Title: String Theory for Mathematicians - Lecture 7

Date: Jun 06, 2017 02:00 PM

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Abstract:

IIB ⇝ B-model
top' string on $\mathbb{C}^5$

D-branes ⟶ B branes

IIA ⇝ $\mathbb{R}^2_A \times \mathbb{C}^4_B$

D-branes ⇝ A/B branes on
$\mathbb{R} \times \mathbb{C}^k$ $k \leq 4$

T-duality

IIA on $\mathbb{R} \times S^1 \times \mathbb{C}^4$

IIB $\mathbb{C}^* \times \mathbb{C}^4$

from T-duality
$\mathbb{R} \times S^1 \longleftrightarrow \mathbb{C}^*$
in top' string.

Remaining SUSYs in physical string:

from T-duality

$\mathbb{R} \times S^1 \longleftrightarrow \mathbb{C}^\times$

in top' string.

Remaining SUSYs in physical string:

IIB, we found 15 remaining SUSY

deformed B-model on $\mathbb{C}^5$

by $-\partial_{z_i} \wedge \partial_{z_j}$ (make non-commutative)

$-z_k$ (linear superpotential)

Remaining SUSYs in physical string:

IIB, we found 15 remaining SUSY
deformed B-model on $\mathbb{C}^5$
by $- \partial_{z_i} \wedge \partial_{z_j}$ (make non-commutative)
$- z_k$ (linear superpotential)

IIA, similarly 10 remaining
$\partial_{z_i} \wedge \partial_{z_j}$, z_k

B-model on $\mathbb{C}^5$

turn on $\partial_{z_1} \wedge \partial_{z_2}$

What happens?

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z_1, z_2 $\mathbb{C}^2$ becomes A-model (+ B-field)

Example Brane on z_2, z_3, z_4, z_5

Fields are $\Omega^{0,*}(\mathbb{C}^4)[\varepsilon] \otimes \mathfrak{gl}_N[1]$

$\partial_{z_1} \wedge \partial_{z_2} \rightsquigarrow \varepsilon \partial_{z_2}$, deforms the differential.

$\partial_{z_1} \wedge \partial_{z_2}$

What happens?

z_1, z_2 $\mathbb{C}^2$

becomes A-model

(+ B-field)

Example Brane on

z_2, z_3, z_4, z_5

Fields are $\Omega^{0,*}(\mathbb{C}^4)[\varepsilon] \otimes \mathfrak{gl}_N[1]$

$\partial_{z_1} \wedge \partial_{z_2}$ $\rightsquigarrow$ $\varepsilon \partial_{z_2}$, deforms the differential.

After turning on $\partial_{z_1} \wedge \partial_{z_2}$, fields become

$\Omega^*(\mathbb{R}^2) \otimes \Omega^{0,*}(\mathbb{C}^3) \otimes \mathfrak{gl}_N[1]$ $\varepsilon = dz_2$

Same as what we find if we treat $\mathbb{C}^2$ as A-model

In Sum

A further twist of IIB is top' string on $\mathbb{R}^4_A \times \mathbb{C}^3_B$

Twist even more

$\mathbb{R}^8_A \times \mathbb{C}_B$

IIA: Various twists are

$\mathbb{R}^2_A \times \mathbb{C}^4_B$

$\mathbb{R}^6_A \times \mathbb{C}^2_B$

$\mathbb{R}^{10}_A$ ← corresponds to a particular $SU(5)$-inv. top twist

Closed-string fields are

$$\ker \bar\partial \subseteq \Omega^{0,*}(\mathbb{C}^5, \Lambda^* T\mathbb{C}^5)$$

If we turn on a background closed-string field α, this changes

$$\bar\partial \longrightarrow \bar\partial + \{\alpha, -\}$$

$\{\ ,\ \}$ = Schouten bracket

$$\delta_i = \frac{d}{dz_i}$$

$$\Omega^{0,*}(\mathbb{C}^5, \Lambda^* T\mathbb{C}^5) = C^\infty(\mathbb{C}^5)[d\bar z_i, \delta_j]$$

$$\{\delta_j, f\} = \frac{df}{dz_j}$$

$$\{\delta_j, z_i\} = \delta_{ij}$$

$$\partial_{z_1} \wedge \partial_{z_2} = \delta_1 \wedge \delta_2$$

$$\{\delta_1 \wedge \delta_2, -\} = \delta_1 \frac{d}{dz_2} - \delta_2 \frac{d}{dz_1}$$

$$\delta_1 = dz_2$$

$$\delta_2 = -dz_1$$

$\{\delta_1 \wedge \delta_2, -\}$ = holomorphic piece of Rham on $\mathbb{C}^2$

Closed-string fields are now

$$\Omega^*(\mathbb{R}^4) \otimes \Omega^{0,*}(\mathbb{C}^3, \Lambda T\mathbb{C}^3)$$

A-model closed string fields.

B-model on $\mathbb{C}^5$ turn on $\partial_{z_1} \wedge \partial_{z_2}$

What happens?

z_1, z_2 $\mathbb{C}^2$ becomes A-model (+ B-field)

Example

Brane on z_2, z_3, z_4, z_5

Fields are $\Omega^{0,*}(\mathbb{C}^4)[\varepsilon] \otimes \mathfrak{gl}_N[1]$

$\partial_{z_1} \wedge \partial_{z_2}$ $\rightsquigarrow$ $\varepsilon \partial_{z_2}$, deforms the differential.

After turning on $\partial_{z_1} \wedge \partial_{z_2}$, fields become

$$\Omega^*(\mathbb{R}^2) \otimes \Omega^{0,*}(\mathbb{C}^3) \otimes \mathfrak{gl}_N[1] \qquad \varepsilon = dz_2$$

$\frac{d}{dz_1}$

$\mathbb{C}^2$

Closed-string fields are now

$$\Omega^*(\mathbb{R}^4) \otimes \Omega^{0,*}(\mathbb{C}^3, \wedge T\mathbb{C}^3)$$

↑ A-model closed string fields.

IIA

$$\mathbb{R}^2_A \times \mathbb{C}^4_B$$

Then there are SUSY NS5 branes on

$$\text{pt} \times \mathbb{C}^3$$

$$\cap$$

$$\mathbb{R}^2_A \times \mathbb{C}^4_B$$

Field Content:

$\ker \partial \subseteq PV^{\bullet,\bullet}(\mathbb{C}^3)[2]$

Fields of ghost # 0 are

$\Omega^{0,1}(\mathbb{C}^3, T\mathbb{C}^3) \xrightarrow{\simeq} \Omega^{3,1}(\mathbb{C}^3)$

$\Omega^0(\mathbb{C}^3, \wedge^2 \mathbb{C}^3)$

$\Omega^{0,2}(\mathbb{C}^3)$

This 3-form corresponds to self-dual 3-form on an NS5-brane

More generally

$\mathbb{R}^2 \times \underbrace{\mathbb{C}^3_{z_i}}_{NS5} \times \mathbb{C}_w$

Linear superpotential

λw

Then, find interacting B-model string on the NS5

λ = string coupling constant

$\Omega^{0,w}(\mathbb{C}^5, \Lambda^{\cdot}$

$= C^{\infty}(\mathbb{C}^5$

$\{\delta_j, f\} =$

$\{\delta_j, z_i\} =$

$\partial_{z_1} \wedge \partial_{z_2} = \delta_1 \wedge \delta_2$

$\{\delta_1 \wedge \delta_2, -\} = \delta_1$

$\delta_1 = dz_2$

$\delta_2 = -dz_1$

IIB, $\mathbb{R}^4_A \times \mathbb{C}^3_B$

$\cup$

$\mathbb{R}^2 \times \mathbb{C}^2$

Can put an NS5-brane

Claim Theory on an NS5 is a top' -string Theory on $\mathbb{R}^2_A \times \mathbb{C}^2_B$

If we use $\mathbb{R} \times S^1$ instead of $\mathbb{R}^2$ this is T-dual to IIA picture.

Closed-string fields are now

$\Omega^*(\mathbb{R}^4) \otimes \Omega^{0,*}(\mathbb{C}^3, \Lambda T\mathbb{C}^3)$

↑ A-model closed string fields.

Take a D-brane on $\mathbb{R}_{\geq 0} \times \mathbb{C}^k \times 0$

$\cap\!|$

$\mathbb{R} \times \mathbb{R} \times \mathbb{C}^3 \times \mathbb{C}$

$k \leq 3$

Fields are

$\Omega^{\cdot}(\mathbb{R}) \otimes \Omega^{0,*}(\mathbb{C}^k)[\underbrace{\xi_1, \ldots, \xi_{3-k}}_{\text{rep. motion parallel to NS5}}, \underbrace{\delta}_{\text{motion perpendicular to NS5}}] \otimes gl_N[1]$

IIA

$\mathbb{R}^2_A \times \mathbb{C}^4_B$

Then there are SUSY NS5 branes on

$pt \times \mathbb{C}^3$

$\cap\!|$

$\mathbb{R}^2_A \times \mathbb{C}^4_B$

on $\mathbb{R}_{\geq 0} \times \mathbb{C}^k \times 0$

$\cap\!|$

$\mathbb{R} \times \mathbb{R} \times \mathbb{C}^3 \times \mathbb{C}$

$k \leq 3$

$\mathbb{R}^2_A \times \mathbb{C}^4_B$

Then there are SUSY NS5 branes on $\text{pt} \times \mathbb{C}^3$

$\cap\!|$

$\mathbb{R}^2_A \times \mathbb{C}^4_B$

Fields are

$\Omega^*(\mathbb{R}) \otimes \Omega^{0,*}(\mathbb{C}^k)[\underbrace{\xi_{1,1}, \ldots, \xi_{3-k}}_{\text{rep. motion parallel to NS5}}, \underbrace{\delta}_{\text{motion perpendicular to NS5}}] \otimes \mathfrak{gl}_N[1]$

Boundary condition

fields involving δ are set to 0.

Boundary fields

$$\Omega^{0,*}(\mathbb{C}^k)[\epsilon_1, \ldots, \epsilon_{3-k}][\mathfrak{gl}_N[1]]$$

$$= \Omega^{0,*}(\mathbb{C}^k, \mathrm{Ext}^*_{\mathcal{O}_{\mathbb{C}^3}}(\mathcal{O}_{\mathbb{C}^*}, \mathcal{O}_{\mathbb{C}^k})) \otimes \mathfrak{gl}_N[1]$$

= fields on a brane in top' B-model on $\mathbb{C}^3$.

[1]
lar to NS5

Single NS5 brane

Claim The theory on a single NS5 brane is the free limit of string field theory for top' B-model on $\mathbb{C}^3$

Turning on λw, we get deformation λd

$\mathcal{A}\delta$

Bulk theory

No operators.

Boundary theory

brane in
on $\mathbb{C}^3$.

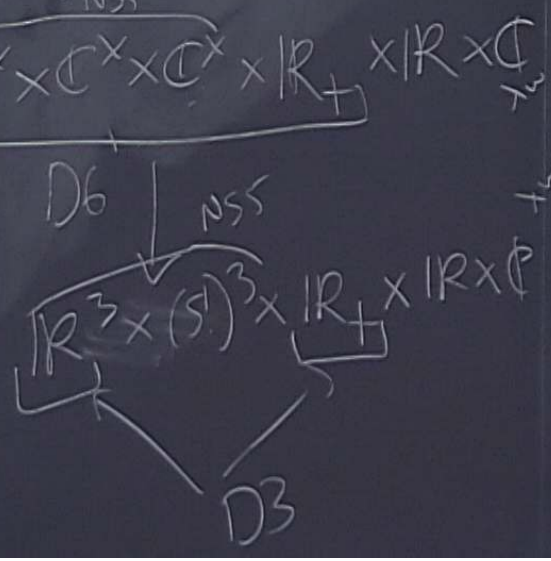
Boundary theory

$\Rightarrow$ D3-ending on NS5 gives CS

Is the theory on a brane in top' string

$k=3$ we find a TFT on $\mathbb{C}^3 \times \mathbb{R}_+$ with holomorphic CS on the boundary.

T-duality:



Linear Quiver Gauge theories

4d $N=2$ quiver gauge theory in the holomorphic twist.

IIA on $\mathbb{R}^2 \times \mathbb{C}^3 \times \mathbb{C}$

D4-branes on $\mathbb{R} \times \mathbb{C}^2 \times 0$

IIB, $\mathbb{R}^4$ $\times$

$\mathbb{R}^2$ $\times$

Can put

Claim Th

is a

on $\mathbb{R}^2$

If we use $\mathbb{R}$

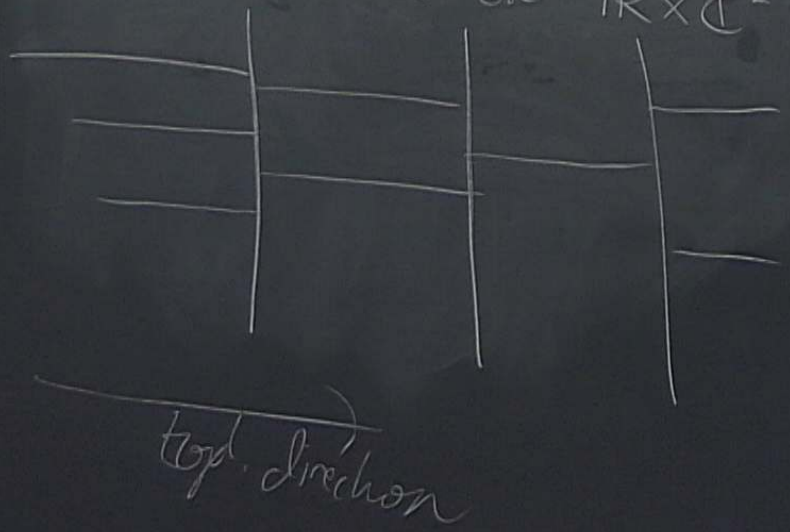
this is T-dua

Linear Quiver Gauge theories

4d $N=2$ quiver gauge theory in the holomorphic twist

IIA on $\mathbb{R}^2 \times \mathbb{C}^3 \times \mathbb{C}$

D4-branes on $\mathbb{R} \times \mathbb{C}^2 \times 0$



IIB, $\mathbb{R}^4_A \times \mathbb{C}^3_B$

$\cup$

$\mathbb{R}^2 \times \mathbb{C}^2$

Can put an NS5

Claim Theory on

is a top' string

on $\mathbb{R}^2_A \times \mathbb{C}^2_B$

If we use $\mathbb{R} \times S^1$ instead

this is T-dual to IIA