\mu\to e} \lesssim 3.5 \cdot 10^{-27}\ e \cdot \mathrm{cm}$$

Sensitive to the heaviest "new physics"

For comparison: electron EDM $d_e < 0.87 \cdot 10^{-28}\, e \cdot \mathrm{cm}$ 10.1126/science.1248213

muon g-2 $d_\mu < 3 \cdot 10^{-22}\, e \cdot \mathrm{cm}$

How to check g_μ-2?

Electron g-2 is likely sensitive to the same New Physics; but at present it is used to determine the fine-structure constant.

A new source of alpha is needed.

How to check g_μ-2?

Nature 442, 516 (2006)
PRA 89, 052118 (2014)

The second best determination of alpha:
from atomic spectroscopy

$$R_\infty = \frac{m_e c \alpha^2}{2h}$$

Needed precision:

$14 \cdot 10^{-11}$

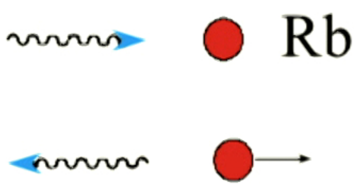
$$\alpha^2 = \frac{2R_\infty}{c} \cdot \frac{u}{m_e} \cdot \frac{M_X}{u} \cdot \frac{h}{M_X}$$

$7 \cdot 10^{-12}$ (but is it for sure?)

$8 \cdot 10^{-11}$ Nature 2014 Sturm et al

$12 \cdot 10^{-11}$ for Rb (better for He)

$124 \cdot 10^{-11}$ improvement needed by factor ~10



gives h/m

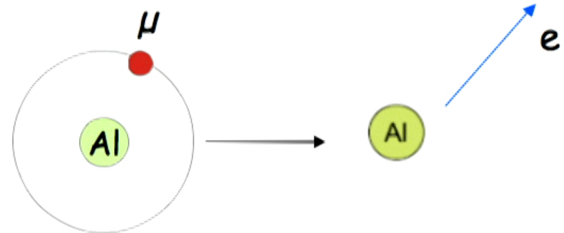
$$\alpha(\mathrm{Rb}) = 1/137.035\,999\,049(90) \quad [66 \cdot 10^{-11}]$$

PRL 106, 080801 (2011)

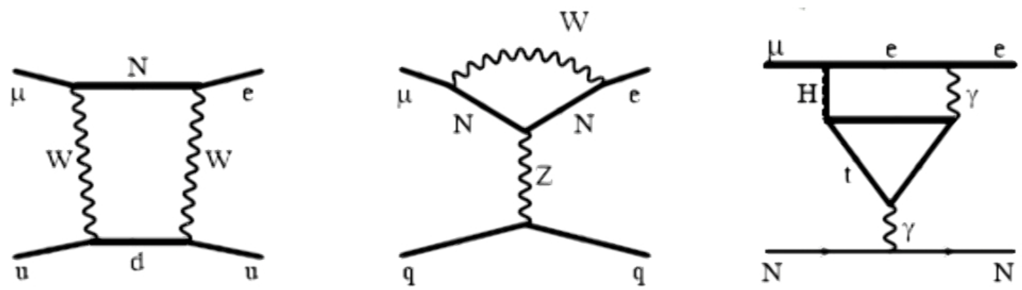
Muon-electron conversion:
probes various types of interactions

New process: muon-electron conversion

(as well as mu --> eee)



Variety of mechanisms:

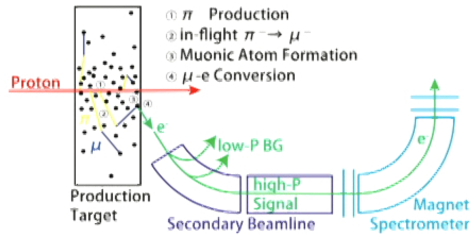


Probes also non-dipole interactions
that are not (directly) probed by
processes with external photons,
by gauge invariance requirements.

Muon-electron conversion plans
(The Next Big Thing in muon physics)

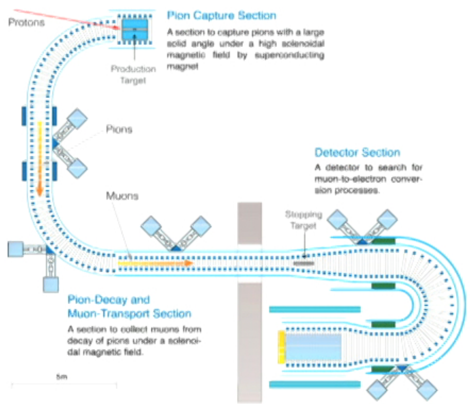
from Hiroaki Natori JPS Conf. Proc. 8, 025017 (2015)

DeeMe
J-PARC



starts ~2018;
aims for 1e-13 (graphite target),
followed by 1e-14 (SiC target)

COMET
Phase 1
J-PARC

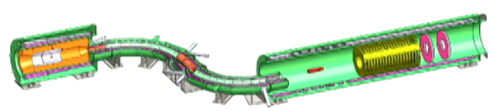


7e-15

COMET
Phase 2
J-PARC

2.6e-17

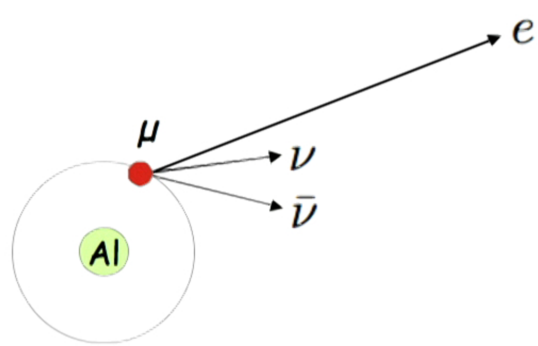
Mu2e
Fermilab



2e-17

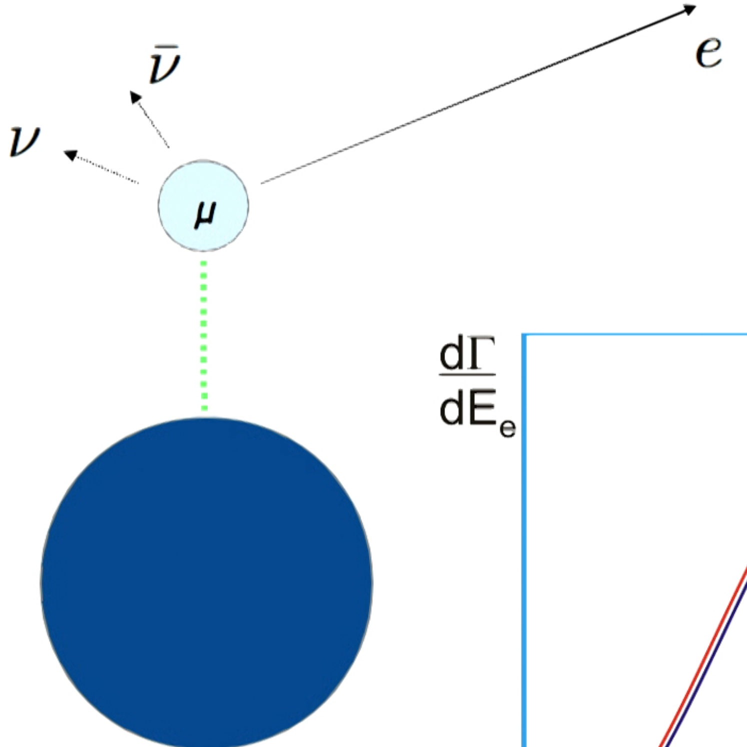
Background for the conversion search

Normal decay of the muon bound in the atom
can produce high-energy electron,



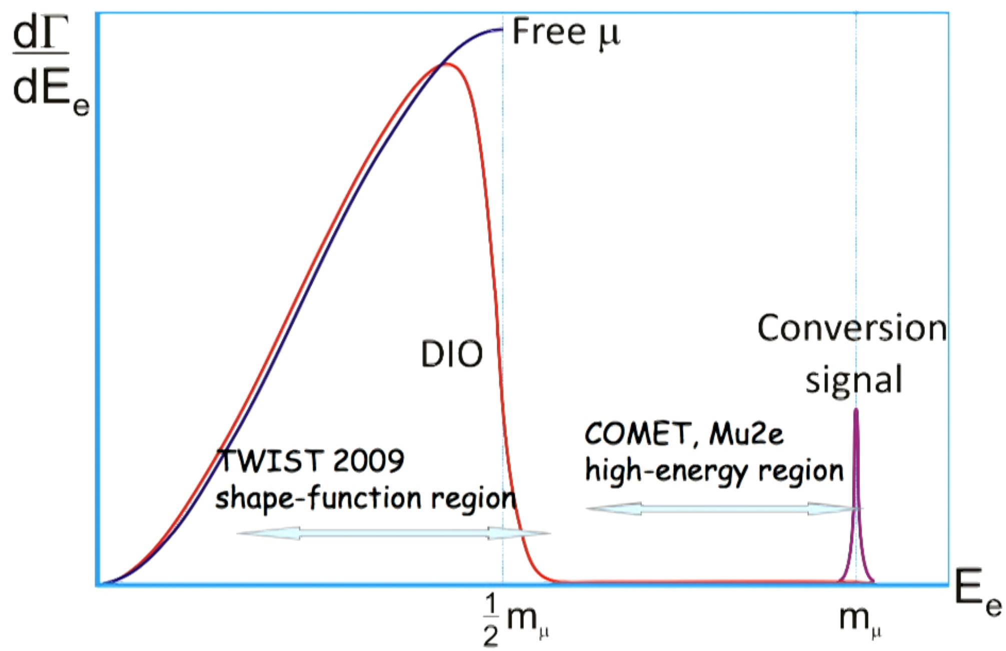
Spectrum has to be well understood.

Electron spectrum in a bound muon decay



Electron energy can be as large as the whole muon mass

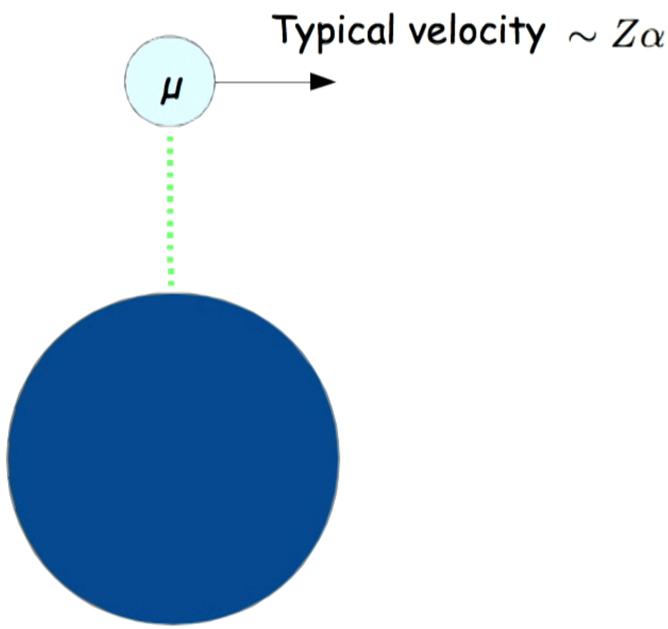
Similar to semileptonic heavy quark decays



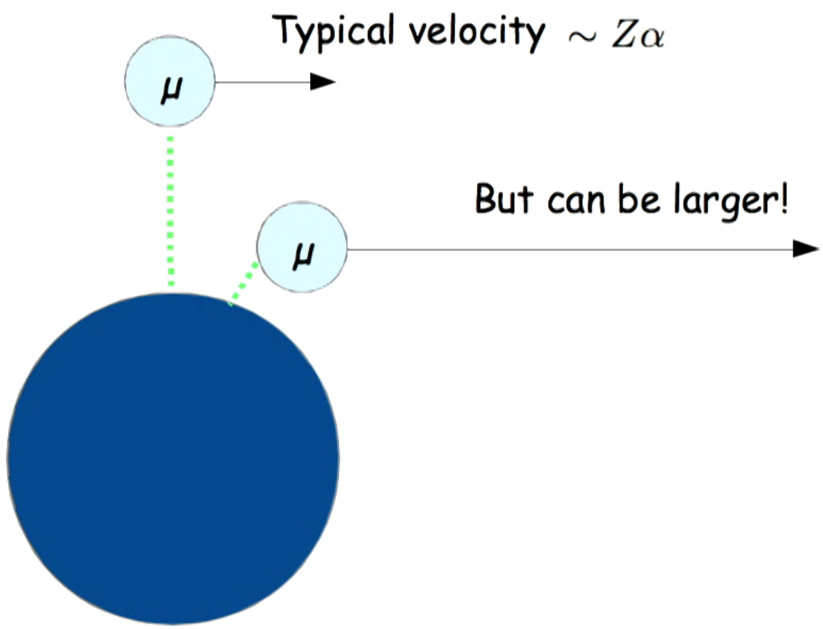
Muon decay-in-orbit spectrum: the shape-function region

Experiment: TWIST

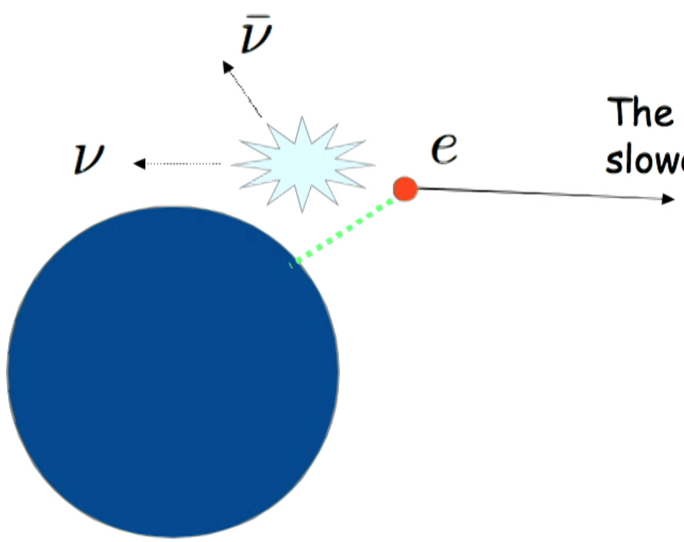
Two effects: muon motion & Coulomb attraction



Two effects: muon motion & Coulomb attraction

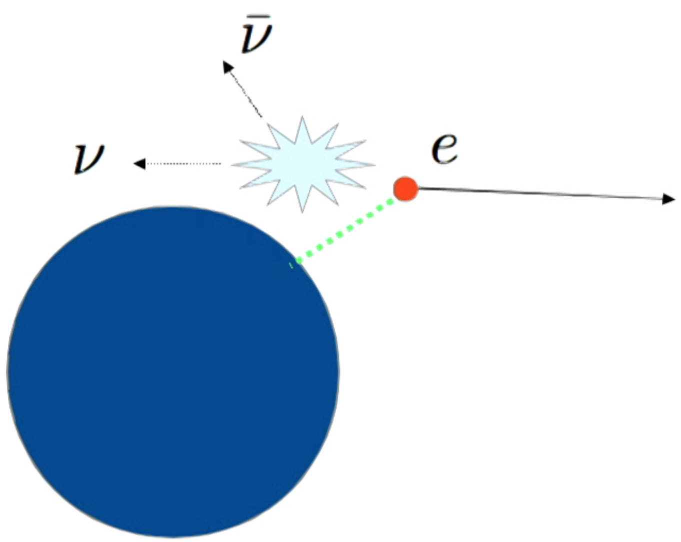


Two effects: muon motion & Coulomb attraction



The boosted daughter electron slowed down by the Coulomb attraction

Two effects: muon motion & Coulomb attraction



Net effect:

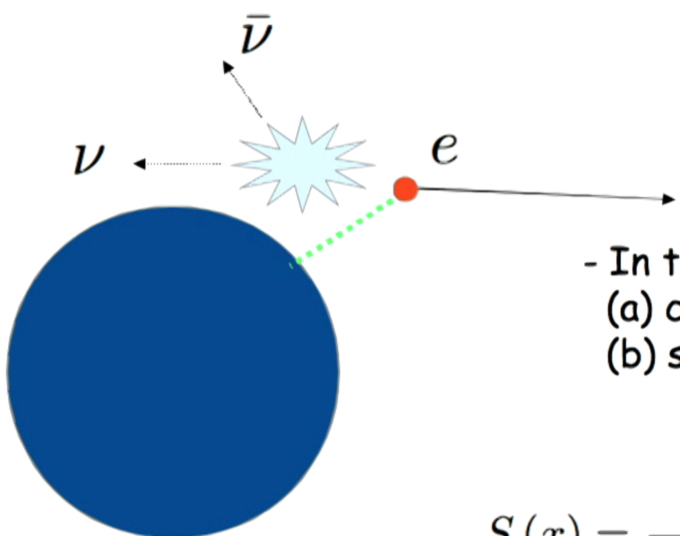
- In the decay rate: almost none; only time dilation

$$\Gamma \to \left(1 - \frac{(Z\alpha)^2}{2}\right)\Gamma$$

Überall, Phys. Rev. 119, 365 (1960)

Two effects: muon motion & Coulomb attraction

Szafron, AC, PRD 92 (2015) 053004

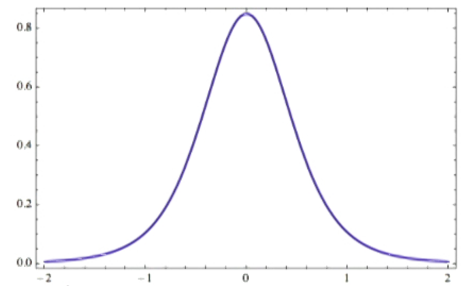


Net effect:
- In the decay rate: almost none; only time dilation

$$\Gamma \to \left(1 - \frac{(Z\alpha)^2}{2}\right)\Gamma$$

- In the electron energy spectrum:
  (a) computable shift
  (b) smearing -> "shape function"

$$S(x) = \frac{8}{3\pi\left[1+x^2\right]^3}$$



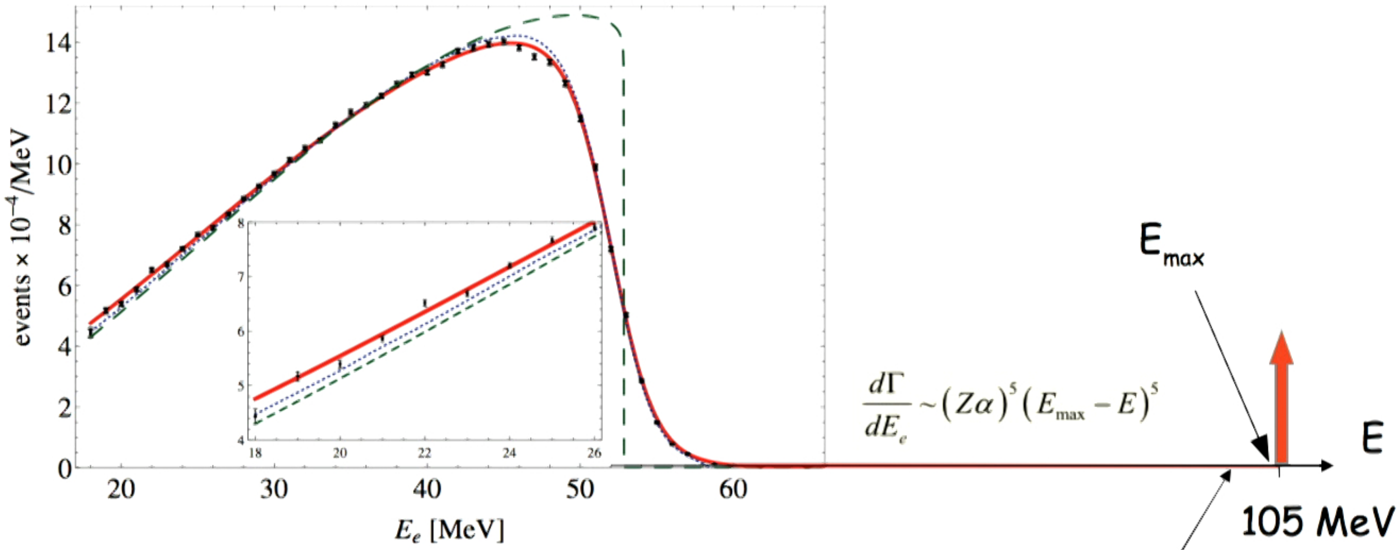
Previously used in heavy mesons, where it cannot be computed from first principles, but can be experimentally accessed.

Mannel, Neubert,
Bigi, Shifman, Uraltsev, Vainshtein

Muon decay-in-orbit spectrum: the high-energy region

Experiments: Mu2e and COMET

Spectrum of the bound muon decay



$$\frac{d\Gamma}{dE_e} \sim (Z\alpha)^5 (E_{max} - E)^5$$

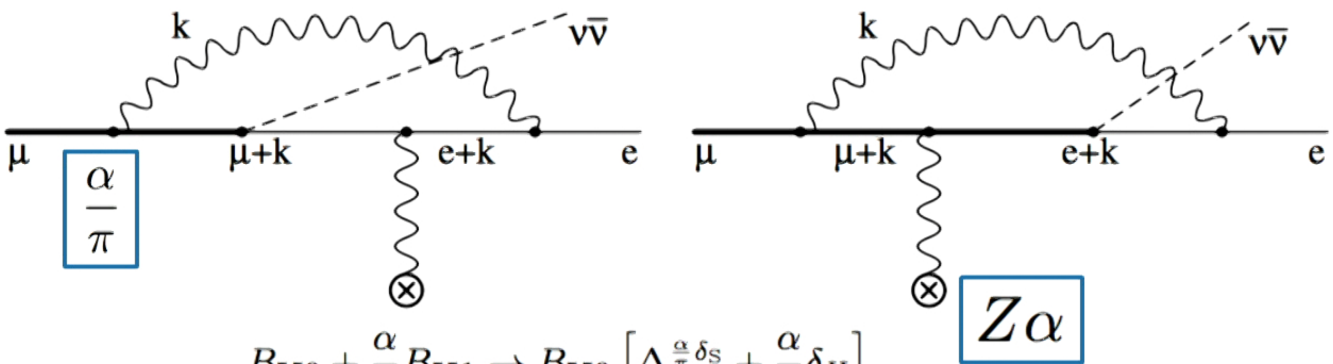
Main background for the conversion signal

AC, M. Dowling, X. Garcia i Tormo, W. Marciano, R. Szafron
R. Szafron, AC

Radiative corrections to the electron spectrum

Expansion near the end-point $\frac{m_\mu}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}E} = \sum_{ijk} B_{ijk}\Delta^i(\pi Z\alpha)^j\left(\frac{\alpha}{\pi}\right)^k$

Three "small" parameters: $\Delta = \frac{E_{\max}-E}{m_\mu}$



$$B_{550} + \frac{\alpha}{\pi}B_{551} \to B_{550}\left[\Delta^{\frac{\alpha}{\pi}\delta_S} + \frac{\alpha}{\pi}\delta_H\right]$$

$$\delta_S = 10.1$$

number of electrons in the end-point bin of 1 (0.1) MeV is reduced by 11% (16%)

Szafron, AC, PLB753, 61 (2016)

Part 2/2: magnetic moment (bound electron)

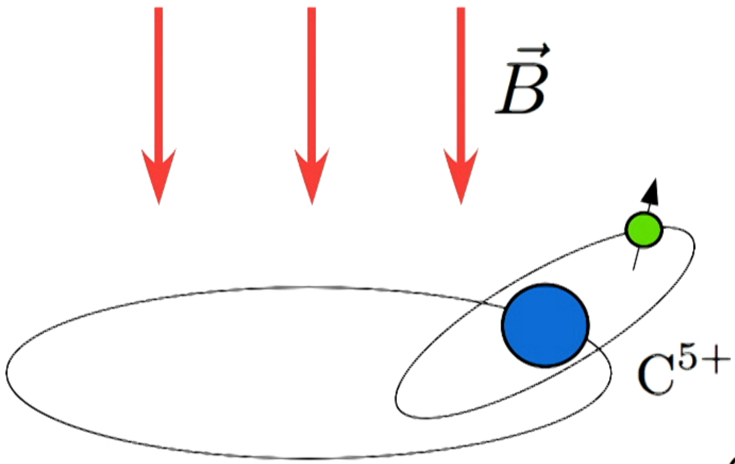
Why useful?

- determination of the electron mass
- future determination of alpha
- indirectly related to muon g-2 (muonium)

Why interesting?

- quantum effects in external field
- simple system, model for more complex ones
- numerical estimates exist for large Z
- should be analytically feasible for small Z (many have tried)

To avoid thermal motion: anchor the electron in an ion

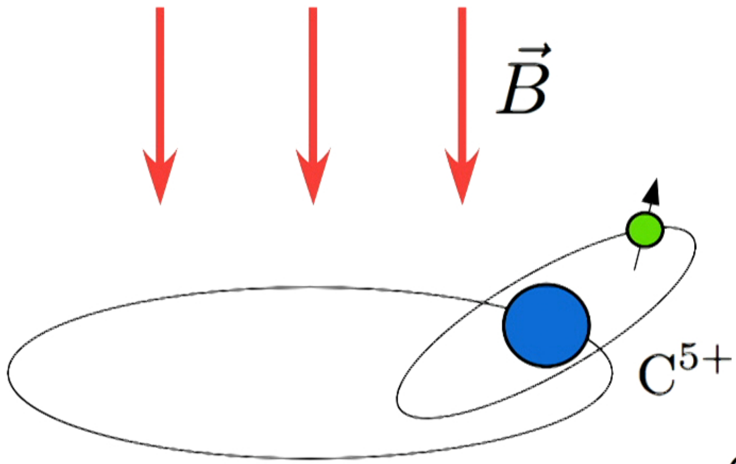


Larmor frequency $\omega_L = \frac{geB}{2m_e}$

Cyclotron frequency $\omega_{\text{cycl}} = \frac{ZeB}{M}$

$$m_e = \frac{g}{2Z} \frac{\omega_{\text{cycl}}}{\omega_L} M$$

To avoid thermal motion: anchor the electron in an ion



Larmor frequency $\omega_L = \frac{geB}{2m_e}$

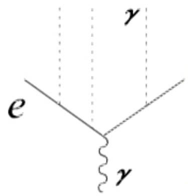
Cyclotron frequency $\omega_{\rm cycl} = \frac{ZeB}{M}$

$$m_e = \frac{g}{2Z}\frac{\omega_{\rm cycl}}{\omega_L}M$$

Side remark: M and m_e have different origins! (QCD vs. Higgs)
Opportunity for searching for time variation.

Bound-electron g-2: the leading effect

Breit 1928: energy correction due to magnetic field in the hydrogen ground state.



$$\delta E = e \int d^3x f^2 v^* \left[1 - i\gamma \mathbf{\Sigma}\cdot\hat{\boldsymbol{r}}\gamma^5\right] \gamma^5 \boldsymbol{A}\cdot\mathbf{\Sigma} \left[1 + i\gamma\mathbf{\Sigma}\cdot\hat{\boldsymbol{r}}\gamma^5\right] v$$

$$g = 2\cdot\frac{1}{3}\left(1+2\sqrt{1-(Z\alpha)^2}\right) \simeq 2\left(1-\frac{(Z\alpha)^2}{3}\right)$$

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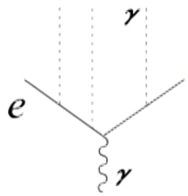
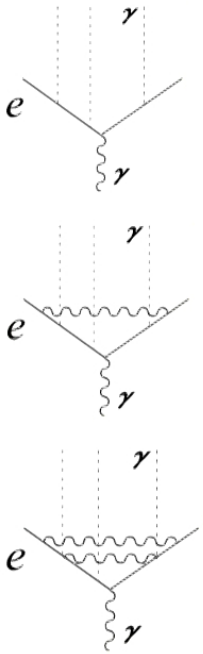


$$\delta E = e \int d^3x f^2 v^* \left[1 - i\gamma \boldsymbol{\Sigma} \cdot \hat{\boldsymbol{r}} \gamma^5\right] \gamma^5 \boldsymbol{A} \cdot \boldsymbol{\Sigma} \left[1 + i\gamma \boldsymbol{\Sigma} \cdot \hat{\boldsymbol{r}} \gamma^5\right] v$$

$$g = 2 \cdot \frac{1}{3}\left(1 + 2\sqrt{1-(Z\alpha)^2}\right) \simeq 2\left(1 - \frac{(Z\alpha)^2}{3}\right)$$

Important: dependence
on alpha; may be exploited
to determine its value.
(Use ions with various Z)

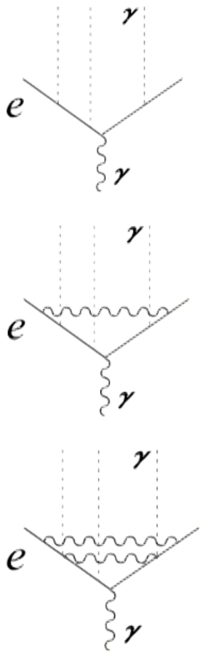
Bound-electron g-2: binding and loops



$$g = 2 - \frac{2\,(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \dots$$
$$+ \frac{\alpha}{\pi}\left[1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4\,(a_{41}\ln Z\alpha + a_{40}) + \dots\right]$$
$$+ \underbrace{\left(\frac{\alpha}{\pi}\right)^2 \left[-0.65..\left(1 + \frac{(Z\alpha)^2}{6}\right) + (Z\alpha)^4\,(b_{41}\ln Z\alpha + b_{40}) + \dots\right]}_{\text{two-loop corrections}}$$

Pachucki,
AC
Jentschura,
Yerokhin
(2005)

Recent experimental improvement



$$g = 2 - \frac{2\,(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \dots$$

$$+ \frac{\alpha}{\pi}\left[1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4\,(a_{41}\ln Z\alpha + a_{40}) + \dots\right]$$

$$+ \left(\frac{\alpha}{\pi}\right)^2 \left[-0.65..\left(1 + \frac{(Z\alpha)^2}{6}\right) + (Z\alpha)^4\,(b_{41}\ln Z\alpha + b_{40}) + \dots\right]$$

$$b_{41} = \frac{28}{9}$$

$$b_{40} = -\cancel{16.4}$$

-18.0

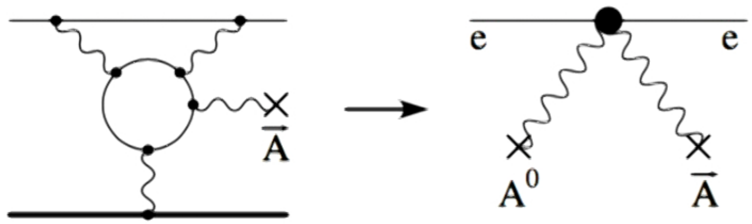
Together, new experiments in Mainz and this theory improved the accuracy of m_e by about a factor 3,

$$\frac{m_e}{u} = 0.000\ 548\ 579\ 909\ 32\,(29)\,(1)$$

$$\frac{m_e}{u} = 0.000\ 548\ 579\ 909\ 067\,(17)$$

Nature 2014
Sturm et al

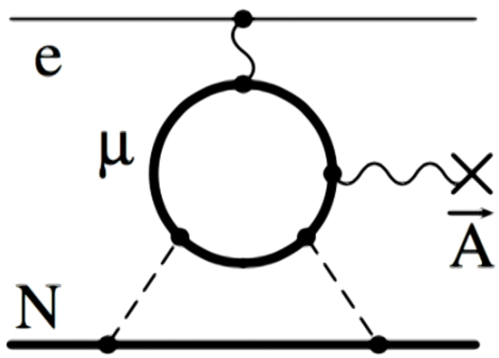
Light-by-light contributions



$$\delta V = \frac{e^2}{2m}\left(2\eta\sigma^{ij}B^{ik}\nabla^j E^k + \xi\sigma^{ij}B^{ij}\nabla^k E^k\right)$$

$$g^{(2)} = \left(\frac{\alpha}{\pi}\right)^2 \frac{(Z\,\alpha)^4}{n^3}\left\{\frac{28}{9}\ln[(Z\,\alpha)^{-2}] + \frac{258917}{19440} - \frac{4}{9}\ln k_0 - \frac{8}{3}\ln k_3 + \frac{113}{810}\pi^2 - \frac{379}{90}\pi^2\ln 2 + \frac{379}{60}\zeta(3)\right.$$
$$\left. + \left(\frac{16-19\pi^2}{108}\right)_{\rm LBL} + \frac{1}{n}\left[-\frac{985}{1728} - \frac{5}{144}\pi^2 + \frac{5}{24}\pi^2\ln 2 - \frac{5}{16}\zeta(3)\right]\right\},$$

Light-by-light: virtual-muon "magnetic loop"



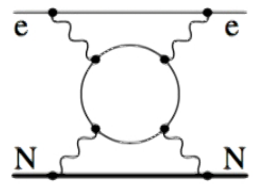
$$g_{\mathrm{ML}}(\mathrm{muon}) = \frac{7}{216}\alpha\,(Z\alpha)^5\left(\frac{m_e}{m_\mu}\right)^3$$

Very recently considered in
Belov et al, 1610.01340, with a result
larger by two orders of magnitude.

we find that this is the only difference
with the virtual-electron loop.
(Karshenboim & Milstein 2002)

Light-by-light contribution to the Lamb shift

We consider two momentum regions in



Light-by-light contribution to the Lamb shift

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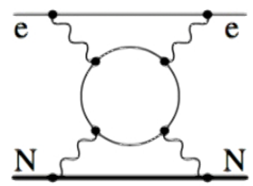
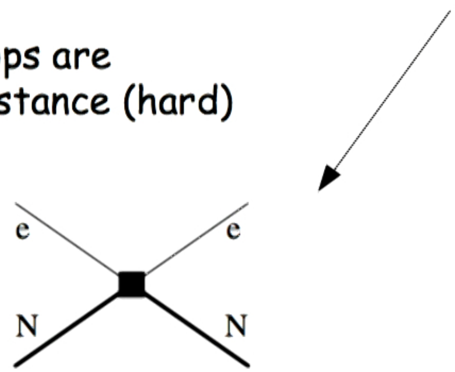


If all loops are
short-distance (hard)



$$\alpha^2 (Z\alpha)^2 \cdot \psi_0^2 \to \alpha^2 (Z\alpha)^5$$

Light-by-light contribution to the Lamb shift

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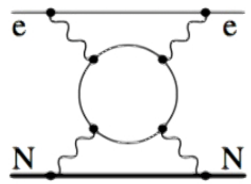
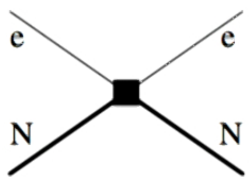


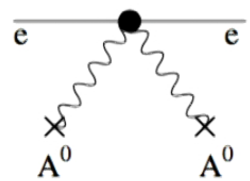
If all loops are short-distance (hard)



$$\alpha^2 (Z\alpha)^2 \cdot \psi_0^2 \to \alpha^2 (Z\alpha)^5$$

Pachucki; Eides+Shelyuto
Dowling, Mondejar, Piclum, AC

If the lowest loop is soft



$$\alpha^2 \langle e^2 \boldsymbol{E}^2 \rangle \sim \alpha^2 \left\langle \frac{(Z\alpha)^2}{r^4} \right\rangle$$

$$\to \alpha^2 (Z\alpha)^6 \ln Z\alpha$$

AC, Szafron
(Jentschura, AC, Pachucki)

How large is the resulting Lamb shift?

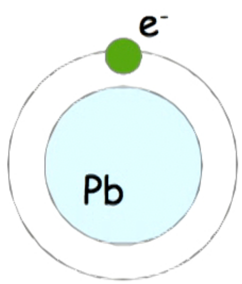
Decreases 1S-2S splitting by about 280 Hz.

For comparison, the experimental error is 10 Hz (~ 10^{-15} relative error)

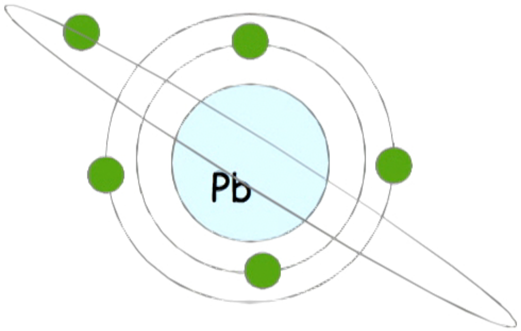
A new source of alpha: highly-charged ions

$$g \simeq 2 - \frac{2\,(Z\alpha)^2}{3} \longrightarrow \frac{\delta\alpha}{\alpha} \sim \frac{1}{(\alpha Z)^2}\sqrt{(\delta g_{\rm exp})^2 + (\delta g_{\rm th})^2}$$

large Z favorable



Hydrogen-like lead



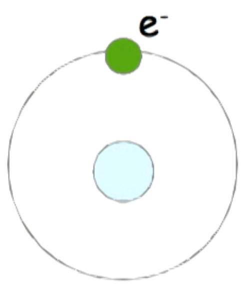
Boron-like lead

There is a combination of g-factors in both ions where the sensitivity to the nuclear structure largely cancels, but the sensitivity to alpha remains.

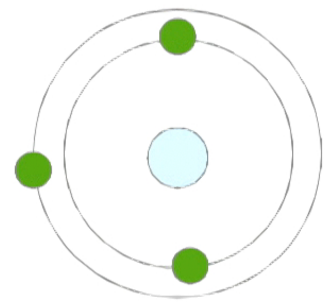
Shabaev, Glazov, Oreshkina, Volotka, Plunien, Kluge, Quint

New idea: medium-charged ions

$$g \simeq 2 - \frac{2\,(Z\alpha)^2}{3}$$



Hydrogen-like ion



Lithium-like ion

Combine H-like and Li-like to remove nuclear dependence;
then combine with a different nucleus, to remove free-g dependence!

Much interesting theoretical work remains to be done!

Yerokhin, Berseneva, Harman, Tupitsyn, Keitel: PRL (2016)

Summary

* Binding modifies the muon decay and the electron g-factor

* Theory of both effects is richer than for free particles

* Synergy with beautiful experiments: lepton-flavor violation, mass of the electron and, in future, the fine structure constant.

* For g: $\alpha(Z\alpha)^5$ effects almost finished; $\alpha^2(Z\alpha)^5$ hopefully soon.

* Opportunities for more theoretical improvement...