Title: String Theory for Mathematicians - Lecture 7

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Abstract:

IIB $\rightsquigarrow$ B-model

top' string on $\mathbb{C}^5$

D-branes $\longrightarrow$ B branes

IIA $\rightsquigarrow$ $\mathbb{R}^2_A \times \mathbb{C}^4_B$

D-branes $\rightsquigarrow$ A/B branes on

$\mathbb{R} \times \mathbb{C}^k \quad k \leq 4$

T-duality

IIA on $\mathbb{R} \times S^1 \times \mathbb{C}^4$

IIB $\quad \mathbb{C}^\times \times \mathbb{C}^4$

from T-duality

$\mathbb{R} \times S^1 \longleftrightarrow \mathbb{C}^\times$

in top' string.

Remaining SUSYs in physical string:

from T-duality

$\mathbb{R}\times S^1 \longleftrightarrow \mathbb{C}^\times$

in top' string.

Remaining SUSYs in physical string:

II B, we found 15 remaining SUSY

deformed B-model on $\mathbb{C}^5$

by $- \ \partial_{z_i} \wedge \partial_{z_j}$ (make non-commutative)

$- \ z_k$ (linear superpotential)

on

$k \leq 4$

Remaining SUSYs in physical string:

IIB, we found 15 remaining SUSY

deformed B-model on $\mathbb{C}^5$

by $-\ \partial_{z_i} \wedge \partial_{z_j}$ (make non-commutative)

$-\ z_k$ ((linear superpotential))

IIA, similarly 10 remaining

$\partial_{z_i} \wedge \partial_{z_j}, \ z_k$

B-model on $\mathbb{C}^5$

turn on

$\partial_{z_1} \wedge \partial_{z_2}$

What happens?

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z_1, z_2 $\quad \mathbb{C}^2$

becomes A-model

(+ B-field)

Example

Brane on

z_2, z_3, z_4, z_5

Fields are $\Omega^{0,*}(\mathbb{C}^4)[\varepsilon] \otimes \mathfrak{g}_N[1]$

$\partial_{z_1} \wedge \partial_{z_2} \rightsquigarrow \varepsilon \partial_{z_2}$, deforms the differential.

$\partial_{z_1} \wedge \partial_{z_2}$

What happens?

$z_1, z_2 \quad \mathbb{C}^2$

becomes A-model (+ B-field)

Example: Brane on z_2, z_3, z_4, z_5

Fields are $\Omega^{0,*}(\mathbb{C}^4)[\varepsilon] \otimes \mathfrak{gl}_N[1]$

$\partial_{z_1} \wedge \partial_{z_2} \rightsquigarrow \varepsilon \partial_{z_2}$, deforms the differential.

After turning on $\partial_{z_1} \wedge \partial_{z_2}$, fields become

$$\Omega^*(\mathbb{R}^2) \otimes \Omega^{0,*}(\mathbb{C}^3) \otimes \mathfrak{gl}_N[1] \qquad \varepsilon = dz_2$$

Same as what we find if we treat $\mathbb{C}^2$ as A-model

In sum

A further twist of IIB is top' string on $\mathbb{R}^4_A \times \mathbb{C}^3_B$

Twist even more

$$\mathbb{R}^8_A \times \mathbb{C}_B$$

IIA: Various twists are

$$\mathbb{R}^2_A \times \mathbb{C}^4_B$$

$$\mathbb{R}^6_A \times \mathbb{C}^2_B$$

$$\mathbb{R}^{10}_A \leftarrow$$ corresponds to a particular SU(5)-inv. top' twist

Closed-string fields are

$\operatorname{Ker} \partial \subseteq \Omega^{0,*}(\mathbb{C}^5, \wedge^* T\mathbb{C}^5)$

If we turn on a background closed-string field α, this changes

$$\partial \longrightarrow \partial + \{\alpha, -\}$$

$\{\ \}$ = Schouten bracket

$$\delta_i = \frac{d}{dz_i}$$

$$\Omega^{0,*}(\mathbb{C}^5, \wedge^* T\mathbb{C}^5) = C^\infty(\mathbb{C}^5)[d\bar{z}_i, \delta_j]$$

$$\{\delta_j, f\} = \frac{df}{dz_j}$$

$$\{\delta_j, z_i\} = \delta_{ij}$$

$$\partial_{z_1} \wedge \partial_{z_2} = \delta_1 \wedge \delta_2$$

$$\{\delta_1 \wedge \delta_2, -\} = \delta_1 \frac{d}{dz_2} - \delta_2 \frac{d}{dz_1}$$

$$\delta_1 = dz_2$$

$$\delta_2 = -dz_1$$

$\{\delta_1 \wedge \delta_2, -\}$ = hol piece of de Rham on $\mathbb{C}^2$

Closed-string fields are now

$$\Omega^*(\mathbb{R}^4) \otimes \Omega^{0,*}(\mathbb{C}^3, \wedge T\mathbb{C}^3)$$

A-model closed string fields.

B-model on $\mathbb{C}^5$
turn on
$\partial_{z_1} \wedge \partial_{z_2}$
What happens?

z_1, z_2 $\mathbb{C}^2$
becomes A-model
(+ B-field)

Example Brane on
z_2, z_3, z_4, z_5
Fields are $\Omega^{0,*}(\mathbb{C}^4)[\varepsilon] \otimes \mathfrak{gl}_N[1]$

$\partial_{z_1} \wedge \partial_{z_2} \rightsquigarrow \varepsilon \partial_{z_2}$, deforms the differential.

After turning on $\partial_{z_1} \wedge \partial_{z_2}$, fields become

$$\Omega^*(\mathbb{R}^2) \otimes \Omega^{0,*}(\mathbb{C}^3) \otimes \mathfrak{gl}_N[1] \qquad \varepsilon = dz_2$$

$\frac{d}{dz_1}$

$\mathbb{C}^2$

Closed-string fields are now

$$\Omega^*(\mathbb{R}^4) \otimes \Omega^{0,*}(\mathbb{C}^3, \wedge T\mathbb{C}^3)$$

A-model closed string fields.

IIA

$$\mathbb{R}^2_A \times \mathbb{C}^4_B$$

Then there are SUSY NS5 branes on

$$\begin{array}{c} pt \times \mathbb{C}^3 \\ \cap \\ \mathbb{R}^2_A \times \mathbb{C}^4_B \end{array}$$

Field Content:

$$\mathrm{Ker}\,\partial \subseteq PV^{*,*}(\mathbb{C}^3)[2]$$

Fields of ghost # 0 are

$$\Omega^{0,1}(\mathbb{C}^3, T\mathbb{C}^3) \xrightarrow{\simeq} \Omega^{2,1}(\mathbb{C}^3)$$

$$\Omega^{0}(\mathbb{C}^3, \Lambda^2\mathbb{C}^3)$$

$$\Omega^{0,2}(\mathbb{C}^3)$$

This 3-form corresponds to self-dual 3-form on an NS5-brane

More generally

$$\mathbb{R}^2 \times \underbrace{\mathbb{C}^3_{z_i}}_{NS5} \times \mathbb{C}_w$$

linear superpotential

λw

Then, find interacting B-model string on the NS5

λ = string coupling constant

$$\Omega^{0,*}(\mathbb{C}^5, \wedge^{\cdot} T$$

$$= C^\infty(\mathbb{C}^5$$

$$\{\delta_j, f\} =$$

$$\{\delta_j, z_i\} =$$

$$\partial_{z_1} \wedge \partial_{z_2} = \delta_1 \wedge \delta_2$$

$$\{\delta_1 \wedge \delta_2, -\} = \delta_1$$

$$\delta_1 = dz_2$$

$$\delta_2 = -dz_1$$

IIB, $\mathbb{R}^4_A \times \mathbb{C}^3_B$

$\cup$|

$\mathbb{R}^2 \times \mathbb{C}^2$

Can put an NS5-brane

Claim Theory on an NS5 is a top'-string theory on $\mathbb{R}^2_A \times \mathbb{C}^2_B$

If we use $\mathbb{R}\times S^1$ instead of $\mathbb{R}^2$ this is T-dual to IIA picture.

Closed-string fields are now

$$\Omega^*(\mathbb{R}^4) \otimes \Omega^{0,*}(\mathbb{C}^3, \wedge T\mathbb{C}^3)$$

A-model closed string fields.

-brane

on NS5

ng Theory

l of $\mathbb{R}^2$

A picture.

Take a D-brane

on $\mathbb{R}_{\geq 0} \times \mathbb{C}^K \times 0$

$$\mathbb{R}_{\geq 0} \times \mathbb{C}^K \times 0 \ \subseteq \ \mathbb{R} \times \mathbb{R} \times \mathbb{C}^3 \times \mathbb{C}$$

$K \leq 3$

Fields are

$$\Omega^*(\mathbb{R}) \otimes \Omega^{0,*}(\mathbb{C}^K)[\underbrace{\varepsilon_1, \dots, \varepsilon_{3-K}}_{\text{rep. motion parallel to NS5}}, \delta] \otimes \mathfrak{gl}_N[1]$$

δ: motion perpendicular to NS5

IIA

$$\mathbb{R}^2_A \times \mathbb{C}^4_B$$

Then there are SUSY NS5 branes on

$$\mathrm{pt} \times \mathbb{C}^3 \ \subseteq \ \mathbb{R}^2_A \times \mathbb{C}^4_B$$

on $\mathbb{R}_{\geq 0} \times \mathbb{C}^K \times 0$

$\cap$

$\mathbb{R} \times \mathbb{R} \times \mathbb{C}^3 \times \mathbb{C}$

$K \leq 3$

Fields are

$\Omega^*(\mathbb{R}) \otimes \Omega^{0,*}(\mathbb{C}^K)[\underbrace{\varepsilon_1, \dots, \varepsilon_{3-K}}_{\text{rep. motion parallel to NS5}}, \delta] \otimes gl_N[1]$

δ: motion perpendicular to NS5

Boundary condition

fields involving δ are set to 0.

$\mathbb{R}^2_A \times \mathbb{C}^4_B$

Then there are SUSY NS5 branes on

$pt \times \mathbb{C}^3$

$\cap$

$\mathbb{R}^2_A \times \mathbb{C}^4_B$

une

NS5

ory

$\mathbb{R}^2$

cture.

Boundary fields

$$\Omega^{0,*}(\mathbb{C}^k)[\varepsilon_1, \ldots, \varepsilon_{3-k}] \otimes \mathfrak{gl}_N[1]$$

$$= \Omega^{0,*}(\mathbb{C}^k, \mathrm{Ext}^{\cdot}_{\mathcal{O}_{\mathbb{C}^3}}(\mathcal{O}_{\mathbb{C}^k}, \mathcal{O}_{\mathbb{C}^k})) \otimes \mathfrak{gl}_N[1]$$

= fields on a brane in top' B-model on $\mathbb{C}^3$.

[1]

lar to NS5

Single NS5 brane

Claim The theory on a single NS5 brane is the free limit of string field theory for top' B-model on $\mathbb{C}^3$

$_{k})\otimes \mathfrak{gl}_N[1]$

$\mathcal{O}_{\mathbb{C}^k}, \mathcal{O}_{\mathbb{C}^k}))\otimes \mathfrak{gl}_N[1]$.

brane in
on $\mathbb{C}^3$.

Turning on λw, we get deformation

$$\lambda \frac{d}{d\delta}$$

Bulk theory:

No operators.

Boundary theory

Boundary theory

Is the theory on a brane in top'l string

$\underline{k=3}$ We find a TFT on $\mathbb{C}^3 \times \mathbb{R}_+$ with holomorphic CS on the boundary.

$\Rightarrow$ D3 ending on NS5 gives CS

T-duality:

$$\underbrace{\overbrace{\mathbb{C}^\times \times \mathbb{C}^\times \times \mathbb{C}^\times}^{NS5} \times \mathbb{R}_+}_{D6} \times \mathbb{R} \times \mathbb{C}$$

$$\downarrow$$

$$\underbrace{\mathbb{R}^3}_{D3} \times \overbrace{(S^1)^3}^{NS5} \times \underbrace{\mathbb{R}_+}_{D3} \times \mathbb{R} \times \mathbb{C}$$

Linear Quiver Gauge theory

4d $N=2$ quiver gauge theory in the holomorphic twist.

IIA on $\mathbb{R}^2 \times \mathbb{C}^3 \times \mathbb{C}$

D4-branes on $\mathbb{R} \times \mathbb{C}^2 \times 0$

IIB, $\mathbb{R}^4_A \times$

$\cup$

$\mathbb{R}^2 \times$

Can put

Claim Th

is a

on $\mathbb{R}^2_A$

If we use $\mathbb{R}\times$

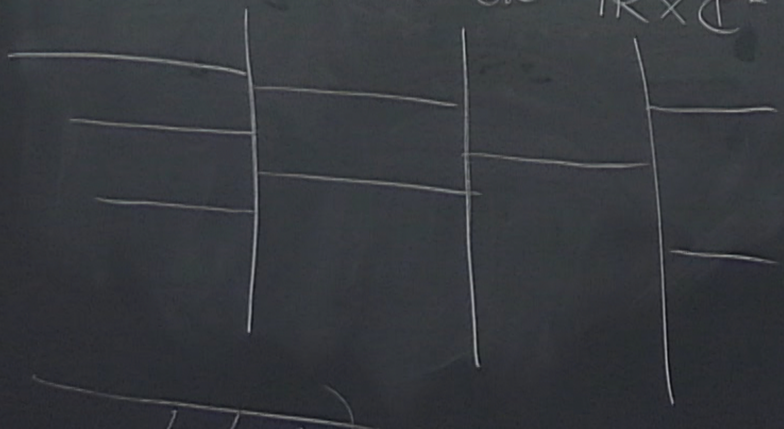
this is T-dua

Linear Quiver Gauge theories

$4d$ $N=2$ quiver gauge theory in the holomorphic twist

IIA on $\mathbb{R}^2 \times \mathbb{C}^3 \times \mathbb{C}$

D4-branes on $\mathbb{R} \times \mathbb{C}^2 \times 0$



top'. direction

IIB, $\mathbb{R}^4_A \times \mathbb{C}^3_B$

$\cup\!\!\mid$

$\mathbb{R}^2 \times \mathbb{C}^2$

Can put an NS5

Claim Theory on is a top' string on $\mathbb{R}^2_A \times \mathbb{C}^2_B$

If we use $\mathbb{R} \times S^1$ instead this is T-dual to IIA